Analyzing an Aging Population—A Dynamic General Equilibrium Approach*

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Abstract

The present paper shows the macroeconomic and welfare implications of an aging population in the United States, using an overlapping-generations model with heterogeneous households. The model uses three population projections in Social Security Administration (2003), and generates economies as equilibrium transition paths from 1961 to 2200. The paper demonstrates how several different population projections and government financing assumptions—to make the Social Security system sustainable—affect households' decisions and welfare. The paper also shows that an immediate increase in the payroll tax might not improve the welfare of future generations.

Journal of Economic Literature Classification Numbers: D9, H3, H6.

Key Words: Social Security, Life Cycle, Overlapping Generations, Heterogeneous Agent, Aging Population.

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1 Introduction

Many proposals for reforming the Social Security system in the United States have been advanced in recent years. For example, Representatives Kolbe and Stenholm proposed the 21st Century Retirement Security Act in 1998 and revised it in 1999 and 2001; and the President's Commission (2001) proposed three possible reform plans of U.S. Social Security.

Although many have already analyzed how much those proposals would improve the actuarial balance, the macroeconomic and welfare implications of those reform plans are still uncertain. However, to evaluate those effects of Social Security reform, a reasonable baseline economy under the current law is also required.

The present paper constructs possible baseline economies with an aging population to analyze Social Security reform plans, using an overlapping generations (OLG) model with heterogeneous households. In this model, households receive idiosyncratic working ability shocks and mortality shocks. Then, the paper shows the effects of simple reform plans as policy experiments.

In this process, the following two aspects are stressed in the present paper:

First, like most other developed countries, the population distribution of the United Sates is aging and, accordingly, the economy cannot be described as a stationary equilibrium. The present paper constructs economies as equilibrium transition paths, solving the model from 1961 through 2200.[1]

Second, under a realistic population projection, the current-law Social Security system is not sustainable. To solve the model for an equilibrium transition path, the model needs to have an additional financing assumption to close the intertemporal budget constraint for the Social Security system.[2]

In Social Security Administration (2003), Trustees of Social Security have used three possible population projections—alternative II (intermediate), alternative I (low cost), and alternative III (high cost)—to evaluate the sustainability of the current Social Security sys-

[1] At the beginning of 1961, households in the model realize an aging population and choose their optimal consumption, labor supply, and savings based on the correct population projection. The model assumes that this adjustment process to an aging population from the initial steady state in 1961 is completed by 2003.

[2] Some examples of government financing assumptions for the Social Security system are whether the payroll tax is increased or benefits are reduced, or both, and when the government changes the payroll tax rate or benefit replacement rates.

tem. The present paper uses the same three population projections by extrapolating those projections beyond 2080.

Regarding the financing assumption, the present paper assumes that the payroll tax is increased and benefits are reduced when the trust funds are depleted, so that each of these policy changes covers a half of the deficit and that the trust funds are kept at zero thereafter.[3]

Then, the paper shows the effect of alternative financing assumptions—the payroll tax is increased when the trust funds are depleted but benefits are kept at the current-law level; alternatively, benefits are reduced when the trust funds are depleted but the payroll tax rate is kept at the current-law level; and, finally, the payroll tax rate is increased immediately in 2004 by 10 percent.

The rest of the government budget, which includes Medicare and Medicaid, are made as simple as possible. The government budget is balanced by adjusting government consumption, which is not in the utility function of the model, so that the per-capita government net wealth grows at the same rate of the productivity growth.[4]

Compared to a balanced growth path, private savings and saving rates in the aging baseline decline throughout the period of 2004-2200, but per-capita private wealth increases because of the improved longevity and larger life cycle savings. Per-capita labor supply also increases until 2013, then, decreases monotonically to a level below that in 2004.

The capital-labor ratio rises in the aging baseline economy, and as a result, the interest rate falls and the wage rate rises significantly as the population ages. How much these factor prices will change depends on the population projection and the financing assumption. One of the interesting findings from the numerical experiments is that an immediate increase in the payroll tax rate might not improve the welfare of future generations, although the trust funds would last much longer.

The present paper is not the first one that calibrates a dynamic general equilibrium OLG model to an aging society. Auerbach and Kotlikoff (1987) analyzed the effects of two stylized

[3]In a stationary economy, the baseline is often an economy under the current law. The present paper chooses the financing assumption for the baseline such that the government keeps the current-law Social Security system as reasonably long as possible without affecting the rest of the government budget, although the trust funds are merely an accounting device and the government does not have to change the law in that timing.

[4]There are some alternative assumptions. For example, the government consumption is population indexed, and individual income tax rates are adjusted so that the rest of the government budget is balanced.

aging populations on the saving rate and Social Security system, using a representative-agent overlapping generations model. More recently, De Nardi, İmrohoroğlu, and Sargent (1999) analyzed Social Security reform plans by solving their model for years between 1975 and 2200.[5] Ríos-Rull.(2000) calibrates a model to the Spanish economy with a stylized aging population. Kotlikoff, Smetters, and Walliser (2001) and Fehr, Jokisch, and Kotlikoff (2003) analyze the effect of demographic changes in the United States.

The rest of the paper is laid out as follows: Section 2 describes the model economy; Section 3 shows the calibration of the model; Section 4 shows the baseline economy with an aging population; Section 5 shows a few simple policy experiments; and Section 6 concludes the paper.

2 Model

The base model used in the present paper is a standard overlapping generations growth model with uninsurable idiosyncratic working ability shocks and mortality shocks.[6] The economy consists of heterogeneous households, a perfectly competitive representative firm, and a government with a full commitment technology. Time is discrete, and a period of the model corresponds to a year. Regarding the openness of the economy, the present paper assumes two polar cases—a closed economy and a small open economy.

The Household's Problem. Households are heterogeneous with respect to their ages, working abilities, asset holdings, and working histories. For simplicity, all households are assumed to be two-earner married couples of the same age, who make their decisions jointly.

Every year, a large number of new households of age 20 enter into the economy. A household of age i receives idiosyncratic working ability shock, e_i, at the beginning of each year t, and chooses its optimal consumption c_i, working hours h_i, and end-of-period wealth holding a_{i+1}, taking a government policy rule $\mathbf{\Psi}_t$, a population projection $\mathbf{\Phi}_t$, and a series

[5] The main differences between De Nardi *et al.* (1999) and the present paper are as follows: The former uses a quadratic utility function so that the household's decision rules and the laws of motion are expressed by linear functions; assumes idiosyncratic endowment shocks rather than working ability shocks; and assumes that labor income tax or consumption tax is adjusted once in 10 years.

[6] The base model is similar to those in Aiyagari (1994), Huggett (1996), and many others, although Aiyagari (1994) assumed infinitely-lived agents. The model is also an extension of that in Nishiyama (2002) and Nishiyama and Smetters (2003).

of factor prices and policy variables $\mathbf{\Omega}_t$, as given.[7] At the end of each year, a fraction $1-\phi_{i,t}$ of households die. Households are possibly alive until age 120, and the survival rate at the end of age 120, $\phi_{120,t}$, is assumed to be zero.

Let $\mathbf{s}_i$ denote the individual state of an age i household,

$$\mathbf{s}_i = (i, e_i, a_i, b_i)\,, \tag{1}$$

where $i \in I = \{(0, ..., 19,)\ 20, ..., 120\}$ is the household's age,[8] $e_i \in E = [e^{\min}, e^{\max}]$ is its working ability (measured by its hourly wage), $a_i \in A = [a^{\min}, a^{\max}]$ is its beginning-of-period asset holding, and $b_i \in B = [b^{\min}, b^{\max}]$ is its average historical earnings.[9] Let $\mathbf{S}_t$ denote the state of the economy at the beginning of year t,

$$\mathbf{S}_t = (x_t(\mathbf{s}_i), W_{S,t}, W_{G,t})\,, \tag{2}$$

where $x_t(\mathbf{s}_i)$ is the measure of households for $\mathbf{s}_i \in I \times E \times A \times B$,[10] $W_{S,t}$ is the beginning-of-period Social Security trust funds, and $W_{G,t}$ is the rest of the government net wealth. Let $\mathbf{\Psi}_t$ denote the government policy rule known at the beginning of year t,[11]

$$\mathbf{\Psi}_t = \{W_{S,t+1}, W_{G,s+1}, \tau_{PO,s}(.), \tau_{PH,s}(.), tr_{SS,s}(.), \tau_{I,s}(.), C_{G,s}\}_{s=t}^{\infty}\,, \tag{3}$$

where $\tau_{PO,s}(.)$ is a payroll tax function for the Old-Age, Survivors, and Disability Insurance (OASDI), $\tau_{PH,s}(.)$ is a payroll tax function for the Hospital Insurance (HI), $tr_{SS,s}(.)$ is an OASDI benefit function, $\tau_{I,s}(.)$ is a progressive income tax function, and $C_{G,s}$ is government's consumption. In the present model, government consumption, $C_{G,t}$, is not in the utility function of a household.

[7] The government does not solve its optimization problem to determine the policy. The government policy rule $\mathbf{\Psi}_t$ is equivalent with its financing rule regarding the Social Security budget and the rest of the government budget, which is assumed to be credible. The population projection Φ_t is deterministic. Because there are no aggregate shocks in this economy, households can perfectly foresight a series of future factor prices and policy variables Ω_t based on the information currently available.

[8] Ages $i \leq 19$ are used to calculate the average number of dependent children and the population of age 20 households in each year t.

[9] The average historical earnings b_i are the approximation of the Average Monthly Indexed Earnings (AIME) multiplied by 12 and used to calculate the household's Social Security benefits in the model.

[10] In other words, $x_t(\mathbf{s}_i)$ is the joint distribution of households in year t multiplied by the population in year t.

[11] At least one of the series in Ψ_t is unknown to the households. In the policy experiments below, the government announces that it will increase the payroll tax and reduce benefits when the trust funds are depleted, so that the trust funds will not be negative. Although the government does not announce explicitly when and how much it will change the payroll tax rate and benefits, households in the model expect rationally the timing and sizes of those changes.

The household's problem is

$$v(\mathbf{s}_i, \mathbf{S}_t; \mathbf{\Psi}_t, \mathbf{\Phi}_t) = \max_{c_i, h_i, a_{i+1}} u_i(c_i, h_i) + \beta\phi_{i,t} E\left[v(\mathbf{s}_{i+1}, \mathbf{S}_{t+1}; \mathbf{\Psi}_{t+1}, \mathbf{\Phi}_{t+1}) | e_i\right] \quad (4)$$

subject to

$$a_{i+1} = \frac{1}{1+\mu}\{w_t e_i h_i + (1+r_t)a_i - \tau_{I,t}(w_t e_i h_i, r_t a_i, tr_{SS,t}(i, b_i)) \quad (5)$$
$$-\tau_{PO,t}(w_t e_i h_i) - \tau_{PH,t}(w_t e_i h_i) + tr_{SS,t}(i, b_i) - c_i\} \geq a^{\min},$$
$$a_{20} = 0, \quad a_{121} \geq 0,$$

where $u_i(.)$ is a period utility function of an age i household, β is the time-preference factor, $\phi_{i,t}$ is the survival rate at the end of age i, w_t is the wage rate, and r_t is the real rate of return to capital.[12] Individual variables, except for working hours, are normalized by the steady-state per capita growth rate μ. Let $\pi_{i,i+1}(e_{i+1} \,|\, e_i)$ denote the conditional probability for the age $i+1$ working ability being e_{i+1} when the age i working ability is e_i. Then,

$$E\left[v(\mathbf{s}_{i+1}, \mathbf{S}_{t+1}) \,|\, e_i\right] = \int_E v(\mathbf{s}_{i+1}, \mathbf{S}_{t+1}) \pi_{i,i+1}(e_{i+1} \,|\, e_i) \, \mathrm{d}\, e_{i+1}. \quad (6)$$

At the beginning of the next period, the state of the household and the state of the economy become

$$\mathbf{s}_{i+1} = (i+1, e_{i+1}, a_{i+1} + q_t, b_{i+1}) \quad \text{with} \quad \pi_{i,i+1}(e_{i+1} \,|\, e_i), \quad (7)$$

$$\mathbf{S}_{t+1} = (x_{t+1}(.), W_{S,t+1}, W_{G,t+1}), \quad (8)$$

where q_t denotes accidental bequests that a household receives at the end of the period, and $W_{S,t+1}$ and $W_{G,t+1}$ are determined by the government budget constraints. The average historical earnings b_i follows

$$b_{i+1} = \begin{cases} 0 & \text{if } i \leq 24 \\ \frac{1}{i-24}\{(i-25)b_i \frac{w_t}{w_{t-1}} + \min(w_t e_i h_i/2, w e h_t^{\max})\} & \text{if } 25 \leq i \leq 59 \\ (1+\mu)^{-1} b_i & \text{if } i \geq 60, \end{cases} \quad (9)$$

[12] The variable w_t is the wage rate per efficiency unit of labor, which is normalized to unity in 2001, and $w_t e_i$ denotes the hourly wage of each household of age i with working ability e_i in year t. The variable r_t is the pre-tax real market rates of return to capital, which is around 6.25 percent in 2003 in the model. The real rates of return to the Social Security trust funds are adjusted to 3.0 percent in 2003 so that the present model generates projections similar to those of the Trustees Report.

where $weh_t^{\max}$ is the OASDI payroll tax cap. Under the current law, the Average Indexed Monthly Earnings (AIME) is calculated from the highest 35 years of earnings. For simplicity, the model assumes that the highest 35 years of earnings correspond to those years of age between 25 and 59.[13]

The decision rule of households is shown as

$$\mathbf{d}\left(\mathbf{s}_i, \mathbf{S}_t; \mathbf{\Psi}_t, \mathbf{\Phi}_t\right) = \left\{c_i\left(\mathbf{s}_i, \mathbf{S}_t; \mathbf{\Psi}_t, \mathbf{\Phi}_t\right), h_i\left(\mathbf{s}_i, \mathbf{S}_t; \mathbf{\Psi}_t, \mathbf{\Phi}_t\right), a_{i+1}\left(\mathbf{s}_i, \mathbf{S}_t; \mathbf{\Psi}_t, \mathbf{\Phi}_t\right)\right\},$$

for $i = 20, ..., 120$.[14]

The Measure of Households. Let $x_t\left(\mathbf{s}_i\right)$ denote the measure of households, and let $X_t\left(\mathbf{s}_i\right)$ be the corresponding cumulative measure. The measure of households is adjusted by the long-run population growth rate ν.

