Offsetting the Incentives: Risk Shifting and Benefits of Benchmarking in Money Management*

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Abstract

Money managers are rewarded for increasing the value of assets under management, and predominantly so in the mutual fund industry. This gives the manager an implicit incentive to exploit the well-documented positive fund-flows to relative-performance relationship by manipulating her risk exposure. In a dynamic portfolio framework, we show that as the year-end approaches, the ensuing convexities in the manager's objective induce her to closely mimic the index, relative to which her performance is evaluated, when the fund's year-to-date return is sufficiently high. As her relative performance falls behind, she chooses to deviate from the index by either increasing or decreasing the volatility of her portfolio. The maximum deviation is achieved at a critical level of underperformance. It may be optimal for the manager to reach such deviation via selling the risky asset despite its positive risk premium. Under multiple sources of risk, with both systematic and idiosyncratic risks present, we show that optimal managerial risk shifting may not necessarily involve taking on any idiosyncratic risk. Costs of misaligned incentives to investors resulting from the manager's policy are economically significant. We then demonstrate how a simple risk management practice that accounts for benchmarking can ameliorate the adverse effects of managerial incentives.

1. Introduction

"The real business of money management is not managing money, it is getting money to manage."[1] Indeed, with the number of mutual funds in the US exceeding the number of stocks, fund managers are increasingly concerned with attracting investors' money.[2] Recent empirical evidence (e.g., Gruber (1996), Chevalier and Ellison (1997), Sirri and Tufano (1998)), offers simple insight to a manager: new money is expected to flow into the fund if the manager has performed well relative to a certain index. With her compensation typically increasing in the value of assets under management, this positive fund-flows to relative-performance relationship creates an *implicit incentive* for the manager to increase the likelihood of future fund inflows, distorting her asset allocation choice. There is, of course, also an *explicit incentive* induced by the manager's compensation: managing assets in line with her own appetite for risk, which need not coincide with those of fund investors. This is another source of conflict between a fund manager and her investors, originally pointed out by Ross (1973).[3] Together, the manager's implicit and explicit incentives shape her asset allocation policy. Understanding this policy is of utmost importance to fund investors who may be hurt by adverse incentive effects. Our objective is to examine the implications of these incentives within a familiar dynamic portfolio choice framework. We will characterize the optimal policies in closed form and argue that a number of conventional views expressed in the literature may not hold.

We consider a dynamic economy and focus on a fund manager, guided by a risk-averse objective. The manager's compensation depends on the total value of the fund at some terminal date (e.g., end of the year). This fund value is determined by the portfolio choice of the manager during the year and by nonmarketable inflows/outflows of new money at the year-end. The rate of flows into the fund, simply low or high for the bulk of our analysis, is driven by the manager's performance over the year relative to an "index" − a reference portfolio of stock and money markets.[4] If her relative performance is above a certain threshold, she gets fund flows at a high rate, otherwise a low rate, leading to local convexities in her objective function. The manager's optimal policy reflects the interplay between implicit incentives given by the flow-performance relationship and her "normal," absent implicit incentives, policy driven by her attitudes towards risk. In particular, as the year-end approaches, if her year-to-date relative return is at the flow threshold, the manager chooses to closely mimic the index. As she starts outperforming the index, she tends to her normal policy. As she falls behind the index, however, she gains an incentive to drastically manipulate her

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[1]As eloquently put by Mark Hurley in the famous Goldman, Sachs and Co. report on the evolution of the investment management industry (see WSJ 11/16/95 and e.g., http://assetmag.com/story/20010601/10438.asp).

[2]At the end of 2001, there were 7177 listed stocks in the US (based on CRSP), and 8307 mutual funds (2002 Mutual Fund Fact Book).

[3]Ross argues that an agency conflict arising due to differences in utility functions when the agent's action is difficult to monitor is the most fundamental delegation problem. This problem is especially relevant in the context of money management, where investors cannot frequently monitor the trading strategies of a fund.

[4]We start our analysis with this simplest form of the fund flows to performance for expositional clarity; later in the paper we consider other, more general, specifications and demonstrate the robustness of our main conclusions.

risk exposure, the pattern of which is shown to depend on whether the manager's normal policy is riskier than the index or not. When the index is less risky, she leverages her portfolio, well beyond the normal level, and keeps increasing the risk exposure further as she falls behind the index. When the index is riskier, she reduces her stock market holdings, eventually selling the market short as she continues falling behind the index. At some critical level of the relative year-to-date return, such large deviations from her normal policy become intolerable to the risk averse manager, and so her risk exposure reaches an extremum, beyond which the normal policy considerations start to weigh in, and her risk exposure tends to her normal.

The manager's policy in the underperformance region near the flow threshold is related to the risk-shifting behavior often arising in corporate finance applications (Jensen and Meckling (1976)) in that she optimally engages in "gambling" due to convexities in her objective. The common argument, made in the related literature, is that risk-taking incentives are maximized at the flow threshold and taper off gradually as the manager falls behind the index. Our predictions are in sharp contrast to the traditional view: risk-taking incentives are minimized around the flow threshold, and maximized at some interior point of the underperformance region. This is because our manager is risk-averse, and her risk aversion always counteracts the tendency to gamble (or manipulate risk exposure). This trade-off between gambling incentives and risk aversion underlying her optimization problem leads to a well-defined optimal risk exposure for a given relative performance, which we take to be the measure of risk-taking incentives. This measure need not agree with that adopted in the earlier work. Moreover, the incentives to gamble may actually force the manager to reduce the volatility of her portfolio, which is also in contrast to the common view based on the traditional risk-shifting intuition. Additionally, our dynamic setting allows us to investigate how the magnitude and the interaction of these risk-taking patterns evolve over time.

Costs of misaligned incentives resulting from the manager's policy are shown to be economically significant.[5] We compare the manager's policy when acting in the best interest of fund investors, fully ignoring her explicit and implicit incentives, with when accounting for incentives optimally (discussed above). The difference is quantified in units of an investor's initial wealth. For example, if the investor's relative risk aversion is 2 and the manager's is 0.5, we find the cost to investor to be nearly 54% of his initial wealth. We demonstrate how the manager's explicit and implicit incentives reinforce each other. The cost due to explicit incentives is particularly severe when the manager's and investor's attitudes towards risk differ substantially, and the cost due to implicit incentives is particularly high when the high and low flow rates deviate significantly or when the index is very risky. The magnitude of these costs motivates us to search for practical instruments

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[5]While we focus on the costs arising from misalignment of objectives, there could also be benefits to hiring a manager. For example, the manager might have lower transactions, market participation or informational costs (Merton (1987)), lower opportunity cost of time (time constraints and other forms of investors' bounded rationality (Rubinstein (1998))), better investments-specific education, or better information or ability. Modeling the benefits together with the costs requires a full-fledged model of interaction between managers and investors, which is beyond the scope of this paper.

aimed at counteracting the manager's adverse incentives.

Our discussion so far does not consider constraining the manager's investment policy. We view the resulting unconstrained solution as a useful theoretical benchmark in evaluating managerial incentives. However, in practice an important feature of the asset management industry is the employment of risk management practices, constraining the investment choice of fund managers. For example, Almazan, Brown, Carlson, and Chapman (2003) present ample evidence on the use of constraints in the mutual fund industry. The need for constraints was recognized shortly after the birth of the first mutual fund in the US, and formalized in the Investment Company Act of 1940. Towards this, we consider risk management practices that account for benchmarking. Establishing an economic role for such widely observed practices is also of independent theoretical interest given the arguments made against them in the academic literature (Roll (1992), Admati and Pfleiderer (1997)). We focus on a simple constraint, referred to as a "minimum performance constraint" or a "benchmarking restriction," which prohibits any year-end shortfall in the manager's return relative to a reference portfolio to exceed a pre-specified level. This simple, yet versatile, constraint is closely related to some popular risk management practices (e.g., portfolio insurance, pure indexing, tracking error constraints), and involves no monitoring costs. We show that a benchmark that is less risky than the index can temper deviations from the investors' desired risk exposure in states where the manager is tempted to deviate the most, and hence is beneficial. For example, as a result of imposing a benchmark consisting of 5% in the stock market and 95% in the money market, most of the loss of 54% in the earlier example can be recouped. Through tailoring the composition of the benchmark and the allowed shortfall, the recouped fraction can be increased further. Our results thus provide a rationale for benchmarking-type restrictions observed in this industry.[6]

For most of our analysis we adopt the simplest possible setting, the Black and Scholes (1973) economy with a single source of risk, to convey our most important insights pertaining to managerial risk shifting. To investigate the manager's portfolio allocation across different stocks and exposure to systematic versus idiosyncratic risk, we extend our baseline model to multiple sources of uncertainty. The overall behavior of the manager is analogous to that in the baseline analysis. However, the manager's incentive to deviate from the index portfolio when underperforming now manifests itself not in increasing or decreasing the fund's volatility but in tilting the weight in each risky security away from the index. We also demonstrate that risk shifting does not necessarily involve taking on idiosyncratic risk. Indeed, when faced with both systematic and idiosyncratic risks, the manager may very well optimally expose herself to no idiosyncratic risk, while engaging

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[6]We, of course, could instead follow a standard theoretical approach of optimal contracting in delegated portfolio management, and offer the manager a contract that will induce the right level of risk exposure. Unfortunately, the manager's risk exposure over the year is not observable, making her asset allocation policy noncontractable. Alternatively, one may consider a dynamic contract based on the fund's year-to-date asset value. However, such contracts are difficult to implement and monitor, leading to their absence in practice. Indeed, the simple linear fee structure is the most prevalent contract in the mutual fund industry, which we take as given. Effectively, instead of altering the manager's compensation structure, we alter her investment opportunity set so as to temper undesirable swings in her risk exposure in the targeted states of the world.

in her optimal risk shifting via systematic risk only.

Related to our work is the literature examining implicit incentive conflicts in money management. To study flows-induced risk taking by mutual fund managers, Chevalier and Ellison (1997) build on the traditional risk-shifting insight and define risk-taking incentive as the sensitivity of a fund's value to its volatility (as is standard in corporate finance). This risk-taking incentive captures the strength of the (value-maximizing) manager's desire to increase her risk exposure relative to some fixed status quo asset allocation. Our measure of risk-taking is the optimal risk exposure as defined in the asset allocation literature: the fraction of the fund optimally invested in the stock market (not necessarily well-defined under risk neutrality). Therefore, our analysis is complementary to Chevalier and Ellison, as we offer considerably different implications for managerial risk taking (see Section 2.3).[7]

Within a dynamic asset allocation framework like ours, Carpenter (2000) examines the risk taking behavior of a fund manager with a convex, option-based compensation. Although it may appear plausible to interpret such a model as reduced form for flow-performance-based implicit incentives, it would lead to the counterfactual implication that fund flows increase as the fund value deteriorates (see Section 2.3). Moreover by design, due to the safety net at poor fund performance, manager's risk aversion considerations are suppressed in the underperformance region, implying unbounded risk exposure. This contrasts to our finding of an interior extremum at poor fund relative-performance, reflecting the tradeoff between the manager's risk-shifting and risk-aversion. Building on the analysis of Carpenter and our paper, Hodder and Jackwerth (2003) analyze a hedge fund manager, incorporating further realistic features in the compensation structure along the lines of Goetzmann, Ingersoll, and Ross (2003). They also provide a thorough comparison of managerial risk taking arising in Carpenter, Goetzmann, Ingersoll, and Ross, and our analysis. Also related are Brennan (1993), Cuoco and Kaniel (2000), and Gomez and Zapatero (2002) who study equilibrium asset prices in an economy with agents compensated based on their performance relative to an index, as well as the fund manager's career concerns problem studied by Arora and Ou-Yang (2000). Hugonnier and Kaniel (2002) endogenize fund flows, which are marketable, in a dynamic economy with a small investor and a noncompetitive fund manager. Under the derived flow function, however, the optimal policy of the manager does not depend on her risk aversion, and does not entail any risk-shifting. In the context of a fund-value-maximizing manager, whose ability in unknown, Berk and Green (2002) derive an empirically plausible fund flows-performance relationship. Lynch and Musto (2003) suggest the convexity in the flows-performance relationship is due to an

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[7]A related area in corporate finance is the work on risk averse managers' risk-taking incentives induced by executive stock options (Lambert, Larcker, and Verrecchia (1991), Hall and Murphy (2000), Hall and Murphy (2002), Lewellen (2002)). Here, a risk-taking incentive is given by the sensitivity of the manager's certainty equivalent wealth to volatility. There are some similarities between the results obtained in this context and ours (see, especially Lewellen (2002)), however unlike in our model the manager is assumed to hold a pre-specified portfolio, and may affect risk exposure only through manipulating the company's stock price. The notion of implicit incentives was introduced by Fama (1980) and Holmstrom (1999), and applied to other related problems in corporate finance by, for example, Zwiebel (1995) and Huddart (1999).

abandonment option (replacement of personnel or technique after bad performance). Finally, Ross (2003) investigates the interaction between the manager's payoff and risk aversion within a general class of preferences and compensation structures, however, he focuses on fee schedules under which the objective function remains globally concave. More in the spirit of our analysis, in a dynamic portfolio choice framework, Cadenillas, Cvitanic, and Zapatero (2003) consider a principal-agent problem in which a risk-averse manager compensated with options chooses the riskiness of the projects she invests in.[8]

Another strand of related literature investigates (adverse) consequences of benchmarking. In a mean-variance setting, Roll (1992) argues that benchmarking a money manager to an index results in her choosing a portfolio that is not mean-variance efficient. Admati and Pfleiderer (1997), in a similar context but with an asymmetrically informed investor and portfolio manager, also advocate against benchmarking the manager, and particularly linking compensation to the types of benchmarks observed in practice. Although this may appear as contradictory to our results, one should exercise caution. Our viewpoint is that there is a well-understood conflict of interest between fund managers and investors, which we accept as a fact of life. The role of our benchmarking restriction is to (partially) alleviate the adverse effects of managerial incentives, thus benefitting investors. There is also a recent literature examining benchmarking absent delegation. In a dynamic setting like ours, Tepla (2001), and Basak, Shapiro, and Tepla (2002) study the optimal policies of an agent subject to a benchmarking restriction.

The rest of the paper is organized as follows. Section 2 describes the model, solves for the optimal risk exposure of the manager, and computes costs of active management to the investor due to managerial explicit and implicit incentives. Section 3 specifies the benchmarking restriction, derives the manager's optimal policy under benchmarking, and evaluates investor's cost/benefit due to risk management. Section 4 discusses the extension of our analysis in Section 2 to multiple sources of uncertainty and multiple stocks. Section 5 concludes, and the Appendix provides the proofs.

2. Fund Manager's Implicit and Explicit Incentives

2.1 The Economic Setting

We adopt the familiar Black and Scholes (1973) economy for the financial investment opportunities. We consider a continuous-time, finite horizon, $[0, T]$ economy, in which uncertainty is driven by a Brownian motion w. Available for investing are a riskless money market account and a risky stock.

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[8]There is a strand of literature, growing out of Bhatacharya and Pfleiderer (1985), investigating optimal contracting (or explicit incentive problems) in the context of delegated portfolio management, where the manager typically has superior information or ability, or expends costly effort. Also in this vein is Starks (1987). A recent example of such a modeling approach is Becker, Ferson, Myers, and Schill (1999), who like us also consider a manager concerned with her relative performance. In a similar sprit is Dybvig, Farnsworth, and Carpenter (2001), the first to include restrictions on the investment opportunity set (trading strategies) as part of an optimal contract.

The money market provides a constant interest rate r. The stock price, S, follows a geometric Brownian motion

$$dS_t = \mu S_t dt + \sigma S_t dw_t,$$

where the stock mean return, μ, and volatility, σ, are constant.[9] Throughout, the notation σ^Z denotes the volatility (instantaneous standard deviation) of an Itô process Z satisfying $dZ_t/Z_t = \mu_t^Z dt + \sigma_t^Z dw_t$.

