Why Does Private Consumption Rise After a Government Spending Shock?[*]

Hafedh Bouakez[†]   Nooman Rebei[‡]

November 2003

Abstract

Recent empirical evidence suggests that private consumption is *crowded-in* by government spending. This outcome violates existing macroeconomic theory, according to which the negative wealth effect brought about by a rise in public expenditure should decrease consumption. In this paper, we develop a simple real business cycle model where preferences depend on private and public spending, and households are habit forming. The model is estimated by the minimum-distance and the maximum-likelihood methods using U.S. data. Estimation results indicate a strong Edgeworth complementarity between private and public spending. This feature enables the model to generate a positive response of consumption following a government spending shock. In addition, the impulse-response functions generated by the estimated model mimic closely those obtained from a benchmark vector autoregression.

JEL classification: E32, E62
Bank classification: Business fluctuations and cycles, Economic models, Fiscal policy.

[†]International Department, Bank of Canada, 234 Wellington St., Ottawa, Canada K1A OG9. Tel.: 1-613-782-8364; Fax: 1-613-782-7658; E-mail: hbouakez@bank-banque-canada.ca.

[‡]Research Department, Bank of Canada, 234 Wellington St., Ottawa, Canada K1A OG9. Tel.: 1-613-782-8871; Fax: 1-613-782-7163; E-mail: nrebei@bank-banque-canada.ca.

1. Introduction

Over the past three years, there has been a renewed interest in the effects of government spending on private consumption. Recent empirical studies based on vector autoregressions (VARs) find that an increase in public spending leads to a significant and persistent increase in private consumption.[1] This result contradicts the basic prediction of the real business cycle (RBC) model that government spending *crowds out* private consumption. Intuitively, an increase in government spending creates a negative wealth effect by lowering the households' permanent income. To prevent a large drop in consumption, households increase their labour supply, but this substitution effect is typically not strong enough to offset the wealth effect. As a result, consumption decreases. This suggests that standard RBC models are not appropriate to examine the macroeconomic implications of fiscal policy shocks.

Departing from the assumption of price flexibility, Linnemann and Schabert (2003) and Galí, López-Salido, and Vallés (2003) recently examined the role of government spending in a New Keynesian framework. Their results show that price stickiness by itself does not overturn the crowding-out effect of public spending on private consumption. However, by amending the New Keynesian model to allow for the presence of non-Ricardian (or rule-of-thumb) consumers along with conventional optimizing consumers, Galí, López-Salido, and Vallés (2003) succeed in generating a positive effect of government spending on consumption. Intuitively, when prices are sticky, an increase in government spending increases aggregate demand, which in turn raises the real wage. Higher current labour income stimulates the consumption of non-Ricardian households, and if the weight of those consumers in the population is large enough, aggregate consumption will also increase.

In this paper, we propose an alternative explanation for the crowding-in effect documented in the VAR literature, and we provide an assessment of its empirical plausibility. Our explanation, which does not require a non-Ricardian environment, emphasizes the complementarity between public and private spending. We formalize this idea within a fully optimizing RBC model augmented with two important features. First, consumer preferences

---

[1]Examples include Fatás and Mihov (2001), Mountford and Uhlig (2002), Perotti (2002), and Galí, López-Salido, and Vallés (2003).

depend on government spending, and second, households are habit forming. Simulation results show that a strong Edgeworth complementarity between public and private spending is necessary to generate a positive effect of government spending on consumption. Intuitively, when the two variables are Edgeworth complements, government spending increases the marginal utility of consumption, which in turn mitigates the negative wealth effect. When the complementarity effect is sufficiently strong, consumption rises in equilibrium. Without habit formation, however, the consumption response to a government spending shock is monotonic and not as persistent as in the data. Habit formation enables the model to generate a persistent and non-monotonic consumption response similar to that obtained from the VAR. Intuitively, habit-forming households smooth both the absolute level of consumption and its rate of change. As a result, consumption response to shocks is typically smaller on impact and more gradual under habit formation than under time-separable preferences.

The question of whether private consumption and public spending are complements or substitutes has been studied by several authors, such as Aschauer (1985), Karras (1994), Amano and Wirjanto (1998), and Okubo (2003). Overall, empirical results yield mixed evidence of complementarity. These earlier studies use a partial-equilibrium approach based on Euler equations to estimate the degree of complementarity between private consumption and government spending. Our paper follows an alternative approach by estimating the parameter governing complementarity within a general-equilibrium framework. This approach has at least two advantages. First, it allows us to deal with the issue of endogeneity, because it takes into account the fact that the model's variables are often simultaneously determined. Second, the general-equilibrium model generates predictions regarding other key variables, such as output, hours worked, and investment, that could be used to test the theory.

The model is estimated by the minimum-distance (MD) and the maximum-likelihood (ML) methods using U.S. data. The first method selects the structural parameters that minimize the distance between the impulse-response functions generated by the model and those obtained from a benchmark VAR. The second method estimates the state-space representation of the model using the Kalman filter to evaluate the likelihood function. Both

methods yield plausible and significant estimates of the deep parameters. The MD and the ML estimates of the elasticity of substitution between private consumption and public spending are virtually identical. Impulse-response functions based on the estimated parameters are remarkably close to their VAR-based counterparts. In particular, the estimated model predicts that a government spending shock leads to a non-monotonic and persistent increase in consumption. In addition, the model replicates the unconditional moments of the data better than a standard RBC model.

The rest of this paper is organized as follows. Section 2 constructs a benchmark VAR to illustrate the crowding-in effect documented by earlier studies. Section 3 develops the theoretical model. Section 4 discusses the model's implications. Section 5 describes the estimation methodologies and the data. Section 6 reports empirical results, discusses the impulse-response functions generated by the estimated model, and evaluates its performance in replicating the second moments of the data. Section 7 concludes.

2. Empirical Evidence

The purpose of this section is to construct and estimate a benchmark VAR to illustrate the macroeconomic effects of a government spending shock. Our benchmark VAR is given by

$$\mathbf{Z}_t = \mathbf{A} + \mathbf{B}(L)\mathbf{Z}_{t-1} + \mathbf{u}_t, \tag{1}$$

where $\mathbf{Z}_t$ is a vector of endogenous variables that includes government spending, consumption, hours, investment, and output; $\mathbf{A}$ is a vector that contains the constant terms; $\mathbf{B}(L)$ is a finite-order vector polynomial in non-negative powers of the lag operator L; and $\mathbf{u}_t$ is a vector of serially uncorrelated shocks.

We use quarterly U.S. data from the DRI database. Owing to data limitations, we restrict our estimation to the sample period 1952Q1 to 2001Q4. Government spending is measured by the sum of local, state, and federal real spending (GGFEQ+GGSEQ).[2] Consumption is measured by real private spending on non-durable goods and services (GCNQ+GCSQ). Hours are measured by the total number of hours worked (LPMHU).

---

[2]This measure of government spending includes public investment but excludes transfers.

Investment is measured by real private non-residential investment (NRIPDC1). Output is measured by real GDP (GDPQ). All variables are converted to per-capita terms by dividing them by the civilian population, age 16 and over (P16). The variables are logged and detrended using the Hodrick-Prescott (H-P) filter. We adopt this strategy to be consistent with the solution of the model presented in section 3, where variables are expressed as percentage deviations from their steady-state values.[3]

We estimate the model (1) using the least-squares method. The Akaike and likelihood-ratio criteria suggest that the optimal number of lags in the VAR is 2. Following Fatás and Mihov (2001) and Galí, López-Salido, and Vallés (2003), we identify the government spending shock using the Cholesky decomposition. The ordering of the variables in the vector $\mathbf{Z}_t$ implies that innovations in government spending affect the remaining variables contemporaneously.

Figure 1 depicts the impulse responses of output, hours, consumption, and investment to a one-standard-deviation increase in government spending. These responses are represented with solid lines, while dotted lines delimit their 95 per cent confidence intervals. Following the shock, output and hours worked increase, but their responses are essentially insignificant. The shock triggers a significant increase in private consumption. That is, consumption is crowded-in by government spending. The consumption response is non-monotonic and persistent, reaching its peak one quarter after the shock. On the other hand, the increase in public spending reduces private investment significantly. The investment response has an inverted hump with a trough occurring six quarters after the shock.

Overall, our empirical findings are similar to those reported by Fatás and Mihov (2001), Mountford and Uhlig (2002), Perotti (2002), and Galí, López-Salido, and Vallés (2003). Thus, the crowding-in effect documented in this paper and in earlier studies seems to be a strong stylized fact that is hardly reconcilable with standard neoclassical models. The next section develops a model that accounts for this puzzle.

