The Equity Premium and the Baby Boom

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Abstract

This paper explores the quantitative impact of the Baby Boom on stock and bond returns. It constructs a neoclassical growth model with overlapping generations, in which agents make a portfolio decision over risky capital and safe bonds in zero net supply. The model has exogenous technology and population shocks that are calibrated to match long run data for the US. With agents allowed to borrow freely by shorting bonds, the model fails to match the historical equity premium by a large margin and generates only small asset market effects over a simulated Baby Boom. When agents are constrained in their ability to borrow, the model comes close to matching the historical equity premium and suggests that there will be a sharp rise in the equity premium when the Baby Boomers retire, driven by a large decline in bond returns as Baby Boomers seek to hold the riskless asset in retirement.

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1. Introduction

This paper explores the quantitative impact of the Baby Boom on stock and bond returns. It builds a neoclassical growth model with four overlapping generations (OLG), in which agents make a portfolio decision over risky capital and safe bonds that are in zero net supply. The model features two exogenous sources of uncertainty, technology shocks and population growth, both of which are calibrated to match long run data for the United States. A numerical solution is obtained and used to simulate asset returns over the Baby Boom.

The paper contrasts two polar opposites of this model. In the first version, the *unconstrained* model, agents are permitted to borrow freely by shorting the riskfree asset. In the second, the *constrained* model, they are restricted in doing so by exogenous borrowing constraints. The unconstrained model generates portfolio behavior whereby agents shift from stocks to bonds as they age. Young workers short the riskless asset and invest the proceeds in risky capital. Old workers hold a mixed portfolio of risky capital and safe bonds. In the constrained model, young workers are allowed to sell-short only a very limited amount of bonds. Because they are raising offspring, they are unwilling to forgo consumption to invest in capital and cease participating in financial markets. As a result, old workers are forced to hold almost the entire capital stock going into retirement. Which portfolio allocation pattern is more realistic? In the unconstrained model, the issuance of bonds by young workers raises the equilibrium riskfree rate to an annualized 7.06 percent on average, just below the expected return on risky capital, which is 7.12 percent per annum. The unconstrained model thus exhibits the classic equity premium or riskfree rate puzzle. In the constrained model, the borrowing constraint prevents young workers from meeting the demand for bonds generated by old workers, reducing the equilibrium riskfree rate to an annualized 4.83 percent, while the expected return on capital, pinned down by the marginal product of capital, is unchanged from the unconstrained model. The resulting risk premium of 2.29 percent per annum is close to the three percent mean equity premium that Constantinides et al. (2002) argue models with only aggregate capital (rather than leveraged equity) should match.

In terms of explaining history, the constrained model is therefore a more realistic baseline for exploring the asset market effects of the Baby Boom. It describes a world where participation by the young in financial markets is restricted by their inability to borrow against future income. In this setting, the borrowing constraints are an admittedly *ad hoc* way of accounting for the fact that human capital alone does not collateralize loans for reasons of moral hazard and adverse selection.[1] Yet stock market participation has grown in recent years to a point where 50 percent of households in the United States now hold equity, possibly the result of

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[1] The constrained model can be seen as an extension of Constantinides et al. (2002) who show that borrowing constraints generate a realistic equity premium in a partial equilibrium OLG model with exogenous production and without demographic risk. Making production endogenous is critical to examining the asset market effects of the Baby Boom, in order to capture general equilibrium effects on capital formation and asset returns.

financial innovation that is making it easier for households to borrow. Simultaneously, the equity premium is estimated to have fallen substantially over the last decade. This suggests that, going forward, the unrestricted model is perhaps not so unrealistic after all. This paper therefore also uses the unconstrained model to simulate the asset market effects of the Baby Boom. How different across both models are the asset return and equity premium effects? And how different are their implications for intergenerational welfare and the ongoing debate over social security reform? These questions are addressed below.

To briefly preview the results, the unconstrained model suggests that the asset market effects of the Baby Boom are relatively modest. It finds that the return Baby Boomers can expect on their retirement saving will be 50 basis points below returns to current retirees. This effect is driven by a rise in capital formation as the Baby Boomers save in preparation for retirement, which pulls down the real interest rate and thus returns on both stocks and bonds. It finds little evidence of a differential effect on risky capital or safe bonds, with the risk premium broadly stable over the transition. In the constrained model, the expected return on capital is still projected to decline by about 50 basis points in the years ahead, due to increased capital formation as the Baby Boomers retire. However, the effects on the bond return are forecast to be much more dramatic. This is because a small cohort of young workers is prevented by the borrowing constraint from meeting the demand for bonds generated by a large cohort of aging Baby Boomers who seek to minimize consumption risk in retirement. The constrained model suggests that this imbalance will cause bond returns to fall 613 basis points below their current level in the years ahead.[2]

In the unconstrained model, the Baby Boomers are better off in lifetime consumption terms than the smaller cohorts around them, notably their parents or children. Because this result obtains in a model without defined-benefit social security, it suggests that the welfare loss from the crowding out of private capital formation through social security may outweigh the ability of such a system to offset movements in the capital-labor ratio that disadvantage large cohorts.[3] In the constrained model, even though the Baby Boomers are worse off in lifetime

[2] The predicted 50 basis point decline in the unconstrained model matches that in other OLG models with perfect foresight and no frictions in financial markets. These models typically use projected population trends to forecast the capital-labor ratio and hence the real return on capital. Boersch-Supan and Winter (2001) generate a decline in the real return on capital for Germany of around 100 basis points and around 20 basis points for the OECD countries as a whole. The European Commission (2001) reviews policy options for pension reform based on a decline of 25 basis points over the next 50 years. In contrast, Kotlikoff et al. (2001) conclude that the US real return on capital will rise 100 basis points by 2030 and 300 basis points by 2100, based on crowding out effects on capital formation from rising tax rates.

[3] Bohn (2001) uses an OLG model with childhood, working-age and retirement to show that movements in the capital-labor ratio generally disadvantage large cohorts. His result is robust to the inclusion of a defined-benefit Social Security system, which taxes smaller cohorts relatively more heavily, provided that the pension system is not unrealistically large. This
(continued)

consumption terms than if the age distribution had remained in equilibrium, they are still better off than their parents or children. Since defined-benefit pay-as-you-go social security only redistributes wealth across adjacent cohorts, it is unclear whether such a system is *ex ante* efficient in terms of intergenerational risk sharing. This is left for future research.

Section 2 describes the OLG economy. Section 3 discusses its calibration, while Section 4 reports on the asset pricing implications of the unconstrained and constrained models once they have been solved numerically. Section 5 simulates the asset market effects of the Baby Boom in both the unconstrained and constrained models. Section 6 concludes. An appendix describes the computational approach for solving the model.

