Chain Reactions, Trade Credit and the Business Cycle

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(Preliminary version)

Abstract

Firms in poor countries often tend to rely on alternative sources of financing different than banks. We show that borrowing constraints lead to financial arrangements between firms that can amplify the effect of liquidity or productivity shocks in the economy. In particular, we focus on the effects of trade credit. Widespread borrowing and lending between firms implies the establishment of relationships that can serve as a way to transmit economic shocks. In other words, trade credit creates a network of firms, all of them linked by the credit given to each other and all of them exposed to the temporary problems the others might have. We develop in this chapter a model based on the ideas of Kiyotaki and Moore (1997). Our model however, is a general equilibrium version of theirs that deals with the aggregate consequences that temporary productivity or liquidity shocks might have on the whole economy. Our results show that in a carefully calibrated model, the effects of these credit chains are quite important. In particular, in an economy like Mexico where 65% of firms claim their main source of financing to be other firms, the impact of productivity shocks is significant and three times bigger than in an economy with levels of trade credit use close to the US case.

1 Introduction

Firms in less developed economies are financially constrained. The lack of development in financial markets makes firms rely either on their own income to finance projects or on other more expensive sources. In particular, in this paper we underline the role of the credit given by suppliers. This is commonly referred to as trade credit. Our main goal is to show that borrowing constraints lead to financial arrangements between firms that can amplify the effect of liquidity or productivity shocks in the economy. Trade credit is such kind of an arrangement. Borrowing and lending between firms imply the establishment of relationships that can serve as a way to transmit economic shocks. In other words, trade credit creates a network of firms, all of them linked by the credit given to each other.

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Intuitively, the transmission mechanism works as follows: imagine an economy where trade credit is widespread and there is some firm heterogeneity. Furthermore, firms can only borrow from other firms and a domestic intermediary (or bank). If the economy overall is hit with a negative productivity shock, firms with profits close to zero will fail to pay their obligations in full. Other agents counting on these payments to cover their own debts will fail to do so too. In other words, the productivity shock triggers a chain reaction by which a larger than usual share of firms is unable to fulfill its obligations. This process is somewhat slowed down by the fact that larger firms will sacrifice some of their profits to pay their debts (compensating for the fact that other firms are not repaying the loans they gave them). In order to guarantee payment to other lenders, profitable domestic firms will absorb not only the negative decrease in income that corresponds to the shock in productivity, but also, they will perceive a lower revenue than the one they expected because other managers are not paying what they were lent. If these larger firms are the ones that make most of the investment in the economy, future output will largely decrease as a consequence of the existence of these credit chains. It follows then, that countries in which these credit relationships are more widespread should also experience larger and longer recessions. In fact, recent empirical data points to the fact that output in less developed economies tends to be relatively more volatile than in rich countries (see Agenor, McDermott and Prasad (2000), Mendoza (1997) and Neumeyer and Perri (2002)). Therefore, in this paper we propose the widespread presence of trade credit in poor economies as a possible explanation of the output variability.

Recent attempts to explain the higher output volatility in less developed economies include Mendoza (1997) who shows that terms of trade changes can explain around 50% of output movements in poor countries. On the other hand, Neumeyer and Perri (2002) explain these facts by noting that interest rates in this type of countries are highly variable. They then develop a model in which the main source of the output volatility are the changes in the interest rate. These papers however put a considerable emphasis on the importance of huge and sustained external shocks (in the form of terms of trade movements and changes in the interest rate). Our goal here is to show that such abrupt changes are not needed to explain why output is so volatile in less developed economies. That is, small productivity or liquidity shocks can also develop into strong output movements through an amplification mechanism. In this particular case, we consider the role of credit chains. Cardoso-Lecourtois (2003) also tries to underline a more subtle transmission mechanism: home production. In both of these two papers then, only small productivity shocks are needed to generate the desired variability in output.

A secondary goal of this paper is to tackle the issue raised by Acemoglu and Scott (1997) and Kocherlakota (2000) that "macroeconomics is looking for an asymmetric amplification and propagation mechanism." Here we show that credit chains can be such a device.

The idea that these types of credit chains can work as a transmission mechanism was first underlined by Kiyotaki and Moore (1997 (a)). They however, do not deal with the general equilibrium consequences of their model, nor with the aggregate effects of the liquidity shocks they impose on their firms. This paper deals with both of these issues.

The paper begins with some data to confirm our claim that trade credit use is more important in less developed economies than in the rest of the world. In particular we concentrate in the cases of Mexico and the US. Then, we lay down a model in which credit

Trade credit use in Mexico*				
	Total value of sales by firm (December 1998) (Millions of dollars)			
Main Source of Financing	Less than 10	10-50	50-500	More than 500
Trade Credit	64.5	55.7	49.7	29.2
Commercial Banks	13.4	18.3	23.2	37.5
Foreign Banks	1.1	1.9	6.5	18.8
Development banks	2.6	2.7	3.2	2.1

*The number in the table gives the percentage of firms in each category that answered that their main source of financing was the one listed in the first column.

Table 1: Trade Credit use in Mexico

chains are established. We define the equilibrium in this economy and its steady state. Later we show the impact of a negative productivity shock at the steady state and how it varies with a higher use of trade credit. We finish with some conclusions and guidelines for future research.

2 Empirical evidence

The main objective of this section is to show some data that can corroborate our claim that trade credit is an important source of financing in less developed economies compared to what happens in more developed ones.

We first present in Tables 1 and 2 data for Mexico and the US respectively. In the case of Mexico, the numbers come from the 1998 Market Evaluation Survey published by the Mexican central bank. The questions asked in this survey are of a qualitative nature. In particular, the survey asks each firm to say which was the main source of financing for the firm during the past quarter. The answers given fluctuate between trade credit, commercial, foreign and government owned banks. The numbers shown in Table 1 give the percentage of firms that answered positively to the question of whether the category listed in the first column was the main source of financing for the firm or not.

From this first Table it is clear that small firms in Mexico heavily rely on trade credit. In particular, firms with sales lower than 10 million dollars report that around 65% of them use trade credit as its main source of financing. On the other hand, only 30 percent of the biggest firms in Mexico use credit given by other producers as its main source of funds. Note that as the total value of sales increases, the importance of commercial banking also goes up.

On the other hand, the data shown for the US in Table 2 comes from the 1998 Survey of Small Firms Finances. This survey does not ask specifically for the main source of financing. In order to calculate a comparable statistic to the one given in the Mexican data we calculate the percentage of firms for which accounts payable surpasses the amount of credit given by banks. We also divide the sample into four groups, each consisting of around 250 firms and ordered according to the total value of sales by each firm. The results are presented in the first row of Table 2.

The most important fact from comparing the numbers in Table 1 and the first row in Table 2 is the marked difference between trade credit use in both countries. In particular, only around 30% of the firms in the US sample rely heavily in their suppliers for financing. We take that as a strong indication that firms in Mexico use relatively more trade credit. Furthermore, we also note that the size of the firms in the Mexican sample seems to be considerably bigger than in the US sample. However, since the tendency in the Mexican data is to have even more trade credit use as firm's size goes down, we would expect these results to be reinforced if we were to have firms of the same size as in the US.

We also present in Table 2 other measurements of trade credit use in the US. The second item in this Table gives trade credit as a percentage of total liabilities for the firms in the sample. Here we find support that the 30% number that we found before might be correct. Furthermore, the medians in the sample are smaller than the averages, suggesting an even smaller (than first thought) role of trade credit in firm financing.

One thing we don't get from the previously shown numbers in the US is that smaller firms depend relatively more on trade credit than larger firms (like we see in the Mexican sample). The last row in Table 2 shows the percentage of cash discounts used by firms. To understand clearly what this means, one has to have certain knowledge of the way these contracts on goods are signed. A typical contract between firms normally establishes that from the moment a supplier delivers the goods, a firm has ten days to pay in cash for them. If the firm does not comply with this, then it has to pay a greater amount 30 days after the first date. That is why it is said that the firm takes advantage of the cash discount if it pays within 10 days. As it can be seen in Table 2, larger firms use trade credit as a means of financing output less often than small firms do. While most of the larger firms pay 80% of their obligations within 10 days, the corresponding number for small firms is only 50%.

