Non-Exclusive Contracts, Collateralized Trade, and a Theory of an Exchange*

Yaron Leitner
Research Department
Federal Reserve Bank of Philadelphia
Ten Independence Mall
Philadelphia, PA 19106-1574
E-mail: yaron.leitner@phil.frb.org

January 2003

Abstract

Liquid markets where agents have limited capacity to sign exclusive contracts, as well as imperfect knowledge of previous transactions by others, raise the following risk: An agent can promise the same asset to multiple counterparties and subsequently default. I show that in such markets an exchange can arise as a very simple type of intermediary that improves welfare. In particular, the only role of the exchange here is to set limits on the number of contracts that agents can report to it. Furthermore, reporting can be voluntary, i.e., pairs of agents can enter contracts without reporting them to the exchange and the exchange cannot observe whether agents enter such contracts. Interestingly, to implement an equilibrium in which agents report all their trades (voluntarily), the exchange may need to set position limits that are non-binding in equilibrium. In addition, the exchange must not make reported trades public (i.e., it is not a bulletin board). An alternative to an exchange is collateralized trade, but this alternative is costly because of the opportunity cost of collateral. I also show that the gains from an exchange increase when markets are more liquid (in the sense that the fixed costs per trade are lower) or when agents have more intangible capital (i.e., reputation).

JEL codes: D82, G20

*This paper is a revised version of the first chapter of my Ph.D. dissertation in finance at the Kellogg Graduate School of Management at Northwestern University. I thank the members of my dissertation committee: Philip Bond, Arvind Krishnamurthy, Adriano Rampini, Asher Wolinsky, and especially my main advisor, Michael Fishman, for guidance and comments. I also thank Mitchell Berlin, seminar participants at Northwestern University, The Philadelphia Fed, and the 2001 Fed's conference on financial structure and regulation (in particular, the discussant, John Weinberg).

1 Introduction

Liquid markets where agents have limited capacity to sign exclusive contracts, as well as imperfect knowledge of previous transactions by others, raise the following risk: An agent can promise the same asset to multiple counterparties and subsequently default. In extreme cases, this risk may eliminate most of the gains from trade. This paper theoretically shows how an exchange can arise in this setting as a very simple type of intermediary that improves welfare. It also illustrates the importance of collateral in enforcing exclusivity.

The exchange here is an entity to which pairs of agents can report the fact that they entered a bilateral contract. The only role of the exchange is to set limits on the number of contracts that agents can report. The main result is that such an exchange can enhance welfare. This holds even if agents can enter contracts without reporting them to the exchange and even though the exchange cannot observe whether agents have entered such contracts. Other results are as follows: (1) Without an exchange, we will see collateralized trade, i.e., agents will put up cash as collateral. The optimal amount of collateral increases when agents have less intangible capital (e.g., reputation) or when markets become more liquid. (2) The gains from an exchange increase when agents have more intangible capital or when markets become more liquid. (3) In some cases the exchange must set position limits that are non-binding in equilibrium. For example, to implement an equilibrium in which every agent enters exactly one contract, the exchange may need to allow each agent to report three contracts. (4) In some cases the exchange must not reveal the exact number of contracts that an agent has already entered. (5) Even an exchange may require that agents post collateral, but less than what would be required without an exchange.

The basic setting is as follows: Agents invest their endowments in two-period projects. They can benefit from bilateral trade because the interim cash flows from their projects are negatively correlated. An agent cannot commit, however, to pay out of interim or final cash flows. But if he defaults, it is possible to terminate his project. Each project has a liquidation value of zero, and the amount that each agent invests in his project is private information. I also assume that the project's assets cannot be pledged exclusively. When an agent can commit to enter exclusive contracts, the threat of losing future income (i.e., future cash flows) is enough to induce him to make the optimal investments and pay what he promised. Otherwise, the agent may strategically choose to enter contracts with multiple counterparties, consume his endowment rather than invest it in his project, and subsequently default on all the contracts he entered.

Both an exchange and collateralized trade can mitigate the problem above. Consider first collateral. Collateral has a dual role in this paper: First, it has the standard role of guaranteeing payments. Second, it has the role of enforcing exclusivity, thereby making agents' future income valuable in backing their promises. Agents in this model can, therefore, promise (credibly) more than the amount of cash that they post as collateral. Collateral is costly, however, because agents forgo investing in their positive NPV projects.[1]

Consider now an exchange. I show that by setting position limits appropriately, the exchange can implement the same outcome as when agents can commit to enter exclusive contracts. I also show that an equilibrium in which all agents report all their trades to the exchange (voluntarily) is immune to self-enforcing deviations by pairs of agents. In particular, a pair of agents cannot gain by not reporting the fact that they entered a bilateral contract, even if there is a small cost associated with reporting. To rule out such deviations, the exchange must permit each member of any deviating pair enough latitude to cheat on his partner. Therefore, in some cases we may see position limits that are non-binding in equilibrium.

The paper relates to the theory of financial intermediation, the market microstructure literature, and the literature on contracting with non-exclusivity. Its main theoretical con-

[1] Telser (1981) focuses on the role of collateral (margins) in guaranteeing payments in futures exchanges, emphasizing its opportunity cost (agents may need to deviate from their optimal portfolio choice). Dubey, Geanakoplos, and Zame (1997) model collateralized trade in a general equilibrium setting. Agents in their model are not allowed, however, to pledge their future income (i.e., they do not lose it if they default), so an agent delivers only if the value of his promises is less than the value of the collateral he put up to secure his promises. In a model of exchange competition, Santos and Scheinkman (2001) focus on the screening role of collateral (see also Bester, 1985), ignoring its opportunity cost. They model collateral as the amount of an agent's future endowment that can be seized.

tribution is that it illustrates a very minimal role that a financial intermediary (an exchange) can have and still improve welfare. The main difference between my paper and the related market microstructure literature is that the latter explored how different trading systems can enhance liquidity,[2] whereas I start with markets that are already very liquid, and show why an exchange may still be needed. In addition, when I introduce an exchange, I keep the matching process as well as the bargaining process unchanged.[3] Unlike Diamond (1984), I do not rely on a diversification argument, and unlike Townsend (1978) the intermediary in my setting arises when the fixed cost involved with each bilateral trade is low rather than high.[4] In the sense that my paper illustrates a negative aspect of liquidity, it also relates to Myers and Rajan (1998).[5]

In a very different framework, Bizer and DeMarzo (1992) and Parlour and Rajan (2001) study the effect of non-exclusivity on equilibrium interest rates and on the competitive structure of a loan market.[6] Bizer and DeMarzo assume that prior contracts are observable and have a priority; so in their setting an exchange cannot improve welfare.[7] In contrast, I assume that previous transactions are not observable. Parlour and Rajan assume that intermediaries simultaneously offer contracts and then a single borrower can accept any subset of contracts. A similar feature between my paper and the paper by Parlour and Rajan is the idea of a strategic default and the fact that an agent who defaults does so on

[2] Examples are Hagerty and McDonald (1996), Pagano and Röell (1996), Benveniste, Marcus and Wilhelm (1992), Glosten (1989, 1994) and Seppi (1997), who focused on the role of anonymity, transparency, one trading floor, a specialist, and a limit order book in mitigating problems of asymmetric information regarding asset values.

[3] Another related work focuses on organized futures exchanges: Telser and Higinbotham (1977) discuss the importance of standard contracts; Edwards (1984) describes the role of a clearing house as a central counterparty that guarantees performance; Baer, France, and Moser (2001) show how a clearing house can save on collateral through netting positions; and Brennan (1986) shows how price limits (not position limits) may serve as a partial substitute for margins in ensuring contract performance (see also Chou and Yu, 2000, who showed that this conclusion is not robust).

[4] See also Brusco and Jackson (1999), who show that a market maker can economize on the fixed costs of trading across periods.

[5] They showed that greater asset liquidity can reduce the firm's capacity to raise external finance because it reduces the firm's ability to commit to a specific course of action.

[6] See also Kahn and Mookherjee (1998), who study more general insurance contracts, and Bisin and Rampini (2002), who suggest a motivation for a bankruptcy institution.

[7] In their model additional contracts impose a negative externality on existing contracts because the agent's hidden effort affects the distribution of his future income.

all the contracts he entered. In their model this can rule out entry even though competing lenders make positive profits. In my paper, this helps sustain an equilibrium in which all agents report to the exchange.

The exchange in this paper has some features of a clearing house, but it has a more minimal role because it does not guarantee performance of contracts.[8] The theory is consistent with recent developments in the over-the-counter market for interest-rate swaps. These swaps have evolved from a customized and illiquid product to a standardized and liquid product (suggesting that standardization and liquidity creation can occur even without an organized exchange). Once this over-the-counter market became liquid, the London Clearing House (LCH) started clearing its products. The LCH neither provides a new trading system, nor does it act as a broker. Instead, it allows its members to register bilaterally negotiated contracts, and then it becomes a central counterparty that guarantees contract performance. According to David Hardy, chief executive of the LCH, "Clearing costs are pretty low anyway....The great benefit to users is that the amount of capital which is tied-up can be reduced."[9] In practice, a clearing house can save on collateral by netting positions. The theory suggests that a clearing house may have a further advantage by being able to require less collateral on the netted position.

The paper proceeds as follows. In Section 2, I use a simple example to illustrate why non-exclusivity may be a problem. In Section 3, I present a formal model of trade between two agents (where non-exclusivity is not an issue) and in Section 4, I extend the model to include a continuum of agents (so that non-exclusivity becomes an issue). I show that the most efficient outcome without an exchange involves collateral and illustrate its dual role. In Section 5, I introduce an exchange and show that an equilibrium in which agents report

[8] Clearing houses around the world set position limits for their members. These limits are capital based and are monitored daily or in real time. Their purpose is to ensure that members maintain positions only within their financial capability. These "capital-based" position limits are different from "speculative" position limits that limit the number of contracts held by a single investor or a group of investors acting in concert. The latter position limits are set both by regulators and by exchanges in order to prevent speculators from manipulating spot prices. In addition, traders who qualify as "bona fide hedgers" can gain exemption.

