The Friedman Rule in a Two Sector Small Open Economy

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Abstract

This paper investigates the properties of optimal monetary and fiscal policy in a two sector small open economy. If the government can optimally select all possible distorting tax rates, then it can implement Pareto efficient outcomes and the Friedman Rule is found to be a necessary condition for this. If the government can select only some of these tax rates, then second best policies may also display the Friedman rule as a feature. However, this last result depends on the set of tax instruments the government can choose from.

Key words: Friedman Rule, optimal taxation, open economy.
JEL: E43, E52, E63.

1 Introduction

In a seminal essay, Friedman [9] suggested that a government should set the nominal interest rate equal to zero to lead the economy to an efficient outcome. He argued that only such a policy would lead to the maximization of the consumer surplus associated to the money demand. That policy prescription became known in the literature as the Friedman rule.

Friedman's argument was a partial equilibrium one. Lucas and Stokey [11] were the first to show that the Friedman rule could also hold in a general equilibrium context. After Lucas and Stokey's paper, a large body of literature was dedicated to the question of whether the Friedman rule is optimal or not. Recent papers on that subject are Carlstrom and Fuerst [1]; Cavalcanti and Villamil [2]; Chari, Christiano and Kehoe [3]; Cole and Kocherlakota [4]; Correia and Teles [5] and [6]; De Fiore and Teles [8]; Ireland [10]; and Schmitt-Grohé and Uribe [14].

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A main lesson that emerges from Chari, Christiano and Kehoe [3]; Correia and Teles [5] and [6]; and De Fiore and Teles [8] is that the optimality of the Friedman rule does not critically rely on any specific type of monetary friction. For standard preferences and technologies, the Friedman rule is optimal in cash-in-advance, money in the utility function and shopping time economies. Correia and Teles [5] also showed that in some models in which a zero nominal interest rate policy was not an efficient choice, the deviations from it were quantitatively small.

The findings listed in the above paragraph apparently make a strong case for the Friedman rule. However, as we shall see, those findings depend on the implicit hypothesis that the government has access to a sufficiently large set of distorting tax instruments.

Cavalcanti and Villamil [2] studied the problem of selecting the optimal monetary and fiscal policy in an economy with an informal (untaxed) sector. They concluded that in such a context the Friedman rule fails to be optimal. Additionally, the deviations from it can be quantitatively large.

A venue to clarify the contradictory findings concerning the optimality of the Friedman rule is to take a deeper look at the set of distorting taxes available to the government. This issue was obscured in other papers because of the simplicity of the models. In more sophisticated economies, the relevance of the number of tax instruments available becomes obvious.

To better understand this point, consider the cash-credit model in Chari, Christiano and Kehoe [3]. In that simple single sector, single input and constant return to scale economy, there exist only two possible distorting taxes: on consumption (τ^c) and on labor income (τ^l). So, when it comes to fiscal policy, either the government can select both or just one of them. If the government can select both τ^c and τ^l, then it can implement the unique Pareto efficient allocation. Whenever the government can pick only one of the two, the second best solution displays the Friedman rule as one of its features.

Consider now a situation in which there exist three possible generic distorting taxes (τ^1, τ^2 and τ^3). Clearly, there exist many more cases to be studied. In fact, there are seven cases to be looked into. Each of these cases is a non-empty subset of $\{\tau^1, \tau^2, \tau^3\}$. One can think of the non optimality of the Friedman rule in Cavalcanti and Villamil [2] simply as its non optimality for a particular subset of all conceivable distorting taxes that could be possibly implemented in their economy.

The above discussion naturally guides us to inquire about the optimality of the Friedman rule when the government is constrained to choose some, but not all, of several possible distorting taxes. This is one of the problems we study in this paper. We investigate the properties of optimal monetary and fiscal policies in a two sector (tradable and non tradable) small open economy. Consumers face a cash-in-advance constraint on fraction of their purchases of non tradables. There exist distorting taxation on labor income, private consumption of each type of good and foreign interest income. As usual, government consumption is exogenous. We performed several exercises in which the government cannot choose all of these taxes. This allows for a deeper understanding of the relation between the optimality of the Friedman rule and the availability of tax instruments.

Of course, not only open economies have many sectors, goods and tax rates. We chose to consider an open economy for two reasons. First, actual economies are open economies. Hence, this choice brings us closer to the actual decisions that policy makers face in the real world. Second, there exist already some papers that studied the optimality of the Friedman rule in open economies, as Carlstrom and Fuerst [1] and Schmitt-Grohé and Uribe [14] and we want to relate our findings to theirs.

Our main findings are the following. If the government can select all possible distorting taxes, it can implement the unique Pareto efficient allocation and the Friedman rule is a feature of any policy associated to that allocation. If the government cannot select all available tax rates, then the Friedman rule may be or not optimal. Its optimality depends on which subset of distorting taxes the planner can choose from.

This paper is organized as follows. Section 2 describes the model economy. Section 3 is devoted to the characterization of the set of competitive equilibria. Section 4 characterizes the Pareto efficient allocations and discuss their implementation. Section 5 discusses the optimal policies when the first best Pareto allocations cannot be implemented. Concluding remarks are found in Section 6. Proofs and some other technical issues are found in the appendix.

2 The economy

Consider a small country populated by a continuum of identical infinitely lived households with Lebesgue measure one and a government. A household is composed by a shopper and a worker, who is endowed with one unit of time.

The country produces two non tradable goods. The first is consumed by households (c_1^N). The second is consumed by households (c_2^N) and government (g^N). The country also produces a tradable good, which is consumed by households (c^T) and a government (g^T). This last good can also be exported (x) or imported ($-x$).

Markets operate in a particular way. At a first stage of each date t, a spot market for goods and labor services operates. At a second stage, after the market for goods and labor service closes, a security and currency market operates.

A domestic currency M circulates in this economy. Two types of securities are traded: a claim B that pays $(1+i)$ units of M and a claim B^F that pays $(1+i_t^F)$ units of some foreign currency. Both claims have maturity of one period. Foreigners do not sell or buy claims to the domestic currency and i_t^F is exogenous.

Workers cannot sell their services outside the country. Shoppers face a cash-in-advance constraint. The purchases of c_1^N must be paid for with the domestic currency. Except for the purchases of that good, all other transactions are liquidated during the security and currency trading session. The date t price, in terms of the foreign currency, of the tradable good is exogenous and equal to one.

Technology is described by $0 \leq y^T \leq (l^T)^{\alpha^T}$ and $0 \leq y^N \leq (l^N)^{\alpha^N}$, where y^T is the tradable output and l^T is the amount of labor allocated to the production of that good. Similar meanings are assigned to y^N and l^N. Both α^T and α^N lie in the set $(0,1]$.

The sequence $\{g_t^T, g_t^N, i_t^F\}_{t=0}^{\infty}$ is contained in the finite set $G^T \times G^N \times I^F$. Additionally, $G^T \subset \mathbb{R}_+$, $G^N \subset \mathbb{R}_+$ and $I^F \subset \mathbb{R}_+$.