2. The Model

Agents live for four periods: childhood, young working-age, old working-age and retirement. In childhood agents are not active decision makers, their consumption, c_t^0, being determined by the next older generation. In young working-age, agents supply labor inelastically. Out of disposable income, w_t^1, they consume for themselves, c_t^1, and their children, $(1+n_t)c_t^0$, and make a portfolio decision over shares of risky capital, s_{et}^1, and safe bonds, s_{bt}^1. η_t is the payroll tax rate.

$$(1+n_t)c_t^0 + c_t^1 + s_{et}^1 + s_{bt}^1 = w_t(1-\eta_t) = w_t^1 \tag{1}$$

In old working-age, agents again supply labor inelastically. They earn returns on their savings in addition to after-tax wage income and consume only for themselves, c_{t+1}^2, their children having left the household. They also decide on what mix of stocks and bonds to hold going into retirement.

$$c_{t+1}^2 + s_{et+1}^2 + s_{bt+1}^2 = w_{t+1}(1-\eta_{t+1}) + (1+r_{et+1})s_{et}^1 + (1+r_{ft})s_{bt}^1 = w_t^2 \tag{2}$$

In retirement, agents no longer supply labor and consume down their savings, there being no bequests. They get a retirement benefit, which is determined by an exogenous replacement rate *b* and financed out of payroll taxes levied on the workforce.

$$c_{t+2}^3 = (1+r_{et+2})s_{et+1}^1 + (1+r_{ft+1})s_{bt+1}^1 + bw_{t+2} \tag{3}$$

Preferences are given by an additively separable utility function. The expected lifetime utility for a young worker born in period *t-1* is:

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model differs from Bohn's principally in that it incorporates an additional working period and a riskfree bond. These modifications allow workers to supply capital in addition to labor.

$$V_t = \lambda(1+n_t)\frac{(c_t^0)^{1-\theta}}{1-\theta} + \frac{(c_t^1)^{1-\theta}}{1-\theta} + \beta E_t\left[\frac{(c_{t+1}^2)^{1-\theta}}{1-\theta}\right] + \beta^2 E_t\left[\frac{(c_{t+2}^3)^{1-\theta}}{1-\theta}\right] \tag{4}$$

Young workers discount their children's utility at rate λ.[4] β is their subjective discount factor and θ is their coefficient of relative risk aversion. Agents maximize expected lifetime utility subject to the budget constraints in (1) through (3) and subject to state-dependent borrowing constraints that limit the amount they can borrow by shorting the riskfree asset:

$$s_{bt}^1 \geq -l_b w_t^1 \tag{5}$$

$$s_{bt}^2 \geq -l_b w_t^2 \tag{6}$$

The intuition behind these borrowing constraints is simple. Young workers are raising a family and thus spending much of their income. They would like to smooth consumption by borrowing against future wages, consuming part of the loan and investing the remainder in higher return capital. The borrowing constraints prevent them from doing so because, as Constantinides et al. (2002) have argued, human capital alone does not collateralize loans for reasons of moral hazard and adverse selection.[5]

Output is generated by a constant-returns-to-scale neoclassical production function. Factor markets are efficient so that capital and labor are rewarded their marginal products:

$$r_{et} = \alpha A_t K_{t-1}^{\alpha-1} L_t^{1-\alpha} - \delta \tag{7}$$

$$w_t = (1-\alpha) A_t K_{t-1}^{\alpha} L_t^{-\alpha} \tag{8}$$

δ is the depreciation rate and α determines the share of output rewarded to capital. The age distribution in period *t* consist of N_{t-1} young workers, N_{t-2} old workers, and N_{t-3} retirees. The period *t* child cohort is determined by $N_t=(1+n_t)N_{t-1}$ where n_t represents cohort growth. The labor force is given by $L_t= N_{t-1} + N_{t-2}$, while the period *t+1* capital stock is determined by the stock holding decisions of young and old workers in period *t*:

$$K_t = N_{t-1} s_{et}^1 + N_{t-2} s_{et}^2 \tag{7}$$

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[4] This specification of preferences is adopted from Higgins and Williamson (1996) and is similar to Bohn (2001).

[5] There are no short sale constraints on risky capital. For the range of parameters considered below, however, restrictions on shorting risky capital are non-binding in any event.

The riskfree rate r_{ft} moves to satisfy the equilibrium condition that bonds are in zero net supply across generations:

$$0 = N_{t-1}s^1_{bt} + N_{t-2}s^2_{bt} \tag{8}$$

The model abstracts from government activity with the exception of a pay-as-you-go social security system. Given an exogenous replacement rate b, payroll taxes move to balance the pension system, rising with the retiree to worker ratio: $\eta_t = b\, N_{t-3}/(N_{t-1} + N_{t-2})$. Effectively, this is a defined-benefit pension system where the retirement benefit to period t retirees is indexed to period t wages.[6]

The model has two exogenous sources of aggregate uncertainty: a technology shock A_t and cohort size N_t. Both are assumed to follow lognormal AR(1) processes such that:

$$\begin{bmatrix} \ln A_t \\ \ln N_t \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \ln A_{t-1} \\ \ln N_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon^a_t \\ \varepsilon^n_t \end{bmatrix} \tag{9}$$

where $\varepsilon_t = [\varepsilon_t^a, \varepsilon_t^n]$ is a two-dimensional i.i.d. process that is $N(0,\Sigma)$.

3. Calibration

The model is calibrated so that each period represents approximately 20 years. The subjective discount factor β and the discount factor applied to the utility of children λ are each set at 0.44, which corresponds to an annual rate of 0.96. In the benchmark version of the model, the risk aversion parameter θ is 2. The share of output rewarded to capital α is 0.33, while depreciation occurs at 5 percent per year so that δ = 0.65. The benchmark model abstracts from social security and therefore sets the payroll tax to zero. In specifications that include social security, the replacement rate in the pay-as-you-go system is set at 30 percent, so that the steady state payroll tax rate η amounts to 15 percent and roughly matches payroll taxes in the U.S.[7]

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[6] This Social Security system is modeled after Bohn (2001) who shows that a defined-benefit system is more efficient *ex ante* in insuring agents against demographic risk. Neither a defined-contribution system (where π is exogenous and b endogenous) nor a privatized pension system (b=π=0) impose higher taxes on the young when the retiree to worker ratio rises, whereas the defined-benefit system does. In testing the sensitivity of the asset market implications of the model, a defined-benefit specification is therefore more interesting.