The measure of new born people in year $t+1$ is calculated, if it is not exogenously determined, from the age-dependent fertility rates, $f_{i,t}$, and age-population distribution, that is

$$x_{t+1}\left(\mathbf{s}_0\right) = \sum_{i=15}^{49} \int_{E \times A \times B} f_{i,t} \mathrm{d}X_t\left(\mathbf{s}_i\right), \tag{10}$$

where $\mathbf{s}_0 = (0, -, 0, 0)$.[15] Then, the measures of people of ages 1, ..., 19 are calculated from the survival rates at the end of each age,

$$x_{t+1}\left(\mathbf{s}_{i+1}\right) = \frac{\phi_{i,t}}{1+\nu} \int_{E \times A \times B} \mathrm{d}X_t\left(\mathbf{s}_i\right) \qquad \text{for } i \leq 19, \tag{11}$$

where $\mathbf{s}_i = (i, -, 0, 0)$. A household of age 20 is assumed to have no initial wealth and working history. So, $\int_E \mathrm{d}X_t\left(20, e_{20}, 0, 0\right)$ is the population of age 20 households in year t. Let $\mathbf{1}_{[a=y]}$ be an indicator function that returns 1 if $a = y$ and 0 if $a \neq y$. The law of motion

[13] Earnings before age 60 are wage indexed and earnings after age 60 are price indexed. The approximation of AIME by the average historical earnings follows previous Social Security literature, for example, Huggett and Ventura (1999) and De Nardi *et. al.* (1999).

[14] The departure from the previous literature is simply that the household's decision depends not only on its own state, the sate of the economy, and the government policy rule, but also the household's belief (perfect foresight) on the population transition.

[15] The measure of new born people is equal to the new born population divided by 2, because the decision unit in the model economy is a married couple. In the present paper, Equation (10) is used to extrapolate the population projections in Social Security Administration (2003) beyond 2080.

of the measure of households is, for $i = 20, ..., 120$,

$$x_{t+1}(\mathbf{s}_{i+1}) = \frac{\phi_{i,t}}{1+\nu} \int_{E\times A\times B} \mathbf{1}_{[a_{i+1}=a_{i+1}(\mathbf{s}_i,\mathbf{S}_t;\mathbf{\Psi}_t,\mathbf{\Phi}_t)+q_t]} \\ \times \mathbf{1}_{[b_{i+1}=b_{i+1}(w_t e_i h_i(\mathbf{s}_i,\mathbf{S}_t;\mathbf{\Psi}_t,\mathbf{\Phi}_t),b_i)]} \pi_{i,i+1}(e_{i+1}|e_i) \mathrm{d}X_t(\mathbf{s}_i) . \tag{12}$$

For simplicity, accidental bequests, q_t, due to uncertain life span are captured by the government and distributed to all surviving working-age households in a lump-sum manner.[16]

The Firm's Problem. National wealth W_t is the sum of total private wealth, the Social Security trust funds $W_{S,t}$, and the rest of the government net wealth $W_{G,t}$. Total labor supply L_t is measured in efficiency units. Then,

$$W_t = \sum_{i=20}^{120} \int_{E\times A\times B} a_i \, \mathrm{d}X_t(\mathbf{s}_i) + W_{S,t} + W_{G,t}, \tag{13}$$

$$L_t = \sum_{i=20}^{120} \int_{E\times A\times B} e_i \, h_i(\mathbf{s}_i, \mathbf{S}_t; \mathbf{\Psi}_t, \mathbf{\Phi}_t) \, \mathrm{d}X_t(\mathbf{s}_i) . \tag{14}$$

In a closed economy, capital stock K_t is equal to national wealth, that is, $K_t = W_t$, and gross national product Y_t is determined by a constant-returns-to-scale production function,[17]

$$Y_t = F(K_t, L_t). \tag{15}$$

The profit-maximizing condition of the representative firm is

$$F_K(K_t, L_t) = r_t + \delta, \tag{16}$$

$$F_L(K_t, L_t) = (1 + \tau'_{PO,t} + \tau'_{PH,t}) w_t, \tag{17}$$

where δ is the depreciation rate of capital, and $\tau'_{PO,t} + \tau'_{PH,t}$ is the marginal payroll tax rate for the representative firm.[18]

[16] Some of the computationally feasible extensions of the treatment of accidental bequests are, first, assuming age-dependent accidental bequest receipts based on the average age difference between parents and children; and, second, making accidental bequest receipts stochastic, i.e., the wealth left by a deceased household is given to another relatively young household by lottery.

[17] In a closed economy, gross national product equals gross domestic product.

[18] Here, $(1 + \tau'_{PO,t} + \tau'_{PH,t})w_t$ denotes the cost of an efficiency unit of labor for the firm. For high income households, whose labor income is above the OASDI payroll tax cap, $\tau'_{PO,t} + \tau'_{PH,t}$ is 0.0145 rather than 0.0765 under the current law. In the calibration below, it is assumed to be 0.0594 so that the goods market clear in 2001.

In a small open economy, factor prices r_t^* and w_t^* are fixed at the international levels of those, and domestic capital stock $K_{D,t}$ and labor supply L_t are determined so that the firm's profit maximizing condition satisfies, that is,

$$F_K(K_{D,t}, L_t) = r_t^* + \delta, \tag{18}$$

$$F_L(K_{D,t}, L_t) = (1 + \tau'_{PO,t} + \tau'_{PH,t}) w_t^* \tag{19}$$

Gross domestic product $Y_{D,t}$ and gross national product Y_t are calculated as

$$Y_{D,t} = F(K_{D,t}, L_t), \tag{20}$$

$$Y_t = (r_t^* + \delta)\, W_t + (1 + \tau'_{PO,t} + \tau'_{PH,t}) w_t^* L_t, \tag{21}$$

respectively.

The Government's Policy Rule. Government tax revenue consists of income tax, $T_{I,t}$, payroll tax for OASDI, $T_{PO,t}$, and payroll tax for HI, $T_{PH,t}$. These revenues are calculated as

$$T_{I,t} = \sum_{i=20}^{120} \int_{E\times A\times B} \tau_{I,t}\left(w_t e_i h_i(\mathbf{s}_i, \mathbf{S}_t; \mathbf{\Psi}_t, \mathbf{\Phi}_t), r_t a_i, tr_{SS,t}\left(i, b_i\right)\right) \, \mathrm{d}X_t\left(\mathbf{s}_i\right), \tag{22}$$

$$T_{PO,t} = 2 \times \sum_{i=20}^{120} \int_{E\times A\times B} \tau_{PO,t}\left(w_t e_i h_i(\mathbf{s}_i, \mathbf{S}_t; \mathbf{\Psi}_t, \mathbf{\Phi}_t)\right) \, \mathrm{d}X_t\left(\mathbf{s}_i\right), \tag{23}$$

$$T_{PH,t} = 2 \times \sum_{i=20}^{120} \int_{E\times A\times B} \tau_{PH,t}\left(w_t e_i h_i(\mathbf{s}_i, \mathbf{S}_t; \mathbf{\Psi}_t, \mathbf{\Phi}_t)\right) \, \mathrm{d}X_t\left(\mathbf{s}_i\right). \tag{24}$$

For the computational convenience, the payroll tax functions, $\tau_{PO,t}(.)$ and $\tau_{PH,t}(.)$, are assumed to show the taxes levied on employees (married couples) only. Because the same taxes are also levied on employers, the aggregate tax revenues in equations (23) and (24) are multiplied by 2.

Total Social Security (OASDI) benefits $Tr_{SS,t}$ equals

$$Tr_{SS,t} = \sum_{i=20}^{120} \int_{E\times A\times B} tr_{SS,t}\left(i, b_i\right) \, \mathrm{d}X_t\left(\mathbf{s}_i\right). \tag{25}$$

The laws of motion of the trust funds and the rest of government net wealth—both of those are normalized by productivity growth and long-run population growth—are

$$W_{S,t+1} = \frac{1}{(1+\mu)(1+\nu)} \left\{(1+r_t)W_{S,t} + T_{PO,t} - Tr_{SS,t}\right\}, \tag{26}$$

$$W_{G,t+1} = \frac{1}{(1+\mu)(1+\nu)} \left\{(1+r_t)W_{G,t} + T_{I,t} + T_{PH,t} - C_{G,t}\right\}. \tag{27}$$

Definition **Recursive Competitive Equilibrium (Equilibrium Transition Path):** Let $\mathbf{s}_i = (i, e_i, a_i, b_i)$ be the individual state of households, let $\mathbf{S}_t = (x_t(\mathbf{s}_i), W_{S,t}, W_{G,t})$ be the aggregate state of the economy, let $\mathbf{\Psi}_t$ be the government policy rule known at the beginning of year t,

$$\mathbf{\Psi}_t = \{W_{S,t+1}, W_{G,s+1}, \tau_{PO,s}(.), \tau_{PH,s}(.), tr_{SS,s}(.), \tau_{I,s}(.), C_{G,s}\}_{s=t}^{\infty},$$

and let $\mathbf{\Phi}_t$ be the perfect-foresight population projection. A series of factor prices, accidental bequests, the government policy variables, and the parameters φ_s of government policy functions,

$$\mathbf{\Omega}_t = \{r_s, w_s, q_s, W_{S,s+1}, W_{G,s+1}, C_{G,s}, \varphi_s\}_{s=t}^{\infty};$$

the value function of households, $\{v(\mathbf{s}_i, \mathbf{S}_s; \mathbf{\Psi}_s, \mathbf{\Phi}_s)\}_{s=t}^{\infty}$; the decision rule of households,

$$\{\mathbf{d}(\mathbf{s}_i, \mathbf{S}_s; \mathbf{\Psi}_s, \mathbf{\Phi}_s)\}_{s=t}^{\infty} = \{c_i(\mathbf{s}_i, \mathbf{S}_s; \mathbf{\Psi}_s, \mathbf{\Phi}_s), h_i(\mathbf{s}_i, \mathbf{S}_s; \mathbf{\Psi}_s, \mathbf{\Phi}_s), a_{i+1}(\mathbf{s}_i, \mathbf{S}_s; \mathbf{\Psi}_s, \mathbf{\Phi}_s)\}_{s=t}^{\infty};$$

and the measure of households, $\{x_s(\mathbf{s}_i)\}_{s=t}^{\infty}$, are in a recursive competitive equilibrium if, in every year $s = t, ..., \infty$, each household solves the utility maximization problem (1) – (5) taking $\mathbf{\Psi}_t$ and $\mathbf{\Phi}_t$ as given; the firm solves the profit maximization problem, and the capital and labor markets clear, that is, (13) – (19) hold; the government policy rules satisfy (22) – (27); and the goods market clears.

3 Calibration

This section explains the procedure, assumption, and parameterization of the model to construct baseline economies with an aging population as equilibrium transition paths.

3.1 The Procedure

Since an economy with an aging population is not stationary, it is constructed as an equilibrium transition path from 1961 through 2200. The final year 2200 of the transition path is assumed to be in a steady state.[19]

1. Calibrate the model described in Section 2 to the 2001 U.S. economy as if it is in a steady state, using the actual age-population distribution and mortality rates in 2001.[20] Choose the time preference factor β and the share parameter for consumption α in the utility function so that the steady-state economy is consistent with the 2001 U.S. economy with respect to the capital-output ratio and the average annual working hours of households. Also choose total factor productivity A so that the wage rate w is normalized to unity.

2. Calibrate the model to the 1961 U.S. economy as if it is in a steady state, using the actual age-population distribution and mortality rates in 1961 and the parameters β, α, and A obtained in the previous step.[21] Choose the parameters in the OASDI payroll tax function and OASDI benefit function so that total OASDI tax revenue and benefit expenditure in the model economy are equal to those in 1961 as percentages of GDP.