---

[3]It is well known that the H-P filter induces spurious correlations between variables (see Cogley and Nason 1995). The purpose of this paper, however, is not to establish new stylized facts, but to construct a benchmark to which the structural model will be compared. By applying the H-P filter, we seek to be consistent with the approach typically used to evaluate RBC models.

3. The Model

We extend the standard RBC model along two dimensions. First, we allow government spending to enter the utility function, and second, we assume that consumer preferences exhibit habit formation.

3.1 The representative household

The economy is populated by a single, infinitely lived, representative household that derives utility from effective consumption ($\tilde{C}$) and leisure $(1 - N)$.[4] Effective consumption is assumed to be a constant-elasticity-of-substitution (CES) index of private consumption (C) and government spending (G):

$$\tilde{C}_t = \left[\phi C_t^{(\nu-1)/\nu} + (1 - \phi) G_t^{(\nu-1)/\nu} \right]^{\nu/(\nu-1)}, \tag{2}$$

where ϕ is the weight of private consumption in the effective consumption index, and $\nu > 0$ is the elasticity of substitution between private consumption and government spending.[5] In the special case where $\nu = 0$, C_t and G_t become perfect complements. As $\nu \to \infty$, they become perfect substitutes. The CES specification captures the idea of diminishing marginal returns to public spending in order to achieve a given level of effective consumption *ceteris paribus*.

We assume that the household's preferences exhibit habit formation in effective consumption.[6] More precisely, the household's instantaneous utility function depends on the current level of effective consumption relative to its previous level. The functional form of the instantaneous utility function is the following:

$$u(\tilde{C}_t, \tilde{C}_{t-1}, N_t) = \frac{1}{1 - \epsilon_1} (\tilde{C}_t/\tilde{C}_{t-1}^\gamma)^{1-\epsilon_1} + \frac{\psi}{1 - \epsilon_2} \left(1 - N_t\right)^{1-\epsilon_2}, \tag{3}$$

---

[4]It is assumed that, in each period, the representative household is endowed with one unit of time that is divided between labour and leisure.

[5]Earlier studies that allow consumer preferences to depend on government spending include Barro (1981), Aschauer (1985), Christiano and Eichenbaum (1992), Baxter and King (1993), Karras (1994), Ahmed and Yoo (1995), Ambler and Cardia (1997), Amano and Wirjanto (1997, 1998), and Cardia, Kozhaya, and Ruge-Murcia (2003).

[6]Burnside, Eichenbaum, and Fisher (2003) also introduce habit formation in consumption to improve the performance of their model in explaining the effects of a fiscal policy shock.

where $\epsilon_1, \epsilon_2,$ and ψ are positive parameters and $\gamma \in (0,1)$ measures the degree of habit formation.[7] The utility function encompasses the standard case where preferences depend only on the current level of private consumption ($\phi = 1$ and $\gamma = 0$).

The representative household supplies labour and capital to firms, and pays a lump-sum tax to the government. It allocates its disposable income to consumption and investment. Investment increases the household's stock of capital according to

$$K_{t+1} = (1 - \delta)K_t + I_t, \tag{4}$$

where K_t is the stock of capital at the beginning of period t, I_t is investment, and $\delta \in (0,1)$ is the depreciation rate of capital. Investment is subject to convex adjustment costs of the following form:

$$\varphi(I_t, K_t) = \frac{\kappa}{2} \left(\frac{I_t}{K_t} - \delta \right)^2 K_t, \tag{5}$$

where κ is a positive parameter. Therefore, the representative household's budget constraint in period t is

$$C_t + I_t + \varphi(I_t, K_t) \le w_t N_t + r_t K_t - T_t, \tag{6}$$

where w_t is the real wage, N_t is the number of hours worked, r_t is the real rental rate of capital, and T_t is a lump-sum tax.

The representative household maximizes its lifetime utility function given by

$$U_t = E_t \sum_{s=t}^{\infty} \beta^{s-t} u(\tilde{C}_s, \tilde{C}_{s-1}, N_s),$$

subject to (2), (4), (5), and (6). The operator E_t denotes the mathematical expectation conditional on the information available up to time t, and the parameter $\beta \in (0,1)$ is the subjective discount factor. First-order conditions associated with the optimal choice of

---

[7]An alternative specification of habit formation, also found in the literature, assumes that the argument that enters the utility function is the difference between the current level of (effective) consumption and the habit stock. A drawback of this specification is that the utility function is not defined when consumption falls below the habit stock. The specification used in this paper does not have this shortcoming.

C_t, N_t, and K_{t+1} are

$$\lambda_t = \phi(\tilde{C}_t/C_t)^{1/\nu}\left\{(1/\tilde{C}_{t-1}^{\gamma})(\tilde{C}_t/\tilde{C}_{t-1}^{\gamma})^{-\epsilon_1} - \beta\gamma E_t\left[(\tilde{C}_{t+1}/\tilde{C}_t^{1+\gamma})(\tilde{C}_{t+1}/\tilde{C}_t^{\gamma})^{-\epsilon_1}\right]\right\}, \quad (7)$$

$$\lambda_t = \psi(1-N_t)^{-\epsilon_2}/w_t, \quad (8)$$

$$\lambda_t = \frac{\beta E_t\left\{\lambda_{t+1}\left[1 + r_{t+1} - \delta + \kappa(I_{t+1}/K_{t+1} - \delta) + (\kappa/2)(I_{t+1}/K_{t+1} - \delta)^2\right]\right\}}{1 + \kappa(I_t/K_t - \delta)}, \quad (9)$$

where λ_t is the Lagrange multiplier associated with the budget constraint at time t. Equation (7) defines the marginal utility of consumption. Equation (8) equates the marginal rate of substitution between consumption and leisure to the real wage. Equation (9) determines the marginal value of capital.

3.2 Firms

Firms hire labour and rent capital to produce a homogeneous final good using the following Cobb-Douglas technology:

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}, \quad (10)$$

where A_t is a stochastic technology shock that follows a first-order autoregressive process given by

$$\ln A_t = (1 - \rho_A)\ln A + \rho_A \ln A_{t-1} + \mu_{At}, \quad (11)$$

where ρ^A is strictly bounded between -1 and 1, A is the steady-state value of A_t, and μ_{At} is a normally distributed zero-mean disturbance with standard deviation σ_{μ_A}. Each firm chooses labour and capital inputs to maximize its profit. Profit maximization yields the following input-demand equations:

$$w_t = (1-\alpha)Y_t/N_t, \quad (12)$$

$$r_t = \alpha Y_t/K_t, \quad (13)$$

which state that each factor must earn its marginal product.

3.3 The government

Government purchases are entirely financed by taxes. That is,

$$G_t = T_t. \quad (14)$$

Because Ricardian equivalence holds in this model, introducing public debt would be redundant. We assume that government spending is stochastic and follows an autoregressive process given by

$$\ln G_t = (1 - \rho_G)\ln G + \rho_G \ln G_{t-1} + \mu_{Gt}, \tag{15}$$

where ρ is strictly bounded between -1 and 1, G is the steady-state level of government spending, and μ_{Gt} is a normally distributed zero-mean disturbance with standard deviation σ_{μ_G}.

3.4 Market clearing and equilibrium

Substituting equations (12), (13), and (14) into the budget constraint (6) yields the following resource constraint

$$Y_t = C_t + I_t + G_t + \varphi(I_t, K_t), \tag{16}$$

which is the national accounting identity augmented with capital adjustment costs.

A competitive equilibrium for this economy is a collection of nine sequences $(\lambda_t, \tilde{C}_t, C_t, N_t, I_t, K_{t+1}, Y_t, w_t, \text{and } r_t)_{t=0}^{\infty}$ that satisfy (i) the definition of effective consumption (2), (ii) the accumulation equation (4), (iii) the household's maximization conditions (equations (7) to (9)), (iv) the production function (10), (v) the profit maximization conditions (equations (12) and (13)), and (vi) the market-clearing condition (16), given the initial stocks of habit and capital, and the exogenous stochastic processes (A_t, G_t).

To solve the model, we log-linearize the equilibrium conditions around a deterministic steady state where all variables are constant. This yields a system of stochastic linear difference equations that can be solved using standard methods.

4. The Model's Implications

In this section, we examine the model's implications regarding the effects of a government spending shock. More precisely, we illustrate the extent to which the model's predictions depart from those of a standard RBC framework, and the role of the key features of the model in accounting for this departure.

