The empirical relevance of the New Keynesian Phillips curve*

Gunnar Bårdsen
Norwegian University of Science and Technology and Norges Bank

Eilev S. Jansen
Norges Bank, University of Oslo,
and Norwegian University of Science and Technology

Ragnar Nymoen
University of Oslo and Norges Bank

31 March 2003

Abstract

The dynamic properties of the The New Keynesian Phillips curve (NPC) is analysed within the framework of a small system of linear difference equations. We evaluate the empirical results of existing studies which uses 'Euroland' and US data. The debate has been centered around the goodness-of-fit, but this is a weak criterion since the NPC-fit is typically well approximated by purely statistical models (e.g., a random walk). Several other parametric tests are then considered, and the importance of modelling a system that includes the forcing variables as well as the rate of inflation is emphasized. We also highlight the role of existing studies in providing new information relative to that which underlies the typical NPC. This encompassing approach is applied to open economy versions of the NPC for UK and Norway.

Keywords: *New Keynesian Phillips curves, US inflation, Euro inflation, UK inflation, Norwegian inflation, Monetary policy, Dynamic stability conditions, Evaluation, Encompassing tests.*

JEL classification: *C22, C32, C52, E31, E52.*

*We are grateful to J. David López-Salido and Jeremy Rudd for supplying us with data for the Euro Area and the US, respectively. Likewise, we are indepted to Nicoletta Batini, Brian Jackson and Stephen Nickell for providing us with their UK data set. This paper has been presented at seminars at the University of Aarhus, Nuffield College Oxford, University of California San Diego, Federal Reserve Board, Norwegian University of Science and Technology, and the Central Bank of Norway. Finally, we would like to thank Øyvind Eitrheim, Graham Elliot, Neil R. Ericsson, Mark Gertler, Clive Granger, Niels Haldrup, David F. Hendry, Steinar Holden, Jesper Lindé, Sophocles Mavroeidis, Atle Seierstad, Anders Rygh Swensen and Ulf Søderstrøm for comments and discussion.

1 Introduction

The New Keynesian Phillips Curve is promising to become a new consensus theory of inflation in modern monetary economics. This position is due to its stringent theoretical derivation, as laid out in Clarida et al. (1999) and Svensson (2000). In addition favourable empirical evidence is accumulating rapidly. For example, the recent studies of Galí and Gertler (1999) and Galí et al. (2001a) —hereafter GG and GGL— report empirical support for the NPC using European as well as US data. In addition Batini et al. (2000) have derived an open economy NPC and estimated the model on UK economy with favourable results for the specification. Nevertheless, in this paper we show that the empirical relevance of the NPC must be considered to be weak. The basis for our conclusion is a critical appraisal of GGL on their Euro area data set, and separate econometric analyses of UK and Norwegian inflation dynamics.

The pure New Keynesian Phillips curve, hereafter NPC, explains current inflation by expected future inflation and a forcing variable, for example a measure of excess demand. Second, in section 3, we investigate the dynamic properties of the NPC, which entails not only the NPC equation, but also specification of a process for the forcing variable. Given that a system of linear difference equations is the right framework for theoretical discussions about stability and the type of solution (forward or backward), it follows that the practice of deciding on these issues on the basis of single equation estimation is non-robust to extensions of the information set. For example, a forward solution may suggest itself from estimation of the NPC equation alone, while system estimation may show that the forcing variable is endogenous, giving rise to a different set of characteristic roots and potentially giving support to a backward solution.

The discussion in section 3 covers both the pure NPC and the so called 'hybrid' model that also includes lags of the rate of inflation, and section 4 discuss estimation issues of the NPC, using Euro area data for illustration. In section 5 we apply several methods for testing and evaluation of the Euro and US NPC, e.g., goodness-of-fit and tests of significance (of forward terms and of imputed present value terms). One of the messages that emerges is that emphasis on goodness-of-fit is of little value, since the fit is practically indistinguishable from modelling inflation following a random walk. Building on the insight from section 3, we also show that it is useful to extend the evaluation from the single equation NPC to a system consisting of the rate of inflation and the forcing variable.

However, in many countries there is a literature (stemming back to the 1980s and 1990s) on wage and price modelling. For example, there are several studies that have found evidence of cointegrating relationships between wages, prices, unemployment and productivity, as well as a particular ordering of causality. In section 6 we show that these existing results represent new information relative to the (theoretically derived) NPC, and how that information can be used for testing the encompassing implications of the NPC. This approach is applied to open economy versions of the NPC, for Norway and UK.

2 The NPC defined

Let p_t be the log of a price level index. The NPC states that inflation, defined as $\Delta p_t \equiv p_t - p_{t-1}$, is explained by $\mathsf{E}_t \Delta p_{t+1}$, expected inflation one period ahead conditonal upon information available at time t, and excess demand or marginal costs x_t (e.g., output gap, the unemployment rate or the wage share in logs):

$$\Delta p_t = b_{p_1} \mathsf{E}_t \Delta p_{t+1} + b_{p2} x_t + \varepsilon_{pt}, \tag{1}$$

where ε_{pt} is a stochastic error term. Roberts (1995) has shown that several New Keynesian models with rational expectations have (1) as a common representation—including the models of staggered contracts developed by Taylor (1979, 1980)[1] and Calvo (1983), and the quadratic price adjustment cost model of Rotemberg (1982). GG have given a formulation of the NPC in line with Calvo's work: They assume that a firm takes account of the expected future path of nominal marginal costs when setting its price, given the likelihood that its price may remain fixed for multiple periods. This leads to a version of the inflation equation (1), where the forcing variable x_t is the representative firm's real marginal costs (measured as deviations from its steady state value). They argue that the wage share (the labour income share) ws_t is a plausible indicator for the average real marginal costs, which they use in the empirical analysis. The alternative, hybrid version of the NPC that uses both $\mathsf{E}_t \Delta p_{t+1}$ and lagged inflation as explanatory variables is also discussed below.

3 A NPC system

Equation (1) is incomplete as a model for inflation, since the status of x_t is left unspecified. On the one hand, the use of the term forcing variable, suggests exogeneity, whereas the custom of instrumenting the variable in estimation is germane to endogeneity. In order to make progress, we therefore consider the following completing system of stochastic linear difference equations[2]

$$\begin{aligned} \Delta p_t &= b_{p1} \Delta p_{t+1} + b_{p2} x_t + \varepsilon_{pt} - b_{p1} \eta_{t+1} && (2) \\ x_t &= b_{x1} \Delta p_{t-1} + b_{x2} x_{t-1} + \varepsilon_{xt} && (3) \\ 0 &\leq |b_{x2}| < 1 \end{aligned}$$

The first equation is adapted from (1), utilizing that $\mathsf{E}_t \Delta p_{t+1} = \Delta p_{t+1} - \eta_{t+1}$, where η_{t+1} is the expectation error. Equation (3) captures that there may be feed-back from inflation on the forcing variable x_t (output-gap, the rate of unemployment or the wage share) in which case $b_{x1} \neq 0$.

In order to discuss the dynamic properties of this system, re-arrange (2) to yield

$$\Delta p_{t+1} = \frac{1}{b_{p1}} \Delta p_t - \frac{b_{p2}}{b_{p1}} x_t - \frac{1}{b_{p1}} \varepsilon_{pt} + \eta_{t+1} \tag{4}$$

[1] The overlapping wage contract model of sticky prices is also attributed to Phelps (1978).

[2] Constant terms are omitted for ease of exposition.

and substitute x_t with the right hand side of equation (3). The characteristic polynomial for the system (3) and (4) is

$$p(\lambda) = \lambda^2 - \left[\frac{1}{b_{p1}} + b_{x2}\right]\lambda + \frac{1}{b_{p1}}\left[b_{p2}b_{x1} + b_{x2}\right]. \tag{5}$$

If neither of the two roots are on the unit circle, unique asymptotically stationary solutions exists. They may be either causal solutions (functions of past values of the disturbances and of initial conditions) or future dependent solutions (functions of future values of the disturbances and of terminal conditions), see Brockwell and Davies (1991, Ch. 3), Gourieroux and Monfort (1997, Ch. 12).

The future dependent solution is a hallmark of the New Keynesian Phillips curve. Consider for example the case of $b_{x1} = 0$, so x_t is a strongly exogenous forcing variable in the NPC. This restriction gives the two roots $\lambda_1 = b_{p1}^{-1}$ and $\lambda_2 = b_{x2}$. Given the restriction on b_{x2} in (3), the second root is always less than one, meaning that x_t is a causal process that can be determined from the backward solution. However, since $\lambda_1 = b_{p1}^{-1}$ there are three possibilities for Δp_t: i) No stationary solution: $b_{p1} = 1$; ii) A causal solution: $b_{p1} > 1$; iii) A future dependent solution: $b_{p1} < 1$. If $b_{x1} \neq 0$, a stationary solution may exist even in the case of $b_{p1} = 1$. This is due to the multiplicative term $b_{p2}b_{x1}$ in (5). The economic interpretation of the term is the possibility of stabilizing interaction between price setting and product (or labour) markets—as in the case of a conventional Phillips curve.

As a numeric example, consider the set of coefficient values: $b_{p1} = 1$, $b_{p2} = 0.05$, $b_{x2} = 0.7$ and $b_{x1} = 0.2$, corresponding to x_t (interpreted as the output-gap) influencing Δp_t positively, and the lagged rate of inflation having a positive coefficient in the equation for x_t. The roots of (5) are in this case $\{0.96, 0.74\}$, so there is a causal solution. However, if $b_{x1} < 0$, there is a future dependent solution since the largest root is greater than one.

Finding that the existence and nature of a stationary solution is a system property is of course trivial. Nevertheless, many empirical studies only model the Phillips curve, leaving the x_t part of the system implicit. This is unfortunate, since the same studies often invoke a solution of the well known form[3]

$$\Delta p_t = \left(\frac{b_{p2}}{1 - b_{p1}b_{x2}}\right) x_t + \varepsilon_{pt} \tag{6}$$

Clearly, (6) hinges on $b_{p1}b_{x2} < 1$ which involves the coefficient b_{x2} of the x_t process.

If we consider the rate of inflation to be a jump variable, there may be a saddle-path equilibrium as suggested by the phase diagram in figure 1. The drawing is based on $b_{p2} < 0$, so we now interpret x_t as the rate of unemployment. The line representing combinations of Δp_t and x_t consistent with $\Delta^2 p_t = 0$ is downward sloping. The set of pairs $\{\Delta p_t,\, x_t\}$ consistent with $\Delta x_t = 0$ are represented by the thick vertical line (this is due to $b_{x1} = 0$ as above). Point a is a stationary situation, but it is not asymptotically stable. Suppose that there is a rise in x represented

[3] I.e., subject to the transversality condition

$$\lim_{n\to\infty} (b_{p1})^{n+1} \Delta p_{t+n+1} = 0$$

by a rightward shift in the vertical curve, which is drawn with a thinner line. The arrows show a potential unstable trajectory towards the north-east away from the initial equilibrium. However, if we consider Δp_t to be a jump variable and x_t as a state variable, the rate of inflation may jump to a point such as b and thereafter move gradually along the saddle path connecting b and the new stationary state c.

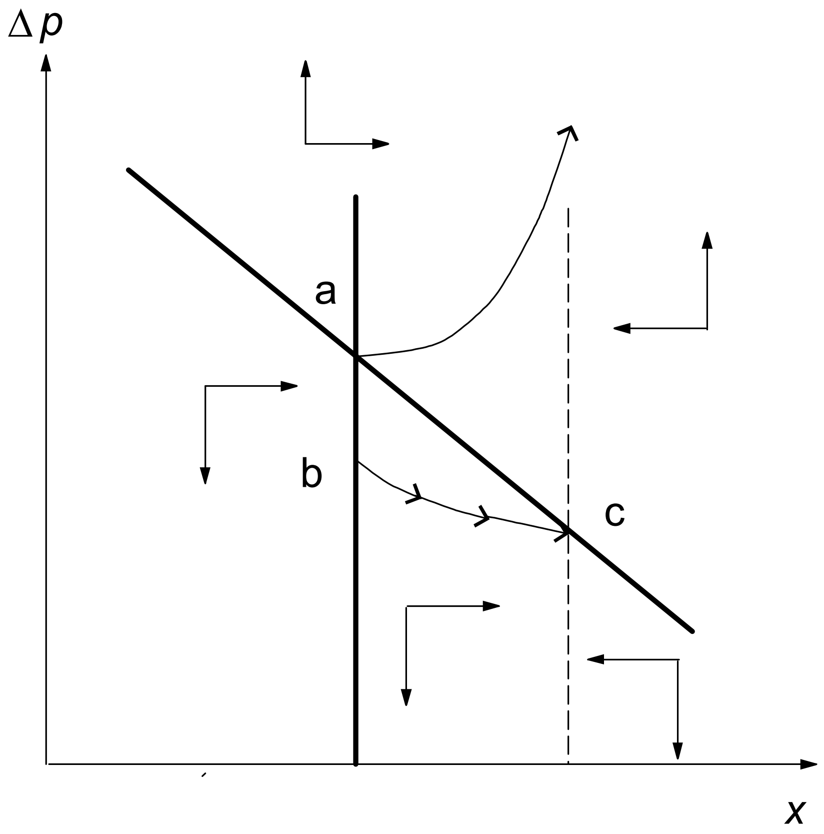

Figure 1: Phase diagram for the system for the case of $b_{p1} < 1$, $b_{p2} < 0$ and $b_{x1} = 0$

The jump behaviour implied by models with forward expected inflation is at odds with observed behaviour of inflation. This have led several authors to suggest a "hybrid" model, by heuristically assuming the existence of both forward- and backward-looking agents, see for example Fuhrer and Moore (1995). Also Chadha et al. (1992) suggest a form of wage-setting behaviour that would lead to some inflation stickiness and to inflation being a weighted average of both past inflation and expected future inflation. Fuhrer (1997) examines such a model empirically and he finds that future prices are empirically unimportant in explaining price and inflation behaviour compared to past prices.

