Investment, Externalities and Industry Dynamics

(Very preliminary and incomplete)

Takashi Kamihigashi
Research Institute of Economics and Business
Kobe University, Kobe, Japan.
E-mail: tkamihig@rieb.kobe-u.ac.jp

Santanu Roy
Department of Economics
Southern Methodist University
Dallas, TX 75206.
E-mail: sroy@smu.edu

January 21, 2004

ABSTRACT. We provide an alternative theoretical explanation for a number of empirical regularities relating to the dynamics of industry structrure (product life cycle) and changes in size and age distribution of firms over time. We explain why entry may continue over a considerable period of time, why shake out of firms occur in mature industries and why exiting firms are likely to be younger and smaller in size than incumbents. Unlike the existing theoretical literature, this explanation is not based on uncertainty, structural non-stationarity or incomplete information. We consider an infinite horizon, complete information, deterministic competitive industry with continuum of firms and stationary market demand. Firms have perfect foresight, may enter or exit the industry at any point of time and active firms undertake investment which reduces their future cost of production. Investment by active firms also leads to the growth of an industry-wide capital that reduces production cost of all firms (externality). The marginal cost curves are upward sloping and firms incur a fixed cost of staying in the industry. While all entering firms earn zero intertemporal net profit, their instantaneous net profit is typically negative when they are young and strictly positive when they mature. Positive profits may persist in the long run. Equilibrium prices decline over time while the level of positive industry-wide externality increases with time.The equilibrium path makes firms indifferent between alternative entry and exit decisions and their investment levels after entry reflects their length of stay and the nature of industry environment (prices, externalities) over their period of stay in the industry. Heterogeneity emerges out of deliberate choice.

1 Introduction.

Economists have long recognized that the dynamics of industry structure as well as firm size & performance are closely related to changes in technology and productivity. The latter, in turn, are related to activities such as investment in learning-by-doing, cost reducing innovations and other forms of capital (including knowledge capital) that take place at the level of individual firms. While some of these activities clearly result in assets that are firm-specific (for example, organizational capital), others lead to creation of assets (such as knowledge) that spillover to other firms in the industry. The stocks of these various forms of capital and the intensity of productivity enhancing investments change over time, causing changes in the size or production "scale" of firms and thereby affecting prices & profitability in the industry. The latter, in turn, determine the incentives for entry and exit of firms and thus, the industry structure. In other words, the dynamics of industry structure, size & performance of firms and productivity changes in the industry are interlinked. In this paper, we analyze the dynamics of a competitive industry where firms may enter or exit the industry over time, carry out investment in capital that reduces firm specific production cost and, at the same times, generates positive industry-wide externalities.

It is now generally understood that industries experience very high turnover of firms and exhibit high degree of variance in size and growth rates across firms.[1] Over the last few decades, empirical studies of technologically progressive manufacturing industries have established certain broad regularities pertaining to the manner in which industries evolve from birth through maturity that have collectively come to be known as the product life cycle[2]. These regularities relate, among other things, to the pattern of entry, exit and growth of firms within industries as well as changes in the size & age distribution of firms. In the early phase of an industry, there is a lot of entry. In some industries, the number of entrants may rise over time or it may peak at the start of the industry and then decline over time. In either case, the number of entrants eventually becomes small and shake-out of firms begins as the industry matures. The number of active of firms grows initially, then reaches a peak, after which it declines steadily despite continued growth in industry output. Eventually, the industry stabilizes.[3]The other set of empirical regularity has to do with age and size distribution of firms. Firms that enter earlier are more likely to grow faster, tend to be larger in size and have a greater chance of survival. On the average, firms that exit the industry are smaller and younger than the incumbents.

Since the early eighties, a wide range of theoretical models of stochastic evolution and selection in competitive industries have been developed in order

[1] For example, Dunne, Roberts and Samuelson (1989) study a sample of US manufacturing industries over a period of 5 years and report rates of entry ranging from 30.7% to 42.7% and an equally dramatic exit rate ranging from 30.8% to 39% across industries. See also, Davis and Haltiwanger (1992).

[2] Gort and Klepper (1982) and Klepper and Grady (1990) examine the annual time path in the number of producers for 46 new products beginning with their commercial inception. See, also later studies by Utterback and Suarez (1993) and Klepper and Simons (1993).

[3] For a nice summary, see Klepper (1996).

to explain the empirical regularities relating to industry dynamics. Thus, Jovanovic (1982) analyzes the dynamics of a competitive industry where firms are uncertain about their productivity and acquire noisy signals about their efficiency as they operate in the industry; incumbent firms afflicted by unfavorable signals conclude they are inefficient and exit the market to be replaced by new entrants - the efficient grow and survive, while the inefficient decline and fail (see also, Lippman and Rumelt, 1982). Pakes and Ericson (1998) discuss the implications of a more general version of this model and compare this with those of a stochastic model of their own, where firms actively undertake investment in order to influence the conditional distribution of future technology shocks affecting them.[4] Klepper and Graddy (1990) discuss an evolutionary model where the number of potential entrants is limited, potential entrants differ in their initial cost and product qualities, receive new information over time which changes their cost and product quality in a stochastic fashion and no further updating of cost and quality occurs after entry.[5] Jovanovic and Lach (1989) consider a model with learning by doing and stochastic diffusion of innovation where potential entrants can gain by learning from incumbent firms but all learning stops after entry; the model generates delayed entry and staggered exit.

In a fairly general model with exogenous firm level technology shocks and allowing for a wide class of firm-level actions (including investment in firm-specific cost reduction), Hopenhayn (1992a,b) shows the possibility of entry and exit as part of the limit behavior of a dynamic stochastic industry. In a similar model, Hopenhayn (1993) related the observed pattern of entry and exit over product life cycle to stochastic demand expansion and technology shocks. Jovanovic and MacDonald (1994) analyze a dynamic competitive industry where innovational opportunities fuel entry and failure to innovate, whose chances are exogenously specified, leads to exit.

The unifying feature of almost the entire existing literature is its reliance on some form of firm-level uncertainty (including uncertainty arising incomplete information & noisy signals) in creating and amplifying heterogeneity among firms. These shocks may affect either potential entrants or incumbents, or both. The process of market selection leads to exit of incumbent firms afflicted by unfavorable shocks (or signals) while entry occurs because of favorable updating of future profitability by potential entrants or simply because the prior belief about future profitability is significantly better than that of the firms that exit.[6] To put

[4] Ericson and Pakes (1995) analyze a general model with stochastic entry and growth of firms that invest in order to improve future profitability & are affected by idiosyncratic shocks; they establish the ergodicity of a rational expectations Markov-perfect equilibrium process for the industry.