[7] Bohn (2001) calibrates his pension system to have a payroll tax rate of 15 percent. Estimates for the average replacement rate of Social Security benefits vary. Aaron et al. (2001) report an average replacement rate of 42 percent.

Figure 1 plots the U.S. population between ages 0 – 19 for the period 1880 to 2040, while Figure 2 plots historical data for U.S. total factor productivity (TFP) over the period 1880 to 2000. Both series are centered 20-year averages and rebased to eliminate scale differences. Annualized cohort growth has fallen steadily from 2 percent in the 1880 period to 0.4 percent in 1980. This decline in cohort growth reflects a long run trend towards lower fertility, which is forecast to continue and cause the U.S. population to stabilize around 2040. Figure 1 shows that the post-war Baby Boom follows a pre-war baby bust in 1940 and is followed in turn by a post-war baby bust. During the pre-war baby bust, annualized cohort growth fell sharply from 1.3 percent around 1920 to 0.3 percent as households postponed having children due to the Great Depression and World War II. The post-war Baby Boom thus represents a catching up, with annualized cohort growth rising to an annualized rate of 1.9 percent in the period around 1960, before resuming its long run trend towards declining fertility, exacerbated temporarily by the post-war baby bust. Figure 2 shows that long run productivity growth has followed a wave-like pattern, as noted by Gordon (2000).[8]

The population shock in the model captures the risk that agents may be born into a large or small cohort. Since the model is stationary, cohort size is defined relative to a long run trend, which is approximated here through a nonlinear trend that captures the long run decline in fertility. Figure 1 shows the detrended population series, which clearly captures the pre-war baby bust in 1940, the Baby Boom in 1960 and the post-war baby bust in 1980. Figure 2 depicts a similar long-run trend for the TFP series, along with the detrended series.[9]

The most serious challenge to this calibration exercise is the estimation of the unconditional moments for the exogenous shocks. With each period representing approximately 20 years, even a century-long time series provides only five non-overlapping observations, resulting in

[8] Both series are rebased from one and shown in natural logs to eliminate scale differences. Annual U.S. population data are spliced together from three sources. Historical data are taken from "Historical Statistics of the United States: Colonial Times to 1970," published by the U.S. Bureau of the Census. For the period 1950 to 1990 data are taken from the United Nations "World Population Prospects: The 1992 Revision." From 1995 on data are from the World Bank "World Population Projections: The 1994-95 Edition." Historical TFP data are spliced together from Kendrick (1961) and Jorgenson and Stiroh (1999). This annual series goes from 1869 to 1996, so that the period 2000 TFP observation only represents 6 annual observations.

[9] The long run trend depicted is generated using the Hodrick-Prescott filter, with the value of the smoothing parameter determined endogenously to capture a very long run trend. For the population series, this implied a smoothing parameter of one, to capture the curvature of the series implied by the long run fertility decline. Since the TFP series is approximately trend stationary, the value for the smoothing parameter was not as important. A value of one was chosen for consistency with the population series.

large standard errors for the point estimates. Standard econometric methods designed to extract more information from the time-series, such as using overlapping observations, only marginally increase the effective number of non-overlapping observations and leave the standard errors large. Against this background, a bivariate first-order autoregression in the detrended series suggests that productivity and demographic shocks are of roughly equal magnitude at long horizons. Setting to zero all estimates that are not significantly different from zero, c_1, c_2, a_{11}, a_{12}, a_{21} and a_{22} are set to zero in the model and the second moment matrix for the shocks is:

$$\Sigma = \begin{bmatrix} 0.07^2 & 0.00 \\ 0.00 & 0.07^2 \end{bmatrix} \tag{10}$$

In the unconstrained model, l_b is set so that the borrowing constraint is non-binding in all states of the world. In the constrained model, agents can borrow a maximum of one percent of their total wealth, so that l_b amounts to 0.01.

4. Model Characteristics

The model is solved numerically using the projection method. How this approach is implemented in the context of both the unconstrained and constrained model is shown in the appendix. This section describes agent behavior and asset returns once the model is solved.

4.1 The Unconstrained Model

Table 1 reports the sample moments of annualized consumption growth and asset returns for the unconstrained model. It is generated by simulating the unconstrained model at the numerical solution for 1,000 periods and then calculating the means, standard deviations and correlations for the variables in question. The annualized mean returns or growth rates in Table 1 are defined as $100 \times [(1 + \mu^{20})^{(1/20)} - 1]$ where μ^{20} are the arithmetic means of the 20-year holding period returns or growth rates. The standard deviations of the annualized returns or growth rates are defined as $100 \times$ [sample standard deviation $\{ (r^{20})^{(1/20)} \}$] where r^{20} are the 20-year holding period returns or growth rates. The average annualized risk premium is defined as the difference between the annualized mean return on capital and the annualized mean return on the bond. The standard deviation of the risk premium is defined as $100 \times$ [sample standard deviation $\{ (r_e^{20})^{(1/20)} - (r_f^{20})^{(1/20)} \}$] where r_e^{20} is the 20-year holding period return on capital and r_f^{20} is the 20-year holding period return on the bond. The reported correlations are constructed using annualized 20-year holding period returns or growth rates.

Table 1 shows that consumption growth from young to old working-age (c_{t+1}^2/c_t^1) and old-working-age to retirement (c_{t+1}^3/c_t^2) amounts to 1.4 percent per annum in the unconstrained model, the same order of magnitude as the 1.8 percent estimate reported in Kocherlakota (1996) for annual consumption data from 1889 to 1978 for the United States. However, the model falls short in generating sufficiently volatile consumption growth, the standard deviation of which is an order of magnitude smaller than the 3.5 percent reported for US data. In addition, though the average return on capital of 7.12 percent annualized comes close to matching the average real return of 6.15 percent annualized on US stocks from 1889 to 1999, it falls far short in matching their volatility. The model generates a standard deviation of only 0.67 percent for the annualized return on capital, compared to 14 percent for US stock returns. And though the return on the bond is riskfree, it is almost as volatile as the return on risky capital, in contrast to the data where bond yields are about half as volatile as stock returns. The model generates a risk premium on risky capital of only 5.54 basis points, compared to an estimate of 5.34 percent for the excess return on the S&P 500 over US long-term government bonds from 1889 to 1999.[10]

One reason why the model fails to generate a sufficiently large risk premium is that capital in the model is a composite asset that aggregates over different claims on productive capital, such as stocks and corporate bonds. As a result, it is not a leveraged asset and therefore not risky enough. One way to control for this deficiency is to compare the risk-adjusted risk premium on capital—the ratio of the average risk premium to its standard deviation, called the *Sharpe* ratio—to that of the historical equity premium. Even in this dimension, the model falls short. It generates a *Sharpe* ratio of 15 percent, compared to a ratio of around 40 percent for the US equity premium.