In the same spirit as these authors, and with particular reference to the empirical assessment in Fuhrer (1997), GG also derive a hybrid Phillips curve that allows a subset of firms to have a backward-looking rule to set prices. The hybrid model contains the wage share as the driving variable and thus nests their version of the NPC as a special case. This amounts to the specification

$$\Delta p_t = b_{p1}^f \mathsf{E}_t \Delta p_{t+1} + b_{p1}^b \Delta p_{t-1} + b_{p2} x_t + \varepsilon_{pt}. \tag{7}$$

GG estimate (7) for the U.S. in several variants —using different inflation measures, different normalization rules for the GMM estimation, including additional lags of inflations in the equation and splitting the sample. They find that the overall picture remains unchanged. Marginal costs have a significant impact on short run inflation dynamics and forward looking behavior is always found to be important.

In the same manner as above, equation (7) can be written as

$$\Delta p_{t+1} = \frac{1}{b^f_{p1}}\Delta p_t - \frac{b^b_{p1}}{b^f_{p1}}\Delta p_{t-1} - \frac{b_{p2}}{b^f_{p1}}x_t - \frac{1}{b_{p1}}\varepsilon_{pt} + \eta_{t+1} \tag{8}$$

and combined with (3). The characteristic polynomial of the hybrid system is

$$p(\lambda) = \lambda^3 - \left[\frac{1}{b^f_{p1}} + b_{x2}\right]\lambda^2 + \frac{1}{b^f_{p1}}\left[b^b_{p1} + b_{p2}b_{x1} + b_{x2}\right]\lambda - \frac{b^b_{p1}}{b^f_{p1}}b_{x2}. \tag{9}$$

Using the typical results (from the studies cited below, see section 5.2) for the expectation and backward-looking parameters, $b^f_{p1} = 0.25$, $b^b_{p1} = 0.75$, together with the assumption of an exogenous x_t process with autoregressive parameter 0.7, we obtain the roots $\{3.0,\ 1.0,\ 0.7\}$.[4] Thus, there is no asymptotically stable stationary solution for the rate of inflation in this case.

This seems to be a common result for the hybrid model as several authors choose to impose the restriction

$$b^f_{p1} + b^b_{p1} = 1, \tag{10}$$

which forces a unit root upon the system. To see this, note first that a 1-1 reparameterization of (8) gives

$$\Delta^2 p_{t+1} = \left[\frac{1}{b^f_{p1}} - \frac{b^b_{p1}}{b^f_{p1}} - 1\right]\Delta p_t + \frac{b^b_{p1}}{b^f_{p1}}\Delta^2 p_t - \frac{b_{p2}}{b^f_{p1}}x_t - \frac{1}{b_{p1}}\varepsilon_{pt} + \eta_{t+1},$$

so that if (10) holds, (8) reduces to

$$\Delta^2 p_{t+1} = \frac{-b_{p0}}{b^f_{p1}} + \frac{(1 - b^f_{p1})}{b^f_{p1}}\Delta^2 p_t - \frac{b_{p2}}{b^f_{p1}}x_t - \frac{1}{b_{p1}}\varepsilon_{pt} + \eta_{t+1}. \tag{11}$$

Hence, the homogeneity restriction (10) turns the hybrid model into a model of the *change* in inflation. Equation (11) is an example of a model that is cast in the difference of the original variable, a so called dVAR, only modified by the driving variable x_t. Consequently, it represents a generalization of the random walk model of inflation that was implied by setting $b^f_{p1} = 1$ in the original NPC. The result in (11) will prove important in understanding the behaviour of the NPC in terms of goodness of fit, see below.

If the process x_t is strongly exogenous, the NPC in (11) can be considered at its own. In that case (11) has no stationary solution for the rate of inflation. A necessary requirement is that there are equilibrating mechanisms elsewhere in

[4]The full set of coefficients values is thus: $b_{x1} = 0$, $b^f_{p1} = 0.25$, $b^b_{p1} = 0.75$, $b_{x2} = 0.7$

the system, specifically in the process governing x_t (e.g., the wage share). This requirement parallels the case of dynamic homogeneity in the backward looking Phillips curve (i.e., a vertical long run Phillips curve). In the present context the message is that statements about the stationarity of the rate of inflation, and the nature of the solution (backward or forward) requires an analysis of the system.

The empirical results of GG and GGL differ from other studies in two respects. First, b_{p1}^f is estimated in the region (0.65, 0.85) whereas b_{p1}^b is one third of b_{p1}^f or less. Second, GG and GGL succeed in estimating the hybrid model without imposing (10). GGL (their Table 2) report the estimates { 0.69, 0.27} and {0.88, 0.025} for two different estimation techniques. The corresponding roots are {1.09, 0.70, 0.37} and {1.11, 0.70, 0.03}, illustrating that as long as the sum of the weights is less than one the future dependent solution prevails.

4 European inflation and the NPC

As mentioned above, GG and GGL use the formulation of the NPC (1) where the forcing variable x_t is the wage share ws_t. Since under rational expectations the errors in the forecast of Δp_{t+1} and ws_t is uncorrelated with information dated $t-1$ and earlier, it follows from (1) that

$$\mathsf{E}\{(\Delta p_t - b_{p1}\Delta p_{t+1} - b_{p2} ws_t) z_{t-1}\} = 0 \tag{12}$$

where z_t is a vector of instruments.

The orthogonality conditions given in (12) form the basis for estimating the model with generalized method of moments (GMM). The authors report results which they record as being in accordance with *a priori* theory. GGL report estimates of (1) for the US as well as for Euroland.[5] Using US data for 1960:1 to 1997:4, they report

$$\Delta p_t = \underset{(0.029)}{0.924}\Delta p_{t+1} + \underset{(0.118)}{0.250} ws_t. \tag{13}$$

Using their aggregate data for the Euro area[6] 1971.3 -1998.1, we replicate their results for Europe:[7]

$$\Delta p_t = \underset{(0.04)}{0.914}\Delta p_{t+1} + \underset{(0.04)}{0.088} ws_t + \underset{(0.06)}{0.14} \tag{14}$$
$$\hat{\sigma} = 0.321 \quad \chi^2_{\mathsf{J}}(9) = 8.21\,[0.51]$$

The instruments used are five lags of inflation, and two lags of the wage share, output gap (detrended output), and wage inflation. $\hat{\sigma}$ denotes the estimated residual standard error, and χ^2_{J} is the statistics of the validity of the overidentifying instruments (Hansen, 1982).

[5] These estimates are - somewhat confusingly - called “reduced form” estimates in GG and GGL, meaning that the ”deep” structural parameters of their model are not identified and hence not estimated in this linear relationsship between Δp_t, $E_t\{\Delta p_{t+1}\}$ and ws_t.

[6] See the Appendix B and Fagan et al. (2001).

[7] That is, equation (13) in GGL. We are grateful to J. David López-Salido of the Bank of Spain, who kindly provided us with the data for the Euro area and a RATS-program used in GGL.

In terms of the dynamic model (3) - (4) the implied roots are $\{1.08, 0.7\}$ for the US and $\{1.09, 0.7\}$ for Europe. Thus, as asserted by GGL, there is a stationary forward solution in both cases. However, since neither of the two studies contain any information about the wage share process, the roots obtained are based on the additional assumption of an exogenous wage share ($b_{x1} = 0$) with autoregressive coefficient $b_{x2} = 0.7$.

In the following, the results in GGL for Euroland will serve as one main point of reference. As background we therefore report some sensitivity analysis of equation (14), which is central in GGL.

First we want to investigate any sensitivity with regards to estimation methodology. The results in (14) were obtained by a GMM procedure which computes the weighting matrix once. When instead iterating over both coefficients and weighting matrix, with fixed bandwidth,[8] we obtain

$$\Delta p_t = \underset{(0.06)}{1.01}\Delta p_{t+1} - \underset{(0.06)}{0.05} ws_t + \underset{(0.09)}{-0.03} \tag{15}$$
$$\text{GMM}, \; T = 104 \; (1972\ (2) \text{ to } 1998\ (1))$$
$$\hat{\sigma} = 0.342 \quad \chi^2_{\mathrm{J}}(9) = 9.83\,[0.36]\,.$$

which reinforces the impression from (14) of the NPC as the equivalent of a random walk in inflation. To further investigate the robustness of the parameters, we have estimated the parameters with rolling regressions, using a fixed window of 80 observations. The upper panels of Figure 2 show that whereas the estimates of the coefficient for the forward variable are fairly robust, but drifting downwards across sub-samples, the significance of the wage share coefficient is fragile. Moreover, we notice that there are increasing instability in the estimated coefficients and the associated uncertainty is much larger in the second half of the time window. An alternative informal test is recursive estimation. Starting with a sample of 40 observations, the lower panels just confirm the impression so far of the NPC as the equivalent of a random walk: the coefficient of expected inflation hovers around unity, while the wage share impact is practically zero.

[8] We used the GMM implementation in Eviews 4.

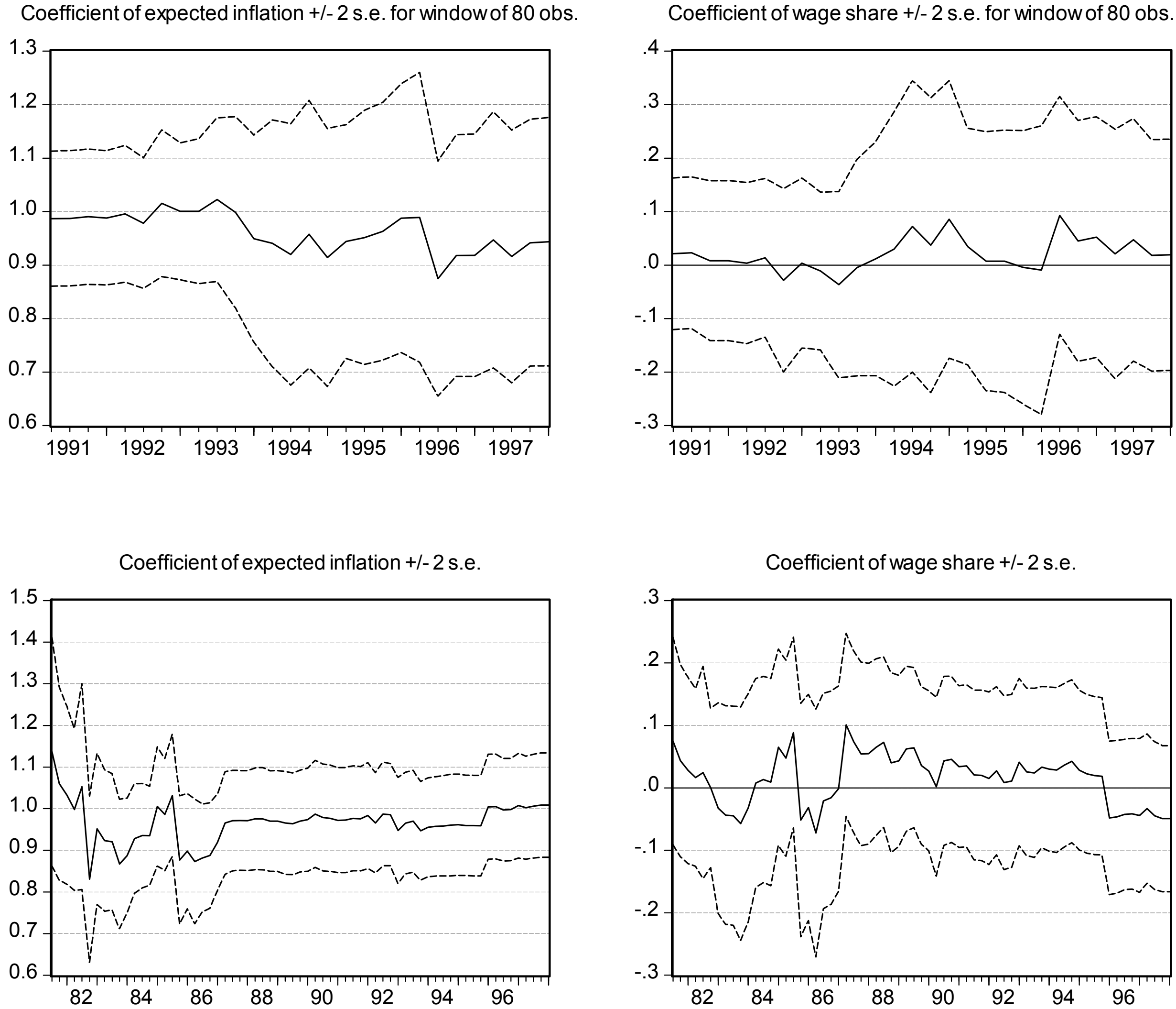

Figure 2: The upper graphs show rolling regression coefficients +/- 2 standard errors of the New Keynesian Phillips curve for the Euro area equation (15), i.e., estimated with GMM. The lower graphs show the recursive estimates of those coefficients.