Each good is produced by a single competitive firm. Let l_t denote the amount of labor supplied by each household at date t. Other variables indexed by t have analogous meaning. Feasibility requires

$$l_t^T + l_t^N = l_t \leq 1,\ c_{1t}^N + c_{2t}^N + g_t^N = (l_t^N)^{\alpha^N},\ c_t^T + g_t^T + x_t = (l_t^T)^{\alpha^T} \ . \tag{1}$$

The government finances the sequence $\{g_t^T, g_t^N\}_{t=0}^{\infty}$ by issuing and withdrawing the domestic currency; by issuing and redeeming claims B of maturity of one period to $1+i$ units of the domestic currency; by purchasing and selling B^F; and by taxing profits, labor income, consumption and interest income on foreign assets. The government budget constraint is

$$\begin{aligned} E_t g_t^T + p_t^N g_t^N + E_t B_{Gt+1}^F + (1+i_t) B_t + M_t = \\ E_t(1+i_t^F) B_{Gt}^F + M_{t+1} + B_{t+1} + \tau_t^l w_t l_t + \tau_t^T E_t c_t^T + \\ \tau_t^N p_t^N (c_{1t}^N + c_{2t}^N) + E_t \delta_t^F i_t^F B_{Ht}^F + \delta_t^T \psi_t^T + \delta_t^N \psi_t^N \ , \end{aligned} \tag{2}$$

where E_t is the nominal exchange rate; p_t^N and w_t are the respective date t monetary prices (in terms of the domestic currency) of the non tradable good and labor services; B_{Gt+1}^F stands for the foreign assets held by the government at the end of date t, while B_{Ht}^F is people's foreign assets at the beginning of the same period; M_{t+1} and B_{t+1} are the amount of domestic currency and public debt held by the households at the end of date t; i_t is the domestic nominal interest rate; ψ_t^T and ψ_t^N are the date t profits; τ_t^l, τ_t^T and τ_t^N are tax rates on labor income and consumption; δ_t^F is the tax rate on households' foreign assets income; and δ_t^T and δ_t^N are the tax rates on profits. A negative value for B_{Gt+1}^F means that the government is borrowing abroad, while a negative value for B_{t+1} means that the government is lending to domestic residents. At $t=0$ the government holds an initial amount B_{G0}^F of foreign assets. To avoid Ponzi schemes, a standard boundedness constraint $\left|B_{G,t+1}^F\right| \leq A < \infty$ is imposed on government foreign assets.

The function $u : \mathbb{R}_+^3 \times [0,1] \to \mathbb{R} \cup \{-\infty\}$,

$$u = u(c^T, c_1^N, c_2^N, l) \ , \tag{3}$$

is the typical household period utility function. It displays local non-satiability and satisfies standard differentiability and Inada conditions. As usual, intertemporal preferences are described by

$$\sum_{t=0}^{\infty} \beta^t u(c_t^T, c_{1t}^N, c_{2t}^N, l_t) \ , \tag{4}$$

where $\beta \in (0,1)$. The date t budget constraint of the typical household is

$$(1+\tau_t^T)E_t c_t^T + (1+\tau_t^N)p_t^N(c_{1t}^N + c_{2t}^N) + M_{t+1} +$$
$$B_{t+1} + E_t B_{Ht+1}^F \leq (1-\tau_t^l)w_t l_t + M_t + (1+i_t)B_t$$
$$E_t[1+(1-\delta_t^F)i_t^F]B_{Ht}^F + (1-\delta_t^T)\psi_t^T + (1-\delta_t^N)\psi_t^N \ . \tag{5}$$

The constraint $\left|\frac{B_{t+1}}{E_{t+1}}\right|, \left|B_{H,t+1}^F\right| \leq A$ prevents Ponzi games. People face the cash-in-advance constraint

$$(1+\tau_t^N)p_t^N c_{1t}^N \leq M_t \ . \tag{6}$$

Given initial cash and bond holdings (M_0, B_0, B_{H0}^F), a household chooses a sequence $\{c_t^T, c_{1t}^N, c_{2t}^N, l_t, M_{t+1}, B_{t+1}, B_{Ht+1}^F\}_{t=0}^\infty$ to maximize (4) subject to the constraints (5), (6), and $l_t \leq 1$. Additionally, the sequences$\{c_t^T\}_{t=0}^\infty$, $\{c_{1t}^N\}_{t=0}^\infty$, $\{c_{2t}^N\}_{t=0}^\infty$ and $\left\{\frac{M_{t+1}}{E_{t+1}}\right\}_{t=0}^\infty$ are required to be bounded.

At each date t, the firm that produces the non tradable good chooses l_t^N to maximize $\psi_t^N = p_t^N(l_t^N)^{\alpha^N} - w_t l_t^N$. In a similar fashion, the other firm chooses l_t^T to maximize $\psi_t^T = p_t^T(l_t^T)^{\alpha^T} - w_t l_t^T$.

3 Competitive equilibrium

A date t policy $(E_t, p_t^N, w_t, i_{t+1}, \tau_t^l, \tau_t^N, \tau_t^T, \delta_{t+1}^F, \delta_t^N, \delta_t^T)$ is denoted by φ_t. A *policy* is an object $\varphi = \{\varphi_t\}_{t=0}^\infty$. End of period t nominal cash and asset holdings $(M_{t+1}, B_{t+1}, B_{Ht+1}^F, B_{Gt+1}^F)$ and period t allocations $(l_t^N, l_t^T, x_t, c_t^T, c_{1t}^N, c_{2t}^N, l_t)$ are denoted, respectively, by ζ_{t+1} and χ_t. Additionally, $\chi = \{\chi_t\}_{t=0}^\infty$ and $\zeta = \{\zeta_{t+1}\}_{t=0}^\infty$.

Definition 1 *A* competitive equilibrium *is an object (φ, χ, ζ) satisfying: (i) given φ, (χ, ζ) provides a solution for the household problem; (ii) given φ_t, l_t^N and l_t^T maximize the respective firm's profit; (iii) (1) and (2) hold. An allocation χ is* attainable *if there exist sequences φ and ζ such that (φ, χ, ζ) is a competitive equilibrium.*

Two points in the above definition should be emphasized. Item (ii) is equivalent to $w_t = p_t^N \alpha^N (l_t^N)^{\alpha^N - 1} = E_t \alpha^T (l_t^T)^{\alpha^T - 1}$. Adding the identities $\psi_t^N + w_t l_t^N = p_t^N(\ c_{1t}^N + c_{2t}^N + g_t^N)$ and $\psi_t^T + w_t l_t^T = E_t(c_t^T + x_t + g_t^T)$ to (2) and (5) taken as equality, one obtains

$$x_t + (1+i_t^F)(B_{Gt}^F + B_{Ht}^F) - B_{Gt+1}^F - B_{Ht+1}^F = 0 \ , \tag{7}$$

which is the balance-of-payments identity for this economy. So, it is not necessary to spell this condition out when defining competitive equilibrium.

Next we characterize the set of competitive equilibrium allocations. We express that characterization in terms of some constraints. As previously mentioned, we will want to study the problem of selecting optimal monetary policy and tax rates when some others tax rates are exogenous. Thus, our characterization will depend on the tax rates.

To simplify the notation, $u(t)$, $u_T(t)$, $u_1(t)$, $u_2(t)$, and $u_l(t)$ will denote, respectively, the value of u and its partial derivatives $\partial u/\partial c^T$, $\partial u/\partial c_1^N$, $\partial u/\partial c_2^N$, and $\partial u/\partial l$ evaluated at the point $(c_t^T, c_{1t}^N, c_{2t}^N, l_t)$. The sum $u_T(t)c_t^T + u_1(t)c_{1t}^N + u_2(t)c_{2t}^N + u_l(t)l_t$ will be denoted by $W(t)$.

