

# Are New Keynesian Phillips Curved Identified?\*

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## Abstract

In this paper we use simple new finite-sample methods to test the empirical relevance of the New Keynesian Phillips curve (NKPC) equation. Unlike generalized method of moments-based methods, these generalized Anderson-Rubin tests are immune to the presence of weak instruments, and allow, by construction, to assess the identification status of a model. Our results are illustrated using the Gali-Gertler (1999) NKPC specifications and data, as well as a survey-based inflation expectation series from the Philadelphia Fed.

Our test rejects the reported Gali-Gertler estimates, conditional on their choice of instruments. Nevertheless, and in contrast to Ma (2002), we do obtain relatively informative confidence sets. This provides support for NKPC equations and illustrates the usefulness of using exact procedures in IV-based estimations. Finally, our results also reveal that the least-well-identified parameter is  $\omega$ ; namely the proportion of firms that do not adjust their prices in period  $t$ .

*Keywords:* NKPC, Anderson-Rubin Test, Weak Instruments.

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# 1. Introduction

The New Keynesian Phillips curve (NKPC) equation resulted from the efforts of recent years to model the short-run dynamics of inflation starting from optimization principles. In its benchmark form, this equation stipulates that inflation at a time  $t$  is a function of expected future inflation and current marginal costs. With its clearly-elucidated theoretical foundations, the NKPC possesses a straightforward structural interpretation and therefore presents a strong theoretical advantage to the traditional reduced-form Phillips curve (which is justified only statistically).

However, confronting the NKPC with the data has raised several issues<sup>1</sup>. In particular, modeling the marginal cost variable is a fundamental problem. Whereas, under some conditions, the output gap series is a natural proxy for this variable, studies using gap measures revealed empirical puzzles; in particular: (i) the coefficient on the output gap was estimated to be negative when theoretically it should be positive, and (ii) adding lagged inflation to the above model in an ad-hoc manner seemed to correct the estimated sign problem; suggesting that, unlike what the theory predicts, past inflation matters<sup>2</sup>.

These results spurred further research on both the theoretical and empirical levels. For instance, Gali-Gertler(1999) modified the standard NKPC theoretical formulation by allowing a proportion of firms to use a rule of thumb when setting prices for their goods (rather than allowing all firms to set prices in a rational manner). The latter modification provides a theoretical justification for the presence of an inflation lag in the first-order-condition. Models which incorporate the above features are referred to as *hybrid* NKPC models.

On empirical grounds, efforts focused on proposing improved proxies for the marginal cost variable. For example, Gali-Gertler(1999) suggested us-

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<sup>1</sup>See, for example, Gali-Gertler(1999), Gali-Gertler-Lopez-Salido(2001), and the references cited therein.

<sup>2</sup>See, for example, Fuhrer-Moore(1995), Roberts(1997) and Fuhrer(1997).

ing measures of marginal cost derived from a production function instead of relying on possibly-badly-measured output gaps. Generalized Method of Moments (GMM) estimation of the hybrid NKPC having these new marginal cost measures yielded the correct sign on that variable and the model was not rejected according to Hansen’s J-test. Moreover, the choice for the marginal cost proxy seemed to affect the estimated weight of the backward- and forward-looking terms in the equation<sup>3</sup>.

While the above outcomes appear encouraging, it is important to note that the recent literature on instrumental variable (IV) based inference casts serious doubts on the reliability of standard inference procedures<sup>4</sup>. These studies demonstrate that standard asymptotic procedures (i.e. procedures which *impose identification away* without correcting for local-almost-identification) are fundamentally flawed and lead to serious overrejections; these problems are not small-sample related and occur with fairly large sample sizes, since they are caused by asymptotics failures. In particular Dufour(1997) shows that usual  $t$ -type tests, based on common IV estimators, have significance levels that may deviate arbitrarily from their nominal levels since it is not possible to bound their null distributions.

To circumvent weak-instruments related difficulties, the above-cited work on IV-based inference has focused on three main directions: (i) refinements in asymptotic analysis which include the local-to-zero or local-to-unity frameworks (e.g. Staiger-Stock(1997), Wang-Zivot(1998), and Stock-Wright(2000)), (ii) proposing asymptotic approximations which hold whether instruments are weak or not (e.g. Kleibergen(2002), Moreira(2003)), and (iii) developing new finite-sample-justified procedures based on proper pivots – that is, finding statistics whose null distributions are either nuisance-parameter-free or are bounded by nuisance-parameter-free distributions (e.g. Dufour(1997), Dufour-Jasiak(2001), Dufour-Khalaf(2002), and Dufour-Taamouti(2003b,c)).

Clearly, the question of whether the NKPC is supported by the data begs

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<sup>3</sup>For example, see Gali-Gertler-Lopez-Salido(2001) and Gagnon-Khan(2001).

<sup>4</sup>see for example Dufour(2003), Stock-Wright-Yogo(2002), and the references cited therein.

re-examination using some of these more reliable econometric methods. Such an exercise was undertaken recently by Ma (2002), who used the asymptotic test statistics proposed in Stock and Wright (2000) to evaluate the empirical relevance of the Gali-Gertler NKPC specifications. He concluded that weak identification was a major concern in these models, and that there was an unreasonably large set of parameter values that were compatible with these models.

In this paper, we focus on the new finite-sample methods to test the empirical relevance of the NKPC. The latter methods allow, by construction, to assess the identification status of a model. Another major advantage they have is that they are valid in samples typical of macroeconomic data – i.e. fairly small. Furthermore, they can provide fairly detailed information regarding the nature of potential under-identification, suggesting useful theory modifications. This is an advantage compared to the Stock and Wright asymptotic methods because the latter do not provide such information directly.

Specifically, we apply the econometric methods presented in Dufour and Jasiak (2001) which are generalizations of the Anderson-Rubin statistics. Like Ma (2002), our results are illustrated using the Gali-Gertler NKPC specifications and data. In the next section we reproduce the NKPC models that were developed by Gali and Gertler (1999), as well as their and Ma's results. In Section 3, we present the generalized Anderson-Rubin (hereafter AR) test. Section 4 documents and discusses the results of the AR test applications to the above NKPC specifications. The last section concludes.

## The Gali-Gertler NKPC models

In Gali-Gertler's benchmark specification all price-setting firms are forward-looking in a monopolistically-competitive environment. Thus, inflation,  $\pi_t$ , is a function of next period's expected inflation,  $E_t\pi_{t+1}$ , and real marginal costs,  $s_t$  (expressed in percent deviation with respect to its steady-state value).

Specifically, the model is given by:

$$\pi_t = \lambda_1 s_t + \beta E_t \pi_{t+1}, \quad (1)$$

with

$$\lambda_1 = \frac{(1 - \theta)(1 - \beta\theta)}{\theta}, \quad (2)$$

and where  $\theta$  is the proportion of firms that do not adjust their prices in period  $t$ ,  $\beta$  is the subjective discount rate, and  $E_t \pi_{t+1}$  is the value of inflation for next period that is expected at time  $t$ .

In contrast, Gali-Gertler's hybrid specification assumes that some of the firms use a rule-of-thumb when setting their prices. The proportion of such firms (referred to as the backward-looking price setters) is given by  $\omega$ . In this case, the model is written as:

$$\pi_t = \lambda_2 s_t + \gamma_f E_t \pi_{t+1} + \gamma_b \pi_{t-1}, \quad (3)$$

with

$$\begin{aligned} \lambda_2 &= \frac{(1 - \omega)(1 - \theta)(1 - \beta\theta)}{\theta + \omega - \omega\theta + \omega\beta\theta} \\ \gamma_f &= \frac{\beta\theta}{\theta + \omega - \omega\theta + \omega\beta\theta} \\ \gamma_b &= \frac{\omega}{\theta + \omega - \omega\theta + \omega\beta\theta}, \end{aligned} \quad (4)$$

and where  $\pi_{t-1}$  is the inflation lag,  $\gamma_f$  is the forward-looking component of inflation, and  $\gamma_b$  is its backward-looking part.

The authors assume rational expectations and re-write the above NKPC models in terms of orthogonality conditions to be estimated by standard two-step GMM. Because of small-sample concerns, each re-written model is normalized in two ways: (i) so as to minimize non-linearities (denoted specification (1)), and (ii) setting the inflation coefficient equal to one (denoted specification (2)).

The data used is quarterly U.S. data, with  $\pi_t$  measured by the percentage change in the GDP deflator, and real marginal costs given by the logarithm of

the labour income share<sup>5</sup>. The instruments used include four lags of inflation, labour share, commodity-price inflation, wage inflation, long-short interest rate spread, and output gap (measured by a detrended log GDP).

For their benchmark model, Gali-Gertler find values of  $(\theta, \beta)$  equal to  $(0.83, 0.93)$  and  $(0.88, 0.94)$  for their specifications (1) and (2) respectively. Constraining  $\beta$  to 1 yields similar results, namely  $\theta = 0.83$  in (1) and  $\theta = 0.92$  in (2). The implied slope coefficients on the marginal cost variable for all these cases is positive and significant – based on the IV-based asymptotic standard errors, and the overidentifying restrictions are not rejected according to the  $J$ -test. For their hybrid model, the same normalizations and instrument set are used. In this case, the obtained values for  $\omega$ ,  $\theta$ , and  $\beta$  are  $(0.27, 0.81, 0.89)$  and  $(0.49, 0.83, 0.91)$  for specifications (1) and (2) respectively. In the restricted cases, these are  $(0.24, 0.80, 1.00)$  and  $(0.52, 0.84, 1.00)$  respectively. Again, the implied slopes are all positive and found to be significant.

Based on these, and some additional GMM estimations carried out for robustness purposes, the authors conclude that there is good empirical support for the NKPC, and furthermore, that the forward-looking component of inflation is more important than the backward-looking part.

Despite these significant outcomes, it is important to be wary of GMM-based outcomes, as the severity of weak-instruments effects is now well-understood in econometrics<sup>6</sup>. Given these concerns, Ma (2002) uses the test statistics developed by Stock and Wright (2000) to re-evaluate the empirical relevance of these NKPC specifications. These asymptotic methods account for the presence of weak instruments and provide corrected confidence intervals for the GMM-estimated parameters.

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<sup>5</sup>They also report results for the case where inflation is measured by the non-farm deflator. These yield similar outcomes to those based on the GDP total deflator measure.

<sup>6</sup>Examples include Dufour(1997), Staiger-Stock(1997), Wang-Zivot(1998), Stock-Wright(2000), Dufour-Jasiak(2001), Stock-Wright-Yogo(2002), Kleibergen-Zivot(2003), Khalaf-Kichian(2002), Dufour-Khalaf(2003), Dufour-Taamouti(2003b,c) and Dufour(2003).

Ma first notes that the benchmark model presents a theoretical identification problem; namely that there is an observational equivalence between the sets  $(\beta, \theta)$  and  $(\beta, 1/\beta\theta)$ . Thus, there is more than one parameter combination that satisfies the GMM minimization criterion. In other words, the objective function being solved by GMM (and which is concentrated with respect to  $\theta$ ) is non-quadratic. Therefore, conventional tests, such as those applied in Gali-Gertler, do not provide accurate information on the precision of GMM estimates.

Turning to the estimates from the hybrid model, Ma calculates the corrected confidence set according to the method proposed in Stock and Wright (2000). He finds that the 90% S-set is particularly large, including all parameter values between  $[0,3]$  for two of the parameters, and  $[0,8]$  for the third. That is, all parameter combinations derived from these value ranges are compatible with the model. This is a clear indication of weak identification in this model.

Thus, the validity of the Gali-Gertler GMM-based estimates is in question. But, the Stock and Wright intervals provide little concrete direction in which theoretical research should be oriented. On the other hand, the recently-advanced finite-sample methods that also deal with the possible presence of weak instruments may be able to point to such directions. In the next section, we present a test strategy belonging in the latter finite-sample category. It is one proposed in Dufour and Jasiak (2001) and is a generalization of the Anderson-Rubin (1949) technique.

## The AR test

The Anderson-Rubin test has recently received renewed interest<sup>7</sup>. In its generalized form – developed by Dufour and Jasiak (2001) – it is applicable to univariate models using limited information, and where one or more of the

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<sup>7</sup>See, for example, Dufour(1997), Staiger-Stock(1997), Dufour-Jasiak(2001), Dufour-Khalaf(2003), and Dufour-Taamouti(2003b,c).

right-hand-side variables are possibly endogenous.