The choices made for GMM estimation is seen to influence the results. This is an argument for checking robustness by also reporting the results of simpler estimation methods. The equation is linear in the parameters, so two stage least squares (2SLS) is an alternative. However, care must be taken when estimating by 2SLS, since we have to correct for induced moving average residuals, see Appendix A. Thus, allowing for an MA term in the residuals, 2SLS produces

$$\Delta p_t = \underset{(0.08)}{0.96}\Delta p_{t+1} + \underset{(0.08)}{0.06} ws_t + \underset{(0.11)}{0.07} \tag{16}$$

$$\text{2SLS, } T = 104 \text{ (1972 (2) to 1998 (1))}$$

$$\hat{\sigma} = 0.30 \quad \text{MA-coeff: } \underset{(0.11)}{-0.40}$$

which comes close to (14) for the coefficient of Δp_{t+1}, whereas the wage share parameter is insignificant in (16).[9] The estimated negative moving average coefficient is only half the magnitude of the coefficient of the forward term. Arguably, these magnitudes should be more or less equal, i.e. if the (1) is the true model, the forward solution applies and the disturbances of the NPC and the x_t process are independent. Importantly, a negative moving average in the NPC residual is not by itself corroborating evidence of the theory. Specifically, such a finding is also consistent with i) a unique causal solution for Δp_t, and ii) $\hat{b}_{p1} \approx 1$ in the estimated NPC (i.e., a form of over-differencing).

We conclude that the significance of the wage share as the driving variable is fragile, depending on the exact implementation of the estimation method used. The close to unity coefficient of the forward variable on the other hand is pervasive and will be a focal point of the following analysis. A moving average residual is another robust feature. However, there is an issue whether the autocorrelated residuals can be taken as evidence of serial correlation in the disturbances of the theoretical NPC, or on the contrary, whether it is a symptom of more general model misspecification.

5 Tests and empirical evaluation: US and 'Euroland' data

The main tools for evaluation of the NPC on US and 'Euroland' data have been the GMM test of validity of the overidentifying restrictions (*i.e.,* the χ^2_{J}-test above) and measures and graphs of goodness-of-fit. In particular the closeness between the fitted inflation of the NPC and actual inflation, is taken as telling evidence of the models relevance for understanding US and 'Euroland' inflation, see GG and GGL. We therefore start with an examination of what goodness-of-fit can tell us about model validity in this area. The answer appears to be: 'very little'. In the following sections we therefore investigate other approaches, with applications to the two large economy data sets.

5.1 Goodness-of-fit

Several papers place conclusive emphasis on the fit of the estimated New Keynesian Phillips curve. Using US data, GG, though rejecting the pure forward-looking model in favour of a hybrid model, nonetheless find that the baseline model remains predominant. In the *Abstract* of GGL the authors state that "the NPC fits Euro data very well, possibly better than US data". However, in the previous section, we saw that statistically speaking, the baseline Euro-area NPC is in fact reducible to a simple random walk, cf. the χ^2_{RW} statistics. In terms of fit, we therefore infer that the Euro-area NPC does as good (or bad) as a simple random walk.

Figure 3 illustrates this point by showing actual and fitted values of (14) together with the fit of a random walk in the left-hand panel. The similarity between the series is obvious, and the right-hand panel shows the cross-plot with regression line of the fitted values.

[9]Robust standard errors are computed using the Newey-West correction

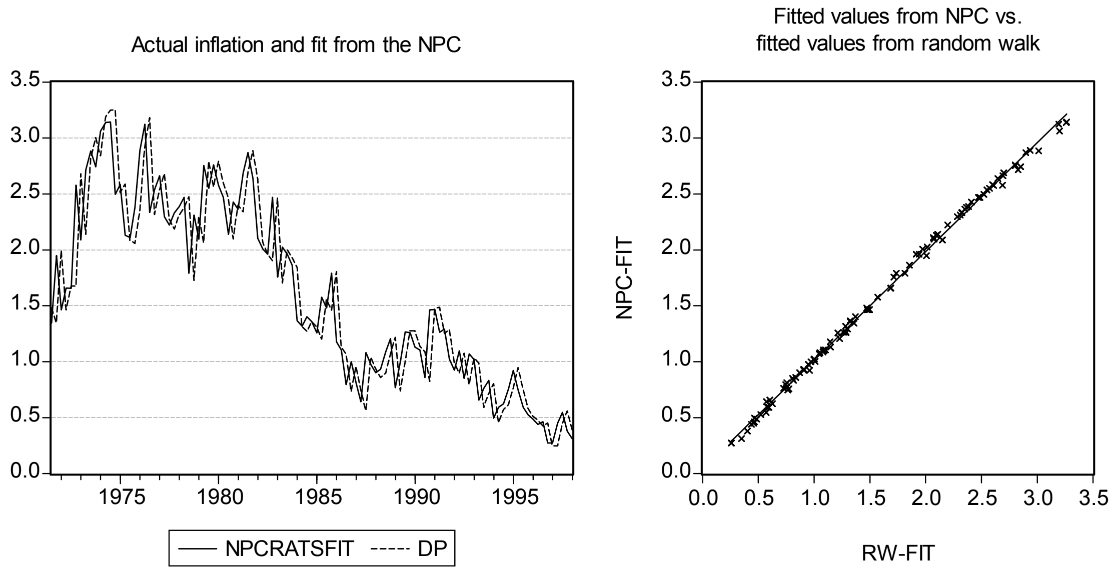

Figure 3: Actual and fitted values of equation (14) together with the fit of a random walk.

Moreover, goodness-of-fit is not well defined for this class of models. It is a question of how to normalize an equation with two endogenous variables, Δp_t and Δp_{t+1}, given that the same set of instruments underlies both rates. As long as the estimated b^f_{p1} is close to one, the normalization does not matter very much. The fits are in any case well approximated by a random walk. However, for hybrid models, the normalization issue becomes a more important concern. Recently, in a paper advocating FIML estimation of a forward looking system of Δp_t, x_t and an interest rate, Lindé (2001) illustrates this point.[10]

Lindé imposes the homogeneity restriction (10), see his Table 6a (column 1). In terms of equation (11), his results for the US GDP inflation rate (Δp_t) yields

$$\Delta \hat{p}_t^{NPCa} = \Delta p_{t-1} + \frac{0.718}{0.282}\Delta\Delta p_{t-1} - \frac{0.048}{0.282} gap_{t-1}. \tag{17}$$

Figure 4**a** shows Δp_t together with $\Delta \hat{p}_t^{NPCa}$. Clearly, the huge coefficient of the acceleration term creates excess variation in the fitted rate of inflation. Using the structural form (*i.e.* normalization on Δp_t) we obtain

$$\Delta \hat{p}_t^{NPCb} = \Delta p_{t-1} + 0.2828\Delta_2\Delta p_{t+1} + 0.048 gap_t \tag{18}$$

which is shown in panel **b** together with the actual Δp_t, showing a very close fit[11]. However, as a possible benchmark for comparison of fit, consider the estimated

[10] Lindé uses data for the US from the study of Rudd and Whelan (2001). We are grateful to Jeromy Rudd of the Federal Reserve Board, who has kindly provided us with the same data set.

[11] For simplicity, actual values of $\Delta_2\Delta p_{t+1}$ have been used, instead of the predicted values from the system estimation (i..e. the displayed fit of the NPC is a little too good).

dVAR[12]

$$(19) \qquad \Delta\hat{p}_t^{dVAR} = \Delta p_{t-1} - 0.248\Delta p_{t-1} - 0.226\Delta p_{t-2},$$

with fit shown in panel **c** of the figure. Characteristically, the dVAR lags one period behind the peaks inflation, nevertheless it is not evident that the NPC adds much in terms of overall fit, cf. the panel **d** which shows the crossplot of $\Delta\hat{p}_t^{NPCb}$ and $\Delta\hat{p}_t^{dVAR}$ with 5 sequentially estimated regression lines drawn.

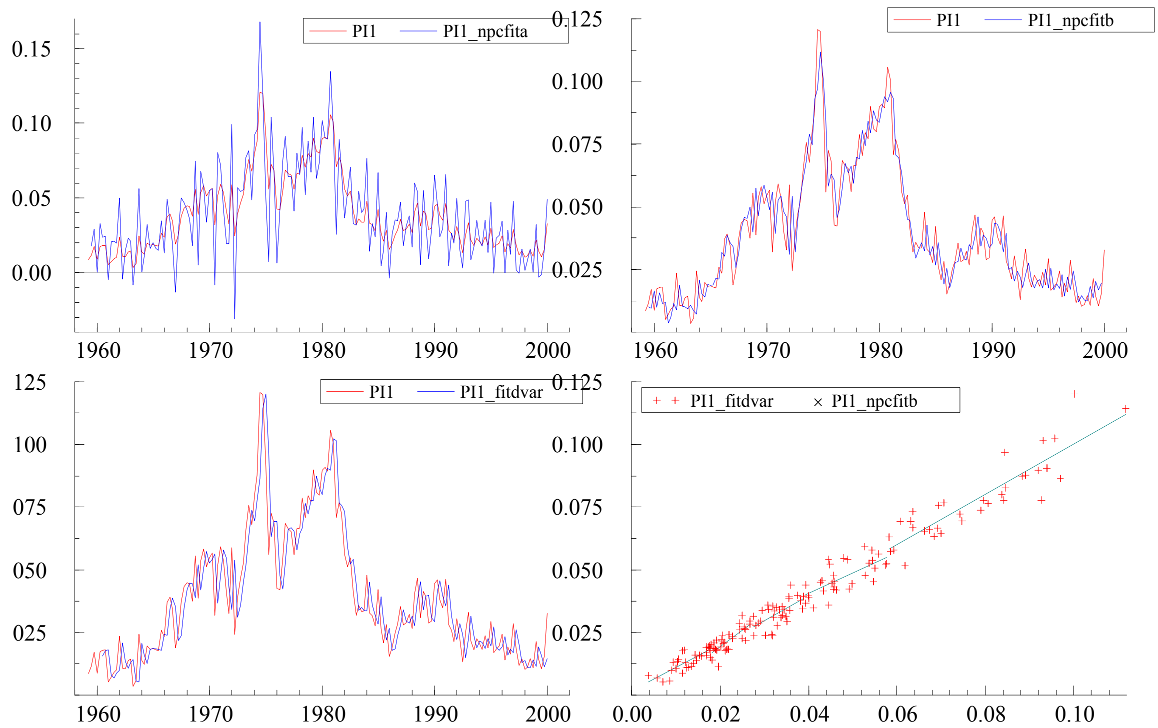

Figure 4: Actual versus fitted values of US (annualized) quarterly rate of inflation. Hybrid NPC with homogeneity and two different normalizations. Panel a: Fit of equation (17). Panel b: fit of equation (18). Panel c: Fit of dVAR in (19). Panel d: Cross plot of fitted values from panel b and c.

Extending the dVAR in (19) with the left hand side of (18) gives

$$\Delta^2\hat{p}_t = -0.245819\Delta p_{t-1} - 0.224157\Delta p_{t-2} - 0.0048\Delta\hat{p}_t^{NPCb}$$

showing that the fitted values from the structural NPC add nothing to the fit of the dVAR.

It seems reasonable to conclude that by themselves, graphs showing the goodness-of-fit tell very little about how well the estimated NPCs approximate reality. At the very least, the NPC fit should be compared to the fit of a simple random walk or of a more general equation in differences (a dVAR). It is interesting to note that, in the cases we have analyzed, the fit of the random walk/dVAR is as good as or better

[12] Estimation period 1960(1)-1997(4).

than the NPC-fit. The goodness-of-fit is not invariant to the chosen normalization, unless the forward term is close to unity, and this creates an 'indeterminateness' about the goodness-of-fit also as a descriptive device.

There is thus a need for other evaluation methods, and in the rest of this section we consider significance testing of the forward term, and of the use of the closed form solution for testing purposes. In section 6 we turn to the testing of the encompassing implications of the NPC for existing econometric models of inflation.

5.2 Tests of significance of the forward term

For the original NPC in equation (1) one would certainly expect to estimate a $\hat{b}_{p_1}$ that leads to rejection of the null hypothesis that $b_{p1} = 0$. This is simply because there is typically a lot of autocorrelation in the rate of inflation, beyond what can be explained by the x_t variable or its lag. Thus, at the outset it seems more sensible to base significance testing of the forward term in the hybrid model.

The empirical evidence based on US data is not unanimous: GG and GGL find that the coefficient of the forward variable outweigh the backward coefficient by 3 to 1 or more, whereas Fuhrer (1997) reports the opposite relative weights. Fuhrer's findings are reinforced by Rudd and Whelan (2001), see below, and by Lindé (2001) who reports the weights 0.75 for the backward variable and 0.25 for the forward variable, while finding the latter significantly different from zero.

However, Rudd and Whelan (2001) in their appraisal of GG, show that care must be taken when interpreting the significance of the forward term in the hybrid model. In particular, they show that the high estimates of b^f_{p1} and low estimates of b^b_{p1} may be expected if the true model is in fact of a conventional (causal) type:

$$\Delta p_t = b_{p0} + b_{p3}\Delta p_{t-1} + b_{p4} z_t + \epsilon_{pt}$$

where z_t is a relevant explanatory variable that is not perfectly correlated with the driving variable of the NPC (i.e., x_t) and ϵ_{pt} is an innovation process.

This occurs when a variable z_t that belongs to the true model is left out of the specification under study. The disturbance (7) is then no longer a pure expectations error, because it includes the influence of z_t on inflation. Rudd and Whelan show that the estimates of b^f_{p1} will be biased upwards as long as Δp_{t+1} and the variables used to instrument it are both correlated with z_t. Moreover, they demonstrate that the bias is likely to be large. Nevertheless, they are pessimistic about the possibility of detecting the bias with Hansen's χ^2_J statistic due to low power. That said, the basic logic of Rudd and Whelan's analysis about biased coefficients of forward variables remains valid, and its empirical relevance should be checked on the specific data sets used in the literature.

