A Location Model with Preference for Variety

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Abstract

We propose a new location model where consumers are allowed to make multiple purchases (i.e., one unit from each firm). This model fits many markets (e.g. newspapers, credit cards, scholarly journals, subscriptions to TV channels, etc.) better than existing models. A common feature of these markets is that some consumers are loyal to one brand, while others consume more than one product. Our model yields predictions consistent with this observation. Moreover, it restores Hotelling's *Principle of Minimum Differentiation*, by generating an equilibrium in pure strategies and with a linear transportation cost, where firms are located at the center and charge prices above marginal cost.

JEL Classification Codes: D43; L10.

Keywords: Hotelling model; Minimum differentiation; Preference for variety.

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1 Introduction

Hotelling (1929), in his seminal paper, introduced a model of spatial competition where two firms are located at the two end points of a busy street producing homogeneous products. Consumers have unit demands, are uniformly distributed on the street and each one buys from the firm which offers the best deal in terms of price and distance that has to be travelled. Later versions of that model have relaxed a number of Hotelling's assumptions, but the one that is firmly maintained is that consumers purchase from one firm exclusively. Implicitly, this says that consumers do not care for diversity. This would be certainly true under the initial hypothesis that the two products are homogenous, but not necessarily under the broader interpretation of the model by which the products are differentiated and the distance that a consumer has to travel serves as a measure of disutility. In many markets, consumers cannot purchase more than one unit of a given brand, but, nevertheless, variety is valued. For example, it does not make sense to buy two copies of the same newspaper, but, for *some* people, it makes perfect sense to read two different newspapers.[1] Other products that fall into this category include credit cards,[2] software[3], subscriptions to TV channels[4], scholarly journals,[5] and magazines. Our aim in this paper is to develop a modeling framework capable of capturing some of the key features of the above examples.[6]

We extend Hotelling's model by allowing consumers to buy from both firms. The model we propose generates a very realistic equilibrium where some consumers remain loyal to one brand, while another group of consumers consumes both brands. We begin by assuming that the firms' horizontal locations are fixed at the two extreme points of the unit interval. Our analysis yields several interesting new insights. We show that when the incremental utility from consuming both brands is not high, then the unique equilibrium is the same as the one in the standard Hotelling

[1] Gentzkow (2003) studies the newspapers market in Washington DC and finds that one third of the consumers in his sample read more than one newspaper.

[2] People usually hold only one credit card from a given issuer, but a second card, from a different issuer, provides to the holder a higher credit limit and consequently a higher utility.

[3] For example, there is a great deal of horizontal preference heterogeneity regarding the typesetting software ScientificWord (or WorkPlace) and Microsoft Word. Nevertheless, utility increases if both are used, since this enhances a person's ability to communicate and collaborate with others.

[4] A baseball fan may have a stronger preference for the New York Yankees than the rival New York Mets, and therefore if he had to choose between subscribing to the TV channel which broadcasts the Yankee games and to the one which broadcasts the Met games, he would certainly choose the former. But at the same time he likes the game of baseball and his utility if he subscribes to both channels is higher than if he subscribes to the Yankee channel exclusively.

[5] Consider two Economics journals differentiated by the mix of applied theory and applied econometrics papers they publish. Although users' strength of preferences for these two products varies, one thing seems to be common among all scholars: appreciation of variety [see McCabe (2002)].

[6] There is a growing interest for empirical applications based on models which combine spatial competition with preference for diversity. For instance, Pinske *et al.* (2002) propose such a model to study the nature of competition in the U.S. wholesale gasoline markets.

model, that is, half of the consumers buy from one firm and half from the other, exclusively (i.e., no consumer, in equilibrium, purchases from both firms). When the incremental utility is in an intermediate range, then there are two equilibria: i) the standard Hotelling outcome remains an equilibrium and ii) a new equilibrium emerges which is characterized by a fraction of consumers that consumes both brands. The first equilibrium Pareto dominates the second one. Finally, when the incremental utility is high, only the second equilibrium from the previous case survives. Overall, this says that the desire for diversity on part of the consumers has to be sufficiently strong for the firms to switch to an equilibrium where some consumers find it beneficial to consume both brands. The presence of a group of consumers who consumes both products acts as a buffer which alters the nature of price competition significantly. When a firm lowers its price the demand for its product increases, but not necessarily at the expense of its rival. This produces interesting comparative statics. For instance, as products become *more* differentiated, equilibrium profits *decrease*, when in equilibrium some consumers consume both brands. The non-cooperative outcome is, for a certain range of parameter values, inefficient. In particular, output is lower than the socially optimal level. This contrasts with the standard Hotelling model, which always yields an efficient equilibrium, for fixed firm locations and a covered market.

Then, we endogenize the locations of the firms. Hotelling claimed that firms will agglomerate in the middle of the unit interval (*Principle of Minimum Differentiation*). d'Aspermont *et al.* (1979) showed that this claim is incorrect. The reason is that when firms are sufficiently close to each other a pure strategy price equilibrium fails to exist. They also demonstrated that if the linear transportation cost is replaced by a quadratic one, then the price equilibrium is restored, but firms locate at the extremes, rather than the middle (*Principle of Maximum Differentiation*).[7] In the present paper, we assume a linear transportation cost and we show that a pure strategy price equilibrium exists regardless of where the firms are located, provided that the incremental utility from buying both brands is above a fixed threshold. Firm profits monotonically increase when they move towards the center. Thus, we retrieve Hotelling's *Principle of Minimum Differentiation.*

Consumers in our model have an elastic demand of a special kind. Although they cannot buy more than one unit from the same firm, they are allowed to purchase up to two units each one from a different firm. A number of papers in the literature [e.g. Anderson *et al.* (1989), Hamilton *et al.* (1994) and Rath and Zhao (2001)] have already introduced models with an elastic demand. In those papers, however, consumers are allowed to buy more than one unit from the same firm, but cannot purchase from other firms (and hence the issue of variety does not arise). The results with regards to product selection are also different from ours. Hamilton *et al.* assume a linear transportation

[7] Other remedies to the problem include, mixed strategy equilibrium, [e.g., Gal-Or (1982)] and heterogeneity in consumers' tastes, [e.g., de Palma *et al.* (1985)]. Gal-Or does not restore the Principle of Minimum Differentiation, while de Palma *et al.* do.

cost paid for every unit of the product and they show that a price equilibrium, in pure strategies, may not exist. Rath and Zhao assume a quadratic lump-sum transportation cost and they show that firms may locate at the extremes or strictly inside the interval (including the middle point) depending upon the ratio of the reservation price and the transportation cost parameter.[8]

Spatial models have been utilized extensively in the literature to address a number of interesting questions related to, price discrimination, entry decisions, product variety, vertical integration and market foreclosure, to mention a few. The modeling framework presented in this paper, fits many markets better than the existing models and it can be adopted to study old and new issues through different lenses. For example, a large body of the spatial price discrimination literature [e.g., Fudenberg and Tirole (2000), Liu and Serfes (2003) and Shaffer and Zhang (2000)], builds on the presumption that firms can easily segment the consumers into two groups: own customers and rival firm's customers. If consumers buy only from one firm, then this distinction is clear, but not when some consumers purchase from both firms. As a second example, consider the incentives of vertically integrated firms in the market for broadband access [e.g. a content provider (upstream firm) and an Internet service provider (downstream firm)] to practice conduit and/or content discrimination, [see Rubinfeld and Singer (2001)]. The major element of differentiation comes from the various contents that a service provider carries. In this case, it seems natural to assume that consumers have heterogeneous preferences for different contents, e.g. music, video games, movies, news etc. At the same time, though, variety is valued. A standard location model would ignore the preference bias towards variety. On the other hand, a representative consumer model [e.g. Singh and Vives (1984)], where preferences are symmetric, would miss the key element of preference heterogeneity.[9]

The rest of the paper is organized as follows. Section 2 presents the model. In section 3, we solve for the Nash equilibrium, assuming that the locations of the firms are fixed at the two endpoints of the unit interval, and in subsection 3.1 we compare the non-cooperative outcome with the social optimum. In section 4, we endogenize product selection, by allowing the firms to choose their horizontal locations. We conclude in section 5.

[8]Caplin and Nalebuff (1991) introduce a general model with multi-dimensionally differentiated products and prove existence of pure strategy price equilibrium, by showing that the profit functions are quasi-concave in a firm's own price [proposition 4, p.39]. Their model encompasses many of the alternative approaches to the theory of differentiated products [e.g. C.E.S. preferences, characteristics approach models and multi-dimensional probabilistic choice models] as special cases. Nevertheless, our framework does not satisfy the assumptions made by Caplin and Nalebuff and therefore it is not nested in theirs. As we demonstrate later, our model yields profit functions which are not quasi-concave.

[9]In Kim and Serfes (2003), we build on the present model to investigate the incentives of two vertically integrated firms to engage in content and/or network discrimination.