3. Solve the model for an equilibrium transition path from 1961 through 2200, using the same parameters β, α, and A, and the population projection through 2200. For years between 1961 and 2002, choose the parameters in the payroll tax function and the benefit function, and the rate of return to the trust funds so that those are consistent with the data as percentages of GDP.[22]

[19] In the final version of the paper, I am planning to solve the model through 2300 for more accurate computation of the economy.

[20] To solve the model for a steady-state equilibrium using an population distribution and mortality rates in 2001, we have to assume the households in the model *falsely* believe that the population distribution is stationary.

[21] Again, to solve the model for a steady-state equilibrium using an population distribution and mortality rates in 1961, we have to assume the households in the model *falsely* believe that the population distribution is stationary. This assumption is probably acceptable because this step is merely a preparation for constructing a model economy in 2004-2200.

[22] For policy experiments, we solve the model for equilibrium transition paths in 2004-2200, using the state of the economy in 2004.

3.2 The Government Policy Rule Ψ_t

Regarding the government policy rule,

$$\mathbf{\Psi}_t = \{W_{S,s+1}, W_{G,s+1}, \tau_{PO,s}(.), \tau_{PH,s}(.), tr_{SS,s}(.), \tau_{I,s}(.), C_{G,s}\}_{s=t}^{\infty},$$

in the baseline economy with an aging population, the present paper makes the following financing assumptions.

3.2.1 The Social Security (OASDI) Budget

The OASDI surplus—the difference between the OASDI payroll tax revenue, $T_{PO,t}$, and the benefit expenditure, $Tr_{SS,t}$—is added to the Social Security trust funds, $W_{S,t}$, as long as the trust funds are positive. That is,

$$W_{S,t+1} = \frac{1}{(1+\mu)(1+\nu)} \max\{(1+r_t)W_{S,t} + T_{PO,t} - Tr_{SS,t}, 0\},$$

for all t if $W_{S,s} > 0$ for all $s \leq t$.[23] Once the trust funds are depleted, the trust funds are kept at zero thereafter, and either the payroll tax rate is raised, or the benefit replacement rates are reduced proportionally, or both by splitting the deficit evenly,[24] to close the intertemporal budget constraint of Social Security. That is,

$$W_{S,t+1} = 0,$$

$$Tr_{SS,t} - T_{PO,t} = (1+r_t)W_{S,t},$$

for all t if there exists $W_{S,s} = 0$ for $s \leq t+1$. If the trust funds are not depleted before 2104 (100 years from now), to obtain the final steady-state equilibrium, the trust funds are kept at the same level (after growth adjustments) thereafter, and either the payroll tax rate or the benefit replacement rates or both of those are changed to close the inter-temporal budget constraint. That is,

$$W_{S,t+1} = W_{S,t},$$

$$Tr_{SS,t} - T_{PO,t} = \{(1+r_t) - (1+\mu)(1+\nu)\} W_{S,t},$$

for all $t \geq 2104$ if $W_{S,s} > 0$ for all $s \leq 2104$.

[23] All aggregate variables in the model are normalized using the steady-state (long-run) growth rate μ and population growth rate ν.

[24] In the baseline economy, the present paper assumes that, when the trust funds are depleted, both the payroll tax rate is raised and the OASDI replacement rates are reduced to cover the deficit evenly.

Table 1: The Source of the Population Projection

<table>
<tr><th></th><th colspan="3">Year</th></tr>
<tr><th></th><th>1941-1960</th><th>1961-2080</th><th>2080-2200</th></tr>
<tr><td>Population of ages:
0-99 by age
100-120 in total</td><td colspan="2">The 2003 Trustees Report
Alternatives I, II, and III</td><td rowspan="2">Extrapolated</td></tr>
<tr><td>100-120 by age</td><td></td><td>Estimated</td></tr>
</table>

3.2.2 The Rest of the Government Budget

The rest of the government net wealth $W_{G,t}$ is simply assumed to grow at the same rate as the long-run growth rate μ and year-by-year population growth rate ν_t, that is,

$$W_{G,t+1} = \frac{1}{(1+\mu)(1+\nu)} \{(1+\mu)(1+\nu_t) W_{G,t}\}.$$

To close the rest of the government intertemporal budget constraint, government consumption is obtained as the residual,[25]

$$C_{G,t} = (1+r_t)W_{G,t} - (1+\mu)(1+\nu) W_{G,t+1} + T_{I,t} + T_{PH,t}.$$

For years before 2004, the present paper assumes that the Social Security budget is combined with the rest of the government budget, or the trust funds are not pre-funded. That is,

$$W_{S,t+1} + W_{G,t+1} = \frac{1}{(1+\mu)(1+\nu)} (1+\mu)(1+\nu_t)(W_{S,t} + W_{G,t}),$$

and

$$\begin{aligned} C_{G,t} &= (1+r_t)(W_{S,t} + W_{G,t}) - (1+\mu)(1+\nu)(W_{S,t+1} + W_{G,t+1}) \\ &\quad + T_{I,t} + T_{PO,t} + T_{PH,t} - Tr_{SS,t}. \end{aligned}$$

3.3 The Population Projection Φ_t

The present paper uses the population projections—alternative II (intermediate), alternative I (low cost), and alternative III (high cost)— in Social Security Administration (2003). The

[25]We can alternatively assume that either income tax or the payroll tax for HI are changed to close the budget constraint. The present paper focuses mainly on the Social Security (OASDI) budget and tries to avoid the policy influence from the rest of the government budget.

Table 2: Age-Specific Fertility Rates (Per 1,000 Females, 2003)

Age of Females	Year 2003	Year 2080-2200
15-19	57.0	57.0 $\times\varphi_{f,t}$
20-24	112.6	112.6 $\times\varphi_{f,t}$
25-29	113.1	113.1 $\times\varphi_{f,t}$
30-34	85.0	85.0 $\times\varphi_{f,t}$
35-39	35.6	35.6 $\times\varphi_{f,t}$
40-44	7.0	7.0 $\times\varphi_{f,t}$
45-49	0.3	0.3 $\times\varphi_{f,t}$
Total Fertility Rate	2.07	

Source: U.S. Census Bureau (2003). In the model, the adjustment factor $\varphi_{f,t}$ of fertility rates and corresponding total fertility rate in 2080-2200 are 1.0581 and 2.19, respectively, in Alternative I, 0.9441 and 1.95 in Alternative II, and 0.8294 and 1.72 in Alternative III. In all three assumptions, the sex ratio at birth (male per female) is 1.0498 in 2080-2200.

Social Security Administration projected the population distributions until 2080. As Table 1 shows, the populations in 2081-2200 are extrapolated, using the age-specific fertility rates of women of ages 15-49 in 2003 and an adjustment factor $\varphi_{f,t} = \varphi_{f,2080}$. (See Table 2.) The adjustment factor is calculated for each population projection to match the total fertility rate in 2080.[26] The age-population distributions in 1961 and 2001, and the projections in selected years (every 25 years from 2003, and 2200) are shown in Figure 1. The long-run annual population growth rates ν of the population projection alternatives I, II, and III are 0.594%, 0.154%, and -0.312%, respectively.[27]

The survival rates $\phi_{i,t}$ of households at the end of age i in year t are simply calculated from the population projection. For people below age 45, $\phi_{i,t}$ tend to be greater than one because the projections include immigrations.

Figure 2 shows the population share of elderly people (aged 65 or above) and the ratio of the working-age (aged 20-64) population to the elderly population for three alternative population projections. The latter ratio rises very slightly until year 2006, then falls monotonically (except for Alternative I) thereafter.

[26]In addition, the populations of people 100-120 years of age in 1961-2080 are estimated, using the population data in 1941-1960 and mortality rates in 1998 with proportionate year adjustments, so that the total populations of ages 100-120 are equal to the numbers in the Trustees' projection.

[27]The program used to calculate the population projections in this paper will be provided upon request.

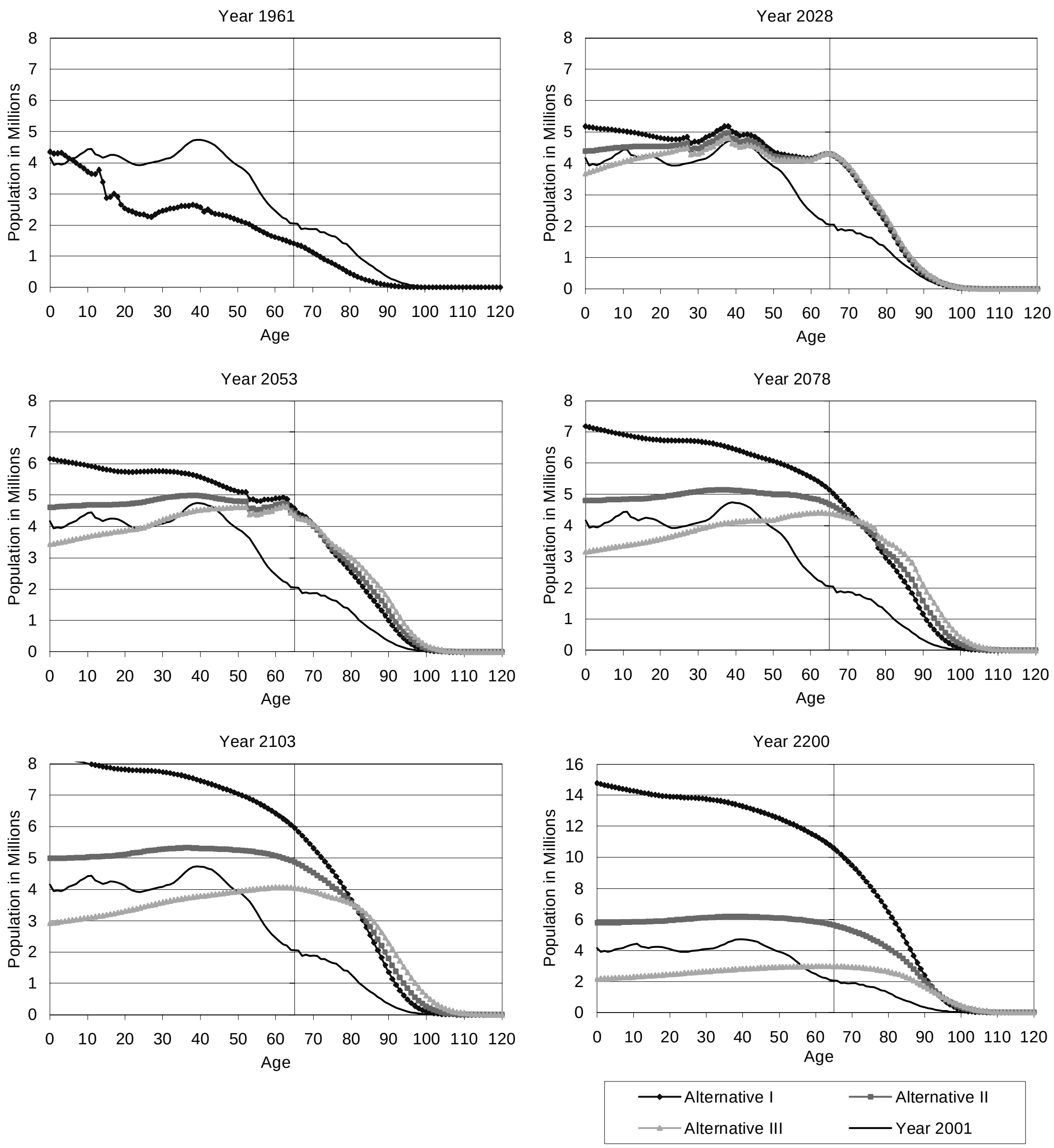

Figure 1: Population Projections in Selected Years: Author's calculation from the data in Social Security Administration (2003)

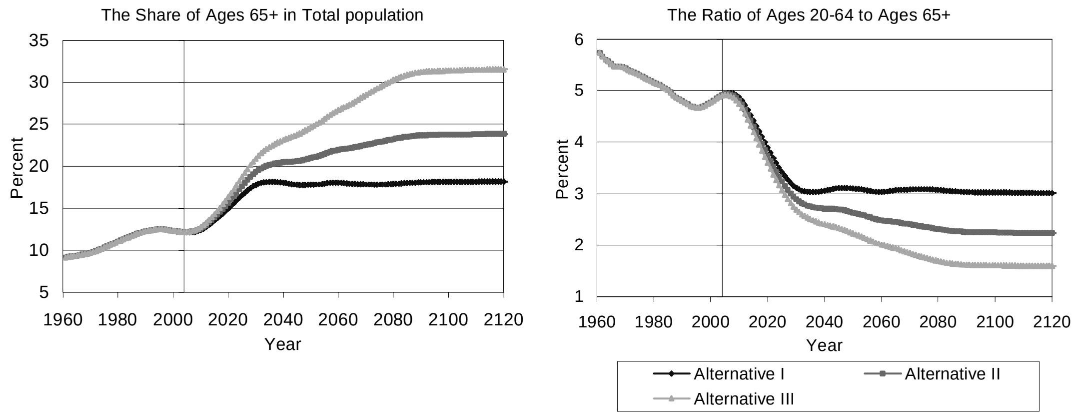

Figure 2: The Share of Elderly People: Author's calculation from the data in Social Security Administration (2003)

Table 3: Parameters

Time preference parameter	β	1.013
Share parameter for consumption	α	0.686
Coefficient of relative risk aversion	γ	4.0
Capital share of output	θ	0.300
Depreciation rate of capital stock	δ	0.047
Long-term real growth rate	μ	0.018
Long-term population growth rate	ν	0.00594, 0.00154, -0.00312
Total factor productivity	A	0.982

3.4 The 2001 Steady-State Economy

Table 3 summarizes the parameter choices. For the 2001 steady-state economy, the degree of time preference β is chosen so that the capital-output ratio is 2.74, total factor productivity A is chosen so that the wage rate w equals unity, and the share parameter for consumption α is chosen so that the average annual working hours of married couples aged between 20 and 64 are 3,368 hours. The capital share parameter θ of the production function and depreciation rate δ are calculated from macroeconomic statistics.