Equation (20) and (21) present the results for the Euro area hybrid model, first using GMM and then 2SLS with a moving average disturbance term:

$$\Delta p_t = \underset{(0.07)}{0.65}\ \Delta p_{t+1} + \underset{(007)}{0.34}\ \Delta p_{t-1} + \underset{(0.05)}{0.03}\ ws_t + \underset{(0.1182)}{0.04} \tag{20}$$

$$\text{GMM, } T = 104 \text{ (1972 (2) to 1998 (1))}$$
$$\hat{\sigma} = 0.277 \quad \chi^2_{\mathsf{J}}(6) = 7.55\,[0.27]$$

$$\Delta p_t = \underset{(0.09)}{0.62}\ \Delta p_{t+1} + \underset{(0.07)}{0.39}\ \Delta p_{t-1} - \underset{(0.03)}{0.008}\ ws_t - \underset{(0.04)}{0.01} \tag{21}$$

$$\text{2SLS, } T = 104 \text{ (1972 (2) to 1998 (1))}$$
$$\hat{\sigma} = 0.19 \quad \text{MA-coeff: } \underset{(0.18)}{-0.75}$$

The same instruments as GGL are used, with some minor changes[13]: First, we use an alternative output-gap measure ($emugap_t$), which is a simple transformation of the one defined in Fagan et al. (2001) as (the log of) real output relative to potential output, measured by a constant-return-to-scale Cobb-Douglas production function with neutral technical progress, see Appendix B. We have also omitted two lags of the growth rate of wage inflation in order to limit the number of equations in the system version of the NPC that we estimate in section 5.5. Despite these modifications, the coefficient estimates of the inflation terms are in good accordance with GGL, and also with the results reported by GG on US data.

In the same way as for the pure NPC, there is clear indication of a unit root (the two inflation coefficients sum to one), and the wage share again plays an unimportant role in the specification. On the other hand, the formal significance of the forward term, and the insignificance of the J-statistic by themselves corroborate the NPC as a relevant model for European inflation dynamics.

However, the overriding question is whether (20) and/or (21), when viewed as statistical models, give rise to valid inference about the significance of the forward coefficient. In other words, the statistical adequacy of the models for testing purposes is a concern. In this perspective, the practice of 'whitening' the residuals (GMM or MA(1) correction in 2SLS), is problematic, since one then tacitly assume that serial correlation in the residuals is symptomatic of serial correlation in the true disturbances. Thus, the specification of the econometric model used for testing a substantive hypothesis (the role of the forward variable), incorporates the alternative hypothesis associated with a misspecification test (i.e., of residual autocorrelation). This is usually not a good idea in econometrics, since the underlying cause of the residual misspecification may be quite different, for example omitted variables, wrong functional form or, in this case, a certain form of over-differencing. Instead, when departures from the underlying assumptions of the statistical model

[13]See below equation (14) in section 4.

have been established, there is need for respecification, with the aim of finding a statistical model which does a better job in capturing the systematic variation in the rate of inflation, see e.g., Harvey (1981, Ch. 8.4), Hendry (1983) and Spanos (1986, Ch. 26) for general discussions.[14].

As an example of this approach, consider first the hybrid NPC estimated by 2SLS without MA(1) correction:

$$\begin{aligned}\Delta p_t \; = \; & \underset{(0.135)}{0.6551}\; \Delta p_{t+1} + \underset{(0.117)}{0.28}\; \Delta p_{t-1} + \underset{(0.086)}{0.071}\; ws_t \\ & + \underset{(0.1182)}{0.1027}\end{aligned} \tag{22}$$

2SLS, T = 104 (1972 (2) to 1998 (1))

$\hat{\sigma}_{IV} = 0.276711$	$RSS = 7.65689519$
$\mathsf{F}_{\mathsf{AR}(1-1)} = 166.93[0.0000]$	$\mathsf{F}_{\mathsf{AR}(2-2)} = 4.7294[0.0320]$
$\mathsf{F}_{\mathsf{ARCH}(1-4)} = 2.4713[0.050]$	$\chi^2_{\mathsf{normality}} = 1.5924[0.4510]$
$\mathsf{F}_{\mathsf{HET}x^2} = 2.6807[0.0191]$	$\mathsf{F}_{\mathsf{HET}x_ix_j} = 2.3445[0.0200]$

The misspecification tests in (22) show that the (uncorrected) hybrid model residuals are characterized not only by first order autocorrelation (which may be induced by the errors in variables estimation method), but also by second order autocorrelation and by heteroscedasticity[15]. Thus, the equation appears to be inadequate as a statistical model of the rate of inflation, and does not provide the basis for reliable inference about parameters of interest (e.g., the coefficient of the forward term and the characteristic roots). As pointed out, the popular route around this is either to 'whiten' the residuals (GMM), and/or to correct the 2SLS coefficient standard errors. However, another way of whitening the residuals is to respecify the model, with the aim of attaining innovation error processes and a firmer basis for testing hypothesis within the respecified model.

In the present case, likely directions for respecification are suggested by pre-existing results from several decades of empirical modelling of inflation dynamics. For example, variables representing capacity utilization (output-gap and/or unemployment) have a natural role in inflation models. Additional lags in the rate of inflation are also obvious candidates. As a direct test of this respecification, we moved the lagged output-gap ($emugap_{t-1}$) and the fourth lag of inflation (Δp_{t-4}) from the list of instruments used for estimation of (21), and included them as explanatory variables in the equation.[16] The results (using 2SLS without corrections)

[14]There is a clear parallel to the fallacy of choosing a common factor model (RALS estimation) on the basis of finding that the residuals of a static model are autocorrelated, Hendry and Mizon (1978), Hendry (1995, Ch. 7.7) and Mizon (1995).

[15]The reported statistics: F distributed tests of autoregressive residual autocorrelation ($\mathsf{F}_{\mathsf{AR}(1-1)}$and $\mathsf{F}_{\mathsf{AR}(2-2)}$), see Godfrey (1978) and Doornik (1996); autoregressive conditional heteroscedasticity ($\mathsf{F}_{\mathsf{ARCH}(1-4)}$), see Engle (1982); and heteroscedasticity due to cross products of the regressors ($\mathsf{F}_{\mathsf{HET}x_ix_j}$), see White (1980) and Doornik (1996), whereas $\mathsf{F}_{\mathsf{HET}x^2}$ is a test of heteroscedasticity due to squares of the regressors. The Chi-square test of residual non-normality ($\chi^2_{\mathsf{normality}}$) is from Doornik and Hansen (1994).

[16]Is the respecified equation only a statistical relationship, or does it have an economic interpre-

are:

$$
(23)\quad
\begin{aligned}
\Delta p_t \;=\;& \underset{(0.284)}{0.06767}\ \Delta p_{t+1} + \underset{(0.167)}{0.248}\ ws_t + \underset{(0.141)}{0.4421}\ \Delta p_{t-1} \\
& + \underset{(0.0939)}{0.1799}\ \Delta p_{t-4} + \underset{(0.0525)}{0.1223}\ emugap_{t-1} + \underset{(0.2997)}{0.5366}
\end{aligned}
$$

2SLS, $T = 104$ (1972 (2) to 1998 (1))

$\hat{\sigma}_{IV} = 0.277027$	$RSS = 7.52092918$
$\chi^2_{\mathsf{ival}}(4) = 4.5194[0.34]$	$\mathsf{F}_{\mathsf{AR}(1-5)} = 1.1117[0.3596]$
$\mathsf{F}_{\mathsf{ARCH}(1-4)} = 0.79776[0.5297]$	$\chi^2_{\mathsf{normality}} = 1.7482[0.4172]$
$\mathsf{F}_{\mathsf{HET}x^2} = 1.4312[0.1802]$	$\mathsf{F}_{\mathsf{HET}x_ix_j} = 1.2615[0.2313]$

When compared to (22), four results stand out:[17]

1. The estimated coefficient of the forward term Δp_{t+1} is reduced by a factor of 10.
2. The diagnostic tests no longer indicate residual autocorrelation or heteroscedasticity, so we can undertake substantive inference in a reliable way. Specifically, we can test the significance of the $\mathsf{E}[\Delta p_{t+1}]$ by a conventional t-test: Clearly, there is no formal evidence of significance of the forward term in this model.
3. The p-value of the Sargan specification test, χ^2_{ival}, is 0.34, and is evidence that (23) effectively represents the predictive power that the set of instruments has about Δp_t.
4. If the residual autocorrelation of the pure and hybrid NPCs above are induced by the forward solution and "errors in variables", there should be a similar moving average process in the residuals of (23). Since there is no detectable residual autocorrelation that interpretation is refuted, supporting instead that the pure and hybrid NPCs are misspecified.

Unlike Rudd and Whelan's conclusions for US inflation, we find that significance testing of the forward term gives a clear answer for the Euro data. This conclusion is however based on the premise that the equation with the forward coefficient is tested *within* a statistically adequate model. This entails thorough misspecification testing of the theoretically postulated NPC, and possible respecification before the test of the forward coefficient is performed. A complementary interpretation follows from a point made by Mavroeidis (2002), namely that the hybrid NPC suffers from underidentification, and that in empirical applications identification is achieved by confining important explanatory variables to the set of instruments with misspecification as a result.

tation? Though it is not in any way our favourite specification, we think that the interpretation is unproblematic, e.g., as a price Phillips curve derived from price setting with the mark-up depending on profitability (approximated by ws, and $emugap_t$).

[17] The specification test χ^2_{ival} is a test of the validity of the instruments as discussed in Sargan (1964).

5.3 Tests based on the closed form solution (Rudd-Whelan test)

The conclusion that significance testing seemed to work rather well in the specific case of Euroland inflation, is not inconsistent with Rudd and Whelan's general concern about possible lack of power. For example, having a large number of instruments (many of them with little predictive power for the rate of inflation) will usually produce an insignificant joint test of instrument validity. Thus, we commend Rudd and Whelan's search for alternative ways of testing the NPC thesis.

They note that the closed form solution to the basic NPC in equation (2), for the case of $b_{p1} < 1$, is

$$\Delta p_t = \ b_{p2} \sum_{i=0}^{\infty} b_{p1}^i x_{t+i} - \sum_{i=0}^{\infty} b_{p1}^{i+1} \varepsilon_{pt+i+1}, \quad b_{p1} < 1, \tag{24}$$

or, when conditioning on $\mathcal{I}_{t-1}$:

$$\Delta p_t = \ b_{p2} \sum_{i=0}^{\infty} b_{p1}^i \mathsf{E}_t[x_{t+i}, \mid \mathcal{I}_{t-1}], \quad b_{p1} < 1. \tag{25}$$

In the solution inflation is a function of the expected present value of the driving variable x_t. In the implementation of this test, Rudd and Whelan truncate the present value calculation at lead 12, and consequently include a lag of the rate of inflation. Finally, they use a representative value of b_{p1} (0.95) from the GG.

Rudd and Whelan use a data set for the U.S. that closely correspond to that used by GG, and they find that such present value terms can explain only a small fraction of observed inflation dynamics. Moreover, inflation plays only a minor role in forecasting future values of the labour share or, alternatively, the output gap. Hence, the importance of lags of inflation found in empirical Phillips curves, which Rudd and Whelan take as a stylized fact for the US economy, cannot be explained by lagged inflation proxying for expected future values of x_t. On the contrary, they claim that the empirical importance of lagged inflation in empirical US Phillips curves should be considered as strong evidence against the New Keynesian model.

It should be noted that the Rudd and Whelan test is only valid for the pure NPC model in equation (2). As pointed out by Galí et al. (2001b) it does not provide a test of the hybrid model, since that model implies a closed form solution of a different form.[18]

Turning to European inflation we set $b_{p1} = 0.91$ in accordance with the findings in GGL. Actually, experiments with other (high) values of b_{p1} did not influence the results. On the other hand, the lead period for the present value calculation *is* very influential, and we next report results based on a 12 quarter lead.

---

[18] Galí et al. (2001b) derive a closed form solution for the hybrid version and find estimates for the US that are in accordance with the results in GG.

$$\Delta p_t = \underset{(0.0198)}{0.0989}\ PVws_t + \underset{(0.108)}{0.4033}\ \Delta p_{t-1} + \underset{(0.00175)}{0.00939} \tag{26}$$

2SLS, $T = 93$ (1972 (2) to 1995 (2))

$\hat{\sigma}_{IV} = 0.299336$	$RSS = 8.06416813$
$\chi^2_{\mathsf{ival}}(7) = 10.977[0.14]$	$\mathsf{F}_{\mathsf{AR}(1-5)} = 2.1225[0.0705]$
$\mathsf{F}_{\mathsf{ARCH}(1-4)} = 1.5658[0.1912]$	$\chi^2_{\mathsf{normality}} = 0.5786[0.7488]$
$\mathsf{F}_{\mathsf{HET}x^2} = 1.8131[0.1338]$	$\mathsf{F}_{\mathsf{HET}x_ix_j} = 1.4480[0.2157]$

We note that the present value term $PVws_t$ is statistically significant with the expected positive sign. Moreover the diagnostics do not suggest that there is residual misspecification in this equation. Thus, the results in (26) is better news for the NPC than what Rudd and Whelan obtained for the US—since they found that the present value term sometimes had the wrong sign or had little statistical significance.