Even more evidence comes from a recent econometric study by Demirguc-Kunt and Maksimovik (2002). This includes data for 39 countries and the authors results are that firm's use of bank debt relative to trade credit is higher in countries with more developed and efficient legal and financial systems. Therefore, they conclude, it seems that trade credit is an important source of financing in less developed economies.

Now that we have shown that there seems to be a relatively high use of trade credit in Mexico than in the US, we proceed in the next section to lay down the model that will allow us to explain how this increased role of credit chains has an impact on output volatility.

3 The Model

We consider an economy where all individuals are infinitely lived, time is discrete and indexed by $t = 1, 2, 3, ...$

3.1 Physical Environment

Within the economy there are J types of individuals and there is a unit measure of agents of each type. Each of these is endowed with one unit of productive time which can be devoted to either managerial activities or to provide unskilled labor. Every individual of type j is

Trade credit use in the US*				
	Total value of sales by firm(December 1998) (Thousands of dollars)			
	Less than 165	165-740	740-3,410	Over 3,410
Main Source of Financing Acc. Payable (% of all firms)	27.2	28.7	32.0	31.0
Trade Credit / Total liab. (%)				
Average	30.1	26.8	28.9	29.0
Median	19.1	18.3	21.8	21.6
% of cash disc. used by firm				
Average	53.5	57.4	60.3	62.3
Median	50.0	75.0	75.0	80.0
*Data from the Survey of Small Firms Finances. Sample consists of 998 firms.				

Table 2: Trade credit use in the US

endowed with a specific ability $z(j)$ in the sense of Lucas (1978). Although we explain this in more detail below, the level of $z(j)$ determines the "managerial talent" of a certain type. Depending on their ability $z(j)$ consumers decide whether to produce or simply work under the orders of other managers. In particular, we assume that $z(1) > ... > z(J)$. If a person of type j decides to be a manager, then she produces an intermediate and a final good.

Individuals only face uncertainty in this model during the first period. Particularly, we assume that A (a parameter describing the state of aggregate productivity) can take on two values during period 1: A^1 with probability $1-p$ and $A^2 < A^1$ with probability p. The rest of the time A equals A^1.

In order to better understand what comes next, we present a brief outline of the timing of transactions in the model during period 1. First, agents are divided into managers and unskilled workers according to their ability $z(j)$. Then the managers go and produce an intermediate good. Next, they purchase and sell intermediate goods from and to other firms and an "intermediary" on credit (the process through which this happens is explained below). Following this, they use their inputs to produce a final good. Then, the state of aggregate technology A is realized and managers pay factors of production. Later, with the remaining income, they settle all debts. Finally, they keep the remaining profits and each agent performs consumption and investment decisions. After $t = 1$, this framework stays the same, the only difference being that no uncertainty is resolved since the value of A is known from the beginning of the period.

Now, we turn to the details of the model beginning with the way goods are produced in this economy. Regarding the production of the intermediate good, each individual of type j can produce one that is exclusive to her type. To produce it, managers can combine capital k_t, and unskilled labor l_t through the following constant returns to scale technology

$$y_{It}(j) = f^j(k_t(j), l_t(j)) \qquad t = 1 \tag{1}$$

$$y_{It}(A,j) = f^j(k_t(A,j), l_t(A,j)) \qquad t \geq 2 \tag{2}$$

where $y_{It}(j)$ is the amount of the intermediate good of type j produced by an individual with managerial ability $z(j)$. Before we proceed further, an explanation about notation is in order. Variables in the model are denoted as $d_t(A)$ to indicate the fact that variable d in period $t \geq 2$ depends on the realization of A in period 1. Variables in period one are written without the dependence on A when the case applies.

On the other hand, technology $f^j(\cdot)$ is only available to producers of type j so that they are the only ones capable of supplying this intermediate good. We make the simplifying assumption that $f^j(\cdot) = f^i(\cdot) = f(\cdot)$ for all $j, i \leq J$. To better understand this last assumption, one can think of each of these types supplying a different color of paint. To produce paint, the same kind of technology is needed, but there is something indistinguishable about each individual that makes the good produced different than what agents of other type can supply.

There is only one final good managers of any type can produce. The technology available to produce this good in $t \geq 2$ is available to everyone and given by

$$y_{Ft}(A,j) = z(j)A^1 g\left[X_t(A,j)\right] \tag{3}$$

where

$$X_t(A,j) = Min\left\{x_t^1(A,j), ..., x_t^{N_t}(A,j)\right\}$$

$y_{Ft}(A,j)$ is the production of the final good supplied by agent of type j, $x_t^n(A,j)$ refers to intermediate goods demanded from producers of type n by manager j, N_t is the market determined number of types that produce an intermediate good in the given economy with $N_t \leq J$, while $g[\cdot]$ has decreasing returns to scale on X_t. Here, we are assuming that in order to produce the final good the agent has to purchase the intermediate good supplied by all other managers currently producing. Note that y_{Ft} rises with z. This is the sense in which z describes an agent's managerial talent. In period 1, output of the final good is given by

$$y_{F1}(A,j) = z(j)Ag\left(X_1(j)\right) \tag{4}$$

where

$$X_1(j) = Min\left\{x_1^1(j), ..., x_1^{N_1}(j)\right\}$$

Note that we assume that managers in the economy make production and credit transactions in period 1 without knowing the value of A. This is only revealed when managers have to pay factors of production and cancel credit obligations. The probabilities of each of the two states however, are known to all individuals in the economy.

We make two important assumptions here: first, individuals take consumption and investment decisions independently from production and credit decisions. Second, individuals manage the two operations (final and intermediate good production) independently from each other up until the point in which they have to pay creditors. Then, if one of the two operations cannot pay its debts in full, the manager uses the profits from the other (if positive) and pays back to lenders the amount borrowed.

3.1.1 Intermediate Goods and "Credit" Markets

The total value of intermediate good output produced by a manager of type j is equal to

$$P_t^j(A) f\left[k_t(A,j), l_t(A,j)\right] \tag{5}$$

where $P_t^j(A)$ is the price of good j in terms of the final good in period $t \geq 2$, while in $t = 1$

$$P_1^j f\left[k_1(j), l_1(j)\right]$$

Each manager can choose to sell its intermediate good output to one of two sources: other producers and an intermediary. Furthermore, the manager delivers these goods under a promise of payment at the end of the period (after the realization of A in the case of $t = 1$). There are no costs of lending nor an alternative use for intermediate goods and they depreciate completely after one period.[1] Therefore, when $t \geq 2$ and there is no uncertainty, the interest rate charged by producers (to either the intermediary or to other managers) equals zero.

In period $t = 1$, however, uncertainty about the aggregate level of technology exists. In particular, it is possible that if the bad state of productivity (A^2) is realized some agents may not be able to pay their credit obligations in full. In this case, we assume that producers always have enough resources to pay back the intermediary (even when they do not have enough to pay back other debtors).[2] In the event in which a manager finds herself without sufficient funds to pay all of her debts, resources are first used to pay the intermediary and the rest is used to cancel obligations with other firms. Intuitively, this assumption can be interpreted as the intermediary being more capable of collecting payment from managers than other producers are.[3]

Therefore, the credit given to other managers is seen as a risky endeavor while credit given to the intermediary is perceived as risk free. In order to make credit to other producers attractive, these offer an interest payment to compensate for the uncertainty. The interest rate paid by producers in period 1 is defined to be R_1. We assume that the market for intermediate goods and credit is competitive. Furthermore, when performing these credit transactions the manager behaves as a risk neutral individual.[4]

To model credit transactions between firms we assume that each manager n chooses a fraction $\psi_t(A, n)$ of her total intermediate good output in $t \geq 2$, to be sold to other firms on credit. The rest, a fraction $(1 - \psi_t(A, n))$ is sold to the intermediary which then sells these goods back to other firms on credit. The same process applies in $t = 1$, the only difference being that the fraction $\psi_1(n)$ is chosen before the realization of A while payment is made after.

This mechanism creates two markets: the one for credit between managers and the market for credit between producers and the intermediary. We first deal with the former.

[1] So managers cannot accumulate them.

[2] This assumption implies that A^2 must be close enough to A^1.

[3] If we think of the intermediary as a bank then what we are saying is that these agents may have more resources and a better collection ability than small firms have.

[4] Although as we'll see later, it behaves as a risk averse individual when performing consumption and investment decisions.