[9] Source: "London Clearing House and Clearnet to Form Pan European Clearing House," fpionline.com

all their trades to the exchange is immune to self-enforcing deviations by pairs of agents. I also illustrate the role of non-binding position limits and show that the exchange is not a bulletin board. I conclude in Section 6. (The appendix contains proofs.)

2 An example of non-exclusivity

The driving force behind the results in this paper is the fact that agents cannot commit to enter exclusive contracts. Before I present a formal model, I will illustrate with a simple example why non-exclusivity may be a problem.

Suppose there are two possible states of nature next year, and consider an agent who is going to receive two dollars in state 1 and zero dollars in state 2. Assume that the agent has limited liability and that today he can enter into a forward contract according to which he should pay one dollar in state 1 and receive one dollar in state 2. If he enters one contract, he is fully hedged (insured) and the amount of dollars he will have next year equals one dollar in both states.

Now suppose that the agent can enter the same contract with n different counterparties, where $n > 2$. In state 2, he receives a total cash flow of n dollars from his counterparties, so his total wealth is n. In state 1, he needs to deliver n dollars. Since he only has two, he defaults, but because of his limited liability, his wealth cannot fall below zero. By increasing n, the agent can increase his wealth in state 1 without decreasing it in the other state. This means that for a wide range of utility functions, the agent can increase his utility as much as he wants by entering multiple contracts and defaulting on all of them.

The following dialogue from the movie "The Producers" is a good illustration of the idea above:

- Blum: "If he were certain that the show would fail, a man could make a fortune...If you were really a bold criminal, you could have raised a million dollars, put on a \$60,000 flop, and kept the rest."
- Bialystock: "But what if the play was a hit?"

- Blum: "Well, then you'd go to jail...Once the play is a hit, you'd have to pay up all the backers, and with so many backers, there could never be enough profits to go around."

Note that in both examples limited liability plays a crucial role. If it were possible to impose infinite penalties (e.g., either pay or die), agents would not enter too many contracts and penalties would not need to be imposed in equilibrium.

3 A two-agent economy

3.1 The model

I start with a simple model of trade between a pair of agents. There are two periods and one divisible good, called cash, or simply dollars. Both agents are risk neutral and obtain an expected utility of $E(c_0 + c_1 + c_2)$ from consuming c_0, c_1 and c_2 dollars at dates $0, 1$ and 2, respectively. They are also protected by limited liability, so $c_t \geq 0$.

At date 0, each agent can invest his endowment of one dollar in a two-period constant-returns-to-scale project. If he invests I dollars at date 0, he receives RI dollars at date 2. Interim (i.e., date-1) cash flows for the two projects are negatively correlated: When the project of one agent yields a positive cash flow, the project of the other agent has a negative cash flow: It requires an additional investment that must be made in full for the project to continue. More specifically, there are two equal probability states, state 1 and state 2, one of which becomes publicly observable at date 1. The project of agent i ($i = 1, 2$) yields εI dollars in state i, but requires an additional investment of εI dollars in the other state, denoted by $-i$. (See Figure 1.)

The two agents can also store between date 0 and date 1 through a third party who can commit not to divert cash (i.e., the risk-free rate is normalized to be zero percent).[10] It is assumed that $R > \varepsilon$, so it is efficient to make the additional investment at date 1 if cash is

[10] The assumption regarding the risk-free rate is a simplifying assumption. In addition, nothing would change if we assumed that storage could also take place between date 1 and date 2.

$$
\begin{array}{ccccc}
\text{date } 0 & & \text{date } 1 & & \text{date } 2 \\
 & \overset{state\, i}{\nearrow} & \varepsilon I & \longrightarrow & RI \\
-I & & & & \\
 & \underset{state\, -i}{\searrow} & -\varepsilon I & \longrightarrow & RI
\end{array}
$$

Figure 1: Project's cash flows for agent i if project operates to maturity

available. It is also assumed that $R > 1$, so in a world without frictions both projects have positive NPVs.

Regarding information, I assume that the amount stored (through a third party) is observable to both agents and can be contracted upon. (One can think of an escrow account.)[11] However, investments in projects, as well as projects' cash flows, are private information and cannot be contracted upon. In particular, an agent can default even if he has enough cash to pay what he promised.[12]

It is assumed that when an agent defaults, the other agent can terminate his project (i.e., shut his business).[13] Thus, agents in this model pay what they promised for fear of losing future cash flows. It is also assumed that each project has a liquidation value of zero at each date. This can be motivated by assuming that each project requires the human capital of the agent who has access to it and that human capital is inalienable.

The assumptions above imply that an agent who suffers a negative liquidity shock at date 1 (i.e., an agent who needs to make an additional investment in his project) cannot borrow at date 1 against future cash flows.[14] However, the two agents can still hedge by entering a forward contract at date 0, according to which the agent whose project yields a positive cash flow at date 1 transfers cash to the other agent. Of course, agents may find it optimal not to hedge. To rule this out, I assume that $\varepsilon < 1$. This is a sufficient condition to

[11] The results will not change if we assume that agents can also store privately and that cash that is stored privately is unobservable (and, therefore, not contractable).

[12] He can always claim that the project yielded no cash flows because no investment has taken place.

[13] Thus, it is observable whether the project operates (i.e., whether the agent is in business), but the level of investment is private information.

[14] This is similar to Holmström and Tirole (1998), who assumed that because of moral hazard, an agent who faces a liquidity shock can only borrow against a fraction of his future income.

ensure that hedging (and bilateral trade) is beneficial.[15] To simplify proofs, I also assume that $R < 2$.

3.2 Optimal contract

A natural contract for the two agents is as follows: At date 0 agent i invests I_i dollars in his project and transfers a_i dollars to a third party for storage. The total amount stored is, therefore, $s = a_1 + a_2$. (In Subsection 3.3, I will clarify the relationship between storage and collateral.) At date 1, if state i is realized, the project of agent i yields εI_i dollars. Out of this amount, agent i pays b_i dollars to the other agent, who realizes a negative liquidity shock. The other agent also receives the total amount stored.[16] While b_i is restricted to be non-negative, there is no restriction on the sign of a_i, so date-0 transfers between the two agents are not ruled out. (Later, we will see, however, that such transfers are suboptimal.)

The set of feasible contracts will be denoted by Ψ, that is, $\Psi = \{(a_1, a_2, b_1, b_2, I_1, I_2) : 0 \leq I_i \leq 1 - a_i$ and $0 \leq b_i \leq \varepsilon I_i$ for $i = 1, 2$, and $s = a_1 + a_2 \geq 0\}$. In principle, a contract could also specify the probability $\lambda_i(\widehat{b}_i|b_i)$ that the project of agent i continues if he delivers $\widehat{b}_i$ when he is required to deliver b_i. One can show, however, that in this model, if an agent defaults (i.e., does not pay what he promised in full), it is optimal to shut down his project with probability 1.[17] In other words,

$$\lambda_i(\widehat{b}_i|b_i) = \begin{cases} 1 & if \quad \widehat{b}_i = b_i \\ 0 & \text{otherwise.} \end{cases}$$

An optimal contract is a feasible contract that maximizes the sum of agents' utilities, subject to the constraint that each agent invests and delivers according to what the contract says. To find it, we first need to derive an expression for $U_i(\widehat{I}_i, \widehat{b}_i \,|\, \psi)$, which is the utility for agent i if he invests $\widehat{I}_i \in [0, 1 - a_i]$ and delivers $\widehat{b}_i \in [0, \varepsilon \widehat{I}_i]$, given that he entered the contract $\psi \in \Psi$, and given that his counterparty, denoted by $-i$, behaves according to the

[15] This is not a necessary condition. A condition that is both necessary and sufficient is given in the proof of Lemma 3.

[16] This is without loss of generality. In other words, if in state i, s_i is transferred to agent i and $s - s_i$ is transferred to the other agent, then there exists an optimal contract such that $s_i = 0$.

[17] It is assumed that it is possible to commit to this continuation/closure policy.

contract, i.e., invests I_{-i} and delivers b_{-i}.

The amount that agent i consumes at date 0 follows from his budget constraint and is given by $c_{0i} = 1 - a_i - \widehat{I}_i$. The total amount that he consumes at dates 1 and 2 depends on the state. If state i is realized, agent i can consume $\varepsilon\widehat{I}_i - \widehat{b}_i + \lambda_i(\widehat{b}_i|b_i)R\widehat{I}_i$. If the other state is realized, the agent can continue his project only if he has enough cash, so the total amount that he consumes at dates 1 and 2 equals $s + b_{-i} + (R-\varepsilon)\widehat{I}_i$, if $s + b_{-i} \geq \varepsilon\widehat{I}_i$, and $s + b_{-i}$ otherwise. The agent's utility is, therefore,

$$\begin{aligned} U_i(\widehat{I}_i, \widehat{b}_i \,|\, \psi) \quad \equiv \quad & 1 - a_i - \widehat{I}_i \\ & + \frac{1}{2}[\varepsilon\widehat{I}_i - \widehat{b}_i + \lambda_i(\widehat{b}_i|b_i)R\widehat{I}_i] \\ & + \frac{1}{2}[s + b_{-i}] + \frac{1}{2}\beta_i(\widehat{I}_i, \widehat{b}_i \,|\, \psi)(R-\varepsilon)\widehat{I}_i \end{aligned}$$

where $\beta_i(\widehat{I}_i, \widehat{b}_i \,|\, \psi) \begin{cases} 1 & \text{if } \; s + b_{-i} \geq \varepsilon\widehat{I}_i \\ 0 & \text{otherwise.} \end{cases}$

In the special case in which the agent behaves according to the contract, i.e., $(\widehat{I}_i, \widehat{b}_i) = (I_i, b_i)$, I will denote $U_i(\widehat{I}_i, \widehat{b}_i \,|\, \psi)$ simply by $U_i(\psi)$.