However, care must be taken before one counts (26) as a success for the NPC. First, the numerical importance of the wage share term need not be very large, even though it is statistically significant. Second, an estimation such as (26) implies very little about the correctness of the underlying theory. This can be demonstrated by changing the value of b_{p1} from 0.91 to 1 which leaves the t-value of $PVws$ (now a pure moving average) almost unchanged, at 5.22. The problem is of course that there is no stationary solution in this case, so using the closed form is inappropriate. In fact in this specific case it seems that 'everything goes' in terms of the significance of the wage share. For example, setting $b_{p1} = 1.1$ meaning that the forward solution no longer applies, still gives a highly significant $PVws$, (t-value of 5.23). Thus without further evidence that can help substantiate the relevance of the forward solution, the finding of a statistically significant present value component is by itself unconvincing.

5.4 Test based on the transformed closed form solution

Given that the forward solution applies, we can invoke the Koyck transformation (lead one period and multiply with b_{p1}^{-1}) and get a relationship of the form

$$\Delta p_t = constant + \frac{1}{b_{p1}}\Delta p_{t-1} - \frac{b_{p2}}{b_{p1}} x_{t-1} + disturbance. \tag{27}$$

where the disturbance term is simply $\varepsilon_{p,t}$ if (24) is taken literally, but may follow a moving average process under the more realistic assumption that there are other disturbances than only the expectation errors.[19] Therefore, after subtracting Δp_{t-1} on both sides of (27), a simple test is to run a regression of $\Delta^2 p_t$ on lagged inflation and the lagged forcing variable.

The NPC implies that the coefficient of Δp_{t-1} ought to be positive, so one should consider a one sided alternative to the null hypothesis of a zero coefficient of

[19] That is, we get back to (4) if the theory holds exactly.

Δp_{t-1}. The following was obtained when (27) was estimated on the GGL data set:

$$\Delta^2 p_t = \underset{(0.066)}{-0.04}\ \Delta p_{t-1} + \underset{(0.072)}{0.024}\ ws_{t-1} + \underset{(0.0011)}{0.0005} \tag{28}$$

NLS, T = 113 (1970 (2) to 1998 (2))

$\hat{\sigma} = 0.331425$ $RSS = 11.972841$ MA-coeff: $\underset{(0.13)}{-0.39}$

	$\mathsf{F}_{\mathsf{AR}(1-5)} = 1.0881[0.37]$
$\mathsf{F}_{\mathsf{ARCH}(1-4)} = 0.80179[0.53]$	$\chi^2_{\mathsf{normality}} = 6.8376[0.033]^*$
$\mathsf{F}_{\mathsf{HET}x^2} = 2.1045[0.06]$	$\mathsf{F}_{\mathsf{HET}x_ix_j} = 1.5504[0.14]$

The moving average coefficient of the residual obtains the expected negative sign, but in other respects the results are difficult to reconcile with the forward solution in (24): A unity restriction on the autoregressive coefficient is clearly statistically acceptable, as is a zero coefficient of ws_{t-1}. Thus the random walk again appears to be as good a model as the model derived from the NPC.

5.5 Evaluation of the NPC system

The nature of the solution for the rate of inflation is a system property, as noted in section 3. Hence, unless one is willing to accept at face value that an operational definition of the forcing variable is strongly exogenous, there is a need to extend the single equation estimation of the 'structural' NPC to a system that also includes the forcing variable as a modelled variable.

For that purpose, Table 1 shows an estimated system for Euro area inflation, with a separate equation (the second in the table) for treating the wage share (the forcing variable) as an endogenous variable. Note that the hybrid NPC equation (first in the table) is similar to (21) above, and thus captures the gist of the results in GGL. This is hardly surprising, since only the estimation method (FIML in Table 1) separates the two NPCs.

Table 1: FIML results for a NPC for the EURO area 1972(2)-1998(1).

$$\Delta p_t = \underset{(0.154)}{0.7696}\ \Delta p_{t+1} + \underset{(0.131)}{0.2048}\ \Delta p_{t-1} + \underset{(0.0930)}{0.0323}\ ws_t + \underset{(0.1284)}{0.0444}$$

$$ws_t = \underset{(0.0296)}{0.8584}\ ws_{t-1} + \underset{(0.0220)}{0.0443}\ \Delta p_{t-2} + \underset{(0.0223)}{0.0918}\ \Delta p_{t-5} + \underset{(0.0067)}{0.0272}\ emugap_{t-2} - \underset{(0.0447)}{0.2137}$$

$$\Delta p_{t+1} = \underset{(0.0988)}{0.5100}\ ws_{t-1} + \underset{(0.0907)}{0.4153}\ \Delta p_{t-1} + \underset{(0.0305)}{0.1814}\ emugap_{t-1} + \underset{(0.1555)}{0.9843}$$

The sample is 1972 (2) to 1998 (1), $T = 104$.

$$\begin{aligned}
\hat{\sigma}_{\Delta p_t} &= 0.290186 \\
\hat{\sigma}_{ws} &= 0.074904 \\
\hat{\sigma}_{\Delta p^e_{t+1}} &= 0.325495 \\
\mathsf{F}^v_{\mathsf{AR}(1-5)}(45, 247) &= 37.100[0.0000]^{**} \\
\mathsf{F}^v_{\mathsf{HET}x^2}(108, 442) &= 0.94319[0.6375] \\
\mathsf{F}^v_{\mathsf{HET}x_ix_j}(324, 247) &= 1.1347[0.1473] \\
\chi^{2,v}_{\mathsf{normality}}(6) &= 9.4249[0.1511]
\end{aligned}$$

An important feature of the estimated equation for the wage share ws_t is the two lags of the rate of inflation, which both are highly significant. The likelihood-ratio test of joint significance gives $\chi^2(2) = 24.31[0.0000]$, meaning that there is clear formal evidence against the strong exogeneity of the wage share. One further implication of this result is that the closed form solution for the rate of inflation cannot be derived from the structural NPC, and representing the solution by equation (25) above is therefore inconsistent with the evidence.

The roots of the system in Table 1 are all less than one (not shown in the table) in modulus and therefore corroborate a forward solution. However, according to the results in the table, the implied driving variable is $emugap_t$, rather than ws_t which is endogenous, and the weights of the present value calculation of $emugap_t$ have to be obtained from the full system. The statistics at the bottom of the table show that the system of equations have clear deficiencies as a statistical model, cf. the massive residual autocorrelation detected by $\mathsf{F}^v_{\mathsf{AR}(1-5)}$.[20] Further investigation indicates that this problem is in part due the wage share residuals and is not easily remedied on the present information set. However, from section 5.2 we already know that another source of vector autocorrelation is the NPC itself, and moreover that this misspecification by and large disappears if we instead adopt equation (23) as our inflation equation.

It lies close at hand therefore to suggest another system where we utilize the

[20]The superscript ^{v} indicates that we report vector versions of the single equation misspecification tests encountered above.

second equation in Table 1, and the conventional Phillips curve that is obtained by omitting the insignificant forward term from equation (23). Table 2 shows the results of this potentially useful model. There are no misspecification detected, and the coefficients appear to be well determined.[21] In terms of economic interpretation the models resembles an albeit 'watered down' version of the modern conflict model of inflation, see e.g. Bårdsen et al. (1998), and one interesting route for further work lies in that direction. That would entail an extension of the information set to include open economy aspects and indicators of institutional developments and of historical events. The inclusion of such features in the information set will also help in stabilizing the system.[22]

Table 2: FIML results for a conventional Phillips curve for the EURO area 1972(2)-1998(1).

$$\Delta p_t = \underset{(0.1202)}{0.2866}\ ws_t + \underset{(0.0868)}{0.4476}\ \Delta p_{t-1} + \underset{(0.091)}{0.1958}\ \Delta p_{t-4} + \underset{(0.0259)}{0.1383}\ emugap_{t-1} + \underset{(0.1823)}{0.6158}$$

$$ws_t = \underset{(0.0298)}{0.8629}\ ws_{t-1} + \underset{(0.0222)}{0.0485}\ \Delta p_{t-2} + \underset{(0.0225)}{0.0838}\ \Delta p_{t-5} + \underset{(0.0068)}{0.0267}\ emugap_{t-2} - \underset{(0.0450)}{0.2077}$$

The sample is 1972 (2) to 1998 (1), $T = 104$.

$$\begin{aligned}
\hat{\sigma}_{\Delta p_t} &= 0.284687 \\
\hat{\sigma}_{ws} &= 0.075274 \\
\mathsf{F}^{v}_{\mathsf{AR}(1-5)}(20, 176) &= 1.4669[0.0983] \\
\mathsf{F}^{v}_{\mathsf{HET}x^2}(54, 233) &= 0.88563[0.6970] \\
\mathsf{F}^{v}_{\mathsf{HET}x_i x_j}(162, 126) &= 1.1123[0.2664] \\
\chi^{2,v}_{\mathsf{normality}}(4) &= 2.9188[0.5715] \\
\textit{Overidentification}\ \chi^2(10) &= 10.709[0.3807]
\end{aligned}$$

6 NPC and inflation in small open economies. Testing the encompassing implications

The above sections reflect that so far the NPC has mainly been used to describe the inflationary process in studies concerning the US economy or for aggregated Euro data. Heuristically, we can augment the basic model with import price growth and other open economy features, and test the significance of the forward inflation rate within such an extended NPC. Recently, Batini et al. (2000) have derived an open economy NPC from first principles, and estimated the model on UK economy.

[21] The *Overidentification* χ^2 is the test of the model in Table 1 against its unrestricted reduced form, see Anderson and Rubin (1949, 1950), Koopmans et al. (1950), and Sargan (1988, p.125 ff.).

[22] The largest root in Table 2 is 0.98.

Once we consider the NPC for individual European economies, there are new possibilities for testing—since preexisting results should, in principle, be explained by the new model (the NPC). Specifically, in UK and Norway, the two economies that we investigate here, there exist models of inflation that build on a different framework than the NPC, namely wage bargaining, monopolistic price setting and cointegration, see e.g., Nickell and Andrews (1983), Hoel and Nymoen (1988), Nymoen (1989) for early contributions, and Blanchard and Katz (1999) for a view on this difference in modelling tradition in the US and Europe. Since the underlying theoretical assumptions are quite different for the two traditions, the existing empirical models define an information set that is wider than the set of instruments that we have seen are typically employed in the estimation of NPCs. In particular, the existing studies claim to have found cointegrating relationships between levels of wages, prices and productivity. These relationships constitute evidence that can be used to test the implications of the NPC.

6.1 The encompassing implications of the NPC

Already there is a literature on the testing of feed-forward (rational expectations based) and feed-back models (allowing data based expectation formation), see Hendry (1988), Engle and Hendry (1993) and Ericsson and Hendry (1999). One of the insights is that the rational expectations hypothesis is inconsistent with the joint finding of a stable feed-back model *and* a regime shift in the process driving the explanatory variable (i.e., x_t above). Essentially, an implication of the feed-forward mode, namely that the Lucas critique applies in the described situation, is refuted by the stability of the feed-back model in the presence of a regime shift.

The test below builds on the same type of logic, namely that certain testable implications follow from the premise that the NPC (with rational expectations) is the correct model. Specifically, the following procedure is suggested[23]:

1. Assume that there exists a set of variables $\mathbf{z} = \begin{bmatrix} \mathbf{z}_1 & \mathbf{z}_2 \end{bmatrix}$, where the sub-set $\mathbf{z}_1$ is sufficient for overidentification of the maintained NPC model. The variables in $\mathbf{z}_2$ are defined by the empirical findings of existing studies.

2. Using $\mathbf{z}_1$ as instruments, estimate the augmented model

$$\Delta p_t = b_{p1}^f \mathsf{E}_t \Delta p_{t+1} + b_{p1}^b \Delta p_{t-1} + b_{p2} x_t + ... + \mathbf{z}_{2,t}\, \mathbf{b}_{p4}$$

under the assumption of rational expectations about forward prices.

3. Under the hypothesis that the NPC is the correct model, $\mathbf{b}_{p4} = \mathbf{0}$ is implied. Thus, non-rejection of the null hypothesis of $\mathbf{b}_{p4} = \mathbf{0}$, corroborates the feed-forward Phillips curve. In the case of the other outcome: non-rejection of $b_{p1}^f = 0$, while $\mathbf{b}_{p4} = \mathbf{0}$ is rejected statistically, the encompassing implication of the NPCs refuted.

The procedure is clearly related to significance testing of the forward term, but there are also notable differences. As mentioned above, the motivation of the test

[23]We thank David F. Hendry for suggesting this test to us.

is that of testing the implication of the rational expectations hypothesis, see Favero and Hendry (1992) and Ericsson and Irons (1995). Thus, we utilize that under the assumption that the NPC is the correct model, consistent estimation of b_{p1}^{f} can be based on $\mathbf{z}_1$, and supplementing the set of instruments by $\mathbf{z}_2$ should not significantly change the estimated b_{p1}^{f}.

In terms of practical implementation, we take advantage of the existing results on wage and price modelling using cointegration analysis which readily imply z_2-variables in the form of linear combinations of levels variables. In other words they represent "unused" identifying instruments that goes beyond the information set used in the Phillips curve estimation. Importantly, if agents are rational as assumed, this extension of the information set should not take away the significance of Δp_{t+1}^{e} in the NPC.