2 The description of the model

There are two firms A and B who produce differentiated brands and are located at the two endpoints of the unit interval $[0, 1]$. A unit mass of consumers is uniformly distributed on the $[0, 1]$ interval. Each consumer can buy at most one unit from a given firm and has the following valuation: $V = \alpha q_i + \beta q_A q_B$, where $i = A, B$ and q_i represents the quantity of brand i that a consumer buys, with $q_i \in \{0, 1\}$. These valuations do not depend on a consumer's particular horizontal location. Hence, if a consumer buys only one brand his valuation is equal to $V(1) = \alpha > 0$, while if he buys both brands his valuation is $V(2) = \alpha + \beta$, with $\beta \geq 0$, or $V(2) \geq V(1)$. Moreover, we assume diminishing incremental (marginal) utility, i.e, $\beta \leq \alpha$, or $V(2) - V(1) \leq V(1)$. Let θ denote the incremental utility, i.e., $\theta = V(2) - V(1)$, with $\theta \in [0, V(1)]$. In addition, consumers incur a disutility from not being able to purchase their "ideal" brand. A consumer who is located at $x \in [0, 1]$ incurs a disutility equal to tx if he buys brand A, a disutility equal to $t(1 - x)$ if he buys brand B and a disutility equal to $tx + t(1 - x) = t$ if he buys both, with $t > 0$.

Firm prices are denoted by p_A and p_B. We assume that the market is covered, i.e., each consumer buys at least one brand. If a consumer who is located at x buys from firm A his indirect utility is, $V(1) - tx - p_A$; if he buys from firm B his indirect utility is, $V(1) - t(1 - x) - p_B$; and if he buys from both firms his indirect utility is, $V(2) - t - p_A - p_B$. There will be two marginal consumers, one denoted by x_1, who is indifferent between buying from firm A only and buying from both firms and the other, denoted by x_2, who is indifferent between buying from firm B exclusively and buying from both firms (see figure 1). The first marginal consumer is located at,

$$V(1) - tx_1 - p_A = V(2) - t - p_A - p_B \Longleftrightarrow x_1 = \frac{t - \theta + p_B}{t}. \tag{1}$$

The second marginal consumer is located at,

$$V(1) - t(1 - x_2) - p_B = V(2) - t - p_A - p_B \Longleftrightarrow x_2 = \frac{\theta - p_A}{t}. \tag{2}$$

Note that,

$$x_2 \geq x_1 \Longleftrightarrow p_A + p_B \leq 2\theta - t. \tag{3}$$

Also,

$$x_1 \geq 0 \Longleftrightarrow p_B \geq \theta - t, \tag{4}$$

and

$$x_2 \leq 1 \Longleftrightarrow p_A \geq \theta - t. \tag{5}$$

When $x_2 = x_1$, then the marginal consumer is indifferent between buying one unit from firm A and one unit from firm B. In this case, no consumer purchases from both firms. Therefore, the marginal consumer is located at,

$$\hat{x} = x_1 = x_2 = \frac{p_B - p_A + t}{2t}.$$

Hence, the consumers who are located in $[0,\, x_1]$ purchase firm A's product exclusively, the consumers in $(x_1,\, x_2)$ purchase from both firms and the ones in $[x_2,\, 1]$ buy only firm B's product (see figure 1).

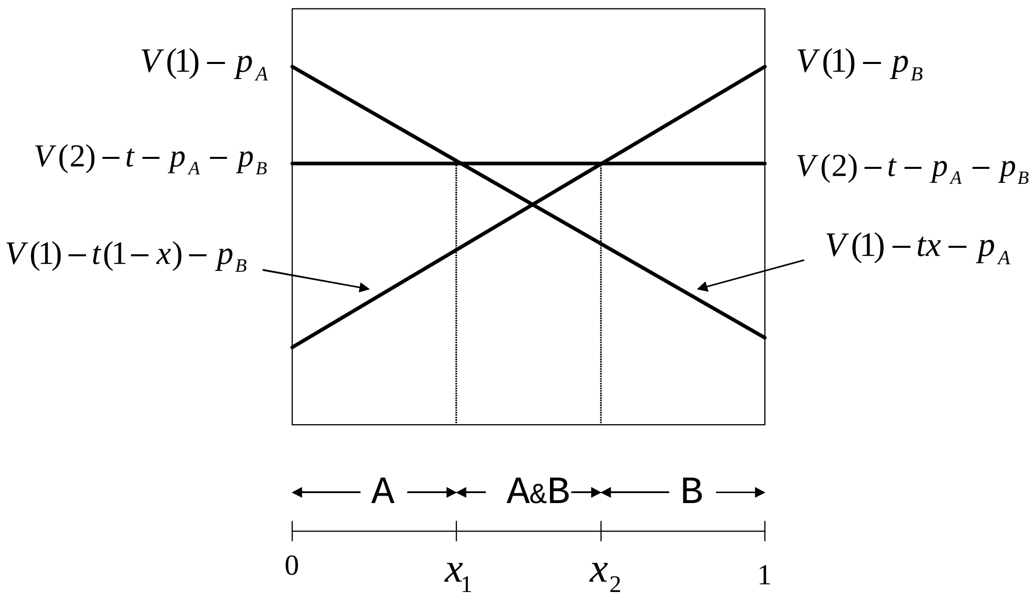

Figure 1

It then follows that firm A's demand function is,

$$d_A = \begin{cases} x_2 = \frac{\theta - p_A}{t}, & \text{if } p_A + p_B \leq 2\theta - t, \text{ i.e., } x_2 \geq x_1 \\ \hat{x} = \frac{p_B - p_A + t}{2t}, & \text{if } p_A + p_B \geq 2\theta - t, \text{ i.e., } x_2 = x_1. \end{cases}$$

Firm B's demand function is,

$$d_B = \begin{cases} 1 - x_1 = \frac{\theta - p_B}{t}, & \text{if } p_A + p_B \leq 2\theta - t, \text{ i.e., } x_2 \geq x_1 \\ 1 - \hat{x} = \frac{p_A - p_B + t}{2t}, & \text{if } p_A + p_B \geq 2\theta - t, \text{ i.e., } x_2 = x_1. \end{cases}$$

Also, $d_i \in [0,1]$, $i = A, B$, a condition we have ignored so far. Until the proof of proposition 1, we implicitly assume that $d_i \in (0,1)$. We deal with the corner solutions in the proof of that proposition.

Note that the demand functions exhibit a kink at $p_A + p_B = 2\theta - t$, i.e., they are non-differentiable. For low prices, each firm is a local monopolist, in the sense that a firm's demand depends only on its own price. In this case, a reduction in price by one firm does not hurt the demand of the rival. Consumers do not switch brands, but simply more consumers find it advantageous to buy both brands (*demand creation* effect). As prices increase and after the kink of the demand function, firms compete head-on for consumers and a reduction in price induces some consumers to switch brands (*business stealing* effect).

We assume that firms have constant and equal marginal costs, which are normalized to zero. Thus, the profit functions are,

$$\pi_A = \begin{cases} \frac{(\theta - p_A)p_A}{t}, & \text{if } p_A + p_B \leq 2\theta - t \text{, i.e., } x_2 \geq x_1 \\ \frac{(p_B - p_A + t)p_A}{2t}, & \text{if } p_A + p_B \geq 2\theta - t \text{, i.e., } x_2 = x_1 \end{cases} \tag{6}$$

and

$$\pi_B = \begin{cases} \frac{(\theta - p_B)p_B}{t}, & \text{if } p_A + p_B \leq 2\theta - t \text{, i.e., } x_2 \geq x_1 \\ \frac{(p_A - p_B + t)p_B}{2t}, & \text{if } p_A + p_B \geq 2\theta - t \text{, i.e., } x_2 = x_1. \end{cases} \tag{7}$$

3 Analysis

First, note that the profit functions are not quasi-concave in the strategic variable. Figure 2, depicts the two functions, as given by (6), when $p_B = 6$, $\theta = 5$ and $t = 1$.

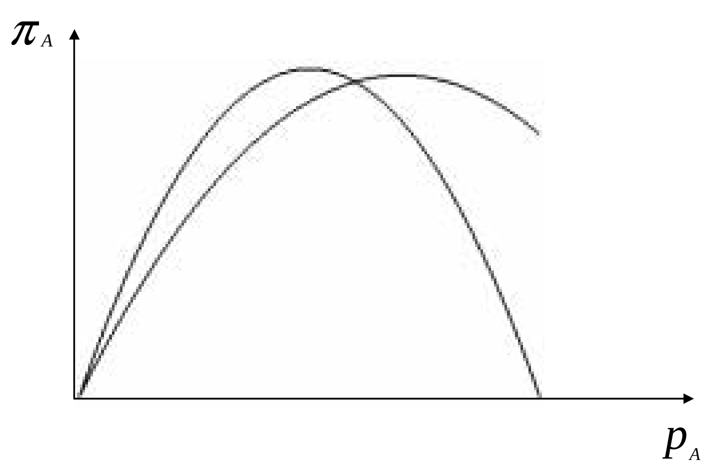

Figure 2: Firm A's profit function.