[5] Klepper (1996) discusses a stochastic evolutionary model where firms differ randomly in innovative capabilities (relating to product innovation) and the value of innovation is proportional to output. The model allows for imitation as well as process innovation that only reduces current cost. The paper generates several observed features of the product life cycle - particularly, the sequencing of product and process innovations.

[6] In a somewhat related exercise, Lambson (1991) analyzes a dynamic competitive model where firms make investment that entails sunk cost and whose profitability is affected by exogenous shocks over time; the equilibrium path exhibits hysterisis and high turnover of plants (see also Dixit, 1989).

it more bluntly, if all uncertainty is taken out of these models, then the industry equilibrium paths hardly generate any kind of interesting dynamics and quite often, reduce to the outcome of a static model.

This, then leads to the following question: is it the case that the patterns of changes in industry structure as observed say, over the product life cycle, arise only through through uncertainty and that there are no other fundamental forces affecting industry dynamics? This paper is an attempt to provide an alternative explanation which is not based on any form of uncertainty, non-stationary demand structure or incomplete information.

We consider a classical, infinite horizon, deterministic, complete information model of a competitive industry with a continuum of firms. All firms are *ex ante* identical, perfectly rational, forward looking and have perfect foresight. There are no strategic factors affecting entry or causing exit. Further, unlike some of the existing literature, there is no dearth of potential entrants at any point of time. We pose the following questions: Can the dynamic path of an industry be consistent with the empirical regularities mentioned earlie?. Would some firms enter later than others? Would some firms exit earlier than others? Would later entrants tend to exit earlier? Would exiting firms be small relative to incumbents that stay on? Would firms become heterogenous over time through deliberate choice even though they are initially identical? Can profits persist in the long run? This paper is an answer to all of these questions. And its an affirmative answer.

We build on earlier work by Petrakis, Rasmusen and Roy (1997) and Petrakis and Roy (1999) that analyze similar deterministic models of dynamic competitive industry. In these papers, firms that enter the industry invest in firm-specific cost reduction through accumulation of capital or experience. The industry equilibrium path is socially optimal and generates shake-out of firms. However, these papers are unable to explain some of the interesting empirical regularities such as the fact that entry continues over a considerable period of time i.e., some firms enter later than others, later entrants tend to be smaller and have a lower survival rate (i.e., exit earlier) so that exiting firms are typically smaller and younger and so on. One reason behind this is the fact that these models rule out spillover from investment by existing firms. Also, the models assume finite time horizon which rules out analysis of long run behavior of industries.

We consider a model which is similar to Petrakis and Roy (1999) - firms invest in cost reduction. However, in our model, the cost of production is also affected by the stock of an industry-wide capital that grows over time according to the total investment effort by all firms in the industry.[7] The introduction of this externality complicates the analysis considerably - particularly, because the properties of the equilibrium path can no longer be related to a social planner's optimization problem.

[7] Stokey (1986) analyzes a model of dynamic oligopoly with industry-wide externality where the unit cost of production depends on cumulative past output of the industry. There is no investment in firm-specific cost reduction nor is there any possibility of entry or exit in her model.

In equilibrium, all entering firms earn zero intertemporal net profit. However, their instantaneous profit, net of investment cost, is typically negative when they are young and strictly positive when they mature. Equilibrium prices decline over time, while the level of positive industry-wide externality increases with time - the former discourages delayed entry while the latter encourages it. The equilibrium path makes firms indifferent between alternative entry and exit decisions. Their investment levels after entry reflects their duration of stay and the nature of industry environment (prices, externalities) over their period of stay. Heterogeneity emerges out of deliberate choice. We show the possibility of delayed entry and shake-out of firms on the equilibrium path and relate them to the technology and demand conditions. Entry may occur even while incumbents earn negative profits. Exit may occur even while incumbents grow in size and earn positive profits. Exiting firms are smaller and younger than incumbents. Under certain conditions, entry and exit decline to zero in the limit and the industry stabilizes. Firms that are active in the long run may earn strictly positive profit in the limit (explains persistence of profits in the long run).

2 Model

We consider a discrete time infinite horizon model of a dynamic competitive industry with a continuum of price taking firms that are free to enter and exit the industry in any period. All firms are *ex ante identical. In any time period,* the cost function of a firm depends on its own stock of firm-specific knowledge and the industry's stock of knowledge (which is an externality). Both stocks of knowledge expand over time through investment in learning by incumbent firms. Firms incur a fixed cost of staying in the industry every period. At any point of time, the stock of knowledge is fixed and so the production technology is subject to decreasing returns i.e., the marginal cost curve slopes upwards. As the stock of firm level and industry level knowledge expands, the marginal cost curve and the fixed cost of staying in the industry shifts downwards (for firms within the industry).

Firms maximize their discounted sum of net profit.

We assume that the good produced is homogenous and non-durable. The market demand function is stationary over time (so that the structural dynamics are not demand-driven).

Notation:

$t = 0, 1, 2, ...\infty$

Z_t =industry wide knowledge in period t.

$x_t(i)$ =increase in firm-specific knowledge (investment) by firm i in period t.

$z_t(i)$ =firm specific knowledge stock of firm i in period t.

$q_t(i)$ =output of firm i in period t.

$\tau(i)$ =entry period for firm i.

$T(i)$ =exit period for firm i.

$A_t = \{i :: \tau(i) \leq t \leq T(i)\}$ = set of active firms in period t.

Q_t =industry output in period t.

$P(Q)$: inverse market demand function, identical every period, strictly decreasing, smooth, $P(Q) \downarrow 0$ as $Q \to \infty$.

$D(p)$: market demand function.

$\overline{p}$: choke price, $D(\overline{p}) = 0$.

p_t :price in period t.

$C(q, z, Z)$:Cost of producing q by a firm (which has entered and has not exited the industry) when its firm-specific knowledge capital is z and when the industry-wide knowledge capital is Z.

Assume: $C(q, z, Z)$ is continuous on $\mathbb{R}^3_+$, continuously differentiable and strictly convex in ($q, z) \in \mathbb{R}^2_+$, strictly increasing in q, strictly decreasing in z and Z; $C(0, z, Z) > 0$ (positive fixed cost of staying in the industry). Further, $C_q(q, z, Z)$ is non-increasing in z and Z. There exists a continuous, strictly increasing function $m(q)$ such that for any $q \geq 0$

$$C_q(q, z, Z) \geq m(q) > 0, \forall (z, Z) \geq (0, 0)$$

and moreover,

$$\lim_{q \uparrow \infty.} m(q) = \infty$$

The latter ensures that the output of and investment by firms are bounded along any feasible path (thus ruling out dynamic increasing returns to scale).