Table 1. The Unconstrained Model: Sample Moments for Annualized Consumption Growth and Asset Returns in Percent (Means, Standard Deviations and Correlations)

	c_{t+1}^2/c_t^1	c_{t+1}^3/c_t^2	r_{et+1}	r_{ft}	$r_{et+1} - r_{ft}$
Mean	1.4307	1.3861	7.1181	7.0627	0.0554
Std Dev	0.5630	0.3212	0.6691	0.5388	0.3596
c_{t+1}^2/c_t^1	.	0.8987	0.8916	0.5125	0.8910
c_{t+1}^3/c_t^2	.	.	0.9996	0.8339	0.6102
r_{et+1}	.	.	.	0.8442	0.5956
r_{ft}	.	.	.	.	0.0722
$r_{et+1} - r_{ft}$	.	.	.	.	.

Even in risk-adjusted terms, the unconstrained model therefore fails to match the historical equity premium. This is not due to the expected return on capital, which roughly matches the mean real return on US stocks over the past century. Instead, the mean bond return of 7.06 percent is much too high, compared to an average real return on long-term US government bonds of only 0.82 percent annually from 1889 to 1999. The unconstrained model thus exhibits the classic equity premium or riskfree rate puzzle.

[10] The estimated means and standard deviations for US stock and bond returns are taken from Constantinides et al. (2002) who report these moments for a period spanning 1889 to 1999 for the S&P 500 total returns series and the Ibbotson Associates US Government Treasury Long-Term Bond yield.

An important feature of the unconstrained model is that agents shift from stocks to bonds as they age. Table 2 contains the unconditional means, standard deviations and correlations of key decision variables and factor returns for the 1,000 period simulation. It shows that young workers on average short the safe bond and invest the proceeds in risky capital. In contrast, old workers hold a balanced portfolio of risky capital and safe bonds. Why do agents behave this way?[11] In the absence of social security, retirees must finance consumption entirely out savings. Because the model generates returns on capital that are much more volatile than wage income, retirees holding all their wealth in capital face much more consumption risk than old workers who still earn wage income. Effectively, agents hold a non-traded asset over the life cycle, human capital, which provides a buffer against adverse technology shocks because of the low volatility of wages. To compensate for running down this non-traded asset as they approach retirement, agents shift their financial wealth from stocks to bonds. The impact of social security in this context is to effectively extend wage income into retirement, thereby flattening the life cycle profile of human capital. The results from an alternative specification of the model, which are not reported for brevity, show that the portfolio shift from stocks to bonds is robust to the inclusion of a pay-as-you-go social security system with a realistic parameterization for the payroll tax rate.[12]

Table 2. The Unconstrained Model: Sample Moments for Decision Variables, the 20-Year Return on Capital and the Wage Rate (Means, Standard Deviations and Correlations)

	r_{et+1}	w_{t+1}	c_{t+1}^1	c_{t+1}^2	s_{et+1}^1	s_{et+1}^2	s_{bt+1}^1	s_{bt+1}^2
Mean	2.9560	0.2087	0.1239	0.1639	0.0293	0.0287	-0.0269	0.0267
Std Dev	0.4994	0.0185	0.0113	0.0173	0.0045	0.0042	0.0057	0.0045
r_{et+1}	.	-0.0234	0.0712	0.2927	0.2010	0.1279	-0.2455	0.3671
w_{t+1}	.	.	0.9194	0.8640	0.9044	0.9189	-0.6327	0.7756
c_{t+1}^1	.	.	.	0.8346	0.8647	0.9383	-0.5892	0.7689
c_{t+1}^2	.	.	.	.	0.9848	0.9358	-0.8521	0.9869
s_{et+1}^1	.	.	.	.	.	0.9731	-0.8784	0.9607
s_{et+1}^2	.	.	.	.	.	.	-0.8279	0.8997
s_{bt+1}^1	.	.	.	.	.	.	.	-0.8864
s_{bt+1}^2	.	.	.	.	.	.	.	.

[11] The portfolio shift towards bonds parallels Jagannathan and Kocherlakota (1996). Storesletten, Telmer and Yaron (2001) generate a u-shaped profile for bond holdings over the life cycle, in an OLG model in which the volatility of persistent idiosyncratic shocks to wage income is negatively correlated with returns on risky capital. When they hold the volatility of idiosyncratic shocks constant, they generate a monotone increasing bond portfolio share over the life cycle. Ameriks and Zeldes (2000) and Heaton and Lucas (2000) find that equity ownership is roughly hump-shaped over the life cycle for US data. With only two working-age periods, the model obviously cannot capture this hump shape.

[12] Brooks (2002) uses a similar model with persistent technology shocks to show that agents in such a setting continue to shift their financial wealth from stocks to bonds as they age.

4.2 The Constrained Model

Table 3 reports the sample moments of annualized consumption growth and stock and bond returns for the constrained model, in which agents are permitted to borrow at most one percent of their wealth by shorting bonds. It is generated by simulating the constrained model at its solution for the same sequence of exogenous shocks used to simulate the unconstrained model. The sample moments are computed as above.

Table 3 shows that the risk premium on capital is 2.29 percent per annum in the constrained model. This is very close to the three percent mean equity premium that Constantinides et al. (2002) argue models with capital rather than leveraged equity should match. In risk-adjusted terms, the risk premium now has a *Sharpe* ratio of 28 percent, close to the 40 percent ratio of the actual equity premium. In other words, the constrained model goes a long way towards resolving the equity premium puzzle in the unconstrained model. It does this not through the expected return on capital, which, pinned down by the capital-labor ratio, is identical to that in the unconstrained model. Instead, the bond return in the constrained model is much lower than in the unconstrained model, averaging 4.83 percent annualized over the simulation. This reflects the fact that the borrowing constraint for young workers is binding 53 percent of the time (it virtually never binds for old workers). More than half of the time, young workers are therefore unable to borrow optimally by shorting bonds, which also means that the demand by old workers for the riskless asset, as they prepare to retire, goes unsatisfied more than half of the time. This unsatisfied demand means that bond prices are on average higher in the constrained than in the unconstrained model or, equivalently, that the average bond return is lower in the constrained than in the unconstrained model. The constrained model therefore also goes in the right direction for resolving the riskfree rate puzzle.