The profit function is the upper envelope of these two functions and it is clearly not quasi-concave. Therefore, the assumptions of Kakutani's fixed point theorem are not satisfied. In particular, the best replies may not be convex-valued. Nevertheless, the game is supermodular[10] and therefore the best replies are increasing. Hence, an equilibrium in pure strategies must exist [see Vives (1999, Theorem 2.5, p.33)]. The analysis which ensues verifies this and furthermore characterizes the equilibrium of the game completely.

We differentiate (6) and (7) with respect to p_A and p_B respectively, set the derivative equal to zero and solve with respect to each firm's strategic variable. This yields,

$$p_A = \begin{cases} \frac{\theta}{2}, & \text{if } p_A + p_B \leq 2\theta - t, \text{ i.e., } x_2 \geq x_1 \\ \frac{p_B + t}{2}, & \text{if } p_A + p_B \geq 2\theta - t, \text{ i.e., } x_2 = x_1, \end{cases}$$

and

$$p_B = \begin{cases} \frac{\theta}{2}, & \text{if } p_A + p_B \leq 2\theta - t, \text{ i.e., } x_2 \geq x_1 \\ \frac{p_A + t}{2}, & \text{if } p_A + p_B \geq 2\theta - t, \text{ i.e., } x_2 = x_1. \end{cases}$$

Let's look at firm A. Firm B's problem will be symmetric. Fix p_B. Firm A has two choices: i) to set $p_A = \frac{\theta}{2}$, provided that $p_A + p_B \leq 2\theta - t$, or ii) to set $p_A = \frac{p_B + t}{2}$, provided that $p_A + p_B \geq 2\theta - t$. The first choice yields profits equal to $\frac{\theta^2}{4t}$ and is valid for,

$$x_2 \geq x_1 \Longleftrightarrow \frac{\theta}{2} + p_B \leq 2\theta - t \Longrightarrow p_B \leq \frac{3\theta}{2} - t.$$

The second choice yields profits equal to $\frac{(p_B + t)^2}{8t}$ and is valid for,

$$x_1 = x_2 \Longleftrightarrow \frac{p_B + t}{2} + p_B \geq 2\theta - t \Longrightarrow p_B \geq \frac{4\theta}{3} - t.$$

Hence for $p_B \in \left[\frac{4\theta}{3} - t, \frac{3\theta}{2} - t\right]$ both choices satisfy the requirements (assuming that $\frac{4\theta}{3} \geq t$). In this case the best response is the one which yields the higher profits. It can be shown that the first choice yields higher profits when $p_B \leq \sqrt{2}\theta - t$, while when $p_B \geq \sqrt{2}\theta - t$, the second choice yields higher profits. Therefore, the best reply correspondences are,

$$p_A = \begin{cases} \frac{\theta}{2}, & \text{if } p_B \leq \sqrt{2}\theta - t, \text{ i.e., } x_2 \geq x_1 \\ \frac{p_B + t}{2}, & \text{if } p_B \geq \sqrt{2}\theta - t, \text{ i.e., } x_2 = x_1, \end{cases} \tag{8}$$

and

$$p_B = \begin{cases} \frac{\theta}{2}, & \text{if } p_A \leq \sqrt{2}\theta - t, \text{ i.e., } x_2 \geq x_1 \\ \frac{p_A + t}{2}, & \text{if } p_A \geq \sqrt{2}\theta - t, \text{ i.e., } x_2 = x_1. \end{cases} \tag{9}$$

[10] See Vives (1999, 2.2.3) for a definition of a supermodular game. It can be easily checked that our game satisfies the conditions of a supermodular game.

Figure 3 presents firm A's best reply correspondence as it is given by Eq.(8). For p_B's less than $\sqrt{2}\theta - t$, firm A's best response is to charge $p_A = \frac{\theta}{2}$. This is the region where firm A has a local monopoly and some consumers purchase both brands. At $p_B = \sqrt{2}\theta - t$, firm A has two optimal prices, $\frac{\theta}{2}$ and $\frac{\sqrt{2}\theta}{2}$. At this point a firm has two equally profitable strategies: i) to offer its product at a low price and sell even to those consumers whose preferences for its brand are not so strong, or ii) to increase the price and focus on the more loyal group of consumers. For any $p_B > \sqrt{2}\theta - t$, firm A's best response is $p_A = \frac{p_B + t}{2}$. This is the region where firms compete head-on for consumers and no consumer buys from both firms. The best reply correspondence indeed never jumps down.

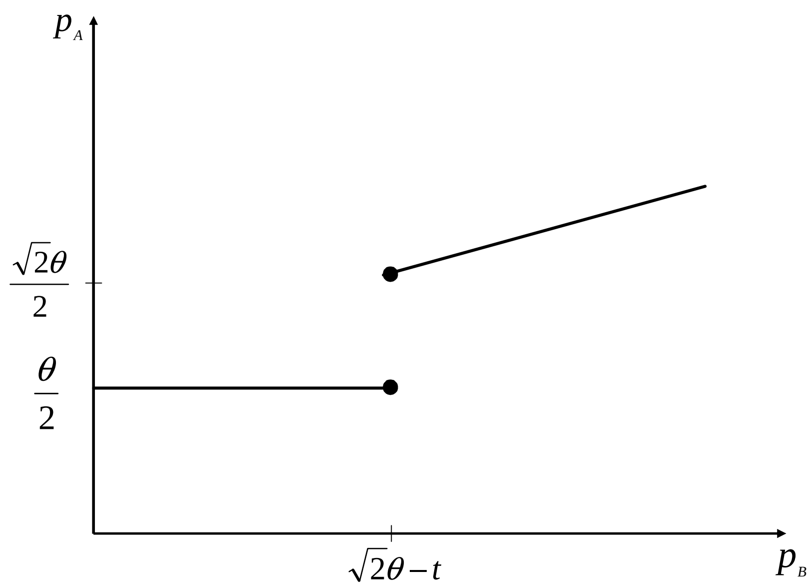

Figure 3: Firm A's best reply correspondence.

Firm B's best reply correspondence can be obtained in an analogous manner. The next proposition summarizes the equilibrium.

Proposition 1. *The Nash equilibrium prices and profits are as follows:*

1. *If* $0 \le \theta < \frac{2t}{2\sqrt{2}-1}$, *then the unique equilibrium is,*

$$(p_A, p_B) = (t, t) \text{ and } (\pi_A, \pi_B) = \left(\frac{t}{2}, \frac{t}{2}\right).$$

2. *If* $\frac{2t}{2\sqrt{2}-1} \le \theta \le \sqrt{2}t$, *then there are two equilibria,*

$$(p_A, p_B) = (t, t) \text{ and } (\pi_A, \pi_B) = \left(\frac{t}{2}, \frac{t}{2}\right),$$

and

$$(p_A,\, p_B) = \left(\frac{\theta}{2},\, \frac{\theta}{2}\right) \text{ and } (\pi_A,\, \pi_B) = \left(\frac{\theta^2}{4t},\, \frac{\theta^2}{4t}\right).$$

3. *If* $\sqrt{2}t < \theta \leq 2t$, *then the unique equilibrium is,*

$$(p_A,\, p_B) = \left(\frac{\theta}{2},\, \frac{\theta}{2}\right) \text{ and } (\pi_A,\, \pi_B) = \left(\frac{\theta^2}{4t},\, \frac{\theta^2}{4t}\right).$$

4. *If* $\theta > 2t$, *then the unique equilibrium is,*

$$(p_A,\, p_B) = (\theta - t,\, \theta - t) \text{ and } (\pi_A,\, \pi_B) = (\theta - t,\, \theta - t)\,.$$

Proof. We know that $\theta \in [0,\, V(1)]$. This interval can be divided into four regions. The proof will be based on the best reply correspondences.

- Case 1: $0 \leq \theta < \frac{2t}{2\sqrt{2}-1}$.