There exist six conditions with obvious economic meaning that must hold in any competitive equilibrium. A trivial condition is (1). The second requirement, ensuring that people's intertemporal marginal rate of substitution is consistent with i_t^F and tax rates, could be expressed as

$$\beta \frac{u_T(t+1)}{u_T(t)} = \frac{1+\tau_{t+1}^T}{1+\tau_t^T} \frac{1}{1+(1-\delta_{t+1}^F)i_{t+1}^F} \ . \tag{8}$$

However, for future convenience, we write that constraint as

$$u_T(t) = \beta^{-t} \frac{1+\tau_t^T}{1+\tau_0^T} \frac{u_T(0)}{\prod_{s=1}^t [1+(1-\delta_s^F)i_s^F]} \ , \tag{9}$$

where the empty product $\prod_{s=1}^0$ is defined to be equal to one. The third constraint is that households' marginal rate of substitution between tradables and non tradables must match the marginal rate of transformation between those types of goods, i.e.,

$$\frac{u_T(t)}{u_2(t)} = \frac{1+\tau_t^T}{1+\tau_t^N} \frac{\alpha^N (l_t^T)^{1-\alpha^T}}{\alpha^T (l_t^N)^{1-\alpha^N}} \ . \tag{10}$$

The fourth

$$-\frac{u_l(t)}{u_2(t)} = \frac{1-\tau_t^l}{1+\tau_t^N} \frac{\alpha^N}{(l_t^N)^{1-\alpha^N}} \ , \tag{11}$$

is an implementability constraint for real wage. The fifth is

$$\begin{aligned} u_1(0)c_{1,0}^N + u_2(0) \left[\frac{(1+i_0)B_0}{(1+\tau_0^N)p_0^N} + \frac{M_0}{(1+\tau_0^N)p_0^N} - c_{1,0}^N \right] + \\ u_T(0) \frac{[1+(1-\delta_0^F)i_0^F]B_{H0}^F}{1+\tau_0^T} = \\ \sum_{t=0}^{\infty} \beta^t \left\{ W(t) + \frac{u_l(t)}{(1-\tau_t^l)} \left[(1-\delta_t^T)\frac{1-\alpha^T}{\alpha^T} l_t^T + (1-\delta_t^N)\frac{1-\alpha^N}{\alpha^N} l_t^N \right] \right\} \ , \end{aligned} \tag{12}$$

which consolidates all date t budget constraints of the households. The sixth is a balance-of-payment constraint

$$-\sum_{t=0}^{\infty} \frac{x_t}{\prod_{s=1}^t (1+i_s^F)} = (1+i_0^F)(B_{H0}^F + B_{G0}^F) \ , \tag{13}$$

which requires imports to be financed by country's initial wealth.

The above constraints are not enough to characterize a competitive equilibrium. The inequalities

$$(1+\tau_0^N)p_0^N c_{1,0}^N \leq M_0 \tag{14}$$

$$u_2(t) \leq u_1(t) \tag{15}$$

are needed to ensure that cash-in-advance constraints hold. Finally, an implementability constraint for a transversality condition is

$$\lim_{t\to\infty} \frac{\beta^t u_1(t) c_{1t}^N}{(1+\tau_t^N)} = 0 \ . \tag{16}$$

Proposition 1 *Let $M_0 > 0$. A bounded sequence χ and a price $p_0^N > 0$ satisfy (1) and (9)-(16) if and only if they are components of a competitive equilibrium (φ, χ, ζ).*

Proof. See appendix.

The proof of Proposition 1 is a long but straightforward exercise. It is enough to modify the techniques discussed by other authors (for instance, Lucas and Stokey [11] or Chari, Christiano and Kehoe [3]) to the model discussed in this essay.

The above set of constraints does not include an implementability condition for the government budget constraint. However, there are implementability conditions for people's budget constraint, resource constraints and balance-of-payments. The government budget constraint is a linear combination of those other constraints.

Let's now consider the constraints the Friedman rule places on the path of the money supply. The Friedman rule specifies that, for all t,

$$i_{t+1} = 0 \ . \tag{17}$$

From people's first order conditions, it is easy to conclude that $i_{t+1} = 0$ if and only if $u_2(t+1) = u_1(t+1)$.

As pointed out by Cole and Kocherlakota [4], Ireland [10] and Wilson [16], (17) has some implications for the money supply in the long run. In the economy considered in this paper, it implies the following two conditions:

$$\lim_{t\to\infty} M_t = 0 \ ; \tag{18}$$

$$\frac{M_t}{\beta^t} \geq \frac{p_0^N(1+\tau_0^N)}{u_2(0)} u_1(t) c_{1t}^N \ . \tag{19}$$

Condition (18) simply states that the nominal quantity of money has to vanish in the long run, while (19) places a bound on its decay rate. Observe that if $u_1(t)c_{1t}^N$ is bounded away from zero, then (19) implies $\inf_t M_t \beta^{-t} > 0$, which is exactly a condition that shows up in Cole and Kocherlakota [4] and Ireland [10]. The next proposition formalizes the above discussion.

Proposition 2 *If a competitive equilibrium* (φ, χ, ζ) *satisfies (17), then it satisfies (18) and (19).*

Proof. See appendix.

Cole and Kocherlakota [4] and Ireland [10] argue that the implementation of the Friedman rule leaves the path of nominal balances undetermined over any finite horizon. The next proposition establishes that the same holds in the economy we consider in this paper.

Proposition 3 *If a competitive equilibrium* (φ, χ, ζ) *satisfies (17), then there exist uncountable many* ζ' *such that* (φ, χ, ζ') *is a competitive equilibrium.*

Proof. See appendix.

The intuition for the aforementioned indeterminacy is extremely simple. If the nominal interest rate is zero, people will be indifferent between domestic bonds and money, provided they carry enough balances to purchase the desired amount of c_1. Thus, the government can carry out open market operations that increase the amount of nominal balances and decrease the domestic public debt by the same amount.

In Cole and Kocherlakota [4] and Ireland [10], the government has access to lump-sum taxation and inflation is the only distorting tax available. Hence, the Friedman rule is a necessary and a sufficient condition for Pareto optimality. So, in those papers, the aforementioned indeterminacy is uniquely associated with the unique Pareto efficient allocation. Proposition 3 shows that the indeterminacy of the quantity of money is not necessarily linked to the efficiency of the underlying allocations. The indeterminacy will arise regardless of the optimality of the policy rule (17).

4 Pareto efficiency

This section discusses the properties and implementation of policies that lead to Pareto efficient outcomes. The first step consists in defining Pareto efficiency for the economy being considered.

Definition 2 *An allocation* χ^* *is* Pareto efficient *if it satisfies (1) and (13) and there is no other allocation* χ *that satisfies these constraints and*

$$\sum_{t=0}^{\infty} \beta^t u(c_t^T, c_{1t}^N, c_{2t}^N, l_t) > \sum_{t=0}^{\infty} \beta^t u(c_t^{T*}, c_{1t}^{N*}, c_{2t}^{N*}, l_t^*).$$

An obvious consequence of the above definition is that an allocation χ^* is Pareto efficient if and only if it maximizes (4) subject to (1) and (13).

Under standard assumptions on u, there exists a unique Pareto efficient allocation. This allocation is characterized by the following set of conditions: (1), (13) plus

$$u_1(t) = u_2(t)\ ; \tag{20}$$

$$\frac{u_T(t)}{u_2(t)} = \frac{\alpha^N (l_t^T)^{1-\alpha^T}}{\alpha^T (l_t^N)^{1-\alpha^N}} \; ; \tag{21}$$

$$-\frac{u_l(t)}{u_2(t)} = \frac{\alpha^N}{(l_t^N)^{1-\alpha^N}} \; ; \tag{22}$$

$$\beta \frac{u_T(t+1)}{u_T(t)} = \frac{1}{1+i_{t+1}^F} \; . \tag{23}$$

The above equalities are first order necessary and sufficient conditions of the problem of maximizing (4) subject to (1) and (13).