In order to simplify notation from here on, we note that we are only interested in cases in which in equilibrium $\psi_t(A,n) = \psi_t(A,m)$, for all $n,m \leq N_t$, $t \geq 2$, and $\psi_1(n) = \psi_1(m)$ for all $n,m \leq N_1$ in $t = 1$, so we abuse notation and assume from now on that $\psi_t(A,n) = \psi_t(A,m) = \psi_t(A)$ and $.\psi_1(n) = \psi_1(m) = \psi_1$.

Now, take any producer m, with $m \leq N_t$. This manager offers in the trade credit market a fraction $\psi_t(A)$ of her intermediate good

$$\psi_t(A)P_t^m(A)f\left[k_t(A,m), l_t(A,m)\right] \tag{6}$$

Again, the manager supplies these goods under a promise from other firms that payment will be received at the end of the period (after the realization of A in $t = 1$).

Regarding the demand for credit, we assume that managers are constrained in their ability to ask for the cheaper intermediary credit. In particular, this entity only lends them a fraction $1 - \varphi$ of the value of their intermediate goods needs. The rest has to be obtained from other firms at the relatively high interest rate. This fraction φ is given and exogenous. In the exercise we perform later, we change this parameter in order to vary the composition of liabilities for managers in this economy. Then, we see the effect of assuming a larger share of loans from other producers on the business cycle.

Therefore, a manager m demands from the producer of intermediate good n

$$\varphi P_t^n(A)x_t^n(A,m) \tag{7}$$

which implies a total demand for trade credit in market n of

$$\sum_{n=1}^{N_t} \varphi P_t^n(A)x_t^n(A,m) \tag{8}$$

We assume that managers do not sign individual debt agreements with other producers but instead they do it with a "union". This includes all producers interested in their good (including themselves). The union receives the "orders" given by (8) from its members and purchases from the suppliers the given quantities demanded. Therefore, equilibrium in the trade credit market for good m requires

$$\psi_t(A)P_t^m(A)f\left[k_t(A,m), l_t(A,m)\right] = \sum_{n=1}^{N_t} \varphi P_t^m(A)x_t^m(A,n) \tag{9}$$

The rest of the manager's input demands constitute purchases made from the intermediary. Total demand for intermediate good m within the "bank credit" market is given by the sum of all managers' demands

$$\sum_{n=1}^{N_t}(1-\varphi)P_t^m(A)x_t^m(A,n) \tag{10}$$

Lastly, the intermediary purchases on credit from each producer m,

$$[1 - \psi_t(A)]\, P_t^m(A) f\,[k_t(A,m), l_t(A,m)]$$

and lends the same amount to other managers since we assume it must have zero profits. Therefore, its role is just that of an intermediary. Equilibrium in the "bank credit" market for good m implies

$$[1 - \psi_t(A)]\, P_t^m(A) f\,[k_t(A,m), l_t(A,m)] = \sum_{n=1}^{N_t} (1-\varphi) P_t^m(A) x_t^m(A,n) \tag{11}$$

Note that if $\psi_t(A) = \varphi$, equations (9) and (11) are exactly the same and therefore the equilibrium price in each market (whether trade or bank credit) will be the same. Indeed, in our definition of equilibrium we only consider those in which such condition is fulfilled.

We now proceed to analyze the problem faced by each of the two operations the manager runs. Income for the intermediate good operation is given by the value of those goods sold to other firms. As explained above, a manager in period 1 chooses a fraction ψ_1 of her total intermediate good production to be sold to other firms on credit. The rest (a fraction $1-\psi_1$) is sold to an intermediary. We further assumed that this last credit is risk free, that is, the intermediary always pays back what it owes in full. However, there exists a probability that other managers will not pay back everything they borrowed. Let P_1^j be the price of intermediate good j in period 1 and $\mu_1(A) \leq 1$ the fraction of the goods lent repaid by other managers in period 1. Then, expected profits at this stage for an intermediate goods operation managed by any agent of type j in period 1 are given by

$$\pi_{I1}(j) = \underset{k_1(j), l_1(j)}{Max} \left\{ (\mu_1^e(1+R_1)\psi_1 + 1 - \psi_1) P_1^j f(k_1(j), l_1(j)) - w_1 l_1(j) - r_1 k_1(j) \right\} \tag{12}$$

where

$$0 \leq \mu_t^e \leq 1$$

$$\mu_1^e = (1-p)\mu_1(A^1) + p\mu_1(A^2)$$

and R_1 is the market determined interest rate on credit to other managers, , w_1 is the wage rate paid to unskilled labor and r_1 the rate of return on capital. All three are denominated in units of the final good.

Moreover, μ_1^e is the fraction of sales made on credit to other producers, that the manager expects to get repaid. The fraction $\mu_1(A)$ depends on the state of aggregate productivity and is determined in equilibrium. The process through which $\mu_1(A)$ is determined is described in the next section. For any other $t \geq 2$ we have that expected profits for the intermediate good operation run by agent j are given by

$$\pi_{It}(A,j) = \underset{k_t(A,j), l_t(A,j)}{Max} \left\{ P_t^j(A) f(k_t(A,j), l_t(A,j)) - w_t(A) l_t(A,j) - r_t(A) k_t(A,j) \right\} \tag{13}$$

The second stage of production implies the transformation of the intermediate goods obtained from the different managers operating in the economy into a final good. For any agent of type j, expected profits from operating the final good technology in period 1 are given by

$$\pi_{F1}(j) = \underset{X_1(j), x_1^1(j), \ldots, x_1^{N_1}(j)}{Max} \left\{ z(j) A^e g\left[X_1(j)\right] - (1 + \varphi R_1) \sum_{n=1}^{N_1} P_1^n(j) x_1^n(j) - w_1 \right\} \tag{14}$$

$$\text{s. to} \quad X_1(j) = Min\left\{x_1^1(j), \ldots, x_1^{N_1}(j)\right\}$$

where

$$A^e = (1-p)A^1 + pA^2$$

and $x_1^n(j)$ is the amount demanded by agent j of intermediate good n. The second term in the maximization problem represents the expenditures made on purchasing the intermediate good from other firms and from the intermediary. We further assume that a wage equal to the one paid to unskilled workers is awarded to the manager and treated as a cost by the firm. This wage is kind of a risk free payment the manager is awarded, while she still has a claim on the rest of the profits. We assume that there is some kind of limited responsibility institution and managers are allowed to use only resources from the profits $\pi_{F1}(j)$ to pay the debts of the firm. Note that this implies that their own wage w_1 cannot be used to pay obligations and it is paid before the firm cancels debts. This, even in the case in which the firm is not able to repay all of its obligations. For $t \geq 2$ we have

$$\pi_{Ft}(A,j) = \underset{X_t(A,j), x_t^1(A,j), \ldots, x_t^{N_t}(A,j)}{Max} \left\{ \begin{array}{c} z(j) A^1 g\left[X_t(A,j)\right] - \\ \left(\sum_{n=1}^{N_t} P_t^n(A) x_t^n(A,j)\right) - w_t(A) \end{array} \right\} \tag{15}$$

$$\text{s. to} \quad X_t(A,j) = Min\left\{x_t^1(A,j), \ldots, x_t^{N_t}(A,j)\right\}$$

3.2 A chain reaction and the determination of $\mu_1(A)$

Our goal here is to explain the process by which $\mu_1(A)$ (the fraction of debts repaid by other firms) is determined and to explore the effect that credit chains might have on transmitting economic shocks. In particular, since managers do not know the exact value of A at the beginning of period 1, they will make production and credit decisions based on their expectations of A. With the actual realization of the productivity shock, payments to be made remain the same for firms but revenues are different than what they expected. In particular, total final output produced is given by $y_{F1}(A^i, m)$ if the value of A equals A^i while the firm expected it to be $y_{F1}(A^e, m)$. This implies that some managers that had close to zero profits, will not be able to pay their obligations in full. Consequently, we could have that the fraction of loans being repaid is lower than the expected value μ_1^e. This brings a second hit that producers have to endure and that further decreases their income (as they perceive lower revenues than expected from the selling of the intermediate good produced). But then, a larger share of managers will not be able to meet their obligations. At one point, the process may stop because managers with higher levels of profits use these to pay their debts.