We consider a fund manager, M, who dynamically allocates the fund's assets, initially valued at W_0, between the risky stock and the money market. Her portfolio value process, W, follows

$$dW_t = [(1 - \theta_t)r + \theta_t \mu] W_t dt + \theta_t \sigma W_t dw_t, \tag{1}$$

where θ denotes the fraction of the portfolio invested in the risky stock, or the *risk exposure*. Consistent with the leading practice, the manager's compensation, due at the horizon T, is proportional to the terminal value of assets under management. Tying of compensation to performance provides the manager with an *explicit incentive* to increase the final value of the portfolio W_T. Perhaps just as significant to the manager's choices are *implicit incentives* underlying the money management industry. There, implicit incentives come in the form of the well-documented fund-flows to relative-performance relationship (see e.g., Chevalier and Ellison (1997)). If the manager does well relative to some index (e.g., the stock market), her assets under management multiply due to the inflow of new investors' money; if she does poorly, a part of assets under her management gets withdrawn. We model this relationship in the simplest possible way. The index relative to which her performance is evaluated, hereafter the *index*, Y, is a value-weighted portfolio with a fraction β invested in the stock market and $(1 - \beta)$ in the money market, following[10]

$$dY_t = (1 - \beta)rY_t dt + \beta(Y_t/S_t)dS_t = [(1 - \beta)r + \beta\mu]Y_t dt + \beta\sigma Y_t dw_t.$$

The (continuously compounded) returns on the manager's portfolio and on the index over the period $[0, t]$ are denoted by $R_t^W = \ln \frac{W_t}{W_0}$ and $R_t^Y = \ln \frac{Y_t}{Y_0}$, respectively, where we normalize $Y_0 = W_0$, without loss of generality. There are two fund flow rates: high, f_H, and low, f_L; $f_H \geq f_L > 0$. At the terminal date, the manager receives fund flows at rate $f_T = f_H$ if $R_T^W - R_T^Y \geq \eta$, and at rate $f_T = f_L$ otherwise. The pivotal difference in returns η, which we will call the *flow threshold*, can be either positive, zero, or negative. The flow rate f_T is understood in the proportion-of-portfolio terms; for example if $f_T > 1$, the manager gets an inflow, otherwise if $f_T < 1$, gets an outflow. It turns out that this simple way of modeling fund flows, adopted for expositional clarity, is able to

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[9]In this Black and Scholes-type setting, the focus is on systematic risk. The extension to multiple sources of uncertainty and multiple stocks where systematic risk is not the only consideration is in Section 4.

[10]Alternatively, the index Y can be interpreted as a peer group performance. The optimal policies we derive in this paper would then constitute the best response of an individual manager. One may in principle attempt to go further and solve for a Nash equilibrium in the game amongst managers evaluated on their performance relative to the peer group. This, however, would require a manager to have full knowledge of other managers' preferences, portfolio compositions, etc.

capture most of the insights pertaining to risk taking incentives of the manager. In Section 2.3, we discuss how our results extend to a general fund-flows to relative-performance relationship, in particular standard option-like payoffs. The manager is guided by constant relative risk aversion (CRRA) preferences, defined over the overall value of assets under management at time T:

$$u_M(W_T f_T) = \frac{(W_T f_T)^{1-\gamma_M}}{1 - \gamma_M}, \ \gamma_M > 0, \tag{2}$$

where f_T directly enters through the utility, and not through the budget constraint, because future (time-T) fund flows are nontradable. We note that this payoff is consistent with a linear fee structure, predominantly adopted by mutual fund companies (e.g., Das and Sundaram (2003), Elton, Gruber, and Blake (2003)) and, albeit in a somewhat modified form, can also be reinterpreted as a fund manager's explicit incentive contract.[11] The manager intertemporally chooses a risk exposure process θ and terminal portfolio value W_T so as to maximize her expected utility (2) subject to the budget constraint (1).

Absent implicit incentive considerations, the manager's optimal risk exposure, θ^N, henceforth the *normal risk exposure*, is given by (Merton (1971)):

$$\theta_t^N = \frac{1}{\gamma_M} \frac{\mu - r}{\sigma^2}.$$

By analogy, we define the risk exposure of the index portfolio, θ^Y, as the fraction of the index invested in the risky asset:

$$\theta_t^Y = \beta.$$

2.2 Manager's Risk Taking Incentives

The optimization problem of the manager is summarized as:

$$\max_{\theta, W_T} E[u_M(W_T f_T)]$$

$$\text{subject to} \quad dW_t = [r + \theta_t(\mu - r)] W_t dt + \theta_t \sigma W_t dw_t, \quad W_0 \text{ given},$$

$$\text{where} \quad f_T = \begin{cases} f_L & \text{if} \quad R_T^W - R_T^Y < \eta, \\ f_H & \text{if} \quad R_T^W - R_T^Y \geq \eta, \quad 0 < f_L \leq f_H, \ \eta \in \mathbb{R}. \end{cases} \tag{3}$$

This problem is nonstandard in that it is nonconcave over a range of W_T, where the range is dependent on the performance of the stochastic index Y, and where the implicit incentives due to fund

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[11]In particular, the fund company may have a linear fee structure, $\alpha W_T f_T$, $\alpha > 0$. Such a linear structure would be optimal in our model absent implicit and explicit incentives, i.e., if $f_T = 1$ and a hypothetical investor had CRRA preferences with γ_M. However, the presence of incentives leads to a considerable cost to investors, as demonstrated in Section 2.4. Our specification does not capture the case of fulcrum fees, which are less common, but the model can be extended to incorporate them. Moreover, our implicit incentive component in the manager's payoff could be consistent with as an explicit performance-based fee, where an individual manager's year-end bonus depends on whether he achieves a performance threshold (much like the bonus structures of US executives (Murphy (1999))).

flows introduce a local convexity. The empirical literature on fund-flows to relative-performance relationship clearly indicates that nonconcavities are inherent in the mutual fund managers' problems. As is well known (e.g., Karatzas and Shreve (1998)), the driving economic state variable in an agent's dynamic investment problem is the so-called state price density. In the complete-markets Black and Scholes (1973) economy, this state price density process, ξ, is given by $d\xi_t = -r\xi_t dt - \kappa \xi_t dw_t$, where $\kappa \equiv (\mu - r)/\sigma$ is the constant market price of risk in the economy. Proposition 1 characterizes the solution to (3) in terms of the state variable ξ.[12]

Proposition 1. *The optimal risk exposure and terminal wealth of a fund manager facing implicit incentives are given by*

(a) for economies with $\theta^N > \theta^Y$, letting $\xi_a > \hat{\xi}$ satisfy $g(\xi_a) = 0$, we have

$$
\begin{aligned}
\hat{\theta}_t &= \theta^N + \left[\mathcal{N}(d(\hat{\kappa}, \xi_a)) - \mathcal{N}(d(\hat{\kappa}, \hat{\xi}))\right](\gamma_M/\hat{\kappa} - 1)\, A\, \theta^N Z(\hat{\kappa})\xi_t^{-1/\hat{\kappa}}/\hat{W}_t \\
&\quad + \Big\{ \left[\phi(d(\hat{\kappa}, \xi_a)) - \phi(d(\hat{\kappa}, \hat{\xi}))\right] A\, Z(\hat{\kappa})\xi_t^{-1/\hat{\kappa}} \\
&\quad + \left[\phi(d(\gamma_M, \hat{\xi}))f_H^{(1/\gamma_M - 1)} - \phi(d(\gamma_M, \xi_a))f_L^{(1/\gamma_M - 1)}\right] Z(\gamma_M)(\hat{y}\xi_t)^{-1/\gamma_M} \Big\} \frac{\gamma_M \theta^N}{\kappa \sqrt{T-t}\,\hat{W}_t}, \\
\hat{W}_T &= \frac{1}{f_H}J_M\left(\frac{\hat{y}}{f_H}\xi_T\right)1_{\{\xi_T < \hat{\xi}\}} + e^{\eta}Y_T 1_{\{\hat{\xi} \le \xi_T < \xi_a\}} + \frac{1}{f_L}J_M\left(\frac{\hat{y}}{f_L}\xi_T\right)1_{\{\xi_a \le \xi_T\}}
\end{aligned}
$$

(b) for economies with $\theta^N < \theta^Y$, letting $\xi_b < \hat{\xi}$ satisfy $g(\xi_b) = 0$, we have

$$
\begin{aligned}
\hat{\theta}_t &= \theta^N + \left[\mathcal{N}(d(\hat{\kappa}, \hat{\xi})) - \mathcal{N}(d(\hat{\kappa}, \xi_b))\right](\gamma_M/\hat{\kappa} - 1)A\, \theta^N Z(\hat{\kappa})\xi_t^{-1/\hat{\kappa}}/\hat{W}_t \\
&\quad + \Big\{ \left[\phi(d(\hat{\kappa}, \hat{\xi})) - \phi(d(\hat{\kappa}, \xi_b))\right] A\, Z(\hat{\kappa})\xi_t^{-1/\hat{\kappa}} \\
&\quad + \left[\phi(d(\gamma_M, \xi_b))f_L^{(1/\gamma_M - 1)} - \phi(d(\gamma_M, \hat{\xi}))f_H^{(1/\gamma_M - 1)}\right] Z(\gamma_M)(\hat{y}\xi_t)^{-1/\gamma_M} \Big\} \frac{\gamma_M \theta^N}{\kappa \sqrt{T-t}\,\hat{W}_t}, \\
\hat{W}_T &= \frac{1}{f_L}J_M\left(\frac{\hat{y}}{f_L}\xi_T\right)1_{\{\xi_T < \xi_b\}} + e^{\eta}Y_T 1_{\{\xi_b \le \xi_T < \hat{\xi}\}} + \frac{1}{f_H}J_M\left(\frac{\hat{y}}{f_H}\xi_T\right)1_{\{\hat{\xi} \le \xi_T\}},
\end{aligned}
$$

where in all economies $\hat{y}$ solves $E[\xi_T \hat{W}_T] = W_0$, $J_M(\cdot)$ is the inverse function of $u'_M(\cdot)$, $\mathcal{N}(\cdot)$ and $\phi(\cdot)$ the standard-normal cumulative distribution and density functions respectively, $\hat{\kappa} = \kappa/(\beta\sigma)$, $\hat{\xi} = (\hat{y}A^{\gamma_M}/f_H^{1-\gamma_M})^{1/(\gamma_M/\hat{\kappa} - 1)}$, $A = W_0 e^{[\eta/T + (1-\beta)r + \beta(\mu - \beta\sigma^2/2 - (r+\kappa^2/2)\sigma/\kappa)]T}$, $g(\xi) = \left(\gamma_M\left(\frac{\hat{y}}{f_L}\xi\right)^{1-1/\gamma_M} - \left(\frac{\xi^{1/\hat{\kappa}}}{Af_H}\right)^{\gamma_M - 1}\right)/(1-\gamma_M) + \hat{y}A\xi^{1-1/\hat{\kappa}}$, $Z(v) = e^{\frac{1-v}{v}\left(r + \frac{\kappa^2}{2v}\right)(T-t)}$, $d(v, x) = \left(\ln\frac{x}{\xi_t} + \left(r + \frac{2-v}{2v}\kappa^2\right)(T-t)\right)/(\kappa\sqrt{T-t})$, and $\hat{W}_t$ is as given in the Appendix. Economies with $\theta^N = \theta^Y$ are described in the Appendix.

Proposition 1 reveals that the manager's optimal behavior has a different pattern depending on whether the index is riskier than her normal policy (economies (a)) or not (economies (b)).

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[12]Our proof of Proposition 1 exploits the dependence of the index Y on the economic state variable ξ. There is an alternative to this direct method, which is presented in the proof of Proposition 3 dealing with multiple sources of uncertainty and multiple stocks.

We note that both types of economies, (a) and (b), are empirically plausible since each economy is identified by conditions involving managerial risk aversion γ_M, which need not equal that of a representative agent. The implications for optimal risk taking are best highlighted by plotting the manager's state-dependent risk exposure as a function of her performance relative to the index.[13]
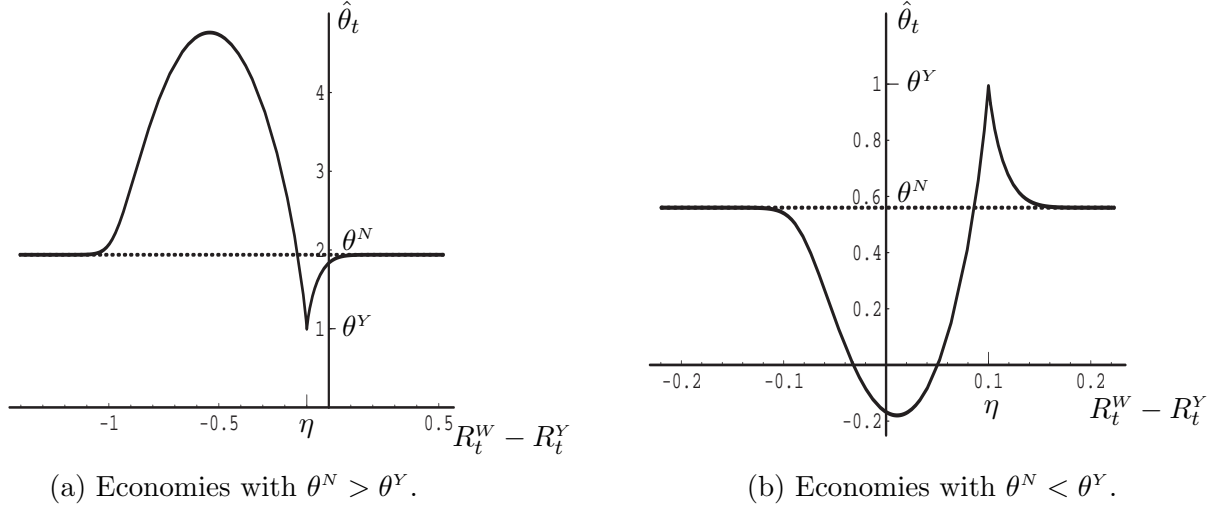

(a) Economies with $\theta^N > \theta^Y$.

(b) Economies with $\theta^N < \theta^Y$.

Figure 1. The manager's optimal risk exposure. The solid plots are for the optimal risk exposure, and the dotted plots are for the manager's normal risk exposure.[14]

There are two considerations affecting the manager's behavior. First is her attitude towards risk, driving the normal policy, second is the risk-shifting incentive induced by nonconcavities due to fund flows. To understand the latter, it is useful to note that the nonconcave payoff to the manager can be expressed as

$$W_T f_T = f_L W_T + (f_H - f_L) W_T 1_{\{R_T^W - R_T^Y \geq \eta\}},$$

where the first term is a linear function of terminal wealth, and the second is a position in $f_H - f_L$ "asset-or-nothing" binary options with a stochastic strike. When the manager is following her normal policy, her optimal wealth process is a geometric Brownian motion, and hence the exact pricing formula for the binary option is readily available. In particular, the volatility of the underlying, W/Y, depends on the volatility of the index, and is given by $|\sigma_t^W - \sigma_t^Y| = \sigma|\theta_t^N - \theta_t^Y|$.[15] As emphasized in the vast risk-shifting literature (originating from Jensen and Meckling (1976)), to increase the value of her compensation, the manager has an incentive to deviate from her normal

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[13]Although in Proposition 1 the optimal risk exposure is expressed as a function of the state price density ξ, we can draw such a plot parametrically provided that the manager's outperformance $R_t^W - R_t^Y$ is monotonic in ξ. We verify in the proof of Proposition 3 that $R_t^W - R_t^Y$ is indeed a valid state variable in the manager's problem (3).

[14]The figure is typical. Parameter values are chosen for demonstrative purposes. In economies (a) parameter values are $\gamma_M = 1.0$, $f_L = 0.85$, $f_H = 1.15$, $\beta = 1.0$, $\eta = -0.1$, $\mu = 0.08$, $r = 0.01$, $\sigma = 0.19$, $W_0 = 1$, $t = 0.8$, $T = 1$; in economies (b) $\gamma_M = 2$, $f_L = 0.99$, $f_H = 1.01$, $\beta = 1.0$, $\eta = 0.1$, $\mu = 0.08$, $r = 0.01$, $\sigma = 0.25$, $W_0 = 1$, $t = 0.95$, $T = 1$.

[15]The binary option with the payoff $W_T 1_{\{R_T^W \geq R_T^Y + \eta\}} = W_T 1_{\{W_T \geq e^\eta Y_T\}}$ is essentially an option on the ratio W/Y. The properties of such an option closely resemble those of an exchange option. For discussion of binary and exchange options, see Hull (2003).

policy by boosting the volatility of the underlying. Note that an increase in the volatility (or risk exposure) of the manager's portfolio W, $\sigma\theta_t$, does not always result in an increase of the volatility of the underlying W/Y.