It can be easily calculated that,

$$t > \frac{\sqrt{2}\theta}{2} \geq \frac{\theta}{2} > \sqrt{2}\theta - t.$$

Figure 4 represents the equilibrium. The Nash equilibrium is,

$$(p_A,\, p_B) = (t,\, t)\,.$$

The equilibrium profits are,

$$(\pi_A,\, \pi_B) = \left(\frac{t}{2},\, \frac{t}{2}\right).$$

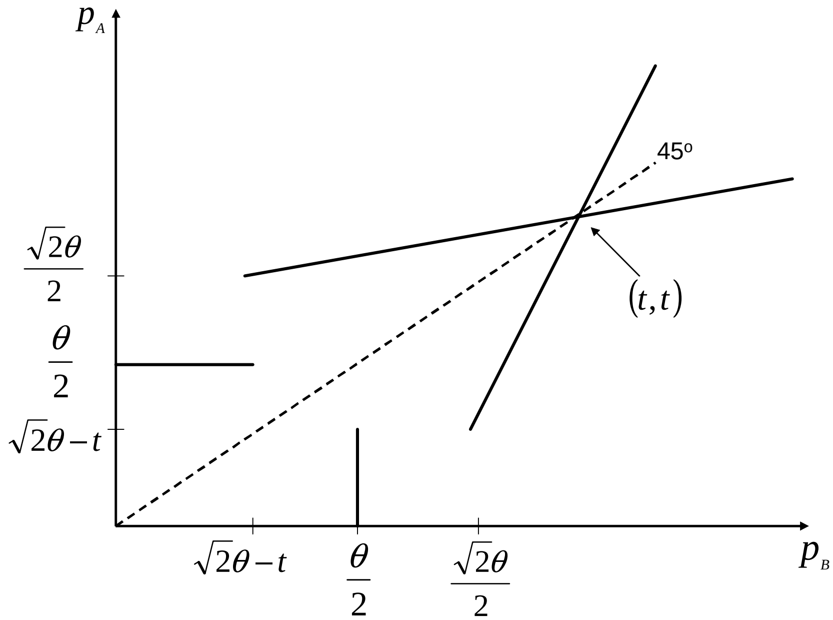

Figure 4

This is the standard Hotelling equilibrium. For this equilibrium to be valid it must be that $x_2 = x_1$, or from (3) we must have, $x_2 = x_1 \iff p_A + p_B \geq 2\theta - t$. This holds since, $t + t > 2\theta - t \implies \frac{3t}{2} > \theta$ a condition which is satisfied in this case. Therefore, no consumer buys from both firms and each firm's market share is $\frac{1}{2}$. It can be easily checked that each consumer's indirect utility is positive provided that $V(1) \geq \frac{3t}{2}$.

- Case 2: $\frac{2t}{2\sqrt{2}-1} \leq \theta \leq \sqrt{2}t$.

It can be easily calculated that,

$$t \geq \frac{\sqrt{2}\theta}{2} \geq \sqrt{2}\theta - t \geq \frac{\theta}{2}.$$

Figure 5 represents the equilibria. There are two Nash equilibria,

$$(p_A, p_B) = (t, t) \text{ and } (p_A, p_B) = \left(\frac{\theta}{2}, \frac{\theta}{2}\right).$$

The associated equilibrium profits are,

$$(\pi_A, \pi_B) = \left(\frac{t}{2}, \frac{t}{2}\right) \text{ and } (\pi_A, \pi_B) = \left(\frac{\theta^2}{4t}, \frac{\theta^2}{4t}\right).$$

It can be easily calculated that the first equilibrium yields higher profits provided that $\theta \leq \sqrt{2}t$, a condition that is satisfied in this case.

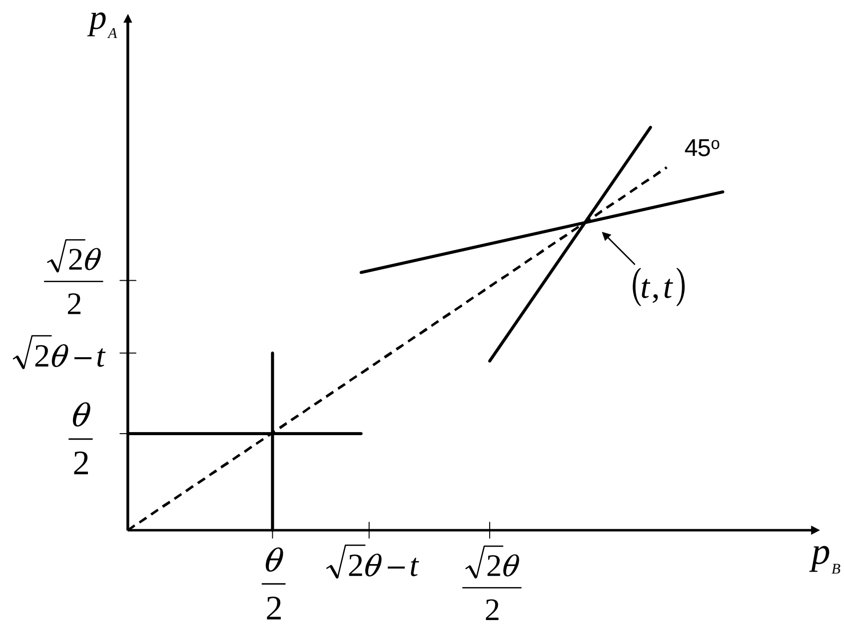

Figure 5

The first equilibrium, following the same logic as in case 1, is valid. For the second one to be valid it must be that $x_2 \geq x_1$. Given the equilibrium prices, and using (1) and (2), we obtain the equilibrium cutoffs,

$$x_1^* = \frac{t - \theta + \frac{\theta}{2}}{t} = \frac{2t - \theta}{2t}, \tag{12}$$

and

$$x_2^* = \frac{\theta - \frac{\theta}{2}}{t} = \frac{\theta}{2t}. \tag{13}$$

Note that $x_2^* > x_1^*$ provided that $\theta > t$ a condition which is satisfied. Hence, the consumers in (x_1^*, x_2^*) buy from both firms. Moreover, if $\theta < 2t$, then $x_1^* > 0$ and $x_2^* < 1$ (the same applies to case 3 below). In the second equilibrium each consumer enjoys a positive indirect utility provided that $V(1) \geq t$.

- Case 3: $\sqrt{2}t < \theta \leq 2t$.

It can be easily calculated that,

$$\sqrt{2}\theta - t > \frac{\sqrt{2}\theta}{2} > t \geq \frac{\theta}{2}.$$

Figure 6 represents the equilibrium. There is one Nash equilibrium,

$$(p_A,\, p_B) = \left(\frac{\theta}{2}, \frac{\theta}{2}\right).$$

The associated equilibrium profits are,

$$(\pi_A,\, \pi_B) = \left(\frac{\theta^2}{4t}, \frac{\theta^2}{4t}\right).$$

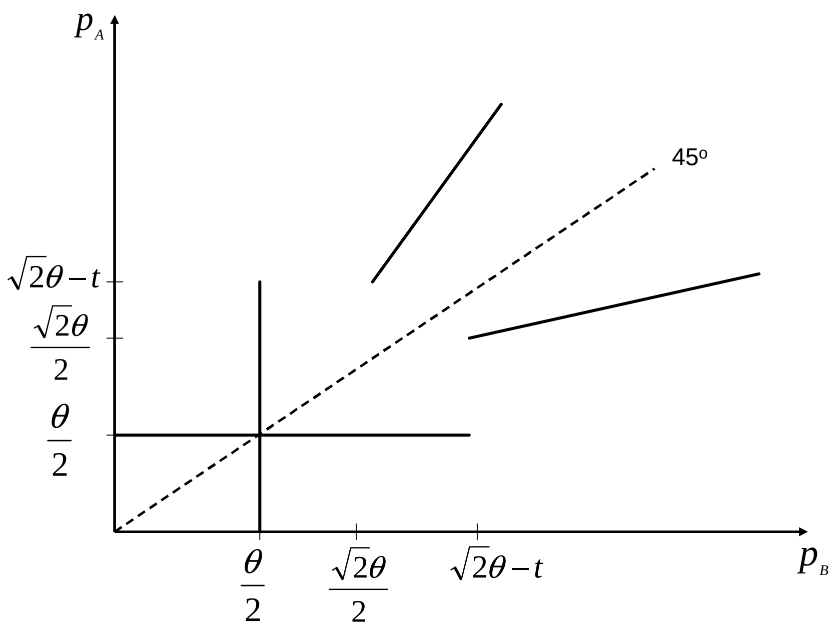

Figure 6

- Case 4: $\theta > 2t$.