For any z, Z, let the minimum average cost of production be denoted by $\underline{A}(z, Z) = \min\{\frac{C(q,z,Z)}{q} : q \geq 0\}$.We assume that:

$$\underline{p} = \inf\{\underline{A}(z, Z) : z \geq 0, Z \geq 0\} > 0$$

Also denote:

$$\overline{p} = \underline{A}(0, 0)$$

$\phi(x)$: cost of investment x in firm specific knowledge capital in any period, ϕ strictly increasing and strictly convex.

There is a measure M of ex ante identical firms - potential entrants - who may enter and exit in any period. M is large enough but finite. With some abuse of notation we shall also denote by M the set of all potential firms. We can think of each potential as points in an interval with Lebesque measure.

A firm i which enters in period $\tau(i)$ and exits in period $T(i)$ ($T(i)$ may be ∞) has cost function $C(q, z_t(i), Z_t)$ where

$$z_{\tau(i)}(i) = 0, z_t(i) = \sum_{j=\tau(i)}^{t-1} x_j(i), t = \tau(i) + 1, .., T(i).$$

and

$$Z_t = Z_{t-1} + \int_{\{i:\tau(i) \leq t \leq T(i)\}} x_{t-1}(i) di, Z_0 = 0$$

This captures the idea that spillover from firm specific learning add to accumulation of industry-wide knowledge which is publicly accessible.[8] Every firm i decides on its period of entry and exit $\tau(i), T(i)$. Not entering is equivalent to setting $\tau(i) = \infty$. In each period $t = \tau(i), ..., T(i)$, an active firm i decides on its output $q_t(i)$ and current investment $x_t(i)$ in firm-specific knowledge capital which generates intertemporal net profit:

$$\sum_{t=\tau(i)}^{T(i))} \delta^t [p_t q_t(i) - C(q_t(i), z_t(i), Z_t) - \phi(x_t(i))].$$

An industry equilibrium path is one where firms maximize profits, every entering firm earns zero intertemporal profit, no firm can make strictly positive profit by any feasible action profile and the market clears every period. Let ν denote the extended set of positive natural numbers including the point $+\infty$.

Definition 1 *An Industry Equilibrium is given by a price sequence* $\{p_t\}_{t=0}^{\infty}$, *a sequence of industry wide knowledge* $\{Z_t\}_{t=0}^{\infty}$*a profile of entry periods for all potential firms* $(\tau(i))_{i \in M}$, $\tau(i) \in \nu$, *a profile of exit periods for all firms that enter* $(T(i))_{i \in M, \tau(i) < \infty}, T(i) \in \nu$, *and a profile of actions for all active firms* $\{q_t(i),\ x_t(i)\}_{t=\tau(i)}^{T(i)}, q_t(i) \geq 0,\ x_t(i) \geq 0, i \in M, \tau(i) < \infty$, *such that:*

(i) $Z_t = Z_{t-1} + \int_{\{i:\tau(i) \leq t-1 \leq T(i)\}} x_{t-1}(i) di, Z_0 = 0$

(i) Every firm maximizes profit over is active lifetime: $\{q_t(i),\ x_t(i)\}_{t=\tau(i)}^{T(i)}$ *solves*

$$\begin{aligned} & max \sum_{t=\tau(i)}^{T(i))} \delta^t [p_t q_t - C(q_t, z_t, Z_t) - \phi(x_t)] \\ \textit{subject to } q_t &\geq 0, x_t \geq 0, z_{\tau(i)} = 0, \\ z_t &= \sum_{j=\tau(i)}^{t-1} x_j, t = \tau(i)+1, .., T(i). \end{aligned}$$

(iii) Every active firm earn zero intertemporal profit:

$$\sum_{t=\tau(i)}^{T(i))} \delta^t [p_t q_t(i) - C(q_t(i), z_t(i), Z_t) - \phi(x_t(i))] = 0$$

(iv) There does not exist any profile of entry, exit, production and investment decisions for a firm which can yield strictly positive intertemporal profit: For

[8] An alternative formulation (industry wide by learning by doing - Marshallian externalities-decreasing cost industry) :

$$Z_\tau = Z_{\tau-1} + \int_{\{i:T(i) \leq \tau \leq T(i)+k(i)\}} q_\tau(i) di = Z_{\tau-1} + Q_t, Z_0 = 0]$$

every $\tau, T, 0 \leq \tau \leq T \leq \infty$.

$$\begin{aligned} 0 &\geq \max \sum_{t=\tau}^{T} \delta^t [p_t q_t - C(q_t, z_t, Z_t) - \phi(x_t)] && (1) \\ \textit{subject to } q_t, x_t &\geq 0 && (2) \\ z_\tau &= 0, z_t = \sum_{j=\tau}^{t-1} x_j, t = \tau + 1, .., T. \end{aligned}$$

(v) Market clears every period:

$$D(p_t) = \int_{A_t} q_t(i) di.$$

.

3 Existence.

To be included later.

4 Characterization of Equilibrium Path.

For the time being assume that a competitive equilibrium exists.

Lemma 2 *If exit occurs at the end of period* t*, then* $p_{t+1} \leq p_t$.

Proof. Suppose to the contrary that $p_{t+1} > p_t$.If firm i exits at the end of period t, then

$$p_t q_t(i) - C(q_t(i), z_t(i), Z_t) \geq 0$$

for otherwise the firm would do strictly better by exiting at the end of period $t - 1$ (or not entering, if $t = 0$). This, in turn, implies that

$$p_{t+1} q_t(i) - C(q_t(i), z_t(i), Z_t) > 0$$

and since $Z_{t+1} \geq Z_t$,

$$p_{t+1} q_t(i) - C(q_t(i), z_t(i), Z_{t+1}) > 0$$

which means the firm can earn strictly positive profit by staying on for one more period with no additional investment. This violates the definition of equilibrium. ∎

Proposition 3 *Every equilibrium price sequence* $\{p_t\}$*is non-increasing over time i.e.,* $p_{t+1} \leq p_t$ *for all* t.

Proof. Suppose to the contrary that $p_{t+1} > p_t$.From the previous lemma, we have that there is no exit at the end of period t. Then,

$$A_t \subset A_{t+1}$$

Observe that since $Z_{t+1} \geq Z_t$ and $z_{t+1}(i) \geq z_t(i)$ for all $i \in A_t$,

$$C_q(q, , z_t(i), Z_t) \geq C_q(q, , z_{t+1}(i), Z_{t+1}) \text{ for all } q \geq 0$$

so that

$$p_t \leq C_q(q_t(i), , z_t(i), Z_t) \text{ and } p_{t+1} = C_q(q_{t+1}(i), , z_{t+1}(i), Z_{t+1})$$

implies

$$q_{t+1}(i) > q_t(i) \text{ for all } i \in A_t \text{ such that } q_{t+1}(i) > 0$$

Further,

$$q_{t+1}(i) = q_t(i) = 0 \text{ for all } i \in A_t \text{ such that } q_{t+1}(i) = 0$$

Therefore,

$$Q_t = \int_{A_t} q_t(i)di < \int_{A_t} q_{t+1}(i)di \leq \int_{A_{t+1}} q_{t+1}(i)di = Q_{t+1}$$

which means that for the market to clear we must have

$$p_t > p_{t+1}$$

a contradiction. ■

Note that the equilibrium prices are bounded below by $\underline{p}$,the lowest possible minimum average cost over all possible levels of learning. For if $p_t < \underline{p}$ for some t, then $p_\tau < \underline{p}$ for all $\tau \geq t$ which makes ALL active firms earn losses every period after t, not compatible with the definition of equilibrium. Also, the prices are bounded above by $\overline{p}$.,It can be shown that, in fact:

Proposition 4 $p_0 \leq \overline{p}, p_t < \overline{p}$ *for all* $t \geq 1$.