Example 1 Suppose that period preferences are given by $u = \log c^T + \log c_1^N + \log c_2^N + \log(1-l)$. Foreign interest rate and preference discount factor satisfy $1+i_{t+1}^F = \beta^{-1}$. Technology is described by $y^T = \sqrt{l^T}$ and $y^N = l^N$. Government consumption of the tradable good satisfies $g_t^T = 0$. Condition (23) implies that c_t^T is constant. Simple algebra shows that conditions (1), (20), (21), (22) can be reduced to

$$c_{2t}^N = 1 - l_t^T - l_t^N \; ;$$
$$2c_{2t}^N + g_t^N = l_t^N \; ;$$
$$c_{2t}^N = 2c^T \sqrt{l_t^T} \; .$$

Schmitt-Grohé and Uribe [14] studied the problem of selecting optimal monetary and fiscal policies in an one sector small open economy. They concluded that innovations on government consumption will have no impact on the optimal levels of consumption and labor. This is not the case here. If c_{2t}^N were constant, both l_t^T and l_t^N would also be constant. That would violate the second equality above. In a multi-sector economy, efficiency requires that innovations on a non-tradable sector not to be fully smoothed out. Similar result shows up in Cunha [7].

The characterization of the policies that are associated with Pareto efficient allocations is discussed next. It will be shown that some necessary and sufficient conditions are

$$\tau_t^N = \tau_t^T = -\tau_t^l \tag{24}$$

and

$$\frac{1}{1+i_{t+1}^F} = \frac{1+\tau_{t+1}^T}{1+\tau_t^T} \frac{1}{1+(1-\delta_{t+1}^F) i_{t+1}^F} \; . \tag{25}$$

The first condition is a standard uniform commodity taxation requirement. It requires consumption and leisure to be taxed at the same rate (recall that a subsidy on labor is a tax on leisure). The second constraint requires that no wedge exists between the prevailing international interest rates and the rate that people can borrow and lend abroad.

Proposition 4 *Let (φ, χ, ζ) be a competitive equilibrium. Then, χ is Pareto efficient if and only if (φ, χ, ζ) satisfies (17), (24), (25) and $u_1(0) = u_2(0)$.*

Proof. See appendix.

Proposition 4 yields a simple way to test the Pareto efficiency of a given competitive equilibrium. However, it does not bring any insight into the issue of implementing a Pareto efficient allocation. Closing the present section, this particular problem is discussed next.

Proposition 5 *If χ is a Pareto efficient allocation, then, there exist sequences φ and ζ such that (φ, χ, ζ) is a competitive equilibrium.*

Proof. See appendix.

Even if some taxes were exogenous, it could still be possible to decentralize a Pareto efficient allocation. For instance, suppose that $\tau_t^N = \tau_t^T = -\tau_t^l = 0$. If the government manages to raise enough tax revenues to balance its budget with the lump sum taxes on profits, then a Pareto efficient allocation can be implemented by setting $\delta_t^F = 0$.

5 Ramsey efficiency

This section considers situations in which a first best Pareto efficient allocation cannot be, for some reason, implemented. For instance, (10) must hold in any competitive equilibrium. If taxes rates on consumption are exogenous and do not satisfy (24), then a competitive equilibrium will not respect (21).

Following the tradition started with the seminal paper of Ramsey [13], the problem of selecting a second best allocation will be considered. That is, an allocation that yields the highest utility among the competitive equilibrium allocations. It turns out that for some class of period utility functions, in several situations the Friedman rule $i_{t+1} = 0$ is still optimal.

Let h be a homogeneous function. Suppose that the period utility function u can be expressed as

$$u(c^T, c_1^N, c_2^N, l) = F(h(c^T, c_1^N, c_2^N), l) \ . \tag{26}$$

Lemma 1, which is found in the appendix, presents some convenient properties of utilities functions that satisfy (26). Most of the period utility functions found in the macroeconomic literature can be expressed as in (26).

The next proposition shows that the Friedman Rule is optimal even if the government cannot optimally select one tax rate.

Proposition 6 *Assume that u satisfies (26). If the government can choose at least three of the four sequences of taxes $\{\tau_t^N\}_{t=0}^{\infty}$, $\{\tau_t^T\}_{t=0}^{\infty}$, $\{\tau_t^l\}_{t=0}^{\infty}$ and $\{\delta_t^F\}_{t=0}^{\infty}$, as well as $\{\delta_t^N\}_{t=0}^{\infty}$ and $\{\delta_t^T\}_{t=0}^{\infty}$, then the optimal policy specifies $i_{t+1} = 0$.*

Proof. See appendix.

Carlstrom and Fuerst [1] argued that in an open economy, the existence of a given international interest rate will impact the behavior of domestic agents in a way that the Friedman rule may fail to be optimal. An obvious way to circumvent such constraint is to tax the income on foreign assets. This possibility is encompassed in Proposition 6. However, the same proposition also shows that even if $\delta_t^F = 0$, the Friedman rule may still be optimal. All that is needed is that the government has access to a sufficient large set of alternative tax instruments.

The proof of the above proposition makes clear that, under some conditions, even if the government cannot select the taxation on profits the Friedman rule is optimal.

Corollary 1 *Assume that u satisfies (26). If $\{\tau_t^l\}_{t=0}^{\infty}$, $\{\delta_t^N\}_{t=0}^{\infty}$ and $\{\delta_t^T\}_{t=0}^{\infty}$ are exogenous and the government can choose $\{\tau_t^N\}_{t=0}^{\infty}$, $\{\tau_t^T\}_{t=0}^{\infty}$, and $\{\delta_t^F\}_{t=0}^{\infty}$, then the optimal policy specifies $i_{t+1} = 0$.*

Proof. See appendix.

The next proposition shows that even under more strict restrictions, the Friedman Rule is optimal.

Proposition 7 *Assume that u satisfies (26). If $\{\delta_t^F\}_{t=0}^{\infty}$ is exogenous and the government can choose at least one of the two sequences of taxes $\{\tau_t^T\}_{t=0}^{\infty}$ and $\{\tau_t^l\}_{t=0}^{\infty}$ and each of the sequences $\{\tau_t^N\}_{t=0}^{\infty}$, $\{\delta_t^N\}_{t=0}^{\infty}$ and $\{\delta_t^T\}_{t=0}^{\infty}$, then the optimal policy specifies $i_{t+1} = 0$.*

Proof. See appendix.

The assumption that the Ramsey planner can choose $\{\tau_t^N\}_{t=0}^{\infty}$ is essential. The example that follows illustrates this fact.

Example 2 As in the previous proposition, assume that u satisfies (26). Further, suppose that $\{\tau_t^N\}_{t=0}^{\infty}$ and $\{\delta_t^F\}_{t=0}^{\infty}$ are exogenous. Let the government select both $\{\tau_t^T\}_{t=0}^{\infty}$ and $\{\tau_t^l\}_{t=0}^{\infty}$. In such a context, the Friedman rule may fail to be optimal. Further details are provided in the appendix.

There is an intuitive explanation to the above finding. The exogeneity of $\{\tau_t^N\}_{t=0}^{\infty}$ will prevent the government from taxing the consumption of non tradables in an optimal fashion. A tax on consumption of the cash good distinct from Friedman's prescription will partially offset that inability of the government to select $\{\tau_t^N\}_{t=0}^{\infty}$.

Next we show that even if consumption taxes are exogenous but satisfy $\tau_t^N = \tau_t^T$, the Friedman rule is still optimal.

Proposition 8 *Assume that u satisfies (26). If consumption tax rates are exogenous and satisfy $\tau_t^N = \tau_t^T$ and the government can select $\{\tau_t^l\}_{t=0}^{\infty}$, $\{\delta_t^F\}_{t=0}^{\infty}$, $\{\delta_t^N\}_{t=0}^{\infty}$ and $\{\delta_t^T\}_{t=0}^{\infty}$, then the optimal policy specifies $i_{t+1} = 0$.*

Proof. See appendix.