6.2 Norway

Consider the NPC (with forward term only) estimated on quarterly Norwegian data:[24]

$$\Delta p_t = \underset{(0.11)}{1.06}\Delta p_{t+1} + \underset{(0.02)}{0.01} ws_t + \underset{(0.02)}{0.04}\Delta pb_t + \text{ dummies} \qquad (29)$$
$$\chi^2_{\mathsf{J}}(10) = 11.93\,[0.29]\,.$$

The closed economy specification has been augmented heuristically with import price growth (Δpb_t) and dummies for seasonal effects as well as the special events in the economy described in Bårdsen et al. (2002). Estimation is by GMM for the period 1972.4 - 2001.1. The instruments used (i.e., the variables in $\mathbf{z}_1$) are lagged wage growth (Δw_{t-1}, Δw_{t-2}), lagged inflation (Δp_{t-1}, Δp_{t-2}), lags of level and change in unemployment (u_{t-1}, Δu_{t-1}, Δu_{t-2}), and changes in energy prices (Δpe_t, Δpe_{t-1}), the short term interest rate (ΔRL_t, ΔRL_{t-1}) and the length of the working day (Δh_t).

The coefficient estimates are similar to GG. Strictly speaking, the coefficient of $\mathsf{E}[\Delta p_{t+1} \mid \mathcal{I}_t]$ suggests that a backward solution is appropriate. But more importantly the estimated NPC once more appears to be a modified random walk model. We checked the stability of the key parameters of the model by rolling regressions with a fixed window of 85 observations. Figure 5 can be compared with Figure 2 for the Euro area showing that the sample dependency is more pronounced in the case of Norway. That said, the standard errors of the estimates are smaller in this case.

[24] Inflation is measured by the official consumer price index, see Appendix B.

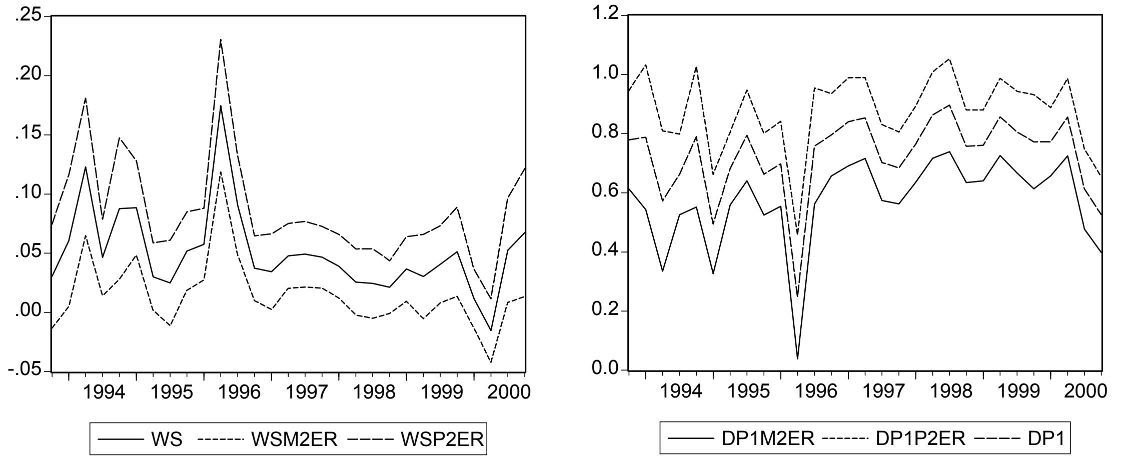

Figure 5: Rolling regression coefficients +/- 2 standard errors of the New Keynesian Phillips curve, estimated on Norwegian data ending in 1993.4 - 2000.4.

Next, we invoke an equilibrium correction term from the inflation model of Bårdsen and Nymoen (2001), which is an update of Bårdsen et al. (1998) and Bårdsen et al. (1999, 2002), and let that variable define the additional instrument, $z_{2,t}$:

$$ecmp_t = p_t - 0.6(w_t - pr_t + \tau 1_t) - 0.4pb_t + 0.5\tau 3_t$$

The results, using GMM, are

$$\Delta p_t = \underset{(0.125)}{-0.02}\Delta p_{t+1} + \underset{(0.025)}{0.04}\, ws_t - \underset{(0.017)}{0.06}\,\Delta pb_t - \underset{(0.020)}{0.10}\, ecmp_{t-1} + \quad \text{dummies}$$
$$\chi^2_{\mathrm{J}}\,(10) = 12.78\,[0.24]\,,$$

showing that the implication of the NPC is refuted by the finding of i) a highly significant (wage) equilibrium correction term defined by an existing study, and ii) the change in the estimated coefficient of Δp_{t+1}, from 1.01 to -0.02, noting thet its statistical significance is lost.

6.3 United Kingdom

As mentioned above, Batini et al. (2000) derive an open economy NPC consistent with optimizing behaviour, thus extending the intellectual rationale of the original NPC. They allow for employment adjustment costs and propose to let the equilibrium mark-up on prices depend on the degree of foreign competition, *com*. In their estimated equations Batini et al. (2000) also include a term for relative price of imports, denoted *rpm* and oil prices *oil*. Equation (30) shows a NPC of this type, using instrumental variables estimation. To facilitate comparison with the GGL type of equation, we abstract from the employment adjustment aspect. This amounts to omitting Δn_{t+1} and Δn_t from the typical equation in Batini et al. (2000). It is our experience that these variables do not obtain significant coefficients, and that omitting them have negligible consequences for the coefficients of the key variables (ws_t and Δp_{t+1}). It also motivates a much needed reduction in the number of instruments.[25]

$$
\begin{aligned}
\Delta p_t \;=\; & \underset{(0.0628)}{0.1748}\; ws_t \;+\; \underset{(0.197)}{0.5203}\; \Delta p_{t+1} \;+\; \underset{(0.0855)}{0.1213}\; gap_{t-1} \\
& -\underset{(0.0139)}{0.005883}\; com_t \;-\; \underset{(0.00519)}{0.01495}\; \Delta oil_t + \underset{(0.0247)}{0.02331}\; rpm_t \qquad (30)\\
& -\underset{(0.00367)}{0.005981}\; SEA_t \;-\; \underset{(0.262)}{0.7259}
\end{aligned}
$$

2SLS, $T = 107$ (1972 (3) to 1999 (1))

$$
\begin{array}{ll}
\hat{\sigma}_{IV} = 0.00962538 & RSS = 00917215334 \\
\chi^2_{\mathsf{ival}}(17) = 24.865[0.0978] & \mathsf{F}_{\mathsf{AR}(1-5)} = 2.1912[0.0616] \\
\mathsf{F}_{\mathsf{ARCH}(1-4)} = 3.1057[0.0191] & \chi^2_{\mathsf{normality}} = 0.28308[0.8680] \\
\mathsf{F}_{\mathsf{HET}x^2} = 3.6057[0.0002] & \mathsf{F}_{\mathsf{HET}x_ix_j} = 2.8960[0.0001]
\end{array}
$$

The estimated coefficients of the three first right hand side variables are in accordance with the results that Batini et al. (2000) report using different variants of GMM (i.e., different methods for pre-whitening the residuals) and IV estimation. The wage share variable used in (30) is the adjusted share preferred by Batini et al. (2000). The terms in the second line represent small open economy features that we noted above. Finally, the impact of the Single European Act on UK inflation is captured by SEA (takes the 1 in 1990(1)-1999(1)), zero elsewhere). The diagnostics show that the specification test χ^2_{ival} is insignificant at the 5% level. The same is true for the joint test of 1-5th order residual autocorrelation and for the normality test. However, all three heteroscedasticity tests are significant. Presumably, pre-whitening the residuals in GMM mops up the heteroscedasticity.[26]

Bårdsen et al. (1998) also estimate a cointegrating wage-price model for the UK. Their two error-correction terms define two z_2-instruments that we can use to test the NPC

$$
\begin{aligned}
ecmw_t &= w_t - p_t - a_t + \tau 1_t + 0.065 u_t \\
ecmp_t &= p_t - 0.6\tau 3_t - 0.89(w + \tau 1_t - a_t) - 0.11 pb_t.
\end{aligned}
$$

The definitions of the cointegrating variables are the same as in Bårdsen et al. (1998), meaning that in the calculation of $ecmp_t$, p_t is log of the retail price index (as explained Δp_t in (30) is for the gross value added price deflator).

Equation (31) shows the results of adding $z_{21,t-1}$ and $z_{22,t-1}$ to the NPC model (30)

[25] Batini et al. (2000) use 40 instruments, while our estimation is based on only 19 instrumental variables: 5 lags of inflation, oil price growth and the wage share, and 4 lags of the output gap since the first lag of the output gap is included in the NPC itself. Inflation is the first difference of the log of gross value added deflator. ws is the adjusted wage share preferred by BJN. The gap variable extracts the Hodrick-Prescott trend, see Data Appendix and Batini et al. (2000) (footnote to Tables 7a and 7b) for more details.

[26] Batini et al. (2000) note that the basic structure of their estimated equations remains broadly unchanged if they remove all pre-whitening.

$$
\begin{aligned}
(31)\quad \Delta p_t \;=\;& -\underset{(0.0679)}{0.003151}\; ws_t \;+\; \underset{(0.217)}{0.1623}\; \Delta p_{t+1} \;+\; \underset{(0.099)}{0.07295}\; gap_{t-1} \\
& +\underset{(0.0233)}{0.0246}\; com_t \;-\; \underset{(0.00613)}{0.001441}\; \Delta oil_t - \underset{(0.0364)}{0.05642}\; rpm_t \\
& -\underset{(0.00345)}{0.005994}\; SEA_t - \underset{(0.567)}{0.574} \\
& -\underset{(0.0942)}{0.3696}\; z_{21,t-1} \;-\; \underset{(0.129)}{0.47}\; z_{22,t-1}
\end{aligned}
$$

2SLS, $T = 80$ (1976 (2) to 1996 (1))

$\hat{\sigma}_{IV} = 0.00712542$	$RSS = 0.00355401187$
$\chi^2_{\mathsf{ival}}(17) = 22.474[0.1672]$	$\mathsf{F}_{\mathsf{AR}(1-5)} = 0.28504[0.9197]$
$\mathsf{F}_{\mathsf{ARCH}(1-4)} = 0.013340[0.9996]$	$\chi^2_{\mathsf{normality}} = 0.68720[0.7092]$
$\mathsf{F}_{\mathsf{HET}x^2} = 0.68720[0.7092]$	$\mathsf{F}_{\mathsf{HET}x_ix_j} = 0.51527[0.9634]$

As in the case of Norway, the results are clear cut: The forward term Δp_{t+1} is no longer significant, and the value of the estimated coefficient is reduced from 0.52 to 0.16. Also ws_t loses its significance and the hypothesis that the two coefficients are zero cannot be rejected statistically: $\chi^2(2) = 0.615211[0.7352]$. Conversely, the two z_2 -terms, which ought to be of no importance if the NPC is the correct model, both obtain t-values above 3 in absolute value. The hypothesis that both have zero coefficients is firmly rejected on the Chi-square statistics: $\chi^2(2) = 15.694[0.0004]$. We also note that these significance tests are well founded statistically, since there is no sign of residual misspecification in (31).

7 Conclusions

Earlier researchers of the New Keynesian Phillips curve have concluded that the NPC represents valuable insight into the driving forces of inflation dynamics. Our evaluation gives completely different results:

First, the fit is no better than a "theory void" dVAR. Hence, the economic content of the NPC adds nothing to goodness-of fit on Euro data. Second, as statistical models, both the pure and hybrid NPC are inadequate, and the significance of the forward term in the hybrid model of GGL is therefore misleading. We show that one simple way of obtaining statistically adequate models (for testing purposes) is to include variables from the list of instruments as explanatory variables. In the respecified model the forward term vanishes. Third, in many countries, empirical inflation dynamics is a well researched area, meaning that studies exist that any new model should be evaluated against. Applying the encompassing principle to UK and Norwegian inflation, leads to clear rejection of the NPC.

Finally, although our conclusion goes against the NPC hypothesis, this does not preclude that forward expectations terms could play a role in explaining inflation dynamics within other, statistically well specified, models. The methodology for obtaining congruent models are already in place, confer the earlier UK debate on testing feed-back vs feed-forward mechanisms, for example in the context of money demand, see Hendry (1988) and Cuthbertson (1988), and the modelling of wages and prices, see Wren-Lewis and Moghadam (1994), and Sgherri and Wallis (1999).

References

Anderson, T. W. and H. Rubin (1949). Estimation of the parameters of a single equation in a complete system of stochastic equations. *Annals of Mathematical Statistics*, *20*, 46–63.

Anderson, T. W. and H. Rubin (1950). The asymptotic properties of estimates of the parameters of a single equation in a complete system of stochastic equations. *Annals of Mathematical Statistics*, *21*, 570–582.

Bårdsen, G., P. G. Fisher and R. Nymoen (1998). Business Cycles: Real Facts or Fallacies? In Strøm, S. (ed.), *Econometrics and Economic Theory in the 20th Century: The Ragnar Frisch Centennial Symposium*, no. 32 in Econometric Society Monograph Series, chap. 16, 499–527. Cambridge University Press, Cambridge.

Bårdsen, G., E. S. Jansen and R. Nymoen (1999). Econometric Inflation Targeting. Arbeidsnotat 1999/5, Research Department, Norges Bank [Central Bank of Norway].

Bårdsen, G., E. S. Jansen and R. Nymoen (2002). Model specification and inflation forecast uncertainty. *Annales d'Économie et de Statistique*, *67*(68), 501–522.

Bårdsen, G. and R. Nymoen (2001). Sammenhengen mellom rente og inflasjon (Interest rates and inflation). *Norsk Økonomisk Tidsskrift*, *115*(2), 125–147.

Batini, N., B. Jackson and S. Nickell (2000). Inflation dynamics and the labour share in the UK. Discussion Paper 2, External MPC Unit, Bank of England.

Blake, D. (1991). The Estimation of Rational Expectations Models: A survey. *Journal of Economic Studies*, *18*(3), 31–70.