Proof. If $p_t > \overline{p}$ a firm could enter for only one period and make strictly positive profit. If $p_t = \overline{p}$ for some $t \geq 1$, then since prices are non-increasing over time, $p_k = \overline{p}, k = 0, ..t$. In that case, a firm can enter in period 0, make an infinitesimal but strictly positive investment and make strictly positive intertemporal profit by exiting in period t. ■

Proposition 5 *For any firm* i *such that* $\tau(i) < \infty$ *and for any* $\widehat{t}$, $\tau(i) < \widehat{t} \leq T(i)$

$$\sum_{t=\widehat{t}}^{T(i))} \delta^t [p_t q_t - C(q_t, z_t, Z_t) - \phi(x_t)] \geq 0$$

Proof. Trivial. Else, firm is better off exiting the market earlier than period $\widehat{t}$. ■

Proposition 6 *Let* $(\widehat{z}_t\ , \widehat{z}_{t+1})$ *and* $(\widetilde{z}_t, \widetilde{z}_{t+1})$ *be the levels of firm specific knowledge for two firms that are active in the market in both periods* t *and* $t+1$*. Then,* $\widehat{z}_t\ < \widetilde{z}_t \Rightarrow \widehat{z}_{t+1}\ < \widetilde{z}_{t+1}$*.*

Proof. Suppose to the contrary that $\widehat{z}_{t+1}\ \geq \widetilde{z}_{t+1}$.Then

$$\widehat{z}_{t+1}\ \geq \widetilde{z}_{t+1} \geq \widetilde{z}_t > \widehat{z}_t$$

For a firm that is active in the market at the beginning of period $t+1$ with firm-specific knowledge stock equal to z, let $V_{t+1}(z)$ denote the (maximum) present value of intertemporal profits from period $t+1$ onwards (net of investment made in such periods). Let $\pi_t(z_t)$ denote the maximum profit in period t (gross of investment cost incurred in period t) for a firm whose current knowledge stock is z_t.Then,

$$\begin{aligned} \widehat{z}_{t+1} &\in \arg\max\{\pi_t(\widehat{z}_t) - \phi(z - \widehat{z}_t) + \delta V_{t+1}(z) : z \geq \widehat{z}_t\} \\ \widetilde{z}_{t+1} &\in \arg\max\{\pi_t(\widetilde{z}_t) - \phi(z - \widetilde{z}_t) + \delta V_{t+1}(z) : z \geq \widetilde{z}_t \end{aligned}$$

Therefore:

$$\begin{aligned} \pi_t(\widehat{z}_t) - \phi(\widehat{z}_{t+1} - \widehat{z}_t) + \delta V_{t+1}(\widehat{z}_{t+1}) &\geq \pi_t(\widehat{z}_t) - \phi(\widetilde{z}_{t+1} - \widehat{z}_t) + \delta V_{t+1}(\widetilde{z}_{t+1}) \\ \pi_t(\widetilde{z}_t) - \phi(\widetilde{z}_{t+1} - \widetilde{z}_t) + \delta V_{t+1}(\widetilde{z}_{t+1}) &\geq \pi_t(\widetilde{z}_t) - \phi(\widehat{z}_{t+1} - \widetilde{z}_t) + \delta V_{t+1}(\widehat{z}_{t+1}) \end{aligned}$$

so that

$$\begin{aligned} -\phi(\widehat{z}_{t+1} - \widehat{z}_t) + \delta V_{t+1}(\widehat{z}_{t+1}) &\geq -\phi(\widetilde{z}_{t+1} - \widehat{z}_t) + \delta V_{t+1}(\widetilde{z}_{t+1}) \\ -\phi(\widetilde{z}_{t+1} - \widetilde{z}_t) + \delta V_{t+1}(\widetilde{z}_{t+1}) &\geq -\phi(\widehat{z}_{t+1} - \widetilde{z}_t) + \delta V_{t+1}(\widehat{z}_{t+1}) \end{aligned}$$

First, consider the case where $\widehat{z}_{t+1}\ > \widetilde{z}_{t+1}$.From the above inequalities, we have

$$\phi(\widehat{z}_{t+1} - \widehat{z}_t) - \phi(\widetilde{z}_{t+1} - \widehat{z}_t) \leq \delta[V_{t+1}(\widehat{z}_{t+1}) - V_{t+1}(\widetilde{z}_{t+1})] \leq \phi(\widehat{z}_{t+1} - \widetilde{z}_t) - \phi(\widetilde{z}_{t+1} - \widetilde{z}_t)$$

which violates strict convexity of ϕ as $\widetilde{z}_t > \widehat{z}_t$ .Next, consider the case where $\widehat{z}_{t+1}\ = \widetilde{z}_{t+1}$. Let i denote the firm whose knowledge stock in period t is $\widetilde{z}_t$ and let j denote the firm whose knowledge stock in period t is $\widehat{z}_t$.As $V_{t+1}(\widetilde{z}_{t+1}) = V_{t+1}(\widehat{z}_{t+1})$, the maximum discounted sum of profits net of investment cost between periods $t+1$ and $T(i)$ for firm i must be exactly equal to the maximum discounted sum of profits net of investment cost between periods $t+1$ and $T(j)$ for firm j. This would mean that from period $t+1$ onwards, the output and investment path of firm j (and firm $j's$ exit period) is also optimal for firm i. The first order condition for firm j with respect to its investment in period t implies (note $\widehat{z}_{t+1} - \widehat{z}_t > 0$)

$$\phi'(\widehat{z}_{t+1} - \widehat{z}_t) = -\sum_{k=t+1}^{T(j)} \delta^{k-t} C_z(q_k(j), z_k(j), Z_k)$$

and since $\widetilde{z}_{t+1} - \widetilde{z}_t < \widehat{z}_{t+1} - \widehat{z}_t$

$$\phi'(\widetilde{z}_{t+1} - \widetilde{z}_t) < -\sum_{k=t+1}^{T(j)} \delta^{k-t} C_z(q_k(j), z_k(j), Z_k)$$

which implies that if firm i invests an amount slightly higher than $\widetilde{z}_{t+1} - \widetilde{z}_t$ in period t and thereafter replicates the action profile of firm j, its net intertemporal profit will be higher than its initial optimal path, a contradiction. //
■

5 Delayed Entry

Main argument: Some firms choose to enter later in order to take advantage of industry wide increase in knowledge capital over time even though equilibrium prices are falling. In particular, in an equilibrium with delayed entry, equilibrium prices are such that firms are indifferent between entering in period 0 and in a later period.