More formally, consider a limited information simultaneous equations system

$$y = Y\delta + X_1\kappa + u, \quad (5)$$

where  $y$  is an  $n \times 1$  dependent variable,  $Y$  is an  $n \times m$  matrix of endogenous variables,  $X_1$  is an  $n \times k_1$  matrix of exogenous variables, and  $u$  is an error term which satisfies standard regularity conditions typical to IV regressions; see Dufour-Jasiak(2001).

In this context, consider hypotheses of the form

$$H_0 : \delta = \delta^0. \quad (6)$$

Define  $\tilde{y} = y - Y\delta^0$  so that under the null hypothesis, (6) implies

$$\tilde{y} = X_1\kappa + u. \quad (7)$$

In view of this, the AR test is based on assessing the exclusion of  $X_2$  (of size  $n \times k_2$ ) in the regression of  $\tilde{y}$  on  $X_1$  and  $X_2$ , which can be conducted using the standard F-test or its chi-square asymptotic variant; see Dufour-Jasiak(2001). Let  $X = (X_1, X_2)$ , and define

$$\begin{aligned} M &= I - X(X'X)^{-1}X' \\ M_1 &= I - X_1(X_1'X_1)^{-1}X_1' \end{aligned}$$

then the statistic takes the form

$$AR = \frac{[(y - Y\delta^0)' M_1 (y - Y\delta^0) - (y - Y\delta^0)' M (y - Y\delta^0)] / k_2}{(y - Y\delta^0)' M (y - Y\delta^0) / (n - k_1 - k_2)}. \quad (8)$$

Under the null hypothesis and imposing strong exogeneity and i.i.d. normal errors,  $AR \sim F(k_2, n - k_1 - k_2)$ ; the normality and i.i.d. hypotheses can be relaxed so under standard regularity conditions and weakly exogenous regressors,  $(k_2 \times AR) \stackrel{asy}{\sim} \chi^2(k_2)$ .

The test can be readily extended to accommodate additional constraints on the exogenous variables coefficients; see Maddala(1974), Dufour-Jasiak(2001),



Dufour-Taamouti(2003b,c) and Dufour(2003). Specifically, consider a hypothesis of the form:

$$H_0 : \delta = \delta^0, \quad \kappa_1 = \kappa_1^0, \quad (9)$$

where  $\kappa_1$  is a subset of  $\kappa$ , i.e.  $\kappa = (\kappa_1', \kappa_2')'$ . Partition the matrix  $X_1$  (into  $X_{11}$  and  $X_{12}$  sub-matrices) accordingly and let

$$\check{y} = y - Y\delta^0 - X_{11}\kappa_1, \quad (10)$$

so the restricted model becomes

$$\check{y} = X_{12}\kappa_{12} + u, \quad (11)$$

and the test may be carried out as above.

While the test in its original form was derived for the case where the first stage regression is linear, Dufour-Taamouti(2003b,c) have shown that it is in fact robust to: (i) the specification of the model for  $Y$ , and (ii) excluded instruments; in other words, the test is valid whether the first stage regression is linear or not, or whether the matrix  $X_2$  includes all available instruments or not. As argued in Dufour(2003), since one is never sure that all instruments have been accounted for, the latter property is quite important. Most importantly, this test (and several variants discussed in Dufour(2003)) is the only truly pivotal statistic whose properties in finite samples are robust to the quality of instruments.

Of course, despite the latter desirable statistical properties, the test as presented provides no guidance for practitioners regarding the choice of instruments. However, simulation studies reported in the above cited references show that the power of AR-type tests may be affected by the number of instruments. To see this, consider the case of (5)-(6): here, the AR test requires assessing (in the regression of  $\tilde{y}$  on  $X_1$  and  $X_2$ ) the exclusion of the  $n \times k_2$  variables in  $X_2$ , even though the number of structural parameters under test is  $m$  ( $\kappa$  is  $m \times 1$ ). On recalling that identification implies  $k_2 \geq m$ , we see that over-identification (or alternatively, the availability of more instruments) leads to degrees-of-freedom losses with obvious implication on power.

To circumvent this problem, Dufour-Taamouti(2003b,c) have shown that for the problem at hand, an optimal instrument (in the sense of *point-optimal* power) may be derived as follows

$$\bar{Z} = X_2\Pi_2$$

where  $\Pi_2$  is the coefficient of  $X_2$  in the first-stage regression, *i.e.* the regression of  $Y$  on  $X_1$  and  $X_2$ . Clearly, the latter optimal instrument involves information reduction, for the associated AR-test amounts to testing for the exclusion of the  $n \times m$  variables in  $\bar{Z}$ , which preserves available degrees-of-freedom even if the model is highly over-identified. In other words, the optimal test can reflect the informational content of all available instruments with no statistical costs.

Unfortunately of course,  $\Pi_2$  is unknown so the optimal instruments needs to be estimated, with obvious implications on feasibility and exactness. Dufour(2003) shows that if the OLS estimator of  $\Pi_2$  in the unrestricted reduced form multivariate regression

$$\begin{bmatrix} y & Y \end{bmatrix} = \begin{bmatrix} X_1 & X_2 \end{bmatrix} \begin{bmatrix} \pi_1 & \Pi_1 \\ \pi_2 & \Pi_2 \end{bmatrix} + \begin{bmatrix} u & V \end{bmatrix} \quad (12)$$

is used in the construction of  $\bar{Z}$ , then the associated AR-test coincides with the LM procedure defined by Wang-Zivot(1998). In addition, Dufour and Khalaf (2003) show that Kleibergen's K-test may be obtained as an optimal AR-test based on  $\bar{Z}$  where  $\Pi_2$  is replaced by its OLS estimates using the reduced form (12) constrained by the structural identification condition

$$\pi_2 = \Pi_2\delta.$$

Dufour and Khalaf (2003) provide simple analytical formula (applying e.g. Berndt-Savin(1977) and Dufour-Khalaf(2002b)) to derive this estimate.

Both tests so obtained are not exact, but their asymptotic validity does not impose identification away. Split sample estimation techniques (where the first sub-sample is used to estimate  $\Pi_2$  and the second to run the optimal AR-test based on the latter estimate) may be easily applied to obtain

exact optimal AR tests, as suggested by Dufour-Taamouti(2003b,c) and Dufour(2003); see also Dufour-Jasiak(2001) regarding split-sample procedures.

## Applications of the AR test

The econometric models that we use for the AR applications are the Gali-Gertler benchmark and hybrid models in equations (1) and (3) respectively, with  $E_t\pi_{t+1}$  given by a survey measure of inflation expectations,  $\tilde{\pi}_{t+1}$ . The Federal Reserve Bank at Philadelphia publishes quarterly mean forecasts of the next quarter US GDP implicit price deflator, which we first difference to obtain our inflation expectations series<sup>8</sup>. A measurement error term  $u_t$  is added to the equation to reflect the fact that the expectations variable is a proxy. Thus, our econometric equivalents of the Gali-Gertler models are:

$$\pi_t = \lambda_1 s_t + \beta \tilde{\pi}_{t+1} + u_t, \quad (13)$$

and

$$\pi_t = \lambda_2 s_t + \gamma_f \tilde{\pi}_{t+1} + \gamma_b \pi_{t-1} + u_t, \quad (14)$$

where  $\lambda_1$ ,  $\lambda_2$ ,  $\gamma_f$ , and  $\gamma_b$  are defined as previously (see equations (2) and (4)).

In this framework, and for both the benchmark and hybrid models,  $y = \pi_t$ ,  $Y = (s_t, \tilde{\pi}_{t+1})'$ , and  $X_2$  is the 24-variable set of instruments used by Gali-Gertler. In addition,  $X_1$  is zero in the benchmark case, and equal to  $\pi_{t-1}$  in the hybrid case.

We test the Gali and Gertler (1999) estimates for the benchmark and hybrid models, and for both specifications, using their instrument set each time<sup>9</sup>. An illustration is as follows: say we want to test their benchmark model specification (1) estimates. We impose  $\theta = 0.83$ ,  $\beta = 0.93$ , and calculate the corresponding slope value, which is  $\lambda_0 = 0.05$ . The null hypothesis for the AR test is then given by:  $H_0 : \lambda_0 = 0.05$  and  $\beta_0 = 0.93$ . Constructing  $\tilde{y}$ , we regress it on all the Gali-Gertler instruments. We also obtain the  $M$

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<sup>8</sup>Source: <http://www.phil.frb.org/econ/spf/index.html>.

<sup>9</sup>Due to the presence of the expectations variable our sample starts in 70:1.

and  $M_1$  matrices as in equations (8). With these, we compute the value of the AR statistic according to equation (8). We have  $n = 112$  observations and  $k_2 = 24$  instruments. The statistic is therefore compared to the  $F(24,88)$  distribution, and in the case where the normality and i.i.d. hypotheses are relaxed,  $24 \times \text{AR}$  is compared to a  $\chi^2(24)$ .

The results are reported in Table 1. From there, we can see that all of the Gali-Gertler GMM estimates are decisively rejected at the 5% level. In other words, given the instrument set that was used by Gali and Gertler, both their benchmark and hybrid models are strongly rejected by the data, whether specification (1) or (2) estimates are used, and whether the  $\beta$  parameter is restricted to equal 1.

Next we ask whether, for the same instrument set, there are any parameter combinations for which the models are not rejected. Thus, we conduct such a grid search for each of the benchmark and hybrid models, allowing the range (0,1) as the admissible space for  $\omega$ ,  $\theta$ , and  $\beta$ , and varying these values with increments of 0.1. We find that all parameter combinations reject the model at the 5% level, whether it is the benchmark or the hybrid equation that's being tested.

This conclusion is in striking contrast with the findings of Ma (2002), although both our results highlight the weak-instrument problem emphatically. That is, while the Stock-Wright asymptotic test finds that all parameter combinations do not reject the model, we find that all of them actually reject it. Therefore, it appears that the AR-test has more power compared to the Stock-Wright methodology<sup>10</sup>.

## Selected Instrument Sets

It is evident that whether a model is rejected or not depends on the instruments that are used to specify it. The issue of which instruments to use is quite difficult and beyond the scope of this study. However, an easy

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<sup>10</sup>Although, we note that there is a slight difference in our two instrument sets: Ma's set includes a constant and has no 4th lag for each of the three variables in levels.

way to understand the relevance of various instrument sets is to specify the model with each and then test it. We consider seven different instrument sets, each comprised of 4 lags of a variable amongst the following: GDP deflator inflation, wage inflation, commodity price inflation, labor income share, long-short interest rate spread, quadratically-detrended output gap, and cubically-detrended output gap<sup>11</sup>. For each of these sets, we conduct grid searches for the benchmark and hybrid models again, always admitting a (0,1) range for each of  $\omega$ ,  $\theta$ , and  $\beta$ , and still varying the parameter values by increments of 0.1.

The results are tabulated in Table 2-6b. Table 2 shows, for the seven instrument sets, those combinations of  $\beta$  and  $\theta$  values that do not reject the tested specification. The remaining Tables are results for the hybrid model. Tables 3a-3b show the outcomes for estimations over the full sample, while Tables 4a-4b, 5a-5b, and 6a-6b in the Appendix tabulate results for non-intersecting sub-samples (70:1-79:4, 80:1-89:4, 90:1-97:4). In each case, we report the results with four of the instrument sets<sup>12</sup>.

First and foremost, the overall results show that there are parameter combinations for which a given model is rejected, and others for which it is not. Second, some instrument sets appear to have more informational content than others (i.e. they yield a smaller set of parameter combinations that do not reject the model). These results are somewhat positive for macroeconomic theorists because they indicate that the NKPC models are not rejected outright. But while the scope of the identification issue is slightly less dramatic with our results than with those suggested by the

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<sup>11</sup>For our output gap measure, and for all the tests we conduct, rather than detrending the log of GDP using the full sample,  $n$ , we proceed iteratively: to obtain the value of the gap at time  $t$ , we detrend GDP with data ending in  $t$ . We then extend the sample by one more observation and re-estimate the trend. This is then used to detrend GDP and yields a value for the gap at time  $t + 1$ . And so on till the end of the sample. In this fashion, our gap measures at time  $t$  do not use information beyond that period and can therefore be used as valid instruments.

<sup>12</sup>This is to save space. The remaining tables are available upon request.

Stock-Wright method, our various tables do indeed indicate the presence of pervasive identification problems. For instance, in the benchmark model, the instrument sets in columns 1-4 and in column 6 show that there are many parameter combinations for which the model is valid. Similarly, in the hybrid model case, there are numerous parameter combinations that do not reject the NKPC specification.