To find an optimal contract, we need to solve the following problem:

$$\max_{\psi \in \Psi} \sum_{i=1}^{2} U_i(\psi)$$

subject to the following incentive constraint:

$$(I_i, b_i) \in \arg \max_{(\widehat{I}_i, \widehat{b}_i) \in B_i(\psi)} U_i(\widehat{I}_i, \widehat{b}_i \,|\, \psi) \quad \text{for} \quad i = 1, 2. \tag{IC-1}$$

where $B_i(\psi)$ denotes the set of feasible choices of $\widehat{I}_i$ and $\widehat{b}_i$ for agent i. The solution to the problem above will be referred to as the *second best.* The solution to the unconstrained problem, i.e., the same problem, but without the incentive constraint, is the *first best.*

One can easily verify that the first-best contract is given by $a_1 = a_2 = 0$, $b_1 = b_2 = \varepsilon$, and $I_1 = I_2 = 1$. In other words, at date 0 each agent invests one dollar in his positive NPV project. Then at date 1, the agent whose project yields a positive cash flow transfers

ε dollars to the other agent, so that the latter will have enough cash to continue his project. This way each agent obtains an expected utility R.

We are now ready to solve for the second-best contract. Two useful observations are as follows:

ob1: An agent will either deliver everything that he promised to deliver or nothing. In addition, if he has enough cash to pay, he will not default.

ob2: If an agent chooses to deviate from the optimal contract, the best thing for him is to invest nothing in his project (and subsequently default when he needs to make a payment).

The logic behind the first observation is that if an agent makes a partial payment, he still loses his project. Therefore, he is better off not paying at all and keeping what he has to himself. In addition, when an agent's project yields εI and he needs to pay $b \leq \varepsilon I$, he is better off paying in full, because otherwise he can keep b, but he loses RI, which is more than εI (and b). To see why the second observation follows, note that if an agent defaults, he obtains εI from his project in one state and at most $(R-\varepsilon)I$ in the other state.[18] The assumption $R < 2$ implies that $\frac{1}{2}\varepsilon I + \frac{1}{2}(R-\varepsilon)I = \frac{1}{2}RI < 1$, so the agent is better off consuming his initial endowment rather than investing.

The second observation implies that we can replace constraint IC-1 with

$$U_i(\psi) \geq \overline{U}_i(\psi) \quad \text{for} \quad i = 1, 2, \tag{IC-1'}$$

where $\overline{U}_i(\psi) = U_i(0, 0 \,|\, \psi)$.

Proposition 1 *If $R \geq 1 + \frac{1}{2}\varepsilon$, the second-best contract equals the first best. Otherwise, it is (uniquely) given by $a_1 = a_2 = 1 - I$, $b_1 = b_2 = (2+\varepsilon)I - 2$ and $I_1 = I_2 = I$ where $I = \frac{1}{2+\frac{1}{2}\varepsilon - R} < 1$.*

[18] We can focus without loss of generality on deviations in which the agent defaults.

The idea behind the proof is as follows: First, the optimal contract is symmetric and can be denoted simply by $\psi = (a, b, I)$. This follows from the symmetric nature of the problem. Second, since it is possible to store and consume at date 1, the contract can be designed so that no consumption takes place at date 0. Therefore,

$$a + I = 1. \tag{1}$$

Third, since the contract is entered for hedging purposes, it should be designed so that each agent will have enough cash to continue his project when he realizes a negative shock. In other words, the contract should satisfy

$$s + b = \varepsilon I. \tag{2}$$

It follows that the utility for agent i (if everyone follows the contract) is given by

$$U_i(\psi) = 1 + (R - 1)I, \tag{3}$$

and the highest utility that agent i can obtain if he deviates from what the contract says is

$$\overline{U}_i(\psi) = 1 + \frac{1}{2}b. \tag{4}$$

Now, one can use equations (1) and (2) (plus the fact that $s = 2a$) to express a and b as a function of I, and then use the incentive constraint IC-1' and equations (3) and (4) to find the optimal level of investment I.

The reason why the first best may not be achieved is that the incentive constraint limits the amount of cash that an agent can credibly promise out of his project's cash flows. In particular, if we substitute equations (1) and (2) in the incentive constraint, we obtain

$$b \leq 2(R - 1)I.$$

If $\varepsilon \leq 2(R - 1)$, which is equivalent to $R \geq 1 + \frac{1}{2}\varepsilon$, the first best can be achieved, because an agent who invests $I = 1$ can credibly promise $b = \varepsilon$. Otherwise, agents need to satisfy some of the demand for liquidity at date 1 by storing cash between date 0 and date 1, and the first-best outcome cannot be achieved.

3.3 Collateral

Denoting $k_i = a_i$ and $x_i = a_i + b_i$, one can interpret a contract as follows: Agent i promises to pay x_i dollars at date 1 if state i happens. When he enters the contract, he also puts up k_i dollars as collateral. At date 1, in the state in which the agent does not need to deliver, he receives his collateral back. Otherwise, the collateral is automatically transferred to the other agent. Thus, agent i can default only on the amount $x_i - k_i$.

Since $a_i = 1 - I_i$, equation (3) can be rewritten as

$$U_i(\psi) = R - (R-1)k_i. \tag{5}$$

The first term represents the first-best utility, and the second term represents the opportunity cost of collateral: By posting collateral, agents forgo investing in their positive NPV projects.

4 Decentralized trade with a continuum of agents

4.1 Trading environment

To create an environment where non-exclusivity may be an issue, I extend the model from the previous section to include a continuum of agents, half of which are type 1 and half of which are type 2, where type $i \in \{1, 2\}$ corresponds to agent i from the previous section.

Trade takes place at date 0 during an infinite (but countable) number of rounds. One can think of many points of time during a trading day. Each round, a continuum of agents arrives to trade for the first time, with an equal mass of both types. Agents can also stay for subsequent rounds, but once an agent leaves the trading process, he cannot come back. Agents who are present in each round are pairwise matched according to their types. Each pair includes one agent of each type.[19] (If the mass of type-1 agents does not equal the mass of type-2 agents, some agents remain unmatched.) After being matched, the two agents offer contracts simultaneously. If they both offer the same contract, they enter this

[19] Thus, the type of each agent is observable.

contract. Otherwise, they do not enter a contract. Finally, after all rounds of trade have ended, agents store through third parties and make date-0 transfers simultaneously. Then each agent makes his individual date-0 investment and consumption decisions.

To have some measure of liquidity, I assume that there is some positive cost δ associated with every match, except for the first one. The lower this cost, the more liquid the market. It is assumed, for simplicity, that δ is not paid in cash. Instead, it is measured in utility terms. It could represent the time and effort involved in entering a contract. I assume that each agent is endowed with one unit of utility that he can spend on entering contracts, so an agent can enter at most $1 + \frac{1}{\delta}$ contracts. This guarantees that the mass of agents present in each round is finite.

To capture the idea of a trading environment in which agents cannot observe previous transactions of their counterparties, I add the following informational assumptions:

1. An agent does not know in what round he arrived to trade. He only knows in how many rounds he was present.

2. Agents do not observe the history of other agents. In particular, an agent cannot observe contracts that other pairs of agents have already entered.

3. Agents cannot observe whether and how much other pairs of agents have stored through third parties.[20]

I also assume that

4. Project's assets cannot be posted as collateral. In other words, the right to terminate an agent's project cannot be promised exclusively.

[20] Thus, storage through a third party is not equivalent to public storage that is observable to all.

4.2 Equilibrium

I analyze the trading process above as an extensive-form game with imperfect information. I focus on symmetric (perfect Bayesian) equilibria in pure strategies.[21] Given the first assumption, it is natural to assume that the same contract is entered in every round. I will refer to this contract as the *equilibrium contract*. In addition, Lemma 5 in the appendix shows that without loss of generality, we can restrict attention to equilibria whose outcome is that every agent enters the equilibrium contract exactly once.

Consider such an equilibrium. Since an agent who is matched does not know previous transactions of his counterparty, he needs to form beliefs. The only beliefs consistent with the equilibrium path are that "my counterparty has just arrived to trade." This is because "all agents who appeared in previous rounds must have entered the equilibrium contract and left." Consequently, when entering a bilateral contract, an agent need not worry about contracts that his counterparty might have entered in the past, but he may need to worry about contracts that his counterparty may enter in the future. In particular, if the cost δ of being matched is small enough and the equilibrium contract does not require too much collateral, an agent can stay for additional rounds and enter the equilibrium contract with multiple counterparties. To rule out such a deviation, the equilibrium contract must be

[21] The outcome of the game is the set of contracts entered, the amount that each agent invests, and the amount that each agent delivers. (Contracts that were offered but not entered are not included because they do not affect payoffs.) The payoff for each agent is his utility. In particular, consider a type-i agent who has entered the sequence of contracts $h \equiv (\psi^1, \psi^2, \ldots, \psi^n)$, where $\psi^j = (a_1^j, a_2^j, b_1^j, b_2^j, I_1^j, I_2^j)$. Suppose that he invests $\widehat{I}_i$ and delivers a total amount $\widehat{b}_i$, and that his $j's$ counterparty delivers $\widehat{b}_{-i}^j$. The agent's utility is $U_i(\widehat{I}_i, \widehat{b}_i \mid \sum_{j=1}^n \widehat{\psi}^j)$, where $\widehat{\psi}^j$ denotes the contract ψ^j in which the element b_{-i}^j is replaced with $\widehat{b}_{-i}^j$.