From the log-linearized version of the model (see Appendix A), it is easy to show that, for a given level of private consumption, the effect of a change in government spending on the marginal utility of consumption is given by

$$\frac{\partial \hat{\lambda}_t}{\partial \hat{G}_t} = (1 - \phi)(G_t/\tilde{C}_t)^{(\nu-1)/\nu} \left\{ 1/\nu - \epsilon_1 - \frac{\beta\gamma(1 - \epsilon_1)(1 + \gamma - \rho_G)}{1 - \beta\gamma} \right\}. \qquad (17)$$

Recall that our model nests the standard RBC case, which can be obtained by imposing the restrictions $\phi = 1$ and $\gamma = 0$.[8] In this case, the right-hand side of equation (17) collapses to zero, so that government spending affects consumption only through the wealth channel.

Figure 2 depicts the impulse responses to a 1 per cent government spending shock, generated by this version of the model. The responses were obtained using a plausible parameterization of the model. Specifically, we set the steady-state level of technology, A, to 1; the elasticity of output with respect to capital, α, to 0.36; and the depreciation rate, δ, to 0.025. The steady-state ratio of government spending to GDP, G/Y, is chosen to be 0.2. Preference parameters β, ϵ_1, and ϵ_2 are calibrated to 0.99, 2, and 1, respectively. The scaling parameter ψ is chosen so that, given the values of A and ϵ_2, the proportion of time allocated to work in the steady state, N, is equal to 0.31. The capital adjustment cost parameter, κ, is set to 0, and the autocorrelation coefficient of the government spending shock, ρ_G, is chosen to be 0.9.

Figure 2 shows that a positive government spending shock increases output and employment and decreases consumption and investment. Intuitively, an increase in government spending means a lower permanent income for the representative household, and thus a lower private consumption. To prevent a large drop in consumption, the household increases its labour supply. But this substitution effect is not strong enough to offset the negative wealth effect on consumption. The increase in labour supply translates into a higher output. Owing to consumption smoothing, consumption decreases less, in absolute value, than disposable income. Thus, the representative household must dissave and, as a consequence, investment decreases. In summary, the standard RBC model is clearly unable to account for the documented increase in private consumption following a government

---

[8]The parameter ν becomes irrelevant in this case.

spending shock.

Now consider a version of the model where we keep abstracting from habit formation ($\gamma = 0$), but where effective consumption depends on government spending ($\phi < 1$). In this case, the derivative $\partial \hat{\lambda}_t / \partial \hat{G}_t$ has the same sign as the term $1/\nu - \epsilon_1$. When the elasticity of substitution, ν, is lower than $1/\epsilon_1$, government spending raises the marginal utility of consumption, *ceteris paribus*. Hence, an increase in government purchases has not only a negative wealth effect on consumption, but also a positive effect that stems from the Edgeworth complementarity between private and public spending. The latter effect is stronger the smaller the value of ν relative to $1/\epsilon_1$. For sufficiently low values of ν, the complementarity effect may actually offset the wealth effect, causing consumption to increase in equilibrium.

Figure 3 illustrates the impact of complementarity between private and public spending on the economy's response to a government spending shock.[9] We consider three different scenarios by setting the elasticity of substitution, ν, to $1, 0.45$, and 0.25, respectively. Figure 3 shows that, when ν is equal to 1, a government spending shock produces a larger crowding-out effect on consumption than that predicted by the standard RBC model. Intuitively, because ν is higher than $1/\epsilon_1$, government spending decreases the marginal utility of consumption (that is, private and public spending are Edgeworth substitutes), which reinforces the negative wealth effect. When ν is set to 0.45, private and public spending become Edgeworth complements as government spending now raises the marginal utility of consumption. Figure 3 clearly shows that the complementarity effect mitigates the wealth effect, but the overall effect of the shock on consumption is still negative. This suggests that the complementarity between private consumption and government spending is rather weak in this case. The last scenario corresponds to the case $\nu = 0.25$. Under this parameterization, the complementarity effect is strong enough to dominate the wealth effect, so that private consumption is crowded-in by government spending.

Hence, the model with strong complementarity between private and public spending

---

[9]As stated earlier, for government spending to play a role in the utility function, the weight of private consumption in the CES aggregator must be strictly less than 1. Hence, we set ϕ to 0.8.

seems to solve the puzzling increase in consumption in response to a government spending shock. Yet, the model's success is only partial, as it fails to reproduce the non-monotonic response of consumption and the inverted hump-shaped response of investment obtained from the VAR.

Finally, consider the model augmented with habit formation in effective consumption $(0 < \gamma \leq 1)$. In this case, the derivative $\partial\hat{\lambda}_t/\partial\hat{G}_t$ has the same sign as the expression in brackets in equation (17). Straightforward calculation shows that the derivative is decreasing in γ. That is, habit formation reduces the effect of government spending on the marginal utility of consumption, which, in turn, dampens the complementarity effect. The intuition behind this result is as follows. Habit-forming households smooth both the absolute level of consumption and its rate of change. As a result, consumption response to shocks is smaller on impact and more gradual under habit formation than under time-separable preferences. This suggests that habit formation could help the model replicate the non-monotonic consumption response predicted by the VAR.

Figure 4 illustrates the effect of habit formation on the impulse-response functions generated by the model. The figure depicts the responses obtained using three values of the parameter γ, $0, 0.5$, and 0.8, holding the intertemporal elasticity of substitution, ν, fixed at 0.25. A comparison of the impulse responses under time-separability $(\gamma = 0)$ and habit-forming preferences $(\gamma = 0.5$ and $0.8)$ shows that habit formation introduces a hump in the responses of consumption and investment to a government spending shock. Note, however, that when habit formation is relatively important $(\gamma = 0.8)$, private consumption decreases on impact. This occurs because large values of γ attenuate the complementarity effect so severely that it fails to dominate the wealth effect. On the other hand, with a moderate amount of habit formation $(\gamma = 0.5)$, the consumption response is positive on impact and reaches its peak one quarter after the shock, exactly as predicted by the VAR. Under this parameterization, however, the model predicts that the investment response reaches its trough much earlier than the six quarters found in the VAR.

In summary, to the extent that complementarity between private and public spending is strong, and habit formation is moderate, the model presented in section 3 is capable of

11

replicating the crowding-in effect of government spending on consumption as well as the non-monotonic responses of consumption and investment predicted by the VAR.

5. Parameter Estimation

The purpose of this section is to use U.S. data to obtain values for the model parameters. In particular, we are interested in measuring the extent of complementarity between public expenditures and private consumption and the degree of habit formation. We estimate the model using two different methods. The first method involves minimizing the distance between the impulse-response functions generated by the model and those obtained from a benchmark VAR. The second method consists in maximizing the likelihood function of the model's solution. Both methods are described in more detail below.

5.1 Minimum-distance estimation

Let θ denote the vector that contains the model's structural parameters. To the extent that a unique equilibrium exists, the choice of θ uniquely determines the impulse-response functions generated by the model. Let $\boldsymbol{\Psi}(\theta)$ denote the mapping from θ to the model-based impulse-response functions to a government spending shock, and let $\bar{\boldsymbol{\Psi}}$ denote the impulse-response functions obtained from the VAR. The MD estimation amounts to selecting the vector $\hat{\theta}$ that minimizes the gap between $\boldsymbol{\Psi}(\theta)$ and $\bar{\boldsymbol{\Psi}}$. Formally:

$$\hat{\theta} = \arg\min \ \left(\boldsymbol{\Psi}(\theta) - \bar{\boldsymbol{\Psi}}\right)' \mathbf{W} \left(\boldsymbol{\Psi}(\theta) - \bar{\boldsymbol{\Psi}}\right),$$

where $\mathbf{W}$ is a weighting matrix.

Earlier studies that used this method include Rotemberg and Woodford (1997), Christiano, Eichenbaum, and Evans (2001), Boivin and Giannoni (2002), and Amato and Laubach (2003). In each of those papers, the authors focus on matching the impulse responses to a monetary policy shock. We are not aware, however, of any previous work that attempts to apply this methodology to the case of a government spending shock.

Because our estimation strategy focuses on impulse responses to a government spending shock, it will remain silent regarding the parameters that describe the technology shock

12

(ρ_A and σ_A). Moreover, some structural parameters turn out to be poorly identified and, therefore, should be calibrated. Thus, we fix ϕ, β, ϵ_1, and δ to the values used in section 4, and we estimate the following vector[10]:

$$\theta = (\nu, \gamma, \epsilon_2, \kappa, \alpha, \rho_G, \sigma_G)'.$$

The model is estimated using the impulse-response functions of government spending, output, consumption, hours worked, and investment. We restrict ourselves to the first 16 elements of each response function. The weighting matrix, $\mathbf{W}$, is a diagonal matrix, the elements of which are the inverse of the sample variances of the VAR-based impulse responses. This weighting strategy puts more weight on the impulse responses that are more precisely estimated.