Let us focus first on the manager's optimal policies at the year-end, right before her receiving fund flows in reward for her performance. We start with economies (a) (Figure 1a), in which the manager's normal risk exposure is higher than that of the index ($\theta^N > \theta^Y$). When the return on the manager's portfolio exceeds the index return by exactly the flow threshold η (the manager's option is at-the-money), she chooses to closely match the index. In these states, the benefits to the manager of a small deviation from following the index, are very small as compared to the potential cost of ending up with an out-of-the-money option. As the manager's portfolio starts outperforming the index, the normal policy considerations weigh in more, and the manager starts taking more and more risk, converging to her normal risk exposure in the limit. The pattern is quite different in the underperformance region. For the states in which her option is slightly out-of-the-money, the manager has an incentive to increase the volatility of the underlying (gamble) for a chance to end up in-the-money. Since the manager's normal policy is riskier than the index, the manager has an incentive to increase her risk exposure above normal. As the underperformance widens, the manager's option dips deeper out-of-the-money, where its value and the likelihood of finishing in-the-money is less sensitive to volatility. For a chance to end up in-the-money, the manager needs to increase the volatility even higher (take a bigger gamble). As the underperformance reaches a critical level where the benefits of increasing risk exposure are equal to the cost due to risk aversion of deviating from her normal policy, the risk exposure achieves the maximum point. Beyond that point, the manager's risk exposure starts to decrease as the underperformance widens, converging to her normal. Analogous intuition is applicable to economies (b) (Figure 1b), in which the manager's normal risk exposure is lower than that of the index ($\theta^N < \theta^Y$). She matches the index when her option is at-the-money, and gradually decreases her risk exposure converging to her normal in the outperformance states. In the underperformance states, again, the manager has an incentive to gamble. However, since $\theta^N - \theta^Y < 0$, to increase the volatility of the underlying, the manager chooses to *decrease* her position in the risky stock. As she falls behind the index more and more, her position decreases further. The manager may even optimally decide to short the risky stock. Again, due to a tradeoff between risk aversion and the payoff-induced incentive to gamble, the risk exposure does not go to (negative) infinity: it reaches an extremum at some point where the option is sufficiently deep out-of-the-money. Beyond that, the risk aversion considerations prevail, and the manager's policy starts converging to her normal.

The described wave-shape optimal policy finances the manager's terminal portfolio value which displays three distinct patterns depending on the state of the world. In the extreme states (low ξ_T or high ξ_T), the manager behaves as if the fund flows were constant at the low f_L or high f_H rate. In addition, there is an extended intermediate region in which the manager mimics the index.

The nonconcavity of the manager's problem gives rise to a discontinuity in the optimal wealth profile at $\xi_T = \hat{\xi}$. This discontinuity is due to the fact that over the range where her preferences are nonconcave in the terminal portfolio value (or returns), the manager exhibits a risk-loving behavior. That is, she will always prefer adding a gamble to her portfolio over a certainty equivalent portfolio value falling into this (suboptimal) range (see Section 2.3 for further elaboration).
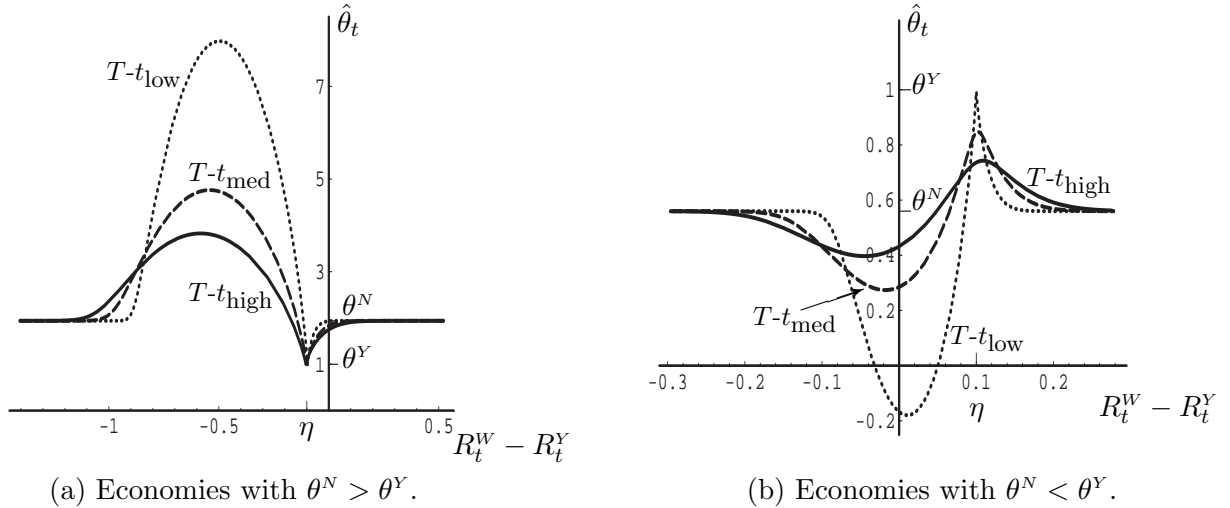

(a) Economies with $\theta^N > \theta^Y$.   (b) Economies with $\theta^N < \theta^Y$.

Figure 2. Dynamics of the manager's optimal risk exposure. $T\text{-}t_{\mathrm{high}} = 0.4$, $T\text{-}t_{\mathrm{med}} = 0.2$, and $T\text{-}t_{\mathrm{low}} = 0.05$. The remaining parameter values are the same as in Figure 1.

The manager's optimal trading strategy earlier in the year reflects the anticipation of the year-end drive to avoid the suboptimal range of the terminal portfolio returns. She does not wait till the year-end to see how her returns play out, to then take a gamble right before the terminal date if necessary. Rather, she starts tilting her risk exposure around the suboptimal range well in advance, displaying a hump in the risk exposure as in Figure 1. However, the more opportunities she has to adjust her portfolio in the future, the less risk exposure she is willing to bear today. The risk aversion (normal policy) considerations dominate early in the year, substantially tempering the risk-shifting considerations and bringing the optimal policy closer to normal, but as time progresses, the risk-shifting motive grows stronger, and hence the magnitude of risk taking around the suboptimal range of portfolio returns grows. Additionally, the range over which the risk exposure displays a risk-shifting-induced hump is wider early in the year, and shrinks monotonically as the horizon approaches. The manager starts engaging in risk shifting when her option is deeper out-of-the-money, and reduces deviations from the index (locks in gains) even if she is still in the outperformance range. For reasonable parameter values, the difference between the outperformance point, at which the manager minimizes her deviations from the index, and the flow threshold η is positive but very small, often not distinguishable to the eye on a plot. The plots in Figure 2 illustrate this discussion.

2.3   Further Discussion and Generalizations

This section offers further insight into the manager's optimal behavior and attempts to generalize our intuition to alternative specifications of the fund-flow to relative-performance relationship.

A.   Risk-Taking in Corporate Finance versus Our Analysis

We think it is useful to contrast our results on the manager's optimal risk taking to measures of risk-taking incentives as defined in the corporate finance literature, typically under the assumption of agents' risk-neutrality. For example, Green and Talmor (1986), in the context of the asset substitution problem, define the risk-taking incentive as the sensitivity of the value of the equityholders' option-like payoff to "changes in investment risk" (variability of the underlying cash flow). A similar measure is adopted by Chevalier and Ellison (1997) whose focus is the closest to ours. In option pricing, this measure is typically referred to as vega, the partial derivative of an option's (portfolio) value with respect to the volatility of the underlying. The risk-taking incentive, as defined in corporate finance, then captures the strength of the (value-maximizing) manager's desire to increase the volatility of her portfolio (risk exposure) relative to some fixed status quo asset allocation. This risk taking incentive will be the strongest (weakest) when vega of the manager's payoff achieves its maximum (minimum). Adopting this measure for the relative-performance-based payoff, as argued in the literature, would imply that the risk-taking incentives are maximized at the flow threshold and taper off gradually as the manager falls further behind the index.

Our conclusions are in sharp contrast to the traditional view: managerial risk taking is minimized around the threshold, and maximized at a point well into the underperformance region. Our measure of risk taking is the manager's optimal risk exposure: the fraction of wealth she optimally invests in the risky asset in response to her incentives. This quantity allows us to formalize an important interaction of the manager payoff's vega with her risk aversion. Risk aversion induces the manager to take as little risk (as small a gamble) as necessary to cross over into the moneyness in the good states of the world. Thus, a small increase in the risk exposure is sufficient near the money, and a much larger increase is needed when the option is deep out-of-the-money. Note that our endogenous wave-shape pattern of risk exposure does not converge to the corporate-finance (bell-shaped) measure of risk-taking incentives even as the risk aversion coefficient of the manager tends to zero. This is because the proper limit of the preferences of our manager is a linear function over the range of positive values of terminal wealth, coupled with a restriction that wealth cannot fall below zero (the negativity of wealth is ruled out by the Inada conditions). This function is (weakly) concave, so we do not get a risk-neutral (linear) objective even in the limit.

B. Alternative Specifications of Flow-Performance Relationship

Below we discuss how our analysis applies more generally, beyond our simple high-low fund flows specification.

(i) *Flow-performance specification of Chevalier and Ellison (1997).* Consider a functional form for the flow-performance relationship along the lines of Chevalier and Ellison:

$$
f_T = \begin{cases}
f_L & \text{if} \quad r_T^W - r_T^Y < 0, \\
f_L + r_T^W - r_T^Y & \text{if} \quad 0 \leq r_T^W - r_T^Y < \eta, \\
f_H = f_L + \eta & \text{if} \quad \eta \leq r_T^W - r_T^Y, \quad 0 < f_L \leq f_H, \ \eta > 0,
\end{cases}
$$

where $1 + r_t^Z = e^{R_t^Z}$ denotes the simple (holding period) return on Z. Chevalier and Ellison highlight the following feature of the flows-performance relationship: a convex "kink" for slight underperformance, followed by an approximately linear segment, then a concave "kink" in the overperformance region. They argue that the manager's risk taking incentives (defined by vega) at time t display a bell shape centered at the first kink followed by an inverted bell centered at the second.[16] Note that we can express the manager's payoff as a linear function of the terminal portfolio value W_T and a portfolio of two call options with stochastic strikes Y_T and $Y_T + \eta W_0$:

$$
W_T f_L + \frac{W_T}{W_0} \max\{W_T - Y_T, 0\} - \frac{W_T}{W_0} \max\{W_T - Y_T - \eta W_0, 0\}.
$$

Of interest to us, is the first call option. Unlike the second call, it introduces a nonconcavity in the manager's payoff. The local behavior around this nonconcavity will resemble that described in Proposition 1. In particular, since the volatility of the underlying is again given by $|\sigma_t^W - \sigma_t^Y| = \sigma|\theta_t - \theta^Y|$, the patterns of manager's behavior will be distinctly different depending on whether the index is riskier than the manager's normal policy or not. The kink at $r_T^W = r_T^Y$ gives the manager an incentive to increase the riskiness of the underlying. The way she achieves this depends on whether the economic environment is of type (a) or (b). Managerial risk aversion will act to counterbalance the option-induced incentive to increase risk. The risk exposure is wave-shaped, displaying a local maximum and a local minimum in the neighborhood of $r_T^W = r_T^Y$.

(ii) *Flow-performance specification of Carpenter (2000).* Carpenter studies the risk taking of a risk-averse manager paid with a call option on the assets she controls. As Carpenter highlights, such a convex compensation structure could potentially arise from the documented fund flow-performance relationship. Here, we attempt to interpret Carpenter's model as reduced form for such an implicit incentive. The closest way to obtain Carpenter's "safety net" at poor fund performance within our model, is to set $f_L = K/W_T$, guaranteeing the floor K for portfolio values below K, and

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[16]Of course, our form of f_T is an overly simplified version of the specification revealed by Chevalier and Ellison's estimation. For example, to simplify exposition we abstract away from two other "kinks" corresponding to the regions of extremely good and extremely bad performance relative to the index. This is because, similarly to Chevalier and Ellison, we are interested in identifying the manager's behavior locally around a given kink. The functional form we adopt thus suits this purpose.

a payoff linear in W_T for levels above K. The index Y is nonstochastic ($\theta^Y = 0$) for most of Carpenter's analysis. Note that this form of fund-flows to relative-performance relationship entails a counterfactual implication that fund flows increase as the fund value decreases when performing poorly (for low W_T, $f_L > f_H$), and tends to (positive) infinity at zero fund value. This behavior highlights the incentive effects of a "safety net", along the lines of the risk-shifting story of Jensen and Meckling (1976). Our manager, who is not rewarded for poor performance but is effectively penalized instead (by low fund flow for low W_T), behaves considerably differently. The risk aversion considerations play an important role in the underperformance region in counteracting risk-shifting, thus ruling out unbounded risk exposure. This feature will also be present in a model where a manager can get fired when the fund return is low, since the manager would optimally choose to limit the size of the gamble she takes to avoid the states in which she gets fired. Another finding of our analysis is uncovering of economies (b). Although a stochastic benchmark is considered in Carpenter, her model is able to uncover only economies of type (a). Our analysis distinguishing between the two economies underscores the fact that the manager's behavior is in part driven by an incentive to manipulate the distance between the risk exposure of the fund and that of the index across states of the world. This provides an alternative explanation for Carpenter's finding that the manager's optimal risk exposure may drop below normal.

(iii) *General flow-performance specification.* We now consider a general payoff structure $W_T f_T$. We conjecture that the bulk of our results holds locally for every region in which $u_M(W_T f_T)$ is nonconcave. If such a region includes $W_T = 0$ or $W_T = \infty$, then at the global maximum (or minimum), the manager's risk exposure can be infinite (a corner solution) or not well-defined. Otherwise, the manager's risk exposure is bounded from above and below for each t, and the emerging wave-shape pattern of risk taking incentives is along the lines of that described in this section.

Remark 1. (Alternative applications) Our analysis and insights may possibly be applicable to address other issues in finance, where nonconcavities are inherent in the payoff structure and where agents can be argued to be (effectively) risk averse. For example, in banking "gambling for resurrection" arises due to nonconcavities introduced by deposit insurance; in corporate finance, the classical asset substitution problem is due to shareholders having a nonconcave payoff. Furthermore, risk-averse managers may have a fee schedule that is nonconcave. Examples include stock-option-based executive compensation and option-like compensation of hedge fund managers. Since in most of these applications nonconcavities in the payoff do not arise over a stochastic range as in our problem with a stochastic relative performance index, it is worth pointing that a riskless index is a special case of our analysis. The pattern of optimal risk exposure will resemble that in our Figure 1a.

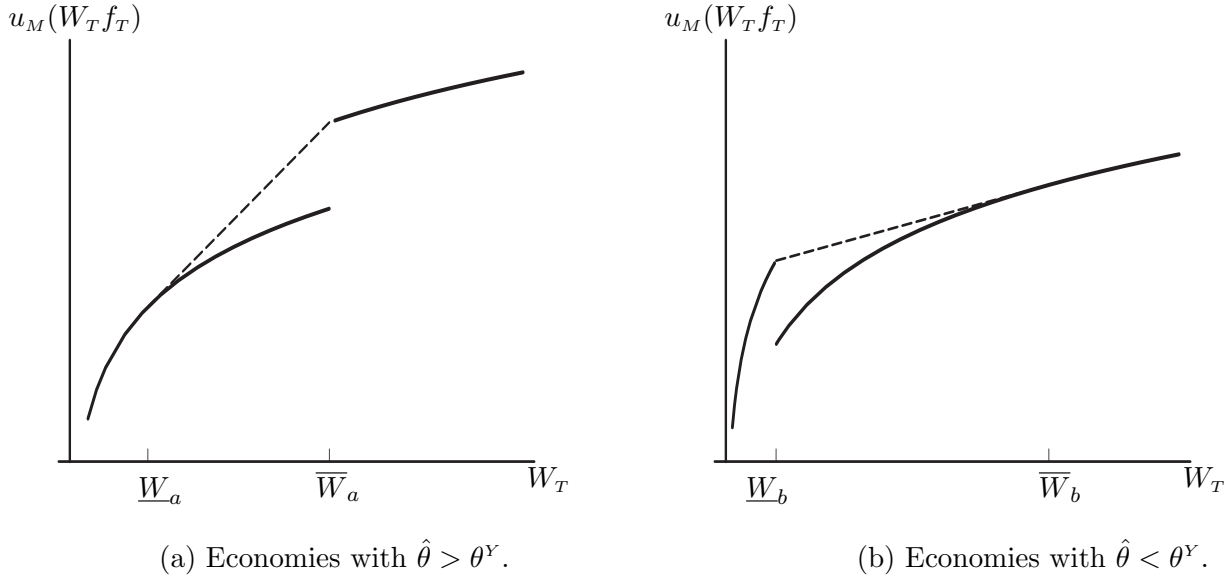## C. A Further Illustration of Nonconcavities and Risk Shifting

(a) Economies with $\hat{\theta} > \theta^Y$.   (b) Economies with $\hat{\theta} < \theta^Y$.

Figure 3. The manager's terminal utility. The solid plots are for the manager's utility, with its concavification superimposed with the dashed line. Quantity $\hat{\theta}$ denotes the risk exposure, which is assumed constant in the plots.