A strategy for a type-i agent is a pair of functions (α_i, β_i), where $\alpha_i(h)$ specifies what contract to offer (or whether to leave) and $\beta_i(h)$ specifies how much to invest and how much to deliver. A belief function $\mu_i(h)$ specifies a distribution function on sequences of contracts that each agent of type $-i$ present in a given round has already entered.

Denote by S the strategy space, and by $V^i(\sigma_i, \sigma_{-i} \mid h, \mu_i)$ the expected payoff for a type-i agent if he follows strategy σ_i and all the agents of the other type follow the strategy σ_{-i}. The profile of strategies $\sigma^* = (\sigma_i^*)_{i=1,2}$ and the system of beliefs $\mu^* = (\mu_i^*)_{i=1,2}$ are a perfect Bayesian equilibrium if for every type i and for every h, the following holds

$$V^i(\sigma_i^*, \sigma^*_{-i} \mid h, \mu_i^*) \geq V^i(\widehat{\sigma}, \sigma_{-i}^* \mid h, \mu_i^*) \quad \text{for every} \quad \widehat{\sigma} \in S.$$

In addition, the beliefs should be consistent with the equilibrium path (i.e., if possible, they should be derived using Bayes' rule).

such that each agent would find it optimal to enter it only once, rather than many times.

As in the previous section, we can assume without loss of generality that an agent who plans to deviate by entering more than one contract will subsequently default on all of them. Such an agent will invest nothing in his project and enter as many contracts as he can. Therefore, the equilibrium contract $\psi = (a_1, a_2, b_1, b_2, I_1, I_2)$ must satisfy incentive constraint IC-1', as well as the following incentive constraint

$$U_i(\psi) \geq \overline{U}_i(\widehat{n}_i\psi) - (\widehat{n}_i - 1)\delta \quad \text{for } i = 1, 2, \tag{IC-2}$$

where $\widehat{n}_i\psi$ denotes the aggregate contract, i.e., the contract $(\widehat{n}_i a_1, \widehat{n}_i a_2, \widehat{n}_i b_1, \widehat{n}_i b_2, \widehat{n}_i I_1, \widehat{n}_i I_2)$, and $\widehat{n}_i$ is the largest integer smaller than or equal to $\min(\frac{1}{a_i}, 1 + \frac{1}{\delta})$.

To solve for the most efficient equilibrium, we need to find a feasible contract $\psi \in \Psi$ that maximizes the sum of agents' utilities subject to constraints IC-1' and IC-2. A solution to this optimization problem will be referred to as *third best.*

To avoid some technical problems that may arise, I drop the restriction that n_i be an integer.[22] As in the previous section, the (unique) optimal contract is symmetric and satisfies equations (1) and (2). Therefore, we can express both a and b in terms of I. If the second-best contract satisfies the additional incentive constraint, i.e., constraint IC-2, the third-best contract equals the second-best; otherwise, the amount I can be found from constraint IC-2.

Proposition 2 *If either $R \geq 1+\frac{1}{2}\varepsilon$ and $\delta \geq \frac{\varepsilon}{2R-\varepsilon}$ or $R < 1+\frac{1}{2}\varepsilon$ and $\delta \geq \frac{R-1}{2+\frac{1}{2}\varepsilon-R}$, the third-best contract equals the second-best. Otherwise, it is (uniquely) given by $a_1 = a_2 = 1 - I$,*

[22] A microeconomics foundation for this is as follows: Assume that instead of one economy, there is an infinite number of economies corresponding to the interval $(0, 1]$, and that agents in economy $\mu \in (0, 1]$ have an initial endowment of μ. The economy to which each agent belongs and his endowment are private information. In the first round agents must trade in their original economy, but afterward agents can switch back and forth among the different economies. The only restriction is that an agent who has an endowment e can trade in economy v only if $v \leq e$ (i.e., an agent can say that he has less than what he has, but he cannot say that he has more). The cost of being matched in economy μ is scaled to be $\mu\delta$. Then if $\mu\psi$ is the equilibrium contract in economy μ, it is possible to enter it n times, where n is not restricted to be an integer.
If n is restricted to be an integer, an optimal contract might not exist because the set of feasible contracts that satisfy constraint IC-2 might be open.

$b_1 = b_2 = (2+\varepsilon)I-2$ *and* $I_1 = I_2 = I$ *where* $I = \frac{1}{4(R-1)}\left(2(R-2)-\rho+\sqrt{8\rho+(2R-\rho)^2}\right)$ *and* $\rho = \varepsilon - 2\delta$.

Proposition 3 *There exists an equilibrium whose outcome is that every agent enters the third-best contract, leaves after one round, and does not default. This equilibrium cannot be Pareto dominated by any other symmetric equilibrium in pure strategies, in which the same contract is entered in every round.*

4.3 The role of collateral

The next proposition shows that when markets become very liquid, collateral becomes essential for trade.

Proposition 4 *If* $\psi = (a, b, I)$ *is an equilibrium contract that specifies no collateral (i.e.,* $a = 0$*), then when markets become liquid (i.e.,* δ *approaches zero), the amount* b *that an agent can promise credibly out of his project's cash flows goes to zero.*

The proof is as follows: It follows from equations (1) and (2) that constraint IC-2 can be rewritten as follows:

$$b \leq \frac{2(R-1)I}{n} + 2\delta - \frac{2\delta}{n}, \tag{6}$$

where $n = \min(\frac{1}{a}, 1+\frac{1}{\delta})$. In the limit case, when there is no collateral ($a = 0$) and no cost of entering contracts ($\delta \longrightarrow 0$), n approaches infinity and the right-hand side of constraint (6) goes to zero. The intuition is simple: When an agent can enter multiple contracts, either because of low cost of being matched, or low margin requirements, his future income R has little value in backing his promises because it is "shared" among many counterparties.

To illustrate the role of collateral, consider the case in which $\delta = 0$ (i.e., markets are very liquid), and denote $k = a$ and $x = a + b$, as in subsection 3.3. Constraint (6) then becomes

$$x \leq k + \frac{2(R-1)I}{(1/k)}.$$

It shows that collateral requirements can increase the amount of cash that agents can promise (credibly) for two reasons: First, an agent cannot default on the amount of cash he posted as collateral (first term). Second, collateral requirements limit the number of contracts he can enter, thereby making his future income valuable in backing his promises (second term). Consequently, agents in this model can promise more than the amount of cash they put up as collateral.

With the interpretation that R represents intangible capital (e.g., reputation, or future profits), the next proposition shows that the level of collateral, as well as the ratio of collateral to the amount promised, decreases when an agent has more intangible capital and increases when markets become more liquid.

Proposition 5 *The optimal level of collateral, as well as the ratio of collateral to the amount promised, increases when markets become more liquid (i.e., δ decreases) or when agents have less intangible capital (i.e., R decreases).*

5 An equilibrium with an exchange

5.1 The role of an exchange

I now introduce an exchange. The exchange here has a very minimal role: It only sets a limit on the numbers of contacts that agents can report (voluntarily). More formally, before trading begins, the exchange sets a position limit $L \in \{1, 2, \ldots\}$. The trading game is as before, but now when an agent offers a contract, he also needs to specify whether he wants to report it to the exchange. The contract is entered only if both agents agree on whether to report. The only restriction that the exchange imposes is that an individual agent can report at most L contracts. Thus, a pair of agents can report a contract only if both agents have entered less than L contracts.[23]

[23] In principle, agents can report even if they don't enter a contract, or if they enter a contract different from the equilibrium contract. Doing so will be suboptimal, however.

Proposition 6 *If the exchange sets a position limit $L = 1$, there is an equilibrium in which every agent enters the second-best contract and reports it to the exchange.*

The idea behind the proof is simple: First, since all agents choose to report their contracts, an agent cannot gain by unilaterally offering not to report. Second, since an agent can report at most one contract, he cannot gain by staying for more than one round.

Since the most efficient outcome without an exchange is the third best, and the exchange can implement the second best, the exchange can improve welfare if and only if the third best cannot achieve the second best. It follows from Proposition 2 that

Proposition 7 *An exchange can improve welfare if either $R \geq 1 + \frac{1}{2}\varepsilon$ and $\delta < \frac{\varepsilon}{2R-\varepsilon}$ or $R < 1 + \frac{1}{2}\varepsilon$ and $\delta < \frac{R-1}{2+\frac{1}{2}\varepsilon-R}$.*

The benefit from an exchange is $U_i(\psi_{sb}) - U_i(\psi_{tb})$ where ψ_{sb} denotes the second-best contract and ψ_{tb} denotes the third-best contract. The next proposition says that this benefit increases when agents have more intangible capital (i.e., R increases) and/or when markets become more liquid (i.e., δ decreases).

Proposition 8 *The benefit from an exchange increases when markets become more liquid (i.e., δ decreases) or when agents have more intangible capital (i.e., R increases).*

5.2 Pair-proofness and non-binding position limits

One way to test whether the equilibrium in Proposition 6 is robust is to ask the following question: "Suppose there was some small cost f involved in reporting a contract to the exchange, could a pair of agents gain by not reporting?" To answer this question we first need to refine the equilibrium concept. A formal definition is in the appendix. An informal definition is as follows:

Definition 1 *An equilibrium is pair-proof if there does not exist a bilateral deviation that is both beneficial and self-enforcing.*

The idea is as follows: When a pair of agents jointly deviates, they need to agree on the contract they enter and whether to report it, but they also need to coordinate their moves afterward (e.g., agree that none of them stays for additional rounds). The joint deviation is *self-enforcing* if neither agent wants to undo it by subsequent unilateral deviations. The deviation is *beneficial* if both agents gain (at least one agent gains strictly). It is assumed that when calculating utilities, both agents take the equilibrium strategies of all other agents as given. In addition, they do not revise their beliefs (regarding other agents as well as regarding one another). In particular, an agent does not automatically assume that his counterparty is a "bad type" (say, he has already entered many contracts) if he offers to deviate.[24]

Note that the fact that I chose a very simple bargaining process of simultaneous offers does not weaken the results. For example, if I changed the bargaining process to "one agent offers a contract, and the other agent can either accept or reject," I could easily rule out potential deviations (and obtain an equilibrium) by defining out-of-equilibrium beliefs that say that "if my counterparty offers a contract different from the equilibrium contract (or if he offers not to report the contract), I believe that he has already entered many contracts and he is going to default." The definition of pair-proofness restricts the set of sensible beliefs by requiring that an agent does not update his beliefs after his counterparty has made an out-of-equilibrium offer.[25]

Is the equilibrium from Proposition 6 pair-proof? The answer is not necessarily. The reason is that by setting limits too low (e.g., $L = 1$), the exchange may permit a pair of agents to deviate without permitting either agent to undo the joint deviation by subsequent unilateral deviations. The next example illustrates this.