Standard errors of the estimated parameters are computed as the square root of the diagonal elements of the matrix $\mathbf{V} \equiv (\mathbf{D'WD})^{-1}/T$, where $\mathbf{D} = \partial g(\theta, \bar{\mathbf{\Psi}})/\partial \theta'$, $g(\theta, \bar{\mathbf{\Psi}}) = (\mathbf{\Psi}(\theta) - \bar{\mathbf{\Psi}})$, and T is the total number of impulse responses used in the estimation.

5.2 Maximum-likelihood estimation

The model's solution can be written in the following state-space form:

$$\mathbf{X}_{t+1} = \mathbf{FX}_t + \mu_{t+1}, \tag{18}$$

$$\mathbf{P}_t = \mathbf{QX}_t, \tag{19}$$

where the circumflex over a variable denotes the percentage deviation of that variable from its steady-state value; $\mathbf{X}_t = (\hat{A}_t, \hat{G}_t, \hat{K}_t, \hat{\bar{C}}_{t-1})'$ is a 4×1 vector that contains the state variables of the model; $\mathbf{P}_t = (\hat{\lambda}_t, \hat{\bar{C}}_t, \hat{C}_t, \hat{N}_t, \hat{I}_t, \hat{Y}_t, \hat{w}_t, \hat{r}_t)'$ is an 8×1 vector that contains the forward-looking variables; $\mu_t = (\mu_{A,t}, \mu_{G,t})'$ is a 2×1 vector that contains the innovations of the shocks; and $\mathbf{F}$ and $\mathbf{Q}$ are, respectively, 4×4 and 8×4 matrices, the elements of which are combinations of the deep parameters of the model.

The *transition* equation (18) and a *measurement* equation that collects a subset of the variables included in $\mathbf{P}_t$ can be estimated via the ML method. The likelihood function

---

[10]The steady-state quantities G/Y and N are also set to their respective values used in section 3.

can be evaluated recursively using the Kalman filter. This method has been used to estimate dynamic stochastic general-equilibrium models by (among others) McGrattan (1994), McGrattan, Rogerson, and Wright (1997), Kim (2000), Ireland (2001), Bouakez, Cardia, and Ruge-Murcia (2002), and Dib (2003). A crucial requirement of this method is that the number of observable variables used in the estimation does not exceed the number of shocks in the model; otherwise, the variance-covariance matrix of the residuals becomes singular, in which case the ML procedure fails. In our case, this would imply estimating the model's parameters using no more than two series, since we have only two structural shocks. One way to circumvent this problem is to add measurement errors to the variables in the measurement equation.[11]

Our intention is to estimate the model's parameters using data on consumption, hours worked, investment, and output. Therefore, our empirical model is given by:

$$\mathbf{X}_{t+1} = \mathbf{F}\mathbf{X}_t + \mu_{t+1}, \tag{20}$$

$$\mathbf{Y}_t = \mathbf{H}\mathbf{X}_t + \eta_t, \tag{21}$$

where $\mathbf{Y}_t = (\hat{C}_t, \hat{N}_t, \hat{I}_t, \hat{Y}_t)'$, $\eta_t = (\eta_{Ct}, \eta_{Nt}, \eta_{It}, \eta_{Yt})'$ is a 4×1 vector that contains the measurement errors, and $\mathbf{H}$ is a 4×4 matrix the elements of which are combinations of structural parameters. These elements are computed from the model's solution in each iteration of the optimization procedure. Thus, the estimation procedure takes into account all the cross-equation restrictions implied by the theoretical model. In addition, we numerically restrict each estimated parameter to lie within its economically meaningful interval.

Standard errors of the estimated parameters are computed as the square root of the diagonal elements of the inverted Hessian of the (negative) log likelihood function evaluated at the maximum.

The ML procedure yields more efficient estimates of the parameters than the minimum-distance method, because the former exploits all the information contained in the data. In particular, it takes into account the dynamics induced by the technology shock, thus enabling us to estimate the parameters ρ_A and σ_{μ_A}. This gain in efficiency, however, comes

---

[11]The addition of measurement errors to get around the singularity problem has been done by McGrattan, Rogerson, and Wright (1997), Ireland (1999), and Bouakez (2002).

at the cost of making convergence very time-consuming, and inducing further difficulties to estimate certain parameters.[12]

6.   Results

6.1   Parameter estimates

Table 1 reports estimates of structural parameters obtained using the two estimation methods described above. Figures between parentheses are standard errors. At the estimated parameters, the condition for existence of a unique solution to the model is satisfied. That is, the number of explosive eigenvalues of the system of linear difference equations equals the number of non-predetermined variables.

The MD and ML methods yield very precise and very similar estimates of the elasticity of substitution, ν, (0.376 and 0.357, respectively). Interestingly, both values are statistically lower than $1/\epsilon_1$. That is, the necessary condition for government spending to increase the marginal utility of consumption is satisfied, meaning that private and public spending are Edgeworth complements. This result contradicts earlier findings by Aschauer (1985), who finds that government spending and private consumption are Edgeworth substitutes. Karras (1994), and Amano and Wirjanto (1998), on the other hand, find that U.S. private and public consumption are best described as unrelated. Examining evidence from a number of countries, however, Karras (1994) concludes that the two aggregates are complementary in the Edgeworth sense.

The MD estimate of the habit-formation parameter, γ, is higher than, but still consistent with, the estimate obtained using the ML method.[13] In each case, the estimated value of γ is significantly different from zero. Both estimates, however, imply a lesser extent of habit formation than those reported by Heaton (1995), Fuhrer (2000), and Bouakez, Cardia, and Ruge-Murcia (2002), for example.

The adjustment cost parameter, κ, is found to be essentially zero regardless of the estimation method. Traditionally, very low values of the parameter κ have been used to

---

[12]We are unable to obtain convergence when we attempt to estimate the parameters ϵ_2 and α by ML.

[13]A 95 per cent confidence interval around the MD point estimate includes the point estimate given by the ML method.

calibrate RBC models. Some models, such as that developed by Hansen (1985), do not even have capital adjustment costs. The MD method yields estimates of the output elasticity with respect to capital, α, and the preference parameter ϵ_2 equal to 0.39 and 0.87, respectively. The parameter α is precisely estimated and is consistent with the range of values used in the RBC literature. The point estimate of ϵ_2 implies that the elasticity of labour supply with respect to the real wage is equal to 0.39. While this value is plausible, the estimate of ϵ_2 is too imprecise to allow reliable conclusions.

The parameters that describe the government spending process, ρ_G and σ_{μ_G}, are precisely estimated by the MD and ML methods. The estimates of ρ_G are quite comparable across the two methods, but the ML estimate of σ_{μ_G} is higher than its MD counterpart. The ML procedure yields plausible estimates of the autocorrelation coefficient and the standard deviation of the technology shock, ρ_A and σ_{μ_A}, respectively.

6.2   Impulse-response functions

In this section, we evaluate the ability of the estimated model to account for the documented dynamic effects of government spending shocks. In particular, we assess whether the model is capable of generating the positive effect of public spending on private consumption observed in the data. Figures 5 and 6 compare the impulse-response functions implied by the estimated model with those obtained from the benchmark VAR. In Figure 5, we use the MD estimates of the structural parameters to compute the model-based responses, whereas in Figure 6 the ML estimates are used.

Regardless of the estimation method, the estimated model is remarkably successful in replicating the impulse responses obtained from the VAR. For each variable, the model-based response lies mostly within the 95 per cent confidence interval around its VAR-based counterpart. The model generates a positive consumption response to a government spending shock with a peak at one quarter after the shock, exactly as predicted by the VAR. In addition, the magnitude and the persistence of the consumption response are very comparable across the model and the VAR, especially in Figure 5. The model-based investment response is also reasonable, although the trough takes place earlier than the six

16

quarters predicted by the VAR.

In summary, the estimated model is capable of explaining the dynamics of the key variables in response to a government spending shock. Note, however, that because the MD estimation focuses only on those parts of the model that depend on the government spending shock, the empirical success of the model does not guarantee that the remaining parts of the model are well specified. In this sense, the fact that the model yields similar results when estimated by ML can be viewed as heuristic evidence that the model is not misspecified.