Blanchard, O. J. and L. Katz (1999). Wage Dynamics: Reconciling Theory and Evidence. NBER Working Paper Series 6924, National Bureau of Economic Research.

Brockwell, P. J. and R. A. Davies (1991). *Time Series: Theory and Methods. Second edition.*. Springer, New York.

Calvo, G. A. (1983). Staggered prices in a utility maximizing framework. *Journal of Monetary Economics*, *12*, 383–398.

Chadha, B., P. R. Masson and G. Meredith (1992). Models of Inflation and the Costs of Disinflation. *IMF Staff Papers*, *39*, 395–431.

Clarida, R., J. Galí and M. Gertler (1999). The Science of Monetary Policy: A New Keynesian Perspective. *Journal of Economic Literature*, *37*(4), 1661–1707.

Cuthbertson, K. (1988). The Demand for M1: A Forward Looking Buffer Stock Model. *Oxford Economic Papers*, *40*, 110—131.

Davidson, J. (2000). *Econometric Theory*. Blackwell, Oxford.

Doornik, J. (1996). Testing Vector Autocorrelation and Heteroscedasticity in Dynamic Models. Working paper, Nuffield College.

Doornik, J. A. and H. Hansen (1994). A Practical Test of Multivariate Normality. Unpublished paper, Nuffield College.

Engle, R. F. (1982). Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation. *Econometrica*, *50*, 987–1007.

Engle, R. F. and D. F. Hendry (1993). Testing Super Exogeneity and Invariance in Regression Models. *Journal of Econometrics*, *56*, 119–139.

Ericsson, N. R. and D. Hendry (1999). Encompassing and Rational Expectations: How Sequential Corroboration Can Imply Refutation. *Empirical Economics*, *24*(1), 1—21.

Ericsson, N. R. and J. S. Irons (1995). The Lucas Critique in Practice: Theory Without Measurement. In Hoover, K. D. (ed.), *Macroeconometrics: Developments, Tensions and Prospects*, chap. 8. Kluwer Academic Publishers.

Fagan, G., J. Henry and R. Mestre (2001). An area-wide model (AWM) for the Euro area. Working Paper 42, European Central Bank.

Favero, C. A. and D. F. Hendry (1992). Testing the Lucas Critique: A Review. *Econometric Reviews*, *11*, 265–306.

Fuhrer, J. C. (1997). The (un)importance of forward-looking behavior in price specifications. *Journal of Money, Credit and Banking*, *29*, 338–350.

Fuhrer, J. C. and G. A. Moore (1995). Inflation persistence. *Quarterly Journal of Economics*, *110*, 127–159.

Galí, J. and M. Gertler (1999). Inflation Dynamics: A Structural Econometric Analysis. *Journal of Monetary Economics*, *44*(2), 233–258.

Galí, J., M. Gertler and J. D. López-Salido (2001a). European Inflation Dynamics. *European Economic Review*, *45*, 1237–1270.

Galí, J., M. Gertler and J. D. López-Salido (2001b). Notes on estimating the closed form of the hybrid new Phillips curve. Unpublished paper, CREI/New York University/Bank of Spain.

Godfrey, L. G. (1978). Testing for higher order serial correlation when the regressors include lagged dependent variables. *Econometrica*, *46*, 1303–1313.

Gourieroux, C. and A. Monfort (1997). *Time Series and Dynamic Models*. Cambridge University Press, Cambridge, New York.

Hansen, L. P. (1982). Large Sample Properties of Generalized Method of Moments Estimators. *Econometrica*, *50*, 1029–1054.

Harvey, A. C. (1981). *The Econometric Analysis of Time Series*. Philip Allan, Oxford.

Hendry, D. (1983). Econometric Modelling: The Consumption Function in Retrospect. *Scottish Journal of Political Economy*, *30*, 193–220.

Hendry, D. F. (1988). The Encompassing Implications of Feedback verus Feedforward Mechanisms in Econometrics. *Oxford Economic Papers*, *40*, 132–149.

Hendry, D. F. (1995). *Dynamic Econometrics*. Oxford University Press, Oxford.

Hendry, D. F. and G. E. Mizon (1978). Serial correlation as a convenient simplification, not a nuisance: a comment on a study of the demand for money by the Bank of England. *Economic Journal*, *88*, 549–563.

Hoel, M. and R. Nymoen (1988). Wage Formation in Norwegian Manufacturing. An Empirical Application of a Theoretical Bargaining Model. *European Economic Review*, *32*, 977–997.

Koopmans, T. C., H. Rubin and R. B. Leibnik (1950). Measuring the equation systems of dynamic economics. In Koopmans, T. C. (ed.), *Statistical inference in dynamic economic models*, Cowles Commission Monograph 10, 53–237. Wiley, New York.

Lindé, J. (2001). Estimating New-Keynesian Phillips Curves on Data With Measurements Errors A Full Information Maximum Likelihood Approach. Working Paper Series 129, Sveriges Rikbank.

Mavroeidis, S. (2002). *Econometric Issues in Forward-Looking Monetary Models*. Ph.D. thesis, Nuffield College, Oxford University.

Mizon, G. M. (1995). Progressive Modelling of Macroeconomic Time Series: The LSE Methodology. In Hoover, K. D. (ed.), *Macroeconometrics: Developments, Tensions and Prospects*, chap. 4. Kluwer Academic Publishers.

Nickell, S. J. and M. Andrews (1983). Unions, Real-Wages and Employment in Britain 1951-79. *Oxford Economic Papers (Supplement)*, *35*, 183–206.

Nymoen, R. (1989). Modelling Wages in the Small Open Economy: An Error-Correction Model of Norwegian Manufacturing Wages. *Oxford Bulletin of Economics and Statistics*, *51*, 239–258.

Phelps, E. S. (1978). Disinflation without recession: Adaptive guideposts and monetary policy. *Weltwirtschaftliches Archiv*, *100*, 239–265.

Roberts, J. M. (1995). New Keynesian economics and the Phillips curve. *Journal of Money, Credit and Banking*, *27*, 975–984.

Rotemberg, J. J. (1982). Sticky prices in the United States. *Journal of Political Economy*, *90*, 1187–1211.

Rudd, J. and K. Whelan (2001). New tests of the New Keynesian Phillips curve. Research and Statistics Discussion Paper 30, Federal Reserve Board of Governors.

Sargan, J. D. (1964). Wages and Prices in the United Kingdom: A Study of Econometric Methodology. In Hart, P. E., G. Mills and J. K. Whitaker (eds.), *Econometric Analysis for National Economic Planning*, 25–63. Butterworth Co., London.

Sargan, J. D. (1988). *Lectures on Advanced Econometric Theory*. Blackwell, Oxford.

Sbordone, A. M. (2002). Prices and unit labor costs: a new test of price stickiness. *Journal of Monetary Economics*, *49*, 265–292.

Sgherri, S. and K. F. Wallis (1999). Inflation Targetry and the Modelling of Wages and Prices. Mimo ESRC Macroeconomic Modelling Bureau, University of Warwick.

Spanos, A. (1986). *Statistical Foundations of Econometric Modelling*. Cambridge University Press, Cambridge.

Svensson, L. (2000). Open Economy Inflation Targeting. *Journal of International Economics*, *50*, 155–183.

Taylor, J. B. (1979). Staggered wage setting in a macro model. *American Economic Review*, *69*, 108–113.

Taylor, J. B. (1980). Aggregate dynamics and staggered contracts. *Journal of Political Economy*, *88*, 1–23.

White, H. (1980). A Heteroskedasticity-Consistent Covariance Matrix Estimator and a Direct Test of Heteroskedasticity. *Econometrica*, *48*, 817–838.

Wren-Lewis, S. and R. Moghadam (1994). Are Wages Forward Looking? *Oxford Economic Papers*, *46*, 403—424.

A Solution and estimation of simple rational expectations models

A.1 Introduction

This note illustrates solution and estimation of simple models with forward looking variables—the illustration being the "New Keynesian Phillips curve". Finally, we comment on a problem with observational equivalence, or lack of identification within this class of models.

A.2 Model with forward looking term only

The model is

$$\Delta p_t = b_{p1} E_t \Delta p_{t+1} + b_{p2} x_t + \varepsilon_{pt} \tag{32}$$

$$\Delta p_{t+1} = E_t \Delta p_{t+1} + \eta_{t+1} \tag{33}$$

$$x_t = b_{x1} \Delta p_{t-1} + b_{x2} x_{t-1} + \varepsilon_{xt} \tag{34}$$

We consider the solution, and estimation, by the "errors in variables" method – where expected values are replaced by actual values and the expectational errors:

$$\Delta p_t = b_{p1} \Delta p_{t+1} + b_{p2} x_t + \varepsilon_{pt} - b_{p1} \eta_{t+1}. \tag{35}$$

This model can be estimated by instrument variable methods.

We start by deriving the roots of the system, then the solution of the model, before we take a closer look at some estimation issues.

A.2.1 Roots

With Δp_{t+1} as the left hand side variable, the system is

$$\begin{aligned} \Delta p_{t+1} &= \frac{1}{b_{p1}} \Delta p_t - \frac{b_{p2}}{b_{p1}} x_t - \frac{1}{b_{p1}} \varepsilon_{pt} + \eta_{t+1} \\ x_{t+1} &= b_{x1} \Delta p_t + b_{x2} x_t + \varepsilon_{xt+1} \end{aligned}$$

or

$$\begin{bmatrix} \Delta p \\ x \end{bmatrix}_{t+1} = \begin{bmatrix} \frac{1}{b_{p1}} & -\frac{b_{p2}}{b_{p1}} \\ b_{x1} & b_{x2} \end{bmatrix} \begin{bmatrix} \Delta p \\ x \end{bmatrix}_t + \cdots,$$

The characteristic polynomial is

$$\begin{vmatrix} \frac{1}{b_{p1}} - \lambda & -\frac{b_{p2}}{b_{p1}} \\ b_{x1} & b_{x2} - \lambda \end{vmatrix} = \lambda^2 - \left(\frac{1}{b_{p1}} + b_{x2} \right) \lambda + \frac{1}{b_{p1}} \left(b_{p2} b_{x1} + b_{x2} \right)$$

with the roots

$$\lambda = \frac{\left(\frac{1}{b_{p1}} + b_{x2} \right) \pm \sqrt{\left(\frac{1}{b_{p1}} + b_{x2} \right)^2 - 4 \frac{1}{b_{p1}} \left(b_{p2} b_{x1} + b_{x2} \right)}}{2}$$

In the case of $b_{x1} = 0$, they simplify to

$$\lambda_1 = \frac{1}{b_{p1}},$$
$$\lambda_2 = b_{x2}.$$

The root λ_2 is smaller than one, while λ_1 is bigger than one if b_{p1} is less than one.

A.2.2 Solution

Consider the simplified system with $b_{x1} = 0$:

$$\begin{aligned} \Delta p_t &= b_{p1}\Delta p_{t+1} + b_{p2}x_t + \varepsilon_{pt} - b_{p1}\eta_{t+1} \\ x_{t+1} &= b_{x2}x_t + \varepsilon_{xt+1} \end{aligned}$$

Following Davidson (2000, p. 109–10), we derive the solution in two steps:

1. Find $E_t\Delta p_{t+1}$
2. Solve for Δp_t

Solving for $E_t\Delta p_{t+1}$ We start by reducing the model to a single equation:

$$\Delta p_t = b_{p1}\Delta p_{t+1} + b_{p2}b_{x2}x_{t-1} + b_{p2}\varepsilon_{xt} + \varepsilon_{pt} - b_{p1}\eta_{t+1}.$$

Solving forwards then produces:

$$\begin{aligned} \Delta p_t &= b_{p1}\left(b_{p1}\Delta p_{t+2} + b_{p2}b_{x2}x_t + b_{p2}\varepsilon_{xt+1} + \varepsilon_{pt+1} - b_{p1}\eta_{t+2}\right) + b_{p2}b_{x2}x_{t-1} + b_{p2}\varepsilon_{xt} + \varepsilon_{pt} - b_{p1}\eta_{t+1} \\ &= \left(b_{p2}b_{x2}x_{t-1} + b_{p2}\varepsilon_{xt} + \varepsilon_{pt} - b_{p1}\eta_{t+1}\right) + b_{p1}\left(b_{p2}b_{x2}x_t + b_{p2}\varepsilon_{xt+1} + \varepsilon_{pt+1} - b_{p1}\eta_{t+2}\right) + (b_{p1})^2\Delta p_{t+2} \\ &= \sum_{j=0}^{n}(b_{p1})^j\left(b_{p2}b_{x2}x_{t+j-1} + b_{p2}\varepsilon_{xt+j} + \varepsilon_{pt+j} - b_{p1}\eta_{t+j+1}\right) + (b_{p1})^{n+1}\Delta p_{t+n+1}. \end{aligned}$$

By imposing the transversality condition:

$$\lim_{n\to\infty}(b_{p1})^{n+1}\Delta p_{t+n+1} = 0$$

and then taking expectations conditional at time t, we get the "discounted solution":

$$\begin{aligned} E_t\Delta p_{t+1} &= \sum_{j=0}^{\infty}(b_{p1})^j\left(b_{p2}b_{x2}E_tx_{t+j} + b_{p2}E_t\varepsilon_{xt+j+1} + E_t\varepsilon_{pt+j+1} - b_{p1}E_t\eta_{t+j+2}\right) \\ &= \sum_{j=0}^{\infty}(b_{p1})^j\left(b_{p2}b_{x2}E_tx_{t+j}\right). \end{aligned}$$

However, we know the process for the forcing variable, so:

$$\begin{aligned}
E_{t-1}x_t &= b_{x2}x_{t-1} \\
E_t x_t &= x_t \\
E_t x_{t+1} &= b_{x2}x_t \\
E_t x_{t+2} &= E_t\left(E_{t+1}x_{t+2}\right) = E_t b_{x2}x_{t+1} = b_{x2}^2 x_t \\
E_t x_{t+j} &= b_{x2}^j x_t.
\end{aligned}$$

We can therefore substitute in:

$$\begin{aligned}
E_t\Delta p_{t+1} &= \sum_{j=0}^{\infty}\left(b_{p1}\right)^j\left(b_{p2}b_{x2}b_{x2}^j x_t\right) \\
&= b_{p2}b_{x2}\sum_{j=0}^{\infty}\left(b_{p1}b_{x2}\right)^j x_t \\
&= \left(\frac{b_{p2}b_{x2}}{1-b_{p1}b_{x2}}\right)x_t
\end{aligned}$$

All that is left to obtain the complete solution is to substitute back into the original equation and rearrange.