Example 1 Suppose that $R > 1 + \frac{1}{2}\varepsilon$, so the second-best contract does not involve collat-

[24] In some sense pair-proofness is a special case of the coalition proof notion a la Bernheim, Peleg, and Whinston (1987) because it deals with coalitions of two agents. As far as I know, however, the existing literature does not define the notion of coalition proofness for extensive-form games with imperfect information. In such games one may want to specify whether and how agents who deviate update their beliefs.

[25] With the restriction that new beliefs must be consistent with the equilibrium path, the results below would hold even if I allowed a deviating pair to change their beliefs.

eral. In particular, assume that $R = 1.6$ and $\varepsilon = 0.3$. Suppose, that $L = 1$, and that when an agent reports a contract to the exchange, he incurs a small cost of f units of utility. To save f, a pair of agents can deviate by entering the second-best contract without reporting it to the exchange. They can also agree not to enter additional contracts afterward. Obviously, this deviation is beneficial to both agents, because they save the cost of reporting. Is such a deviation self-enforcing? The answer is yes. While it is true that given the equilibrium strategies of all other agents, the two agents cannot enter additional contracts without reporting, it is also true that if they do not report the current contract, each one of them can then enter one additional contract and report it to the exchange. An agent who does so (i.e., enters a total of two contracts - one reported and one that is not) and subsequently defaults can obtain a utility of $1 + \frac{1}{2}(2\varepsilon) - f - \delta = 1.3 - f - \delta$. But this is less than R, so the agent cannot gain by undoing the joint deviation. Since the deviation is both beneficial and self-enforcing, the equilibrium is not pair-proof.

Can an exchange implement the second-best outcome through an equilibrium that is pair-proof? The answer is yes. To do so, the exchange may need to set position limits that are non-binding in equilibrium. For example, it may need to set $L = 4$, even though every agent enters only one contract in equilibrium. The reason is that the exchange must permit each member of any deviating pair enough latitude to cheat on his partner, so that the initial joint deviation is unprofitable. The following example illustrates this.

Example 2 Go back to Example 1 and suppose the exchange sets $L = 4$. Now after a pair of agents enters a contract without reporting, each one of them can enter four additional contracts that are reported to the exchange, for a total of five contracts. Therefore, an agent who undoes the joint deviation can obtain a utility of $1 + \frac{1}{2}(5\varepsilon) - 4f - 4\delta = 1.75 - 4f - 4\delta$. When f and δ are small enough, this is more than R. This means that the original deviation is not self-enforcing. Also note that while a position limit of $L = 4$ rules out joint deviations, it also rules out unilateral deviations in which an agent enters too many contracts and subsequently defaults. The reason is that if an agent enters four contracts and defaults, he can obtain $1 + \frac{1}{2}(4\varepsilon) - 4f = 1.6 - 4f$, but this is less than the utility of $1.6 - f$ that he obtains if he enters only one contract and reports it to the exchange.

In general, to implement an equilibrium that is pair-proof, the exchange needs to pay attention to two things: First, it needs to make sure that each agent would rather enter the second-best contract only once (and report it) rather than L times (and report all). That is,

$$U(\psi_{sb}) \geq \overline{U}(L\psi_{sb}) - (L-1)(\delta + f). \tag{7}$$

Second, the exchange needs to make sure that a bilateral deviation in which a pair of agents enters the second-best contract without reporting is not self-enforcing. In particular, each agent should gain if he cheats on his counterparty by entering L additional contracts that he reports. That is,

$$U(\psi_{sb}) < \overline{U}(\psi_{sb} + L\psi_{sb}) - L(\delta + f). \tag{8}$$

The next proposition shows that if f is low enough, the two conditions above are not only necessary but also sufficient.

Proposition 9 *If f is small enough, an equilibrium in which every agent enters the second-best contract and reports it to the exchange is pair-proof if and only if the position limit L satisfies conditions (7) and (8).*

Interestingly, the exchange in this paper is not a "bulletin board." In other words, in some cases it should not make reported trades public. The logic is as follows: When position limits must be non-binding in equilibrium (as is the case in the examples above), an agent who has already reported a contract but has not exhausted his limit should be allowed to enter additional contracts even though he is going to default. Since no agent would enter a contract knowing that his counterparty is going to default, the exchange should not reveal information about the exact number of contracts an agent has entered.

Proposition 10 *When position limits are non-binding in equilibrium, the exchange should not reveal the exact number of contracts an agent has already reported. It should reveal only whether the limit was reached or not.*

Remark 1 One can show that the equilibrium from Proposition 3 is pair-proof. This is because ψ_{sb} solves the problem $\max_{\psi \in \Psi} \sum_{i=1}^{2} U_i(\psi)$ subject to $U_i(\psi) \geq \overline{U}_i(\psi + n_i \psi_{tb}) - n_i \delta$ for $i = 1, 2$ where $n_i = \min(\frac{1-a_i}{a_{tb}}, \frac{1}{\delta})$.

6 Conclusion

I developed a theory of an exchange showing that it can increase welfare even if its only role is to limit the number of contracts that agents can report to it *voluntarily.* I also illustrated the role of collateral in enforcing exclusivity and showed how it can make other types of assets valuable in backing promises even though these assets cannot be pledged exclusively. While putting up cash as collateral may have an economic cost (e.g., in my model, agents forgo investing in their positive NPV projects), collateral may be essential when agents cannot commit to enter exclusive contracts and punishments for default are finite. One can think of the number of contracts agents are allowed to report to the exchange as a special form of collateral created by the exchange, and the exchange can be viewed as a mechanism that provides agents with "cheap" collateral. In my setting, since an agent can make his future income worthless by his hidden action (he can choose not to invest), the exchange may also require that agents put up cash as collateral, so that they will have a future income, which, in turn, will induce them to pay what they promised.

I showed that an equilibrium in which agents report all their trades to the exchange is immune to self-enforcing deviations by pairs of agents and that, to implement such an equilibrium, the exchange may need to set position limits that are non-binding in equilibrium. The idea is that to rule out bilateral deviations, the exchange needs to permit each member of any deviating pair enough latitude to cheat on his partner, so that the initial joint deviation is unprofitable. I also showed that the exchange is not a "bulletin board."

In other words, it should not reveal the exact number of contracts an agent has entered.

I showed that the benefits from an exchange increase when markets become more liquid or when agents have more intangible capital. The theory is consistent with the evolution of the over-the-counter market for interest rate swaps. Once this market became liquid, the London Clearing House started clearing its products without becoming a broker or providing a new trading system.

Appendix

Proof of Proposition 1: Denote the second-best contract that is given in Proposition 1 by $\psi_{sb} = (a_{sb}, b_{sb}, I_{sb})$. We need to show that ψ_{sb} is an optimal solution to the following problem (referred to as P_1):

$$\max_{\psi \in \Psi} \sum_{i=1}^{2} U_i(\psi)$$
$$\text{s.t.} \quad U_i(\psi) \quad \geq \quad \overline{U}_i(\psi) \quad \text{for} \quad i = 1, 2 \tag{IC-1'}$$

Consider problem P_1' that is defined as follows:

$$\max_{\psi \in \Psi'} \sum_{i=1}^{2} U_i(\psi)$$
$$\text{s.t.} \quad \sum_{i=1}^{2} U_i(\psi) \quad \geq \quad \sum_{i=1}^{2} \overline{U}_i(\psi) \tag{9}$$

where $\Psi' = \{\psi = (a_1, a_2, b_1, b_2, I_1, I_2) : \sum_{i=1}^{2} a_i + \sum_{i=1}^{2} I_i \leq 2, \sum_{i=1}^{2} I_i \geq 0$, and $\sum_{i=1}^{2} a_i \geq 0\}$.

It is easy to verify that $\Psi \subset \Psi'$, and that every $\psi \in \Psi$ that satisfies constraint IC-1' satisfies also constraint 9. In addition $\psi_{sb} \in \Psi$. Therefore, to show that ψ_{sb} is optimal for P_1, it is enough to show that it is an optimal solution for P_1'.

Lemma 1 *There exists an optimal solution to P_1' that satisfies the following properties:*

1. $s + \sum_{i=1}^{2} I_i = 2$.

2. Either $s + b_{-i} \geq \varepsilon I_i$ for $i \in \{1, 2\}$, or $s + b_{-i} < \varepsilon I_i$ for $i \in \{1, 2\}$.

3. $a_1 = a_2$, $b_1 = b_2$ and $I_1 = I_2$.