The following sections describe the choice of functional forms and parameter values, and the choice of target variables and values.

Household's Utility Function. The model has elastic labor supply and uses the following Cobb-Douglas utility function with constant relative risk aversion (CRRA), which is compatible with the existence of a steady state,

$$u_i(c_i, h_i) = \frac{\left\{((1+n_i/2)^{-\zeta}\, c_i)^{\alpha}(h_i^{\max} - h_i)^{1-\alpha}\right\}^{1-\gamma}}{1-\gamma},$$

where γ is the coefficient of relative risk aversion, n_i is the number of dependent children, ζ is the consumption adjustment parameter, and $h_i^{\max}$ is the maximum working hours. The coefficient of relative risk aversion is assumed to be 4.0, following Auerbach and Kotlikoff (1987).[28] In this setting, the growth-adjusted β becomes $\beta(1+\mu)^{\alpha(1-\gamma)}$, which is 0.976 in the calibration. The numbers of dependent children by parents' age are calculated from the Panel Study of Income Dynamics (PSID) 1993 Family Data (see Table 4). Multiplying these numbers by a time-variant adjustment factor $\varphi_{n,t}$, the model calculates the average number of children of ages 0-19, n_i, in an age i household, which is consistent with each of three population projections. The consumption adjustment parameter is assumed to be 0.6.[29]

The annual working hours in the model are the sum of the working hours of a husband and a wife. The average working hours of married households between ages 20 and 64 are 3,368 hours in the 1998 Survey of Consumer Finances (SCF). The maximum working hours $h_i^{\max}$ are set to be 5,460 for all households, which is the 95th percentile in the same survey. In this calibration, the parameter α is chosen to be 0.686 so that average working hours of age 20 and age 64 become 3,368 hours in the 2001 steady-state economy.[30]

Working Ability. The working ability in this calibration corresponds to the hourly wage (labor income per hour) of each household in the 1998 SCF. The average hourly wage of a

[28]There is no strong consensus about the coefficient of relative risk aversion γ. For the estimates of γ in previous literature, see Auerbach and Kotlikoff (1987) and Prescott (1986). Citing these two papers, Huggett (1996) used γ of 1.5 and 3.0. More recently, Gourinchas and Parker (2002) estimated γ varying between 0.5 and 1.4, and Laitner (2003) estimated γ at 2.3 or 2.0.

[29]Since $2^{0.6} = 1.516$, a married couple with two dependent children consume about 52 percent more than a married couple with no children if other things are equal. This increase is slightly smaller than the assumption in Elmendorf and Sheiner (2000) but larger than the estimates in Laitner (2003).

[30]In a separate policy experiment not shown in the present paper, we found that the uncompensated wage elasticity of labor supply is about 0.15 in the short run under this utility parameter setting.

Table 4: Number of People Under 18 Years of Age in a Married Household

Age of parents	Number of people under age 18	Age of parents	Number of people under age 18	Age of parents	Number of people under age 18
20	1.02	35	1.83	50	0.61
21	0.96	36	1.87	51	0.50
22	0.98	37	1.90	52	0.42
23	0.89	38	1.96	53	0.35
24	0.96	39	1.85	54	0.29
25	1.08	40	1.76	55	0.23
26	1.12	41	1.75	56	0.22
27	1.15	42	1.66	57	0.19
28	1.19	43	1.51	58	0.15
29	1.29	44	1.43	59	0.15
30	1.36	45	1.30	60	0.13
31	1.49	46	1.13	61	0.09
32	1.60	47	0.96	62	0.10
33	1.68	48	0.82	63	0.10
34	1.77	49	0.70	64	0.09

Source: Author's calculations from the Panel Study of Income Dynamics 1993 Family Data. In the model, these numbers are multiplied by an adjustment factor $\varphi_{n,t}$to be consistent with the population of ages 0-19 in each year t.

married couple (family members #1 and #2 in SCF) used for the calibration is calculated by

$$\text{Hourly Wage} = \frac{\text{Regular and Additional Salaries (\#1 + \#2) + Welfare or Assistance}}{\max\{\text{Working Hours (\#1 + \#2), 520}\}}.$$

To capture the earnings risk a household is exposed to more precisely, unemployment or worker's compensation, Temporary Assistance for Needy Families (TANF), food stamps, and other forms of welfare or assistance are added to the salaries before calculating the hourly wage. Table 5 shows the eight discrete levels of working abilities of five-year age cohorts.[31] Using a shape-preserving cubic spline interpolation, the working ability of each age from 20 to 79 is obtained. The average hourly earnings of production workers have increased by 16.7 percent during the years from 1997 to 2001.[32] In the calibration, the numbers in the table are multiplied by 1.167 to convert the hourly wages in 1997 into those in 2001.

[31] Here, the hourly wage of a household that works less than 520 hours (10 hours a week per couple) is assumed to be zero. In the real economy, some households have fairly high working ability but choose not to work (for example, because of schooling). One observation of the age 20-24 cohort, which has an hourly wage of $193.01, is ignored.

[32] Source: Bureau of Labor Statistics.

Table 5: Working Abilities of a Household (in U.S. Dollars per Hour)

	Percentile	Age cohorts					
		20-24	25-29	30-34	35-39	40-44	45-49
e^1	0-20th	3.83	5.42	5.42	6.93	6.12	6.59
e^2	20-40th	7.07	8.64	9.76	11.28	11.36	12.70
e^3	40-60th	8.68	10.91	13.46	15.01	15.59	17.22
e^4	60-80th	10.67	14.01	18.08	19.96	22.09	23.22
e^5	80-90th	14.05	17.52	27.17	25.27	30.89	31.58
e^6	90-95th	18.20	22.48	33.71	33.38	48.59	44.31
e^7	95-99th	28.43	32.64	54.11	52.16	76.13	86.50
e^8	99-100th	36.81	46.09	167.15	186.47	221.34	301.99
	Percentile	Age cohorts					
		50-54	55-59	60-64	65-69	70-74	75-79
e^1	0-20th	5.48	3.52	0.00	0.00	0.00	0.00
e^2	20-40th	11.53	10.06	4.54	0.00	0.00	0.00
e^3	40-60th	16.16	14.26	11.18	2.82	0.00	0.00
e^4	60-80th	23.44	21.28	18.16	10.37	1.81	0.00
e^5	80-90th	32.14	30.93	28.56	19.48	12.57	0.00
e^6	90-95th	43.01	44.10	59.36	27.68	29.03	1.96
e^7	95-99th	78.61	85.29	96.22	59.34	64.91	14.25
e^8	99-100th	314.59	379.44	421.55	299.25	195.73	146.14

Source: Nishiyama and Smetters (2003). The authors' calculations from the 1998 SCF data.

Markov Transition Matrix. The Markov transition matrix, Γ, of working ability is calculated from the hourly wage of people ages 30-39 in 1991 in the PSID individual data. To make the working ability process more persistent, the matrix is calculated as the transition from the average of years 1989 and 1990 to the average of years 1990 and 1991.

$$\Gamma = \begin{pmatrix} 0.7674 & 0.2049 & 0.0183 & 0.0045 & 0.0049 & 0.0000 & 0.0000 & 0.0000 \\ 0.1810 & 0.6033 & 0.1844 & 0.0129 & 0.0000 & 0.0086 & 0.0046 & 0.0052 \\ 0.0388 & 0.1517 & 0.6768 & 0.1220 & 0.0011 & 0.0046 & 0.0050 & 0.0000 \\ 0.0126 & 0.0361 & 0.1039 & 0.7210 & 0.0980 & 0.0139 & 0.0145 & 0.0000 \\ 0.0000 & 0.0081 & 0.0332 & 0.2360 & 0.6306 & 0.0676 & 0.0145 & 0.0100 \\ 0.0000 & 0.0000 & 0.0000 & 0.0582 & 0.3224 & 0.5303 & 0.0891 & 0.0000 \\ 0.0007 & 0.0000 & 0.0000 & 0.0354 & 0.0000 & 0.2827 & 0.6433 & 0.0379 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.3553 & 0.6447 \end{pmatrix},$$

where $\Gamma(j,k) = \pi(e_{i+1} = e_{i+1}^k \,|\, e_i = e_i^j)$.

The Firm's Production Function. Production takes the Cobb-Douglas form,

$$F(K_t, L_t) = A_t K_t^{\theta} L_t^{1-\theta}.$$

To compute GNP, the model uses the sum of working hours in efficiency units as total labor supply L_t. The capital share of output θ is chosen by

$$\theta = 1 - \frac{\text{Compensation of Employees } + (1-\theta) \times \text{Proprietors' Income}}{\text{National Income } + \text{ Consumption of Fixed Capital}}.$$

The number of θ in 2000 is 0.30.[33] The annual growth rate μ is assumed to be 1.8 percent. Total factor productivity A is chosen to be 0.982 so that the wage per unit of efficient labor is normalized to be unity.

Fixed Capital and Private Wealth. Fixed capital K is the sum of private fixed assets and government fixed assets. In 2000, private fixed assets are $21,165 billion, government fixed assets are $5,743 billion, and the government debt held by the public is $3,410 billion.[34] From these numbers, the government net wealth is set to a 9.5 percent of total private wealth in the initial steady-state economy. In 2000, the capital-GDP ratio is 2.74. The time preference parameter β is chosen so that the capital-GDP ratio of the steady state economy (a balanced growth path) is 2.74.

The depreciation rate of fixed capital δ is chosen by the steady-state condition,

$$\delta = \frac{\text{Total Gross Investment}}{\text{Fixed Capital}} - \mu - \nu.$$

In 2000, private gross fixed investment accounted for 17.2 percent of GDP, and government (federal and state) gross investment accounted for 3.3 percent of GDP.[35] When the capital-output ratio is 2.74, the ratio of gross investment to fixed capital is 7.5 percent. Subtracting the productivity and population growth rates, the annual depreciation rate is assumed to be 4.7 percent.

[33] Source: Bureau of Economic Analysis. The average of θ in years between 1996 and 2000 is 0.31.

[34] *ibid.*.and Congressional Budget Office (2001).

[35] *ibid.*

Table 6: Marginal Payroll Tax Rates in 2001

Labor Income per worker $(w_t e_i h_i/2)$			Marginal Tax Rate (%)	
			OASDI	HI
\$0	–	\$80,400	$6.2 \times \varphi_{\tau_{PO}}$	1.45
\$80,400	–		$0.0 \times \varphi_{\tau_{PO}}$	1.45

The same amount of tax is levied to employers.

Table 7: OASDI Replacement Rates in 2001

AIME (b_{65}/12)			Marginal Replacement Rate (%)
\$0	–	\$561	$90.0 \times \varphi_{tr_{SS}}$
\$561	–	\$3,381	$32.0 \times \varphi_{tr_{SS}}$
\$3,381	–		$15.0 \times \varphi_{tr_{SS}}$

The adjustment factor also reflects the DI and survivors insurance.

The Current Law Social Security System. The tax rate levied on both employers and employees for OASDI is 6.2 percent, and the tax rate for HI is 1.45 percent. In 2001, employee wages above \$80,400 were not taxable for OASDI. (See Table 6.) So, the firm's profit-maximization problem becomes

$$w \times (1 + \text{Marginal Payroll Tax Rate}) = AF_L(K, L),$$

where the marginal payroll tax rate is either 0.0765 or 0.0145 for high-earnings workers. Because the marginal payroll tax rates are not uniform across households, the calibration uses the average payroll tax rate (total payroll tax paid by employers divided by total labor income) instead, so that the Walras' law holds. Social Security benefits are calculated from each worker's Average Indexed Monthly Earnings (AIME), $b_{65}/12$, and the replacement rate schedule in the United States. (See Table 7.)

In 2001, the OASDI payroll tax revenue was 5.25 percent of GDP, and OASDI benefit expenditure was 4.28 percent of GDP. In the model, the ratio of statutory payroll tax revenue to GDP is higher because the model assume all households are two-earner married couples. The payroll tax in the model is multiplied by an adjustment factor $\varphi_{\tau_{PO}}$, which is equal to 0.812, so that the size of payroll tax revenue as a percentage of GDP equals 5.25 percent. (See Table 9.) In the model, survivors' benefits and disability insurance are simply assumed to be proportional to the benefits for old-age workers. The statutory old age benefits in the

Table 8: Marginal Individual Income Tax Rates in 2001 (Married Household, Filed Jointly)

Taxable Income			Marginal Income Tax Rate (%)
\$0	–	\$45,200	$15.0 \times \varphi_{\tau_I}$
\$45,200	–	\$109,250	$28.0 \times \varphi_{\tau_I}$
\$109,250	–	\$166,500	$31.0 \times \varphi_{\tau_I}$
\$166,500	–	\$297,350	$36.0 \times \varphi_{\tau_I}$
\$297,350	–		$39.6 \times \varphi_{\tau_I}$

The standard deduction is \$7,600 and exemption per person is \$2,900.

model are multiplied by an adjustment factor $\varphi_{tr_{SS}}$, which equals 1.199, so that the size of OASDI benefits equals 4.28 percent of GDP in 2001.