Table 3. The Constrained Model: Sample Moments for Annualized Consumption Growth and Asset Returns in Percent (Means, Standard Deviations and Correlations)

	c_{t+1}^2/c_t^1	c_{t+1}^3/c_t^2	r_{et+1}	r_{ft}	$r_{et+1} - r_{ft}$
Mean	1.3999	1.4013	7.1182	4.8331	2.2851
Std Dev	0.4156	0.4240	0.6109	8.1585	8.0936
c_{t+1}^2/c_t^1	.	0.9661	0.9383	-0.0138	0.0848
c_{t+1}^3/c_t^2	.	.	0.9488	0.1917	-0.1217
r_{et+1}	.	.	.	0.1432	-0.0688
r_{ft}	.	.	.	.	-0.9972
$r_{et+1} - r_{ft}$	.	.	.	.	.

The constrained model still fails to match the volatility of observed equity returns. Indeed, the standard deviation of the return on capital is now slightly below that in the unconstrained model. Given the aggregated nature of the capital stock, however, this comes as no surprise. The bond return is now much more volatile however. With a standard deviation of 8.16 percent annually, it is close to the estimate of 7.40 percent annualized by Constantinides et al. (2002) for the standard deviation of real long-term bond yields from 1889 to 1999.

Table 4. The Constrained Model: Sample Moments for Decision Variables, the 20-Year Return on Capital and the Wage Rate (Means, Standard Deviations and Correlations)

	r_{et+1}	w_{t+1}	c_{t+1}^{1}	c_{t+1}^{2}	s_{et+1}^{1}	s_{et+1}^{2}	s_{bt+1}^{1}	s_{bt+1}^{2}
Mean	2.9561	0.2084	0.1241	0.1632	0.0032	0.0543	-0.0013	0.0013
Std Dev	0.4546	0.0177	0.0106	0.0128	0.0021	0.0066	0.0010	0.0010
r_{et+1}	.	0.1042	0.1758	0.2029	0.0600	0.2023	0.0514	-0.0468
w_{t+1}	.	.	0.9119	0.8858	0.3184	0.9004	0.1642	-0.1782
c_{t+1}^{1}	.	.	.	0.8321	0.3465	0.9024	0.1957	-0.2014
c_{t+1}^{2}	.	.	.	.	0.0461	0.9753	-0.0613	0.0460
s_{et+1}^{1}	.	.	.	.	.	0.0384	0.5866	-0.5143
s_{et+1}^{2}	.	.	.	.	.	.	0.0305	-0.0564
s_{bt+1}^{1}	.	.	.	.	.	.	.	-0.9912
s_{bt+1}^{2}	.	.	.	.	.	.	.	.

In the unconstrained model, young workers borrow on average 13 percent of their wages by shorting bonds. In the constrained model, this ratio falls to 0.65 percent on average. As a result, young workers effectively cease to participate in financial markets, while old workers are forced to hold virtually the entire capital stock going into retirement.

5. Simulating the Asset Market Effects of the Baby Boom

This section uses the unconstrained and the constrained models to simulate the asset market effects of a population shock, which is calibrated to closely match the observed Baby Boom. The simulated population shift begins with a population bust in 1940, followed by the Baby Boom in 1960 and another population bust in 1980. This bust-boom-bust is calibrated to closely match the U.S. experience, though it assumes that the age distribution is in equilibrium before and after the transition. The baby bust occurs in 1940 when the realization of the shock to $\ln(N_t)$ is -0.07, which amounts to one standard deviation of the population shock, the same order of magnitude as the negative population shock in 1940 in Figure 1. In 1960, this shock is +0.07. Thereafter it returns to its steady state value of zero. The impact of this bust-boom-bust on asset returns is simulated by holding the realization of the technology shock constant at its expected value over the transition. This approach amounts to generating the expected stock and bond returns over the population shift. Figure 3 plots the size of the child cohort over the bust-boom-bust. Figure 4 shows how the different generations make their way through the age distribution.

Figure 5 plots the expected stock and bond returns over the simulated bust-boom-bust in the unconstrained model. Though the pre-war baby bust hits in 1940, returns on both assets are in equilibrium because the capital stock and labor supply are predetermined. That same period, however, the birth of the pre-war baby busters reduces youth dependency below steady state. This increases capital formation, causing expected returns on stocks and bonds to fall below steady state in 1960, pulled down by the lower real interest rate. The Baby Boomers are born in 1960. Youth dependency in the model rises, which reduces capital formation and causes the capital-labor ratio to fall 2.18 percent below steady state in 1980. Both asset returns are above their stochastic steady states in 1980 as a result. The capital-labor ratio bottoms out at 8.52 percent below steady state in 2000, pushing both the stock and the bond return to their respective peaks (30 basis points above steady state . During 2020, the Baby Boomers are in retirement. The associated rise in the supply of capital and the decline in the labor force push the capital-labor ratio 7.15 percent above steady state. Returns on both assets fall by 50 basis points below their 2000 levels. Beyond 2020, returns gradually return to equilibrium as the age distribution returns to steady state. Overall, the unconstrained model suggests that the asset market effects of the Baby Boom will be modest. Furthermore, it finds that there is little differential effect on stocks versus bonds: the risk premium is roughly stable over the simulated transition. Instead, it appears that the capital-labor ratio is the main driving force for both asset returns.

In terms of lifetime consumption, the unconstrained model shows that the Baby Boomers actually do better than smaller cohorts around them, notably their parents or their children. Figure 6 shows that the Baby Boomers are 0.84 percent better off in lifetime consumption terms than if the age distribution had remained in steady state. In contrast, their parents are 1.24 percent worse off, while their children are 2.26 percent worse off. Bohn (2001) has argued that a defined-benefit pay-as-you-go social security system can offset movements in the capital-labor ratio that go against large cohorts who supply labor when wages are low and accumulate capital when returns are low. This is because a defined-benefit system taxes smaller cohorts more heavily than large cohorts. But with the Baby Boomer generation better off than smaller cohorts, notably in a model without social security, this suggests that the welfare losses associated with the crowding out of private capital formation from having a pay-as-you-go social security system may outweigh the ability of such a system to offset movements in the capital-labor ratio that disadvantage large cohorts.