This is the boundary case. All consumers buy both products. Firms set their prices such that $x_2 = 1$ and $x_1 = 0$. Using (1) and (2), this yields,

$$(p_A,\, p_B) = (\theta - t,\, \theta - t)\,.$$

Since all consumers buy both products the equilibrium profits are the same as the prices,

$$(\pi_A,\, \pi_B) = (\theta - t,\, \theta - t)\,.$$

Each consumer enjoys a positive indirect utility, provided that $\theta \le V(1) + t$. This inequality is satisfied given our assumption of diminishing marginal utility. ¥

From proposition 1, when $\frac{2t}{2\sqrt{2}-1} \leq \theta \leq \sqrt{2}t$, there are two Nash equilibria in pure strategies, $(p_A, p_B) = (t, t)$ and $(p_A, p_B) = \left(\frac{\theta}{2}, \frac{\theta}{2}\right)$. Moreover, the first one is the Pareto better equilibrium and consequently it will be the one preferred by the firms. This is a property of supermodular games, when the payoff to a player is increasing in the strategy of the other player, as it is the case in our model [see Vives, Remark 14, p.34]. We assume that firms are able to coordinate their play on the better equilibrium. To summarize, the non-cooperative outcome is: i) if $0 \leq \theta \leq \sqrt{2}t$, then $(p_A, p_B) = (t, t)$, $(\pi_A, \pi_B) = \left(\frac{t}{2}, \frac{t}{2}\right)$ and $x_1^* = x_2^* = \frac{1}{2}$ [*no consumer purchases both brands*], ii) if $\sqrt{2}t < \theta \leq 2t$, then $(p_A, p_B) = \left(\frac{\theta}{2}, \frac{\theta}{2}\right)$, $(\pi_A, \pi_B) = \left(\frac{\theta^2}{4t}, \frac{\theta^2}{4t}\right)$ and [from (12) and (13)] $0 < x_1^* = \frac{2t-\theta}{2t} < x_2^* = \frac{\theta}{2t} < 1$ [*some consumers purchase both brands*] and iii) if $\theta > 2t$, then $(p_A, p_B) = (\theta - t, \theta - t)$, $(\pi_A, \pi_B) = (\theta - t, \theta - t)$ and $0 = x_1^* < x_2^* = 1$ [*all consumers consume both brands*].

There are two types of equilibria. The first is the standard Hotelling outcome where no consumer purchases from both firms. In the second type of equilibrium some consumers purchase both brands. Each type of equilibrium behaves differently when products become more differentiated, i.e., as t increases. Profits associated with the first type of equilibrium increase, while, somewhat surprisingly, profits associated with the second type decrease. Moreover, in case 4 prices also decrease as products become more differentiated. In the second type of equilibrium, firms benefit when products move closer to each other. Demand increases since consumers find it less costly to consume one more product and for this to happen a firm does not have to lower its price. The consumers in (x_1, x_2) act as a buffer which lessens, up to a certain extent, the intensity of price competition. When a firm lowers its prices, for example, the demand for its product increases but not at the expense of its rival. Simply, more consumers find it beneficial to incur the incremental transportation cost and consume both products instead of one. This changes the nature of price competition.

Next, we calculate the socially optimal outcome and we compare it with the non-cooperative equilibrium.

3.1 Welfare analysis

The socially optimal outcome can be found as follows. Since a price is only a transfer and marginal cost is assumed to be zero, a social planner will choose the locations of the two marginal consumers to maximize the utility minus the transportation cost,

$$\begin{aligned} & \max_{(x_1,x_2)} \int_0^{x_1} [V(1) - tx]\, dx + \int_{x_1}^{x_2} [V(2) - t]\, dx + \int_{x_2}^{1} [V(1) - t(1-x)]\, dx \\ = & \max_{(x_1,x_2)} V(1)x_1 - \frac{tx_1^2}{2} + V(2)x_2 - V(2)x_1 + tx_1 - \frac{t}{2} - \frac{tx_2^2}{2} + V(1) - V(1)x_2. \qquad (14) \end{aligned}$$

The locations which maximize social welfare are,

$$x_1^{so} = \frac{t-\theta}{t} \text{ and } x_2^{so} = \frac{\theta}{t}. \tag{15}$$

Note that $x_2^{so} > x_1^{so}$ if and only if $\theta > \frac{t}{2}$, otherwise it is socially optimal that no consumer purchases both brands, i.e., $x_1 = x_2$. Next, we compare the non-cooperative outcome with the social optimum.

- $0 \leq \theta \leq \frac{t}{2}$.

The non-cooperative outcome is efficient. It is socially optimal that no consumer purchases both brands, which coincides with the non-cooperative outcome.

- $\frac{t}{2} < \theta \leq \sqrt{2}t$.

The non-cooperative outcome is inefficient. In the socially optimal outcome some consumers buy from both firms, but in the non-cooperative equilibrium each consumer still buys exclusively from one firm. Output is below its efficient level.

- $\sqrt{2}t < \theta < 2t$.

The non-cooperative outcome is inefficient. Although, in the non-cooperative outcome, some consumers consume both products now, they are fewer relative to the number that is socially desired, since $x_2^{so} > x_2^*$ and $x_1^{so} < x_1^*$. Again, output is below its efficient level.

- $\theta \geq 2t$.

The non-cooperative outcome is efficient. Efficiency dictates that all consumers should buy from both firms, a situation which is supported when firms behave non-cooperatively.

We see that for intermediate values of θ, there is a deadweight loss associated with the Nash equilibrium, unlike the standard Hotelling model which yields an efficient outcome (if the firm locations are fixed and the market is covered).

4 Product selection

We consider a two-stage game. In stage 1 firms choose their locations on the unit interval. Let a (the location of firm A) and b (the location of firm B) denote the distance from 0. We assume that $1 \geq b \geq a \geq 0$. In stage 2, they choose prices. As we showed in the previous sections, when firms are located at the two extremes, there are, in general, two types of equilibria: one where no consumer buys from both firms and the other where some consumers purchase both brands. The former case cannot yield an equilibrium for *any* a and b. As firms move close to the middle, a price equilibrium will not exist (see d'Aspermont *et al.*). Therefore, the only hope to find a pure strategy price equilibrium (for any a and b) is to focus on the latter case where some consumers buy from both firms, i.e., $x_2 > x_1$. This is what we do next. We then use the result of proposition 2 to argue that firms will agglomerate towards the center (*Principle of Minimum Differentiation*).[11]

Proposition 2. *A pure strategy price equilibrium exists for any* a and b if and only if $\theta \geq \hat{\theta} = \frac{2t}{2\sqrt{2}-1}$ (i.e., $\approx 1.0938t$). *The equilibrium prices and profits are,*

- *(interior solution; $x_2 < 1$) if $\theta < t(2-a)$, then,*

$$p_A = \frac{\theta + ta}{2} \text{ and } \pi_A = \frac{(\theta + ta)^2}{4t}$$

- *(corner solution; $x_2 = 1$) if $\theta \geq t(2-a)$, then,*

$$p_A = \theta - t\,(1-a) \text{ and } \pi_A = \theta - t\,(1-a)\,.$$

- *(interior solution; $x_1 > 0$) if $\theta < t(1+b)$, then,*

$$p_B = \frac{\theta + t\,(1-b)}{2} \text{ and } \pi_B = \frac{(\theta + t\,(1-b))^2}{4t}$$

- *(corner solution; $x_1 = 0$) if $\theta \geq t(1+b)$, then,*

$$p_B = \theta - tb \text{ and } \pi_B = \theta - tb.$$

The locations of the marginal consumers are given by,

$$0 \leq x_1 = \frac{tb - \theta + t}{2t} < x_2 = \frac{\theta + ta}{2t} \leq 1.$$

Moreover, the equilibrium we have described above is unique for any a and b if and only if $\theta > \bar{\theta} = \sqrt{2}t$.

[11] If a consumer who is located at x, with $x < a < b$, buys from both firms, the distance he travels is $(a-x)+(b-x)$. This is more consistent with the view that the distance is a measure of disutility, rather than a representation of geographical distance. Under the latter interpretation, it seems more reasonable to assume that the consumer could go first to the firm located at a and then go directly to the one located at b making the total distance that he has to travel equal to $(b-x)$ (although one can think of situations where this is not true). Previous models did not have to make this distinction, since consumers in those models buy from one firm exclusively.

Proof. See appendix. ¥

For θ's in $\left[\hat{\theta},\, \bar{\theta}\right)$, the second stage price equilibrium that is presented in proposition 2, is not unique for all a and b. For example, as we know from proposition 1, when $a = 0$ and $b = 1$, $(t,\, t)$ is another equilibrium. Nevertheless, as we argued at the beginning of this section, this type of outcome which yields exclusive purchases, cannot be an equilibrium for any a and b. That is why we have focused on the outcome which yields some non-exclusive purchases and guarantees a price equilibrium regardless of the locations of the two rivals. Moreover, this equilibrium becomes unique, for any a and b, when θ exceeds $\bar{\theta}$. This is because a high θ makes a deviation, from a pair of prices which induces exclusive sales only to one where some consumers purchase both brands, profitable for any location configuration and not only when firms are positioned close to each other (as it is the case when θ is relatively low).

Now we move up to stage 1 where firms choose their locations. Since $\frac{d\pi_A}{da} > 0$ and $\frac{d\pi_B}{db} < 0$, firms will have the tendency to agglomerate in the middle, i.e., $a = b = \frac{1}{2}$. The intuition is as follows. When a firm is moving towards the middle, it increases its demand by making its product more attractive to those who prefer the rival's brand more. This, however, does not lead to an all-out competition, since the increase in demand does not automatically imply lower sales for the rival firm, provided of course that consumers value variety, i.e., the incremental utility is above a fixed threshold. As a consequence the rival has no incentive to lower its price since its customers are not switching brands but simply are buying both, and in a way it accommodates the firm's movement towards the center.