However, an important fact is that additional information can be gained from the examination of these tables regarding the direction in which theoretical research should be oriented. In particular, with the hybrid model case (Tables 3a-6b), some patterns emerge: (i) as the value of  $\theta$  increases, the values for  $\beta$  decrease, and (ii) results are more restrictive when  $\omega$  is not too high or too low. Based on these, we can see that if one is willing to assume a range for the subjective discount rate that is economically meaningful (say, values ranging from 0.8 to 1), then the space of admissible parameter values is greatly reduced: thus  $\theta$  is almost never above 0.4, and it is lower when  $\omega$  is either high or low. That is, the  $\omega$  parameter is less well-identified than  $\theta$ , which implies that research needs to find better ways of characterizing the inertia in inflation dynamics.

The information in the above tables has been summarised in a number of graphs. For each model, we show the graphs for the parameter combinations that do not reject the model when the latter is specified using four different instrument sets. Figure 1 presents the graphs for the benchmark model for the instrument sets: lags 1 to 4 of inflation, lags 1 to 4 of wage inflation, lags 1 to 4 of the long-short spread, and lags 1 to 4 of labor income share. Figure 2 depicts graphs for the hybrid model with the same instrument sets. Column 1 shows the graphs for  $\theta$  and  $\beta$  for all values of  $\omega$  considered, while the subsequent columns show  $\theta$  and  $\beta$  for  $\omega = 0.2$ ,  $\omega = 0.5$ , and  $\omega = 0.8$  respectively. Figures 4, 6 and 8 in the Appendix depict each similar graphs to Figure 2, but for the subsamples 1970:1-1979:4, 1980:1-1989:4, and 1990:1-1997:4 respectively. In these cases, however, we show results with lags 1 to 4 for the quadratically-detrended output gap rather than lags 1 to 4 for the

labor income share as the former are more interesting. Finally, Figures 3, 5, 7, and 9 depict the graphs corresponding to figures 2, 4, 6 and 8 respectively, but with  $\beta$  constrained to be equal to or above 0.8 which are economically more meaningful.

The graphs show more clearly the patterns in the results that were observed from the tables. Furthermore, it is clear from the sub-sample graphs that there is evidence of parameter instability. In particular, whereas the quadratically-detrended output instrument set results show all parameter combinations to reject the model in the seventies, results with the same instrument set show non-rejections for the eighties and nineties.

## **Optimal Instrument Sets**

## **Conclusion**

Table 1: AR test results on Gali-Gertler models - US Data

Tested model	spec.	restr	data sample	D.F.	Fstat (p-value)	Chi-stat (p-value)
Benchmark	(1)	-	70:1-97:4	88	8.77 (<)	210.54 (<)
Benchmark	(2)	-	70:1-97:4	88	9.31 (<)	223.61 (<)
Benchmark	(1)	yes	70:1-97:4	88	11.57 (<)	277.59 (<)
Benchmark	(2)	yes	70:1-97:4	88	11.55 (<)	277.27 (<)
Hybrid	(1)	-	70:1-97:4	87	7.98 (<)	199.42 (<)
Hybrid	(2)	-	70:1-97:4	87	13.55 (<)	338.83 (<)
Hybrid	(1)	yes	70:1-97:4	87	11.05 (<)	276.26 (<)
Hybrid	(2)	yes	70:1-97:4	87	17.41 (<)	435.28 (<)
Hybrid	(1)	-	70:1-89:4	55	4.73 (<)	118.21 (<)
Hybrid	(1)	yes	70:1-89:4	55	8.19 (<)	204.72 (<)
Hybrid	(2)	-	80:1-97:4	47	7.01 (<)	175.29 (<)
Hybrid	(2)	yes	80:1-97:4	47	14.13 (<)	353.19 (<)

Note: Table abbreviations are: D.F. is degrees of freedom, spec is specification, restr is restricted, and the symbol " < " indicates values that are less than  $10^{-5}$ .

Table 2: Benchmark Model - Parameter Grid Search Results - US Data  
Parameter Combinations that Do Not Reject  $H_0$  - Full Sample

	$dw_1 - dw_4$	$sp_1 - sp_4$	$dp_1 - dp_4$	$s_1 - s_4$	$gq_1 - gq_4$	$dc_1 - dc_4$	$gc_1 - gc_4$
$\theta = 0.0$	(0 - 0.8)	(0 - 1)	(0 - 0.5)	-	-	(0 - 1)	-
$\theta = 0.1$	(0.3 - 0.6)	(0.6 - 1)	(0.3 - 0.6)	-	-	(0 - 1)	-
$\theta = 0.2$	(0.5 - 0.6)	(0.7 - 1)	(0.5 - 0.6)	(0 - 0.1)	-	(0 - 1)	-
$\theta = 0.3$	0.6	(0.7 - 0.9)	0.6	(0 - 0.3)	-	(0 - 1)	-
$\theta = 0.4$	0.6	(0.7 - 0.8)	0.6	(0 - 0.5)	-	(0 - 1)	-
$\theta = 0.5$	0.6	(0.7 - 0.8)	0.6	(0 - 0.5)	-	(0 - 1)	-
$\theta = 0.6$	0.6	(0.7 - 0.8)	0.6	(0 - 0.6)	-	(0 - 1)	-
$\theta = 0.7$	0.6	(0.7 - 0.8)	0.6	(0 - 0.7), 1	-	(0 - 1)	-
$\theta = 0.8$	-	(0.7 - 0.8)	-	(0 - 1)	-	(0 - 1)	-
$\theta = 0.9$	-	(0.7 - 0.8)	-	(0 - 1)	-	(0 - 1)	-
$\theta = 1.0$	-	(0.7 - 0.8)	-	(0 - 1)	-	(0 - 1)	-

Note: Reported values are for the parameter  $\beta$ . Instrument sets are lags 1 to 4 of wage inflation (dw), long-short interest spread (sp), inflation (dp), labor income share (s), quadratically-detrended output gap (gq), commodity price inflation (dc), cubically-detrended output gap (gc).



Table 3a: Hybrid Model - Parameter Grid Search Results - US Data  
 Parameter Combinations that Do Not Reject  $H_0$  - Full Sample

	dw <sub>1</sub> - dw <sub>4</sub>										
	$\omega = 0$	$\omega = 0.1$	$\omega = 0.2$	$\omega = 0.3$	$\omega = 0.4$	$\omega = 0.5$	$\omega = 0.6$	$\omega = 0.7$	$\omega = 0.8$	$\omega = 0.9$	$\omega = 1.0$
$\theta = 0.0$	(0 - 1)	-	-	-	-	-	-	-	-	-	-
$\theta = 0.1$	(0.4 - 0.7)	(0.9 - 1)	-	-	-	-	-	-	-	-	-
$\theta = 0.2$	(0.5 - 0.6)	0.8	(0.9 - 1)	-	-	-	-	-	-	-	-
$\theta = 0.3$	0.6	0.7	0.8	-	-	-	-	-	-	-	-
$\theta = 0.4$	0.6	0.7	0.7	-	-	-	-	-	-	-	-
$\theta = 0.5$	0.6	0.7	0.7	-	-	-	-	-	-	-	-
$\theta = 0.6$	0.6	-	-	-	-	-	-	-	-	-	-
$\theta = 0.7$	0.6	0.6	0.6	-	-	-	-	-	-	-	-
$\theta = 0.8$	0.6	0.6	0.6	-	-	-	-	-	-	-	-
$\theta = 0.9$	0.6	0.6	0.6	-	-	-	-	-	-	-	-
$\theta = 1.0$	-	0.6	-	-	-	-	-	-	-	-	-

	sp <sub>1</sub> - sp <sub>4</sub>										
$\theta = 0.0$	-	-	-	-	-	-	-	-	-	-	-
$\theta = 0.1$	-	-	-	-	-	-	-	-	-	-	-
$\theta = 0.2$	-	-	-	-	-	-	-	-	-	-	-
$\theta = 0.3$	-	-	-	-	-	-	-	-	-	-	-
$\theta = 0.4$	-	-	-	-	-	-	-	-	-	-	-
$\theta = 0.5$	-	-	-	-	0.6	-	-	-	-	-	-
$\theta = 0.6$	-	-	-	0.6	0.5	-	-	-	-	-	-
$\theta = 0.7$	-	0.6	0.6	0.6	0.5	-	-	-	-	-	-
$\theta = 0.8$	0.6	0.6	0.6	-	0.5	0.4	-	-	-	-	-
$\theta = 0.9$	0.6	0.6	0.6	0.5	-	0.4	0.3	-	-	-	-
$\theta = 1.0$	-	-	-	-	-	-	-	-	-	-	-

Note: Reported values are for the parameter  $\beta$ . Instrument sets are lags 1 to 4 of wage inflation (dw) and of long-short interest spread (sp).

Table 3b: Hybrid Model - Parameter Grid Search Results - US Data  
 Parameter Combination that Do Not Reject  $H_0$  - Full Sample

	dp <sub>2</sub> - dp <sub>5</sub>										
	$\omega = 0$	$\omega = 0.1$	$\omega = 0.2$	$\omega = 0.3$	$\omega = 0.4$	$\omega = 0.5$	$\omega = 0.6$	$\omega = 0.7$	$\omega = 0.8$	$\omega = 0.9$	$\omega = 1.0$
$\theta = 0.0$	(0-1)	-	-	-	-	-	-	-	-	-	-
$\theta = 0.1$	(0.3-0.7)	(0.8-1)	-	-	-	-	-	-	(0.7-1)	(0.3-0.5)	-
$\theta = 0.2$	(0.5-0.6)	(0.7-0.8)	(0.9-1)	1	1	(0.9-1)	(0.8-1)	(0.6-0.8)	(0.4-0.6)	0.2	-
$\theta = 0.3$	0.6	0.7	(0.7-0.8)	(0.8-0.9)	(0.7-0.9)	(0.7-0.8)	(0.6-0.7)	(0.4-0.6)	(0.3-0.4)	0.11	-
$\theta = 0.4$	0.6	0.7	0.7	0.7	(0.6-0.7)	(0.6-0.7)	(0.5-0.6)	0.4	(0.2-0.3)	0.11	-
$\theta = 0.5$	0.6	(0.6-0.7)	(0.6-0.7)	(0.6-0.7)	0.6	(0.5-0.6)	(0.4-0.5)	(0.3-0.4)	0.2	0.11	-
$\theta = 0.6$	0.6	0.6	0.6	0.6	(0.5-0.6)	(0.4-0.5)	0.4	0.3	0.2	-	-
$\theta = 0.7$	0.6	0.6	0.6	(0.5-0.6)	0.5	(0.4-0.5)	(0.3-0.4)	(0.2-0.3)	0.1	-	-
$\theta = 0.8$	-	0.6	0.6	0.5	0.5	0.4	0.3	0.2	0.1	-	-
$\theta = 0.9$	-	0.6	0.6	0.5	(0.4-0.5)	0.4	0.3	0.2	0.1	-	-
$\theta = 1.0$	-	0.6	0.6	0.5	0.4	(0.3-0.4)	0.3	0.2	0.1	-	-

	s1 - s4										
$\theta = 0.0$	-	-	-	-	-	-	-	-	-	0.3	0.8
$\theta = 0.1$	-	-	-	-	-	-	-	-	(0.6 - 1)	(0.1 - 0.6)	0
$\theta = 0.2$	-	-	-	-	-	1	0.9	(0.6 - 0.8)	(0.4 - 0.5)	(0.1 - 0.3)	0
$\theta = 0.3$	-	-	-	-	-	0.7	(0.6 - 0.7)	0.5	(0.3 - 0.4)	(0.1 - 0.2)	0
$\theta = 0.4$	-	-	-	0.7	0.7	0.6	0.5	0.4	(0.2-0.3)	0.1	0
$\theta = 0.5$	-	-	-	-	0.6	0.5	0.4	0.3	0.2	(0-0.1)	0
$\theta = 0.6$	-	-	0.6	0.6	-	0.5	0.4	0.3	0.2	(0-0.1)	0
$\theta = 0.7$	-	0.6	0.6	-	0.5	0.4	0.3	0.2	0.1	0	0
$\theta = 0.8$	-	0.6	0.6	-	0.5	0.4	0.3	0.2	0.1	0	0
$\theta = 0.9$	-	0.6	0.6	0.5	-	0.4	0.3	0.2	0.1	0	-
$\theta = 1.0$	-	0.6	-	0.5	-	-	0.3	0.2	0.1	0	-

Note: Reported values are for the parameter  $\beta$ . Instrument sets are lags 2 to 4 of inflation (dp) and lags 1 to 4 of labor income share (s).