What follows now is an explanation of how $\mu_1(A)$ is determined. We start by assuming each manager thinks that it is being repaid a fraction μ_1^e of the loans given to other producers after the realization of A. As explained above, this is not likely if the realized value of A

is low enough. However, this serves as a starting point from which we will able to see if $\mu_1(A)$ will be higher or lower than μ_1^e after the realization of A. Much like the Walrasian auctioneer trying prices to make the excess demand for a given good equal to zero, we try several values of $\mu_1(A)$ until one satisfies the criteria set below.

First, assume that the realized value of A equals A^i. If for some manager $m \leq N_t$ we have that

$$\pi_{F1}(m) + \pi_{I1}(m) + \frac{(A^i - A^e)}{A^i} y_{F1}(A^i, m) < 0 \tag{16}$$

then that particular producer will not be able to pay all of her debts. The first two terms of this last expression account for the profits obtained from the production of the intermediate and final goods. The third term represents the loss in revenue in terms of units of the final good the manager experiences when $A = A^i$, compared to what she expected to obtain before the realization of A. The fact that the sum of these three terms is negative implies that costs are higher than the income generated by manager m when the state of aggregate productivity is equal to A^i. Then, this means that at least one manager (m) will not be able to pay all of her obligations. For notational simplicity, let's call the third term in the above inequality $\Delta y_{Ft}(A^i, z_m)$.

With less income than what they expected, managers will have to pay some of their creditors less than what they promised. In this case, remember we assumed that managers first cancel obligations with workers, owners of capital and the intermediary. Lastly, with any remaining income they have, they pay what they can of what is owed to other producers. Furthermore, we assume that once managers see they cannot pay their obligations in full they are able to hide a fraction θ of the goods borrowed from their debtors. Therefore, the remaining income manager m has (and uses to pay debts with other producers) is given by

$$\pi_{F1}(m) + \pi_{I1}(m) + \Delta y_{F1}(A^i, m) + \varphi\,(1 + R_1)\,(1 - \theta)\left(\sum_{n=1}^{N} P_1^n x_1^n(m)\right) \tag{17}$$

Each manager m for which condition (16) is true, pays to the union an amount equal to (17). All other managers, on the other hand, pay their debts in full. After the union has received these payments it proceeds to pay managers the goods lent. We assume this is done on a "pro-rated" manner. What this implies is the following: define the fraction of its trade credit debt that a producer with managerial ability $z(m)$ will be able to pay as $\lambda(m)$. If the firm generates enough income to pay all of their debts, that is, if profits after the realized shock A^i are bigger than or equal to zero, then $\lambda(m) = 1$. If not,

$$\lambda(m) = \frac{\pi_{F1}(m) + \pi_{I1}(m) + \Delta y_{F1}(A^i, m) + \varphi\,(1 + R_1)\,(1 - \theta)\left(\sum_{n=1}^{N} P_1^n x_1^n(m)\right)}{\varphi\,(1 + R_1)\left(\sum_{n=1}^{N} P_1(n) x_1^n(m)\right)} < 1 \tag{18}$$

Define v_1 the manager with $z(v_1)$ such that for all $n \geq v_1$, $\pi_{F1}(n) + \pi_{I1}(n) + \Delta y_{F1}(A^i, n) \geq 0$ and for all $n < v_1$ we have that opposite is true.That is, all firms with managerial ability equal or above $z(v_1)$ have enough goods to cover their obligations. With this notation, we define now the fraction of debts actually paid by the union to other firms $\mu_1^1(A)$ as

$$\frac{(1+R_1)\left[\sum_{m=v_1+1}^{N}\lambda(m)\varphi\left(\sum_{n=1}^{N}P_1^n x_1^n(m)\right)+\sum_{m=1}^{v_1}\varphi\left(\sum_{n=1}^{N}P_1^n x_1^n(m)\right)\right]}{(1+R_1)\sum_{m=1}^{N}\varphi\left(\sum_{n=1}^{N}P_1^n x_1^n(m)\right)}$$

If $\mu_1^1(A) < \mu_1^e$ then further resources would have to be used from profits to cover the expenses of the intermediate good firms. In particular we have that for any producer n

$$\left[\mu_1^1(A)(1+R_1)\psi_1\right] P_1^n f\left[k_1(n), l_1(n)\right] < \left[\mu_1^e(1+R_1)\psi_1\right] P_1^n f\left[k_1(n), l_1(n)\right]$$

which implies that the remaining income after having paid all obligations but those owed to other firms by any firm m is now not given by (17), but by

$$\pi_{F1}(m) + \pi_{I1}(m) + \Delta y_{F1}(A^i, m) + \varphi\left(1+R_1\right)\left(1-\theta\right)\left(\sum_{n=1}^{N}P_1^n x_1^n(m)\right) +$$
$$\psi_1(\mu_1^1(A) - \mu_1^e)(1+R_1)P_1^m f\left[k_1(m), l_1(m)\right]$$

and therefore we might have that, if v_1 is close enough to N

$$\pi_{F1}(v_1)+\pi_{I1}(v_1)+\frac{(A^i - A^e)}{A^i}y_{F1}(A^i, v_1)+\psi_1(\mu_1^1(A)-\mu_1^e)(1+R_1)P_1^{v_1} f\left[k_1(v_1), l_1(v_1)\right] < 0 \quad (19)$$

This implies that there is a new group of managers who will not be able to pay their obligations in full. In particular, this includes all firms for which (19) holds. Let manager v_2 be the one such that for all producers with $m \leq v_2$ we have that

$$\pi_{F1}(m) + \pi_{I1}(m) + \Delta y_{F1}(A^i, m) + \psi_1(\mu_1^1(A) - \mu_1^e)(1+R_1)P_1^m f\left[k_1(m), l_1(m)\right] \geq 0$$

Then, we have to recalculate the fraction of loans repaid since producers v_2 to v_1 are now not paying in full. Using the intuition above we have

$$\mu_1^2(A) = \frac{(1+R_1)\left[\sum_{m=v_2+1}^{N}\lambda(m)\varphi\left(\sum_{n=1}^{N}P_1^n x_1^n(m)\right)+\sum_{m=1}^{v_2}\varphi\left(\sum_{n=1}^{N}P_1^n x_1^n(m)\right)\right]}{(1+R_1)\sum_{m=1}^{N}\varphi\left(\sum_{n=1}^{N}P_1^n x_1^n(m)\right)} \quad (20)$$

such that $\mu_1^2(A) < \mu_1^1(A) < \mu_t^e$ and therefore a new batch of managers who cannot pay all their obligations in full has to be found. It can be proved that because $\psi_1 < 1$ this process converges to a fraction

$$\mu_1(A) = \frac{1}{\Theta}\frac{\sum_{m=v+1}^{N}\left(\Lambda_1(A, m)\right)+\sum_{m=1}^{v}\varphi\left(1+R_1\right)\left(\sum_{n=1}^{N}P_1^n x_1^n(m)\right)}{\sum_{m=1}^{N}\varphi\left(1+R_1\right)\left(\sum_{n=1}^{N}P_1^n x_1^n(m)\right)} \quad (21)$$

where

$$\Theta = 1 + \frac{\sum_{m=v+1}^{N} \varphi\left(1+R_1\right)\left(\sum_{n=1}^{N} P_1^n x_1^n(m)\right)}{\sum_{n=1}^{N} \varphi\left(1+R_1\right)\left(\sum_{n=1}^{N} P_1^n x_1^n(m)\right)}$$

$$\begin{aligned}\Lambda_1(A,m) &= \pi_{F1}(m) + \pi_{I1}(m) + \Delta y_{F1}(A,m) + \\ &\quad \varphi\left(1+R_1\right)\left(1-\theta\right)\left(\sum_{n=1}^{N} P_1^n x_1^n(m)\right) + \psi_1 \mu_1^e (1+R_1) P_1^m f\left[k_1(m), l_1(m)\right]\end{aligned}$$

and for all managers of type m such that $m \leq v$ we have

$$\pi_{F1}(m) + \pi_{I1}(m) + \Delta y_{F1}(A,m) + \psi_1(\mu_1(A) - \mu_1^e)(1+R_1)P_1^m f\left[k_1(m), l_1(m)\right] \geq 0$$

Finally, we have to deal with the case in which after the realization of the shock, all managers have enough income to cover their obligations. Here, $\mu_1(A) = 1$.