From (A21), the marginal consumers (when $a = b = \frac{1}{2}$) are located at,

$$x_1^{**} = \frac{\frac{3t}{2} - \theta}{2t} \text{ and } x_2^{**} = \frac{\theta + \frac{t}{2}}{2t}. \tag{16}$$

Note that if $\theta < \frac{3t}{2}$, then $x_1^{**} > 0$ and $x_2^{**} < 1$. If $\theta \geq \frac{3t}{2}$, then $x_1^{**} = 0$ and $x_2^{**} = 1$, i.e., all consumers purchase both brands. The socially optimal locations are given by (15). As we saw in section 3.1, when firms are located at the extremes, $x_1^{so} \leq x_1^*$ and $x_2^{so} \geq x_2^*$ and the non-cooperative outcome is (weakly) inefficient. Moreover, $x_1^{so} \leq x_1^{**} \leq x_1^*$ and $x_2^{so} \geq x_2^{**} \geq x_2^*$. Since the welfare function [see (14)] is concave in the locations, efficiency improves when firms are free to choose their positions on the unit interval (i.e., more output is produced).

5 Conclusion

We introduce a model of differentiated products with two firms, each producing one brand. Consumers cannot buy more than one unit from each brand, but they can purchase two brands, one from each firm. This model is capable of generating a very realistic equilibrium where some consumers remain loyal to one brand, while another group of consumers consumes both brands. Products that fit this description include, credit cards, software, subscriptions to magazines, newspapers, scholarly journals and TV channels. First, we ignore the issue of product selection, by fixing the firms' locations at the two extremes of the unit interval. There are two types of equilibria: i) the standard Hotelling one, where no consumer buys from both firms and equilibrium profits decrease as products become less differentiated (i.e, as the transportation cost parameter t decreases) and ii) an equilibrium where some consumers purchase from both firms and equilibrium profits increase as products become less differentiated. When the magnitude of the incremental utility from purchasing both brands is low, then the equilibrium is of the first type. For medium values of the incremental utility both types of equilibria emerge and for high values of the incremental utility the unique equilibrium is of the second type.

Then, we allow the firms to choose their locations on the horizontal dimension. In particular, we analyze a two-stage game where firms position themselves in stage 1 and in stage 2 they compete in prices. If the incremental utility exceeds a fixed threshold, then a price equilibrium in pure strategies (with a linear transportation cost) exists regardless of the firm locations. Equilibrium profits monotonically increase as firms move towards the center. Hence, this model restores Hotelling's *Principle of Minimum Differentiation*. It also provides a new explanation as to why retailers, in many cases, choose to locate very close to each other [e.g. big shopping centers and malls].

The model presented in this paper can be extended to shed light on the issue of product variety in a monopolistically competitive setting. When competition is localized, as in Salop's circular model [Salop (1979)], then the number of brands (firms) is excessive from the social point of view. The reason is the lack of global competition, which tends to boost profits and consequently entry. When consumers can buy from more than one firm, as we have postulated in this paper, then firms compete not only with their neighbors, but also with rivals which are not adjacent to them. This retains the spatial differentiation aspect, but injects into the model a realistic dose of non-localized competition. This can potentially change the qualitative features of the equilibrium.

APPENDIX

Proof of proposition 2. We search for an equilibrium where some consumers buy both brands, i.e., $x_2 > x_1$. As in the proof of proposition 1, we assume that $V(1)$ is sufficiently high so that every consumer buys at least one brand. The proof is divided into two parts. First we show that a price equilibrium exists (for any a and b) if and only if $\theta \geq \hat{\theta}$. Second we show that this equilibrium is unique (for any a and b) if and only if $\theta > \bar{\theta}$.

EXISTENCE. The marginal consumer x_1 satisfies,

$$V(1) - t\,|a - x_1| - p_A = V(2) - t\,|a - x_1| - t\,|b - x_1| - p_A - p_B.$$

First note that $b \geq x_1$. To see this, suppose by way of contradiction that $b < x_1$. The consumer who is located at x_1 is, by definition, indifferent between purchasing brand A exclusively and purchasing both brands, i.e.,

$$\begin{aligned} V(1) - t\,(x_1 - a) - p_A &= V(2) - t\,(x_1 - a) - t\,(x_1 - b) - p_A - p_B \Longrightarrow \\ V(1) &= V(2) - t\,(x_1 - b) - p_B. \end{aligned} \tag{*}$$

Consider now a consumer located at $x \in (x_1, x_2)$ (recall that we maintain the assumption that $x_2 > x_1$). This consumer must strictly prefer to purchase both brands, i.e.,

$$\begin{aligned} V(2) - t\,(x - a) - t\,(x - b) - p_A - p_B &> V(1) - t\,(x - a) - p_A \Longrightarrow \\ V(1) &< V(2) - t\,(x - b) - p_B. \end{aligned} \tag{**}$$

By combining $(*)$ and $(**)$ we can conclude that,

$$V(2) - t\,(x - b) - p_B > V(2) - t\,(x_1 - b) - p_B \Longrightarrow x < x_1,$$

a contradiction to the assumption that $x \in (x_1, x_2)$.

Then, the first marginal consumer, x_1, is given by the following expression,

$$x_1 = \frac{tb - \theta + p_B}{t}. \tag{A1}$$

The second marginal consumer, x_2, satisfies,

$$V(1) - t\,|b - x_2| - p_B = V(2) - t\,|b - x_2| - t\,|a - x_2| - p_A - p_B.$$

By following the same steps as the ones to prove that $b \geq x_1$, we can show that $x_2 \geq a$. Then,

$$x_2 = \frac{\theta + ta - p_A}{t}. \tag{A2}$$

Note that as long as $x_2 > x_1$ the indifferent consumers always have measure zero. Hence, we avoid the problem that Hotelling faced, which caused the non-existence of price equilibrium.[12] Note that,

$$x_2 > x_1 \Longleftrightarrow p_A + p_B < 2\theta - t\,(b-a)\,. \tag{A3}$$

Also,

$$x_1 > 0 \Longleftrightarrow p_B > \theta - tb, \tag{A4}$$

and

$$x_2 < 1 \Longleftrightarrow p_A > \theta - t\,(1-a)\,. \tag{A5}$$

Firm A's demand function is $d_A = \frac{\theta + ta - p_A}{t}$ and firm B's is $d_B = \frac{\theta + t(1-b) - p_B}{t}$. The profit functions are, $\pi_A = p_A d_A$ and $\pi_B = p_B d_B$.

Interior solution. Assuming that $x_1 > 0$ and $x_2 < 1$, the prices which maximize the profit functions are,

$$p_A = \frac{\theta + ta}{2} \text{ and } p_B = \frac{\theta + t\,(1-b)}{2}. \tag{A6}$$

The maximized profits are,

$$\pi_A = \frac{(\theta + ta)^2}{4t} \text{ and } \pi_B = \frac{(\theta + t\,(1-b))^2}{4t}. \tag{A7}$$

Based on the optimal prices, this case is valid provided that,

$$x_2 > x_1 \Longleftrightarrow \theta > \frac{t}{2} + \frac{t\,(b-a)}{2}. \tag{A8}$$

This case is guaranteed to hold, for any a and b, provided that $\theta > t$ (i.e., at $a = 0$ and $b = 1$ where $\frac{t}{2} + \frac{t(b-a)}{2}$ is maximized).

Moreover,

$$x_1 > 0 \Longleftrightarrow \theta < t(1+b) \tag{A9}$$

and

$$x_2 < 1 \Longleftrightarrow \theta < t\,(2-a)\,. \tag{A10}$$

[12] Nevertheless, the indifferent consumers may have a strictly positive measure in the deviation we consider later in this proof, where $x_1 = x_2$.