Figure 7 depicts the expected stock and bond returns over the simulated bust-boom-bust in the constrained model. The effects on the return on capital are still modest. In particular, it is still projected to decline by about 50 basis points from 2000 to 2020, due to increased capital formation as the Baby Boomers retire. In contrast, the impact on the bond return is now much more dramatic. In 1980, the bond return falls 276 basis points below steady state, driven by the fact that young workers in the 1960 period (the pre-war baby busters) are raising a large cohort of children (the post-war Baby Boom), causing their borrowing to hit the constraint. Old workers, a comparatively large generation, are therefore unable to invest as much as they would like in bonds, which pushes the riskfree rate down in 1980. The opposite occurs in 2000, when the bond return is 169 basis points above its steady state level. This is because in 1980 a small cohort of old workers (the pre-war baby busters) is trading on the bond market with a large cohort of young workers (the Baby Boomers). The bond return rises to clear the bond market of this imbalance. The most pronounced effect comes in 2020, when the riskfree rate falls 444 basis point below its steady state and 613 basis points below its level in 2000. This decline is driven by the aging Baby Boomers who want to hold the riskless asset going into retirement but are unable to, because the borrowing constraint is binding for young workers. Overall, the constrained model suggests that the effects of the Baby Boom will be much more pronounced for the riskfree bond. This is because the return on capital is pinned down by the marginal product of capital, which prevents sudden movements. This is not so for the bond return, especially when the borrowing constraint is binding for young workers.

The asset market effects of the Baby Boom are therefore more pronounced in a scenario that replicates the historically low degree of stock market participation in the population. What implications does the constrained model have for intergenerational welfare? Figure 8 shows that, in lifetime consumption terms, the Baby Boomers are now 0.18 percent worse off than if the age distribution had remained in equilibrium. This is because the risk premium on capital now moves against them, when before it was neutral. However, they are still better off than their parents or their children, who are 0.20 and 0.60 percent worse off than if the age distribution had remained unchanged. Because defined-benefit pay-as-you-go social security redistributes wealth only across adjacent cohorts, it is not clear that such a system is *ex ante* optimal even in this environment.

6. Conclusion

This paper explores the quantitative impact of the Baby Boom on stock and bond returns. It builds a neoclassical growth model with overlapping generations, in which agents make a portfolio decision over risky capital and safe bonds in zero net supply. The model has exogenous technology and population shocks that are calibrated to match long run data for the US. With agents allowed to borrow freely by shorting bonds, the model fails to match the historical equity premium by a large margin and generates only small asset market effects over a simulated Baby Boom. When agents are constrained in their ability to borrow, the model comes close to matching the historical equity premium and suggests that there will be a sharp rise in the equity premium when the Baby Boomers retire, driven by a large decline in bond returns as Baby Boomers seek to hold the riskless asset in retirement.

In the unconstrained model, the Baby Boomers are better off in lifetime consumption terms than the smaller cohorts around them, notably their parents or children. Because this result obtains in a model without defined-benefit social security, it suggests that the welfare loss from the crowding out of private capital formation through social security may outweigh the ability of such a system to offset movements in the capital-labor ratio that disadvantage large cohorts. In the constrained model, even though the Baby Boomers are worse off in lifetime consumption terms than if the age distribution had remained in equilibrium, they are still better off than their parents or children. Since defined-benefit pay-as-you-go social security only redistributes wealth across adjacent cohorts, it is unclear whether such a system is *ex ante* efficient in terms of intergenerational risk sharing. This is left for future research.

Appendix

Maximizing expected utility, period t young workers choose c_t^0, c_t^1, s_{et}^1, and s_{bt}^1 such that

$$c_t^0 = \lambda^{1/\theta} c_t^1 \tag{A1}$$

$$\left(c_t^1\right)^{-\theta} = \beta E_t\left[\left(c_{t+1}^2\right)^{-\theta}\left(1 + r_{et+1}\right)\right] \tag{A2}$$

$$\left(c_t^1\right)^{-\theta}\left(s_{et}^2\right)^2 = \beta\left(1 + r_{ft}\right)E_t\left[\left(c_{t+1}^2\right)^{-\theta}\left(s_{et}^2\right)^2\right] \quad \text{and} \quad s_{bt}^1 > -l_b w_t^1 \tag{A3.1}$$

or

$$\left(c_t^1\right)^{-\theta}\left(s_{et}^2\right)^2 \geq \beta\left(1 + r_{ft}\right)E_t\left[\left(c_{t+1}^2\right)^{-\theta}\left(s_{et}^2\right)^2\right] \quad \text{and} \quad s_{bt}^1 = -l_b w_t^1 \tag{A3.2}$$

$$\left(1 + n_t\right)c_t^0 + c_t^1 + s_{et}^1 + s_{bt}^1 = w_t\left(1 - \eta_t\right) = w_t^1 \tag{A4}$$

are satisfied, taking factor returns and the return on the riskless asset as given. Period t old workers choose c_t^2, s_{et}^2, and s_{bt}^2 such that

$$\left(c_t^2\right)^{-\theta} = \beta E_t\left[\left(c_{t+1}^3\right)^{-\theta}\left(1 + r_{et+1}\right)\right] \tag{A5}$$

$$\left(c_t^2\right)^{-\theta}\left(s_{et}^1\right)^2 = \beta\left(1 + r_{ft}\right)E_t\left[\left(c_{t+1}^3\right)^{-\theta}\left(s_{et}^1\right)^2\right] \quad \text{and} \quad s_{bt}^2 > -l_b w_t^2 \tag{A6.1}$$

or

$$\left(c_t^2\right)^{-\theta}\left(s_{et}^1\right)^2 \geq \beta\left(1 + r_{ft}\right)E_t\left[\left(c_{t+1}^3\right)^{-\theta}\left(s_{et}^1\right)^2\right] \quad \text{and} \quad s_{bt}^2 = -l_b w_t^2 \tag{A6.2}$$

$$c_t^2 + s_{et}^2 + s_{bt}^2 = w_t\left(1 - \eta_t\right) + \left(1 + r_{et}\right)s_{et-1}^1 + \left(1 + r_{ft-1}\right)s_{bt-1}^1 = w_t^2 \tag{A7}$$

are satisfied, again taking factor returns and the return on the riskless asset as given. Consumption of the period t retiree cohort is given by:

$$c_t^3 = \left(1 + r_{et}\right)s_{et-1}^2 + \left(1 + r_{ft-1}\right)s_{bt-1}^2 + bw_t \tag{A8}$$

(A1) through (A8) represent a system of eight equations that characterize individual consumption *(c_t^0, c_t^1, c_t^2, c_t^3)* and investment behavior *(s_{et}^1, s_{bt}^1, s_{et}^2, s_{bt}^2)* for given wage and return distributions. In equilibrium, the consumption and investment decision rules that maximize expected utility at the individual level must be consistent with the equilibrium conditions for the stock and bond markets.[13]