3.3 Preferences, Budget and Resource Constraints

There is a single consumption good and all agents, regardless of their type, have identical preferences given by

$$\sum_{t=1}^{\infty} \beta^{t-1} \left\{(1-p)U\left[c_t(A^1, j)\right] + pU\left[c_t(A^2, j)\right]\right\} \tag{22}$$

where U is the one period utility function, β is the discount factor and $c_t(A^i, j)$ is the level of consumption of the final good by an agent of type j according to the realization of A in period 1.

Each of these individuals is endowed with one unit of productive time which can be devoted to either managerial activities or to provide unskilled labor. In any case they receive a market determined wage $w_t(A)$ for their services when $t \geq 2$ and of w_1 in period 1.

As it was explained above, individuals that decide to manage a firm (and produce intermediate and final goods) have a right on profits after paying factors of production and canceling debts. An individual of type j decides to be a manager (vs. a worker) if and only if the sum of the profits that come with producing the final and intermediate goods are bigger than or equal to zero.

Individuals have three sources of income: first, wages $w_t(A)$ collected whether they are managers or unskilled workers. Second, the profits obtained from producing the intermediate and final goods. Third, agent number 1 (the one with $z = z(1)$), is assumed to be the only one with the capacity of owning capital and accumulate it.[5] Agent number 1 is also endowed with an initial level of capital k_1. The other $J-1$ individuals dedicate all their income only to purchase the consumption good. Define $\chi_t(A, j)$ for $t \geq 2$ to be an indicator function that

[5]This assumption is not essential to the results shown below, but it makes computation a little easier. Certainly, the model can be modified to allow for all firms with positive profits to accumulate capital. The results of the paper will not be changed in essence.

agents of type j choose to be equal to 1 if they want to be managers, or, in terms of profits

$$\pi_{Ft}(A,j) + \pi_{It}(A,j) \geq 0$$

and equal to zero otherwise. Therefore, the budget constraint for any agent of type $j \neq 1$ looks like follows

$$c_t(A,j) = (1 - \chi_t(A,j))w_t(A) + \chi_t(A,j)\left[w_t(A) + \pi_{Ft}(A,j) + \pi_{It}(A,j)\right] \tag{23}$$

while for agent 1 we have

$$\begin{aligned} c_t(A,1) + k_{t+1}(A,1) &= (1 - \chi_t(A,1))w_t + \chi_t(1)\left[w_t + \pi_{Ft}(A,1) + \pi_{It}(A,1)\right] + \\ &\quad (1 - \delta + r_t)k_t(A,1) \end{aligned} \tag{24}$$

where r_t is the rate of return on capital, k_t is capital, δ is the depreciation rate.

We now determine the budged constraint when $t = 1$. During this period, there exists the possibility that some managers will not pay back the loans they received. Therefore, some losses could appear that the manager did not expect to have. Define $\chi_1(j)$ to be an indicator function that agents of type j choose to be equal to 1 if they want to be managers in period 1, or in other terms

$$\pi_{F1}(j) + \pi_{I1}(j) \geq 0$$

Then, budget constraints for agents with $j \neq 1$ look as follows

$$\begin{aligned} c_1(A,j) &= (1 - \chi_1(j))w_1 + \\ &\quad \chi_1(j)\left\{w_1 + \xi_1(A,j)\left[\pi_{F1}(j) + \pi_{I1}(j) + T_1(A,j)\right]\right\} + && (25) \\ &\quad \left[1 - \xi_1(A,j)\right]\varphi\theta\left(\sum_{n=1}^{N} P_1^n x_1^n(j)\right) && (26) \end{aligned}$$

where

$$T_1(A,j) = \Delta y_{F1}(A,j) + \psi_1(\mu_1(A) - \mu_1^e)(1 + R_1)P_1^j f\left[k_1(j), l_1(j)\right]$$

and $\xi_1(A,j)$ is an indicator function that equals one if

$$\pi_{F1}(j) + \pi_{I1}(j) + T(A,j) \geq \varphi\theta\left(\sum_{n=1}^{N} P_1^n x_1^n(j)\right) \tag{27}$$

and zero otherwise. For agent 1, on the other hand, we have

$$\begin{aligned} c_1(A,1) + k_2(A) &= (1 - \chi_1(1))w_1 + \\ &\quad \chi_1(1)\left\{w_1 + \xi_1(A,1)\left[\pi_{F1}(1) + \pi_{I1}(1) + T_1(A,1)\right]\right\} + && (28) \\ &\quad \left[1 - \xi_1(A,1)\right]\varphi\theta\left(\sum_{n=1}^{N} P_1^n x_1^n(1)\right) + (1 - \delta + r_1)k_1(A) && (29) \end{aligned}$$

Note that there is no between periods borrowing or lending. There is, however, within the same period debt transactions between firms. The resource constraint for the whole economy satisfies that the sum of all consumption made by each of the types, plus the investment made by agent of type 1 has to be equal to the total final output produced when the realized state of aggregate technology is A, or

$$\sum_{j=1}^{J} c_t(A,j) + k_{t+1}(A) - (1-\delta)k_t(A) = \sum_{j=1}^{J} \chi_t(A,j)\left[z_j A^1 g\left(Min\left\{x_{It}^{j1},...,x_{It}^{jN_t}\right\}\right)\right] \quad (30)$$

for $t \geq 2$ and in period 1

$$\sum_{j=1}^{J} c_1(A,j) + k_1(A) - (1-\delta)k_1(A) = \sum_{j=1}^{J} \chi_1(j)\left[z_j A g\left(Min\left\{x_{It}^{j1},...,x_{It}^{jN_t}\right\}\right)\right] \quad (31)$$

4 Equilibrium

An equilibrium for this economy is given by quantities $\left\{\widehat{c}_1(A,j),\widehat{k}_2(A)\right\}$, indicator functions $\left\{\widehat{\chi}_1(j),\widehat{\xi}_1(A,j)\right\}$ number of producers and managers that pay all their obligations in full $\left\{\widehat{N}_1,\widehat{v}\right\}$, firm's allocations $\left\{\widehat{\psi}_1(n),\widehat{k}_1(n),\widehat{l}_1(n),\widehat{x}_1^1(n),...,\widehat{x}_1^{\widehat{N}_1}(n)\right\}_{n=1}^{\widehat{N}_1}$, prices, wages and rents paid $\left\{\widehat{P}_1^n,\widehat{w}_1,\widehat{r}_1,\widehat{R}_1\right\}_{n=1}^{\widehat{N}_1}$ and the fraction of trade credit loans repaid $\{\widehat{\mu}_1(A)\}$ for $t=1$, along with allocations

$$\left\{\widehat{c}_t(A,j),\widehat{k}_{t+1}(A,j),\widehat{\chi}_t(A,j)\right\}_{t=2,3,...}$$

and

$$\left\{\widehat{N}_t,\widehat{\psi}_t(A,n),\widehat{k}_t(A,n),\widehat{l}_t(A,n),\widehat{x}_t^1(A,n),...,\widehat{x}_t^{\widehat{N}_t}(A,n)\right\}_{n=1;\ t=2,3,...}^{\widehat{N}_t}$$

and prices wages and rents paid

$$\left\{\widehat{P}_t^n(A),\widehat{w}_t(A),\widehat{r}_t(A)\right\}_{n=1;\ t=2,3,...}^{\widehat{N}_1}$$

such that

1. Given $\{\widehat{w}_1,\widehat{r}_1\}$, and $\left\{\widehat{P}_t^n(A),\widehat{w}_t(A),\widehat{r}_t(A)\right\}_{n=1;\ t=2,3,...}^{\widehat{N}_1}$,

   $$\left\{\widehat{c}_1(A,j),\widehat{k}_2(A),\widehat{\chi}_1(j),\widehat{\xi}_1(A,j)\right\}$$

   and

   $$\left\{\widehat{c}_t(A,j),\widehat{k}_{t+1}(A,j),\widehat{\chi}_t(A,j)\right\}_{t=2,3,...}$$

   maximize (22) subject to (23), (26), with $\widehat{k}_1 = k_1$.