Federal and Local Income Taxes. Federal income tax and state and local taxes are assumed to be the level in year 2001 before the Bush tax cuts, "Economic Growth and Tax Relief Reconciliation Act of 2001 (EGTRRA)." Every household in the model is assumed to be a married couple, which is subject to the standard deduction and exemptions. In 2001 the standard deduction for a married household was \$7,600, and the exemption was \$2,900 per person. The exemptions for dependent children follow Table 4. Table 8 shows the statutory marginal tax rates before EGTRRA.

All of the tax brackets, standard deduction, and exemption are assumed to be growth adjusted so that there is no real bracket creep throughout the transition path. Because the economic income of a household is in general larger than taxable income, the effective tax rates are much lower. In 2000, the ratio of total private income tax to nominal GDP was 0.102 and that of corporate income tax was 0.021. The statutory federal income tax is multiplied by the adjustment factor φ_{τ_I}, which is equal to 0.775, so that income tax revenue (including corporate income tax) is 12.3 percent of GDP in the 2001 steady-state equilibrium. Also, since the effective capital income tax rates are lower than labor income tax rates, the tax function is adjusted so that the tax rate on capital income (including corporate income) is about 30 percent lower than that of labor income. State and local income tax in the model is simply a 4.0 percent flat tax for an income (excluding Social Security benefits) above the same standard deduction and exemptions.

3.5 Equilibrium Transition Paths in 1961–2200

The present paper solves the model for transition paths from 1961 to construct baseline economies in 2004-2200 under several different assumptions. Rather than implementing all relevant policy changes in 1961-2003, the present paper considers only the changes in the OASDI payroll tax, benefits, and the trust funds in this period for simplicity. Table 9 shows the sizes of the OASDI payroll tax revenue and benefit expenditure as percentages of GDP in 1961-2002. In the model, the adjustment factors, $\varphi_{\tau_{PO},t}$ and $\varphi_{tr_{SS},t}$, in the payroll tax function and benefit function, respectively, are changed in each year so that the sizes of payroll tax revenue and benefit expenditure in the model are consistent with those in the U.S. historical data.

The adjustment factors of the OASDI payroll tax function in 2002 are 0.810 in a closed economy and 0.808 in a small open economy. These numbers are smaller than 1.0, because the model assumes all households as two-earner married couples and this assumption pushes up the payroll tax revenue without any adjustments. The adjustment factors of the OASDI benefit function in 2002 are 1.231 and 1.218. These are about 51-52 percent larger than those of the payroll tax function, because both survivors' benefits and disability insurance are included into the benefits.[36] For 2003-2200, the adjustment factors are simply assumed to be the average of those in 1998-2002.

The Rate of Return to the Trust Funds. Because the historical rates of return to the Social Security trust funds in 1961-2002 are in general different from those generated in the model, the sizes of the trust funds are adjusted so that those are also consistent with the data as percentages of GDP. The nominal rates of return to the Social Security trust funds that the Social Security Administration assumes from 2003 to 2041 are between 5.93 percent and 6.09 percent.[37] The Social Security Administration also assumes the CPI inflation rate of 3.0 percent from 2007 and 2041.[38]

[36]Because there are no health shocks in the model, disabilities insurance benefits are assumed to be entitled to households 65 years of age and older proportionally to their OASDI benefits.

[37]These numbers are calculated from the projections of interest income and Social Security assets in Table VI.F9 in the Trustees Report (available at http://www.ssa.gov/OACT/TR/TR03/lr6F9-2.html).

[38]The CPI inflation rates in 2003-2006 are 2.35, 2.39, 2.70, and 2.92 percent. See Table VI.F7 (available at http://www.ssa.gov/OACT/ TR/TR03/lr6F7-2.html).

Table 9: OASDI Payroll Tax Revenue and Benefit Expenditure

	OASDI Payroll Tax Revenue			OASDI Benefits Expenditure		
	As a Percentage of GDP	Adjustment Factor $\varphi_{\tau_{PO}}$		As a Percentage of GDP	Adjustment Factor $\varphi_{tr_{SS}}$	
		Closed	Small Open		Closed	Small Open
1961	2.26	0.345	0.344	2.34	0.740	0.729
1962	2.23	0.341	0.340	2.47	0.771	0.761
1963	2.53	0.387	0.386	2.49	0.770	0.759
1964	2.54	0.389	0.387	2.44	0.749	0.739
1965	2.39	0.366	0.364	2.54	0.774	0.764
1966	2.87	0.440	0.439	2.54	0.767	0.756
1967	3.06	0.470	0.468	2.57	0.772	0.762
1968	3.01	0.461	0.460	2.74	0.818	0.810
1969	3.25	0.498	0.497	2.72	0.807	0.800
1970	3.39	0.520	0.518	3.07	0.904	0.897
1971	3.45	0.528	0.527	3.30	0.963	0.956
1972	3.50	0.535	0.534	3.36	0.973	0.967
1973	3.78	0.578	0.577	3.72	1.069	1.064
1974	3.96	0.606	0.604	3.90	1.113	1.108
1975	3.96	0.605	0.603	4.10	1.161	1.157
1976	3.96	0.604	0.603	4.15	1.166	1.163
1977	3.91	0.595	0.594	4.17	1.163	1.161
1978	3.90	0.593	0.592	4.05	1.122	1.121
1979	4.04	0.614	0.613	4.07	1.120	1.121
1980	4.20	0.638	0.637	4.31	1.178	1.179
1981	4.48	0.681	0.680	4.50	1.225	1.226
1982	4.50	0.684	0.683	4.79	1.298	1.300
1983	4.61	0.700	0.699	4.72	1.271	1.273
1984	4.66	0.708	0.707	4.47	1.201	1.202
1985	4.77	0.725	0.724	4.42	1.184	1.184
1986	4.78	0.726	0.725	4.42	1.177	1.178
1987	4.76	0.723	0.722	4.30	1.137	1.138
1988	5.00	0.761	0.760	4.25	1.120	1.120
1989	5.04	0.768	0.767	4.21	1.108	1.108
1990	5.14	0.784	0.783	4.27	1.122	1.121
1991	5.14	0.784	0.783	4.48	1.177	1.177
1992	5.02	0.766	0.765	4.53	1.190	1.190
1993	4.93	0.753	0.752	4.55	1.195	1.194
1994	4.96	0.758	0.757	4.49	1.184	1.182
1995	4.93	0.755	0.754	4.49	1.189	1.186
1996	4.94	0.757	0.756	4.44	1.183	1.178
1997	4.98	0.764	0.763	4.35	1.170	1.164
1998	5.01	0.770	0.769	4.27	1.160	1.153
1999	5.08	0.782	0.781	4.16	1.144	1.136
2000	5.13	0.792	0.790	4.15	1.151	1.141
2001	5.25	0.812	0.810	4.28	1.199	1.188
2002	5.23	0.810	0.808	4.34	1.231	1.218
2003-2200		0.793	0.791		1.177	1.167

Author's calculation from the data in Table 4A3, Social Security Administration (2002), also available at http://www.ssa.gov/OACT/STATS/t4a3Income.html and http://www.ssa.gov/OACT/STATS/t4a3Outgo.html. OASDI payroll tax revenue used in this table excludes interest income. The adjustment factors in 1961-2002 are calculated so that the sizes of revenue and expenditure are consistent with the historic data as percentages of GDP. The adjustment factors in 2003-2200 are the averages of those in 1998-2002.

In this paper, the real market rate of return to capital in a small open economy is fixed at about 6.25 percent (before capital income and corporate income taxes). The market of return in 2003 in a closed economy is 6.22 percent, which is roughly equal to that in a small open economy. So, the model assumes a risk premium of 3.25 percent between the market rate of return and the rate of return to the trust funds, so that the real rate of return to the trust funds in 2003 is around 3.0 percent in the model economy.

4 Baseline Economy

Baseline economies are obtained as Equilibrium transition paths with three population projections, intermediate (alternative II), low cost (alternative I), and high cost (alternative III) in Social Security Administration (2003).

Table 10 shows aging baseline economies, a closed economy and a small open economy, with the intermediate population projection (alternative II), in which the long-run population growth rate ν is 0.154 percent. The Social Security trust funds are depleted in 2053 [2056] in a closed [small open] economy.[39] The model assumes that the OASDI payroll tax rate is increased and benefit replacement rates are reduced when the trust funds are depleted, so that each of those policy changes finances a half of the deficit and that the trust funds are kept at zero thereafter. The numbers in the table are either percent changes or changes in percentage points from the balanced growth path through the economy in 2004 with population adjustments.

Table 11 shows aging economies with the low cost population projection (alternative I), in which the long-run population growth rate ν is 0.594 percent. The trust funds are depleted in 2082 [2100] in a closed [small open] economy. Finally, Table 12 shows aging economies with the high cost population projection (alternative III), in which the long-run population growth rate ν is -0.312 percent. With the same government financing assumption, the trust funds are depleted in 2044 [2046] in a closed [small open] economy.

[39]The trust funds last longer in the present paper than those in Social Security Administration (2003) with all population projections, partly because the current Social Security system uses the consumer price index (CPI) for the cost of living adjustment of benefits, and CPI inflation rates are higher than personal consumption deflator growth rates. When the cost of living adjustment was modified in the model, the trust funds would be depleted in 2045 [2047] under the population projection alternative II in a closed [small open] economy.

Table 10: The Equilibrium Transition Path with an Aging Population Alternative II (Changes from the population-adjusted balanced growth path through 2004)

	Year				Average			
	2028	2053	2078	2200	04-28	29-53	54-78	04-78
Closed Economy								
%ch(National Wealth)	21.4	22.7	26.1	26.1	13.3	21.6	24.9	19.9
%ch(Labor Supply)	-1.0	-2.7	-4.1	-4.9	1.2	-2.2	-3.4	-1.5
%ch(GNP)	5.3	4.3	4.1	3.5	4.6	4.4	4.3	4.4
%ch(Consumption)	6.9	7.3	7.2	7.1	4.2	7.3	7.2	6.2
ch(Private Savings / GDP%)	-1.98	-1.81	-2.69	-3.03	-0.90	-1.74	-2.42	-1.69
ch(OASDI Payroll Tax / GDP%)	-0.17	-0.21	0.72	0.85	-0.11	-0.16	0.60	0.11
ch(OASDI Benefits / GDP%)	1.33	1.79	1.84	1.97	0.35	1.94	1.73	1.34
ch(Trust Funds / GDP%)	13.1	-13.5	-14.0	-14.0	9.9	0.2	-14.0	-1.3
ch(Interest Rate%)	-1.44	-1.63	-1.90	-1.95	-0.80	-1.54	-1.79	-1.38
%ch(Wage Rate)	6.3	6.9	7.9	8.0	3.4	6.8	7.4	5.9
Small Open Economy								
%ch(National Wealth)	25.3	29.8	34.4	35.4	14.8	27.7	32.5	25.0
%ch(Labor Supply)	-2.9	-4.9	-7.0	-7.9	0.1	-4.4	-6.1	-3.5
%ch(GNP)	5.6	5.6	5.5	5.2	4.6	5.3	5.5	5.1
%ch(Consumption)	6.5	8.2	8.5	8.8	3.7	7.6	8.2	6.5
ch(Private Savings / GDP%)	-0.87	-0.01	-1.44	-1.88	-0.23	-0.68	-1.05	-0.65
ch(OASDI Payroll Tax / GDP%)	-0.12	-0.13	0.95	1.07	-0.07	-0.12	0.73	0.18
ch(OASDI Benefits / GDP%)	1.64	2.48	2.06	2.18	0.51	2.25	2.02	1.59
ch(Trust Funds / GDP%)	17.3	-8.1	-14.0	-14.0	11.6	5.7	-13.6	1.2

Percent changes or changes in percentage points from the balanced growth path of 1.8% per-capita real growth through the 2004 economy. Years 2028, 2053, and 2078 are the 25th, 50th, and 75th years, respectively, from 2004. The trust funds are depleted in 2053 in a closed economy and 2056 in a small open economy. When the trust funds are depleted, both the payroll tax rate is raised and benefits are cut to balance the OASDI thereafter.

With all population projections, per-capita national wealth is larger in aging economies than in the balanced growth path. With population projections alternative II and alternative III, per-capita national wealth grows faster than per-capita GDP. Because households need to accumulate larger life-cycle savings for the period after retirement, expecting lower mortality rates, lower Social Security benefits, and higher payroll tax rates. Contrary to the forecasts calculated from a model with fixed saving rates by age cohort, the severer aging population results in larger private wealth per household.[40]

Labor supply tends to be smaller in aging economies than in the balanced growth path.