Figure 1: Benchmark Model - Full Sample

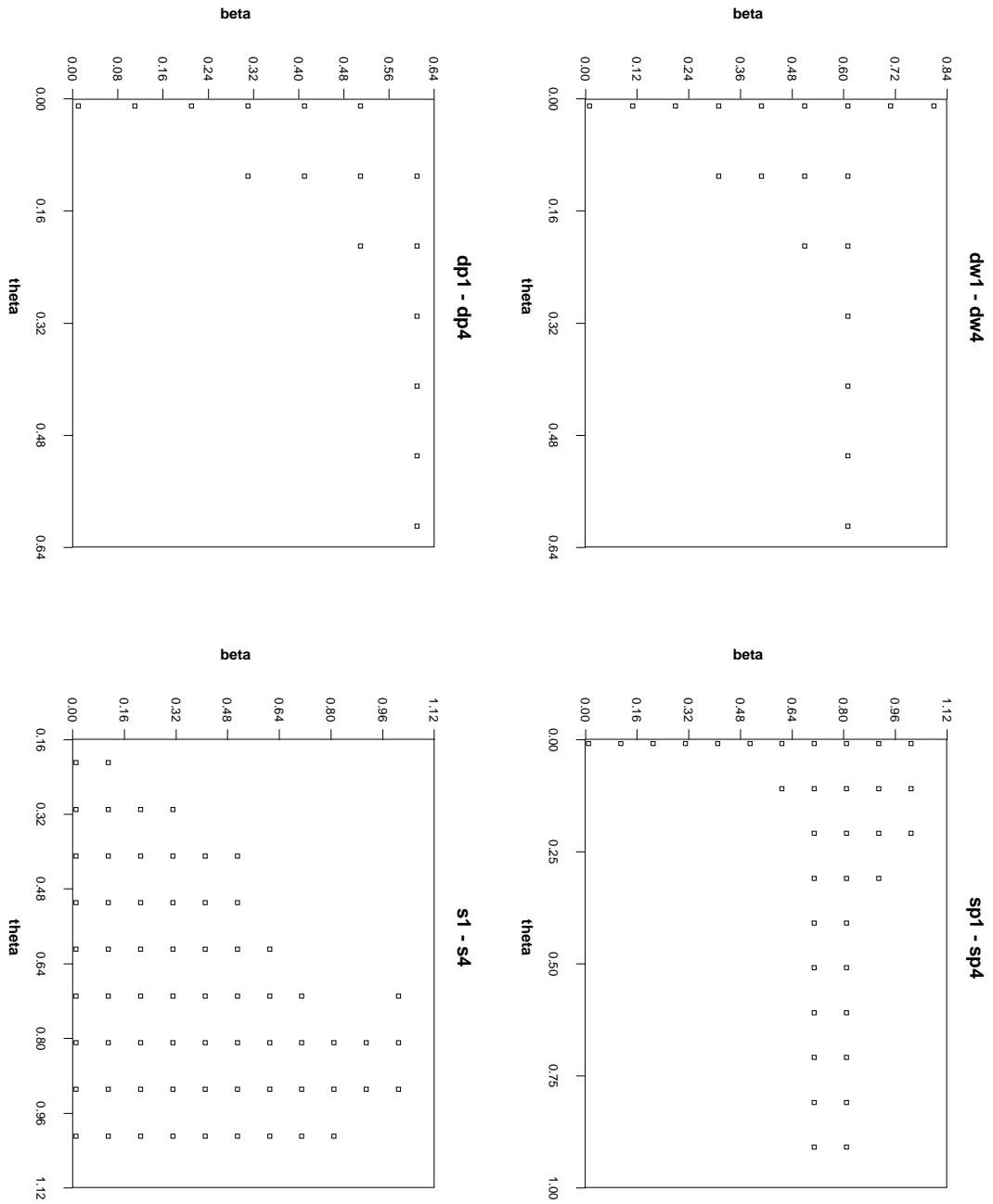


Figure 2: Hybrid Model - Full Sample

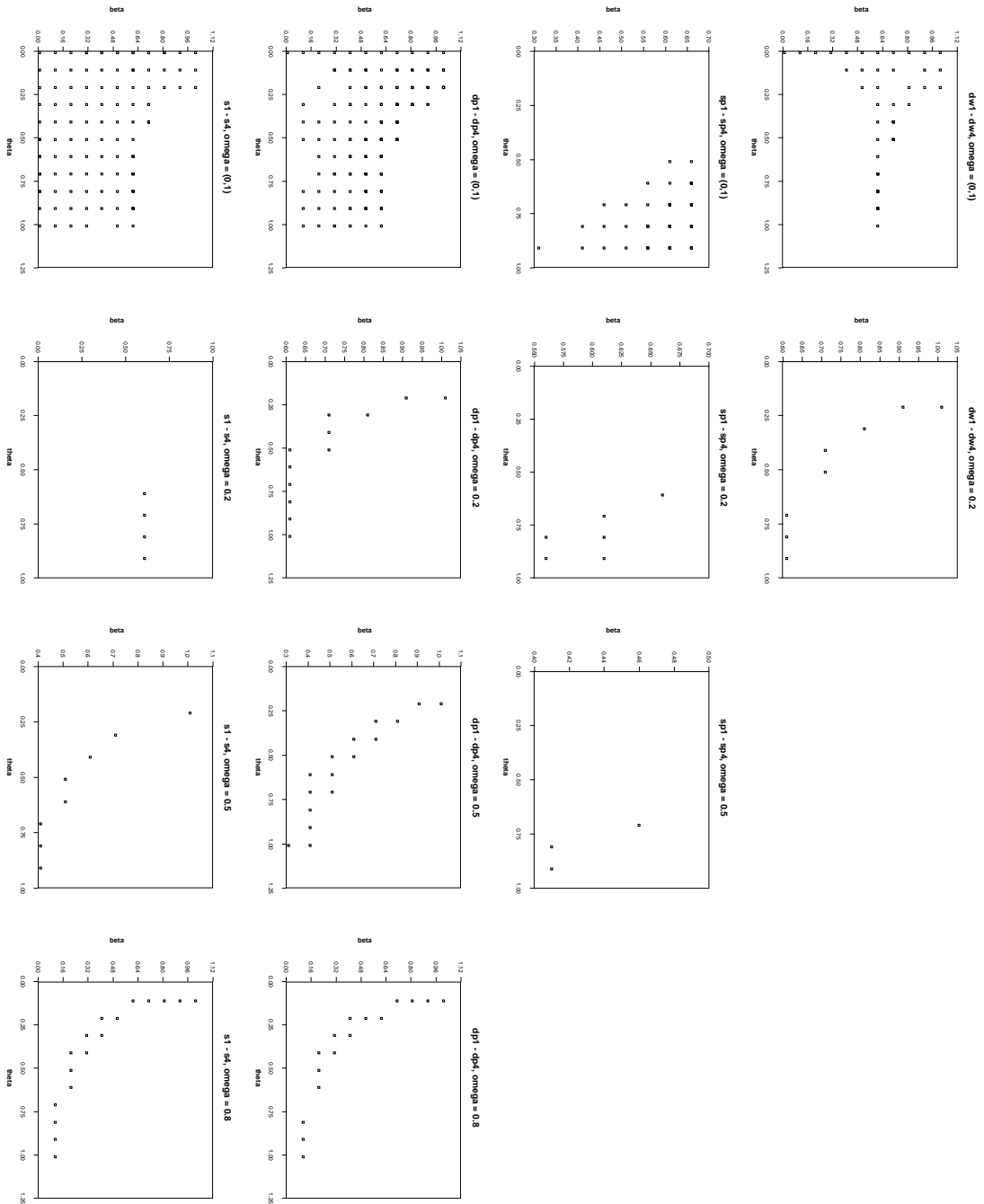
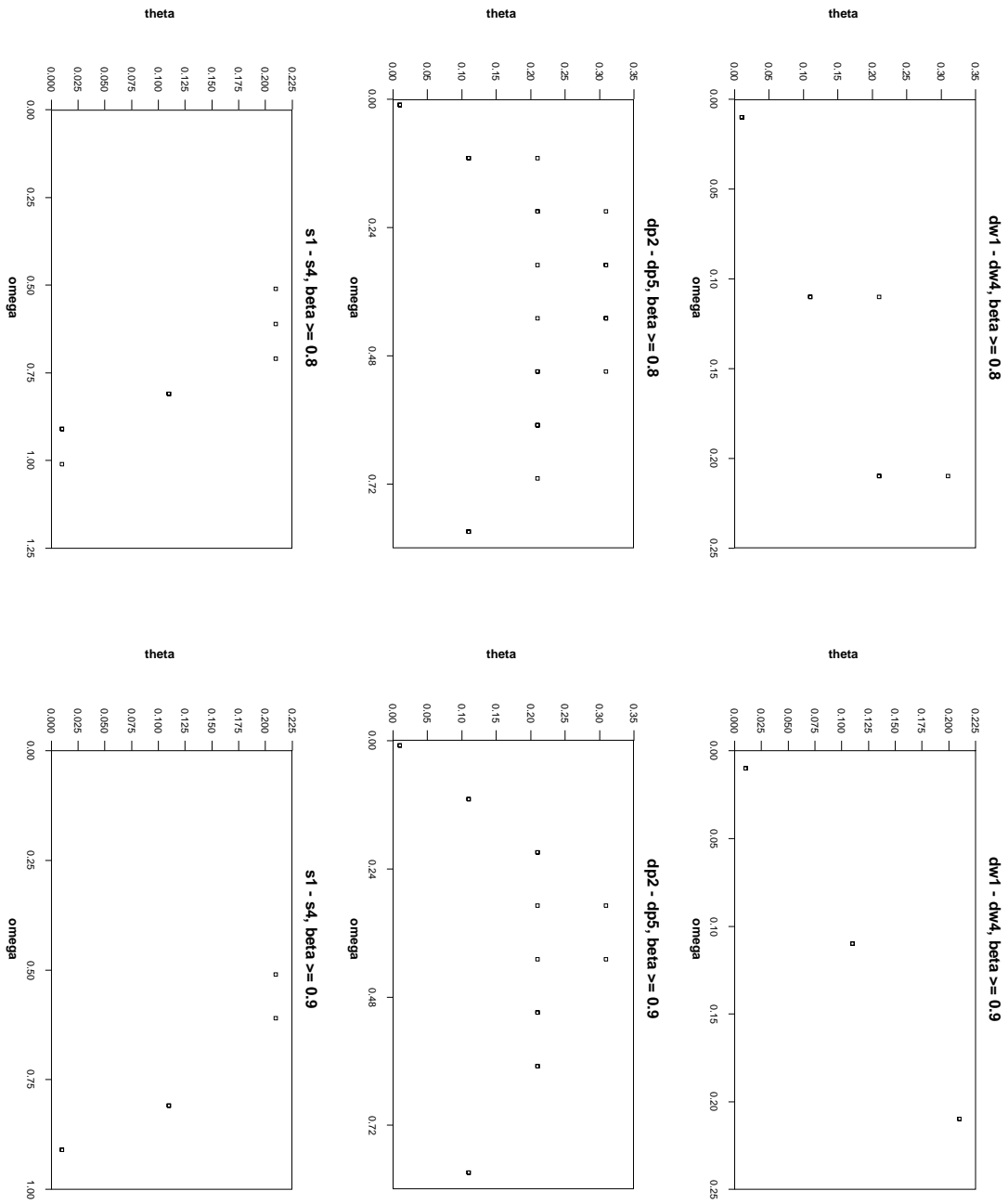


Figure 3: Constrained Hybrid Model - Full Sample  
 (no parameter combinations exist with the sp instruments.)