Clearly, the above proposition encompasses the case in which $\tau_t^N = \tau_t^T = 0$. Schmitt-Grohé and Uribe [14] studied the optimality of the Friedman rule in an one sector small open economy. They concluded that if the Ramsey planner does not have access to consumption taxes (i.e., such taxes are necessarily equal to zero) but has to labor income taxes, the Friedman rule is not optimal. Our conclusion is different from theirs because we allowed the Ramsey planner to pick $\{\delta_t^F\}_{t=0}^{\infty}$ in an optimal fashion.

The assumption that $\tau_t^N = \tau_t^T$ is essential in the above proposition. Had it not been imposed, the Friedman rule would not be optimal. The next example discusses this case.

Example 3 Consider now the situation in which tax rates satisfy all hypothesis of Proposition 8, except that now $\tau_t^N \neq \tau_t^T$. This non-uniform taxation of consumption goods will prevent the Friedman Rule from being optimal. Further details are provided in the appendix.

6 Conclusion

The Friedman rule (i.e., zero nominal interest rates) has been the focus of a large body of literature on optimal monetary and fiscal policy. This paper investigated the optimality of that policy prescription in a two sector small open economy.

We showed that if the government can choose all possible distorting tax rates in an efficient manner, then it can implement Pareto efficient allocations. Any policy that implements a Pareto efficient allocation will satisfy the Friedman rule.

We considered the case in which not all distorting tax rates can be selected in an efficient manner. In such a context, the second best solution may respect or not the Friedman rule. It turns out that the optimality of that prescription will depend on the set of constraints the government faces when selecting the distorting tax rates.

7 Appendix

7.1 Households' first order conditions

If M_0 is positive, the first order necessary and sufficient conditions for a typical household are

$$\beta^t u_T(t) = \lambda_t(1+\tau_t^T)E_t \ ; \tag{27}$$

$$\beta^t u_1(t) = (\lambda_t + \mu_t)(1+\tau_t^N)p_t^N \ ; \tag{28}$$

$$\beta^t u_2(t) = \lambda_t(1+\tau_t^N)p_t^N \ ; \tag{29}$$

$$-\beta^t u_l(t) = \lambda_t(1-\tau_t^l)w_t \ ; \tag{30}$$

$$\lambda_t = \lambda_{t+1} + \mu_{t+1} \ ; \tag{31}$$

$$\lambda_t = \lambda_{t+1}(1+i_{t+1}) \ ; \tag{32}$$

$$\lambda_t E_t = \lambda_{t+1} E_{t+1}[1+(1-\delta^F_{t+1})i^F_{t+1}] \ ; \tag{33}$$

$$M_t \geq (1+\tau^N_t)p^N_t c^N_{1t} \ \& \ \mu_t[M_t - (1+\tau^N_t)p^N_t c^N_{1t}] = 0 \ ; \tag{34}$$

$$(1+\tau^T_t)E_t c^T_t + (1+\tau^N_t)p^N_t(c^N_{1t}+c^N_{2t}) + M_{t+1} + B_{t+1} + E_t B^F_{Ht+1} =$$

$$(1-\tau^l_t)w_t l_t + M_t + (1+i_t)B_t + E_t[1+(1-\delta^F_t)i^F_t]B^F_{Ht} + (1-\delta^T_t)\psi^T_t + (1-\delta^N_t)\psi^N_t \ ; \tag{35}$$

$$\lim_{t\to\infty} \lambda_t M_{t+1} = \lim_{t\to\infty} \lambda_t B_{t+1} = \lim_{t\to\infty} \lambda_t E_t B^F_{Ht+1} = 0 \ ; \tag{36}$$

$$c^T_t, c^N_{1t}, c^N_{2t}, l_t, M_t, \lambda_t, \mu_t \geq 0 \ , \ l_t \leq 1 \ ; \tag{37}$$

where λ_t and μ_t are Lagrange multipliers for, respectively, budget and cash-in-advance constraints. The notation $u_T(t)$, $u_1(t)$, $u_2(t)$ and $u_l(t)$ was introduced on page 6.

7.2 Proofs

Proof of Proposition 1: For the 'if part', suppose that (φ, χ, ζ) is a competitive equilibrium. It is needed to show that (1) and (9)-(16) hold. Constraint (1) is trivially satisfied.

We will now show that (9) holds. Forward (27) by one period and divide it by itself. Then, combine the resulting equation with (33). This yields (8), from which (9) can be trivially obtained.

To obtain (10), divide (27) by (29) and combine the resulting equation with firms' first order conditions. For (11), divide (30) by (29) and again combine the resulting equation with firms' first order conditions..

Consider now (12). Multiplying (35) by λ_t, using (27)-(34) and the equalities $\psi^T_t = \frac{1-\alpha^T}{\alpha^T} w_t l^T_t$ and $\psi^N_t = \frac{1-\alpha^N}{\alpha^N} w_t l^N_t$ one obtains

$$\beta^t W(t) + (\lambda_{t+1} + \mu_{t+1})M_{t+1} + \lambda_t B_{t+1} +$$

$$\lambda_t E_t B^F_{Ht+1} = (\lambda_t + \mu_t)M_t + \lambda_t(1+i_t)B_t + \lambda_t E_t[1+(1-\delta^F_t)i^F_t]B^F_{Ht} -$$

$$\frac{\beta^t u_l(t)}{(1-\tau^l_t)}\left[(1-\delta^T_t)\frac{1-\alpha^T}{\alpha^T}l^T_t + (1-\delta^N_t)\frac{1-\alpha^N}{\alpha^N}l^N_t\right] \ .$$

Adding up from date 0 to some date k and using (32) and (33) to cancel the identical terms out one gets

$$\sum_{t=0}^{k} \beta^t \left\{ W(t) + \frac{u_l(t)}{(1-\tau^l_t)}\left[(1-\delta^T_t)\frac{1-\alpha^T}{\alpha^T}l^T_t + (1-\delta^N_t)\frac{1-\alpha^N}{\alpha^N}l^N_t\right]\right\} +$$

$$\lambda_k M_{k+1} + \lambda_k B_{k+1} + \lambda_k E_k B^F_{Hk+1} =$$

$$\mu_0 M_0 + \lambda_0 \left\{ M_0 + (1+i_0)B_0 + E_0[1+(1-\delta^F_0)i^F_0]B^F_{H0} \right\} \ . \tag{38}$$

But $\mu_0 M_0 = \mu_0(1+\tau_0^N)p_0^N c_{1,0}^N = u_1(0)c_{10}^N - \lambda_0(1+\tau_0^N)p_0^N c_{1,0}^N$. Plug this last equality into (38). Then, use the fact that $\lambda_0 = \frac{u_T(0)}{(1+\tau_0^T)E_0} = \frac{u_2(0)}{(1+\tau_0^N)p_0^N}$ to eliminate λ_0 from (38). Make $k \to \infty$ and use the transversality condition in (36) to obtain (12).

Equation (7) has to hold in a competitive equilibrium. Divide it by the factor $\prod_{s=1}^t (1+i^F)$. Then, add up from date 0 to some date k to obtain

$$\sum_{t=0}^{k} \frac{x_t}{\prod_{s=1}^t (1+i_s^F)} + (1+i_0^F)(B_{H0}^F + B_{G0}^F) = \frac{B_{Hk+1}^F + B_{Gk+1}^F}{\prod_{s=1}^t (1+i_s^F)} \ . \tag{39}$$

Recall that both B_{Ht}^F and B_{Gt}^F are bounded and i_t^F belongs to a finite set of positive numbers. Thus, as $k \to \infty$ the right hand side of (39) vanishes and (13) holds.