To further illustrate the role of nonconcavities, we consider a simple static setting with two periods $(0, T)$. Figure 3 depicts the manager's utility function, with its concavification superimposed on the plot. The solid plot represents the manager's utility over terminal period wealth under a given constant risk exposure $\hat{\theta}$ chosen at the initial period. The figure illustrates the effects of correlation of the fund portfolio with the index.[17] In panel (a) $(\hat{\theta} > \theta^Y)$, the fund receives flows at the low rate for low fund values and at the high rate for high values $(W_T > \overline{W}_a)$. Conversely, in panel (b) $(\hat{\theta} < \theta^Y)$, fund flows are high for low fund values and low for high values $(W_T > \underline{W}_b)$. This is because the driving factor in receiving flows is not absolute performance, but relative. In panel (a), the fund risk exposure is higher than that of the index, and hence a percent return on the index is accompanied by more than a percent return on the fund. So, in relative terms, the fund beats the index in good states (high W_T) and lags in bad states (low W_T). In panel (b), the fund risk exposure is lower than that of the index, and hence a percent return on the index is matched by less than a percent return on the fund. Then in good states (high W_T), the fund is actually underperforming the index, while outperforming in bad states (low W_T). Accordingly, the incentives to adjust risk exposure relative to the status quo level $\hat{\theta}$ are distinctly different in the two scenarios, akin to the two distinct behavior presented in Proposition 1.

The figure highlights the existence of a finite range of portfolio values over which the utility is

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[17]The role of the figure is to illustrate our results by appealing to a simple two-period intuition. To construct the figure, we fixed the manager's risk exposure, $\hat{\theta}$, to be a constant above that of the index, θ^Y in economies (a), and to be below in economies (b). This is analogous to Figure 1, where $\hat{\theta}_t$ is above (below) θ^Y in economies (a) (economies (b)).

nonconcave, and hence the manager has an incentive to gamble. That is, she would always prefer adding a zero present value gamble $\{+\varepsilon_1$ with probability 50%, $-\varepsilon_2$ with probability 50%$\}$ to her status quo portfolio (defined by fixed $\hat{\theta}$) over ending up with a value in the suboptimal $(\underline{W}_a, \overline{W}_a)$ or $(\underline{W}_b, \overline{W}_b)$ ranges. An additional layer of complexity in our setup is that the nature of the nonconcavity changes dynamically, and so the "direction" of the risk-taking patterns (increased vs. decreased volatility) in Proposition 1 and Figure 1, cannot be deduced from the static illustration in Figure 3. Nevertheless, the optimal terminal fund portfolio value derived in Proposition 1 does feature a discontinuity, responsible for the manager's risk-shifting behavior depicted in Figure 1. How would the manager, whose investment opportunity set consists of assets with continuous distributions, achieve a discontinuous optimal wealth profile? Simply, by taking advantage of continuous trading and thus synthetically replicating a 50/50 gamble, or its close substitute, a binary option. One can see from the expressions for the optimal trading strategies that they indeed contain binary option-type components. What if, perhaps more realistically, the manager is unable to synthetically create a binary option, as would be the case, for example, in the popular two-period model with continuous state space but with a finite number of securities available for investment? The above argument indicates that in such an economy the manager would clearly benefit from introduction of specific securities into her investment opportunity set: binary options. We note that this discussion applies generally to any preferences exhibiting a nonconcavity, and is not driven by the fact that in our setting the manager's payoff essentially includes a binary option.

2.4 Costs of Active Management

In this Section, we assess the economic significance of the manager's adverse behavior. Towards this, we consider an investment policy associated with the manager acting in the best interest of fund investors, fully ignoring her explicit and implicit incentives. A hypothetical fund investor is assumed to have CRRA preferences, $u_I(W_T) = \frac{W_T^{1-\gamma_I}}{1-\gamma_I}$, $\gamma_I > 0$, over the horizon wealth W_T. The investor is passive in that he delegates all his initial wealth, W_0, to the manager to invest. The decision to delegate, exogenous here, captures in a reduced form the choice to abstain from active investing due to various imperfections associated with money management (participation and information costs, time required to implement a dynamic trading strategy, transaction costs, behavior limitations), or simply because he believes that the manager has better information or ability. The manager's investment policy that maximizes the investor's utility is given by $\theta^I = \frac{1}{\gamma_I} \frac{\mu - r}{\sigma^2}$.

In order to evaluate the economic significance of the manager's incentives, we compute the utility loss to the investor of the manager's deviating from the policy θ^I. Following Cole and Obstfeld (1991), we define a cost-benefit measure, $\hat{\lambda}$, reflecting the investor's gain/loss quantified in units of his initial wealth:

$$V^I((1+\hat{\lambda})W_0) = \hat{V}(W_0),$$

where $V^I(\cdot)$ denotes the investor's indirect utility under the policy absent incentives θ^I, and $\hat{V}(\cdot)$

16

his indirect utility under the optimal policy accounting for incentives $\hat{\theta}$. In order to disentangle the implications of explicit and implicit incentives of the manager, we decompose the total cost-benefit measure into two components: λ^N and λ^Y. The former captures the effects of the manager's attitude towards risk driving her normal policy, while the latter the effects of implicit incentives. In particular, λ^N solves $V^I((1+\lambda^N)W_0) = \hat{V}(W_0; f_T = 1)$, where $\hat{V}(W_0; f_T = 1)$ denotes the investor's indirect utility absent implicit incentives only, and λ^Y solves $1 + \hat{\lambda} = (1 + \lambda^N)(1 + \lambda^Y)$.

The main parameter governing the gain/loss due to explicit incentives is the manager's risk aversion, γ_M. Absent implicit incentives, the further γ_M deviates from the investor's risk aversion, γ_I, the larger the discrepancy between the optimal risk exposure of the manager, θ^N, and that optimal for the investor, θ^I, and consequently the higher the loss to the investor. As reported in Tables 1a and 1b, the loss due to explicit incentives, λ^N, is zero when the manager and the investor have the same attitude towards risk, $\gamma_M = \gamma_I (= 2)$. However, for $\gamma_M = 0.5$ such a loss can be quite significant: 32.16% in economies (a) and 9.96% in economies (b).[18]

The strength of the implicit incentives is dependent upon the value of the option component due to fund flows relative to the total value of the compensation package of the manager. Absent explicit incentives, the more important this option component is, the more the manager engages in the gambling behavior in the underperformance region, deviating further from the investor's desired risk exposure. The value of the option component is increasing in the implicit reward for outperformance, $f_H - f_L$. Accordingly, the loss to the investor due to implicit incentives, λ^Y, increases with $f_H - f_L$. For the largest implicit reward we consider in Table 1, $f_H - f_L = 1.0$, the loss is 8.56% in economies (a) and 8.09% in economies (b). Additionally, this value inherits the properties of the binary "asset-or-nothing" option with a stochastic strike Y. In particular, the value is sensitive to the volatility of the underlying, which is related to $|\hat{\theta}_t - \theta^Y|$. This observation renders insights into identifying economic environments, in which the effects of implicit incentives would be most pronounced. For example, in Table 1a we report the cost due to implicit incentives of 12.17% for the highest risk exposure of the index we consider ($\theta^Y = 1.5$), and in Table 1b a corresponding cost of 8.45%.

The explicit and implicit incentives effects reinforce each other in harming of the investor. Table 1 reports the total cost due to both explicit and implicit incentives, $\hat{\lambda}$ ranging from 0.71% to 53.68%. The magnitude of these costs motivates us to search for practical instruments aimed at counteracting the manager's adverse incentives.

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[18]The values reported in Table 1 are for the model parameters, calibrated to conform with the observed market dynamics and roughly capturing the observed flow-performance relationship for mutual funds. The market parameters in economies (b) represent "unfavorable" market conditions designed to temper the manager's normal risk exposure below that of the index assumed to be the stock market. Although we do not frequently observe mutual fund managers holding a leveraged portfolio, the standard argument (Merton (1971) applied to parameter estimates based on historical data) predicts that they should. This observation is related to the discussion whether very high historical equity premium can be reconciled with a typical agent's preferences initiated by Mehra and Prescott (1985).

3. Unwinding the Manager's Incentives with Benchmarking

3.1 A Benchmarking Restriction

Our analysis of the manager's behavior so far has not imposed any constraints on the manager's investment policy. We view the resulting unconstrained solution as a useful theoretical benchmark in evaluating managerial incentives. However, in practice an important feature of the asset management industry is the presence of risk management practices, constraining the investment choice of fund managers. Some of these practices may appear inappropriate in light of the theoretical work cautioning against the use of risk management and in particular benchmarking a manager to a certain portfolio (Roll (1992), Admati and Pfleiderer (1997)). In this Section, we intend to establish a rationale for such practices by showing that an appropriately specified risk management restriction can benefit fund investors.

Towards that end, we look for a restriction which if imposed on the fund manager, will reduce her implicit-incentives-induced risk taking behavior as well as bring the manager's effective risk aversion closer to that of the investor. Such a restriction has to be state-dependent: we would like it to affect the manager's risk exposure in the underperformance states, in which she engages in gambling to a greater extent than in the outperformance states where her risk exposure is within well-defined bounds. Unlike, for example, a simple solution such as a short sale constraint, which will work towards this goal in economies (b) but not in economies (a), where the manager optimally never wants to short the stock, we would also want our restriction to apply uniformly to all economies. We thus consider *benchmarking* the manager to a value-weighted portfolio X, with a fraction δ invested in the stock market and $(1 - \delta)$ in the money market (where we normalize $X_0 = W_0$, without loss of generality). Anticipating our results, to achieve reduction of the effects of implicit incentives, we require that the risk exposure of the benchmark, $\theta^X = \delta$, is less than that of the index, θ^Y. To simplify presentation in the proposition below, we further restrict the risk exposure of the benchmark to be below the manager's normal exposure. In the sequel, we comment on the manager's optimal behavior when these two conditions are violated. The *benchmarking restriction* can be formally stated as

$$R_T^W - R_T^X \geq \varepsilon, \qquad \theta^X \leq \min\{\theta^N, \theta^Y\}, \tag{4}$$

where ε is the manager's *allowed shortfall*. For example, $\varepsilon = -5\%$ means that the maximal shortfall of the manager's return over that of the benchmark may not exceed 5%. This restriction nests several popular risk management practices, including portfolio insurance ($\delta = 0$, $\varepsilon < 0$) and stock market indexing ($\delta = 1$, $\varepsilon = 0$). Our hard constraint can easily be generalized to a softer tracking error-type constraint, which may permit exceeding the allowed shortfall with some probability or allow violating the benchmarking restriction at a cost, capturing a more widespread industry practice. Such generalizations would not alter our main conclusions, but may make some of the effects

less pronounced. Absent delegation and implicit incentives, such a dynamic investment problem has recently been studied by Tepla (2001) and Basak, Shapiro and Tepla (2002). The advantage of this restriction is that the contracted quantities are easily observable. In that, a benchmarking restriction has a clear advantage over constraints imposed on dynamic trading strategies, unless a fund's positions are frequently monitored. The benchmarking restriction does not necessitate frequent monitoring: it is sufficient to just observe the horizon return.

Proposition 2 characterizes the solution to (3)–(4) in terms of the primitive economic state variable ξ.

Proposition 2. *The optimal risk exposure and terminal wealth of a fund manager facing implicit incentives and a benchmarking restriction are given by*

(a) for economies with $\theta^N > \theta^Y$:

$$
\begin{aligned}
\theta_t^* &= \theta^N + \left[\mathcal{N}(d(\hat{\kappa}, \check{\xi})) - \mathcal{N}(d(\hat{\kappa}, \hat{\xi})) \right] \left(\tfrac{\gamma_M}{\hat{\kappa}} - 1 \right) A\, \theta^N Z(\hat{\kappa}) \xi_t^{-1/\hat{\kappa}} / W_t^* \, 1_{\{a_2, a_3, a_4\}} \\
&\quad + \mathcal{N}(-d(\check{\kappa}, \check{\xi}))(\gamma_M / \check{\kappa} - 1)\, B\, \theta^N Z(\check{\kappa}) \xi_t^{-1/\check{\kappa}} / W_t^* \\
&\quad + \Big\{ \left[\phi(d(\hat{\kappa}, \check{\xi})) - \phi(d(\hat{\kappa}, \hat{\xi})) \right] A\, Z(\hat{\kappa}) \xi_t^{-1/\hat{\kappa}} 1_{\{a_2, a_3, a_4\}} \\
&\quad + \left[\phi(d(\gamma_M, \check{\xi})) f_H^{(1/\gamma_M - 1)} + \left(\phi(d(\gamma_M, \check{\xi})) - \phi(d(\gamma_M, \xi_a)) \right) f_L^{(1/\gamma_M - 1)} 1_{\{a_4\}} \right] \frac{Z(\gamma_M)}{(y^* \xi_t)^{1/\gamma_M}} \\
&\quad - \phi(d(\check{\kappa}, \check{\xi})) B\, Z(\check{\kappa}) \xi_t^{-1/\check{\kappa}} \Big\} \frac{\gamma_M \theta^N}{\kappa\, \sqrt{T-t}\, W_t^*} \,, \\
W_T^* &= \tfrac{1}{f_H} J_M \left(\tfrac{y^*}{f_H} \xi_T \right) 1_{\{\xi_T < \tilde{\xi}\}} + e^\eta Y_T 1_{\{\tilde{\xi} \le \xi_T < \check{\xi}; a_2, a_3, a_4\}} + \tfrac{1}{f_L} J_M \left(\tfrac{y^*}{f_L} \xi_T \right) 1_{\{\check{\xi} \le \xi_T < \check{\xi}; a_4\}} \\
&\quad + e^\varepsilon X_T 1_{\{\check{\xi} \le \xi_T\}}
\end{aligned}
$$

(b) for economies with $\theta^N < \theta^Y$:

$$
\begin{aligned}
\theta_t^* &= \theta^N + \left[\mathcal{N}(d(\hat{\kappa}, \check{\xi})) - \mathcal{N}(d(\hat{\kappa}, \xi_b)) \right] \left(\tfrac{\gamma_M}{\hat{\kappa}} - 1 \right) A\, \theta^N Z(\hat{\kappa}) \xi_t^{-1/\hat{\kappa}} / W_t^* \, 1_{\{b_2, b_3, b_4, b_5\}} \\
&\quad + \left[\left(\mathcal{N}(d(\check{\kappa}, \xi_b)) - \mathcal{N}(d(\check{\kappa}, \xi_1)) \right) 1_{\{b_2, b_3\}} + \mathcal{N}(-d(\check{\kappa}, \check{\xi})) \right] \left(\tfrac{\gamma_M}{\check{\kappa}} - 1 \right) B\, \theta^N Z(\check{\kappa}) \xi_t^{-1/\check{\kappa}} / W_t^* \\
&\quad + \Big\{ \left[\phi(d(\hat{\kappa}, \check{\xi})) - \phi(d(\hat{\kappa}, \xi_b)) \right] A\, Z(\hat{\kappa}) \xi_t^{-1/\hat{\kappa}} 1_{\{b_2, b_3, b_4, b_5\}} \\
&\quad + \left[\phi(d(\gamma_M, \check{\xi})) f_L^{(1/\gamma_M - 1)} + \left(\phi(d(\gamma_M, \check{\xi})) - \phi(d(\gamma_M, \check{\xi})) \right) f_H^{(1/\gamma_M - 1)} 1_{\{b_3, b_5\}} \right] \frac{Z(\gamma_M)}{(y^* \xi_t)^{1/\gamma_M}} \\
&\quad + \left[\left(\phi(d(\check{\kappa}, \xi_b)) - \phi(d(\check{\kappa}, \xi_1)) \right) 1_{\{b_2, b_3\}} - \phi(d(\check{\kappa}, \check{\xi})) \right] B\, Z(\check{\kappa}) \xi_t^{-1/\check{\kappa}} \Big\} \frac{\gamma_M \theta^N}{\kappa\, \sqrt{T-t}\, W_t^*} \,, \\
W_T^* &= \tfrac{1}{f_L} J_M \left(\tfrac{y^*}{f_L} \xi_T \right) 1_{\{\xi_T < \tilde{\xi}\}} + e^\eta Y_T 1_{\{\xi_b \le \xi_T < \check{\xi}; b_2, b_3, b_4, b_5\}} + \tfrac{1}{f_H} J_M \left(\tfrac{y^*}{f_H} \xi_T \right) 1_{\{\check{\xi} \le \xi_T < \check{\xi}; b_3, b_5\}} \\
&\quad + e^\varepsilon X_T 1_{\{(\xi_1 \le \xi_T < \xi_b; b_2, b_3) \text{ or } (\check{\xi} \le \xi_T)\}}
\end{aligned}
$$

where in all economies y^* solves $E[\xi_T W_T^*] = W_0$, with $J_M(\cdot)$, $\mathcal{N}(\cdot)$, $\phi(\cdot)$, $g(\cdot)$, $Z(\cdot)$, $d(\cdot)$, $\hat{\xi}$, ξ_a, ξ_b, A, $\hat{\kappa}$ are as given in Proposition 1, and $\check{\kappa} = \kappa/(\delta\sigma)$, $\xi_1 = (y^* B^{\gamma_M} / f_L^{1-\gamma_M})^{1/(\gamma_M/\check{\kappa} - 1)}$, $\xi_2 = (y^* B^{\gamma_M} / f_H^{1-\gamma_M})^{1/(\gamma_M/\check{\kappa} - 1)}$, $\xi_3 = (A/B)^{k/(\sigma(\beta - \delta))}$, $B = W_0 e^{[\varepsilon/T + (1-\delta)r + \delta(\mu - \delta\sigma^2/2 - (r + \kappa^2/2)\sigma/\kappa)]T}$.