# Appendix

Table 4a: Hybrid Model - Parameter Grid Search Results - US Data  
Parameter Combination that Do Not Reject  $H_0$  - Sample period (70:1 - 79:4)

	dw1 - dw4										
	$\omega = 0$	$\omega = 0.1$	$\omega = 0.2$	$\omega = 0.3$	$\omega = 0.4$	$\omega = 0.5$	$\omega = 0.6$	$\omega = 0.7$	$\omega = 0.8$	$\omega = 0.9$	$\omega = 1.0$
$\theta = 0.0$	(0 - 1)	-	-	-	-	-	-	-	-	-	-
$\theta = 0.1$	0.8	-	-	-	-	-	-	-	-	-	-
$\theta = 0.2$	-	-	-	-	-	-	-	-	-	-	-
$\theta = 0.3$	-	0.9	1	1	-	-	-	-	-	-	-
$\theta = 0.4$	0.7	0.8	(0.8 - 0.9)	0.9	0.8	-	-	-	-	-	-
$\theta = 0.5$	(0.7 - 0.8)	0.8	0.8	0.8	0.7	-	-	-	-	-	-
$\theta = 0.6$	(0.7 - 0.8)	(0.7 - 0.8)	(0.7 - 0.8)	0.7	0.7	-	-	-	-	-	-
$\theta = 0.7$	(0.7 - 0.8)	(0.7 - 0.8)	0.7	0.7	0.6	-	-	-	-	-	-
$\theta = 0.8$	(0.7 - 0.8)	(0.7 - 0.8)	0.7	(0.6 - 0.7)	0.6	0.5	-	-	-	-	-
$\theta = 0.9$	(0.7 - 0.8)	0.7	(0.6 - 0.7)	0.6	0.5	-	-	-	-	-	-
$\theta = 1.0$	(0.7 - 0.8)	0.7	(0.6 - 0.7)	0.6	0.5	0.4	-	-	-	-	-

	sp1 - sp4										
	$\omega = 0$	$\omega = 0.1$	$\omega = 0.2$	$\omega = 0.3$	$\omega = 0.4$	$\omega = 0.5$	$\omega = 0.6$	$\omega = 0.7$	$\omega = 0.8$	$\omega = 0.9$	$\omega = 1.0$
$\theta = 0.0$	(0-1)	-	-	-	-	-	-	-	-	-	(0-1)
$\theta = 0.1$	(0.7-0.8)	-	-	-	-	-	-	-	(0.9-1)	(0.2 - 0.7)	(0-0.1)
$\theta = 0.2$	(0.7-0.8)	(0.9-1)	-	-	-	-	1	(0.8-1)	(0.5-0.7)	(0.1-0.3)	0
$\theta = 0.3$	(0.7-0.8)	(0.8-0.9)	(0.9-1)	(0.9-1)	(0.9-1)	(0.9-1)	(0.7-0.9)	(0.5-0.7)	(0.3-0.5)	(0.1-0.2)	0
$\theta = 0.4$	(0.7-0.8)	0.8	(0.8-0.9)	(0.8-0.9)	(0.8-0.9)	(0.7-0.8)	(0.6-0.7)	(0.4-0.5)	0.3	(0.1-0.2)	0
$\theta = 0.5$	(0.7-0.8)	(0.7-0.8)	0.8	(0.7-0.8)	(0.7-0.8)	(0.6-0.7)	(0.5-0.6)	0.4	(0.2-0.3)	0.1	0
$\theta = 0.6$	(0.7-0.8)	(0.7-0.8)	(0.7-0.8)	0.7	(0.6-0.7)	(0.5-0.6)	(0.4-0.5)	(0.3-0.4)	0.2	0.1	0
$\theta = 0.7$	(0.7-0.8)	(0.7-0.8)	0.7	(0.6-0.7)	0.6	0.5	0.4	0.3	0.2	0.1	0
$\theta = 0.8$	(0.7-0.8)	0.7	0.7	(0.6-0.7)	(0.5-0.6)	0.5	0.4	0.3	0.2	0.1	0
$\theta = 0.9$	(0.7-0.8)	0.7	(0.6-0.7)	0.6	(0.5-0.6)	(0.4-0.5)	(0.3-0.4)	0.2	0.2	-	0
$\theta = 1.0$	(0.7-0.8)	0.7	(0.6-0.7)	0.6	0.5	0.4	0.3	0.2	0.1	-	0

Note: Reported values are for the parameter  $\beta$ . Instrument sets are lags 1 to 4 of wage inflation (dw) and of long-short interest spread (sp).

Table 4b: Hybrid Model - Parameter Grid Search Results - US Data  
 Parameter Combination that Do Not Reject  $H_0$  - Sample period (70:1 - 79:4)

		dp2 - dp5									
	$\omega = 0$	$\omega = 0.1$	$\omega = 0.2$	$\omega = 0.3$	$\omega = 0.4$	$\omega = 0.5$	$\omega = 0.6$	$\omega = 0.7$	$\omega = 0.8$	$\omega = 0.9$	$\omega = 1.0$
$\theta = 0.0$	(0-1)	-	-	-	-	-	-	-	-	(0-1)	(0-1)
$\theta = 0.1$	0.7	-	-	-	-	-	-	(0.9-1)	(0.4-1)	(0-1)	(0-0.6)
$\theta = 0.2$	0.7	(0.9-1)	-	-	-	1	(0.8-1)	(0.5-1)	(0.2-1)	(0-0.6)	(0-0.3)
$\theta = 0.3$	0.7	(0.8-0.9)	(0.9-1)	(0.9-1)	(0.8-1)	(0.7-1)	(0.6-1)	(0.4-0.9)	(0.2-0.7)	(0-0.4)	(0-0.2)
$\theta = 0.4$	(0.7-0.8)	(0.8-0.9)	(0.8-0.9)	(0.7-1)	(0.7-1)	(0.6-0.9)	(0.5-0.8)	(0.3-0.7)	(0.1-0.5)	(0-0.3)	(0-0.1)
$\theta = 0.5$	(0.7-0.8)	(0.7-0.8)	(0.7-0.9)	(0.7-0.9)	(0.6-0.8)	(0.5-0.8)	(0.5-0.7)	(0.3-0.6)	(0.1-0.4)	(0-0.2)	(0-0.1)
$\theta = 0.6$	(0.7-0.8)	(0.7-0.8)	(0.7-0.8)	(0.6-0.8)	(0.5-0.8)	(0.5-0.7)	(0.3-0.6)	(0.2-0.5)	(0.1-0.3)	(0-0.2)	(0-0.1)
$\theta = 0.7$	(0.7-0.8)	(0.7-0.8)	(0.6-0.8)	(0.6-0.8)	(0.5-0.7)	(0.4-0.6)	(0.3-0.5)	(0.2-0.4)	(0.1-0.3)	(0-0.2)	0
$\theta = 0.8$	(0.7-0.8)	(0.7-0.8)	(0.6-0.8)	(0.6-0.7)	(0.5-0.7)	(0.4-0.6)	(0.3-0.5)	(0.2-0.4)	(0.1-0.3)	(0-0.1)	0
$\theta = 0.9$	(0.7-0.8)	(0.7-0.8)	(0.6-0.7)	(0.5-0.7)	(0.5-0.6)	(0.4-0.5)	(0.3-0.4)	(0.2-0.3)	(0.1-0.2)	(0-0.1)	0
$\theta = 1.0$	(0.7-0.8)	(0.6-0.7)	(0.6-0.7)	(0.5-0.7)	(0.4-0.6)	(0.3-0.5)	(0.3-0.4)	(0.2-0.3)	(0.1-0.2)	(0-0.1)	0

		gq1 - gq4									
$\theta = 0.0$	-	-	-	-	-	-	-	-	-	-	-
$\theta = 0.1$	-	-	-	-	-	-	-	-	-	-	-
$\theta = 0.2$	-	-	-	-	-	-	-	-	-	-	-
$\theta = 0.3$	-	-	-	-	-	-	-	-	-	-	-
$\theta = 0.4$	-	-	-	-	-	-	-	-	-	-	-
$\theta = 0.5$	-	-	-	-	-	-	-	-	-	-	-
$\theta = 0.6$	-	-	-	-	-	-	-	-	-	-	-
$\theta = 0.7$	-	-	-	-	-	-	-	-	-	-	-
$\theta = 0.8$	-	-	-	-	-	-	-	-	-	-	-
$\theta = 0.9$	-	-	-	-	-	-	-	-	-	-	-
$\theta = 1.0$	-	-	-	-	-	-	-	-	-	-	-

Note: Reported values are for the parameter  $\beta$ . Instrument sets are lags 2 to 4 of inflation (dp) and lags 1 to 4 of quadratically-detrended output gap (gq).



Table 5a: Hybrid Model - Parameter Grid Search Results - US Data  
Parameter Combination that Do Not Reject  $H_0$  - Sample period (80:1 - 89:4)

		dw <sub>1</sub> - dw <sub>4</sub>										
		$\omega = 0$	$\omega = 0.1$	$\omega = 0.2$	$\omega = 0.3$	$\omega = 0.4$	$\omega = 0.5$	$\omega = 0.6$	$\omega = 0.7$	$\omega = 0.8$	$\omega = 0.9$	$\omega = 1.0$
$\theta = 0.0$	(0-1)		(0.5-1)	-	-	-	-	-	-	-	(0.6-1)	-
$\theta = 0.1$	(0.1-0.6)	(0.6-1)	(0.9-1)	(0.7-0.9)	(0.6-0.7)	(0.7-1)	(0.7-0.9)	1	(0.8-1)	(0.5-0.9)	(0.1-0.4)	-
$\theta = 0.2$	(0.4-0.6)	(0.5-0.7)	(0.7-0.9)	(0.7-0.9)	(0.6-0.7)	(0.6-0.7)	(0.5-0.7)	(0.6-0.8)	(0.5-0.6)	(0.3-0.4)	(0.1-0.2)	-
$\theta = 0.3$	0.5	(0.5-0.6)	(0.6-0.7)	(0.6-0.7)	(0.5-0.6)	(0.5-0.6)	(0.4-0.5)	0.4	(0.3-0.4)	(0.2-0.3)	0.1	-
$\theta = 0.4$	0.5	(0.5-0.6)	(0.5-0.6)	(0.5-0.6)	(0.5-0.6)	(0.4-0.5)	(0.4-0.5)	0.4	0.3	0.2	0.1	-
$\theta = 0.5$	0.5	(0.5-0.6)	(0.5-0.6)	(0.5-0.6)	(0.5-0.6)	(0.4-0.5)	(0.4-0.5)	(0.3-0.4)	(0.2-0.3)	(0.1-0.2)	-	-
$\theta = 0.6$	0.5	0.5	0.5	0.5	(0.4-0.5)	(0.4-0.5)	(0.3-0.4)	0.3	0.2	0.1	-	-
$\theta = 0.7$	0.5	0.5	0.5	0.5	(0.4-0.5)	0.4	(0.3-0.4)	0.3	0.2	0.1	-	-
$\theta = 0.8$	0.5	0.5	(0.4-0.5)	(0.4-0.5)	(0.4-0.5)	0.4	0.3	0.2	0.2	0.1	-	-
$\theta = 0.9$	0.5	0.5	(0.4-0.5)	(0.4-0.5)	0.4	(0.3-0.4)	0.3	0.2	-	0.1	-	-
$\theta = 1.0$	0.5	0.5	(0.4-0.5)	(0.4-0.5)	0.4	(0.3-0.4)	0.3	0.2	-	0.1	-	-

		sp1 - sp4										
$\theta = 0.0$	(0-1)	(0.7-1)	-	-	-	-	-	-	-	-	(0-1)	(0-1)
$\theta = 0.1$	(0.2-0.6)	(0.6-1)	(0.9-1)	(0.7-0.9)	(0.7-1)	(0.7-1)	(0.6-1)	(0.9-1)	(0.6-1)	(0.3-1)	(0-0.6)	(0-0.1)
$\theta = 0.2$	(0.4-0.6)	(0.5-0.7)	(0.6-0.9)	(0.6-0.7)	(0.5-0.7)	(0.5-0.7)	(0.6-1)	(0.5-0.9)	(0.4-0.7)	(0.2-0.5)	(0-0.3)	0
$\theta = 0.3$	0.5	(0.5-0.6)	(0.6-0.7)	(0.5-0.6)	(0.5-0.6)	(0.5-0.6)	(0.5-0.7)	(0.4-0.6)	(0.3-0.5)	(0.1-0.3)	(0-0.1)	0
$\theta = 0.4$	0.5	(0.5-0.6)	(0.5-0.6)	(0.5-0.6)	(0.4-0.5)	(0.4-0.5)	(0.4-0.5)	(0.3-0.5)	(0.2-0.4)	(0.1-0.3)	(0-0.1)	0
$\theta = 0.5$	0.5	(0.5-0.6)	(0.5-0.6)	(0.4-0.5)	(0.4-0.5)	(0.4-0.5)	(0.4-0.5)	(0.3-0.4)	(0.2-0.3)	(0.1-0.2)	(0-0.1)	0
$\theta = 0.6$	0.5	0.5	0.5	(0.4-0.5)	(0.4-0.5)	0.4	(0.3-0.4)	0.3	(0.2-0.3)	(0.1-0.2)	(0-0.1)	0
$\theta = 0.7$	0.5	0.5	0.5	(0.4-0.5)	(0.4-0.5)	(0.3-0.4)	(0.3-0.4)	(0.2-0.3)	0.2	0.1	(0-0.1)	0
$\theta = 0.8$	0.5	0.5	(0.4-0.5)	(0.4-0.5)	(0.4-0.5)	(0.3-0.4)	0.3	(0.2-0.3)	0.2	0.1	0	0
$\theta = 0.9$	0.5	0.5	(0.4-0.5)	(0.4-0.5)	0.4	(0.3-0.4)	0.3	0.2	(0.1-0.2)	0.1	0	0
$\theta = 1.0$	0.5	0.5	(0.4-0.5)	(0.4-0.5)	0.4	(0.3-0.4)	0.3	0.2	(0.1-0.2)	0.1	0	0

Note: Reported values are for the parameter  $\beta$ . Instrument sets are lags 1 to 4 of wage inflation (dw) and of long-short interest spread (sp).