6.3 Second moments

The previous section has shown that the structural model is capable of replicating the conditional covariance of government spending and key economic variables. A more demanding test is to ask whether the model is able to match the unconditional moments of the data. To answer this question, we report in Table 2 some selected historical moments and their counterparts predicted by the model.[14] To statistically evaluate the model's success in matching the data, we estimate the historical moments using generalized methods of moments and report their standard deviations. The model's performance is compared with that of a standard RBC model.

Table 2 shows that the standard RBC model and the one developed in this paper (called an augmented RBC model) are both successful in accounting for the volatility of consumption, investment, and, to some extent, government spending, relative to that of output.[15] Both models, however, generate little volatility in hours worked. The augmented model does marginally better than the standard model in replicating the first-order autocorrelation of the variables, but both models significantly overpredict the autocorrelation of consumption. The correlation of each variable with output predicted by the augmented model is similar to that predicted by the standard model and, except for investment, is fairly close to its

---

[14]The predicted moments are evaluated using the MD estimates of the structural parameters. Broadly similar results are obtained when the parameters are calibrated to their ML estimates.

[15]We parameterize the standard deviation and the autocorrelation of the technology shock so that the standard deviation and the first-order autocorrelation of output are the same as in the data.

actual counterpart. The standard and augmented models diverge sharply in matching the correlation between consumption and government spending. This correlation is positive in the data (although statistically insignificant), but the standard model generates a negative value. On the other hand, the correlation predicted by the augmented model has the correct sign and is reasonably close to the actual one.

In view of these results, we may conclude that the model developed in this paper captures the covariance of consumption and government spending better than a standard RBC model, without worsening the dynamics of remaining variables.

7. Conclusion

The purpose of this paper was to explain the puzzling crowding-in effect of government spending on private consumption. Departing from standard RBC models, we have assumed that public expenditures affect consumer preferences, and that those preferences exhibit habit formation in consumption. Rather than using calibration to assess the relevance of these two novel features, we estimated the model's structural parameters using U.S. data. Estimation results reveal a strong complementarity between public and private spending, and a fairly limited extent of habit formation. The estimated model performs remarkably well in replicating the observed impulse responses of the key variables to a government spending shock. In particular, the model is able to account for the documented crowding-in effect on consumption. Moreover, the consumption response generated by the model has similar dynamics to the response obtained from a benchmark VAR, both in terms of magnitude and persistence. Finally, the model outperforms a standard RBC model in replicating the unconditional moments of the data.

Three important remarks about our results are worth noting. First, as emphasized by Karras (1994), the fact that private and public spending are found to be complements *should not be considered valid for all types of publicly provided goods but only as holding in the aggregate.*[16]

---

[16]Yet, one can think of a variety of specific cases where public spending can increase private consumption. Examples include education, transportation, and communication.

Second, in contrast to the explanation provided by Galí, López-Salido, and Vallés (2003) for the crowding-in effect, the explanation proposed in this paper does not rely on departures from Ricardian equivalence. The two explanations, however, will be mutually reinforcing if their New Keynesian model with non-Ricardian consumers is extended to allow for complementarity between public and private spending.

Finally, throughout the paper, we have assumed that government spending is an exogenous variable. While this assumption is plausible from the households' perspective, the fact that government spending affects the utility function implies that there is scope for optimal fiscal policy. Therefore, a natural extension of the model would be to allow for an optimizing government that chooses public spending endogenously to maximize the households' welfare. This is left for future research.

References

Ahmed, S. and B.S. Yoo. 1995. "Fiscal Trends and Real Economic Aggregates." *Journal of Money, Credit and Banking* 27: 985–1001.

Amano, R.A. and T.S. Wirjanto. 1997. "Intertemporal Substitution and Government Spending." *The Review of Economics and Statistics* 1: 605–09.

———. 1998. "Government Expenditures and the Permanent-Income Model." *Review of Economic Dynamics* 1: 719–30.

Amato, J.D. and T. Laubach. 2003. "Estimation and Control of an Optimization-Based Model with Sticky Prices and Wages." *Journal of Economic Dynamics and Control* 27: 1181–215.

Ambler, S. and E. Cardia. 1997. "Optimal Public Spending in a Business Cycle Model." *Business Cycle and Macroeconomic Stability: Should We Rebuild Built-in Stabilizers?*, edited by J.-O. Hairault, P.-Y. Hénin, and F. Portier. Kluwer Academic Press.

Aschauer, D. 1985. "Fiscal Policy and Aggregate Demand." *American Economic Review* 75: 117–27.

Barro, R. 1981. "Output Effects of Government Purchases." *Journal of Political Economy* 89: 1086–21.

Baxter, M. and R.G. King. 1993. "Fiscal Policy in General Equilibrium." *American Economic Review* 83: 315–34.

Boivin, J. and M.P. Giannoni. 2002. "Has Monetary Policy Become More Effective?" Photocopy.

Bouakez, H. 2002. "Nominal Rigidity, Desired Markup Variations, and Real Exchange Rate Persistence." Bank of Canada Working Paper No. 2002-26.

Bouakez, H., E. Cardia, and F. Ruge-Murcia. 2002. "Habit Formation and the Persistence of Monetary Shocks." Bank of Canada Working Paper No. 2002-27.

Burnside, C., M. Eichenbaum, and J. Fisher. 2003. "Fiscal Shocks and Their Consequences." NBER Working Paper No. 9772.

Cardia, E., N. Kozhaya, and F. Ruge-Murcia. 2003. "Distortionary Taxation and Labor Supply." *Journal of Money, Credit and Banking* 35: 763–85.

Christiano, L. and M. Eichenbaum. 1992. "Cyclical Effects of Government's Employment and Good Purchases." *American Economic Review* 82: 430–50.

Christiano, L., M. Eichenbaum, and C.L. Evans. 2001. "Nominal Rigidities and the Dynamics of a Shock to Monetary Policy." NBER Working Paper No. 8403.

Cogley, T. and J. Nason. 1995. "Effects of the Hodrick-Prescott Filter on Trends and Difference Stationary Time-Series: Implications for Business Cycle Research." *Journal of Economic Dynamics and Control* 19: 353–78.

Dib, A. 2003. "An Estimated Canadian DSGE Model with Nominal and Real Rigidities." *The Canadian Journal of Economics* 36(4): 949–72.

Fatás, A. and I. Mihov. 2001. "The Effects of Fiscal Policy on Consumption and Employment: Theory and Evidence." INSEAD. Photocopy.

Fuhrer, J. 2000. "Habit Formation in Consumption and Its Implications for Monetary-Policy Models." *American Economic Review* 90: 367–90.

Galí, J., J.D. López-Salido, and J. Vallés. 2003. "Understanding the Effects of Government Spending on Consumption." Photocopy.

Hansen, G.D. 1985. "Indivisible Labor and the Business Cycle." *Journal of Monetary Economics* 16: 309–27.

Heaton, J. 1995. "An Empirical Investigation of Asset Pricing with Temporally Dependent Preference Specifications." *Econometrica* 63: 681–717.

Ireland, P. 1999. "A Method of Taking Models to the Data." Boston College. Photocopy.

———. 2001. "Sticky-Price Models of the Business Cycle: Specification and Stability." *Journal of Monetary Economics* 47: 3–18.

Karras, G. 1994. "Government Spending and Private Consumption: Some International Evidence." *Journal of Money, Credit and Banking* 26: 9–22.

Kim, J. 2000. "Constructing and Estimating a Realistic Optimizing Model of Monetary Policy." *Journal of Monetary Economics* 45: 329–59.

Linnemann, L. and A. Schabert. 2003. "Fiscal Policy in the New Neoclassical Synthesis." *Journal of Money, Credit and Banking* 35: 911–29.

McGrattan, E.R. 1994. "The Macroeconomic Effects of Distortionary Taxation." *Journal of Monetary Economics* 33: 573–601.

McGrattan, E.R., R. Rogerson, and R. Wright. 1997. "An Equilibrium Model of the Business Cycle with Household Production and Fiscal Policy." *International Economic Review* 38: 267–90.

Mountford, A. and H. Uhlig. 2002. "What are the Effects of Fiscal Policy Shocks?" CEPR Discussion Paper No. 3338.

Okubo, M. 2003. "Intratemporal Substitution between Private and Government Consumption: The Case of Japan." *Economics Letters* 79: 75–81.

Perotti, R. 2002. "Estimating the Effects of Fiscal Policy in OECD Countries." ISOM conference, Frankfurt.