Corner solution. If $\theta \geq t(1+b)$, then $x_1 = 0$ and if $\theta \geq t(2-a)$ then $x_2 = 1$. The optimal prices are found by setting (A1) and (A2) equal to 0 and 1 respectively and solving with respect to prices. This yields,

$$p_A = \theta - t(1-a) \text{ and } p_B = \theta - tb. \tag{A11}$$

The profits are,

$$\pi_A = \theta - t(1-a) \text{ and } \pi_B = \theta - tb. \tag{A12}$$

Now let's look at firm A's deviation. Firm B's deviation problem will be symmetric. First, we begin from the interior pre-deviation solution. Firm B's price is fixed at $\hat{p}_B = \frac{\theta + t(1-b)}{2}$ [see (A6)]. Clearly, firm A would not find it profitable to deviate by lowering its price, since that would still yield $x_2 > x_1$ [see (A3) and (A8)] and under this assumption (A6) is optimal. Hence, the only other deviation for firm A is to increase its price so that $x_1 = x_2$ (i.e., no consumer buys from both firms). This is the standard Hotelling case. When $x_1 = x_2$, firm A's demand function is given by,

$$d_A = \begin{cases} 1, & \text{if } p_A < \hat{p}_B - t(b-a), \text{ or } p_A < \frac{\theta+t-3tb}{2} + ta \\ \frac{\hat{p}_B - p_A + t(a+b)}{2t}, & \text{if } |p_A - \hat{p}_B| \leq t(b-a), \text{ or } p_A \in \left[\frac{\theta+t-3tb}{2} + ta, \frac{\theta+t+tb}{2} - ta\right] \\ 0, & \text{if } p_A > \hat{p}_B + t(b-a), \text{ or } p_A > \frac{\theta+t+tb}{2} - ta. \end{cases} \tag{A13}$$

Denote the two thresholds (where the discontinuities occur) by,

$$thresh1 = \frac{\theta + t - 3tb}{2} + ta \text{ and } thresh2 = \frac{\theta + t + tb}{2} - ta. \tag{A14}$$

Clearly, $thresh2 \geq thresh1$. The deviation price, p_A^d, must satisfy the following two constraints.

First, it must yield an outcome where $x_2 = x_1$, or in other words,

$$p_A^d + \hat{p}_B \geq 2\theta - t(b-a) \Longleftrightarrow p_A^d \geq thresh3 = \frac{3\theta - t - tb}{2} + ta. \tag{A15}$$

Note that $thresh1$ becomes now irrelevant if $\theta > t$. This is due to the fact that,

$$thresh3 > thresh1 \Longleftrightarrow \theta > t - tb. \tag{A16}$$

Hence, if $\theta > t$ (i.e., at $b = 0$, where $t - tb$ is maximized) and since as we argued above, firm A should not lower its price, the first threshold (i.e., $thresh1$) will never be crossed when firm A deviates by increasing its price.

Second, $p_A^d \leq thresh2$, since otherwise deviation profits will be zero. Thus, a deviation price must satisfy,

$$p_A^d \in [thresh3, thresh2]. \tag{A17}$$

Of course, a deviation will be meaningful only if $thresh2 \geq thresh3$. It turns out that $thresh2 \geq thresh3$, if and only if $\theta \leq t + tb - 2ta$.

Therefore, firms A's demand following its deviation is given by $\frac{\hat{p}_B - p_A + t(a+b)}{2t}$ [i.e., the second part of (A13)]. The deviation profit function is,

$$\pi_A^d = \frac{\left[\hat{p}_B - p_A^d + t\,(a+b)\right] p_A^d}{2t}. \tag{A18}$$

By solving the foc we obtain,

$$p_A^d = \frac{\theta + t + tb + 2ta}{4}. \tag{A19}$$

If $\theta \leq \frac{3t+3tb-2ta}{5}$, then $p_A^d \geq thresh3$. Moreover, if $\theta \geq 6ta - t - tb$, then $p_A^d \leq thresh2$.

First, we compare the highest deviation profits with the ones before deviation [i.e., (A20) with (A7)] ignoring the constraints [i.e., (A17)] that the deviation price, p_A^d, must satisfy (see case 1). We find a threshold, $\hat{\theta}$, such that for any θ above $\hat{\theta}$, this unconstrained deviation is unprofitable for any a and b. Clearly, this threshold is an upper bound. Then, we show that it is also the lower bound. We do so, by incorporating the constraints that p_A^d must satisfy (see cases 2 and 3) and we show that for θ's below $\hat{\theta}$ a deviation is profitable for some values of a and b.

Case 1. Using (A18) and (A19), the maximized deviation profits are,

$$\pi_A^d = \frac{(\theta + t + tb + 2ta)^2}{32t}. \tag{A20}$$

Deviation is profitable, i.e., $(A20) > (A7)$, if,

$$\theta \in \left(\frac{\left[-6a + 1 + b - 2\sqrt{2}\,(1 + a + b)\right] t}{7}, \frac{\left[-6a + 1 + b + 2\sqrt{2}\,(1 + a + b)\right] t}{7} \right).$$

Zero belongs in this interval for any permissible values of a and b. Therefore, deviation is not profitable if,

$$\theta \geq \frac{\left[-6a + 1 + b + 2\sqrt{2}\,(1 + a + b)\right] t}{7}.$$

Note that when $a = 0$ and $b = 1$, the above condition boils down to $\theta \geq \frac{2t}{2\sqrt{2}-1}$, which is the same as the condition for the existence of the second type of equilibrium in proposition 1. Moreover, it can be easily checked that $\frac{\left[-6a+1+b+2\sqrt{2}(1+a+b)\right]t}{7}$ increases as a decreases and as b increases. Hence, its highest value is attained if we set $a = 0$ and $b = 1$ and is equal to $\frac{2t}{2\sqrt{2}-1}$. This implies that if,

$$\theta \geq \hat{\theta} = \frac{2t}{2\sqrt{2} - 1},$$

then the pair of prices given by (A6), constitute an equilibrium regardless of the firms' locations.

Case 2. Suppose first that $\theta < 6ta - t - tb$. In this case, $p_A^d = thresh2$. Note that $\min_{(a,b)} 6ta - t - tb = -2t$, at $(a = 0,\ b = 1)$ and $\max_{(a,b)} 6ta - t - tb = 4t$, at $(a = b = 1)$.

Case 3. Suppose second that $\theta > \frac{3t+3tb-2ta}{5}$. In this case, $p_A^d = thresh3$. Note that $\min_{(a,b)} \frac{3t+3tb-2ta}{5} = \frac{3t}{5}$, at $(a = 0,\ b = 0)$ and $\max_{(a,b)} \frac{3t+3tb-2ta}{5} = \frac{6t}{5}$, at $(a = 0,\ b = 1)$.

By combining the above three cases and noting that $\frac{2t}{2\sqrt{2}-1} > t$ (so that p_A^d never falls below $thresh1$), we conclude that the pair of prices given by (A6), constitute an equilibrium for any permissible values that a and b might take if and only if $\theta \geq \hat{\theta}$. The if part is proved in case 1. The only if part can be seen as follows. First of all, we know from case 1 that if $\theta < \hat{\theta}$, $a = 0$ and $b = 1$, then an unconstrained deviation is profitable. Nevertheless, it could be the case that one of the two constraints that p_A^d must satisfy [i.e., (A17)] is violated and this is not taken into consideration in case 1. This is what we check next. Suppose that $\theta < \hat{\theta}$, $a = 0$ and $b = 1$. From cases 2 and 3, it follows that if $\theta \in \left(-2t,\ \frac{6t}{5}\right)$, then the deviation price satisfies the constraints and the pre-deviation solution, from (A9) and (A10), is interior. Hence, and since $-2t < t < \frac{2t}{2\sqrt{2}-1} < \frac{6t}{5}$, for those θ's less than $\frac{2t}{2\sqrt{2}-1}$ and greater than t a deviation price indeed satisfies the constraints. This should also be true for a's and b's in the neighborhood of 0 and 1 respectively. Therefore, if $\theta < \hat{\theta}$, for some a's and b's, (A6) is not an equilibrium.

Next, we check the profitability of firm A's deviation when the pre-deviation solution is corner. For this to be the case, it must be that $\theta \geq t(2-a)$ [see (A10)]. The pre-deviation profits are given by (A12). First observe, as we discussed above, that as long as $\theta > t - tb$, then $thresh3 > thresh1$ [see (A16)]. As θ increases $\hat{p}_B$ increases as well until it reaches the boundary solution. The boundary solution is reached when $\theta \geq t(1+b)$ [see (A9)]. Since $t(1+b) > t - tb$, then $thresh3 > thresh1$ even when $\hat{p}_B$ is at the boundary. In addition, $t(2-a) > t - tb$, so when the pre-deviation solution for firm A is corner, then $thresh3 > thresh1$. Hence, we can ignore the first part of the demand function [see (A13)] and focus on the second part of it. We look at the unconstrained deviation as it is given by (A20). Deviation is profitable (i.e., (A20) vs. π_A from (A12)) if,

$$\theta < 15t - tb - 2ta - 4t\sqrt{12 - 2b - 2a}.$$

It turns out that $t(2-a) > 15t - tb - 2ta - 4t\sqrt{12 - 2b - 2a}$, for any a and b. Therefore a deviation from a corner solution will never be profitable.