[40]The present paper does not consider the increase in government debt due to the increase in Medicare spending caused by an aging population. The results will change if we assume the financing rule for the rest of the government budget differently.

Table 11: The Equilibrium Transition Path with an Aging Population Alternative I (Changes from the population-adjusted balanced growth path through 2004)

	Year				Average			
	2028	2053	2078	2200	04-28	29-53	54-78	04-78
Closed Economy								
%ch(National Wealth)	15.4	9.2	6.4	5.9	10.7	11.8	7.5	10.0
%ch(Labor Supply)	-3.0	-5.6	-5.8	-6.2	0.3	-4.9	-5.8	-3.5
%ch(GNP)	2.2	-1.3	-2.3	-2.7	3.2	-0.2	-2.0	0.3
%ch(Consumption)	4.8	3.0	2.6	2.4	3.3	3.9	2.7	3.3
ch(Private Savings / GDP%)	-2.64	-2.52	-2.05	-2.47	-1.19	-2.73	-2.27	-2.06
ch(OASDI Payroll Tax / GDP%)	-0.15	-0.12	-0.11	0.18	-0.09	-0.13	-0.11	-0.11
ch(OASDI Benefits / GDP%)	1.09	1.47	1.49	1.30	0.26	1.45	1.53	1.08
ch(Trust Funds / GDP%)	15.7	1.9	-11.7	-14.0	10.7	8.6	-5.4	4.6
ch(Interest Rate%)	-1.25	-1.06	-0.90	-0.89	-0.71	-1.17	-0.97	-0.95
%ch(Wage Rate)	5.4	4.5	3.7	3.5	3.0	5.0	4.1	4.0
Small Open Economy								
%ch(National Wealth)	19.0	15.4	12.3	9.4	12.1	17.1	13.7	14.3
%ch(Labor Supply)	-4.5	-6.9	-7.0	-7.3	-0.6	-6.4	-7.1	-4.7
%ch(GNP)	2.5	-0.2	-1.2	-2.3	3.2	0.7	-0.9	1.0
%ch(Consumption)	4.4	3.4	3.2	2.8	2.9	4.0	3.2	3.4
ch(Private Savings / GDP%)	-1.91	-2.23	-1.99	-2.17	-0.70	-2.27	-2.10	-1.69
ch(OASDI Payroll Tax / GDP%)	-0.10	-0.08	-0.08	0.24	-0.07	-0.09	-0.08	-0.08
ch(OASDI Benefits / GDP%)	1.33	1.57	1.56	1.35	0.39	1.61	1.61	1.20
ch(Trust Funds / GDP%)	19.6	10.2	-1.4	-14.1	12.2	14.9	4.2	10.4

See footnotes of Table 10.

The trust funds are depleted in 2082 in a closed economy and 2100 in a small open economy.

With the population projection alternative II, per-capita labor supply increases relative to the balanced growth path until 2013 [2012] in a closed [small open] economy, then, it decreases relative to the balanced growth path thereafter. Although the retirement decision is endogenous in the model, the increase in the population share of aged 65 or older decreases the average working hours of households in the long run. Under the most severe assumption of population aging (alternative III), per-capita labor supply increases slightly on average for the first 75 years in a closed economy, because households have to work more to prepare for the earlier and larger reduction of OASDI benefits. But, this is not the case in a small open economy, in which the wage rate is fixed.

There are two effects of an aging population on the capital-labor ratio. First, an aging population increases life-cycle savings accumulated for longer periods after retirements, and an aging population reduces per-capita labor supply because a larger share of households are

Table 12: The Equilibrium Transition Path with an Aging Population Alternative III (Changes from the population-adjusted balanced growth path through 2004)

	Year				Average			
	2028	2053	2078	2200	04-28	29-53	54-78	04-78
Closed Economy								
%ch(National Wealth)	29.4	42.8	51.8	50.5	16.7	35.7	49.0	33.8
%ch(Labor Supply)	1.3	0.2	-3.9	-6.0	2.2	0.5	-1.7	0.4
%ch(GNP)	9.0	11.4	10.2	8.3	6.3	10.0	11.4	9.2
%ch(Consumption)	8.7	10.8	10.8	10.0	4.9	10.2	11.0	8.7
ch(Private Savings / GDP%)	-0.84	-1.34	-2.91	-3.62	-0.36	-0.77	-2.04	-1.06
ch(OASDI Payroll Tax / GDP%)	-0.20	0.77	1.49	1.79	-0.12	0.12	1.14	0.38
ch(OASDI Benefits / GDP%)	1.55	1.89	2.61	2.92	0.43	2.09	2.26	1.60
ch(Trust Funds / GDP%)	10.1	-13.9	-13.9	-13.9	9.0	-5.6	-13.9	-3.5
ch(Interest Rate%)	-1.71	-2.38	-2.97	-3.04	-0.93	-2.05	-2.73	-1.90
%ch(Wage Rate)	7.6	10.4	13.2	13.4	4.0	9.1	12.1	8.4
Small Open Economy								
%ch(National Wealth)	33.8	52.2	66.5	69.9	18.4	43.0	61.1	40.9
%ch(Labor Supply)	-1.2	-4.0	-9.0	-11.4	0.9	-2.8	-6.4	-2.8
%ch(GNP)	9.5	13.1	13.9	13.3	6.2	11.2	14.1	10.5
%ch(Consumption)	8.3	12.3	14.5	15.5	4.4	10.9	13.5	9.6
ch(Private Savings / GDP%)	0.91	1.18	0.21	-0.98	0.58	1.50	0.93	1.00
ch(OASDI Payroll Tax / GDP%)	-0.13	1.15	2.74	2.30	-0.08	0.22	1.62	0.59
ch(OASDI Benefits / GDP%)	1.96	2.25	3.14	3.40	0.63	2.59	2.72	1.98
ch(Trust Funds / GDP%)	14.7	-14.0	-14.0	-14.0	10.9	-2.4	-14.0	-1.8

See footnotes of Table 10.

The trust funds are depleted in 2044 in a closed economy and 2046 in a small open economy.

retired. This effect increases the capital-labor ratio of the economy.

Second, under the financing assumption, in which the payroll tax rate is increased to make the Social Security system sustainable, an aging population increases the lifetime payroll tax payments and lifetime benefit receipts of households and, accordingly, it reduces private savings.[41] This effect tends to decrease the capital-labor ratio.

When the initial size of the Social Security system—the average replacement rate of OASI benefits—is relatively small, the first effect is larger than the second one, and the capital-labor ratio rises as a population ages. When the initial size of Social Security is relatively large, the capital-labor ratio possibly falls.

In the present model and parameter settings, the first effect on the capital-labor ratio turns out to be larger than the second one, and the capital-labor ratio goes up with all three pop-

[41] Under the financing assumption, where the payroll tax rate is kept at the current-law level and only future benefits are reduced, there are no effects of increasing the lifetime tax payments and benefit receipts.

ulation projections—up by 32.6 percent under alternative II, 12.9 percent under alternative I, and 60.1 percent under alternative III in the long run. The second effect is largest when the government is assumed to raise the payroll tax, keeping the benefits at the current-law level. Even under this financing assumption, however, the capital-labor ratio increases by 24.2 percent in the long run under alternative II.[42]

In the model economy, the household saving rate is above the steady-state level in 2004 and, accordingly, national wealth grows faster than both GDP and labor supply. But, household savings decrease most of the years in 2004-2200. The saving rate, measured by the ratio of savings to GDP, declines by 2.69 [1.44] percentage points for the first 75 years and by 3.03 [1.88] percentage points in the long run in a closed [small open] economy with the intermediate population projection, returning to the steady-state saving rate.[43]

Before the Social Security trust funds are depleted, the OASDI payroll tax revenue is simply determined by household earnings—labor supply multiplied by the wage rate—and it declines slightly as labor supply decreases. The OASDI benefit expenditure increases rapidly, but its pace depends on the population projection. Although the replacement rates of benefits are cut when the trust funds are depleted, the benefit expenditure increases faster than GDP in the long run.

With the intermediate population projection, the OASDI payroll tax rate is 19.3 [22.2] percent higher and the benefit replacement rates are 13.9 [15.4] percent lower in 2078 (after 75 years) than the current-law levels in a closed [small open] economy[44] (These numbers are not in Table 10.) Also, in a closed economy, the interest rate falls by 1.90 percentage points and the wage rate rises by 7.9 percent during the first 75 years, due to the increase in national

[42]Fehr, Jokisch, and Kotlikoff (2003) calculate equilibrium transition paths with an aging population. In their "Base Case" of the U.S. economy, the capital-labor ratio falls by 53 percent from 2000 to 2100 as the population ages Kotlikoff, Smetters, and Walliser (2001) obtain similar results. These authors assumed that the OASDI benefits are kept at the current-law level and the payroll tax rate is increased to balance the Social Security account, and that all of the government expenditure, which is population indexed, is financed with a wage tax rather than an income tax or a consumption tax.

[43]Unfortunately, the present model does not predict the decline in the private saving rates in 1990s. The further refinement of the model is needed. Gokhale, Kotlikoff, and Sabelhaus (1996) show that the government transfers from young and future households to current old households, such as the increase in Medicare benefits, explain a large part of this decline.

[44]When the trust funds are depleted, a half of the OASDI deficit is covered by the increase in payroll tax rates and the rest of the deficit is covered by the reduction of benefits. The numbers in percent changes are different because payroll tax revenue is larger than benefit expenditure in 2004.

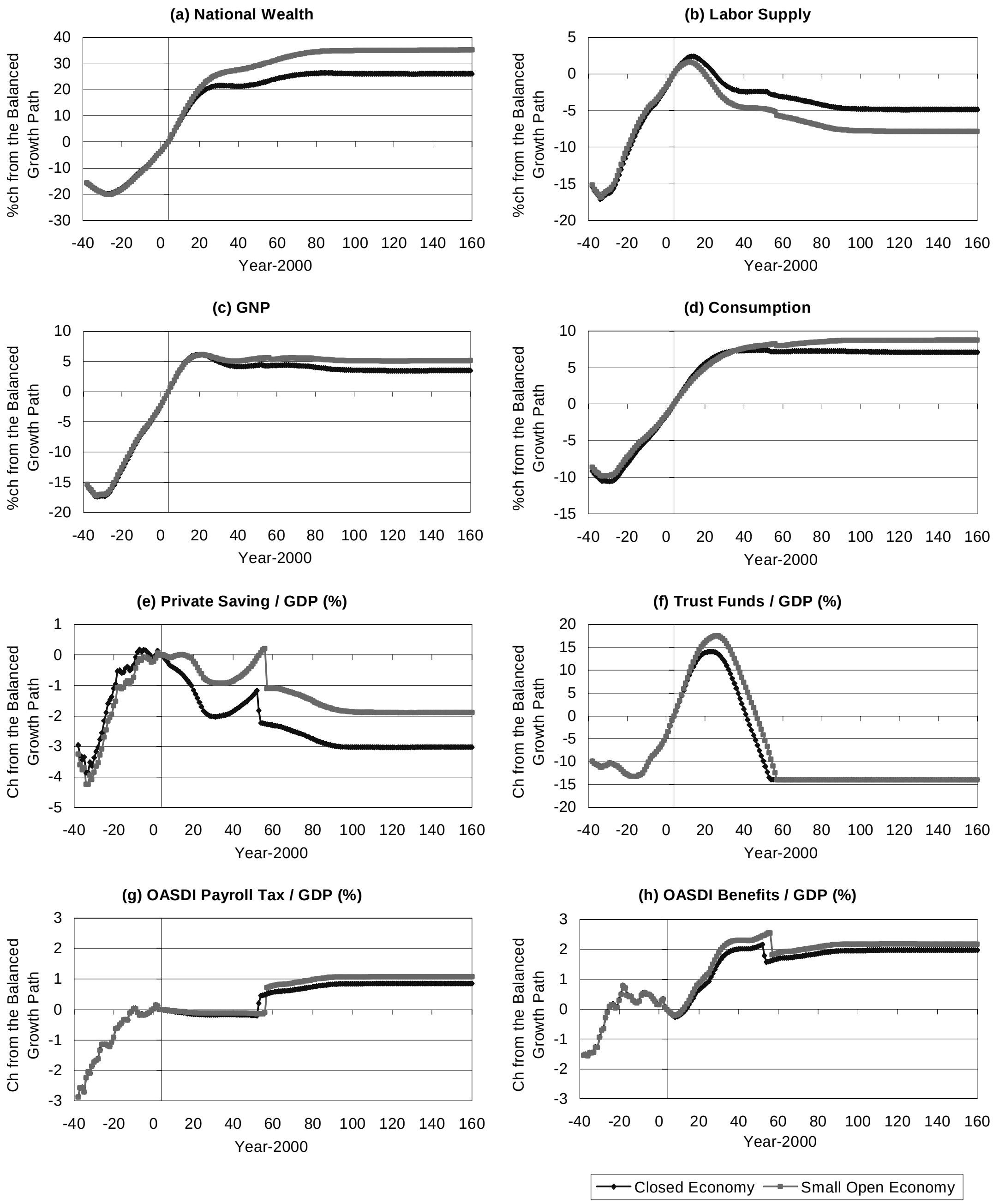

Figure 3: The Baseline Economy with the Population Projection Alternative II (Changes from the population-adjusted balanced growth path through year 2004; Payroll tax is increased and benefits are cut when the trust funds are depleted)

wealth and the decrease in labor supply.