2. For all $n \leq \widehat{N}_1$ and in $t=1$

$$\widehat{r}_1 = (\mu_1^e(1+R_1)\psi_1 + 1 - \psi_1)\widehat{P}_1^n f_1(\widehat{k}_1(n), \widehat{l}_1(n)) \tag{32}$$

$$\widehat{w}_1 = (\mu_1^e(1+R_1)\psi_1 + 1 - \psi_1)\widehat{P}_1^n f_2(\widehat{k}_1(n), \widehat{l}_1(n)) \tag{33}$$

and for all $n \leq \widehat{N}_t$ and $t = 2, 3, ...$

$$\widehat{r}_t = \widehat{P}_t^n(A) f_1(\widehat{k}_t(A,n), \widehat{l}_t(A,n)) \tag{34}$$

$$\widehat{w}_t = \widehat{P}_t^n(A) f_2(\widehat{k}_t(A,n), \widehat{l}_t(A,n)) \tag{35}$$

3. Given $\widehat{N}_1$ and $\left\{\widehat{P}_1^n, \widehat{w}_1\right\}_{n=1}^{\widehat{N}_1}$ in $t=1$

$$\left\{\widehat{x}_t^1(n), ..., \widehat{x}_t^{\widehat{N}_1}(n)\right\}_{n=1}^{\widehat{N}_1}$$

solves for each manager j

$$\underset{x_1^1(j),...,x_1^{N_1}(j)}{Max} \left\{ \begin{array}{c} z_n A^e g\left(Min\left\{x_1^1(j), ..., x_1^{N_1}(j)\right\}\right) - \\ (1+\varphi R_1)\left(\sum_{n=1}^{N_1} P_1^n x_1^n(j)\right) - w_1 \end{array} \right\}$$

and given $\left\{\widehat{N}_t\right\}_{t=2,3,...}$ and

$$\left\{\widehat{P}_t^n(A), \widehat{w}_t(A)\right\}_{n=1;\ t=2,3,...}^{\widehat{N}_t} \left\{\widehat{x}_t^1(n), ..., \widehat{x}_t^{\widehat{N}_t}(n)\right\}_{n=1;\ t=2,3,...}^{\widehat{N}_t}$$

solves for each manager j

$$\underset{x_t^1(A,j),...,x_t^{N_t}(A,j)}{Max} \left\{ \begin{array}{c} z_j A^1 g\left(Min\left\{x_t^1(A,j), ..., x_t^{N_t}(A,j)\right\}\right) - \\ \left(\sum_{n=1}^{N_t} P_t^n(A) x_t^n(A,j)\right) - w_t(A) \end{array} \right\}$$

4. Equilibrium in the capital market requires

$$\sum_{n=1}^{\widehat{N}_1} \widehat{k}_1(A,n) = \widehat{k}_1(A), \text{ for } t=1$$

$$\sum_{n=1}^{\widehat{N}_t} \widehat{k}_t(A,n) = \widehat{k}_t(A), \text{ for } t \geq 2$$

5. Equilibrium in the capital market requires

$$\sum_{n=1}^{\widehat{N}_1} \widehat{l}_1(A,n) = J - \widehat{N}_1, \text{ for } t = 1$$

$$\sum_{n=1}^{\widehat{N}_t} \widehat{l}_t(A,n) = J - \widehat{N}_t, \text{ for } t \geq 2$$

6. Equilibrium in the intermediate goods market for requires that for each good m

$$\widehat{P}_1^m f\left[\widehat{k}_1(m), \widehat{l}_1(m)\right] = \sum_{n=1}^{\widehat{N}_1} \widehat{P}_1^m \widehat{x}_1^m(n), \text{ for } t = 1 \tag{36}$$

$$\widehat{P}_t^m(A) f\left[\widehat{k}_t(A,m), \widehat{l}_t(A,m)\right] = \sum_{n=1}^{\widehat{N}_t} \widehat{P}_t^m(A) \widehat{x}_t^m(A,n), \text{ for } t \geq 2 \tag{37}$$

7. Economy's aggregate resource constraint is satisfied

$$\sum_{j=1}^{J} \widehat{c}_1(A,j) + \widehat{k}_2(A) - (1-\delta)\widehat{k}_1(A) = \sum_{j=1}^{J} \widehat{\chi}_1(j) \left[z_j A g\left(Min\left\{\widehat{x}_1^1(j), ..., \widehat{x}_1^{\widehat{N}_1}(j)\right\}\right)\right] \tag{38}$$

$$\sum_{j=1}^{J} \widehat{c}_t(A,j) + \widehat{k}_{t+1}(A) - (1-\delta)\widehat{k}_t(A) = \sum_{j=1}^{J} \widehat{\chi}_t(A,j) \left[z_j A^1 g\left(Min\left\{\widehat{x}_{It}^{j1}, ..., \widehat{x}_{It}^{jN_t}\right\}\right)\right] \tag{39}$$

8. The fraction $\widehat{\mu}_1(A)$ is given by equation (21) if condition (16) is fulfilled for at least one manager in period 1 and $\widehat{\mu}_1(A)$ equals one otherwise.

9. $$\widehat{R}_1 = \frac{1}{(1-p)\widehat{\mu}_t(A^1) + p\widehat{\mu}_t(A^2)} - 1$$

10. $\widehat{\psi}_1 = \varphi$, and $\widehat{\psi}_t(A) = \varphi$ for $t \geq 2$ with $0 < \varphi < 1$.

11. For all managers of type m such that $m \leq \widehat{v}$ we have

$$\pi_{F1}(m) + \pi_{I1}(m) + \Delta y_{F1}(A,m) + \psi_1(\mu_1(A) - 1)P_1^m f\left[k_1(m), l_1(m)\right] \geq 0$$

5 The zero probability case

In this section we explore the case in which the probability of the bad shock being realized is equal to zero, that is, the value A^2 comes completely unexpected to all managers in the economy. In particular, as the probability of the bad shock approaches zero, the economy presented above resembles one not facing any uncertainty and having a constant aggregate level of productivity equal to A^1. Furthermore, this deterministic economy has a steady state where all variables are constant and that satisfies the equilibrium conditions given in our previous definition (when $p = 0$). In the following sections, we use this steady state to calibrate the parameters of the model presented above. Then, we simulate the economy using as our initial level of capital the one the deterministic economy approaches at the steady state. Later, we assume that people assign a zero probability of occurrence to the realization of A^2 in period 1. Finally, we see the effects on the economy when the actual realization of the shock is equal to A^2.

This case is of special interest because it allows us to see the effect credit chains can have on the business cycle.

5.1 Calibration

In order to be able to simulate the model, we assume the following functional forms for the production functions of the intermediate and final goods and for the utility function

$$f(k,l) = k^{\alpha} l^{1-\alpha}, \qquad 0 < \alpha < 1$$

$$g(x) = A\,(X)^{\phi} \qquad 0 < \phi < 1$$

$$U(c) = \ln c$$

To facilitate calculations, we assume that the economy is composed of only three types with $z_1 > z_2 > z_3$. Furthermore, we assume that agent 2 has managerial ability of such degree that it has exactly zero profits at the deterministic steady state. Moreover, this implies that 1 and 2 produce the final good and 3 provides the unskilled labor. We further assume that $z_1 = A^1 = 1$ to ease computations. Using the profit function specified above, these functional forms and our assumption that agent 2 has zero profits at the deterministic steady state, it is possible to obtain a formula for z_2 in terms of parameters α and ϕ which is given by

$$z_2 = \left[\frac{(1-\alpha)\phi}{1-\phi(2-\alpha)} \right]^{1-\phi} \tag{40}$$

Using the first order conditions for the firms we obtain that the labor's share of income equals in our model

$$\frac{J w_{ss}}{y_{Fss}} = J(1-\alpha)\phi \tag{41}$$

where y_{Fss} is total output produced at the deterministic steady state and J is the number of individuals in the economy, each of which, receive a wage regardless of their activity. We set this value to be equal to 2/3 as it is usually done in the Business Cycle literature.