In order to illustrate the possibility of delayed entry, we first consider the situation where the marginal cost curve is independent of investment in learning as well as industry-wide externalities i.e., the cost function is of the form:

$$C(q, z, Z) = f(z, Z) + g(q)$$

where $f(z, Z)$ represents the fixed cost (of being active in the industry) while $g(q)$ represents the variable cost. Assume that $g'(q) > 0, g"(q) > 0$.Further, $f_Z(0, 0) < 0$ and $1 + \delta f_z(0, Z) < 0$ for all $Z > 0$. (This can be relaxed).

Proposition 7 *A positive measure of firms necessarily enter after period* 0*.*

Proof. Suppose not. Then, $A_{t+1} \subset A_t$ for all t. We first claim it must be the case that $p_t = p_{t+1}$ for all t. To see this, note that the equilibrium price sequence is non-increasing and if $p_t > p_{t+1}$ for some t,then $q_t(i) > q_{t+1}(i)$ for all $i \in A_{t+1}$ and therefore

$$Q_t = \int_{A_t} q_t(i)di \geq \int_{A_{t+1}} q_t(i)di > \int_{A_{t+1}} q_{t+1}(i)di = Q_{t+1}$$

which means that for the market to clear we must have

$$p_t < p_{t+1}$$

a contradiction. So, the equilibrium prices are constant over time, equal to p^*, say. Every firm produces q^* every period where

$$g'(q^*) = p^*$$

This also implies that for the market to clear, $A_{t+1} = A_t$ a.e. for all t (no exit).Now, since $1 + \delta f_z(0, Z) < 0$, $x_0(i) > 0$ for all $i \in A_0$. This implies that $Z_1 > 0$. From the definition of equilibrium:

$$\sum_{t=0}^{\infty} \delta^t [p^* q^* - g(q^*) - f(z_t(i), Z_t) - \phi(x_t(i))] = 0 \text{ for all active } i$$

Since $f_Z(0, 0) < 0$, for any active i

$$\sum_{t=0}^{\infty} \delta^t [p^* q^* - g(q^*) - f(z_t(i), \widehat{Z}_t) - \phi(x_t(i))] > 0 \text{ where } \widehat{Z}_t = Z_{t+1}$$

The above expression also gives the intertemporal net profit earned by a firm j that enters in period 1 and sets $q_t(j) = q^*, x_t(j) = x_{t-1}(i)$ for all $t \geq 1$. This violates the definition of equilibrium. The proof is complete. ■

The next step is to extend the possibility of delayed to a more general situation where both marginal cost & fixed cost can fall over time with increase in learning and externalities.

Recall that the equilibrium prices are bounded above by $\overline{p}$. Define an upper bound $\overline{q}$ on the quantity produced by any firm in any period by:

$$m(\overline{q}) = \overline{p}$$

Then, for any firm entering the industry, an upper bound on its present value of production cost is given by:

$$\frac{C(\overline{q}, 0, 0)}{1 - \delta}$$

Let $\overline{x}$ be defined by

$$\frac{C(\overline{q}, 0, 0)}{1 - \delta} = \phi(\overline{x})$$

Then, $\overline{x}$ is an upper bound on x_t and z_t It might be possible to find a much tighter upper bound and work with that instead.

The next proposition shows that if the MC curves decline at a much smaller rate than the average cost (i.e., the fixed cost), then delayed entry occurs.

Proposition 8 *Suppose that for any* $\widehat{q} \in [0, \overline{q}], \widehat{z} \in [0, \overline{x}], Z \geq 0$ *and* $h > 0$

$$\begin{aligned} & \max_{0 \leq q \leq \overline{q}, 0 \leq z \leq \overline{x}} [C_q(q, z, Z) - C_q(q, z + \overline{x}, Z + h)] \\ < \quad & [\frac{C(\widehat{q}, z, Z)}{\widehat{q}} - \frac{C(\widehat{q}, z, Z + h)}{\widehat{q}}] \end{aligned}$$

then delayed entry must occur on the equilibrium path.

Proof. Consider an industry equilibrium and suppose that late entry does not occur. Let $\{p_t, Z_t\}_{t=o}^{\infty}$ be the associated sequence of prices and industry-wide knowledge on the equilibrium path. Note that in any such equilibrium, there must exist a strictly positive measure of active firms i such that $T(i) = \infty$. This is because the total industry output along the equilibrium path is bounded below by $D(\overline{p})$ while the output of any individual firm in any period is bounded above by $\overline{q} > 0$. Choose any firm i such that $T(i) = \infty$. Let the optimal sequence of actions (output and investment) chosen by this firm be given by $\{q_t, x_t\}_{t=0}^{\infty}$ and let $\{z_t\}_{t=0}^{\infty}$ denote the resulting sequence of firm-specific knowledge. Then

$$\sum_{t=0}^{\infty} \delta^t [p_t q_t - C(q_t, z_t, Z_t) - \phi(x_t)] = 0$$

Now. consider a firm which enters in period 1, never exits and chooses actions $\widetilde{q}_t = q_{t-1}, \widetilde{x}_t = x_{t-1}$ (with firm specific knowledge $\widetilde{z}_t = z_{t-1}$), $t = 1, ..., \infty$. The net intertemporal profit earned by such a firm is given by:

$$\begin{aligned} & \sum_{t=1}^{\infty} \delta^t [p_t q_{t-1} - C(q_{t-1}, z_{t-1}, Z_t) - \phi(x_{t-1})] \\ = \quad & \delta[\sum_{t=1}^{\infty} \delta^{t-1} [(p_t - p_{t-1} + p_{t-1}) q_{t-1} \\ & -\{C(q_{t-1}, z_{t-1}, Z_t) - C(q_{t-1}, z_{t-1}, Z_{t-1}) + C(q_{t-1}, z_{t-1}, Z_{t-1})\} \\ & -\phi(x_{t-1})] \\ = \quad & \delta[\{\sum_{t=1}^{\infty} \delta^{t-1} (p_{t-1} q_{t-1} - C(q_{t-1}, z_{t-1}, Z_{t-1}) - \phi(x_{t-1}))\} \\ & +\{\sum_{t=1}^{\infty} \delta^{t-1} ((p_t - p_{t-1}) q_{t-1} - (C(q_{t-1}, z_{t-1}, Z_t) - C(q_{t-1}, z_{t-1}, Z_{t-1}))\}] \end{aligned}$$