19

Economies (a) have four subcases: in a_1 $\xi_3 \leq \xi_2 \leq \hat{\xi}$, in a_2 $\hat{\xi} < \xi_2 < \xi_3$, in a_3 $\xi_1 \leq \xi_a \leq \hat{\xi}$, in a_4 $\xi_a < \xi_1 < \xi_3$, and then $\tilde{\xi} = \xi_2$ in a_1 and $\tilde{\xi} = \hat{\xi}$ otherwise; $\check{\xi} = \xi_3$ in a_2 and $\check{\xi} = \xi_a$ in a_3, a_4; $\check{\xi} = \xi_2, \xi_3, \xi_a, \xi_1$, in a_1, a_2, a_3, a_4 respectively;
Economies (b) have five subcases: in b_1 $\xi_1 < \xi_3 \leq \xi_b$, in b_2 $\xi_1 < \xi_b < \xi_3 < \hat{\xi}$, in b_3 $\xi_1 < \xi_b < \hat{\xi} < \xi_3$, in b_4 $\xi_b \leq \xi_3 < \hat{\xi}$ and $\xi_b \leq \xi_1$, in b_5 $\xi_b < \hat{\xi} < \xi_3$ and $\xi_b \leq \xi_1$, and then $\tilde{\xi} = \xi_1$ in b_1, b_2, b_3 and $\tilde{\xi} = \xi_b$ otherwise; $\check{\xi} = \xi_3$ in b_2, b_4 and $\check{\xi} = \hat{\xi}$ in b_3, b_5; $\check{\xi} = \xi_1$ in b_1, $\check{\xi} = \xi_3$ in b_2, b_4, $\check{\xi} = \xi_2$ in b_3, b_5.

The benchmarking restriction renders a much richer structure to the solution, with multiple subcases appearing for each economy. Figure 4 highlights the implications of the benchmarking restriction for the manager's optimal risk exposure by superimposing the optimal policies revealed by Proposition 2 on pertinent panels of Figure 1 from Section 2.
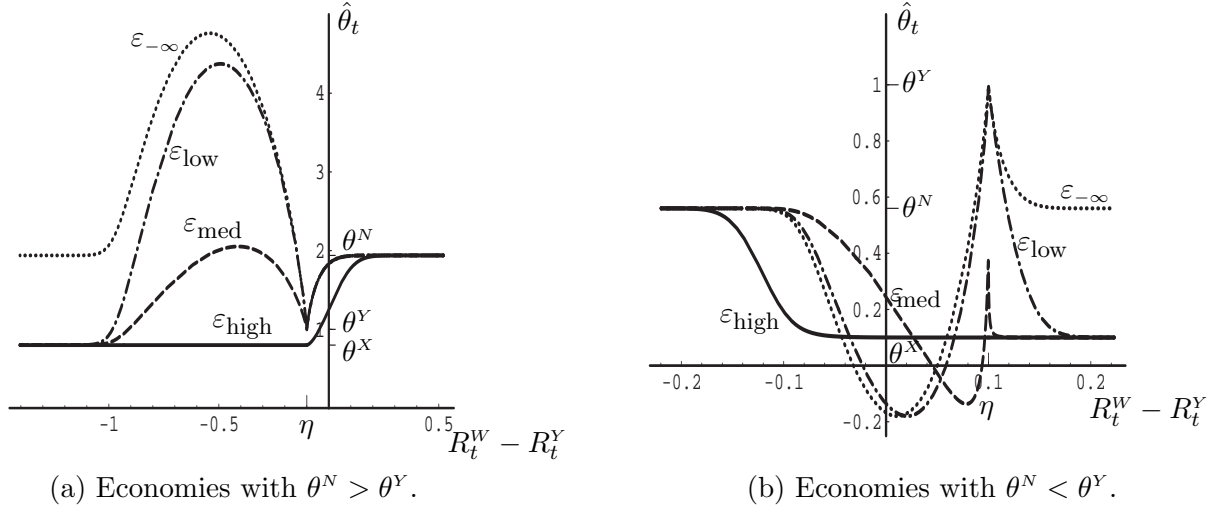

(a) Economies with $\theta^N > \theta^Y$.　　　　　(b) Economies with $\theta^N < \theta^Y$.

Figure 4. The effects of the benchmarking restriction. In economies (a) $\varepsilon_{\text{low}} = -1.0$, $\varepsilon_{\text{med}} = -0.5$, $\varepsilon_{\text{high}} = -0.05$, and $\delta = 0.8$; in (b) $\varepsilon_{\text{low}} = -0.2$, $\varepsilon_{\text{med}} = -0.04$, $\varepsilon_{\text{high}} = -0.01$, and $\delta = 0.1$. The remaining parameter values are the same as in Figure 1.

The figure underscores the importance of imposing a state-dependent restriction on the manager for the purposes of reducing her implicit incentive-induced tendencies to gamble. In the states in which the manager is outperforming the index, the benchmarking restriction does not drastically affect her behavior. In contrast, in the underperformance states, in both types of economies, the benchmark has a significant effect by forcing the manager to tilt her risk exposure closer towards the risk exposure of the benchmark X. Since by construction, the benchmark we propose is safer than both the manager's normal policy and the index, it will always act in the direction of reducing the manager's risk exposure. The lever controlling how much power the benchmarking restriction has in reducing the risk exposure is the allowed shortfall ε. An allowed shortfall of close to $-\infty$ essentially removes the benchmarking restriction; as ε increases, the manager's risk exposure is forced to approach that of the benchmark, converging to the latter when ε reaches its upper bound ($\varepsilon = 0$). Loosely speaking, it is this lever ε that gives rise to a range of subcases in

Proposition 2. For the subcases corresponding to a very low ε (economies a_4, b_4, b_5), the manager is allowed to underperform the benchmark by a large amount, and so the benchmarking restriction has practically no effect in the range where the manager gambles (dotted plots in Figure 4). By increasing ε, we move to subcases a_3, b_2 and b_3, for which the benchmarking restriction is strong enough to target the risk-exposure humps induced by implicit incentives (dashed plots). Finally, for high enough ε (economies a_1, a_2, b_1), we reach the subcases where the gambling behavior is no longer present (solid plots). The benchmarking restriction effectively limits the size of the gamble that the manager would be willing to take. Although the constraint is imposed at the terminal date T, at no earlier date $t < T$ the manager would be willing to allow the return on her portfolio to drop far below R_t^X (the shortfall range will depend on ε). Otherwise, there is a positive probability that the benchmarking restriction may be violated. Absent the benchmarking restriction, the manager engages in excessive risk- taking in low relative return (underperformance) states – these are exactly the states where the benchmarking restriction would be most effective.

Perhaps of no lesser importance to investors are also explicit incentives the manager faces. The benchmarking restriction can be very effective in aligning those as well. Absent implicit incentives, the general rule is very simple: the manager's risk exposure decreases if she is benchmarked to a portfolio X that is less risky than her normal policy, otherwise increases if benchmarked to X that is riskier than her normal policy. The overall effect of the benchmarking restriction on the manager's incentives reflects the interaction of the two mechanisms described above. We assess it quantitatively in the following section, and discuss the cost-benefit implications for the investor.

The expressions for the optimal terminal portfolio value revealed by Proposition 2 make the distinction between the subcases we discussed very precise. The parameter space is subdivided into two (in a_1, b_1) to five (in b_3) regions of distinct behavior of the manager. Although the expressions for the subcases offer additional insights into the subtleties of the manager's economic behavior, we do not present the details here in the interest of preserving space.

Finally, we comment that our choice of a benchmark that is safer than both the manger's normal policy and the index, $\theta^X \leq \min\{\theta^N, \theta^Y\}$, was for expositional purposes, and is also most likely to be a choice that will favorably resonate with regulators and investors. Indeed, Figure 5 examines the scenario in which the benchmark is riskier than both the normal policy and the index, $\theta^X > \max\{\theta^N, \theta^Y\}$. The contrast with Figure 4 is striking. The risk taking incentives are not reduced, on the contrary, the risk exposure is amplified as the manager tilts her portfolio towards the riskier benchmark X.
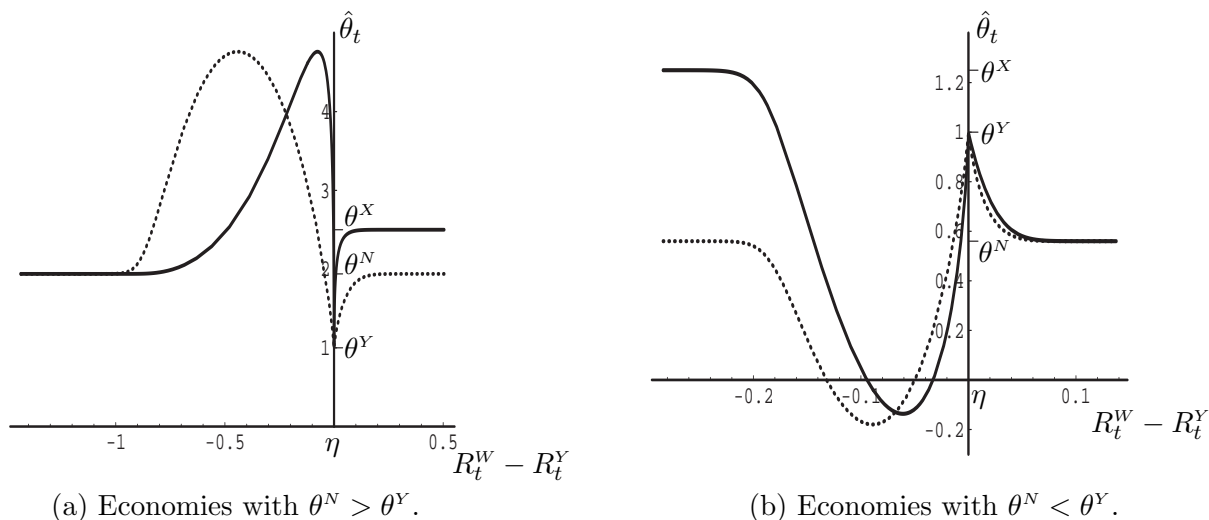
(a) Economies with $\theta^N > \theta^Y$.

(b) Economies with $\theta^N < \theta^Y$.

Figure 5. The effects of the benchmarking restriction with $\theta^X > \max\{\theta^N, \theta^Y\}$. The solid plots are for the risk exposure of the manager facing a benchmarking restriction, and the dotted plots are for the unconstrained manager. In economies (a) $\delta = 2.5$, and in economies (b) $\delta = 1.25$. In all economies $\varepsilon = -0.25$, $\eta = 0$, and the remaining parameter values are the same as in Figure 1.

3.2 Cost-Benefit Implications of Benchmarking

To quantify the effects of imposing a benchmarking restriction, we need to define a measure of an incremental increase in the hypothetical investor's utility due to restraining the manager, λ^*:

$$V^I((1 + \lambda^*)(1 + \hat{\lambda})W_0) = V^*(W_0),$$

where $\hat{\lambda}$ is the utility loss to the investor absent the benchmarking restriction as defined in Section 2.4, and $V^*(\cdot)$ is the indirect utility of the investor under managerial incentives and benchmarking. A positive λ^* means that the benchmarking restriction benefits the investor.

At the outset, one rarely thinks of investment restrictions as being beneficial. This would certainly be impairing if the manager were acting in the investor's best interest. However, in the context of active money management, risk management restrictions can be economically justified. Consider, for example, the case of a highly risk averse investor (more precisely, consider the case of $\theta^I < \min\{\theta^N, \theta^Y\}$). Suppose now that we benchmark the fund manager to a low-risk portfolio X, along the lines of Section 3.1. As one can infer from Figure 4, by tightening the benchmarking restriction (increasing ε), the investor or a regulator can effectively reduce the risk exposure of the manager, bringing her policy closer to that optimal for the investor. Indeed, the corresponding gains reported in Table 2 for this scenario are all positive and can be very large in magnitude: for example, in the top left entry, an increase of 111.98% (most of the loss is recouped) in economies (a) and 12.66% in economies (b).

The surprising result is that even a risk tolerant investor may benefit from benchmarking a less risk tolerant manager to a safer portfolio. One could argue that such an investor would simply

desire to increase the manager's risk exposure, as the latter is normally below the investor's desired policy, by benchmarking the manager to a riskier portfolio. Instead, Table 2 illustrates that the reverse can be true. In Table 2, the benchmark portfolio X is safer than the optimal risk exposure of both the investor and the manager, but nevertheless all entries for the cost-benefit measure λ^* (including those in which the manager is less risk tolerant than the investor, $\gamma_M > \gamma_I$) are positive (except for the case of $\gamma_M = 4$ in Table 2a). These results show that the simple argument in favor of a riskier benchmark fails in the context of real-life mutual fund managers whose policies may be driven by implicit incentives to a larger degree than by their attitudes towards risk.

Once we have demonstrated that a benchmarking restriction reduces the cost of active management, the natural next step is to ask how such a restriction needs to be designed for the highest benefit to the investor. The guideline can be inferred from Table 2. There are two parameters of the restriction one can adjust: the risk exposure of the benchmark θ^X and the allowed shortfall ε. Table 2 shows an optimum for both. In economies (a), the optimal benchmarking restriction calls for selecting the risk exposure of around 0.4 (40% stock/ 60% money market) and the allowed shortfall of about 0.20. In economies (b), these numbers are 0.4 and 0.05, respectively. In both economies, it is beneficial to the investor to benchmark the manager to a relatively safe portfolio. The benchmarking restriction is quite loose in economies (a) and very tight, close to the upper bound on ε, in economies (b).

4. Multiple Sources of Risk and Multiple Stocks

Until now, we have adopted the Black and Scholes (1973) specification for the financial investment opportunities featuring one riskless and one risky asset. Consequently, the decision of the fund manager has been the allocation of assets between the risky and riskless securities. This setting has served as the simplest possible setting, which allowed us to highlight the most important insights pertaining to risk taking incentives of the fund manager. In real life, however, the decision of the manager often involves allocating her portfolio between different stocks, rather than between stocks versus bonds. Moreover, unlike in our baseline model, managers may wish to adjust their portfolio riskiness through taking on idiosyncratic rather than systematic risk. Thus, it is of interest to examine a setup in which one can make a distinction between the effects of systematic and idiosyncratic risks on the manager's decisions.

Towards this end, we extend our model of Section 2 to multiple sources of uncertainty and multiple stocks. Each stock price, S_i, follows

$$dS_{it} = \mu_i S_{it} dt + \sigma_i S_{it} dw_t, \qquad i = 1, \ldots, n,$$

where the stock mean returns $\mu \equiv (\mu_1, \ldots, \mu_n)^\top$ and the nondegenerate volatility matrix $\sigma \equiv \{\sigma_{ij}, \ i, j = 1, \ldots n\}$ are constant, and $w = (w_1, \ldots, w_n)^\top$ is an n-dimensional standard Brownian

motion. The index relative to which the manager is evaluated, Y, is now a value-weighted portfolio with fractions $\beta \equiv (\beta_1, \ldots, \beta_n)^\top$ invested in the stocks and $1\text{-}\beta^\top \mathbf{1}$ in the money market, where $\mathbf{1} \equiv (1, \ldots, 1)^\top$. The manager's optimization problem, as before, is given by (3).

Allowing for multiple sources of uncertainty increases the dimensionality of the problem, introducing certain technical difficulties, and our proof of Proposition 1 does not readily extend. The proof of Proposition 3 (in the Appendix) employs a method to reduce the multi-state variable problem to a single-state variable one via a change of variable (accounting for indexing) and a change of measure (accounting for risk aversion). A by-product of this procedure is to verify that $R^W - R^Y$ is indeed a valid state variable in the manager's problem. As in Proposition 1, however, we present Proposition 3 in terms of the state-price density process ξ, following $d\xi_t = -\xi_t r dt - \xi_t \kappa^\top dw_t$, where now the market price of risk $\kappa \equiv \sigma^{-1}(\mu - r\mathbf{1})$ is n-dimensional.