Rotemberg, J. and M. Woodford. 1997. "An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy." *NBER Macroeconomics Annual 1997* 297–346.

Appendix A: The Log-Linearized Model

In this appendix, variables without time subscripts denote steady-state values, and the circumflex denotes percentage deviation from steady state. Linearizing equations (2), (4), (7), (8), (9), (10), (12), (13), and (16) yields

$$\hat{\tilde{C}}_t = \phi(C/\tilde{C})^{(\nu-1)/\nu}\hat{C}_t + (1-\phi)(G/\tilde{C})^{(\nu-1)/\nu}\hat{G}_t,$$

$$\hat{K}_{t+1} = (1-\delta)\hat{K}_t + \delta\hat{I}_t,$$

$$\hat{\lambda}_t = \frac{\beta\gamma(\epsilon_1-1)}{1-\beta\gamma}E\hat{\tilde{C}}_{t+1} - \frac{\beta\gamma(\gamma(\epsilon_1-1)+1)+\epsilon_1-(1-\beta\gamma)/\nu}{1-\beta\gamma}\hat{\tilde{C}}_t + \frac{\gamma(\epsilon_1-1)}{1-\beta\gamma}\hat{\tilde{C}}_{t-1} - \frac{1}{\nu}\hat{C}_t,$$

$$\hat{\lambda}_t = \frac{N\epsilon_2}{1-N}\hat{N}_t - \hat{w}_t,$$

$$\hat{\lambda}_t = E_t\hat{\lambda}_{t+1} + \beta q E_t\hat{r}_{t+1} - \kappa(1+\beta\delta)\hat{K}_{t+1} + \kappa\hat{K}_t + \beta\kappa\delta E_t\hat{I}_{t+1},$$

$$\hat{Y}_t = \alpha\hat{K}_t + (1-\alpha)\hat{N}_t + \hat{A}_t,$$

$$\hat{w}_t = \hat{Y}_t - \hat{N}_t,$$

$$\hat{r}_t = \hat{Y}_t - \hat{K}_t,$$

$$\hat{Y}_t = \frac{C}{Y}\hat{C}_t + \frac{I}{Y}\hat{I}_t + \frac{G}{Y}\hat{G}_t.$$

The stochastic processes of the shocks, (11) and (15), are already linear. Using the same notation as above, they are rewritten as

$$\hat{A}_{t+1} = \rho_A\hat{A}_{t-1} + \mu_{At},$$

$$\hat{G}_{t+1} = \rho_G\hat{G}_{t-1} + \mu_{Gt}.$$

Table 1. Estimates of Structural Parameters[†]

Description	Parameter	Minimum-distance estimate	Maximum-likelihood estimate
Elasticity of substitution	ν	0.3764	0.3573
		(0.0494)	(0.0136)
Habit-formation parameter	γ	0.4092	0.2985
		(0.0938)	(0.0537)
Adjustment cost parameter	κ	0.0000	0.0000
		(4.8314)	(0.0370)
Elasticity of output w.r.t capital	α	0.3983	0.3600
		(0.1310)	−
Preference parameter	ϵ_2	0.8702	1.000
		(1.1929)	−
Autocorrelation coefficient of A_t	ρ_A	−	0.7945
			(0.0251)
Autocorrelation coefficient of G_t	ρ_G	0.8161	0.8317
		(0.0165)	(0.0264)
Standard deviation of μ_{At}	σ_{μ_A}	−	0.0046
			(0.0002)
Standard deviation of μ_{Gt}	σ_{μ_G}	0.0117	0.0211
		(0.0009)	(0.0024)

† Figures between parentheses are standard errors. The restrictions imposed on the parameters are γ, $\alpha \in (0,1)$, ρ^A, $\rho^G \in (-1,1)$, and ν, κ, ϵ_2, σ_{μ_A}, $\sigma_{\mu_G} \in (0,\infty)$.

Table 2. Actual and Predicted Moments

Statistic	Data[†]		Standard	Augmented
	Estimate	Std. error	RBC model[‡]	RBC model[‡]
Std. Deviation Relative to Y				
C	0.58	(0.06)	0.44	0.46
H	0.97	(0.04)	0.57	0.60
I	3.35	(0.41)	2.93	3.01
G	1.39	(0.26)	1.26	1.26
Autocorrelation				
C	0.80	(0.15)	0.99	0.96
H	0.88	(0.17)	0.84	0.86
I	0.87	(0.19)	0.80	0.81
G	0.84	(0.34)	0.82	0.82
Correlation with Y				
(Y, C)	0.69	(0.15)	0.62	0.60
(Y, H)	0.87	(0.17)	0.55	0.50
(Y, I)	0.28	(0.07)	0.95	0.92
(Y, G)	−0.01	(0.11)	0.05	0.08
Correlation with G				
(G, C)	0.24	(0.19)	−0.15	0.33
(G, H)	0.05	(0.11)	0.22	0.33
(G, I)	−0.15	(0.08)	−0.19	−0.26

† The statistics are based on logged and H-P filtered quarterly data for the period 1952Q2–2001Q4. Figures between parentheses are standard errors.

‡ The standard deviation and the autocorrelation of the technology shock are chosen so that the standard deviation and the first-order autocorrelation of output are the same as in the data: 0.0162 and 0.83, respectively.
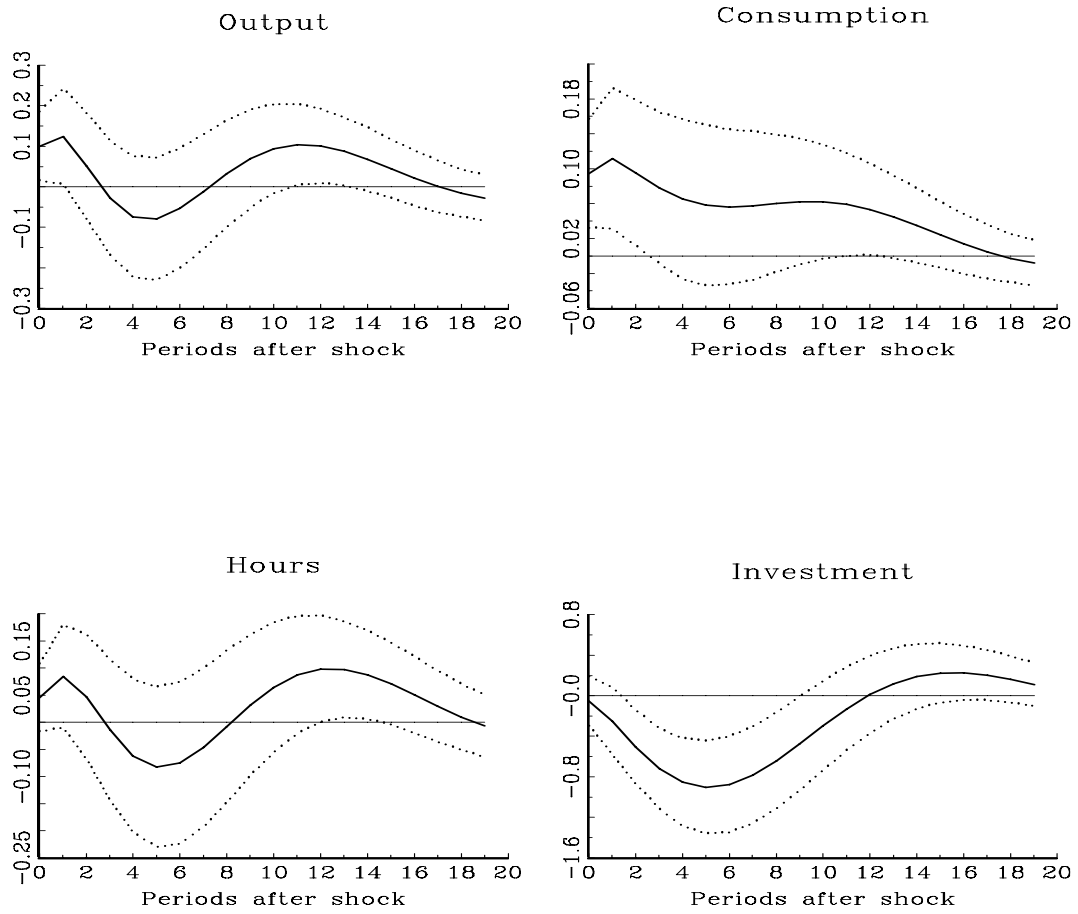
Figure 1: VAR-based impulse responses to a 1 per cent government spending shock.
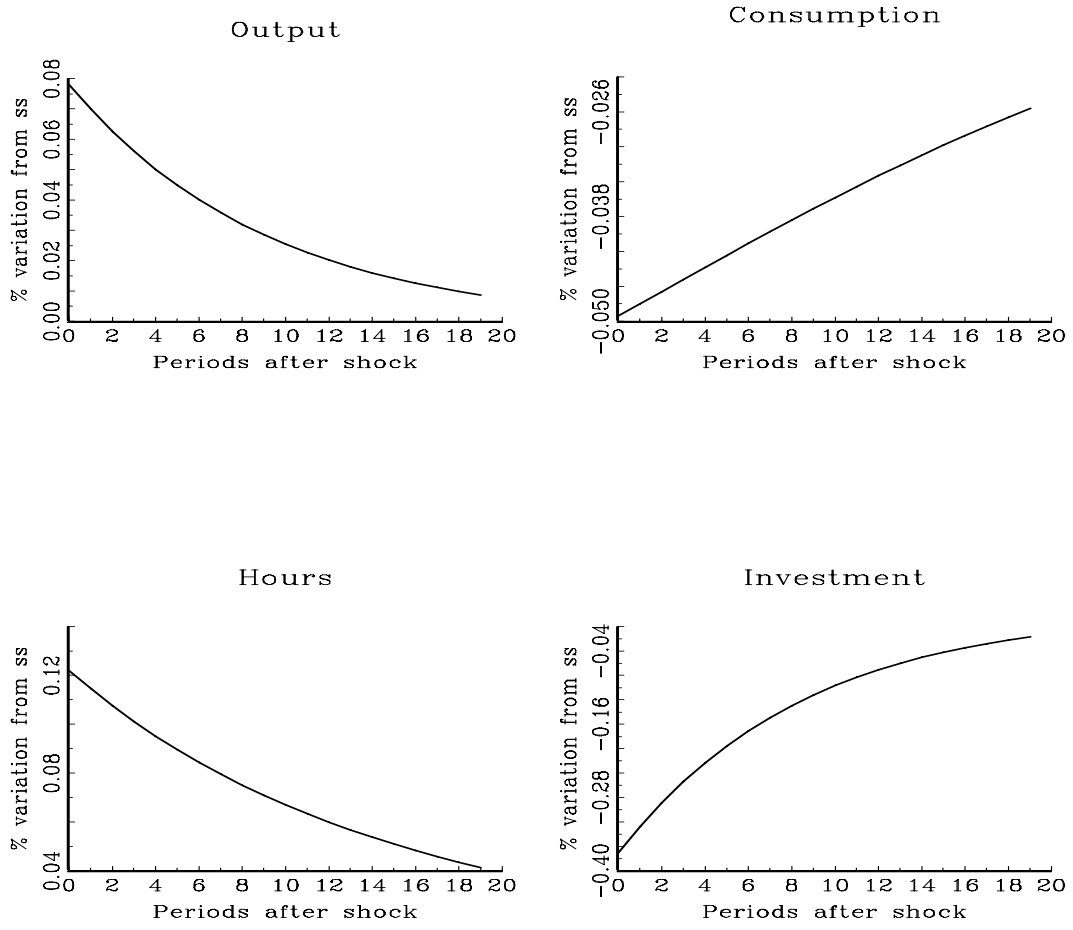Solid lines: impulse responses, dotted lines: error bands