Constraint (14) is obviously satisfied. Concerning (15), divide (28) by (29) to obtain $\frac{u_1(t)}{u_2(t)} = 1 + \frac{\mu_t}{\lambda_t} \geq 1$.

The 'if part' of the proof will be concluded by showing that (16) is satisfied. From (28), (31), (34) and (36),

$$\lambda_{t-1}M_t = (\lambda_t + \mu_t)M_t = \frac{\beta^t u_1(t) M_t}{(1+\tau_t^N)p_t^N} \geq \frac{\beta^t u_1(t) c_{1t}^N}{(1+\tau_t^N)} \geq 0 \ . \tag{40}$$

Now make $t \to \infty$ and apply (36) to obtain (16).

For the 'only if part', take an initial price $p_0^N > 0$ and an object χ satisfying (1) and (9)-(16). It must be shown that there exist sequences φ and ζ such that (φ, χ, ζ) satisfies all conditions of a competitive equilibrium.

Recall that p_0^N is given. Thus, it is possible to define the sequence $\{p_t^N\}_{t=1}^\infty$ recursively. Set those prices according to the kernel

$$p_{t+1}^N = p_t^N \beta \frac{1+\tau_t^N}{1+\tau_{t+1}^N} \frac{u_1(t+1)}{u_2(t)} \ . \tag{41}$$

Set λ_t as in (29), μ_t as in (28), E_t as in (27), i_t as in (32) and w_t as in (30). Define cash holdings as $M_t = (1+\tau_t^N)p_t^N c_{1t}^N$. Let $B_{Ht+1}^F = 0$. Define B_1 to balance household's budget constraint at date 1. The entire sequence $\{B_{t+1}\}_{t=0}^\infty$ is constructed in this recursive way, while $\{B_{Gt+1}^F\}_{t=0}^\infty$ is defined recursively to balance the budget constraint of the government.

It remains to show that the proposed (φ, χ, ζ) is a competitive equilibrium. Item (iii) of definition 1 is clearly satisfied. For item (i) it is enough to prove that (27)-(37) are satisfied. The variables were defined so that (27)-(30) hold. Concerning (31), (41) implies that

$$\frac{\beta^t u_2(t)}{(1+\tau_t^N)p_t^N} = \frac{\beta^{t+1} u_1(t+1)}{(1+\tau_{t+1}^N)p_{t+1}^N} \ .$$

This last equality, combined to (28) and (29), yields (31). The sequence $\{i_{t+1}\}_{t=0}^\infty$ was defined in such a way that (32) is satisfied. Observe that (9) implies (8).

Combine this last equation to (27) to obtain (33). Money balances were defined so that (34) holds, while the definition of $\{B_{t+1}\}_{t=0}^{\infty}$ ensures that (35) is satisfied. The last transversality condition in (36) is trivially satisfied. To verify that the other two also hold, observe that (38) can be obtained exactly as in the first part of the proof. Plus, (12) ensures that the series in (38) converges in $\mathbb{R}$. So, making $k \to \infty$, one concludes that $\lambda_k M_{k+1} + \lambda_k B_{k+1} \to 0$. Thus, it remains to show that either $\lambda_k M_{k+1}$ or $\lambda_k B_{k+1}$ goes to zero. Observe that (40) holds as an equality. So, (16) ensures that $\lambda_k M_{k+1} \to 0$. With the exception of $\mu_t \geq 0$, all inequalities in (37) are trivially true. To show that $\mu_t \geq 0$, divide (28) by (29) and use (15).

To finish the 'only if part', it remains to show that each firm is maximizing its respective date t profit. Combine (29), (30) and (11) to conclude that $w_t = p_t^N \alpha^N (l_t^N)^{\alpha^N - 1}$. Finally, combine this last equality to (27), (29) and (10) to obtain $w_t = E_t \alpha^T (l_t^T)^{\alpha^T - 1}$. ■

Proof of Proposition 2: Let (φ, χ, ζ) be a competitive equilibrium that satisfies (17). This equation and (32) implies $\lambda_t = \lambda_{t+1}$. So, (36) implies (18). First order conditions (28), (29) and (31) yield (41) and $u_1(t+1) = u_2(t+1)$. Therefore,

$$M_t \geq p_t^N (1 + \tau_t^N) c_{1t}^N = p_0^N (1 + \tau_0^N) \beta^t \left[\prod_{s=1}^{t} \frac{u_1(t)}{u_2(t-1)} \right] c_{1t}^N \ .$$

Use the fact that $u_1(t+1) = u_2(t+1)$ to obtain (19). ■

Proof of Proposition 3: Let (φ, χ, ζ) be a competitive equilibrium that satisfies (17). Let $\{M'_{t+1}\}_{t=0}^{\infty}$ be any sequence satisfying $M'_{t+1} \geq M_{t+1}$ and (18). Define $B'_{t+1} = M_{t+1} + B_{t+1} - M'_{t+1}$ and $\zeta'_{t+1} = (M'_{t+1}, B'_{t+1}, B^F_{Ht+1}, B^F_{Gt+1})$. It is a straightforward exercise to show that (φ, χ, ζ') satisfies all conditions of a competitive equilibrium. ■

Proof of Proposition 4: For the 'if part', assume that a competitive equilibrium (φ, χ, ζ) satisfies (17), (24), (25) and $u_1(0) = u_2(0)$. It is needed to show that it also respects (1), (13), (20), (21), (22), and (23). The first two are trivially satisfied. It was shown in the previous proof that

$$i_{t+1} = 0 \Rightarrow u_1(t+1) = u_2(t+1) \ .$$

Since it was assumed that $u_1(0) = u_2(0)$, (20) is satisfied. Equations (10) and (24) imply (21). Similarly, (11) and (24) imply (22), while (23) can be obtained from (9) and (25).

For the 'only if part', let (φ, χ, ζ) be a competitive equilibrium and assume that χ is Pareto efficient. We need to show that (17), (24), (25) and $u_1(0) = u_2(0)$ hold. The last condition follows directly from (20). Concerning (17), (20) and people's first order conditions imply $\mu_t = 0$. This last equality leads to $\lambda_t = \lambda_{t+1}$, which in its turn implies $i_{t+1} = 0$. Equalities (10), (11), (21), and (22) yield (24). Finally, (9) and (23) imply that (25) holds. ■

Proof of Proposition 5: Let χ be a Pareto efficient allocation. From Proposition 1, it suffices to show that there exist an initial price level p_0^N and a sequence $\{i_{t+1}, \tau_t^l, \tau_t^N, \tau_t^T, \delta_{t+1}^F, \delta_t^N, \delta_t^T\}_{t=0}^{\infty}$ that satisfy (1) and (9)-(16).

There are several such p_0^N and $\{i_{t+1}, \tau_t^l, \tau_t^N, \tau_t^T, \delta_{t+1}^F, \delta_t^N, \delta_t^T\}_{t=0}^{\infty}$. Of course, it suffices to establish that there exists one. Set i_{t+1}, τ_t^N, τ_t^T, τ_t^l and δ_t^F to be constant and to satisfy (17), (24) and (25). Note that this implies $\delta_t^F = 0$. Set $\delta_t^N = \delta_t^T$ and constant too. It will soon become clear how to pin down these constant values of δ^N and τ^N. Set p_0^N so that $M_0 = p_0^N(1+\tau^N)c_{1,0}^N$. It remains to show that (1) and (9)-(16) hold.

Feasibility constraint (1) is trivially satisfied. Since χ is Pareto efficient, it satisfies (21), (22) and (23). Combine these three constraints with (24) and the fact that tax rates are constant to obtain (9), (10) and (11).