Table 5b: Hybrid Model - Parameter Grid Search Results - US Data  
 Parameter Combination that Do Not Reject  $H_0$  - Sample period (80:1 - 89:4)

	dp2 - dp5										
	$\omega = 0$	$\omega = 0.1$	$\omega = 0.2$	$\omega = 0.3$	$\omega = 0.4$	$\omega = 0.5$	$\omega = 0.6$	$\omega = 0.7$	$\omega = 0.8$	$\omega = 0.9$	$\omega = 1.0$
$\theta = 0.0$	(0-1)	(0.8-1)	-	-	-	-	-	-	-	(0.2-1)	(0-0.2)
$\theta = 0.1$	(0.2-0.6)	(0.6-7)	1	-	-	-	-	(0.8-1)	(0.5-0.9)	(0.1-0.4)	0
$\theta = 0.2$	(0.4-0.5)	0.6	(0.7-0.8)	0.8	(0.8-0.9)	(0.7-9)	(0.6-0.8)	(0.5-0.6)	(0.3-0.4)	(0-0.2)	0
$\theta = 0.3$	0.5	0.5	0.6	(0.6-0.7)	(0.6-0.7)	(0.5-0.6)	0.5	(0.3-0.4)	(0.2-0.3)	(0-0.1)	-
$\theta = 0.4$	0.5	0.5	0.6	0.6	0.5	0.5	0.4	0.3	0.2	(0-0.1)	-
$\theta = 0.5$	0.5	0.5	0.5	0.5	0.5	0.4	(0.3-0.4)	0.2	(0.1-0.2)	(0-0.1)	-
$\theta = 0.6$	0.5	0.5	0.5	0.5	0.4	0.4	0.3	0.2	0.1	0	-
$\theta = 0.7$	0.5	0.5	0.5	0.4	0.4	0.3	0.3	0.2	0.1	0	-
$\theta = 0.8$	0.5	0.5	0.5	0.4	0.4	0.3	0.2	0.2	0.1	0	-
$\theta = 0.9$	0.5	0.5	0.5	0.4	(0.3-0.4)	0.3	0.2	0.2	0.1	0	-
$\theta = 1.0$	0.5	0.5	0.5	0.4	0.3	0.3	0.2	-	0.1	0	-

	gq1 - gq4										
$\theta = 0.0$	-	-	-	-	-	-	-	-	-	(0-1)	(0-1)
$\theta = 0.1$	-	-	-	-	-	-	-	(0.9-1)	(0.4-0.8)	(0-0.5)	(0-0.1)
$\theta = 0.2$	-	-	-	-	-	-	-	0.5	(0.2-0.4)	(0-0.2)	0
$\theta = 0.3$	-	-	-	-	-	-	-	0.4	(0.2-0.3)	(0-0.1)	0
$\theta = 0.4$	-	-	-	-	-	-	-	0.3	0.2	(0-0.1)	0
$\theta = 0.5$	-	-	-	-	-	-	-	-	0.1	(0-0.1)	0
$\theta = 0.6$	-	-	-	-	-	-	-	0.2	0.1	0	0
$\theta = 0.7$	-	-	-	-	-	-	-	0.2	0.1	0	0
$\theta = 0.8$	-	-	-	-	-	-	-	-	0.1	0	0
$\theta = 0.9$	-	-	-	-	-	-	-	-	0.1	0	0
$\theta = 1.0$	-	-	-	-	-	-	-	-	0.1	0	0

Note: Reported values are for the parameter  $\beta$ . Instrument sets are lags 2 to 4 of inflation (dp) and lags 1 to 4 of quadratically-detrended output gap (gq).

Table 6a: Hybrid Model - Parameter Grid Search Results - US Data  
Parameter Combination that Do Not Reject  $H_0$  - Sample period (90:1 - 97:4)

	dw1 - dw4										
	$\omega = 0$	$\omega = 0.1$	$\omega = 0.2$	$\omega = 0.3$	$\omega = 0.4$	$\omega = 0.5$	$\omega = 0.6$	$\omega = 0.7$	$\omega = 0.8$	$\omega = 0.9$	$\omega = 1.0$
$\theta = 0.0$	(0-1)	(0-1)	-	-	-	-	-	-	(0-1)	(0-1)	(0-1)
$\theta = 0.1$	(0-1)	(0.5-1)	1	-	-	-	(0.9-1)	(0.4-1)	(0-1)	(0-1)	(0-0.5)
$\theta = 0.2$	(0.4-0.9)	(0.6-1)	(0.8-1)	(0.9-1)	(0.8-1)	(0.7-1)	(0.5-1)	(0.3-1)	(0-1)	(0-0.6)	(0-0.2)
$\theta = 0.3$	(0.6-0.8)	(0.7-1)	(0.7-1)	(0.7-1)	(0.7-1)	(0.5-1)	(0.4-1)	(0.2-0.9)	(0-0.7)	(0-0.4)	(0-0.1)
$\theta = 0.4$	(0.6-0.8)	(0.7-0.9)	(0.7-1)	(0.6-1)	(0.6-1)	(0.5-0.9)	(0.3-0.8)	(0.2-0.7)	(0-0.5)	(0-0.3)	(0-0.1)
$\theta = 0.5$	(0.6-0.8)	(0.6-0.9)	(0.6-0.9)	(0.6-0.9)	(0.5-0.9)	(0.4-0.8)	(0.3-0.7)	(0.1-0.6)	(0-0.4)	(0-0.2)	(0-0.1)
$\theta = 0.6$	(0.7-0.8)	(0.6-0.8)	(0.6-0.8)	(0.5-0.8)	(0.5-0.8)	(0.4-0.7)	(0.3-0.6)	(0.1-0.5)	(0-0.3)	(0-0.2)	0
$\theta = 0.7$	(0.7-0.8)	(0.6-0.8)	(0.6-0.8)	(0.5-0.8)	(0.4-0.7)	(0.3-0.6)	(0.2-0.5)	(0.1-0.4)	(0-0.3)	(0-0.2)	0
$\theta = 0.8$	(0.7-0.8)	(0.6-0.8)	(0.6-0.8)	(0.5-0.7)	(0.4-0.7)	(0.3-0.6)	(0.2-0.5)	(0.1-0.4)	(0-0.2)	(0-0.1)	0
$\theta = 0.9$	(0.7-0.8)	(0.6-0.8)	(0.6-0.8)	(0.5-0.7)	(0.4-0.6)	(0.3-0.6)	(0.2-0.4)	(0.1-0.3)	(0-0.2)	(0-0.1)	0
$\theta = 1.0$	(0.7-0.8)	(0.6-0.8)	(0.5-0.7)	(0.5-0.7)	(0.4-0.6)	(0.3-0.5)	(0.2-0.4)	(0.1-0.3)	(0-0.2)	(0-0.1)	0

	sp1 - sp4										
$\theta = 0.0$	(0-1)	(0.7-1)	(0.8-1)	-	-	-	-	-	(0-1)	(0-1)	(0-1)
$\theta = 0.1$	(0.3-0.8)	(0.7-1)	(0.7-1)	-	-	-	(0.8-1)	(0.3-1)	(0-1)	(0-1)	(0-0.7)
$\theta = 0.2$	(0.5-0.8)	(0.7-0.9)	(0.7-1)	(0.9-1)	(0.8-1)	(0.7-1)	(0.5-1)	(0.2-1)	(0-1)	(0-0.7)	(0-0.3)
$\theta = 0.3$	(0.6-0.8)	(0.7-0.9)	(0.7-0.9)	(0.7-1)	(0.6-1)	(0.5-1)	(0.4-1)	(0.1-1)	(0-0.7)	(0-0.4)	(0-0.2)
$\theta = 0.4$	(0.6-0.8)	(0.7-0.8)	(0.5-0.9)	(0.6-1)	(0.6-1)	(0.4-0.9)	(0.3-0.8)	(0.2-0.7)	(0-0.5)	(0-0.3)	(0-0.1)
$\theta = 0.5$	(0.6-0.8)	(0.7-0.8)	(0.6-0.8)	(0.6-0.9)	(0.5-0.8)	(0.5-0.8)	(0.3-0.7)	(0.1-0.6)	(0-0.4)	(0-0.3)	(0-0.1)
$\theta = 0.6$	(0.7-0.8)	(0.7-0.8)	(0.6-0.8)	(0.6-0.8)	(0.5-0.7)	(0.4-0.7)	(0.2-0.6)	(0.1-0.5)	(0-0.3)	(0-0.2)	(0-0.1)
$\theta = 0.7$	0.7	(0.7-0.8)	(0.6-0.7)	(0.5-0.7)	(0.4-0.7)	(0.3-0.6)	(0.2-0.5)	(0.1-0.4)	(0-0.3)	(0-0.2)	(0-0.1)
$\theta = 0.8$	0.7	(0.7-0.8)	(0.6-0.7)	(0.5-0.7)	(0.4-0.6)	(0.3-0.6)	(0.2-0.5)	(0.1-0.4)	(0-0.3)	(0-0.1)	0
$\theta = 0.9$	0.7	(0.6-0.7)	(0.6-0.7)	(0.5-0.7)	(0.4-0.6)	(0.3-0.5)	(0.2-0.4)	(0.1-0.3)	(0-0.2)	(0-0.1)	0
$\theta = 1.0$	0.7	(0.6-0.7)	(0.6-0.7)	(0.5-0.7)	(0.4-0.6)	(0.3-0.5)	(0.2-0.4)	(0.1-0.3)	(0-0.2)	(0-0.1)	0

Note: Reported values are for the parameter  $\beta$ . Instrument sets are lags 1 to 4 of wage inflation (dw) and of long-short interest spread (sp).

Table 6b: Hybrid Model - Parameter Grid Search Results - US Data  
 Parameter Combination that Do Not Reject  $H_0$  - Sample period (90:1 - 97:4)

	dp2 - dp5										
	$\omega = 0$	$\omega = 0.1$	$\omega = 0.2$	$\omega = 0.3$	$\omega = 0.4$	$\omega = 0.5$	$\omega = 0.6$	$\omega = 0.7$	$\omega = 0.8$	$\omega = 0.9$	$\omega = 1.0$
$\theta = 0.0$	(0 - 1)	-	-	-	-	-	-	-	-	-	-
$\theta = 0.1$	(0 - 1)	(0.5 - 1)	-	-	-	-	-	-	-	-	-
$\theta = 0.2$	(0.4 - 0.8)	(0.7 - 1)	(0.9 - 1)	-	-	-	-	-	-	-	-
$\theta = 0.3$	(0.6 - 0.7)	0.8	0.9	-	-	-	-	-	-	-	-
$\theta = 0.4$	-	-	0.9	-	-	-	-	-	-	-	-
$\theta = 0.5$	-	-	0.9	-	-	-	-	-	-	-	-
$\theta = 0.6$	-	-	-	-	-	-	-	-	-	-	-
$\theta = 0.7$	-	-	-	-	-	-	-	-	-	-	-
$\theta = 0.8$	-	-	-	-	-	-	-	-	-	-	-
$\theta = 0.9$	-	-	-	-	-	-	-	-	-	-	-
$\theta = 1.0$	-	-	-	-	-	-	-	-	-	-	-

	gq1 - gq4										
	$\omega = 0$	$\omega = 0.1$	$\omega = 0.2$	$\omega = 0.3$	$\omega = 0.4$	$\omega = 0.5$	$\omega = 0.6$	$\omega = 0.7$	$\omega = 0.8$	$\omega = 0.9$	$\omega = 1.0$
$\theta = 0.0$	-	-	-	-	-	-	-	-	-	(0-1)	(0-0.9)
$\theta = 0.1$	-	-	-	-	-	-	-	1	(0.5-1)	(0-0.7)	0
$\theta = 0.2$	-	-	1	-	(0.8-1)	(0.7-1)	(0.8-1)	(0.5-1)	(0.3-0.7)	(0-0.4)	0
$\theta = 0.3$	0.7	(0.8-0.9)	(0.8-1)	(0.8-1)	(0.6-0.9)	(0.6-0.9)	(0.5-0.9)	(0.4-0.7)	(0.2-0.5)	(0-0.2)	0
$\theta = 0.4$	(0.7-0.8)	(0.7-0.9)	(0.7-0.9)	(0.7-1)	(0.6-0.8)	(0.6-0.9)	(0.4-0.7)	(0.3-0.6)	(0.1-0.4)	(0-0.2)	0
$\theta = 0.5$	(0.7-0.8)	(0.7-0.8)	(0.7-0.9)	(0.6-0.9)	(0.6-0.8)	(0.5-0.7)	(0.4-0.6)	(0.2-0.5)	(0.1-0.3)	(0-0.1)	0
$\theta = 0.6$	(0.7-0.8)	(0.7-0.8)	(0.6-0.8)	(0.6-0.8)	(0.5-0.7)	(0.4-0.7)	(0.3-0.5)	(0.2-0.4)	(0.1-0.3)	(0-0.1)	0
$\theta = 0.7$	(0.7-0.8)	(0.7-0.8)	(0.6-0.8)	(0.6-0.7)	(0.5-0.7)	(0.4-0.6)	(0.3-0.5)	(0.2-0.4)	(0.1-0.2)	(0-0.1)	0
$\theta = 0.8$	(0.7-0.8)	(0.7-0.8)	(0.6-0.7)	(0.5-0.7)	(0.5-0.6)	(0.4-0.6)	(0.3-0.4)	(0.2-0.3)	(0.1-0.2)	(0-0.1)	-
$\theta = 0.9$	0.7	(0.6-0.7)	(0.6-0.7)	(0.5-0.7)	(0.4-0.6)	(0.3-0.5)	(0.3-0.4)	(0.2-0.3)	(0.1-0.2)	(0-0.1)	-
$\theta = 1.0$	0.7	(0.6-0.7)	(0.6-0.7)	(0.5-0.6)	(0.4-0.6)	(0.3-0.5)	(0.2-0.3)	(0.2-0.3)	(0.1-0.2)	0	-