Proposition 3. *The optimal fractions of the manager's portfolio invested in risky assets and her terminal wealth are given by*

$$
\begin{aligned}
\hat{\theta}_t \;=\;& \theta^N + (\theta^N - \theta^Y) \left\{ \left[\mathcal{N}(d_2(\gamma_M, \pi_*)) - \mathcal{N}(d_2(\gamma_M, \pi^*)) \right] e^\eta \right. \\
& + \left[(\pi^{*1/\gamma_M} \phi(d_1(\gamma_M, \pi^*)) - \pi_*^{1/\gamma_M} \phi(d_1(\gamma_M, \pi_*))) e^\eta \right. \\
& \left. + (f_H^{(1/\gamma_M - 1)} \phi(d_1(\gamma_M, \pi_*)) - f_L^{(1/\gamma_M - 1)} \phi(d_1(\gamma_M, \pi^*))) y^{-1/\gamma_M} \right] \left. \frac{\gamma_M \xi_t^{-1/\gamma_M} \hat{Z}(\gamma_M)}{||\kappa^\top - \gamma_M \sigma^\top \beta|| \sqrt{T-t}\, \hat{W}_t} \right\},
\end{aligned}
$$

$$
\hat{W}_T \;=\; \frac{1}{f_H} J_M \left(\frac{y}{f_H} \xi_T \right) 1_{\{\xi_T Y_T^{\gamma_M} < \pi_*\}} + e^\eta Y_T 1_{\{\pi_* \le \xi_T Y_T^{\gamma_M} < \pi^*\}} + \frac{1}{f_L} J_M \left(\frac{y}{f_L} \xi_T \right) 1_{\{\pi^* \le \xi_T Y_T^{\gamma_M}\}},
$$

where y solves $E[\xi_T \hat{W}_T] = W_0$, $J_M(\cdot)$, $\mathcal{N}(\cdot)$, $\phi(\cdot)$ are as given in Proposition 1, $\pi_* = f_H^{1-\gamma_M} e^{-\gamma_M \eta}/y$, $\pi^* > \pi_*$ satisfies $\hat{g}(\pi) = 0$, $\hat{g}(\pi) = \left(\gamma_M \left(\frac{y}{f_L} \pi \right)^{1-1/\gamma_M} - (f_H e^\eta)^{1-\gamma_M} \right)/(1-\gamma_M) + e^\eta y \pi$, $\hat{Z}(\gamma) = e^{\frac{1-\gamma}{2\gamma^2}||\kappa - \gamma \sigma^\top \beta||^2 (T-t)}$, $\hat{d}_1(\gamma, \hat{\pi}) = \frac{\ln \frac{\hat{\pi}}{\xi_t Y_t^\gamma} + \frac{2-\gamma}{2\gamma}||\kappa - \gamma \sigma^\top \beta||^2 (T-t)}{||\kappa - \gamma \sigma^\top \beta|| \sqrt{T-t}}$, $\hat{d}_2(\gamma, \hat{\pi}) = \hat{d}_1(\gamma, \hat{\pi}) - \frac{1}{\gamma}||\kappa - \gamma \sigma^\top \beta|| \sqrt{T-t}$, $\theta^N = \frac{1}{\gamma_M}(\sigma^\top)^{-1}\kappa$, $\theta^Y = \beta$, and $\hat{W}_t$ is as given in the Appendix. The case of $\theta^N = \theta^Y$ is described in the Appendix.

Proposition 3 reveals that our earlier insights go through component-by-component, which can be viewed as "tilting" positions in individual stocks in response to incentives. Thus, at the flow threshold η the manager optimally chooses to select portfolio weights in individual stocks close to those of the index portfolio. When outperforming the index, the manager tilts each portfolio weight away from the index and in the direction of her normal policy, converging to it in the limit. When underperforming, she deviates from the index by tilting the investment in each stock i in the direction dictated by the sign of $\theta_i^N - \theta_i^Y$. For each stock's portfolio weight, we now obtain two typical investment patterns, where the underperforming manager either increases or decreases her weight in the stock, analogous to economies (a) and (b) of Proposition 1. Figure 6 illustrates this for the case of two risky stocks with parameters chosen such that $\theta_1^N > \theta_1^Y$, $\theta_2^N > \theta_2^Y$ (panel (a)) and $\theta_1^N < \theta_1^Y$, $\theta_2^N > \theta_2^Y$ (panel (b)). The remaining cases are mirror images of the ones presented.
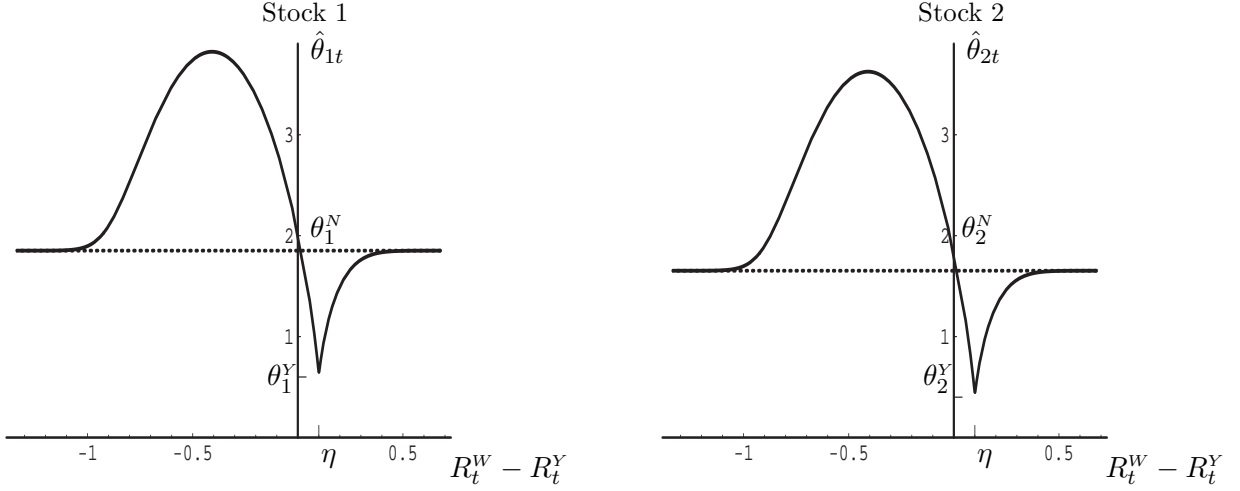
Stock 1

Stock 2

(a) Economies with $\theta_1^N > \theta_1^Y$ and $\theta_2^N > \theta_2^Y$.
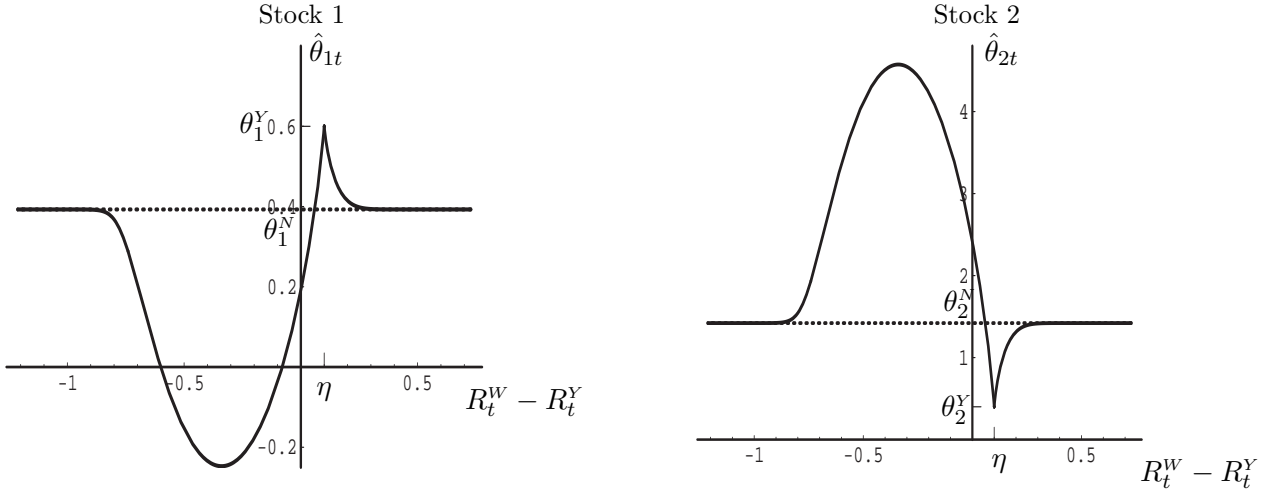
Stock 1

Stock 2

(b) Economies with $\theta_1^N < \theta_1^Y$ and $\theta_2^N > \theta_2^Y$.

Figure 6. The manager's optimal portfolio weights in Stocks 1 and 2. The solid plots are for the optimal portfolio weights, and the dotted plots are for the manager's normal weights. The parameter values are $\gamma_M = 1.0$, $f_L = 0.85$, $f_H = 1.15$, $\beta = (0.6, 0.4)^\top$, $\eta = 0.1$, $\mu = (0.07, 0.09)$, $r = 0.01$, $\sigma_1 = (0.18, 0.0)$ (panel (a)), $\sigma_1 = (0.18, 0.13)$ (panel (b)), $\sigma_2 = (0.0, 0.22)$, $t = 0.85$, $T = 1$.

It is also of interest to investigate whether the manager achieves her optimal risk-taking profile by taking on systematic versus idiosyncratic risk, given the current discussion in the literature (e.g., Chevalier and Ellison (1997)). When underperforming, does the manager really take on systematic risk as suggested by our baseline model, or rather, opts for idiosyncratic gambles, more in line with the perceived wisdom? To shed some light on this issue, we present a special case of our model, in which the manager has a choice between systematic and idiosyncratic risks. We consider

a two-stock economy with the following parameterization:

$$\mu = \begin{pmatrix} \mu_1 \\ r \end{pmatrix}, \quad \sigma = \begin{pmatrix} \sigma_{11} & 0 \\ 0 & \sigma_{22} \end{pmatrix}, \quad \beta = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

Here, the relative performance index Y is given by stock 1, which is driven solely by the Brownian motion w_1. In contrast, stock 2 is purely driven by w_2 and does not command any risk premium, with mean return equal to the riskless rate. Consequently, the market price of risk is given by $\kappa = \left(\frac{\mu_1 - r}{\sigma_{11}}, 0 \right)^\top$. In other words, by investing in stock 1 the manager is exposed to systematic risk, and by investing in stock 2 the manager takes on idiosyncratic risk. We report the ensuing optimal portfolio weights in stocks 1 and 2 in Figure 7.
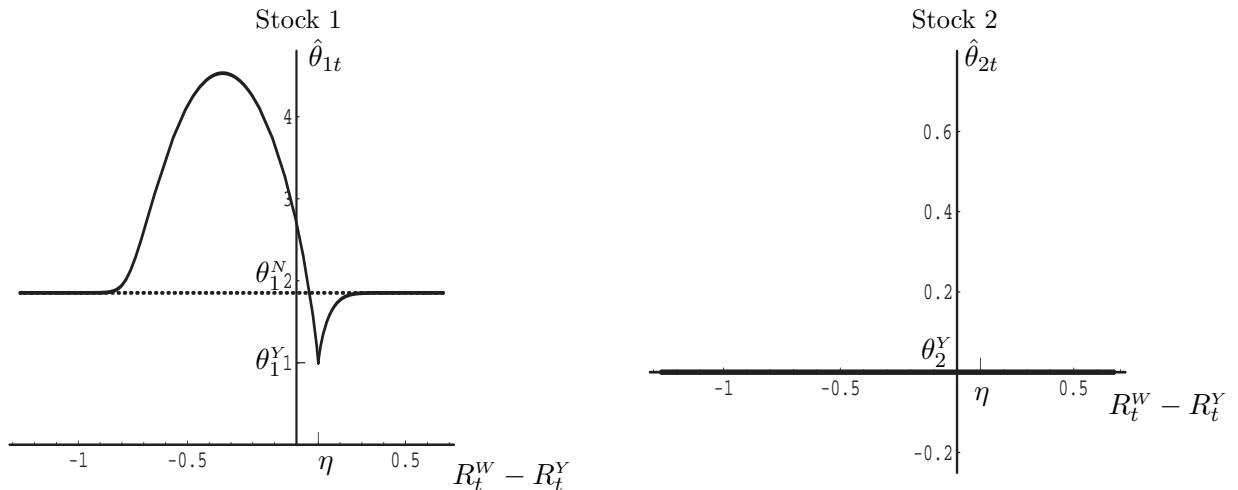

Figure 7. The manager's optimal portfolio weights in Stocks 1 and 2. Stock 1 is driven by systematic risk, while stock 2 is driven by idiosyncratic risk. The solid plots are for the optimal portfolio weights, and the dotted plots are for the manager's normal weights. The parameter values are $\gamma_M = 1.0$, $f_L = 0.85$, $f_H = 1.15$, $\beta = (1.0, 0.0)^\top$, $\eta = 0.1$, $\mu = (0.07, 0.01)$, $r = 0.01$, $\sigma_1 = (0.18, 0.0)$, $\sigma_2 = (0.0, 0.22)$, $t = 0.75$, $T = 1$.

As evident from Figure 7, the manager chooses to optimally invest *nothing* in stock 2, and hence is not exposed to any idiosyncratic risk. Instead, she is only exposed to systematic risk and engages in optimal risk-shifting via adjusting her position in stock 1, in a manner consistent with our results of Section 2. We note that the above example is not pathological. We have considered a sequence of economies parameterized by μ_2^m, with $\mu_2^m \to r$ as $m \to \infty$. In each economy m, the market price of risk due to w_2 is nonzero, and the weights in each risky asset are similar to those presented in Figure 6. As we increase m, the investment in the second stock uniformly converges to zero.

Remark 2. (Empirical applicability) Some support for our model's implications can be drawn from existing empirical literature on risk-shifting behavior of mutual funds. For example, Brown,

Harlow, and Starks (1996) find that underperforming managers increase the volatility of their portfolios towards the year-end. Employing a different data set, Busse (2001) revisits their test and refines their findings. However, the above papers do not offer direct evidence in favor of our model because the measure of portfolio riskiness adopted in these studies is the portfolio's total variance, and not a measure that accounts for the differential between the volatilities of the portfolio and the index (tracking error volatility) as our model implies. Chevalier and Ellison (1997) also find that the risk-taking incentives of the underperforming managers are higher than those who overperform. Their paper, in contrast, can be viewed as providing a stronger support for our conclusions, because they do use the sample variance of a fund's excess return over the market as a measure of the fund's riskiness.

A more precise way to test the implications of our model, however, would be to use data on mutual funds holdings, rather then estimates of funds' volatilities or tracking errors. Moreover, tests suggested by our analysis would differ from the standard approach in that by explicitly modeling a multi-asset environment, we have sharper predictions regarding investment in individual securities. The data on holdings is publicly available for a large crossection of funds at quarterly frequency. This data set needs to be matched with returns on the funds and the appropriate indices. It would appear reasonable to group funds by industry and use an industry benchmark as an appropriate index. For each quarter, one can compute a measure of deviation of portfolio weights of say top m holdings of each fund i from the corresponding weights in the relevant index Y: say, $Dev_{ti} = \sqrt{(\hat{\theta}_{1ti} - \theta_{1t}^Y)^2 + \ldots + (\hat{\theta}_{mti} - \theta_{mt}^Y)^2}$. We can then estimate the form of Dev as a function of relative performance. A possible way to do so is to use series estimation − a nonlinear regression of Dev on returns in excess of the index. One can then examine whether the function indeed has a wave shape as predicted by the model. To test our prediction that the risk-shifting behavior intensifies during the year, one can add quarter dummies and include the interaction terms with the dummies in the regression described above.

The outlined test does not require any estimation of the parameters of the investment opportunity set. One can then refine the test further and look at the directions in which the manager tilts her portfolio weights. This approach would require estimates of θ^N for each fund. One caveat of the above procedure is that mutual fund holdings sampled at quarterly frequency may be subject to window dressing, which would add errors in variables to the estimation, hence, if possible, it would be desirable to use data on holdings sampled at higher frequencies.

5. Conclusion

In this paper we have attempted to isolate the two most important adverse incentives of a fund manager: an implicit incentive to perform well relative to an index, and an explicit incentive to manage the fund in accordance with her own appetite for risk. Implicit incentives introduce a

nonconcavity in the manager's problem, akin to nonconcavities observed in many corporate finance applications (e.g., asset substitution problem, "gambling for resurrection," executive compensation, hedge fund managers compensation). It has been argued that in some of these applications, agents do not behave as though they are risk-neutral, and may be effectively risk averse. Our methodology of dealing with nonconcavities in the presence of risk aversion may then help shed some light on these and other issues of interest. We solve the manager's problem within a standard dynamically-complete Black and Scholes (1973) framework. The complete markets assumption offers considerable tractability, allowing us to derive the manager's optimal policy in closed-form, and also establishes a useful reference point for future research. In many real world applications, nonconcavities in the payoff structure go hand-in-hand with capital markets frictions, for example with restrictions against trading the underlying security designed to induce the "right" incentives to the manager. Our story of the optimal interaction of risk-shifting with risk aversion would then be further compounded by the effects of such frictions.