The model has only two active decision makers: young and old workers. Both make their consumption-investment decision based on total wealth, which for young workers is simply after-tax wage income.

$$w_t^1 = w_t\left(1-\eta_t\right) \tag{A9}$$

Total wealth of old workers consists of the after-tax wage, in addition to stock and bond holdings plus returns.

$$w_t^2 = w_t\left(1-\eta_t\right) + \left(1+r_{et}\right)s_{et-1}^1 + \left(1+r_{ft-1}\right)s_{bt-1}^1 \tag{A10}$$

w_t^1 and w_t^2 are the distribution of wealth across working-age cohorts and are endogenous state variables. In addition the age distribution with the exception of the retiree cohort, which will not live to see the next period, represents an exogenous state variable. Assuming that A_t is *iid*, the set of period t state variables is then:

$$\Theta_t = \left[w_t^1, w_t^2, N_t, N_{t-1}, N_{t-2}\right] \tag{A11}$$

The paper solves for the endogenous variables in the model in two steps. In the first step, it assumes that none of the borrowing constraints are binding and parameterizes the conditional expectations in (A2), (A3.1), (A5) and (A6.1) as functions of the state variables in period t:

$$\left(c_t^1\right)^{-\theta} = \Psi\left(\Theta_t, \tau\right) \tag{A12}$$

$$\left(c_t^1\right)^{-\theta}\left(s_{et}^2\right)^2 = \beta\left(1+r_{ft}\right)\Omega\left(\Theta_t, \gamma\right) \tag{A13}$$

$$\left(c_t^2\right)^{-\theta} = \Lambda\left(\Theta_t, \xi\right) \tag{A14}$$

$$\left(c_t^2\right)^{-\theta}\left(s_{et}^1\right)^2 = \beta\left(1+r_{ft}\right)\Gamma\left(\Theta_t, \omega\right) \tag{A15}$$

[13] Following Marcet and Singleton (1999), (A3) and (A6) are multiplied through by functions of the share holdings of risky capital. This approach ensures that the parameterized system of equations is invertible with respect to the set of endogenous variables and addresses an indeterminacy that arises in models that solve for equilibrium holdings of two or more assets.

Given the states in period t and starting values for τ, γ, ξ, and ω, it is then possible to solve out for the endogenous variables in periods t and $t+1$. In the second step, it checks if either s_{bt}^1 or s_{bt}^2 in this unconstrained solution are outside their bounds. If $s_{bt}^1 < -l_b w_t^1$, then (A3.2) holds instead of (A3.1) and the bond holdings of young workers are given by $s_{bt}^1 = -l_b w_t^1$. Solving out again for the endogenous variables, it can be shown that the inequality in (A3.2) holds unambiguously and the Kuhn-Tucker conditions are therefore satisfied. If instead $s_{bt}^2 < -l_b w_t^2$, then (A6.2) holds rather than (A6.1) and the bond holdings of old workers are given by $s_{bt}^2 = -l_b w_t^2$. Solving out for the endogenous variables in this case, the inequality in (A6.2) can be shown to hold unambiguously and the Kuhn-Tucker conditions are again satisfied.

This paper uses the projection method to solve for the parameterized expectations. This approach is based on approximating a functional equation, such as the expected marginal utility in period $t+1$, by a polynomial in the state variables in period t. The projection from current state variables to expected endogenous variables is possible because the forcing variables are assumed to follow Markov processes.[14]

In the case of (A12), for example, the projection method maps the expected marginal utility in period $t+1$, $F_t = \beta E_t[(c_{t+1}^2)^{-\theta}(1+r_{et+1})]$, into a polynomial in the period t state variables:

$$\Psi(\Theta_t,\tau) = \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{m=1}^{M}\tau_{ijklm}\left(\ln w_{t,u}^1\right)^i\left(\ln w_{t,v}^2\right)^j\left(\ln N_{t,w}\right)^k\left(\ln N_{t-1,x}\right)^l\left(\ln N_{t-1,y}\right)^m \quad \text{(A16)}$$

where u, v, w, x and y denote grid points to be specified below. The parameter vector τ is then determined by minimizing the residual function:

$$\varepsilon\left(w_{t,u}^1, w_{t,v}^2, N_{t,w}, N_{t-1,x}, N_{t-1,y};\tau\right) = F_t - \Psi(\Theta_t,\tau) \quad \text{(A17)}$$

where F_t denotes the expected marginal utility at a particular grid point and $\Psi(\Theta_t,\tau)$ is the corresponding polynomial approximation. The vector τ is determined by minimizing the sum of squared residuals $(\varepsilon(w_{t,u}^1,\ w_{t,v}^2,\ N_{t,w},\ N_{t-1,x},\ N_{t-2,y};\tau))^2$ over all $U\times V\times W\times X\times Y$ grid points where the number of coefficients is $(I+1)\times(J+1)\times(K+1)\times(L+1)\times(M+1) < U\times V\times W\times X\times Y$. The grid points consist of all combinations of $\{\ln w_{t,u}^1,\ \ln w_{t,v}^2,\ \ln N_{t,w},\ \ln N_{t-1,x},\ \ln N_{t-2,y}\}$ that are based on U values for $\ln w_{t,u}^1$ in the interval $[\ln w^{1,down}, \ln w^{1,up}]$, V values for $\ln w_{t,v}^2$ in the interval $[\ln w^{2,down}, \ln w^{2,up}]$ and so on for other state variables. Each grid point is referenced by u, v, w, x and y and can be thought of as representing a possible state in period t.

This paper uses orthogonal regressors when minimizing the sum of squared residuals, which is more efficient in problems with many state variables. The orthogonality of the regressors

[14] See Christiano and Fisher (2000), Helbling (2002) and Judd (1998) for more detailed descriptions of the projection method.

stems from the fact that in the polynomial function (A16), Chebyshev polynomials are used in place of simple polynomials:

$$\Psi(\Theta_t,\tau)=\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{m=1}^{M}\tau_{ijklm}\Phi^i\left[\chi\left(\ln w_{t,u}^1\right)\right]..\Phi^m\left[\chi\left(\ln N_{t-1,y}\right)\right] \tag{A18}$$

where the function $\chi(s)$ maps the variable s from the interval [s^{down}, s^{up}] into [-1, 1] and where $\Phi^i(s)$ stands for the i'th order Chebyshev polynomial evaluated at s. The grid points in the interval over [-1, 1] are given by the corresponding number of Chebyshev roots. The length of the intervals [s^{down}, s^{up}] are given respectively by 20 percent above and below the steady state values for w^1 and w^2 in a deterministic version of the unconstrained model and by 3.5 times the unconditional standard deviation of ln(N_t) for the other state variables. Fourth-order polynomials are used to approximate the parameterized expectations. This approximation generates an accurate solution according to an accuracy criterion described below.