Note: Reported values are for the parameter  $\beta$ . Instrument sets are lags 2 to 4 of inflation (dp) and lags 1 to 4 of quadratically-detrended output gap (gq).

Figure 4: Hybrid Model - 1970:1-1979:4  
 (all parameter combinations reject with the gq instruments.)

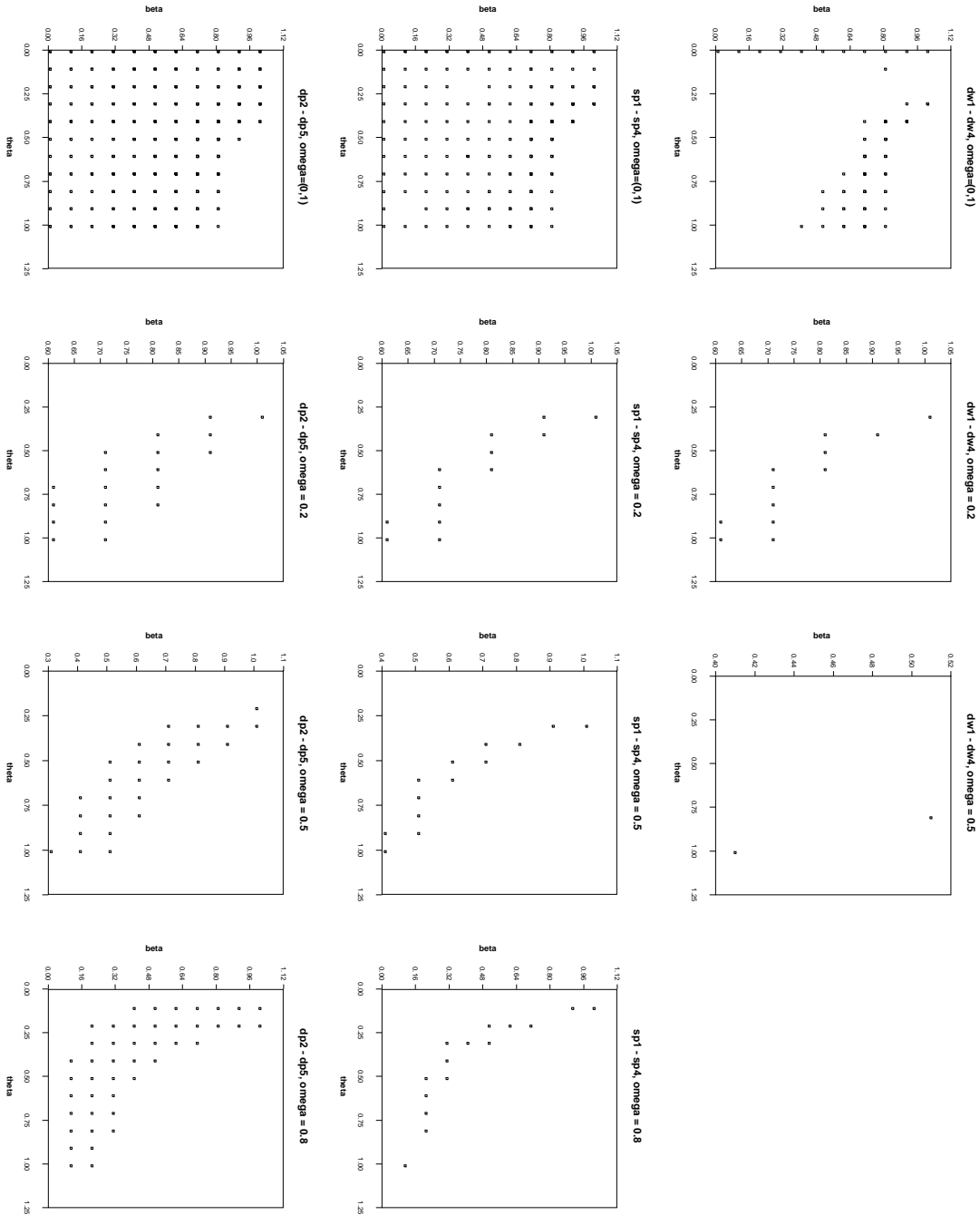


Figure 5: Constrained Hybrid Model - 1970:1-1979:4  
 (all parameter combinations reject with the gq instruments.)