and this last expression is in turn equal to

$$\begin{aligned}
&= \delta[\{\sum_{t=0}^{\infty}\delta^t(p_tq_t - C(q_t, z_t, Z_t) - \phi(x_t))\} \\
&\quad +\{\sum_{t=0}^{\infty}\delta^t((p_{t+1}-p_t)q_t - (C(q_t,z_t,Z_{t+1}) - C(q_t,z_t,Z_t))\}] \\
&= \delta[0+\sum_{t=0}^{\infty}\delta^t((p_{t+1}-p_t)q_t - (C(q_t,z_t,Z_{t+1}) - C(q_t,z_t,Z_t))] \\
&= \delta\sum_{t=0}^{\infty}\delta^t[\{C(q_t,z_t,Z_t) - C(q_t,z_t,Z_{t+1})\} - q_t(p_t - p_{t+1})]
\end{aligned}$$

We shall show that the last expression is strictly positive by showing that for $t = 0, 1, ...\infty$

$$\{C(q_t,z_t,Z_t) - C(q_t,z_t,Z_{t+1})\} - q_t(p_t - p_{t+1}) > 0$$

To see this let

$$\alpha = \max_{0\leq q\leq\overline{q}, 0\leq z\leq\overline{x}}[C_q(q,z,Z_t) - C_q(q,z+\overline{x},Z_{t+1})]$$

Under the condition outlined in the proposition:

$$\frac{C(q_t,z_t,Z_t) - C(q_t,z_t,Z_{t+1})}{q_t} > \alpha$$

It is sufficient to show that

$$p_t - p_{t+1} \leq \alpha$$

Suppose to the contrary that $p_t - p_{t+1} > \alpha$.Then, for any firm i which is active in both periods t and $t+1$,it must be the case that $q_t(i) > q_{t+1}(i)$. For if $q_t(i) \leq q_{t+1}(i)$

$$\begin{aligned}
p_{t+1} &= C_q(q_{t+1}(i), z_{t+1}(i), Z_{t+1}) \\
&\geq C_q(q_t(i), z_{t+1}(i), Z_{t+1}) \\
&\geq C_q(q_t(i), z_t(i)+\overline{x}, Z_{t+1}) \\
&\geq C_q(q_t(i), z_t(i), Z_t) - \alpha = p_t - \alpha
\end{aligned}$$

a contradiction. But if $q_t(i) > q_{t+1}(i)$ for all firms active in periods t and $t+1$ and no new entry occurs in period $t+1$, then the total industry output in period t is greater than that in period $t+1$ which violates the market clearing condition since $p_t - p_{t+1} > \alpha$ implies $p_t > p_{t+1}$. Thus, it must be the case that $p_t - p_{t+1} \leq \alpha$. The proof is complete. ■

6 Shake-out.

What are the conditions under which some firms exit the industry over time on the equilibrium path?

Main argument: If the marginal cost curve declines (firms' supply curve expands) sharply with increase in knowledge, then in the absence of shake-out, the industry prices would have to fall drastically. But if the prices fall too sharply, firms cannot recover their investment through future profits - prices faced by mature firms need to be more than their minimum average cost and their optimal output in later periods exceeds their minimum efficient scale.

Another way to see it is that the market is "restricted efficient" in the sense that taking as given any equilibrium path of industry wide knowledge $\{Z_t\}$, the industry path of output, investment, entry and exit maximizes the discounted sum of social surplus; such a "restricted" social planner would want a lot of firms around initially (steep MC curve initially) in order to reduce industry production cost but would want only some of them to grow "big" and produce at low MC - while dispensing with the rest (saving on the fixed cost). Recall. that for any z, Z, the minimum average cost of production is denoted by $\underline{A}(z, Z) = \min\{\frac{C(q,z,Z)}{q} : q \geq 0\}$ and that $\overline{p} = \underline{A}(0.0)$.Let $q^m(z, Z)$ be the output at which average cost of production is minimized when knowledge levels are (z, Z)

$$C_q(q^m(z, Z), z, Z) = \underline{A}(z, Z)$$

Proposition 9 *Suppose that for any $(z, Z) >> 0$,*

$$\frac{D(\overline{p})}{q^m(0,0)} > \frac{D(\underline{A}(z, Z))}{q^m(z, Z)}$$

then there exists a positive measure of firms who exit in finite time.

Proof. Suppose that no (positive measure of) exit occurs along the equilibrium path. Then the set of active firms in every period includes A_0 and the measure of active firms is at least as large as n_0.

Since $\{p_t\}$ is a bounded decreasing sequence, it converges to, say, $\widehat{p}$. Note that $\{z_t(i)\}$ and $\{Z_t\}$ are bounded sequences along any equilibrium path (to see the latter observe that the total discounted sum of industry cost of production using cost function $C(q, 0, 0)$ and assuming output is equal to $D(\underline{p})$ every period is bounded; the total investment of all firms cannot exceed that). For $i \in A_0$,let $\widehat{z}(i) = \lim_{t\to\infty} z_t(i)$ and let $\widehat{z} = \inf_{i\in A_0} \widehat{z}(i)$. Further, let $\widehat{Z} = \lim_{t\to\infty} Z_t$.Then, it must be true that

$$\widehat{p} \geq \underline{A}(\widehat{z}, \widehat{Z})$$

To see this note that if $\widehat{p} < \underline{A}(\widehat{z}, \widehat{Z})$, there exists a positive measure of firms $i \in A_0$ and time periods $t_0(i)$, such that $t \geq t_0(i)$ implies $p_t < \underline{A}(z_t(i), Z_t)$ which implies that such firms are better off exiting before time $t_0(i)$ and the latter contradicts the hypothesis that zero measure of exit occurs along the equilibrium path. For $i \in A_0$,let $\widehat{q}(i) = \lim_{t\to\infty} q_t(i)$ and let $\widehat{q} = \inf_{i\in A_0} \widehat{q}(i)$. It must be true that

$$\widehat{q} \geq q^m(\widehat{z}, \widehat{Z})$$

for otherwise, there would be a positive measure of firms $i \in A_0$ and time periods $t_0(i)$, such that $t \geq t_0(i)$ implies $q_t(i) < q^m(z_t(i), Z_t)$ which wouldn't be optimal unless $p_t < \underline{A}(z_t(i), Z_t)$, a contradiction. Choose any $\epsilon > 0$. Then, there exists t_1 large enough such that for all $t \geq t_1$ and $i \in A_0$, $q_t(i) \geq \widehat{q} - \epsilon$ and for all such t

$$n_0 \leq \frac{D(p_t)}{\inf_{i\in A_0} q_t(i)} \leq \frac{D(p_t)}{\widehat{q} - \epsilon}$$

so that taking limit as $t \to \infty$

$$n_0 \leq \frac{D(\widehat{p})}{\widehat{q} - \epsilon}$$

and since ϵ is arbitrary we have

$$n_0 \leq \frac{D(\widehat{p})}{\widehat{q}}$$

and using the fact that $\widehat{p} \geq \underline{A}(\widehat{z}, \widehat{Z})$ and $\widehat{q} \geq q^m(\widehat{z}, \widehat{Z})$ we have

$$n_0 \leq \frac{D(\underline{A}(\widehat{z}, \widehat{Z})}{q^m(\widehat{z}, \widehat{Z})}$$