Since only agent 1 earns profits (because of our assumption of zero profits for agent 2), the manager's share on output in the model is given by

$$\frac{\pi_{Fss}(z_1)}{y_{Fss}} = \frac{1-\phi}{1+(z_2)^{1/(1-\phi)}} - (1-\alpha)\phi \tag{42}$$

Quintin (2001) finds this share in the US to be between 0.09 and 0.145. We use the latter

Parameter	Value
α	0.4587
ϕ	0.4106
δ	0.0120
β	0.9950
θ	0.2000
ψ	to be determined

Table 3: Parameter Values

value. Note then that equations (40) through (42) give a system with three unknowns that we can solve for. From the first order conditions for the consumer's problem with managerial ability z_1 we obtain the following equation at the steady state

$$r_{ss} = \frac{1}{\beta} - 1 + \delta \tag{43}$$

and we assume an annual interest rate on assets equal to 7% as in Prescott (1986). We transform this into quarterly numbers since that will be our time unit. Furthermore, the corresponding value obtained for the depreciation rate of capital by Cooley and Prescott (1995) is 4.8% which gives a quarterly rate of 1.2%. We plug these values into the interest rate equation and obtain the discount factor β. Noting that the rate of return on capital is also given by

$$r_{ss} = \alpha\phi \frac{y_{Fss}}{k_{ss}} \tag{44}$$

and that total output produced in the economy is

$$y_{Fss} = \left[1 + {z_2}^{1/(1-\phi)}\right]^{1-\phi} \left[\frac{k_{ss}^{\alpha}}{2}\right]^{\phi}$$

we can obtain the level of capital at the steady state. Moreover, from the budget constraints for each individual we can obtain consumption levels at the steady state.

The last thing we need to talk about is the amount of income people will be able to hide from other firms in the case in which they cannot pay their debts. We assumed that this quantity is proportional to the amount lent. In particular, we let people that cannot fulfill their obligations hide and keep for themselves, 20% of the assets that people lent them. That is, we assume $\theta = 0.20$. Since we do not have any empirical data that corroborates this assumption, we later present simulations of the model where we change the value of this parameter and see whether it has any impact on the results.

Finally, we note that in equilibrium, people are indifferent between borrowing from the intermediary or from other firms. Therefore, any holdings of the two kinds of debt would be consistent with the definition given in the previous section. In particular, the exercise we perform below is to define the level of ψ exogenously and to change it in order to see the impact of an increased use of trade credit in the economy. Table 3 includes a summary of the values of all parameters in the model.

5.1.1 Results

As explained before, we assume here that the starting values for period 1 are those obtained from the steady state in a deterministic economy when $p = 0$. Before we go to the simulations, it is worth taking a look to the budget constraints. Specifically, for the three types of individuals we have that in period 1

$$c_1(A,1) + k_2(A) = w_1 + \pi_{F1}(1) + \pi_{I1}(1) + T_1(A,1) + (1-\delta+r_1)k_1(A) \tag{45}$$

$$c_1(A,2) = w_1 + \varphi\theta\left(\sum_{n=1}^{N} P_1^n x_1^n(2)\right) \tag{46}$$

$$c_1(A,3) = w_1 \tag{47}$$

where

$$T_1(A,1) = \Delta y_{F1}(A,1) + \psi_1(\mu_1(A) - \mu_1^e)(1+R_1)P_1^1 f\left[k_1(1), l_1(1)\right] \tag{48}$$

Any change in income perceived by agent one comes from this last equation. Regarding the first term in (48) note that it includes the decrease in productivity with respect to the expected level of output. Remember that

$$\Delta = \frac{(A^i - A^e)}{A^i}$$

In this case, $A^e = A^1 = 1$ and $A^i = A^2 < A^1$. Therefore, the first term in equation (48) is negative. This is standard. The second term however, introduces the quantity lost by firms because other managers did not pay the amount expected. This depends on the percentage of intermediate good production lent to other managers ψ_1, the difference between the fraction of loans repaid and the one the firm expected $(\mu_1(A) - \mu_1^e)$, and the value of those goods lent. In this exercise we have that $\mu_1(A) < \mu_1^e$ and therefore the second term is also negative.

At first sight an increase in trade credit use in the economy (a shift upwards in parameter ψ_1) seems to have a larger negative impact on income: the fact that a large proportion of the intermediate good produced is given to other firms on credit and that these firms do not pay in full implies a higher negative impact on profits for manager 1.

However, increases in the use of trade credit also have a positive impact on $\mu_1(A)$ as it is evident from Figure (1). The graph is done using a negative shock in productivity equal to 1% of total output produced at the steady state. Intuitively, this happens because as firms finance most of their input purchases with trade credit, the amount of disposable income available to them to pay back firms increases as ψ_1 goes up: since the amount paid to the intermediary becomes relatively small as ψ_1 increases, firms are able to repay a larger fraction to other firms and $\mu_1(A)$ goes up.

Summarizing the previous discussion, it is not possible to know before hand the effect that an increase in ψ_1 will have on the income of manager 1. Figure (2) shows the effect of changes in ψ_1 on the product $\psi_1(\mu_1(A) - \mu_1^e)$ when the shock in productivity equals 1% of the steady state output. As it can be seen in the figure, as trade credit use increases the shock will be amplified.

In order to see whether or not the use of trade credit amplifies the original shock we take a