In our setup, the need for risk management restrictions arises as a natural consequence of altering the manager's incentives so as to make her act more in the interest of fund investors. The benchmarking restriction we advocate clearly benefits fund investors. However, our analysis leaves aside many possible constraints that may also be beneficial. We believe that endogenizing investment restrictions in the context of active money management is a fruitful area for future research. It would also be of interest to endogenize within our model the fund-flows to relative-performance relationship that we have taken as given.

There is some empirical work, such as Brown, Harlow, and Starks (1996) documenting that managers increase their portfolio variance halfway during the year when underperforming. Our model suggests that such a test may not fully capture managerial risk-shifting incentives. Even in the simple single-stock setting that we have considered, it may be optimal to decrease portfolio volatility. In a more general setting with multiple risky stocks, we demonstrate how the manager optimally tilts the portfolio weight in each stock in response to incentives. It would be of interest to investigate whether this is indeed the case in practice in future empirical work. While we focus on the fund manager's changing her portfolio composition in response to incentives, this may not be the only way the manager can increase her relative performance. As Christoffersen (2001) documents, about half of money fund managers voluntarily waive fees so as to increase their net return, and variation in these fee waivers is largely driven by relative performance. It would be interesting to incorporate such a feature into our model.

Appendix: Proofs

Proof of Proposition 1. Before proceeding with the proof, we present for completeness the results that for brevity were not included in the body of the proposition. First, note that for $\gamma_M = 1$, $g(\cdot)$ takes the form: $g(\xi) = -\left(\ln\frac{\hat{y}\xi}{f_L} + 1 - \ln\frac{\xi^{1/\hat{\kappa}}}{Af_H}\right) + yA\xi^{1-1/\hat{\kappa}}$. Second, since $\hat{W}_t\xi_t$ is a martingale (given the dynamics of $\hat{W}_t$ and ξ_t), the time-t wealth is obtained by evaluating the conditional expectation of $\hat{W}_T\xi_T$. In the economies described in (a):

$$
\begin{aligned}
\hat{W}_t &= E_t\left[\hat{W}_T\xi_T/\xi_t\right] \\
&= \left[\mathcal{N}(d(\gamma_M,\hat{\xi}))f_H^{(1/\gamma_M-1)} + \mathcal{N}(-d(\gamma_M,\xi_a))f_L^{(1/\gamma_M-1)}\right]Z(\gamma_M)(\hat{y}\xi_t)^{-1/\gamma_M} \\
&\quad + \left[\mathcal{N}(d(\hat{\kappa},\xi_a)) - \mathcal{N}(d(\hat{\kappa},\hat{\xi}))\right]AZ(\hat{\kappa})\xi_t^{-1/\hat{\kappa}}.
\end{aligned}
\tag{A1}
$$

Similarly, in the economies described in (b):

$$
\begin{aligned}
\hat{W}_t &= \left[\mathcal{N}(d(\gamma_M,\xi_b))f_L^{(1/\gamma_M-1)} + \mathcal{N}(-d(\gamma_M,\hat{\xi}))f_H^{(1/\gamma_M-1)}\right]Z(\gamma_M)(\hat{y}\xi_t)^{-1/\gamma_M} \\
&\quad + \left[\mathcal{N}(d(\hat{\kappa},\hat{\xi})) - \mathcal{N}(d(\hat{\kappa},\xi_b))\right]AZ(\hat{\kappa})\xi_t^{-1/\hat{\kappa}}.
\end{aligned}
\tag{A2}
$$

Finally, when $\theta^N = \theta^Y$, if $\eta \leq 0$ (so that $e^\eta Y_T$ and hence $R_T^Y + \eta$ are feasible) or $\eta \geq \bar{\eta}$ (so that $e^\eta Y_T$ and hence $R_T^Y + \eta$ are above a critical level of infeasibility), then $\hat{\theta}_t = \theta^N$ and $\hat{W}_T = J_M(\hat{y}\xi_T)$; otherwise $\hat{W}_T = \left\{\frac{1}{f_L}J_M\left(\frac{\bar{y}}{f_L}\xi_T\right) \quad or \quad e^\eta Y_T\right\}$, with the indifference solution alternating between the two values in any way that satisfies the budget constraint, where (using the steps outlined below) one can show that $\bar{\eta}$ solves $g\left(\xi = 1, y = f_L^{1-\hat{\kappa}}e^{(1-\hat{\kappa})(r+\beta(\mu-r)/2)T}/W_0^{\hat{\kappa}}, \eta\right) = 0$, and $\bar{y} > f_H^{1-\gamma_M}/A^{\gamma_M}$ solves $g(\xi = 1, y) = 0$.

We now proceed with the steps of the proof. To obtain the risk exposure expressions in the proposition, note that from (1), the diffusion term of the manager's optimal portfolio value process is $\hat{\theta}_t\sigma\hat{W}_t$. Equating the latter term with the diffusion term obtained by applying Itô's Lemma to (A1) and (A2) yields the expressions for $\hat{\theta}_t$ in economies (a) and (b), respectively. Therefore, to complete the proof, it is sufficient to establish optimality of the given terminal wealth $\hat{W}_T$. Methodologically, most related to this proof is the proof of Proposition 2 of the constrained model in Basak, Shapiro, and Tepla (2002), but the setting here is notably different and the optimization problem is unconstrained. The non-concavity of the problem, arising due to (3), has an extra dimension of complexity, as not only does the expression we employ for the convex conjugate has a discontinuous non-concavity at a stochastic location, but the magnitude of the discontinuity is stochastic as well. Therefore, to our knowledge, the way the proof below adapts the martingale representation and the convex-duality techniques (see, e.g., Karatzas and Shreve (1988)) to a non-concave problem, has not been previously used in the literature.

Appealing to the martingale representation approach, the dynamic budget constraint (1) of the manager's optimization problem can be restated using the terminal value of the state price density

process as $E[\hat{W}_T \xi_T] = W_0$. The manager thus effectively solves a static variational problem (in which, without loss of generality, $\xi_0 = 1$). For a given value of θ^N and θ^Y, let W_T be any candidate optimal solution, satisfying the static budget constraint $E[W_T \xi_T] \leq W_0$. Consider the following difference in the manager's expected utility:

$$E[u_M(\hat{W}_T f_T)] - E[u_M(W_T f_T)] = E[u_M(\hat{W}_T f_T)] - \hat{y}W_0 - (E[u_M(W_T f_T)] - \hat{y}W_0)$$

$$\geq E[u_M(\hat{W}_T f_T)] - E[\hat{y}\hat{W}_T \xi_T] + (E[u_M(W_T f_T)] - E[\hat{y}W_T \xi_T]) = E[v(\hat{W}_T, \xi_T) - v(W_T, \xi_T)], \quad \text{(A3)}$$

where the inequality is due to $\hat{W}_T$ satisfying the budget constraint with equality, while W_T satisfying the budget constraint with inequality, and where

$$v(W, \xi) = u_M(W f_L 1_{\{R^W - R^Y < \eta\}} + W f_H 1_{\{R^W - R^Y \geq \eta\}}) - \hat{y}W\xi. \quad \text{(A4)}$$

To show optimality of $\hat{W}_T$, it is left to show that the right-hand side of (A3) is non-negative. Under the geometric Brownian motion dynamics of Y_T and ξ_T, and using the normalization of Y_0 and ξ_0, it is straightforward to verify that $Y_T = A e^{-\eta} \xi_T^{-\beta\sigma/\kappa}$. The expression in (A4) is thus simplified to

$$v(W, \xi) = u_M(W f_L 1_{\{W < A\xi^{-\beta\sigma/\kappa}\}} + W f_H 1_{\{W \geq A\xi^{-\beta\sigma/\kappa}\}}) - \hat{y}W\xi. \quad \text{(A5)}$$

Given the manager's CRRA preferences, to establish the non-negativity of (A3), one needs to account for the relation between the parameters γ_M, β, σ, and κ in (A5). To avoid repetition of technical details, we provide the proof for optimality of $\hat{W}_T$ for the economies in (a) with $\gamma_M > 1$. The logic of the proof applies to the remaining subdivisions of the parameter space, as identified in the Proposition. Therefore, we now show that for the case in which $\kappa/(\beta\sigma) > \gamma_M > 1$,

$$\arg\max_W v(W, \xi) = f_H^{1/\gamma_M - 1}(\hat{y}\xi)^{-1/\gamma_M} 1_{\{\xi < \hat{\xi}\}} + A\xi^{-\beta\sigma/\kappa} 1_{\{\hat{\xi} \leq \xi < \xi_a\}} + f_L^{1/\gamma_M - 1}(\hat{y}\xi)^{-1/\gamma_M} 1_{\{\xi_a \leq \xi\}}.$$

Indeed, in the above convex conjugate construction, there are three local maximizers of $v(W, \xi)$: $W_H \equiv \frac{1}{f_H} J_M\left(\frac{\hat{y}}{f_H}\xi\right) = f_H^{1/\gamma_M - 1}(\hat{y}\xi)^{-1/\gamma_M}$, $W_L \equiv \frac{1}{f_L} J_M\left(\frac{\hat{y}}{f_L}\xi\right) = f_L^{1/\gamma_M - 1}(\hat{y}\xi)^{-1/\gamma_M}$, and $W_A \equiv A\xi^{-\beta\sigma/\kappa}$, where each of the three can become the global maximizer of $v(W, \xi)$ for different values of ξ. When $\xi = \hat{\xi}$, then $W_H(\hat{\xi}) = W_A(\hat{\xi})$. When $\xi < \hat{\xi}$, then $W_L > W_H > W_A$ holds under the given subdivision of the parameter space, and so for $W \in \{W_L, W_H, W_A\}$, we get $v(W, \xi) = u_M(W f_H) - \hat{y}W\xi$, establishing W_H as the global maximizer. When $\xi > \hat{\xi}$, then $W_H < \min(W_A, W_L)$, and $v(W_H, \xi) = u_M(W_H f_L) - \hat{y}W_H\xi$, establishing that W_H cannot be the global maximizer, because for $W_L < W_A$ by the local optimality of W_L, we have $u_M(W_H f_L) - \hat{y}W_H\xi < u_M(W_L f_L) - \hat{y}W_L\xi = v(W_L, \xi)$, and for $W_L \geq W_A$ accounting for the local optimality of W_A due to the stochastic non-concavity, we have $u_M(W_H f_L) - \hat{y}W_H\xi < u_M(W_A f_L) - \hat{y}W_A\xi = v(W_A, \xi)$. Moreover, for $\xi > \underline{\xi}$, where $\underline{\xi} = (\hat{y}A^{\gamma_M}/f_L^{1-\gamma_M})^{1/(\gamma_M/\hat{\kappa}-1)} > \hat{\xi}$, we have $W_L < W_A$; whereas for the range $\hat{\xi} \leq \xi < \underline{\xi}$, we get $W_L > W_A$, and for this range W_A is the global maximizer. Finally, note that for $\xi > \underline{\xi}$, we obtain $v(W_L, \xi) = v(W_A, \xi) + g(\xi)$. Then, using $\kappa/(\beta\sigma) > \gamma_M > 1$, and the fact that $g(\underline{\xi}) < 0$, $g(\infty) = \infty$, it is straightforward to verify that $g(\xi) > 0$ if and only if $\xi > \xi_a$, where $g(\xi_a) = 0$, and

$\xi_a > \underline{\xi}$, thereby completing the proof for the case of interest in the parameter space. Note that since $W_L(\underline{\xi}) = W_A(\underline{\xi})$, having $\hat{W} = W_A$ for $\hat{\xi} < \xi < \xi_a$ gives rise to a discontinuity in $\hat{W}_T$ as a function of ξ_T at ξ_a. The discontinuity arises in the other subcases in (a) as well, and analogously, under the parameter values in (b), the optimal policy is discontinuous at ξ_b. ∎

Proof of Proposition 2. In states in which the benchmarking restriction in (4) is binding, the manager's terminal wealth is given by $W_T^* = e^\varepsilon X_T$, where similarly to the case with the index Y, the benchmark level X_T is given by $X_T = Be^{-\varepsilon}\xi_T^{-\delta\sigma/\kappa}$. In states in which the restriction is not binding, the Lagrange multiplier associated with (4) is zero, and hence $W_T^* = \hat{W}_T(y^*)$, where y^* is the Lagrange multiplier associated with the static budget constraint of the restricted manager. Therefore, the terminal wealth is given by $W_T^* = \max\{\hat{W}_T, e^\varepsilon X_T\}$. For $\varepsilon = -\infty$, we have $W_T^* = \hat{W}_T$, while as ε increases to zero, the maximum operator generates the economies described in the proposition. The time-t wealth can then be obtained as a conditional expectation of the terminal wealth. In the economies described in (a):

$$
\begin{aligned}
W_t^* &= \mathcal{N}(d(\gamma_M, \tilde{\xi})) f_H^{(1/\gamma_M - 1)} Z(\gamma_M)(y^*\xi_t)^{-1/\gamma_M} \\
&\quad + \left[\mathcal{N}(d(\hat{\kappa}, \breve{\xi})) - \mathcal{N}(d(\hat{\kappa}, \hat{\xi}))\right] A Z(\hat{\kappa})\xi_t^{-1/\hat{\kappa}} 1_{\{a_2, a_3, a_4\}} \\
&\quad + \left[\mathcal{N}(d(\gamma_M, \breve{\xi})) - \mathcal{N}(d(\gamma_M, \xi_a))\right] f_L^{(1/\gamma_M - 1)} Z(\gamma_M)(y^*\xi_t)^{-1/\gamma_M} 1_{\{a_4\}} \\
&\quad + \mathcal{N}(-d(\breve{\kappa}, \breve{\xi})) B Z(\breve{\kappa})\xi_t^{-1/\breve{\kappa}} .
\end{aligned}
\tag{A6}
$$

In the economies described in (b):

$$
\begin{aligned}
W_t^* &= \mathcal{N}(d(\gamma_M, \tilde{\xi})) f_L^{(1/\gamma_M - 1)} Z(\gamma_M)(y^*\xi_t)^{-1/\gamma_M} \\
&\quad + \left[\mathcal{N}(d(\hat{\kappa}, \breve{\xi})) - \mathcal{N}(d(\hat{\kappa}, \xi_b))\right] A Z(\hat{\kappa})\xi_t^{-1/\hat{\kappa}} 1_{\{b_2, b_3, b_4, b_5\}} \\
&\quad + \left[\mathcal{N}(d(\gamma_M, \breve{\xi})) - \mathcal{N}(d(\gamma_M, \breve{\xi}))\right] f_H^{(1/\gamma_M - 1)} Z(\gamma_M)(y^*\xi_t)^{-1/\gamma_M} 1_{\{b_3, b_5\}} \\
&\quad + \left[(\mathcal{N}(d(\breve{\kappa}, \xi_b)) - \mathcal{N}(d(\breve{\kappa}, \xi_1)))1_{\{b_2, b_3\}} + \mathcal{N}(-d(\breve{\kappa}, \breve{\xi}))\right] B Z(\breve{\kappa})\xi_t^{-1/\breve{\kappa}} .
\end{aligned}
\tag{A7}
$$

The optimal risk exposure is derived for each economy using (A6) and (A7), following the steps outlined in Proposition 1. ∎

Proof of Proposition 3. To establish optimality of the stated terminal wealth for the manager's optimization problem in (3) under n sources of uncertainty and n risky assets, it is still sufficient to establish the non-negativity of the right-hand side of (A3), as in the proof of Proposition 1. However, because for $n > 1$, ξ and Y each span a different subspace of the state space, $v(W, \xi)$ as given in (A4) does not in general simplify to the parametric form in (A5). Therefore, to gain further tractability, note that in (A4), $v(W, \xi) = Y^{1-\gamma_M}\hat{v}(\frac{W}{Y}, \xi Y^{\gamma_M})$, where $Y^{1-\gamma_M} \geq 0$, and

$$
\hat{v}(V, \pi) = u_M(V f_L 1_{\{V < e^\eta\}} + V f_H 1_{\{V \geq e^\eta\}}) - y\pi V .
$$

Using this change of variables ($V \equiv \frac{W}{Y}$, $\pi \equiv \xi Y^{\gamma_M}$), one obtains:

$$\arg\max_V \hat{v}(V,\pi) = f_H^{1/\gamma_M - 1}(y\pi)^{-1/\gamma_M} 1_{\{\pi < \pi_*\}} + e^\eta 1_{\{\pi_* \le \pi < \pi^*\}} + f_L^{1/\gamma_M - 1}(y\pi)^{-1/\gamma_M} 1_{\{\pi^* \le \pi\}} \quad \text{(A8)}$$

$$= \left[\frac{1}{f_H} J_M\left(\frac{y}{f_H}\xi\right) 1_{\{\pi < \pi_*\}} + e^\eta Y_T 1_{\{\pi_* \le \pi < \pi^*\}} + \frac{1}{f_L} J_M\left(\frac{y}{f_L}\xi\right) 1_{\{\pi^* \le \pi\}} \right] \frac{1}{Y}. \quad \text{(A9)}$$

The equality in (A8) is readily verified by following steps analogous to those in Proposition 1, to show that each of the three local maximizers in the above convex conjugate construction is indeed a global maximizer in designated ranges of π. The equality in (A9) holds due to the change of variables. The expression in brackets in (A9) is the terminal wealth, $\hat{W}$, stated in the Proposition, thereby verifying its optimality. Note from (A8)–(A9) that $\hat{V}_T \equiv \frac{\hat{W}_T}{Y_T}$ is a function of only $\pi_T \equiv \xi_T Y_T^{\gamma_M}$. Also note that the diffusion component of $\xi_T Y_T^{\gamma_M}$ is a function of

$$-\kappa^\top w_T + \gamma_M \beta^\top \sigma w_T = -\gamma_M w_T^\top \sigma^\top \left(\frac{1}{\gamma_M}(\sigma^\top)^{-1}\kappa - \beta\right) = -\gamma_M w_T^\top \sigma^\top (\theta^N - \theta^Y).$$

The latter indicates that the optimal policy is driven by the component-wise relation between the manager's normal weights in each stock, θ_i^N, compared with the index weights θ_i^Y, $i = 1, \ldots, n$. With $n = 1$, Proposition 1, for expositional purposes, separately examines economies (a) ($\theta^N < \theta^Y$) and (b) ($\theta^N > \theta^Y$), to highlight the economic intuition of each case. With $n > 1$, Proposition 3 does not refine the expressions to account for all possible relations between γ_M, β, σ, and κ. We present results in their general form, and discuss how the basic intuition, with $n = 1$, extends to the case where various components of the optimal policy behave according to their counterparts in economies (a) or (b). Similarly, when for some i, $\theta_i^N = \theta_i^Y$, then $\hat{\theta}_i = \theta_i^N$. However, in the case where $\theta_i^N = \theta_i^Y$, for *all* i, then as discussed in Proposition 1, $\hat{W}_T$ behaves either as in the normal case, or alternates between the normal and the index levels.