Because the intermediate population projection is used as the baseline economy for the policy experiments in Section 5, Figure 3 also shows the equilibrium transition path of selected variables with the projection alternative II. The figures before 2004 (the vertical line) do not necessarily show the forecast of the model, but those indicate the adjustment process from the 1961 steady-state economy to the 2003 aging economy.

5 Policy Experiments

In Section 4, the financing assumption for the OASDI budget is that the payroll tax rate and benefit replacement rates are kept at the current-law levels as long as the trust funds last and that, when the trust funds are depleted, the payroll tax rate is increased and benefit replacement rates are reduced to keep the trust funds at zero thereafter. One the trust funds are depleted, the OASDI becomes the pay-as-you-go system.

In this section, alternative financing assumptions for the OASDI budget are examined.

The first alternative assumption is that, when the trust funds are depleted, benefit replacement rates are kept at the current-law levels and the payroll tax rate is increased to make the OASDI budget balanced (and to keep the trust funds at zero) thereafter.[45]

The second alternative is that, when the trust funds are depleted, the payroll tax rate is kept at the current-law levels and benefit replacement rates are reduced to make the OASDI budget balanced thereafter.

The last policy experiment assumes that the payroll tax rate is increased immediately in 2004 by 10 percent and that, when the trust funds are depleted, the payroll tax rate is increased and benefit replacement rates are reduced to make the OASDI budget balanced thereafter.

Due to limits of space, all of those policy experiments use the intermediate population projection (alternative II), and the results are shown as changes from the baseline economy, which assumes both the payroll tax rate and benefits are changed when the trust funds are

[45] As explained briefly in Section 4, Kotlikoff, Smetters, Walliser (2001) and Fehr, Jokisch, and Kotlikoff (2003) use this type of financing assumptions. But, those authors assume the pay-as-you-go Social Security system without any trust funds, and the payroll tax rates are increased immediately.

depleted, with the same population projection.

5.1 Alternative Government Financing Assumptions

The financing assumption for the OASDI budget affects the size of the Social Security system—the lifetime payroll tax payments and the lifetime benefit receipts per household. Accordingly, the assumption affects the life-cycle savings and working hours of households even before the payroll tax rate and benefit replacement rates are changed.

The first alternative financing assumption and the second one are symmetric. The policy experiments under those two assumptions show the possible range of macroeconomic variables and social welfare in the economy with the population projection alternative II.

5.1.1 The Payroll Tax Rate Is Increased When the Trust Funds Are Depleted

Table 13 shows the result of this policy experiment. The trust funds are depleted in 2053 [2056] in a closed [small open] economy. In the long run, the payroll tax rate has to be increased by 45.3 [48.9] percent in a closed [small open] economy. (The numbers are not shown in Table 13.)

Compared to the baseline economy, in the long run, the payroll tax revenue and benefit expenditure increase by 1.16 [1.20] percent as a percentage of GDP in a closed [small open] economy. The increase in the lifetime payroll tax and benefits reduces private wealth and labor supply, because households expect larger benefits compared to the baseline economy, and they have to accumulate less life-cycle savings for the period after retirement. The higher pay roll tax rate reduces the disposable income and savings of households. In the long run, national wealth decreases by 7.2 [8.2] percent in a closed [small open] economy. The saving rate is lower throughout the transition path. Private consumption increases in the short run because of the lower saving rate, but it decreases in the log run due to the lower disposable income.

The effect on labor supply is ambiguous in a small open economy. Before the payroll tax is increased, labor supply is increased by the intertemporal substitution effect and decreased by the income effect from the increase in the lifetime benefits. Once the payroll tax is increased, labor supply is possibly decreased by the substitution effect, but this effect does

Table 13: The Payroll Tax Rate Is Increased and Benefits Are Kept at the Current-Law Level When the Trust Funds Are Depleated (Changes from the Baseline Economy with Alternative II)

	Year				Average			
	2028	2053	2078	2200	04-28	29-53	54-78	04-78
Closed Economy								
%ch(National Wealth)	-0.8	-4.1	-6.3	-7.2	-0.3	-2.3	-5.4	-2.7
%ch(Labor Supply)	-0.3	-0.9	-0.9	-0.9	-0.1	-0.4	-0.8	-0.5
%ch(GNP)	-0.4	-1.8	-2.5	-2.8	-0.2	-1.0	-2.2	-1.1
%ch(Consumption)	0.2	0.0	-0.5	-0.9	0.1	0.3	-0.3	0.0
ch(Private Savings / GDP%)	-0.28	-1.05	-0.46	-0.30	-0.12	-0.52	-0.53	-0.39
ch(OASDI Payroll Tax / GDP%)	0.00	0.96	1.04	1.16	0.00	0.05	0.91	0.32
ch(OASDI Benefits / GDP%)	0.02	0.47	1.04	1.16	0.01	0.07	0.91	0.33
ch(Trust Funds / GDP%)	0.0	-0.5	0.0	0.0	0.0	-0.1	0.0	0.0
ch(Interest Rate%)	0.04	0.21	0.36	0.42	0.01	0.13	0.30	0.15
%ch(Wage Rate)	-0.2	-1.6	-2.3	-2.7	0.0	-0.6	-2.0	-0.9
Small Open Economy								
%ch(National Wealth)	-0.6	-3.8	-6.7	-8.2	-0.2	-2.0	-5.5	-2.6
%ch(Labor Supply)	-0.1	-0.0	-0.2	0.0	-0.1	-0.1	-0.2	-0.1
%ch(GNP)	-0.3	-1.4	-2.7	-3.2	-0.1	-0.8	-2.2	-1.0
%ch(Consumption)	0.2	0.4	-0.7	-1.4	0.1	0.3	-0.3	0.0
ch(Private Savings / GDP%)	-0.28	-0.98	-0.85	-0.65	-0.11	-0.65	-0.91	-0.56
ch(OASDI Payroll Tax / GDP%)	0.00	0.00	1.09	1.20	0.00	0.00	0.88	0.29
ch(OASDI Benefits / GDP%)	0.01	0.00	1.09	1.20	0.00	0.00	0.88	0.29
ch(Trust Funds / GDP%)	0.0	-0.1	0.0	0.0	0.0	-0.1	0.0	0.0

Percent changes or changes in percentage points from the baseline economy with both payroll tax increases and benefit cuts. The trust funds are depleted in 2053 in a closed economy and 2056 in a small open economy.

not seem to be large because of the higher benefit replacement rates. In a closed economy, however, labor supply decreases slightly throughout the transition path because of the lower wage rate. In a closed economy, the interest rate is 0.42 percentage points higher in the long run than the baseline, and the wage rate is 2.7 percent lower than the baseline.

The welfare gains and losses from the policy experiment are measure by compensating variations in wealth. That is, the welfare gain [loss] of a household is calculated as once-in-a-lifetime wealth tax [transfer] (which is made when the policy change is announced for current households or when the future household becomes age 20) that makes the household as better off as the baseline economy.

The welfare gains and losses differ according to the state of each household. Figure 4 (a) shows the average gains and losses by age cohort, and Figure 4 (b), (c), and (d) show the average gains and losses by age cohort for a specific temporary (not lifetime) working ability

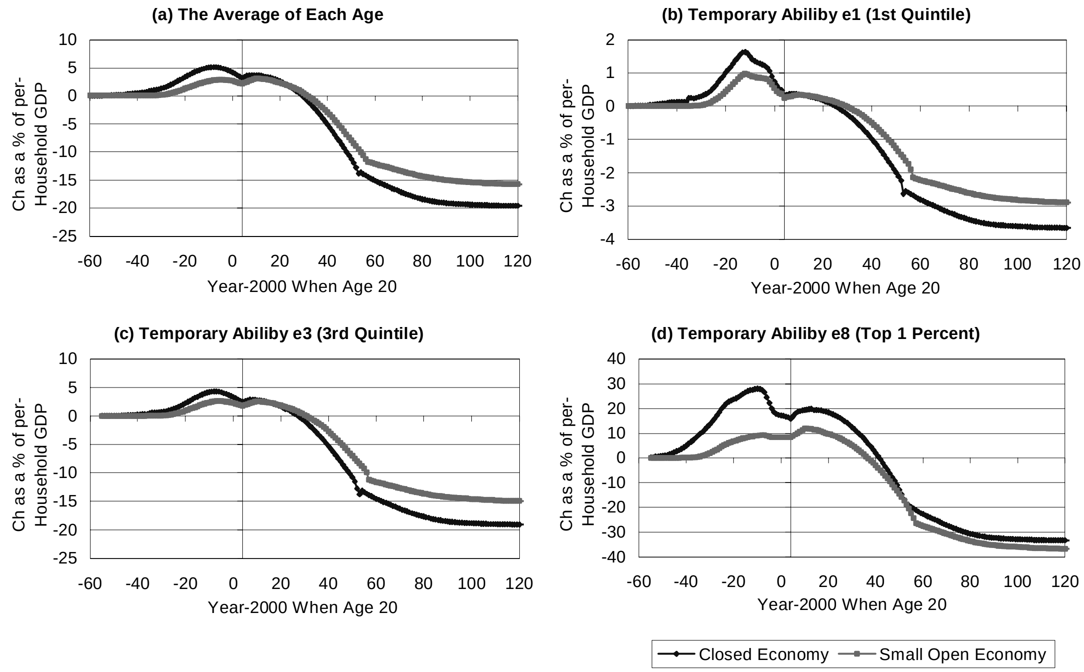

Figure 4: Welfare Gains and Losses from the Baseline Economy—The Payroll Tax Rate Is Increased When the Trust Funds Are Depleted (Compensating Variation in Wealth as Percentages of Per-Household GDP)

class—the bottom 20 percent, the mid 20 percent, and the top 1 percent, respectively.

According to Figure 4 (a), households that enter to the economy before 2030 [2032] in a closed [small open] economy are on average better off by the policy change, and future households that enter to the economy after 2030 [2032] are worse off. If the new households belong to the top 1 percent temporary working ability class, households that enter to the economy before 2042 [2037] in a closed [small open] economy are still better off.

5.1.2 Benefit Replacement Rates Are Reduced When the Trust Funds Are Depleted

Table 14 shows the result of this policy change. The trust funds are depleted in almost the same years as before—2053 [2057] in a closed [small open] economy. In the long run, benefit replacement rates have to be reduced by 30.2 [33.3] percent in a closed [small open] economy. (These numbers are not shown in Table 14.)

Compared to the baseline economy, in the long run, the payroll tax revenue and benefit

Table 14: Benefit Replacement Rates Are Reduced and the Payroll Tax Rate Is Kept at the Current-Law Level When the Trust Funds Are Depleated (Changes from the Baseline Economy with Alternative II)

	Year				Average			
	2028	2053	2078	2200	04-28	29-53	54-78	04-78
Closed Economy								
%ch(National Wealth)	0.9	4.6	7.2	8.3	0.3	2.6	6.2	3.0
%ch(Labor Supply)	0.3	0.7	0.9	0.9	0.1	0.4	0.8	0.5
%ch(GNP)	0.5	1.9	2.7	3.1	0.2	1.1	2.4	1.2
%ch(Consumption)	-0.2	-0.2	0.4	0.9	-0.1	-0.3	0.2	-0.1
ch(Private Savings / GDP%)	0.31	1.20	0.51	0.34	0.13	0.59	0.59	0.44
ch(OASDI Payroll Tax / GDP%)	0.00	-0.39	-0.96	-1.10	0.00	-0.02	-0.83	-0.29
ch(OASDI Benefits / GDP%)	-0.02	0.13	-0.96	-1.10	-0.01	-0.05	-0.83	-0.30
ch(Trust Funds / GDP%)	0.0	0.5	0.0	0.0	0.0	0.1	0.0	0.0
ch(Interest Rate%)	-0.04	-0.25	-0.38	-0.43	-0.01	-0.14	-0.32	-0.16
%ch(Wage Rate)	0.2	1.4	2.5	2.9	0.1	0.7	2.1	0.9
Small Open Economy								
%ch(National Wealth)	0.8	4.3	7.7	9.4	0.3	2.3	6.3	3.0
%ch(Labor Supply)	0.1	0.0	0.1	-0.1	0.1	0.1	0.2	0.1
%ch(GNP)	0.3	1.6	3.0	3.6	0.1	0.9	2.5	1.2
%ch(Consumption)	-0.3	-0.4	0.7	1.5	-0.1	-0.4	0.3	-0.1
ch(Private Savings / GDP%)	0.33	1.11	0.99	0.75	0.13	0.74	1.05	0.64
ch(OASDI Payroll Tax / GDP%)	0.00	0.00	-1.10	-1.22	0.00	0.00	-0.87	-0.29
ch(OASDI Benefits / GDP%)	-0.01	0.01	-1.10	-1.22	0.00	0.00	-0.87	-0.29
ch(Trust Funds / GDP%)	0.0	0.1	0.0	0.0	0.0	0.1	0.0	0.0

Percent changes or changes in percentage points from the baseline economy with both payroll tax increases and benefit cuts. The trust funds are depleted in 2053 in a closed economy and 2057 in a small open economy.

expenditure decrease by 1.10 [1.22] percent as a percentage of GDP in a closed [small open] economy. Expecting smaller benefits, households accumulate larger life-cycle savings for their retirements.