Given the chosen tax rates and price levels, (12) can be written as

$$(1+\tau^N)\left[\sum_{t=0}^{\infty}\beta^t W(t) - u_1(0)c_{10}^N - u_2(0)\frac{(1+i_0)B_0 c_{1,0}^N}{M_0}\right] =$$

$$u_T(0)(1+i_0^F)B_{H0}^F - (1-\delta^N)\sum_{t=0}^{\infty}\beta^t\left(\frac{1-\alpha^T}{\alpha^T}l_t^T + \frac{1-\alpha^N}{\alpha^N}l_t^N\right) .$$

Thus, it is enough to pick values for τ^N and δ^N that satisfy the above equation. Definition 2 ensures that (13) holds. The initial price level and taxes rates were chosen so that (14) holds, while (15) follows from (20). Concerning (16), it is enough to recall that c_{1t}^N is bounded and Pareto efficiency requires $\limsup_{t\to\infty} u_1(t) < \infty$. ■

Lemma 1 *If the period utility function u satisfies (26), then it satisfies the following conditions:*

$$\frac{u_{1l}}{u_1} = \frac{u_{2l}}{u_2}\ ,\ \frac{u_{T1}}{u_1} = \frac{u_{T2}}{u_2}; \tag{42}$$

$$\frac{u_{T1}}{u_1}c^T + \sum_{j=1,2}\frac{u_{1j}}{u_1}c_j^N = \frac{u_{T2}}{u_2}c^T + \sum_{j=1,2}\frac{u_{2j}}{u_2}c_j^N\ ; \tag{43}$$

$$\frac{u_{T1}}{u_1}c^T + \sum_{j=1,2}\frac{u_{1j}}{u_1}c_j^N + \frac{u_{1l}}{u_1}l = \frac{u_{T2}}{u_2}c^T + \sum_{j=1,2}\frac{u_{2j}}{u_2}c_j^N + \frac{u_{2l}}{u_2}l\ . \tag{44}$$

Proof. Differentiate both sides of (26) with respect to l and c_1^N. This yields

$$\frac{u_{1l}}{u_1} = \frac{\frac{\partial^2 F}{\partial l \partial h}}{\frac{\partial F}{\partial h}}\ .$$

Then, differentiate (26) with respect to l and c_2^N. This establishes the first equality in (42). Moreover,

$$\frac{u_{Ti}}{u_i} = \frac{\frac{\partial^2 F}{\partial h^2}h_T}{\frac{\partial F}{\partial h}} + \frac{h_{Ti}}{h_i},\ i = 1,2.$$

On the other hand, the homogeneity of h ensures that there exists a function f that satisfies

$$h_1 = h_2 f\left(\frac{c_1^N}{c_2^N}\right) \Rightarrow h_{T1} = h_{T2} f\left(\frac{c_1^N}{c_2^N}\right) \Rightarrow \frac{h_{T1}}{h_1} = \frac{h_{T2}}{h_2}\ ,$$

and the second equality in (42) is established. Concerning (43), we closely follow Chari, Christiano and Kehoe [3]. If a is any positive real number, the homogeneity of h implies

$$\frac{u_1(ac^T, ac_1^N, ac_2^N, l)}{u_1(c^T, c_1^N, c_2^N, l)} = \frac{u_2(ac^T, ac_1^N, ac_2^N, l)}{u_2(c^T, c_1^N, c_2^N, l)} .$$

Differentiate each fraction in the above expression with respect to a and set $a = 1$ to obtain (43). Equality (43) follows directly from (42) and (43). ■

All proofs that follow are very similar. They built on the arguments of Chari, Christiano and Kehoe [3] and rely on the previous lemma.

Proof of Proposition 6: Suppose that $\{\tau_t^l\}_{t=0}^{\infty}$ is exogenous and $\{\tau_t^N\}_{t=0}^{\infty}$, $\{\tau_t^T\}_{t=0}^{\infty}$, and $\{\delta_t^F\}_{t=0}^{\infty}$ are endogenous. Thus, given some allocation χ, the government can pick $\{\tau_t^N\}_{t=0}^{\infty}$ so that (11) is satisfied, $\{\tau_t^T\}_{t=0}^{\infty}$ so that (10) holds and $\{\delta_t^F\}_{t=0}^{\infty}$ to satisfy (9). Thus, these constraints can be disregarded when choosing a second best allocation. Of course, the same is true regardless of which sequence is exogenous.

Consider the problem of selecting a best competitive equilibrium allocation. Observe that if such an allocation can be implemented by policies that satisfy $M_0 > (1+\tau_0^N)p_0^N c_{1,0}^N$ or $\delta_t^N < 1$ or $\delta_t^T < 1$ for some t or $\delta_0^F < 1$, that amounts to say that the available lump-sum tax revenues have not been fully used up. Then, the allocation in question must be Pareto efficient and the proposition is established. So, in what follows, it will be assumed that $M_0 = (1+\tau_0^N)p_0^N c_{1,0}^N$ and $\delta_t^N = \delta_t^T = \delta_0^F = 1$.

Plug the above equalities into (12). This procedure yields

$$\sum_{t=0}^{\infty} \beta^t W(t) = u_1(0)c_{1,0}^N + u_2(0)\frac{(1+i_0)B_0}{M_0}c_{1,0}^N + u_T(0)\frac{B_{H0}^F}{1+\tau_0^T} . \tag{45}$$

Consider the problem of maximizing (4) subject to (1), (45) and (13). If it were not for constraints (16) and (15), the solution to this problem would be a best competitive equilibrium allocation. The argument used in the proof of Proposition 5 establishes that the solution will satisfy (16). So, if the solution respects (15), such a solution will yield the highest attainable utility level. This turns out to be exactly the case.

Let Γ and $\beta^t \xi_t^N$ be Lagrange multipliers for, respectively, (45) and the resource constraint for the non-tradable sector. For $t \geq 1$, the respective first order conditions for c_{1t}^N and c_{2t}^N are

$$\frac{\xi_t^N}{u_1(t)} = (1+\Gamma) + \Gamma\left[\frac{u_{T1}(t)}{u_1(t)}c_t^T + \frac{u_{11}(t)}{u_1(t)}c_{1t}^N + \frac{u_{12}(t)}{u_1(t)}c_{2t}^N + \frac{u_{1l}(t)}{u_1(t)}l_t\right]$$

and

$$\frac{\xi_t^N}{u_2(t)} = (1+\Gamma) + \Gamma\left[\frac{u_{T2}(t)}{u_2(t)}c_t^T + \frac{u_{12}(t)}{u_2(t)}c_{1t}^N + \frac{u_{22}(t)}{u_2(t)}c_{2t}^N + \frac{u_{2l}(t)}{u_2(t)}l_t\right] .$$