Figure 2: Impulse responses to a 1 per cent government spending shock: Standard RBC model
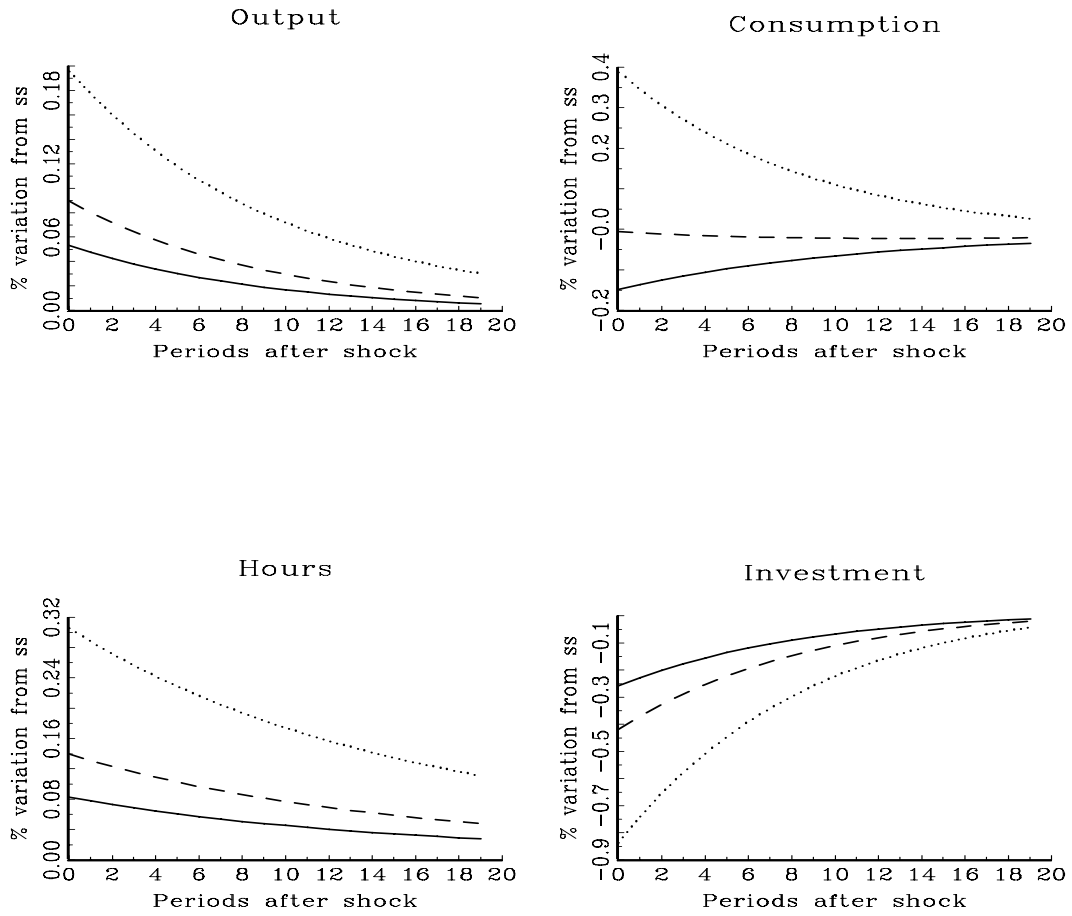
Figure 3: Impulse responses to a 1 per cent government spending shock for different values of ν ($\gamma = 0$). Solid lines: $\nu = 1$, dashed lines: $\nu = 0.45$, dotted lines: $\nu = 0.25$
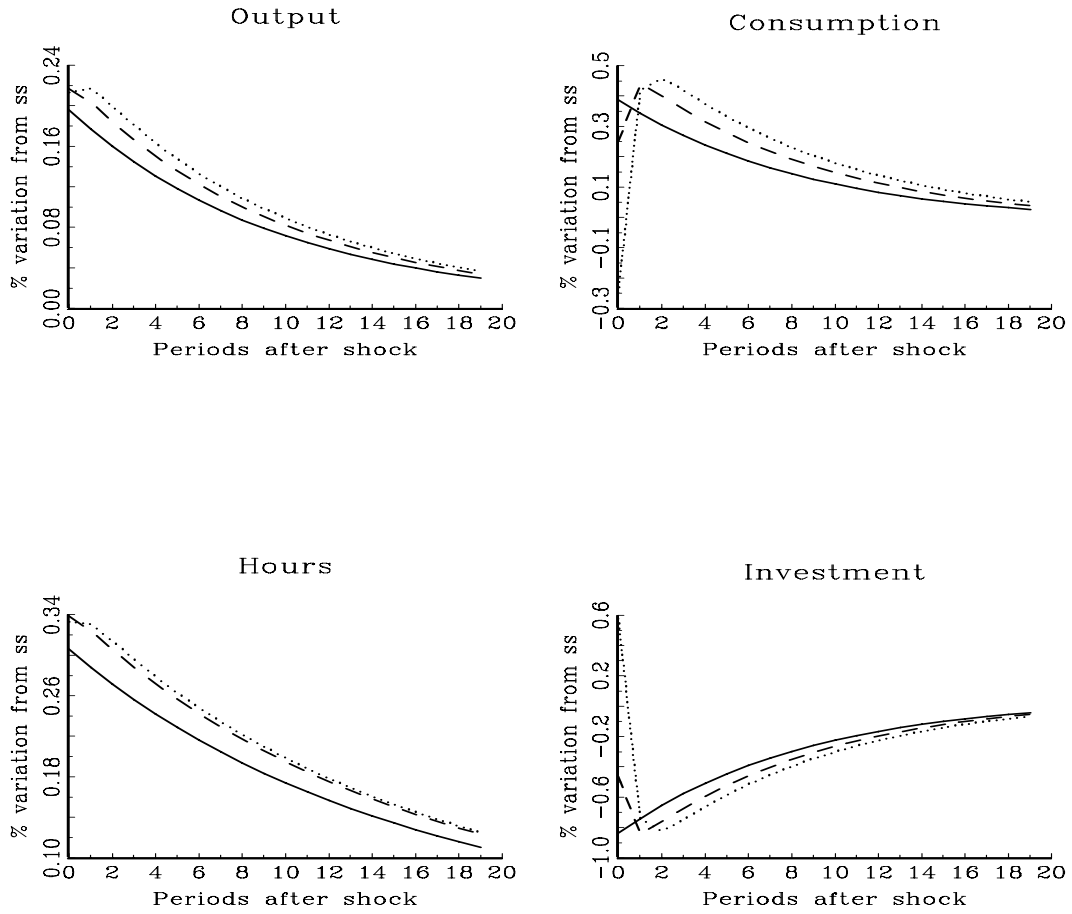
Figure 4: Impulse responses to a 1 per cent government spending shock for different values of γ ($\nu = 0.25$). Solid lines: $\gamma = 0$, dashed lines: $\gamma = 0.5$, dotted lines: $\gamma = 0.8$