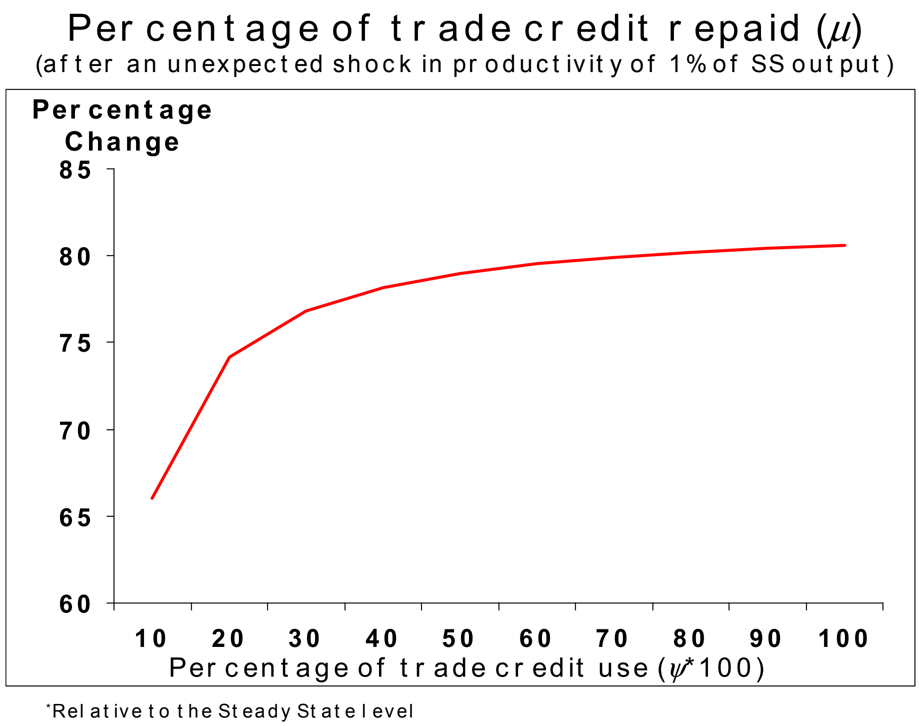

Figure 1: Percentage of Trade Credit repaid

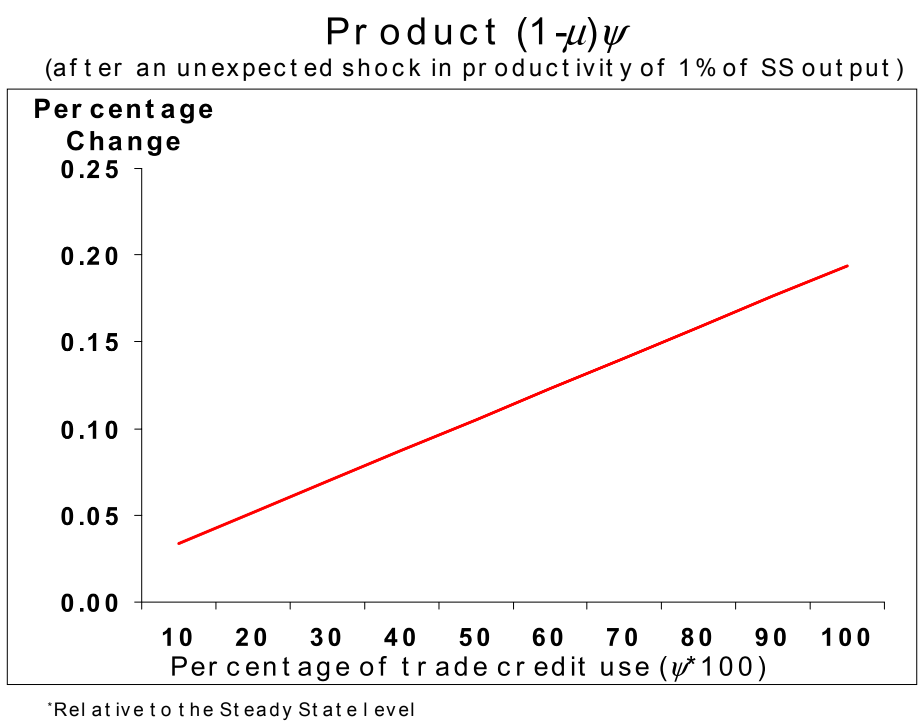

Figure 2: Product (1-mu)*psi

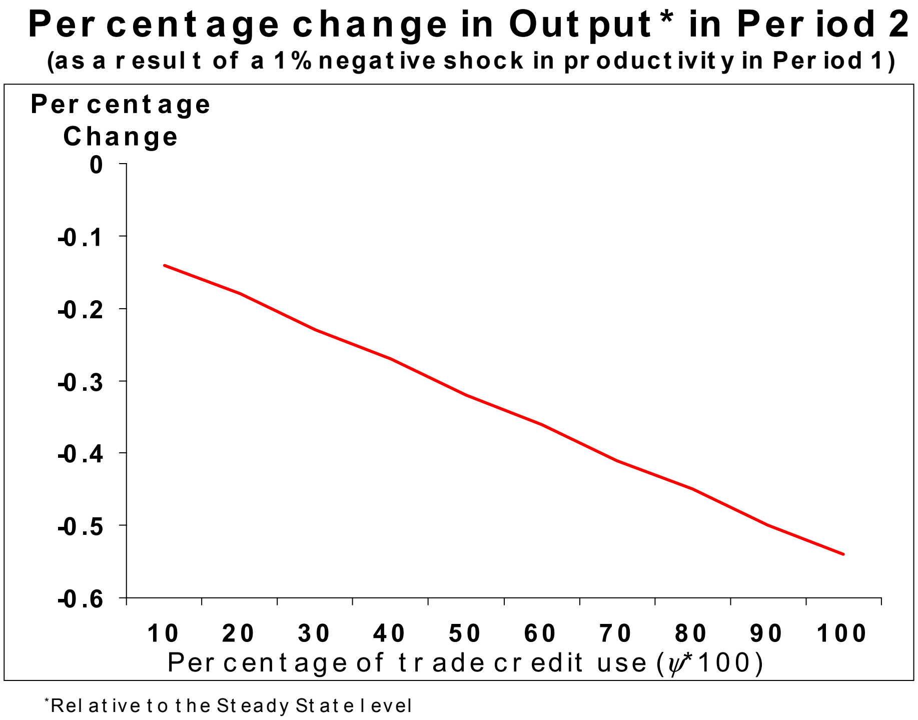

Figure 3: Effects on output of trade credit use

look at value of total output produced in period 2 when the level of aggregate productivity has returned to A^1. Furthermore, out of concern that the amplification of the shock might imply that the linear approximation explained above might not be a good fit we use nonlinear methods and specifically weighted residual methods to approximate them. This type of methods are explained in McGrattan (1999).[6] We use the Galerkin method to set the weight functions. We simulate the model hitting the economy at the steady state with a negative shock equal to one percent of the steady state output so that $\Delta = -0.01$. Thereafter, productivity goes back to its steady state level A^1 and people completely anticipate this. The results are given in Figure (3). Here we see that if the percentage of trade credit use is equal to 10%, output 1 period after the shock (when the level of productivity has gone back to A^1), is 0.14% less than the steady state value (the economy recovers rather quickly). We also see that as we increase the percentage of trade credit use, the effect on output becomes considerably stronger every time. For example, at the extreme, where everyone uses trade credit, output the period after the shock is 0.55% lower than the steady state value. That is almost four times more the impact we see in an economy with low percentages of trade credit use.

Since our choice of the parameter θ (the percentage of the loan kept by any firm not capable of repaying debt in full) was arbitrary we show in table (4) results of changing the value of θ and how this affects output in the second period. Particularly, we see that as firms

[6]Also, there has recently been some research that has found linear methods to be far less accurate than non linear. See in particular Boragan, Fernandez-Villaverde and Rubio-Ramirez (2003).

Effect on Output* of a negative productivity shock (Percentage change with respect to SS level)				
	Percentage of loans kept (θ%)			
Trade Credit Use (ψ%)	10	20	50	90
20	-0.14	-0.18	-0.32	-0.50
40	-0.18	-0.27	-0.54	-0.91
60	-0.23	-0.36	-0.77	-1.32
80	-0.27	-0.45	-1.00	-1.74
100	-0.32	-0.54	-1.23	-2.16

*The percentage change refers to output 1 period after the shock.

Table 4: Effects of changing theta

not repay a higher percentage of the loan, the negative effect of the credit chains in output is reinforced.

As it was shown at the beginning of this chapter, trade credit use seems to be considerably higher in Mexico than in the US. In particular, 65% of all firms in Mexico say that trade credit is their main source of financing. On the other hand, only 20% of the US firms seem to rely heavily on trade credit. Furthermore, we might expect the value of θ in Mexico to be higher than the one we see in the US: due to a lack of law enforcement, firms are able to cheat and hide a larger percentage of assets in less developed economies than what we see in rich ones. Let's assume these values of 65 and 20% for ψ and see what happens if $\theta = 10\%$ in the US and 20% in Mexico. The result is that the effect of the productivity shock is more than double in an economy with parameters closer to the ones seen in Mexico relative to what we see in the economy with parameters close to those observed in the US. Therefore, we conclude, the use of trade credit and the degree of law enforcement can have a tremendous impact on the transmission, propagation and amplification of productivity shocks in the economy.

5.2 Persistence

Regarding persistence note that after the shock the economy returns to its steady state along a transition path and that this takes more time as the economy uses more trade credit. However, this persistence is given mainly by the traditional neoclassical model and it only varies from one simulation to the other because of the size of the shock on manager's 1 first period income (which depends on the degree of trade credit use).

One manner in which trade credit might have a way to explain persistent economic shocks is through the fact that these contracts are signed in advance. The next step here would be to introduce non renegotiable contracts that would last for different periods of time implying that the inability to pay would be transmitted to other periods. This will be a topic of future research.

5.3 Asymmetry

Now imagine that the shock in income is a positive one. Then, equations (45) through (47) are exactly as before

$$c_1(A,1) + k_2(A) = w_1 + \pi_{F1}(1) + \pi_{I1}(1) + T_1(A,1) + (1 - \delta + r_1)k_1(A) \tag{49}$$

$$c_1(A,2) = w_1 + \varphi\theta \left(\sum_{n=1}^{N} P_1^n x_1^n(2) \right) \tag{50}$$

$$c_1(A,3) = w_1 \tag{51}$$

where

$$T_1(A,1) = \Delta y_{F1}(A,1) + \psi_1(\mu_1(A) - \mu_1^e)(1 + R_1)P_1^1 f\left[k_1(1), l_1(1)\right]$$

But now, since the shock in income is positive, $\mu_1(A) = \mu_1^e = 1$. That is, all producers pay back in full their debts to other managers. In this way is that the effect of credit chains is asymmetric in the model: when you have a positive income shock no firms go bankrupt and therefore, there is no transmission mechanism.

6 Conclusions

We have shown a model with trade credit that is able to create amplification of productivity shocks through the chains established when firms borrow from each other. When firms are unable to pay their debts, they decrease the value of accounts receivable for other agents in the economy. This in turn increases the amount of money that has to come from inside financing (own revenue) to pay current outstanding debts. As a consequence this has a negative impact on investment made by these firms. As trade credit use increases the size of the negative impact on these firms also goes up.

Furthermore, we have shown that the effect of credit chains in the economy can be asymmetric: strong during recessions and relatively weak during expansions. The model, however, fails at providing a new explanation for how persistent this shocks might be. This happens mainly because if a firm declares itself unable to pay its debts, it can return to the economy next period and produce as if nothing had happened. Further research should focus on relaxing this assumption.

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