Given the optimal terminal wealth, we proceed to derive the wealth dynamics and the trading strategy. Relying on the martingale property of ξW, $\hat{W}_t = E_t\left[\hat{W}_T \xi_T / \xi_t\right]$, but unlike in Proposition 1, one should account here for the joint distribution of Y^{γ_M} and ξ, under the physical probability measure P. To circumvent this, it is helpful to employ a change of measure. The new measure, G, is defined by a Radon-Nikodym derivative, which accounts for the manager's risk aversion and the composition of the index:

$$\frac{dG}{dP} \equiv e^{-\frac{1}{2}(1-\gamma_M)^2 ||\beta^\top \sigma||^2 T + (1-\gamma_M)\beta^\top \sigma w_T}.$$

Combining this change of measure with the change of variables, the martingale property of ξW implies that

$$\hat{V}_t \equiv \frac{\hat{W}_t}{Y_t} = E_t\left[\frac{\xi_T Y_T}{\xi_t Y_t}\hat{V}(\pi_T)\right] = E_t^G\left[\rho_t \frac{\pi_T}{\pi_t}\hat{V}(\pi_T)\right], \quad \text{(A10)}$$

where E^G is the expectation under the new measure, and $\rho_t = e^{(1-\gamma_M)(r+\beta^\top \sigma \kappa - \gamma_M ||\beta^\top \sigma||^2/2)(T-t)}$ is a deterministic function of time. The first equality in (A10) emphasizes the aforementioned fact that $\frac{\hat{W}_T}{Y_T}$ is only a function of π_T (π_T is log-normally distributed). The second equality in (A10)

32

follows using $Y_T = Y_t e^{(r+\beta^\top \sigma \kappa - ||\beta^\top \sigma||^2/2)(T-t)+\beta^\top \sigma (w_T - w_t)}$ and $w_t = (1-\gamma_M)\sigma^\top \beta t + w_t^G$, where w_t^G is an n-dimensional standard Brownian motion under G. After restating the problem in terms of π, the time-t wealth is straightforwardly obtained by evaluating the conditional expectation in (A10) under G:

$$
\begin{aligned}
\hat{W}_t &= Y_t E_t^G \left[\pi_T \hat{V}(\pi_T) (\rho_t / \pi_t) \right] \\
&= \left[\mathcal{N}(\hat{d}_1(\gamma_M, \pi_*)) f_H^{(1/\gamma_M - 1)} + \mathcal{N}(-\hat{d}_1(\gamma_M, \pi^*)) f_L^{(1/\gamma_M - 1)} \right] \hat{Z}(\gamma_M)(y\xi_t/\rho_t)^{-1/\gamma_M} \\
&\quad + \left[\mathcal{N}(\hat{d}_2(\gamma_M, \pi^*)) - \mathcal{N}(\hat{d}_2(\gamma_M, \pi_*)) \right] e^\eta Y_t .
\end{aligned}
\tag{A11}
$$

Note from (A11) that $\frac{\hat{W}_t}{Y_t}$, and hence $R_t^W - R_t^Y$, is a function of only $\pi_t \equiv \xi_t Y_t^{\gamma_M}$.

The expression for $\hat{\theta}_t$ in the Proposition follows directly from an application of Itô's Lemma to (A11). It is helpful to note the fact that like $\frac{\hat{W}_t}{Y_t}$, $\hat{\theta}_t$ is also a function of only π_t, which is evident since $\hat{\theta}_t$ depends on π_t via the $\hat{d}$ terms and via $\frac{\xi_t^{-1/\gamma_M}}{\hat{W}_t} = (\xi_t Y_t^{\gamma_M})^{-1/\gamma_M} \frac{Y_t}{\hat{W}_t} = \pi_t^{-1/\gamma_M}/\hat{V}(\pi_t)$. Finally, we observe that (A10) can be restated as

$$
\hat{V}_t(\pi_t) = \int_{-\infty}^{\infty} \rho_t e^{h_1(t,z)} \hat{V}_T(\pi_t e^{h_1(t,z)}) h_2(t,z) dz ,
$$

where, $h_1(t,z) = e^{-\frac{1}{2}||k-\gamma_M \sigma^\top \beta||(T-t)-z} \geq 0$, $h_2(t,z) \geq 0$ is a probability density function of a normal random variable with zero mean and a variance that is a deterministic function of time, and $\hat{V}_T(\cdot)$ is the non-increasing function in (A8) with a zero derivative over $[\pi_*, \pi^*)$ and a negative derivative over $(0, \pi_*) \cup (\pi^*, \infty)$. Differentiating under the integral with respect to π_t (assuming appropriate regularity conditions), establishes that $\forall \, 0 < t < T$, $\hat{V}_t(\pi_t)$ is a monotonically decreasing function of π_t. Therefore, regardless of the dimensionality of the structure of uncertainty in the economy, we can use the isomorphism between π_t and $\frac{\hat{W}_t}{Y_t}$ to plot $\hat{\theta}_t$ as a function of $R_t^W - R_t^Y$. ∎

Table 1a
Costs and benefits of active management in economies (a)

The gain/loss quantified in units of a hypothetical investor's initial wealth, $\hat{\lambda}$, solves $V^I((1+\hat{\lambda})W_0) = \hat{V}(W_0)$, where $V^I(\cdot)$ denotes the investor's indirect utility under his desired policy absent incentives θ^I, and $\hat{V}(\cdot)$ his indirect utility under the manager's incentives. The gain due to explicit incentives alone, λ^N, solves $V^I((1+\lambda^N)W_0) = \hat{V}(W_0; f_T = 1)$, where $\hat{V}(W_0; f_T=1)$ denotes the investor's indirect utility absent implicit incentives only. The gain due to implicit incentives alone, λ^Y, solves $1 + \hat{\lambda} = (1 + \lambda^N)(1 + \lambda^Y)$. The fixed parameter values are (where applicable) $\gamma_M = 1.0$, $\gamma_I = 2.0$, $f_L = 0.7$, $f_H = 1.3$, $(f_L + f_H)/2 = 1$, $\beta = 0.5$, $\eta = 0.1$, $\mu = 0.08$, $r = 0.01$, $\sigma = 0.17$, $W_0 = 1$, $T = 1$.

Effects of		Cost-benefit measures λ^Y, λ^N $\hat{\lambda}$ (%)				
Managerial risk	γ_M	0.5	1.00	2.00	3.00	4.00
aversion		-32.16, -31.71	-4.98, -4.15	-2.70, 0.00	-0.77, -0.47	0.35, -1.05
		-53.68	-8.92	-2.70	-1.24	-0.71
Implicit reward	f_H-f_L	0.2	0.4	0.6	0.8	1.0
for outperformance		-1.79, -4.15	-3.33, -4.15	-4.98, -4.15	-6.72, -4.15	-8.56, -4.15
		-5.86	-7.35	-8.92	-10.59	-12.35
Risk exposure	θ^Y	0.50	0.75	1.00	1.25	1.50
of the index		-4.98, -4.15	-6.08, -4.15	-7.71, -4.15	-9.78, -4.15	-12.17, -4.15
		-8.92	-9.98	-11.54	-13.52	-15.81
Flow threshold	η	-0.10	-0.05	0.00	0.05	0.10
		3.10, -4.15	2.89, -4.15	1.51, -4.15	-1.47, -4.15	-4.98, -4.15
		-1.18	-1.38	-2.71	-5.56	-8.92

Table 1b
Costs and benefits of active management in economies (b)

The gain/loss quantified in units of a hypothetical investor's initial wealth, $\hat{\lambda}$, solves $V^I((1+\hat{\lambda})W_0) = \hat{V}(W_0)$, where $V^I(\cdot)$ denotes the investor's indirect utility under his desired policy absent incentives θ^I, and $\hat{V}(\cdot)$ his indirect utility under the manager's incentives. The gain due to explicit incentives alone, λ^N, solves $V^I((1 + \lambda^N)W_0) = \hat{V}(W_0; f_T = 1)$, where $\hat{V}(W_0; f_T{=}1)$ denotes the investor's indirect utility absent implicit incentives only. The gain due to implicit incentives alone, λ^Y, solves $1 + \hat{\lambda} = (1 + \lambda^N)(1 + \lambda^Y)$. The fixed parameter values are (where applicable) $\gamma_M = 1.0$, $\gamma_I = 2.0$, $f_L = 0.7$, $f_H = 1.3$, $(f_L + f_H)/2 = 1$, $\beta = 1.0$, $\eta = 0.1$, $\mu = 0.06$, $r = 0.02$, $\sigma = 0.29$, $W_0 = 1$, $T = 1$.

Effects of		Cost-benefit measures λ^Y, λ^N $\hat{\lambda}$ (%)				
Managerial risk	γ_M	0.5	1.00	2.00	3.00	4.00
aversion		-9.96, -4.19	-5.61, -0.47	-3.24, 0.00	-2.37, -0.05	-1.93, -0.11
		-13.74	-6.06	-3.24	-2.42	-2.05
Implicit reward	f_H-f_L	0.2	0.4	0.6	0.8	1.0
for outperformance		-2.91, -0.47	-4.34, -0.47	-5.61, -0.47	-6.84, -0.47	-8.09, -0.47
		-3.37	-4.80	-6.06	-7.29	-8.53
Risk exposure	θ^Y	0.50	0.75	1.00	1.25	1.50
of the index		-7.35, -0.47	-5.85, -0.47	-5.61, -0.47	-6.69, -0.47	-8.45, -0.47
		-7.79	-6.30	-6.06	-7.13	-8.88
Flow threshold	η	-0.10	-0.05	0.00	0.05	0.10
		-0.91, -0.47	-1.78, -0.47	-4.26, -0.47	-4.47, -0.47	-5.61, -0.47
		-1.38	-2.25	-4.72	-4.92	-6.06

Costs and benefits of benchmarking to fund investors in economies (a)

The gain/loss quantified in units of a hypothetical investor's initial wealth, $\hat{\lambda}$, solves $V^I((1+\hat{\lambda})W_0) = \hat{V}(W_0)$, where $V^I(\cdot)$ denotes the investor's indirect utility under his desired policy absent incentives θ^I, and $\hat{V}(\cdot)$ his indirect utility under the manager's incentives. The incremental increase in the investor's utility due to restraining the manager, λ^*, solves $V^I((1+\lambda^*)(1+\hat{\lambda})W_0) = V^*(W_0)$, where $V^*(\cdot)$ is the indirect utility of the investor in the presence of benchmarking. The fixed parameter values are (where applicable) $\gamma_M = 1.0$, $\gamma_I = 2.0$, $\delta = 0.05$, $\varepsilon = -0.2$, $f_L = 0.7$, $f_H = 1.3$, $(f_L + f_H)/2 = 1$, $\beta = 0.5$, $\eta = 0.1$, $\mu = 0.08$, $r = 0.01$, $\sigma = 0.17$, $W_0 = 1$, $T = 1$.

Effects of		Cost-benefit measures $\hat{\lambda}, \lambda^*$				
Managerial risk	γ_M	0.5	1.00	2.00	3.00	4.00
aversion		-53.68, 111.98	-8.92, 8.82	-2.70, 1.85	-1.24, 0.34	-0.71, -0.17
Implicit reward	f_H-f_L	0.2	0.4	0.6	0.8	1.0
for outperformance		-5.86, 5.32	-7.35, 6.99	-8.92, 8.82	-10.59, 10.84	-12.35, 13.06
Risk exposure	θ^Y	0.50	0.75	1.00	1.25	1.50
of the index		-8.92, 8.82	-9.98, 10.02	-11.54, 11.57	-13.52, 13.48	-15.81, 15.71
Flow threshold	η	-0.10	-0.05	0.00	0.05	0.10
		-1.18, 0.12	-1.38, 0.52	-2.71, 1.78	-5.56, 5.06	-8.92, 8.82
Risk exposure	θ^X	0.1	0.2	0.3	0.4	0.5
of the benchmark		-8.92, 8.86	-8.92, 8.92	-8.92, 8.95	-8.92, 8.95	-8.92, 8.92
Allowed shortfall	ε	-0.25	-0.20	-0.15	-0.10	-0.05
		-8.92, 8.74	-8.92, 8.82	-8.92, 8.75	-8.92, 8.48	-8.92, 7.78

Table 2b
Costs and benefits of benchmarking to the investor in economies (b)

The gain/loss quantified in units of a hypothetical investor's initial wealth, $\hat{\lambda}$, solves $V^I((1+\hat{\lambda})W_0) = \hat{V}(W_0)$, where $V^I(\cdot)$ denotes the investor's indirect utility under his desired policy absent incentives θ^I, and $\hat{V}(\cdot)$ his indirect utility under the manager's incentives. The incremental increase in the investor's utility due to restraining the manager, λ^*, solves $V^I((1+\lambda^*)(1+\hat{\lambda})W_0) = V^*(W_0)$, where $V^*(\cdot)$ is the indirect utility of the investor in the presence of benchmarking. The fixed parameter values are (where applicable) $\gamma_M = 1.0$, $\gamma_I = 2.0$, $\delta = 0.05$, $\varepsilon = -0.2$, $f_L = 0.7$, $f_H = 1.3$, $(f_L + f_H)/2 = 1$, $\beta = 1.0$, $\eta = 0.1$, $\mu = 0.06$, $r = 0.02$, $\sigma = 0.29$, $W_0 = 1$, $T = 1$.

Effects of		Cost-benefit measures $\hat{\lambda}, \lambda^*$				
Managerial risk	γ_M	0.5	1.00	2.00	3.00	4.00
aversion		-13.74, 12.66	-6.06, 3.46	-3.24, 0.89	-2.42, 0.98	-2.05, 0.98
Implicit reward	f_H-f_L	0.2	0.4	0.6	0.8	1.0
for outperformance		-3.37, 0.94	-4.80, 2.09	-6.06, 3.46	-7.29, 4.83	-8.53, 6.25
Risk exposure	θ^Y	0.50	0.75	1.00	1.25	1.50
of the index		-7.79, 5.98	-6.30, 4.16	-6.06, 3.46	-7.13, 4.12	-8.88, 5.58
Flow threshold	η	-0.10	-0.05	0.00	0.05	0.10
		-1.38, 0.08	-2.25, 0.07	-4.72, 2.54	-4.92, 2.55	-6.06, 3.46
Risk exposure	θ^X	0.1	0.2	0.3	0.4	0.5
of the benchmark		-6.06, 3.58	-6.06, 3.76	-6.06, 3.84	-6.06, 3.87	-6.06, 3.86
Allowed shortfall	ε	-0.25	-0.20	-0.15	-0.10	-0.05
		-6.06, 2.77	-6.06, 3.46	-6.06, 4.18	-6.06, 4.90	-6.06, 5.60

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