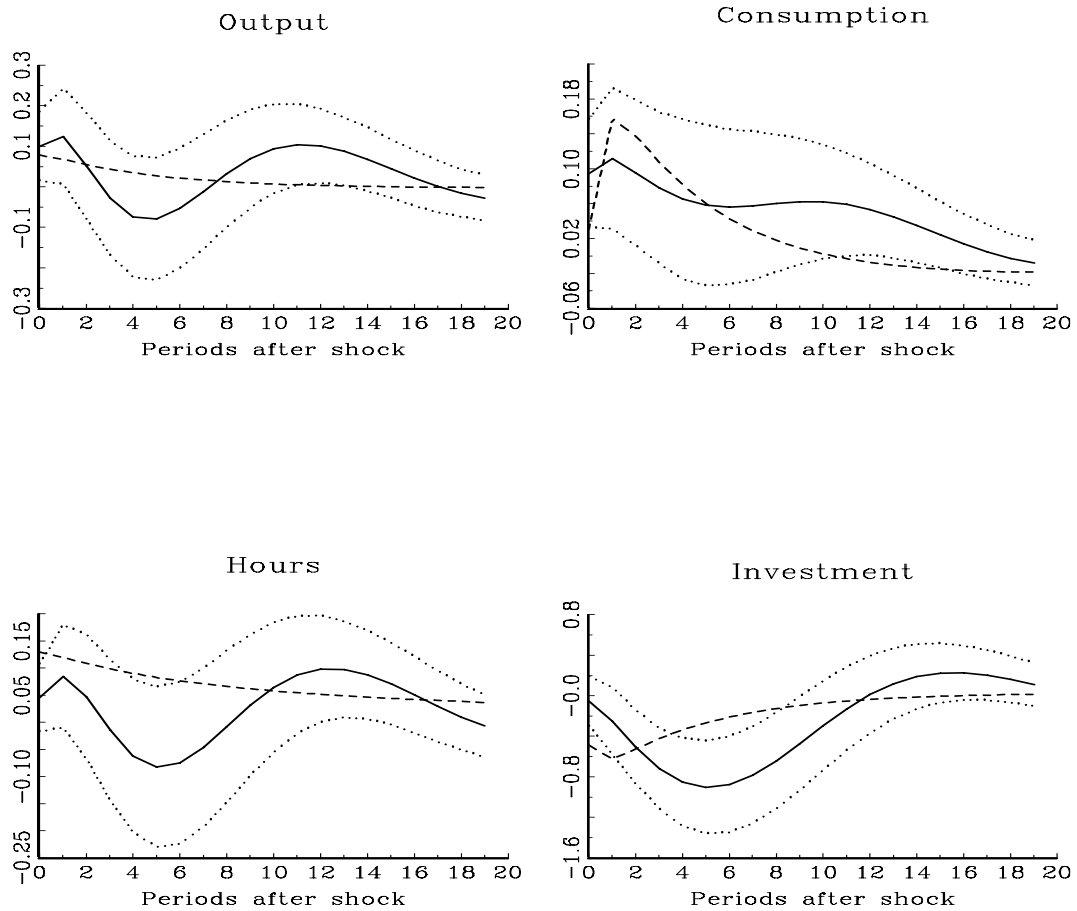
Figure 5: Impulse-response functions generated by the VAR and by the estimated model (MD method). Solid lines: VAR-based responses, dotted lines: error bands, dashed lines: model-based responses
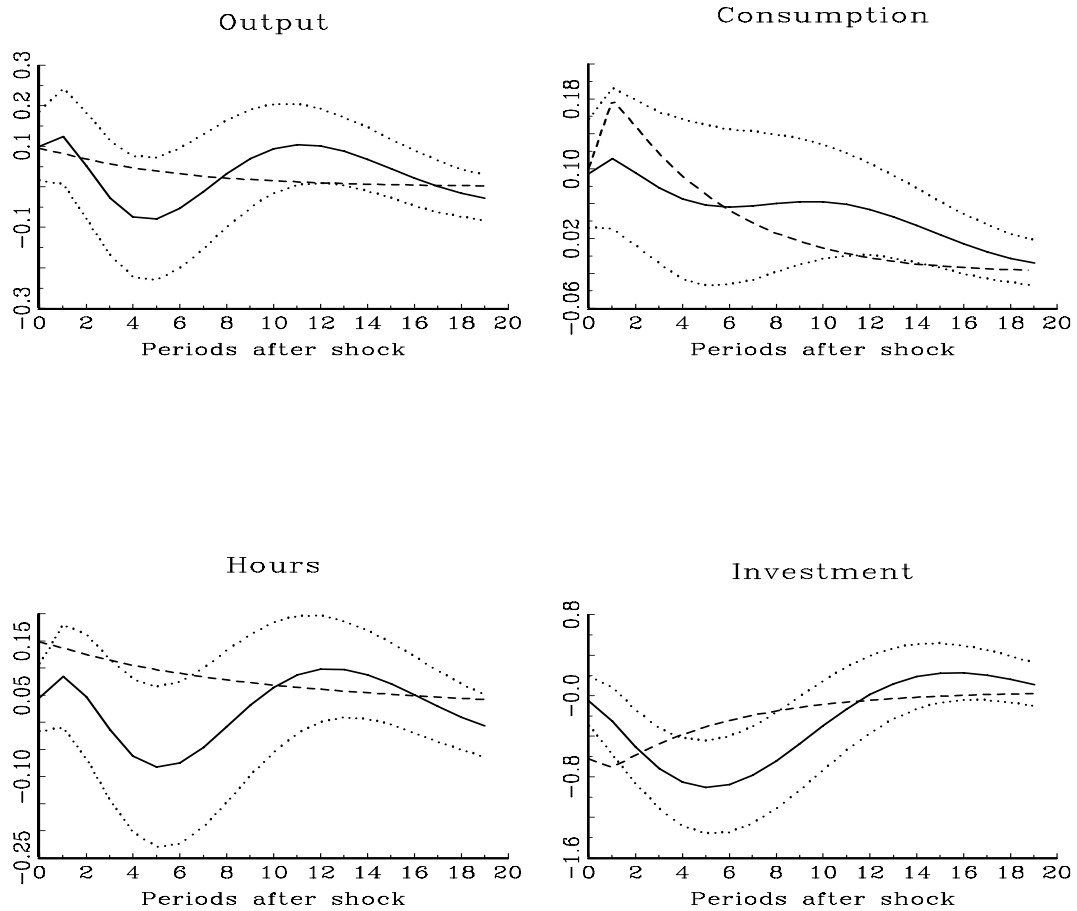
Output

Consumption

Hours

Investment

Figure 6: Impulse-response functions generated by the VAR and by the estimated model (ML method).
Solid lines: VAR-based responses, dotted lines: error bands, dashed lines: model-based responses

31