Proof. After simple algebra, we obtain $\sum_{i=1}^{2} \overline{U}_i(\psi) = 1 + \frac{1}{2}(b_1 + b_2)$ and

$$\begin{aligned}\sum_{i=1}^{2} U_i(\psi) \quad = \quad & 2 - 2(I_1 + I_2) + \frac{1}{2}(R + \varepsilon)(I_1 + I_2) \\ & + \frac{1}{2}(R - \varepsilon) I_1 \beta_1(\widehat{I}_1, \widehat{b}_1 \,|\, \psi) + \frac{1}{2}(R - \varepsilon) I_2 \beta_2(\widehat{I}_2, \widehat{b}_2 \,|\, \psi)\end{aligned}$$

where $\beta_i(\widehat{I}_i, \widehat{b}_i \mid \psi) \begin{cases} 1 & \text{if } s + b_{-i} \geq \varepsilon \widehat{I}_i \\ 0 & \text{otherwise.} \end{cases}$ Suppose that $\psi = (a_1, a_2, b_1, b_2, I_1, I_2)$ is an optimal solution to P_1'.

1. If $s + \sum_{i=1}^{2} I_i < 2$, one can increase s by increasing either a_1 and/or a_2. The contract remains in Ψ', does not violate the incentive constraint, and does not decrease the value of the objective function.

2. Suppose by contradiction that there exists $i \in \{1, 2\}$ such that $s + b_{-i} \geq \varepsilon I_i$ but $s + b_i < \varepsilon I_{-i}$. Consider the contract $\psi' = (a_1', a_2', b_1', b_2', I_1', I_2')$ that is defined as follows: $a_1' = a_1$, $a_2' = a_2$, $b_i' = b_i - \Delta$, $b_{-i}' = b_{-i} + \Delta$, $I_i' = I_i + \frac{\Delta}{\varepsilon}$ and $I_{-i}' = I_{-i} - \frac{\Delta}{\varepsilon}$. Let $s' = a_1' + a_2'$. Note that $s' + b_{-i}' \geq \varepsilon I_i'$, $s' + b_i' < \varepsilon I_{-i}'$, $I_1' + I_2' = I_1 + I_2$, and $I_i' > I_i$. Thus, $\sum_{i=1}^{2} U_i(\psi') > \sum_{i=1}^{2} U_i(\psi)$. In addition, $\sum_{i=1}^{2} \overline{U}_i(\psi') = \sum_{i=1}^{2} \overline{U}_i(\psi)$, so the incentive constraint still holds. Finally, $\psi' \in \Psi'$. This contradicts the fact that the contract ψ is optimal.

3. Consider the contract $\psi' = (a_1', a_2', b_1', b_2', I_1', I_2')$ that is defined as follows: $a_1' = a_2' = \frac{a_1 + a_2}{2}$, $b_1' = b_2' = \frac{b_1 + b_2}{2}$ and $I_1' = I_2' = \frac{I_1 + I_2}{2}$. Let $s' = a_1' + a_2'$. First, since $\psi \in \Psi$, it follows that $\psi' \in \Psi'$. Second, if $s + b_2 \geq \varepsilon I_1$ and $s + b_1 \geq \varepsilon I_2$, it follows that $s' + \frac{b_1 + b_2}{2} \geq \varepsilon(\frac{I_1 + I_2}{2})$; if $s + b_2 < \varepsilon I_1$ and $s + b_1 < \varepsilon I_2$, it follows that $s + (\frac{b_1 + b_2}{2}) < \varepsilon(\frac{I_1 + I_2}{2})$. Therefore, because $I_1' + I_2' = I_1 + I_2$, there is no change in the value of the objective function. Finally, since $b_1' + b_2' = b_1 + b_2$, it follows that $\sum_{i=1}^{2} \overline{U}_i(\psi') = \sum_{i=1}^{2} \overline{U}_i(\psi)$, so the incentive constraint still holds. Therefore, ψ' is an optimal solution. $\blacksquare$

Lemma 1 implies that when solving P_1', we can restrict attention to symmetric contracts of the form (a, b, I), where $I = 1 - a$ and $a \geq 0$.

Lemma 2 *If P_1' has an optimal solution (a, b, I) that satisfies $s + b \geq \varepsilon I$, it also has an optimal solution that satisfies $s + b = \varepsilon I$.*

Proof. Suppose $s + b > \varepsilon I$. If $s < \varepsilon I$, one can decrease b so that $s + b = \varepsilon I$. If $s > \varepsilon I$, one can choose $b = 0$ and decrease s so that $s = \varepsilon I$. In both cases, the contract remains feasible, the value of the objective function does not change, and no constraint is violated. $\blacksquare$

Lemma 3 *ψ_{sb} is an optimal solution to P_1'.*

Proof. By the previous lemmas, we can compare between two cases:

Case 1: $s+b<\varepsilon I$: In this case it is optimal to choose $a=b=0$ and $I=1$. The utility for each agent is $U_i(\psi)=\frac{1}{2}(R+\varepsilon)$.

Case 2: $s+b=\varepsilon I$: Since $s=2a=2(1-I)$, it follows that $b=(2+\varepsilon)I-2$. In addition, $U_i(\psi)=1+(R-1)I$ and $\overline{U}_i(\psi)=1+\frac{1}{2}b$. Thus, to find an optimal solution to P_1', we need to solve $\max_{\psi\in\Psi'} I$ subject to

$$1+(R-1)I\geq 1+\frac{1}{2}[(2+\varepsilon)I-2] \tag{10}$$

Since $a\geq 0$, we must have $I\leq 1$. If we substitute $I=1$ in constraint 10 and simplify, we obtain $R\geq 1+\frac{1}{2}\varepsilon$. Therefore, when $R\geq 1+\frac{1}{2}\varepsilon$, it is optimal to choose $I=1$. Otherwise, we obtain that $I\leq\frac{1}{2+\frac{1}{2}\varepsilon-R}$. Therefore, the optimal solution is $I^*=\min(1,\frac{1}{2+\frac{1}{2}\varepsilon-R})=\begin{cases}1 & if\ R\geq 1+\frac{1}{2}\varepsilon\\ \frac{1}{2+\frac{1}{2}\varepsilon-R} & otherwise,\end{cases}$ and the utility for each agent is $U_i(\psi)=1+(R-1)I^*$.

To choose between the two cases we need to compare utilities. When $I^*=1$, it follows (since $R>\varepsilon$) that $1+(R-1)I^*=R>\frac{1}{2}(R+\varepsilon)$, so the second case is optimal. Suppose now that $I^*<1$. Since $2+\frac{1}{2}\varepsilon-R<1+\varepsilon$, it follows that $I^*\geq\frac{1}{1+\varepsilon}$ and $1+(R-1)I^*\geq 1+(R-1)\frac{1}{1+\varepsilon}=\frac{R+\varepsilon}{1+\varepsilon}$. The assumption $\varepsilon<1$ guarantees that this is more than $\frac{1}{2}(R+\varepsilon)$.

Note that a necessary and sufficient condition for case 2 to be optimal is $1+(R-1)\frac{1}{2+\frac{1}{2}\varepsilon-R}\geq\frac{1}{2}(R+\varepsilon)$, which is equivalent to $\frac{1}{2}\frac{4-2\varepsilon-4R+R\varepsilon+2R^2-\varepsilon^2}{4+\varepsilon-2R}\geq 0$. Since $\dot{R}<1+\frac{1}{2}\varepsilon$ implies that $4+\varepsilon-2R>0$, the condition is equivalent to $4-2\varepsilon-4R+R\varepsilon+2R^2-\varepsilon^2\geq 0$.

■

It follows that ψ is an optimal solution to P_1. To show uniqueness, denote $s_{sb}=2a_{sb}$, and consider another contract $(a_1,a_2,b_1,b_2,I_1,I_2)$. To achieve the optimal value for the objective, the contract must satisfy $s+b_{-i}\geq\varepsilon I_i$ for $i=1,2$, and $I_1+I_2=2I^*$. In addition, feasibility implies that $s\leq 2-(I_1+I_2)=2-2I^*=s_{sb}$. Suppose that $I_1=I^*-\Delta$ and $I_2=I^*+\Delta$, where $\Delta>0$. To satisfy $s+b_1\geq\varepsilon I_2$ for $i\in\{1,2\}$, we must have $b_1\geq\varepsilon I_2-s\geq\varepsilon I^*+\varepsilon\Delta-s_{sb}=b_{sb}+\varepsilon\Delta$. But then the incentive constraint for agent 1 is violated. ■

Proof of Proposition 2:

Lemma 4 *The third-best contract is symmetric and satisfies $a = 1-I$ and $b = (2+\varepsilon)I - 2$.*

Proof. Follow the same logic as in Proposition 1, but add the constraint $\sum_{i=1}^{2} U_i(\psi) \geq \sum_{i=1}^{2} \overline{U}_i(\widehat{n}\psi) - 2(\widehat{n}-1)\delta$ to problem P_1', where $\widehat{n} = \min(\frac{1}{\min(a_1,a_2)}, 1+\frac{1}{\delta})$. Also, in step 3 of Lemma 1, use the fact that $\frac{1}{\min(a_1,a_2)} \geq \frac{1}{\frac{a_1+a_2}{2}}$. ■

Denote the second-best contract by $\psi_{sb} = (a_{sb}, b_{sb}, I_{sb})$. The third-best contract equals the second-best contract if and only if ψ_{sb} satisfies constraint IC-2.

Assume first that $R \geq 1 + \frac{1}{2}\varepsilon$. Substituting $\psi_{sb} = (0,0,1)$ in constraint IC-2, we obtain $R \geq 1 + \frac{1}{2}(1+\frac{1}{\delta})\varepsilon - (1+\frac{1}{\delta}-1)\delta$. This is equivalent to $R \geq \frac{1}{2}(\frac{1}{\delta}+1)\varepsilon$, which is equivalent to $\delta \geq \frac{\varepsilon}{2R-\varepsilon}$.