Using the condition in the statement of the proposition and the fact that $\widehat{z}, \widehat{Z} >> 0$ we have

$$n_0 < \frac{D(\overline{p})}{q^m(0,0)}$$

However, as noted in an earlier proposition, $p_1 \leq \overline{p}$ and therefore, $q_1(i) \leq q^m(0,0)$ so that

$$n_0 \geq \frac{D(\overline{p})}{q^m(0,0)}$$

a contradiction. The proof is complete.// ■

7 Firms that exit are younger.

In this section, we show that later entrants exit earlier. This implies that, at the point of exit, the age of firms that exit is lower than that of other incumbent firms.It will be generalized later.

Proposition 10 *If firm i exits in period t, then for every firm j that is active in the market in period t and does not exit that period, $\tau(j) \leq \tau(i)$.*

Proof. Since $T(j) > T(i)$, it must be true that $z_t(i) \leq z_t(j)$ - for otherwise, firm i can earn strictly positive net intertemporal profit by producing in the market till period $T(j)$ which contradicts the definition of equilibrium. Now, suppose $\tau(j) > \tau(i)$. In period $\tau(j)$,firm $j's$ stock of firm specific knowledge $z_{\tau(j)}(j) = 0$, while firm i's stock of knowledge $z_{\tau(j)}(i) > 0$. Using Proposition (?), we can see that this implies that $z_t(i) > z_t(j)$,a contradiction. ∎

8 Firms that exit are smaller.

Proposition 11 *If $T(i) < \infty$, then for any firm $j \in A_{T(i)}$ such that $T(j) > T(i)$, $z_{T(i)}(j) \geq z_{T((i)}(i)$ and $q_{T(i)}(j) \geq q_{T((i)}(i)$ In other words, at its point of exit a firm's stock of firm-specific knowledge (its accumulated investment) and its output are no larger than that of any incumbent firm which does not exit in that period.*

Proof. Since the MC curve (firm's supply curve) is non-increasing in z, it is sufficient to show that $z_{T(i)}(j) \geq z_{T((i)}(i)$.Suppose to the contrary that $z_{T(i)}(j) < z_{T((i)}(i)$. Note that since firm j does not exit in period $T(i)$

$$\sum_{t=T(i)+1}^{T(j)} \delta^t [(p_t q_t(j) - C(q_t(j), z_t(j), Z_t) - \phi(x_t(j))] \geq 0$$

On the other hand firm i earns zero discounted sum of net profit at its point of exit and so if it stays on till period $T(j) > T(i)$ and replicates the investment and output path of firm j in periods $t = T(i) + 1, .., T(j)$ then it can earn strictly positive intertemporal net profit (as its existing stock of firm-specific knowledge in period $T(i)$ is higher than that of j and will be higher in every period thereafter). This is a contradiction. // ∎

9 Long Run Behavior of the Industry.

Our first result in this section shows that as observed empirically, both entry and exit decline to negligible levels in the long run and the industry "stabilizes". Note that as $\{p_t\}$ is non-increasing and bounded below by $\underline{p} > 0$, it converges to some $p^* > 0$. Similarly, $\{Z_t\}$ is an non-decreasing bounded sequence and converges to some fiinite $Z^* > 0$.

Proposition 12 *The measure of firms that enter the industry as well as measure of firms that exit the industry in period t converge to zero as $t \longrightarrow \infty$.*

Proof. Choose any small $\epsilon > 0$.There exists $T(\epsilon)$, such that for all $t \geq T(\epsilon)$, $p_t \in [p^*, p^* + \epsilon]$, $Z_t \in [Z^* - \epsilon, Z^*]$, the industry output $Q_t \in [D(p^* + \epsilon), D(p^*)]$ and

$$Q_{t+1} - Q_t \leq [D(p^*) - D(p^* + \epsilon)]$$

. For any period $t \geq T(\epsilon)$ in which a measure $m_t > 0$ of firms enter, each entrant firm produces at least an amount $\widetilde{q} > 0$ where $C_q(\widetilde{q}, 0, 0) = p^*$ and since both entry and exit cannot occur in the same period (argued earlier) , it must be the case that

$$Q_{t+1} - Q_t \geq m_t \widetilde{q}$$

so that

$$m_t \leq \frac{[D(p^*) - D(p^* + \epsilon)]}{\widetilde{q}} \longrightarrow 0 \text{ as } \epsilon \longrightarrow 0.$$

Therefore, the measure of entering firms converges to zero as $t \longrightarrow \infty$. Next, we show that the measure of exiting firms must also converge to zero as $t \longrightarrow \infty$. Suppose not. Then there exists $\zeta > 0$ and a subsequence of time periods $\{t'\}$ along which the measure of exiting firms is at least as large as ζ. Once again, note that entry and exit cannot occur simultaneously. Therefore, if a measure $m_{t'} \geq \zeta$of firms exit occurs at the beginning of period t',then this by itself would lead to a decline in industry output (over the previous period) by an amount of at least $\underline{q}\zeta = h(say) > 0$ where $\underline{q}$ is a uniform lower bound on firm level output defined by $C_q(\underline{q}, 0, 0) = \underline{p}$. However, since prices are non-increasing, industry output must be non-decreasing over time. Thus, active firms in period t'that do not exit the market must be expanding their total output over the previous period by at least h.Let $\widehat{S}_{t'}$ denote the set of active firms in period $t' - 1$ that do not exit at the beginning of period t'.Let $\widehat{n}_{t'}$ be the measure of such firms. Then,

$$\int_{\widehat{S}_{t'}} [q_{t'}(i) - q_{t'-1}(i)] di \geq h$$

For any $z \in [0, \overline{z}], Z \in [0, \overline{Z}], p \in [\underline{p}, \overline{p}]$ let an individual firm's supply function $q^S(p; z, Z)$ be implicitly defined by:

$$C_q(q^S(p; z, Z), z, Z) = p$$

It is easy to check that $\frac{dq^S}{dz}, \frac{dq^S}{dZ}$ are strictly positive and bounded on the domain $[\underline{p}, \overline{p}] \times [0, \overline{z}] \times [0, \overline{Z}]$; let α_1 and α_2 be the upper bounds on this domain. Note that $\overline{n} = \frac{D(p)}{\underline{q}}$ is an upper bound on the measure of active firms in any period.