In the long run, national wealth increases by 8.3 [9.4] percent in a closed [small open] economy. The saving rate is higher throughout the transition path, and private consumption decreases in the short run but increases in the log run. The effect on labor supply is ambiguous in a small open economy, but labor supply will increase in a closed economy because of the higher wage rate. In a closed economy, the interest rate falls by 0.43 percentage points in the long run, and the wage rate rises by 2.9 percent.

Figure 5 (a) shows the average gains and losses by age cohort, and Figure 5 (b), (c), and (d) show the average gains and losses by age cohort and by temporary working ability class.

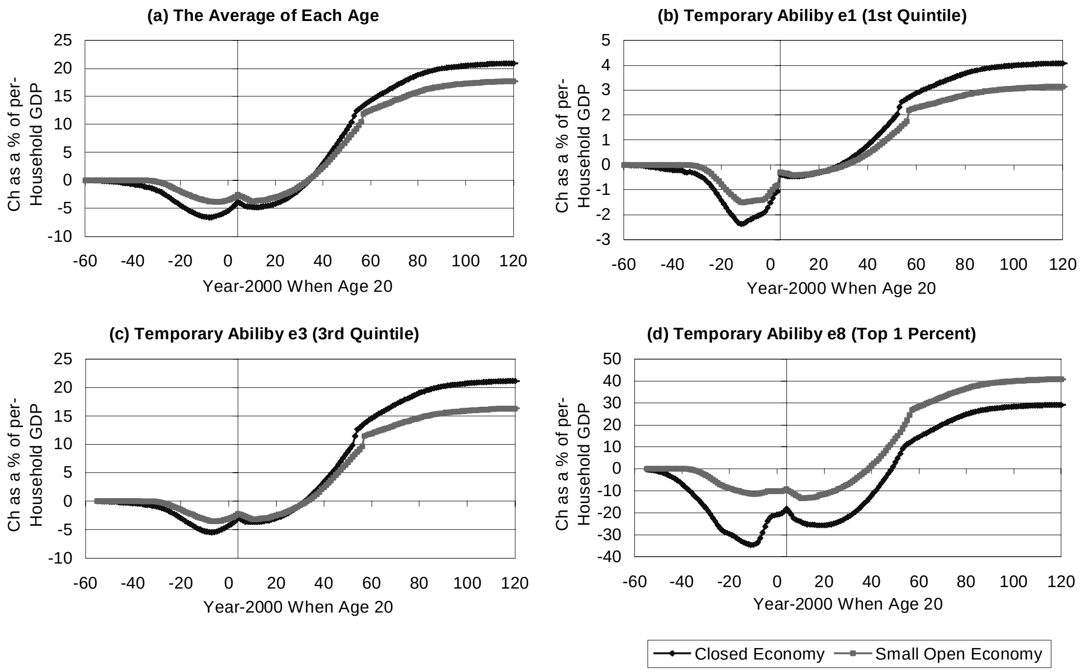

Figure 5: Welfare Gains and Losses from the Baseline Economy—Benefit Replacement Rates Are Reduced When the Trust Funds Are Depleted (Compensating Variation in Wealth as Percentages of Per-Household GDP)

According to Figure 5, households that enter to the economy before 2035 [2034] in a closed [small open] economy are on average worse off by the policy change, and households that enter to the economy after 2035 [2034] are on average worse off.

Although both of those two alternative economies are not Pareto superior to the baseline economy, the second one with reducing future benefits seems to be more efficient than the first one with increasing payroll tax rates, according to Figures 4 and 5. Further evaluations of welfare gains are left for future research.[46]

[46] Nishiyama and Smetters (2003) introduce a mechanism called the lump-sum redistribution authority, which was originally developed in Auerbach and Kotlikoff (1987), to their heterogeneous-agent lifecycle model, and they analyze the efficiency gain or loss of a policy change in an equilibrium transition path.

Table 15: The Payroll Tax Rate Is Increased Immediately by 10 Percent (Changes from the Baseline Economy with Alternative II)

	Year				Average			
	2028	2053	2078	2200	04-28	29-53	54-78	04-78
Closed Economy								
%ch(National Wealth)	1.6	1.0	-1.4	-1.8	0.9	1.5	-0.5	0.6
%ch(Labor Supply)	0.0	0.1	-0.2	-0.2	0.0	0.0	0.0	0.0
%ch(GNP)	0.5	0.3	-0.6	-0.7	0.2	0.4	-0.2	0.2
%ch(Consumption)	-0.4	-0.1	-0.1	-0.2	-0.4	-0.3	0.1	-0.2
ch(Private Savings / GDP%)	-0.43	-0.33	-0.14	-0.07	-0.30	-0.66	0.28	-0.23
ch(OASDI Payroll Tax / GDP%)	0.47	0.10	0.26	0.25	0.48	0.46	-0.10	0.28
ch(OASDI Benefits / GDP%)	-0.03	0.36	0.26	0.25	-0.01	-0.01	0.60	0.19
ch(Trust Funds / GDP%)	10.1	19.0	0.0	0.0	5.2	14.8	6.7	8.9
ch(Interest Rate%)	-0.11	-0.06	0.08	0.10	-0.07	-0.10	0.03	-0.05
%ch(Wage Rate)	0.2	0.2	-0.5	-0.7	0.0	0.1	-0.1	0.0
Small Open Economy								
%ch(National Wealth)	2.2	3.1	0.0	-1.8	1.2	2.9	1.7	1.9
%ch(Labor Supply)	-0.1	-0.1	0.3	0.0	-0.1	-0.1	0.2	0.0
%ch(GNP)	0.7	1.1	0.1	-0.7	0.3	1.0	0.8	0.7
%ch(Consumption)	-0.5	-0.2	0.5	-0.3	-0.5	-0.4	0.3	-0.2
ch(Private Savings / GDP%)	-0.37	-1.21	0.94	-0.14	-0.23	-0.74	0.19	-0.26
ch(OASDI Payroll Tax / GDP%)	0.48	0.48	-0.62	0.24	0.49	0.48	-0.39	0.19
ch(OASDI Benefits / GDP%)	0.00	-0.01	1.09	0.24	0.00	-0.00	0.85	0.28
ch(Trust Funds / GDP%)	12.5	29.0	6.5	0.0	6.0	20.7	21.7	16.1

Percent changes or changes in percentage points from the baseline economy with both payroll tax increases and benefit cuts. The trust funds are depleted in 2070 in a closed economy and 2081 in a small open economy.

5.2 The Payroll Tax Rate Is Increased Immediately by 10 Percent

With a reasonable population projection, such as the intermediate population projection alternative II, the current-law Social Security system is not sustainable. The trust funds are eventually depleted, and the government has to increase payroll tax, reduce benefits, or increase transfers from the rest of the government budget. One of the simplest reform plans is increasing the payroll tax rate immediately rather than waiting for the depletion of the trust funds.

In this policy experiment, the payroll tax rate is increased immediately (in 2004) by 10 percent and, when the trust funds are depleted, the payroll tax rate is increased further and benefit replacement rates are reduced so that each of those changes covers a half of the deficit thereafter.

Table 15 shows the result of this policy change. The trust funds last until 2070 [2081]

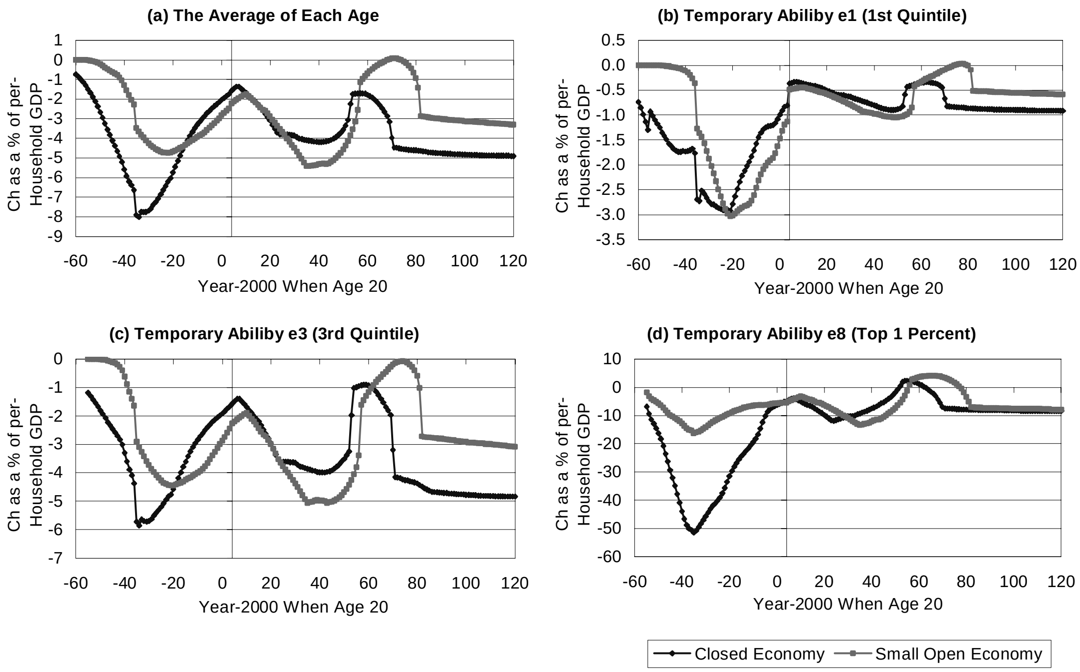

Figure 6: Welfare Gains and Losses from the Baseline Economy—The Payroll Tax Rate Is Increased Immediately by 10 Percent (Compensating Variation in Wealth as Percentages of Per-Household GDP)

in a closed [small open] economy. Compared to the baseline economy, in the long run, the payroll tax rate is higher by 4.4 percent and benefit replacement rates are higher by 3.9 percent, compared to the baseline economy. Also the payroll tax revenue and benefit expenditure are larger than the baseline economy by 0.25 [0.24] percent as a percentage of GDP in a closed [small open] economy.

Because of the lower after-tax income, private savings as a percentage of GDP are smaller throughout the transition path, and private wealth decreases from the baseline economy. In the short run, however, national wealth increases because of the increase in the trust funds. As explained before, the effect on labor supply in a small open economy is ambiguous. In a closed economy, labor supply increases slightly in the short run because of the higher wage rate, but it decreases in the long run. The interest rate rises 0.10 percentage points from the baseline in the long run, and the wage rate falls by 0.7 percent.

Intuitively, an immediate increase in the payroll tax rate to postpone the depletion of the trust funds would likely hurt current working-age households and benefit future households. Then, one of the interesting questions is whether those future households would be better off more than current working-age households would be worse off.

Figure 6 (a) shows the average welfare gains and losses from the baseline economy by age cohort, and Figure 6 (b), (c), and (d) show the average gains and losses by age cohort for selected temporary working ability classes.

According to Figure 6, households are on average worse off in almost all age cohorts, although welfare losses by this policy change are modest. Those households whose earnings ability is at their peak in 2004-2053 [2056] in a closed [small open] economy tend to be hurt more than others. Those households whose ability is peak in 2053-2070 [2056-2081] tend to be hurt less. But, except for a few households in the top 1 percent working ability class, all households are worse off from the policy change, because of the reduction in private wealth (and accidental bequest receipts) and the long-run decline in the wage rate.[47]

6 Concluding Remarks

The present paper extends a heterogeneous-agent overlapping generations model with idiosyncratic working ability shocks and mortality shocks by introducing an aging population of the United States. The paper constructs baseline economies as equilibrium transition paths with three population projections in Social Security Administration (2003). Then, the alternative economies with other government financing assumptions are examined as policy experiments.

There are two main findings in the present paper. First, under a reasonable parameter setting and population projection, private wealth per household is likely to increase and labor supply per capita is likely to decrease as the population ages. However, this result depends possibly on the initial size of the Social Security system and the financing assumption for both the Social Security budget and the rest of the government budget.

[47] According to Cutler, Poterba, Sheiner, and Summers (1990), the optimal policy response to an anticipated demographic change is almost certainly a reduction in the national saving rate. Elmenmdorf and Sheiner (2000) also argue that the optimal response to the aging of the U.S. population is to allow future cohorts to bear much or all that burden. The last policy experiment in the present paper is confirmative with those conclusions.

Second, according to the present model, an earlier Social Security reform may not necessarily improve the efficiency of the economy and the welfare of future households. The last policy experiment demonstrates that an immediate increase in the payroll tax rate would possibly be worse than waiting for the increase in the tax rate until the trust funds are depleted.

Overall, those numerical exercises show how the economy with an aging population looks like and how the aging population projection and the government financing assumption affect macro-economic and welfare implications.

Although policy experiments performed in the present paper are very simple, more complex experiments on Social Security reform can easily be done, since the present model has already been equipped with a detailed Social Security system. More realistic reform plans, including those with individual (personal savings) accounts, will be examined in the separate paper, using one of baseline economies constructed in the present paper.

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