Assume now that $R < 1 + \frac{1}{2}\varepsilon$. Then $U_i(\psi_{sb}) = \overline{U}_i(\psi_{sb})$. In addition, $a_{sb} < 1$, so $\min(\frac{1}{a_{sb}}, 1+\frac{1}{\delta}) > 1$. Now note that

$$\begin{aligned}
& \overline{U}_i(n_i\psi_{sb}) - (n_i-1)\delta \\
= \; & 1 + \frac{1}{2}n_i b_{sb} - (n_i-1)\delta \\
= \; & \overline{U}_i(\psi_{sb}) + (n_i-1)(\frac{1}{2}b_{sb} - \delta) \\
= \; & U_i(\psi_{sb}) + (n_i-1)(\frac{1}{2}b_{sb} - \delta).
\end{aligned}$$

Thus, it follows that constraint IC-2 can be satisfied if and only if $\frac{1}{2}b_{sb} - \delta \leq 0$. This condition is equivalent to $\delta \geq \frac{1}{2}b_{sb} = \frac{1}{2}[(2+\varepsilon)I_{sb} - 2] = \frac{1}{2}(2+\varepsilon)\frac{1}{2+\frac{1}{2}\varepsilon - R} - 1 = \frac{R-1}{2+\frac{1}{2}\varepsilon-R}$.

Suppose now that ψ_{sb} does not satisfy constraint IC-2. The optimal level of investment I can be found from constraint IC-2. We can assume without loss of generality that $\frac{1}{a} \leq 1+\frac{1}{\delta}$, because otherwise one could decrease a, thereby increasing the value of the objective function without violating the constraints. Therefore, since $b = (2+\varepsilon)I - 2$ and $n = \min(\frac{1}{a}, 1+\frac{1}{\delta}) = \frac{1}{a}$, constraint IC-2 can be written as follows:

$$1 + (R-1)I \geq 1 + \frac{1}{2}\frac{1}{a}[(2+\varepsilon)I - 2] - (\frac{1}{a} - 1)\delta$$

After substituting $a = 1 - I$ and doing some simple algebra, we obtain:

$$2(R-1)I^2 + (4 + \varepsilon - 2\delta - 2R)I - 2 \leq 0$$

Solving for I, we obtain

$$\frac{1}{4\,(R-1)}\left(2(R-2) - \rho - \sqrt{8\rho + (2R-\rho)^2}\right) \leq I \leq \frac{1}{4\,(R-1)}\left(2(R-2) - \rho + \sqrt{8\rho + (2R-\rho)^2}\right)$$

where $\rho = \varepsilon - 2\delta$. Therefore, it is optimal to choose $I = \frac{1}{4(R-1)}\left(2(R-2) - \rho + \sqrt{8\rho + (2R-\rho)^2}\right)$
■

Proof of Proposition 3: For $h = (\psi^1, \psi^2, \ldots \psi^n)$, denote by $F_i(h, n')$ the highest utility that an agent who has already entered the sequence of contracts h can obtain if he enters the contract ψ_{tb} with n' additional counterparties, given that none of his counterparties defaults, and denote by $N_i(h) = \{n' \geq 0 : \sum_{j=1}^{n} a_i^j + n' a_{tb,i} \leq 1,\ (n + n' - 1)\delta \leq 1\}$ the set of feasible choices of n'. Let $n_i^*(h) = \arg\max_{n' \in N_i(h)} F_i(h, n')$ and let $(I_i'(h), b_i'(h)) = \arg\max_{(\widehat{I}_i, \widehat{b}_i) \in B_i(\sum_{i=1}^{n} \psi^i)} U_i(\widehat{I}_i, \widehat{b}_i \mid \sum_{i=1}^{n} \psi^i)$.

Consider the following strategy for a type i agent: In the first round offer ψ_{sb}. Then given that the sequence of contracts h was entered and $n_i^*(h) = 0$, leave and choose $I_i'(h), b_i'(h)$. Otherwise, stay and offer ψ_{sb}.

One can verify that if all agents follow the strategies above, the outcome of the game is that each agent enters the third-best contract, leaves after one round, and then chooses I_i^* and b_i^*. Thus, the only beliefs consistent with the equilibrium path are that "my counterparty has just arrived for trade."

One can also verify that the strategies are optimal for each agent given the beliefs and given that his counterparties follow the equilibrium strategies. In particular, note that since all other agents offer the third-best contract, there is no point to offer a different contract, and that entering the third-best contract is preferred to autarky. Pareto dominance follows from the definition of the third-best contract and from Lemma 5 below. ■

Lemma 5 *An equilibrium in which each agent enters more than one contract cannot improve welfare.*

Proof. Consider an equilibrium in which each agent enters the contract ψ with n different counterparties, so each agent obtains a utility $U_i(n\psi) - (n-1)\delta$. As in Proposition 2, we can assume without loss of generality that ψ is symmetric and is given by (a, b, I) where a, b, I satisfy equations (1) and (2). We can also assume without loss of generality that $\frac{1}{a} \leq 1 + \frac{1}{\delta}$, because otherwise one could decrease a, thereby increasing the value of the objective function without violating the constraints. Suppose that $U_i(n\psi) - (n-1)\delta > U_i(\psi^*)$ where $\psi^* = (a^*, b^*, I^*)$ denotes the third-best contract. Consider the contract $\widehat{\psi} = n\psi = (na, nb, nI)$. It is easy to verify that $\widehat{\psi} \in \Psi$. In addition, if it is possible to enter $\widehat{\psi}$ with n' counterparties, it is possible to enter ψ with nn' counterparties. This is because $n'(na) \leq 1$ implies that $nn' \leq \frac{1}{a} \leq 1 + \frac{1}{\delta}$. Since ψ is entered in equilibrium n times, it must be the case that $\widehat{\psi}$ satisfies the two incentive constraints IC-1' and IC-2. But then the fact that $U_i(\widehat{\psi}) = U_i(n\psi) > U_i(\psi^*)$ contradicts the optimality of ψ^*. ∎

Proof of Proposition 5: When the third best achieves the second best, $I = \min(1, \frac{1}{2+\frac{1}{2}\varepsilon - R})$, and it is easy to see that $\frac{\partial I}{\partial R} > 0$. Assume now that the third best does not achieve the second best. It follows from the proof of Proposition 2 that I solves the equation $H(I, R, \delta) = 0$ where $H(I, R, \delta) = 2(R-1)I^2 + (4 + \varepsilon - 2\delta - 2R)I - 2$, so by the envelope theorem, $\frac{\partial I}{\partial R} = -\frac{\partial H/\partial R}{\partial H/\partial I}$ and $\frac{\partial I}{\partial \delta} = -\frac{\partial H/\partial \delta}{\partial H/\partial I}$. Since $b \leq \varepsilon$ and $\frac{1}{2}b - \delta > 0$, it must be the case that $\varepsilon > 2\delta$. In addition, it follows from the proof of Lemma 3 that $I \geq \frac{1}{1+\varepsilon}$, and since $\varepsilon < 1$, it follows that $I \geq \frac{1}{2}$. Therefore, $\frac{\partial H}{\partial I} = 4(R-1)I + (4 + \varepsilon - 2\delta - 2R) \geq 4(R-1)\frac{1}{2} + (4 + \varepsilon - 2\delta - 2R) = \varepsilon - 2\delta + 2 > 0$, $\frac{\partial H}{\partial R} = 2I^2 - 2I = 2I(I-1) < 0$ and $\frac{\partial H}{\partial \delta} = -2I < 0$. It follows that $\frac{\partial I}{\partial R} > 0$ and $\frac{\partial I}{\partial \delta} > 0$. Since $k = a = 1 - I$, it follows that $\frac{\partial k}{\partial R} < 0$ and $\frac{\partial k}{\partial \delta} < 0$. Since $b = (2+\varepsilon)I - 2$, it follows that $\frac{\partial b}{\partial R} > 0$ and $\frac{\partial b}{\partial \delta} > 0$. Finally, let $\theta = \frac{k}{x} = \frac{k}{k+b} = \frac{1}{1+\frac{b}{k}}$. Then it follows that $\frac{\partial \theta}{\partial R} < 0$ and $\frac{\partial \theta}{\partial \delta} < 0$. ∎

Proof of Proposition 8: Suppose that $\psi_{sb} \neq \psi_{tb}$. The benefit from an exchange is

$$\begin{aligned} H(R,\delta) &\equiv U_i(\psi_{sb}) - U_i(\psi_{tb}) = (R-1)I_{sb} - (R-1)I_{tb} \\ &= (R-1)\min(1, \frac{1}{2+\frac{1}{2}\varepsilon - R}) - \frac{1}{4}\left(2(R-2) - \rho + \sqrt{8\rho + (2R-\rho)^2}\right) \end{aligned}$$

where $\rho = \varepsilon - 2\delta$.

Sensitivity to δ : $\frac{\partial H}{\partial \delta} < 0$ because $\frac{\partial H}{\partial \delta} = \frac{\partial H}{\partial \rho}\frac{\partial \rho}{\partial \delta}$, $\frac{\partial H}{\partial \rho} = \frac{1}{4}[1 - \frac{4-(2R-\rho)}{\sqrt{8\rho+(2R-\rho)^2}}] > 0$, and $\frac{\partial \rho}{\partial \delta} < 0$.

Sensitivity to R :

Case 1: $R \geq 1+\frac{1}{2}\varepsilon$. In this case, $H(R,\delta) = (R-1) - \frac{1}{4}\left(2(R-2) - \rho + \sqrt{8\rho + (2R-\rho)^2}\right)$ and $\frac{\partial H}{\partial R} = 1 - \frac{1}{2}\frac{\sqrt{(8\rho+4R^2-4R\rho+\rho^2)}+2R-\rho}{\sqrt{(8\rho+4R^2-4R\rho+\rho^2)}} = \frac{\sqrt{8\rho+(2R-\rho)^2}-(2R-\rho)}{2\sqrt{(8\rho+4R^2-4R\rho+\rho^2)}} > 0$.