Solving for Δp_t

$$\begin{aligned}
\Delta p_t &= b_{p1}E_t\Delta p_{t+1} + b_{p2}x_t + \varepsilon_{pt} \\
&= b_{p1}\left(\frac{b_{p2}b_{x2}}{1-b_{p1}b_{x2}}\right)x_t + b_{p2}x_t + \varepsilon_{pt} \\
&= \left(\frac{b_{p2}}{1-b_{p1}b_{x2}}\right)x_t + \varepsilon_{pt}
\end{aligned}$$

A.2.3 Estimation

The implications of estimating the model by means of the "errors in variables" method is to induce moving average errors. Following Blake (1991), this can be readily seen as follows:

1. Lead (32) one period and subtract the expectation to find the RE error:

$$\begin{aligned}
\eta_{t+1} &= \Delta p_{t+1} - E_t\Delta p_{t+1} \\
&= b_{p1}E_{t+1}\Delta p_{t+2} + b_{p2}x_{t+1} + \varepsilon_{pt+1} - E_t\Delta p_{t+1} \\
&= \left(\frac{b_{p2}}{1-b_{p1}b_{x2}}\right)x_{t+1} + \varepsilon_{pt+1} - \left(\frac{b_{p2}}{1-b_{p1}b_{x2}}\right)b_{x2}x_t \\
&= \varepsilon_{pt+1} + \left(\frac{b_{p2}}{1-b_{p1}b_{x2}}\right)\varepsilon_{xt+1}
\end{aligned}$$

2. Substitute into (35):

$$\Delta p_t = b_{p1}\Delta p_{t+1} + b_{p2}x_t + \varepsilon_{pt} - b_{p1}\varepsilon_{pt+1} - \left(\frac{b_{p1}b_{p2}}{1-b_{p1}b_{x2}}\right)\varepsilon_{xt+1}.$$

So even though the original model has white noise errors, the estimated model will have first order moving average errors.

A.3 Hybrid model with both forward and backward looking terms

The model is now

$$\Delta p_t = b^f_{p1}E_t\Delta p_{t+1} + b^b_{p1}\Delta p_{t-1} + b_{p2}x_t + \varepsilon_{pt} \tag{36}$$

$$\Delta p_{t+1} = E_t\Delta p_{t+1} + \eta_{t+1} \tag{37}$$

$$x_t = b_{x1}\Delta p_{t-1} + b_{x2}x_{t-1} + \varepsilon_{xt} \tag{38}$$

A.3.1 Roots

With Δp_{t+1} as the left hand side variable, the system is

$$\begin{aligned}
\Delta p_{t+1} &= \frac{1}{b^f_{p1}}\Delta p_t - \frac{b^b_{p1}}{b^f_{p1}}\Delta p_{t-1} - \frac{b_{p2}}{b^f_{p1}}x_t - \frac{1}{b^f_{p1}}\varepsilon_{pt} + \eta_{t+1}\\
x_{t+1} &= b_{x1}\Delta p_t + b_{x2}x_t + \varepsilon_{xt+1}
\end{aligned}$$

with companion form

$$\begin{bmatrix}\Delta p_{t+1}\\ x_{t+1}\\ \Delta p_t\end{bmatrix} = \begin{bmatrix}\frac{1}{b^f_{p1}} & -\frac{b_{p2}}{b^f_{p1}} & -\frac{b^b_{p1}}{b^f_{p1}}\\ b_{x1} & b_{x2} & 0\\ 1 & 0 & 0\end{bmatrix}\begin{bmatrix}\Delta p_t\\ x_t\\ \Delta p_{t-1}\end{bmatrix} + \cdots,$$

and characteristic polynomial

$$\lambda^3 - \left(\frac{1}{b^f_{p1}} + b_{x2}\right)\lambda^2 + \frac{1}{b^f_{p1}}\left(b^b_{p1} + b^b_{p1}b_{x1} + b_{x2}\right)\lambda - \frac{b^b_{p1}}{b^f_{p1}}b_{x2}.$$

In the simplified case of $b_{x1}=0$, the roots are

$$\begin{aligned}
\lambda_1 &= b_{x2},\\
\lambda_2 &= \tfrac{1}{2}\tfrac{1}{b^f_{p1}}\left(1-\sqrt{1-4b^b_{p1}b^f_{p1}}\right),\\
\lambda_3 &= \tfrac{1}{2}\tfrac{1}{b^f_{p1}}\left(1+\sqrt{1-4b^b_{p1}b^f_{p1}}\right).
\end{aligned}$$

With the further restriction $b^b_{p1}+b^f_{p1}=1$, the roots simplify to

$$\begin{aligned}
\lambda_1 &= b_{x2},\\
\lambda_2 &= \tfrac{1}{b^f_{p1}} - 1,\\
\lambda_3 &= 1.
\end{aligned}$$

A.4 Does the MA(1) process prove that the forward solution applies?

Assume that the true model is

$$\Delta p_t = b_{p1}\Delta p_{t-1} + \varepsilon_{pt}, \quad |b_{p1}| < 1$$

but that the following model is estimated by instrument variable methods

$$\Delta p_t = b_{p1}^f \Delta p_{t+1} + \varepsilon_{pt}^f$$

What are the properties of ε_{pt}^f?

$$\varepsilon_{pt}^f = \Delta p_t - b_{p1}^f \Delta p_{t+1}$$

Assume, as is common in the literature, that we find that $b_{p1}^f \approx 1$. Then

$$\begin{aligned} \varepsilon_{pt}^f &\approx \Delta p_t - \Delta p_{t+1} = -\Delta^2 p_{t+1} \\ &= -\left[\varepsilon_{pt+1} + (b_{p1} - 1)\varepsilon_{pt} + ...\right]. \end{aligned}$$

So we obtain a model with a moving average residual, but this time the reason is not forward looking behaviour but rather model misspecification.

B Data Appendix. Data definitions

B.1 The Euroland data

B.1.1 The basic series from ECB

The data for Euroland is derived from a set of data base for the Area Wide model of the ECB, see Fagan et al. (2001), which can be downloaded with the paper at http://www.ecb.int/. The following series are used from that data base:
YEN - Nominal GDP.
YER - Real GDP.
WIN - Compensation to employees.
LNN - Total employment.
YGA - Output gap, defined as actual real output relative to potential real output (YET) measured by a constant returns to scale production function with neutral technical progress.

B.1.2 Data used to replicate GGL (sections 3 and 5.1)

$p_t = 100 \cdot \log(YEN/YER)$ - GDP price level.
$ws_t = 100 \cdot (\log(WIN/(YEN \cdot 6.1333333)) - mean(\log(WIN/(YEN \cdot 6.1333333))$ - Wage share corrected for a constant (due to Sbordone (2002)) minus the sample mean of this corrected wage share.
gap_t = Output gap, defined as the deviation of log of the actual real output (100·log(YER)) from a quadratic trend.
$w_t = 100 \cdot \log(WIN/LNN)$ - wages. The annualised growth in wages $(4 \cdot (w_t - w_{t-1}))$ is used as an instrument by GGL.

B.1.3 Data used in sections 5.3 - 5.5

$emugap_t = 100 \cdot (YGA - 1)$.

B.2 The U.S. data

B.2.1 The source data

PGDP = Total GDP chain index (1996=100).
GDP = Real GDP, total U.S. economy.
Sources: U.S. National Income accounts.
For later use:
COMPHR = Hourly Non-Farm Business (NFB) compensation.
LABSHR= Labour share in Non-Farm Business (NFB) income.
Sources: LABSHR and COMPHR is from Bureau of Labor Statistics' Productivity and Cost release.

B.2.2 Data used in section 5.1

$p_t = \log(PGDP)$.
gap_t = Output gap, defined as the deviation of the log of actual real GDP output ($\log(GDP_t)$) from a quadratic trend.

For later use:
$ws_t = \log(LABSHR) - \log(mean)$ - Demeaned labour share.

B.3 The Norwegian data

B.3.1 Notes

1. Unless another source is given, all data are taken from RIMINI, the quarterly macroeconometric model used in Norges Bank (The Central Bank of Norway). The data are seasonally unadjusted.

2. For each RIMINI-variable, the corresponding name in the RIMINI-database is given by an entry [RIMINI: variable name] at the end of the description. (The RIMINI identifier is from Rikmodnotat 140, Version 3.1415, dated 11. December 2001, Norges Bank, Research Department.)

3. Several of the variables refer to the *mainland economy*, defined as total economy minus oil and gass production and international shipping.

4. Baseyear is 1997 (for the sample 1972.4 - 2001.1).

B.3.2 Definitions

H Normal working hours per week (for blue and white colour workers). [RIMINI: NH].

P Consumer price index. Baseyear =1. [RIMINI: CPI].

PB Deflator of total imports. Baseyear =1. [RIMINI: PB].

PE Consumer price index for electricity. Baseyear = 1 [RIMINI: CPIEL].

PR Mainland economy value added per man hour at factor costs, fixed baseyear (1991) prices. Mill. NOK. [RIMINI: ZYF].

RL Average interest rate on bank loans. [RIMINI: R.LB].

$\tau 1$ Employers tax rate. $\tau 1 = WCF/WF - 1$.

$\tau 3$ Indirect tax rate. [RIMINI: T3].

U Rate of unemployment. registered unemployed plus persons on active labour market programmes as a percentage of the labour force, calculated as employed wage earners plus unemployment. [RIMINI: UTOT].

W Nominal mainland hourly wages. This variable is constructed from RIMINI-database series as a weighted average of hourly wages in manufactures and construction and hourly wages in private and public service production, with total number of man-hours (corrected for vacations, sick hours, etc) as weights:

$$W = (WIBA * TWIBA + WOTVJ * (TWTV + TWO + TWJ))/TWF.$$

Finally, the NPC model (29) for Norway includes seasonal dummies and

$Pdum$ Composite dummy for introduction and lift of direct price regulations. 1 in 1971.1, 1971.2,1976.4,1977.4, 1979.1,1996.1. -1 in 1975.1,1980.1,1981.1,1982.1, 1986.3. Zero otherwise.

B.4 The UK data

B.4.1 The source data for the Bank of England study

Batini et al. (2000) use the following series from UK Office of National Statistics:
ABML - Gross value added at basic prices (excluding taxes less subsidies on products).
ABMM - Gross value added (constant prices) at basic prices.
AJFA - British pounds - US dollar exchange rate.
BCAJ - Number of employee in the work-force (seasonally adjusted).
DYZN - Number of selfemployment work-force jobs (seasonally adjusted).
HAEA - Compensation to employees, including the value of social contributions payable by the employer.
IKBI - Total imports (to the UK) at current prices.
IKBL - Total imports (to the UK) at constant prices.
NMXS - Compensation of employees paid by general government.

Moreover, Batini et al. (2000) calculate and collect from other sources:
GGGVA - A measure of the part of gross value added attributable to general government.
PETSPOT - Oil spot price (average of Brent Crude, West Texas and Dubai Light). Source: Bloomberg.
WPX - Weighted average of export prices from the G7 countries (excluding UK), with effective exchange rates as weights. Sources: Datastream and International Finance Statistics, IMF.
EER - UK nominal effective exchange rate. Source: International Finance Statistics, IMF.
A = (BCAJ + DYZN) / BCAJ

B.4.2 Bank of England variables used in section 6.3

Small letters denote natural logarithms (log) :
$ws_t = \log[\{((HAEA_t - NMXS_t) \cdot A_t)/(ABML_t - GGGVA_t)\} \cdot 100]$.

$p_t = \log[(ABML_t/ABMM_t)\cdot 100]$.
$oil_t = \log[PETSPOT_t/AJFA_t)\cdot 100]$.
$gap_t =$ Output gap defined as output ($\log(ABMM_t)$) deviations from trend, where the trend is estimated by a Hodrick-Prescott filter.
$com_t = \log[(WPX_t/EER_t)\cdot 100]$.
$rpm_t = \log[(IKBI_t/IKBL_t)\cdot 100]$.
$n_t = \log(BCAJ_t + DYZN_t)$
$SEA_t =$ Dummy variable for the Single European Act, which takes on the value 1 after 1990.1, zero otherwise.

B.4.3 Variables used to calculate the error correction terms in section 6.3

W_t- Wages, index of actual quarterly earnings.
P_t- Retail price index.
PR_t- Non -North Sea productivity.
PB_t- Price deflator for expenditure on imported goods and services.
U_t- Unemployment rate: registered number of unemployed.
$\tau 1_t-$ Employers' tax rate.
$\tau 3_t-$ Tax rate on the retail price index basket, excluding mortgage interest payments.

For more precise definitions and for sources, see Bårdsen et al. (1998).