The locations of the two marginal consumers, after plugging (A6) into (A1) and (A2), are given by,

$$0 \leq x_1 = \frac{tb - \theta + t}{2t} < x_2 = \frac{\theta + ta}{2t} \leq 1. \tag{A21}$$

UNIQUENESS. Now we show that there does not exist an equilibrium with $x_1 = x_2$, if and only if $\theta > \bar{\theta} = \sqrt{2}t$. Let x be the consumer who is indifferent between buying from firm A and from firm B. The firms' demand functions are given by,

$$d_A = \begin{cases} 1, & \text{if } p_A < p_B - t\,(b-a), \\ \frac{p_B - p_A + t(a+b)}{2t}, & \text{if } |p_A - p_B| \leq t\,(b-a), \\ 0, & \text{if } p_A > p_B + t\,(b-a), \end{cases} \tag{A22}$$

and

$$d_B = \begin{cases} 1, & \text{if } p_B < p_A - t\,(b-a), \\ \frac{p_A - p_B + t(2-a-b)}{2t}, & \text{if } |p_A - p_B| \leq t\,(b-a), \\ 0, & \text{if } p_B > p_A + t\,(b-a). \end{cases} \tag{A23}$$

The profit functions are $\pi_A = p_A d_A$ and $\pi_B = p_B d_B$. We begin by noting that an equilibrium must satisfy the condition $|p_A - p_B| < t\,(b-a)$ [for a proof see d'Aspermont *et al.* p.1147]. Hence, an equilibrium must maximize,

$$\pi_A = \frac{[p_B - p_A + t\,(a+b)]\,p_A}{2t} \text{ and } \pi_B = \frac{[p_A - p_B + t\,(2-a-b)]\,p_B}{2t}.$$

By taking first order conditions we obtain,

$$p_A = \frac{(2+a+b)\,t}{3} \text{ and } p_B = \frac{(4-a-b)\,t}{3}. \tag{A24}$$

The profits are,

$$\pi_A = \frac{t\,(2+a+b)^2}{18} \text{ and } \pi_B = \frac{t\,(4-a-b)^2}{18}. \tag{A25}$$

d'Aspermont *et al.*, p.1146, state that (A24) is not an equilibrium if and only if the two firms are located close to each other. The same applies to our model. It remains to be shown that, in our model, the pair of prices given by (A24) is not an equilibrium even when the two firms are not located close to each other. For this case to be valid, i.e., $x_2 = x_1$, it must be, from (A3) and after using (A24), that,

$$p_A + p_B \geq 2\theta - t\,(b-a) \Longleftrightarrow \theta \leq t\left(1 + \frac{b-a}{2}\right). \tag{A26}$$

First, we assume that (A26) is satisfied and we look at firm A's deviation from (A24). Fix firm B's price at $\hat{p}_B = \frac{(4-a-b)t}{3}$. In particular, we check the deviation which yields $x_2 > x_1$. For this to happen (A3) must first of all be satisfied. That is,

$$p_A^d + \hat{p}_B < 2\theta - t(b-a) \Longleftrightarrow p_A^d < 2\theta + \frac{t(4a-2b-4)}{3}. \tag{A27}$$

We know that if $x_2 > x_1$, then the unconstrained optimal price is given by $p_A = \frac{\theta+ta}{2}$ (see A6). It turns out that,

$$\frac{\theta+ta}{2} < 2\theta + \frac{t(4a-2b-4)}{3} \Longleftrightarrow \theta > \frac{t(4b-5a+8)}{9}.$$

At $b=1$ and $a=0$, $\frac{t(4b-5a+8)}{9}$ attains its maximum value of $\frac{4t}{3}$. Hence, if $\theta > \frac{4t}{3}$, then $p_A^d = \frac{\theta+ta}{2}$ and the deviation profits are given by (A7), i.e.,

$$\pi_A^d = \frac{(\theta+ta)^2}{4t}. \tag{A28}$$

Deviation is profitable, i.e., $\pi_A^d > \pi_A$ [(A28) vs. (A25)], if,

$$\theta > \frac{\left[-3a+\sqrt{2}(2+a+b)\right]t}{3}.$$

Now observe that $\max_{(a,b)} \frac{\left[-3a+\sqrt{2}(2+a+b)\right]t}{3} = \sqrt{2}t$ at $(a=0,\, b=1)$. Thus, if $\theta > \sqrt{2}t$ (which is also greater than $\frac{4t}{3}$), then such a deviation is guaranteed to be profitable for any a and b.

Second, we turn to the case where (A26) is not satisfied, i.e., $\theta > t\left(1+\frac{b-a}{2}\right)$. This implies that for the prices, as given by (A24), $p_A + p_B < 2\theta - t(b-a)$. In this case, an equilibrium must satisfy $p_A + p_B = 2\theta - t(b-a)$. By rewriting the equality as, $p_B - p_A = 2\theta - t(b-a) - 2p_A$, firm A's pre-deviation profit function becomes,

$$\pi_A = \frac{(\theta+ta-p_A)\,p_A}{t}.$$

The maximum possible profits for firm A are $\tilde{\pi}_A = \frac{(\theta+ta)^2}{4t}$ at $\tilde{p}_A = \frac{\theta+ta}{2}$. Let's first look at firm A's deviation which leads to $x_2 > x_1$. We know that when some consumers consume both brands the optimal price is $\frac{\theta+ta}{2}$. Hence, when firm A deviates from p_A, it must do so by lowering its price to $p_A^d = \frac{\theta+ta}{2}$, in order to enter the region where $x_2 > x_1$ (see figure A1). To this end, assume that $p_A > \frac{\theta+ta}{2}$ (southeast of point C in figure A1) and firm A deviates to $p_A^d = \frac{\theta+ta}{2}$. Such a deviation is profitable since,

$$\pi_A^d = \frac{(\theta+ta)^2}{4t} = \tilde{\pi}_A > \pi_A.$$

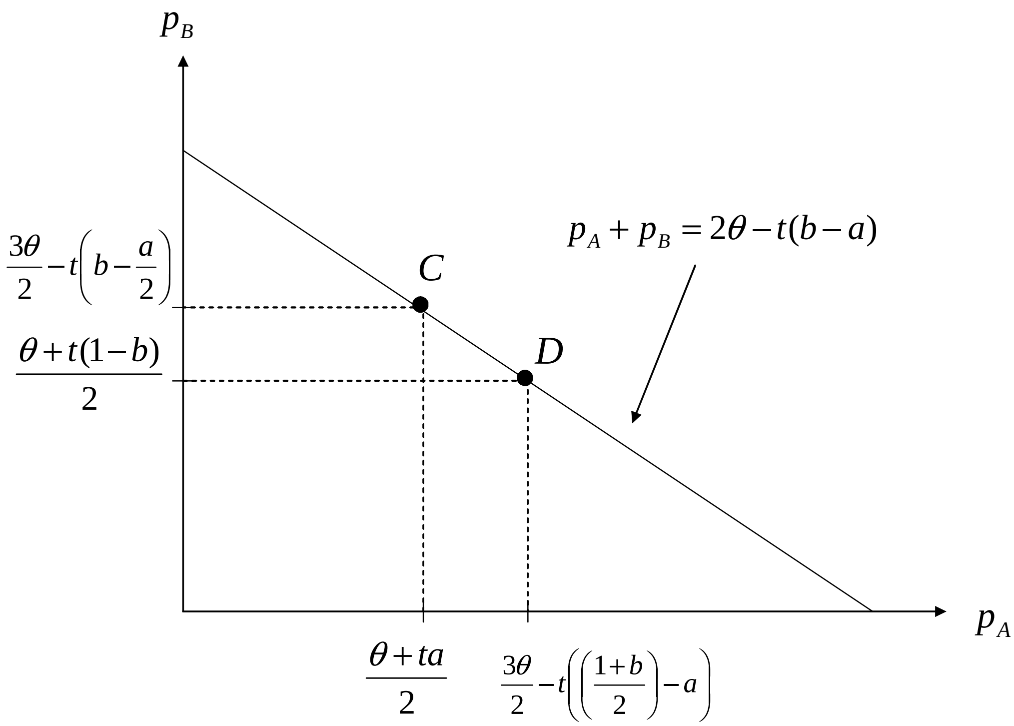

Figure A1

Similarly, we can show that firm B will deviate to $p_B^d = \frac{\theta+t(1-b)}{2}$, from any $p_B > p_B^d$ (northwest of point D in figure A1). This clearly covers all possible price pairs on the constraint $p_A + p_B = 2\theta - t(b-a)$ and therefore no pair of prices on this constraint is an equilibrium (note that point D is located southeast of point C provided that $\theta > \frac{tb-ta+t}{2}$, a condition which is clearly satisfied if $\theta > \sqrt{2}t$).

So far we demonstrated that if $\theta > \sqrt{2}t$, then no pair of prices that yields $x_1 = x_2$, constitutes an equilibrium for any a and b. We will show that this condition is also necessary. This follows easily from proposition 1 where we showed that if $\theta \leq \sqrt{2}t$, and $(a = 0,\ b = 1)$ an equilibrium where no consumers purchases both brands exists. ¥

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