Case 2: $R < 1+\frac{1}{2}\varepsilon$. In this case $H(R,\delta) = (R-1)(I_{sb} - I_{tb})$ where I_{tb} is the solution to $2(R-1)I^2 + (4+\varepsilon-2\delta-2R)I - 2 = 0$ and I_{sb} is the solution to $(R-1)I - \frac{1}{2}(2+\varepsilon)I - 1 = 0$. Since $R > 1$ and $I_{sb} > I_{tb}$, to show that $\frac{\partial H}{\partial R} > 0$, it is enough to show that $\frac{\partial I_{tb}}{\partial R} > \frac{\partial I_{tb}}{\partial R}$. From the envelope theorem, $\frac{\partial I_{tb}}{\partial R} = \frac{I_{sb}}{2+\frac{1}{2}\varepsilon-R}$ and $\frac{\partial I_{tb}}{\partial R} = -\frac{2I_{tb}^2-2I_{tb}}{4(R-1)I_{tb}+(4+\varepsilon-2\delta-2R)} = \frac{I_{tb}(1-I_{tb})}{2(R-1)I_{tb}+(2+\frac{1}{2}\varepsilon-\delta-R)}$. Since $I_{tb}(1-I_{tb}) < I_{sb}$, it remains to show that $2(R-1)I_{tb} + (2 + \frac{1}{2}\varepsilon - \delta - R) > 2 + \frac{1}{2}\varepsilon - R$. This is equivalent to $2(R-1)I_{tb} - \delta > 0$ which follows from the incentive constraint and the fact that $\delta < b_{tb}$. ■

A definition of pair-proofness

Consider a profile of strategies $\sigma^* = (\sigma_1^*, \sigma_2^*)$ and a system of beliefs μ^* that are a perfect Bayesian equilibrium. Consider a pair of agents, say agent 1 and agent 2, who were matched, and suppose that agent i $(i = 1,2)$ has already entered the sequence of contracts h_i. The pair (σ^*, μ^*) and the individual histories (h_1, h_2) induce an extensive form game, referred to as $\Upsilon(\sigma^*, \mu^*, h_1, h_2)$, in which the only players are agent 1 and agent 2. Denote by $F^i(\sigma_1, \sigma_2 \,|\, \sigma^*, \mu^*, h_i)$ the utility for agent i in this game if he follows the strategy σ_i and the other agent follows the strategy σ_{3-i} (given the beliefs μ^*).

Definition 2 *A deviation for agents* 1 *and* 2 *is a pair of strategies* (σ_1, σ_2).

Denote by $M(\sigma^*)$ the set of individual histories (h_1, h_2) that are consistent with σ^* and with the fact that agents 1 and 2 were matched.

Definition 3 *A deviation (σ_1, σ_2) is* ***beneficial*** *given (σ^*, μ^*) and given $(h_1, h_2) \in M(\sigma^*)$ if it satisfies the following inequalities*

1. $F^1(\sigma_1, \sigma_2 \,|\, \sigma^*, \mu^*, h_1) > F^1(\sigma_1^*, \sigma_2^* \,|\, \sigma^*, \mu^*, h_1)$

2. $F^2(\sigma_1, \sigma_2 \,|\, \sigma^*, \mu^*, h_2) > F^2(\sigma_1^*, \sigma_2^* \,|\, \sigma^*, \mu^*, h_2)$

Definition 4 *A deviation (σ_1, σ_2) is* ***self-enforcing*** *given (σ^*, μ^*) and given $(h_1, h_2) \in M(\sigma^*)$ if the following two conditions hold:*

1. *$F^1(\sigma_1, \sigma_2 \,|\, \sigma^*, \mu^*, h) \geq F^1(\widehat{\sigma}_1, \sigma_2 | \, \sigma^*, \mu^*, h)$ for every $\widehat{\sigma}_1 \in S$ and for every h that starts with h_1.*

2. *$F^2(\sigma_1, \sigma_2 \,|\, \sigma^*, \mu^*, h) \geq F^2(\sigma_1, \widehat{\sigma}_2 | \, \sigma^*, \mu^*, h)$ for every $\widehat{\sigma}_2 \in S$ and for every h that starts with h_2.*

Definition 5 *A perfect Bayesian equilibrium (σ^*, μ^*) is* ***pair-proof*** *if there does not exist a pair of strategies (σ_1, σ_2) and histories $(h_1, h_2) \in M(\sigma^*)$ such that (σ_1, σ_2) is a deviation that is both self-enforcing and beneficial given (σ^*, μ^*) and given (h_1, h_2).*

Proof of Theorem 9. First note that if L satisfies condition (7), there is an equilibrium in which every agent enters the second-best contract and reports it to the exchange. As shown in the text, if the equilibrium is pair-proof, condition (8) needs to be satisfied as well. To show that the two conditions are sufficient, it is enough to show that the best bilateral deviation that is self-enforcing is not beneficial. Suppose that the two agents enter ψ without reporting. The deviation is self-enforcing if

$$U_i(\psi) \geq \overline{U}_i(\psi + L\psi_{sb}) - L(\delta + f) \text{ for } i = 1, 2, \tag{11}$$

and the best deviation solves: $\max_\psi \sum_{i=1}^{2} U_i(\psi)$ subject to constraint (11). As before, the optimal contract is symmetric and satisfies equations (1) and (2). Denote it by $\psi^* = (a^*, b^*, I^*)$. Inequality (8) implies that the second-best contract violates constraint (11). Therefore, $U_i(\psi^*) < U_i(\psi_{sb})$. The equilibrium is pair-proof if the deviation is not beneficial, i.e., if $U_i(\psi^*) \leq U_i(\psi_{sb}) - f$ for $i = 1, 2$. This holds when f is small enough. Finally, note that if the two agents report, the best thing is to enter ψ_{sb}. ■

References

[1] Baer H. L., V.G. France, and J.T. Moser (2001), "Opportunity Cost and Prudentiality: An Analysis of Collateral Decisions in Bilateral and Multilateral Settings," Working Paper 01-26, Federal Reserve Bank of Chicago.

[2] Benveniste, L., A. Marcus, and W. Wilhelm (1992), "What's Special About the Specialist?" *Journal of Financial Economics,* 32, 61-86.

[3] Bernheim B., B. Peleg, and M. Whinston (1987), "Coalition-Proof Nash Equilibria I: Concepts," *Journal of Economic Theory,* 42, 1-12.

[4] Bester, H. (1985), "Screening vs. Rationing in Credit Markets with Imperfect Information," *American Economic Review,* 75, 850-855.

[5] Bisin, A., and A. Rampini (2002), "Exclusive Contracts and the Institution of Bankruptcy," Working Paper no. 270, Finance Department, Northwestern University.

[6] Bizer, D., and P. DeMarzo (1992), "Sequential Banking," *Journal of Political Economy,* 100, 41-60.

[7] Brennan, M. (1986), "A Theory of Price Limits in Futures Markets," *Journal of Financial Economics,* 16, 213-233.

[8] Brusco, S., and M. Jackson (1999), "The Optimal Design of a Market," *Journal of Economic Theory,* 88, 1-39.

[9] Chou, P., M. Lin, and M. Yu (2000), "Price Limits, Margin Requirements, and Default Risk," *Journal of Futures Markets,* 20, 573-602.

[10] Diamond, D. (1984), "Financial Intermediation and Delegated Monitoring," *Review of Economic Studies,* 51, 393-414.

[11] Dubey, P., J. Geanakoplos, and W. Zame (1997), "Collateral, Default and Market Crashes," manuscript.

[12] Edwards, F. (1984), "The Clearing Association in Futures Markets: Guarantor and Regulator," *The Industrial Organization of Futures Markets: Structure and Organization*, Lexington Books, Lexington, MA.

[13] Glosten, L. (1989), "Insider Trading, Liquidity, and the Role of the Monopolist Specialist," *Journal of Business,* 62, 211-235.

[14] Glosten, L. (1994), "Is the Electronic Open Limit Order Book Inevitable?" *Journal of Finance,* 49, 1127-1161.

[15] Hagerty, K., and R. McDonald (1996), "Brokerage, Market Fragmentation, and Securities Market Regulation," *The Industrial Organization and Regulation of the Securities Industry*, edited by Andrew W. Lo, The University of Chicago Press, 35-56.

[16] Holmström, B., and J. Tirole (1998), "Private and Public Supply of Liquidity," *Journal of Political Economy,* 106, 1-40.

[17] Kahn, C., and D. Mookherjee (1998), "Competition and Incentives with Nonexclusive Contracts," *RAND Journal of Economics,* 29, 443-465.

[18] Myers, S., and R. Rajan (1998), "The Paradox of Liquidity," *Quarterly Journal of Economics,* 113, 733-771.

[19] Pagano, M., and A. Röell (1996), "Transparency and Liquidity: A Comparison of Auction and Dealer Markets with Informed Trading," *Journal of Finance,* 51, 579-611.

[20] Parlour, C.A., and U. Rajan (2001), "Competition in Loan Contracts," *American Economics Review,* 91, 1311-1328

[21] Santos, T., and J. Scheinkman (2001), "Competition Among Exchanges," *Quarterly Journal of Economics, 116,* 1027-1061.

[22] Seppi, D. (1997), "Liquidity Provision with Limit Orders and a Strategic Specialist," *The Review of Financial Studies,* 10, 103-150.

[23] Telser, L., and H. Higinbotham (1977), "Organized Futures Markets: Costs and Benefits," *Journal of Political Economy,* 85, 969-1000.

[24] Telser, L. (1981), "Margins and Futures Contracts," *Journal of Futures Markets,* 1, 225-253.

[25] Townsend, R. (1978), "Intermediation with Costly Bilateral Exchange," *Review of Economic Studies,* 55, 417-425.