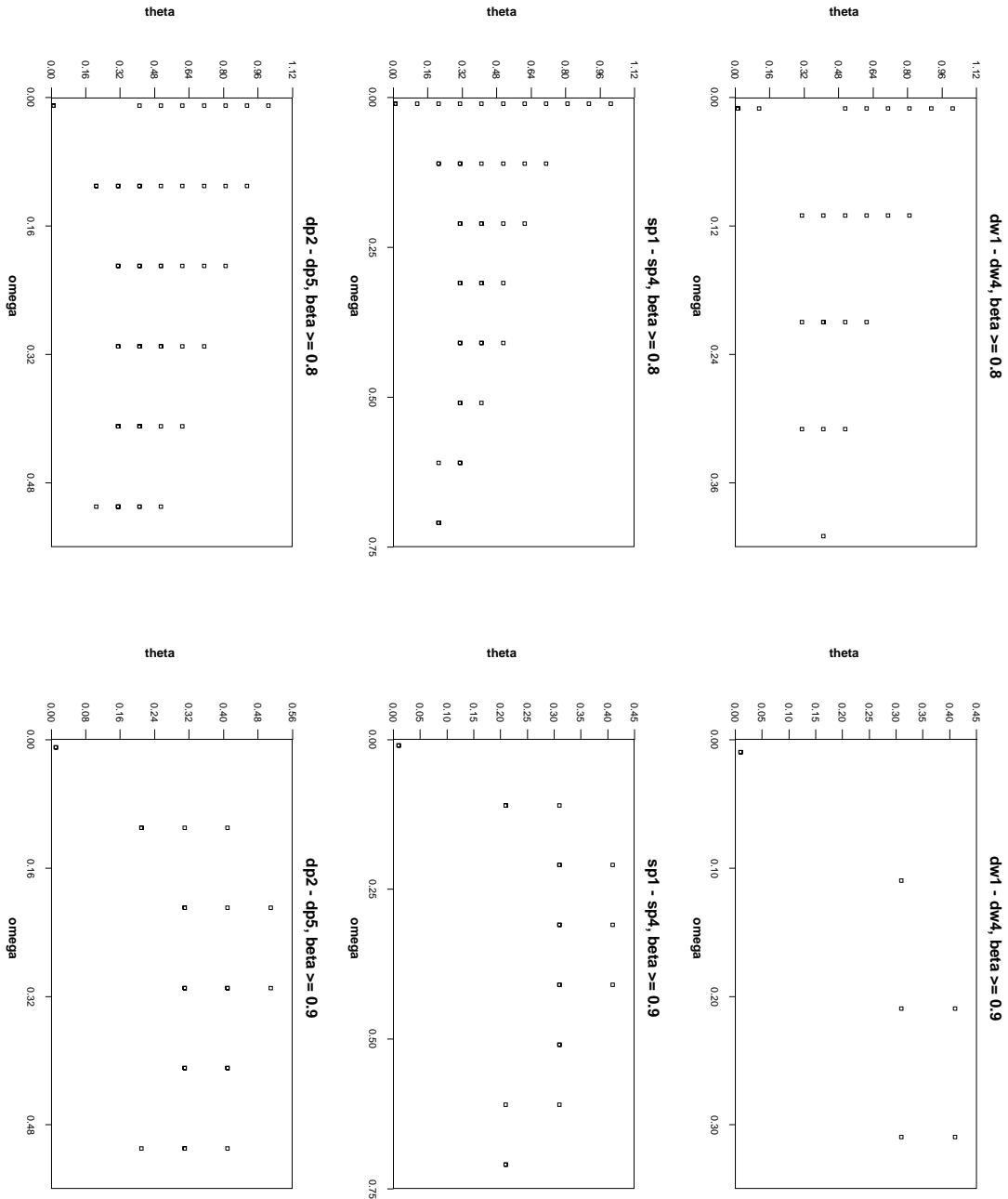


Figure 6: Hybrid Model - 1980:1-1989:4

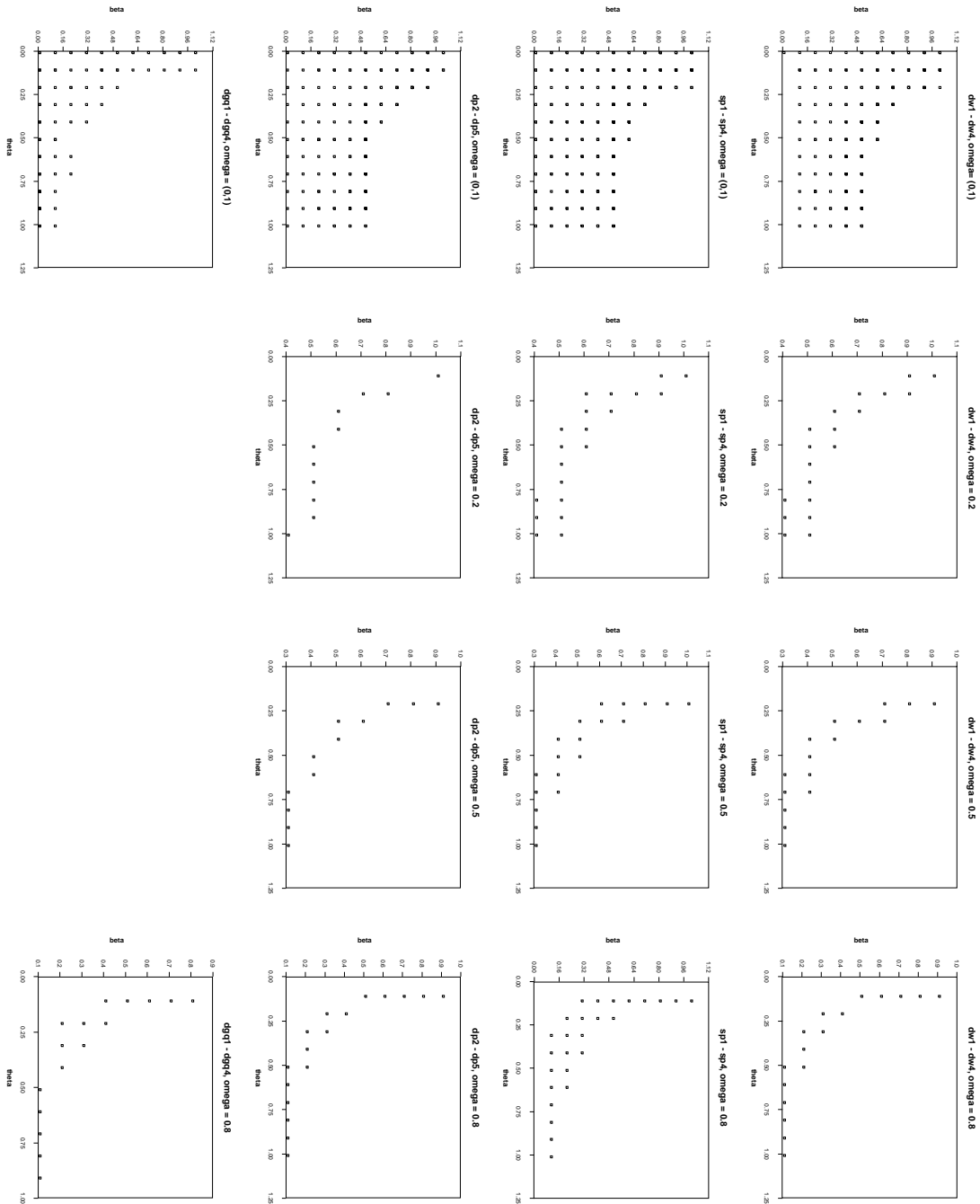


Figure 7: Constrained Hybrid Model - 1980:1-1989:4

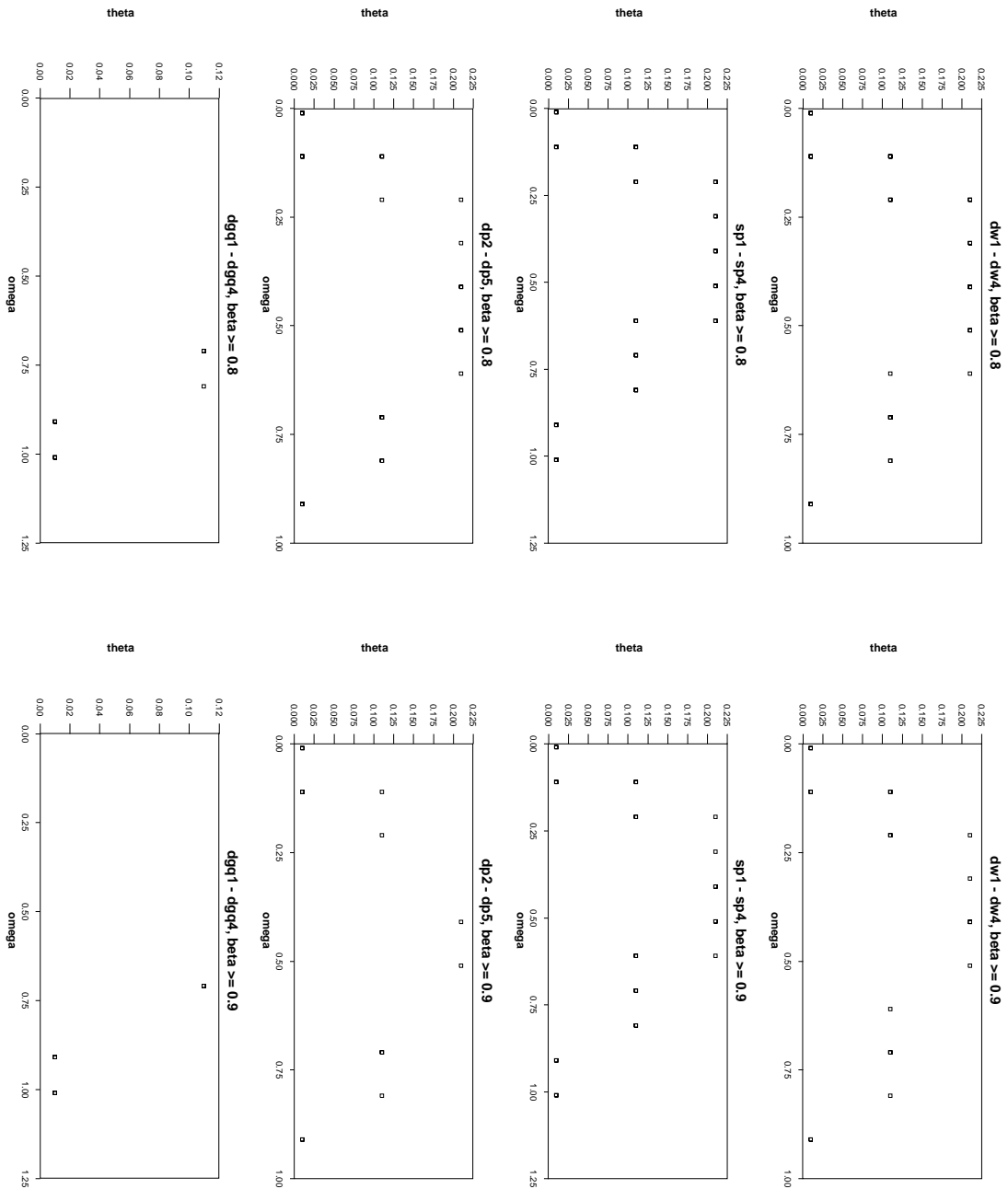




Figure 8: Hybrid Model - 1990:1-1997:4

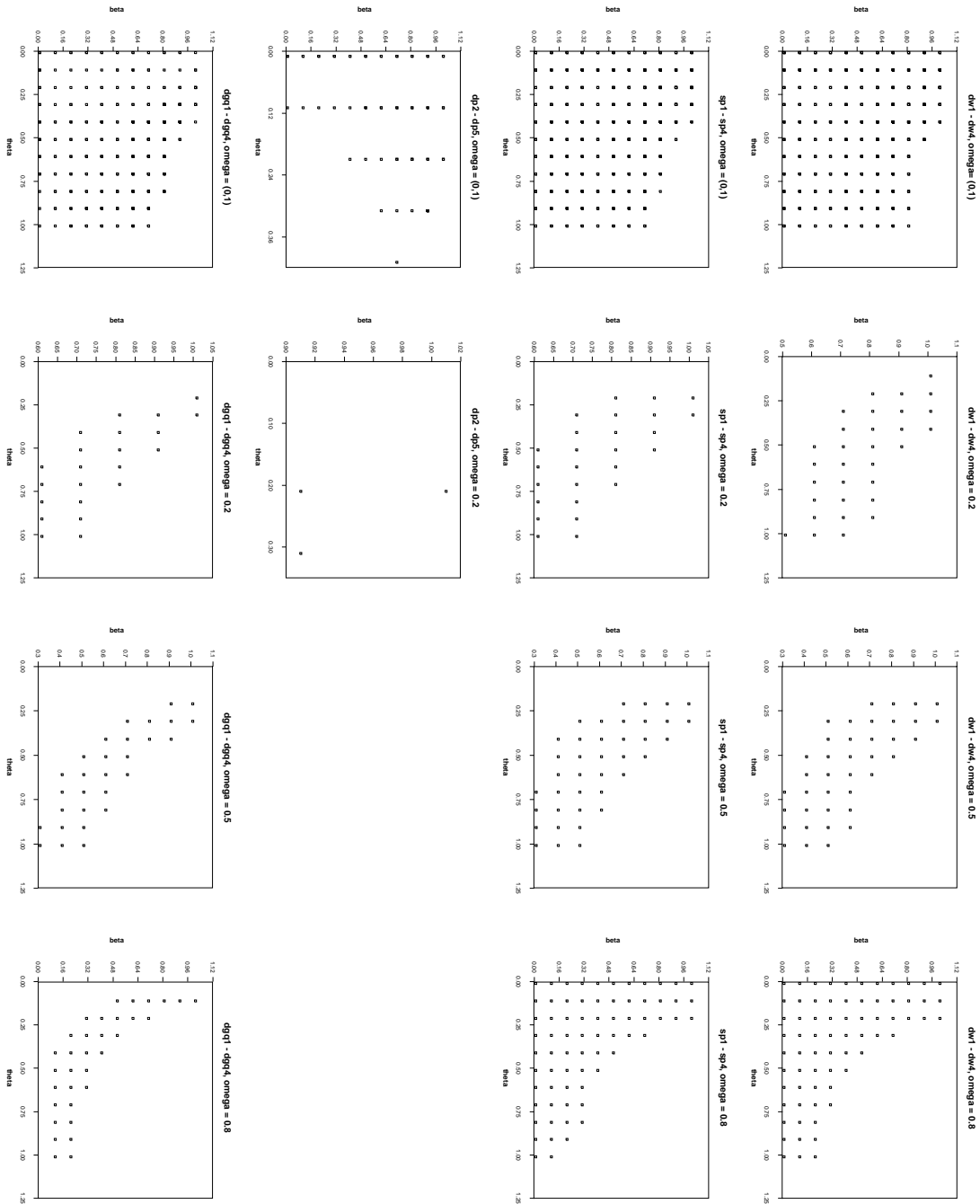
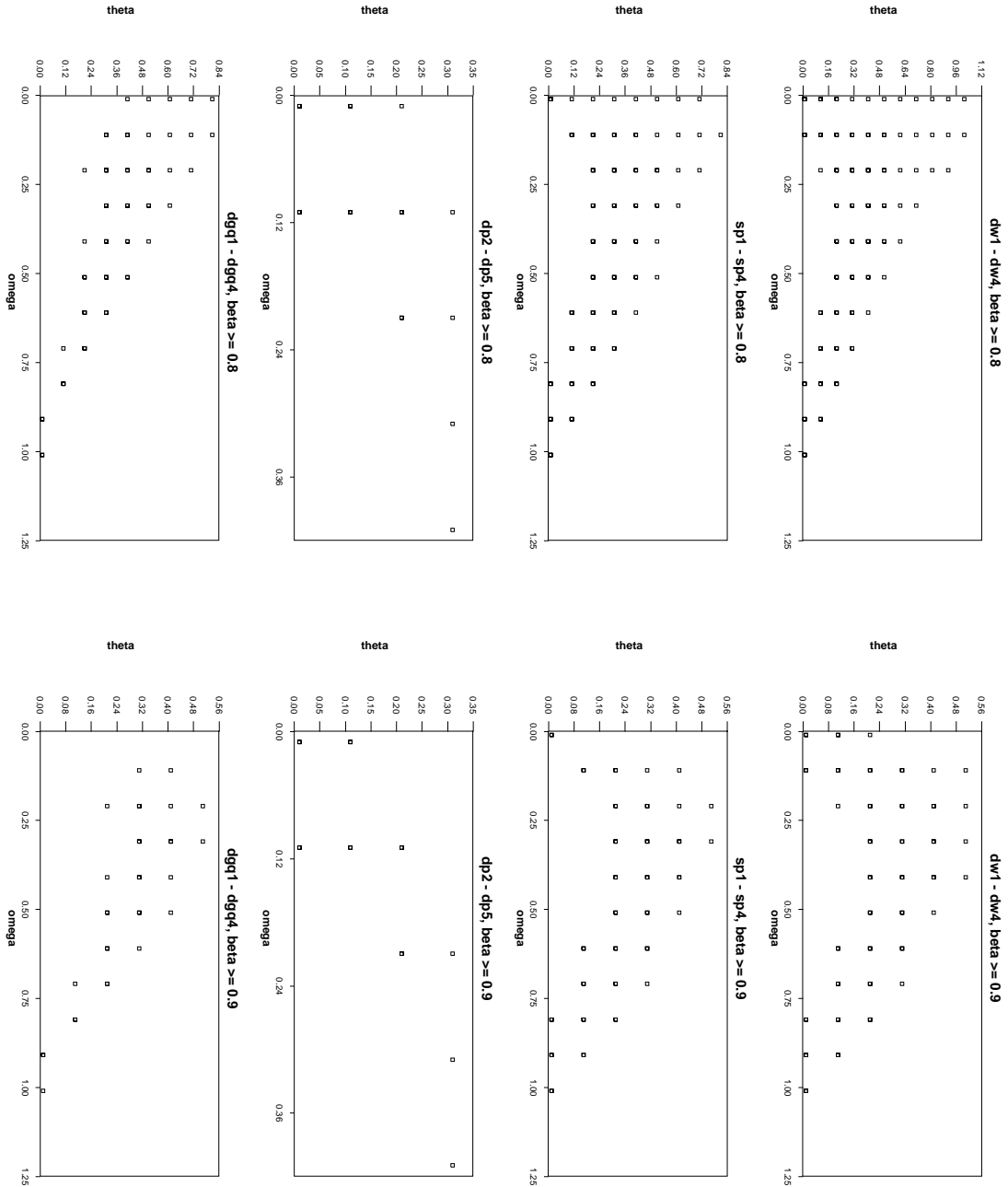


Figure 9: Constrained Hybrid Model - 1990:1-1997:4



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