Then, for all terms t' in the candidate subsequence,

$$\begin{aligned}
0 &< h \leq \int_{\widehat{S}_{t'}} [q_{t'}(i) - q_{t'-1}(i)]di \\
&= \int_{\widehat{S}_{t'}} [q^S(p_{t'}, z_{t'}(i), Z_{t'}) - q^S(p_{t'-1}, z_{t'-1}(i), Z_{t'-1})]di \\
&\leq \int_{\widehat{S}_{t'}} [q^S(p_{t'-1}, z_{t'}(i), Z_{t'}) - q^S(p_{t'-1}, z_{t'-1}(i), Z_{t'-1})]di \\
&\leq \int_{\widehat{S}_{t'}} [\alpha_1(z_{t'}(i) - z_{t'-1}(i)) + \alpha_2(Z_{t'} - Z_{t'-1})]di \\
&= \int_{\widehat{S}_{t'}} [\alpha_1 x_{t'-1}(i)]di + \alpha_2(Z_{t'} - Z_{t'-1})]\widehat{n}_{t'} \\
&= \alpha_1(Z_{t'} - Z_{t'-1}) + \alpha_2(Z_{t'} - Z_{t'-1})]\widehat{n}_{t'} \\
&\leq (\alpha_1 + \overline{n}\alpha_2)(Z_{t'} - Z_{t'-1}), \\
&\longrightarrow 0 \text{ as } t' \longrightarrow \infty, \text{ since } \{Z_t\} \text{ is convergent.}
\end{aligned}$$

This leads to a contradiction. // ■

Let $S^* = \{i : \tau(i) < \infty, T(i) = \infty\}$ be the set of firms that produce in the long run.

For $i \in S^*$, let $z^*(i) = \lim_{t \to \infty} z_t(i)$.

Questions:

1. Possibility of persistence of positive profits in the long run: $p^* > \underline{A}(z^*(i), Z^*)$ for all $i \in$ subset of S^* of strictly positive measure?

2. Possibility of persistence of heterogeneity in the long run: $z^*(i) \neq \frac{1}{m(S^*)} \int_{S^*} z^*(j)dj$ for all $i \in$ subset of S^* of strictly positive measure?

Other questions:

1. Condition for no shakeout

2. Condition for no late entry

3. Characterization of first best path - is late entry possible on first best path?

4. How does equilibrium path differ from first best?

References

[1] Bergin, J. and D. Bernhardt, "Industry dynamics with stochastic demand," Mimeo., Queens University, Toronto, 2003.

[2] Dasgupta, P. , "The theory of technological competition" in J. Mathewson and J.E. Stiglitz, eds., *Modern Approaches to Industrial Economics* (London: Macmillan, 1986), 519-47.

[3] Dasgupta, P. and J.E. Stiglitz, " Industrial structure and the nature of innovative activity", *Economic Journal* 90 (1980), 266-93.

[4] Davis, S. and J. Haltiwanger, "Gross job-creation, gross job destruction and employment reallocation", *Quarterly Journal of Economics* 107 (1992), 819-863.

[5] Dixit, A., "Entry and exit decisions under uncertainty", *Journal of Political Economy* 97 (1989), 620-38.

[6] Dunne, T., M. Roberts and L. Samuelson, "The growth and failure of U.S. manufacturing plants", *Quarterly Journal of Economics* 104 (1989), 671-698.

[7] Ericson, R. and A. Pakes, "Markov perfect industry dynamics: a framework for empirical work," *Review of Economic Studies* 62 (1995), 53-82.

[8] Flaherty, M.T., "Industry structure and cost reducing investment," *Econometrica* 48 (1980), 1187 - 1209.

[9] Geroski, P.A. and M. Mazzucato, "Modelling the dynamics of industry populations," *International Journal of Industrial Organization* 19 (2001), 1003 - 22.

[10] Gort, M. and S. Klepper, "Time paths in the diffusion of product innovations," *Economic Journal* 92 (1982), 630-53.

[11] Hopenhayn, H., "Entry, exit and firm dynamics in long run equilibrium," *Econometrica* 60 (1992a), 1127-150.

[12] Hopenhayn, H., "Exit, selection and the value of firms", *Journal of Economic Dynamics and Control* 16 (1992b), 621-53.

[13] Hopenhayn, H., "The shake-out", Working Paper # 33, Dept. of economics, University of Pompeu Fabra, 1993.

[14] Jovanovic, B., "Selection and the evolution of industry", *Econometrica* 50 (1982), 649-70.

[15] Jovanovic, B. and S. Lach, "Entry, exit and diffusion with learning by doing," *American Economic Review* 79 (1989), 690-99.

[16] Jovanovic, B. and G. MacDonald, "The life cycle of a competitive industry,"*Journal of Political Economy* 102 (1994), 322-47.

[17] Klepper, S., "Entry, exit, growth, and innovation over the product life cycle," *American Economic Review* 86 (1996), 562-83.

[18] Klepper, S. and E. Graddy, "The evolution of new industries and the determinants of market structure," *Rand Journal of Economics* 21 (1990), 27 -44.

[19] Lach, S. and R. Rob, "R&D investment and industry dynamics", *Journal of Economics and Management Strategy* 5 (1996), 217 - 49.

[20] Lambson, V.E., "Industry evolution with sunk costs and uncertain market conditions," *International Journal of Industrial Organization* 9 (1991), 171-96.

[21] Lippman, S.A. and R.P. Rummelt, "Uncertain imitability: an analysis of interfirm differences in efficiency and competition", *Bell Journal of Economics* 13 (1982), 418-38

[22] Lucas, R. and E.C. Prescott, "Investment under uncertainty," *Econometrica* 39 (1971), 659-81.

[23] Pakes, A. and R. Ericson, " Empirical implications of alternative models of firm dynamics", *Journal of Economic Theory* 79 (1998), 1-46.

[24] Petrakis, E. and S. Roy, "Cost-reducing investment, competition and industry dynamics", *International Economic Review* 40 (1999), 381 -401.

[25] Petrakis, E., E. Rasmusen and S. Roy, "The learning curve in a competitive industry," *Rand Journal of Economics* 28 (1997), 248-68.

[26] Stokey, N., "The dynamics of industrywide learning, " in *Equilibrium Analysis: Essays in Honor of Kenneth J. Arrow, Volume II* (W. Heller, R. Starr and D. Starrett, Eds.), Cambridge University Press, 1986, 81-104.

[27] Spence, M., "Cost reduction, competition and industry performance", *Econometrica* 52 (1984), 101-21.