Apply Lemma 1 to conclude that $u_1(t) = u_2(t)$, which implies $i_{t+1} = 0$. ■

Proof of Corollary 1: Except for the fact that constraint (45) should be replaced by

$$\sum_{t=0}^{\infty}\beta^t\left\{W(t)+\frac{u_l(t)}{(1-\tau_t^l)}\left[(1-\delta_t^T)\frac{1-\alpha^T}{\alpha^T}l_t^T+(1-\delta_t^N)\frac{1-\alpha^N}{\alpha^N}l_t^N\right]\right\}=$$

$$u_1(0)c_{1,0}^N+u_2(0)\frac{(1+i_0)B_0}{M_0}c_{1,0}^N+u_T(0)\frac{B_{H0}^F}{1+\tau_0^T}\ ,$$

the same argument used in the previous proof applies. ■

Proof of Proposition 7: Again, there is no loss of generality in assuming that $\delta_t^T=\delta_t^N=1$. Consider first the case in which $\{\tau_t^T\}_{t=0}^{\infty}$ is exogenous and $\{\tau_t^l\}_{t=0}^{\infty}$ is not. Given any allocation, it is always possible to choose $\{\tau_t^N\}_{t=0}^{\infty}$ and $\{\tau_t^l\}_{t=0}^{\infty}$ so that (10) and (11) hold. The Ramsey problem is to maximize (4) subject to (1), (9),(12) and (13). As in Proposition 6, its solution will satisfy (16). It remains to show that it satisfies $u_1(t)=u_2(t)$. Let Γ and $\beta^t\xi_t^N$ be as previously defined and $\beta^t\xi_t^F$ be a Lagrange multiplier for (9). The first order conditions with respect c_{1t}^N and c_{2t}^N can be written as

$$\frac{\xi_t^N}{u_1(t)}=(1+\Gamma)+\Gamma\left[\frac{u_{T1}(t)}{u_1(t)}c_t^T+\frac{u_{11}(t)}{u_1(t)}c_{1t}^N+\frac{u_{12}(t)}{u_1(t)}c_{2t}^N+\frac{u_{1l}(t)}{u_1(t)}l_t\right]+\xi_t^F\frac{u_{T1}(t)}{u_1(t)}$$

and

$$\frac{\xi_t^N}{u_2(t)}=(1+\Gamma)+\Gamma\left[\frac{u_{T2}(t)}{u_2(t)}c_t^T+\frac{u_{12}(t)}{u_2(t)}c_{1t}^N+\frac{u_{22}(t)}{u_2(t)}c_{2t}^N+\frac{u_{2l}(t)}{u_2(t)}l_t\right]+\xi_t^F\frac{u_{T2}(t)}{u_2(t)}\ .$$

Then, apply Lemma 1 to conclude that $u_1(t)=u_2(t)$. A similar reasoning is used for the case in which $\{\tau_t^l\}_{t=0}^{\infty}$ is exogenous and $\{\tau_t^T\}_{t=0}^{\infty}$ is not. First, combine (9), (10) and (11) to obtain

$$u_l(t)=\beta^{-t}\frac{1-\tau_t^l}{1-\tau_0^l}\left(\frac{l_0^T}{l_t^T}\right)^{1-\alpha^T}\frac{u_l(0)}{\prod_{s=1}^t[1+(1-\delta_t^F)i_t^F]}\ . \tag{46}$$

Then, maximize (4) subject to (1), (46),(12) and (13). Except for the factors $\frac{u_{T1}}{u_1}$ and $\frac{u_{T2}}{u_2}$ being replaced by $\frac{u_{l1}}{u_1}$ and $\frac{u_{l2}}{u_2}$, this procedure leads to the same pair of equalities as in the previous case. Hence, $u_1(t)=u_2(t)$ again. ■

Details of Example 2: We repeat the steps of the second part of the last proof. A competitive equilibrium must satisfy

$$u_2(t)=\beta^{-t}\frac{1+\tau_t^N}{1+\tau_0^N}\left(\frac{l_0^T}{l_t^T}\right)^{1-\alpha^T}\left(\frac{l_t^N}{l_0^N}\right)^{1-\alpha^N}\frac{u_2(0)}{\prod_{s=1}^t[1+(1-\delta_t^F)i_t^F]}\ .$$

Then, maximize (4) subject to the above equality, (1), (12) and (13). Simple manipulation of the first order conditions shows that

$$\frac{\xi_t^N}{u_1(t)}-\frac{\xi_t^N}{u_2(t)}=\xi_t^F\left[\frac{u_{12}(t)}{u_1(t)}-\frac{u_{22}(t)}{u_2(t)}\right]\ .$$

Clearly, for several utilities functions we may have $u_1(t) \neq u_2(t)$. For instance, this will happen whenever $u_{12} \geq 0$ and $u_{22} < 0$.

Proof of Proposition 8: Given the hypothesis on the tax rates, the government can decentralize any attainable allocation by setting δ_t^F and τ_t^l to satisfy (9) and (11). Consider now the problem of maximizing (4) subject to (1), (45) and (13). As in Proposition 6, its solution will satisfy (16) and $u_1(t) = u_2(t)$. Hence, it remains to show that it satisfies (10). Let Γ and $\beta^t \xi_t^N$ be as previously defined and $\beta^t \xi_t^T$ and $\beta^t \xi_t^l$ be Lagrange multipliers for, respectively, the resource constraint in the tradable sector and labor resource constraint. The reasoning used in the proof of Proposition 6 shows that

$$\frac{u_T(t)}{u_2(t)} = \frac{\xi_t^T}{\xi_t^N} \ .$$

The first order conditions for l_t^N and l_t^T imply

$$\frac{\xi_t^T}{\xi_t^N} = \frac{\alpha^N (l_t^T)^{1-\alpha^T}}{\alpha^T (l_t^N)^{1-\alpha^N}} \ .$$

Since $\tau_t^T = \tau_t^N$, these two equalities yield (10). ∎

Details of Example 3: From the previous proof, it should be clear that constraint (10) cannot be omitted from the maximization problem. Write that constraint as

$$(1+\tau_t^N)\alpha^T (l_t^N)^{1-\alpha^N} u_T(t) = (1+\tau_t^T)\alpha^N (l_t^T)^{1-\alpha^T} u_2(t) \tag{47}$$

Now, maximize (4) subject to (45), (13) and (47). Denote the Lagrange multiplier of this last constraint by $\beta^t \eta_t$, while Γ and $\beta^t \xi_t^N$ have the previous meaning. The first order conditions for c_{1t}^N and c_{2t}^N can be written as

$$\frac{\xi_t^N}{u_1(t)} = (1+\Gamma) + \Gamma \left[\frac{u_{T1}(t)}{u_1(t)} c_t^T + \frac{u_{11}(t)}{u_1(t)} c_{1t}^N + \frac{u_{12}(t)}{u_1(t)} c_{2t}^N + \frac{u_{1l}(t)}{u_1(t)} l_t \right] -$$
$$\eta_t \left[(1+\tau_t^N)\alpha^T (l_t^N)^{1-\alpha^N} \frac{u_{T1}(t)}{u_1(t)} - (1+\tau_t^T)\alpha^N (l_t^T)^{1-\alpha^T} \frac{u_{12}(t)}{u_1(t)} \right]$$

and

$$\frac{\xi_t^N}{u_2(t)} = (1+\Gamma) + \Gamma \left[\frac{u_{T2}(t)}{u_2(t)} c_t^T + \frac{u_{12}(t)}{u_2(t)} c_{1t}^N + \frac{u_{22}(t)}{u_2(t)} c_{2t}^N + \frac{u_{2l}(t)}{u_2(t)} l_t \right] -$$
$$\eta_t \left[(1+\tau_t^N)\alpha^T (l_t^N)^{1-\alpha^N} \frac{u_{T2}(t)}{u_2(t)} - (1+\tau_t^T)\alpha^N (l_t^T)^{1-\alpha^T} \frac{u_{22}(t)}{u_2(t)} \right] .$$

Apply Lemma 1 to obtain

$$\frac{\xi_t^N}{u_1(t)} - \frac{\xi_t^N}{u_2(t)} = \eta_t (1+\tau_t^T)\alpha^N (l_t^T)^{1-\alpha^T} \left[\frac{u_{12}(t)}{u_1(t)} - \frac{u_{22}(t)}{u_2(t)} \right]$$

As in Example 2, there can be several circumstances in which $u_1(t) \neq u_2(t)$.

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