The laws of motion in (9) for ln(A_t) and ln(N_t) determine the values that these variables take in period $t+1$. Since the shocks to ln(A_t) and ln(N_t) are normally distributed, F_t can be approximated using bivariate Gaussian-Hermite quadrature. 15 nodes are used in this step. This paper then follows Den Haan and Marcet (1990) in taking a first-order approximation of $\Psi(\Theta_t, \tau_n)$ around τ_n. After rearranging terms, minimizing the sum of squares in (A17) becomes an OLS regression where τ_e is the coefficient vector. At the n'th iteration a new value τ_{n+1} is generated according to $\tau_{n+1} = \lambda\tau_n + (1-\lambda)\tau_e$ where τ_e. Given τ_{n+1}, γ_n, ξ_n, and ω_n, the system is solved out again and the algorithm fits the other conditional expectations in turn. This procedure is repeated until the algorithm reaches a fixed point in τ, γ, ξ, and ω, which is assumed to have been reached when $(\tau_e - \tau_n)'\,(\tau_e - \tau_n)$ is less than 1e-05.

For the unconstrained model, the accuracy of the solution is checked using an accuracy test developed by Den Haan and Marcet (1994). The intuition behind this test is to check if the prediction errors that agents make are orthogonal to their information set at t. The accuracy test therefore checks for orthogonality between the Euler equation residuals and a vector v_t of variables in agents' period t information set.

$$\begin{bmatrix} \left(c_{t+1}^2\right)^{-\theta}\left(1+r_{et+1}\right)-\Psi\left(\Theta_t,\tau^*\right) \\ \left(c_{t+1}^2\right)^{-\theta}\left(s_{et}^2\right)^2-\Omega\left(\Theta_t,\gamma^*\right) \\ \left(c_{t+1}^3\right)^{-\theta}\left(1+r_{et+1}\right)-\Lambda\left(\Theta_t,\xi^*\right) \\ \left(c_{t+1}^3\right)^{-\theta}\left(s_{et}^1\right)^2-\Gamma\left(\Theta_t,\omega^*\right) \end{bmatrix}=\varepsilon_{t+1} \tag{A19}$$

where τ^*, γ^*, ξ^*, and ω^* are the parameterized expectation estimates at convergence. For any $m\times 1$ vector v_t in agents' period t information set, the statistic

$$G = (T-1)\left[\frac{\sum_{t=1}^{T-1}\left(\varepsilon_{t+1}\otimes v_t\right)}{T-1}\right]'\left[\frac{\sum_{t=1}^{T-1}\left(\varepsilon_{t+1}\otimes v_t\right)\left(\varepsilon_{t+1}\otimes v_t\right)'}{T-1}\right]^{-1}\times$$

$$\left[\frac{\sum_{t=1}^{T-1}\left(\varepsilon_{t+1}\otimes v_t\right)}{T-1}\right] \tag{A20}$$

has an asymptotic χ^2 distribution with degrees of freedom given by $4\times m$. The vector of state variables Θ_t is chosen for v_t. This test is implemented in the following manner. Given τ^*, γ^*, ξ^* and ω^* at convergence, the model is simulated N times, each time for different draws of the technology shock and the age distribution. For these N simulations, the frequency with which the G statistic is greater than the critical value of the 95th percentile of a χ^2_{20} is reported. If the percentage of G statistics above the critical value of the 95th percentile is substantially greater than five percent, this is evidence against accuracy of the solution.

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Figure 1. Normalized U.S. Population Ages 0-19

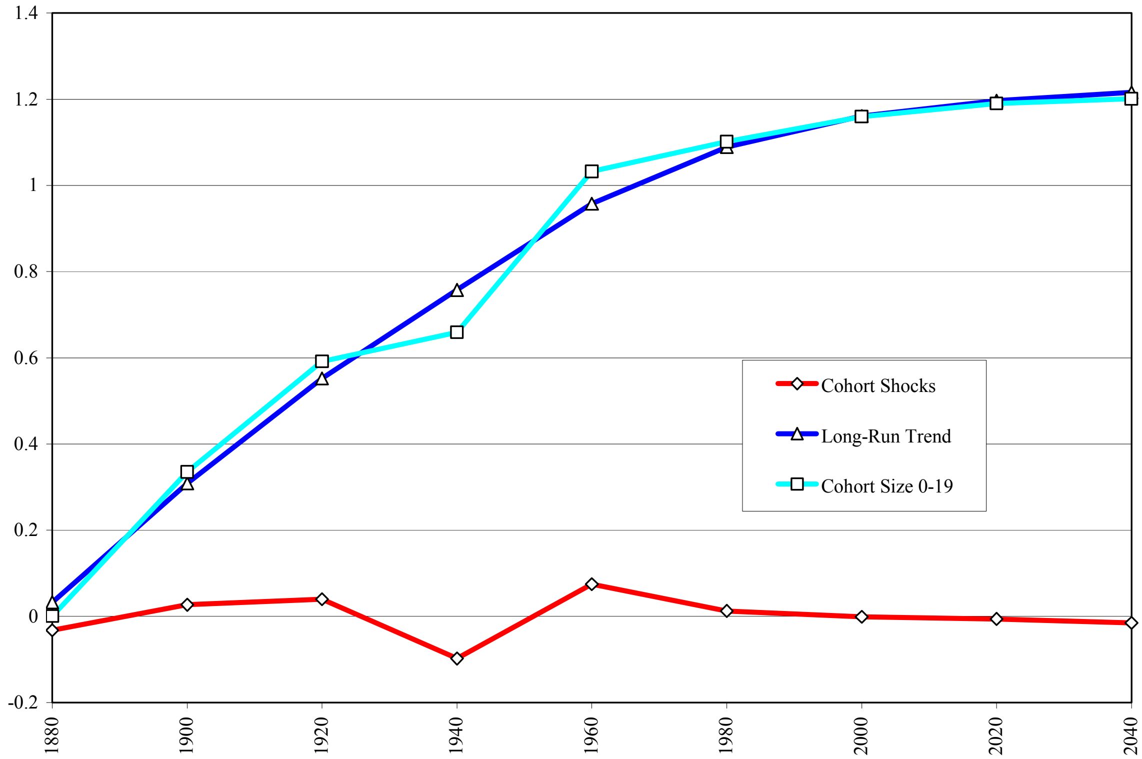

Figure 2. Normalized U.S. Total Factor Productivity

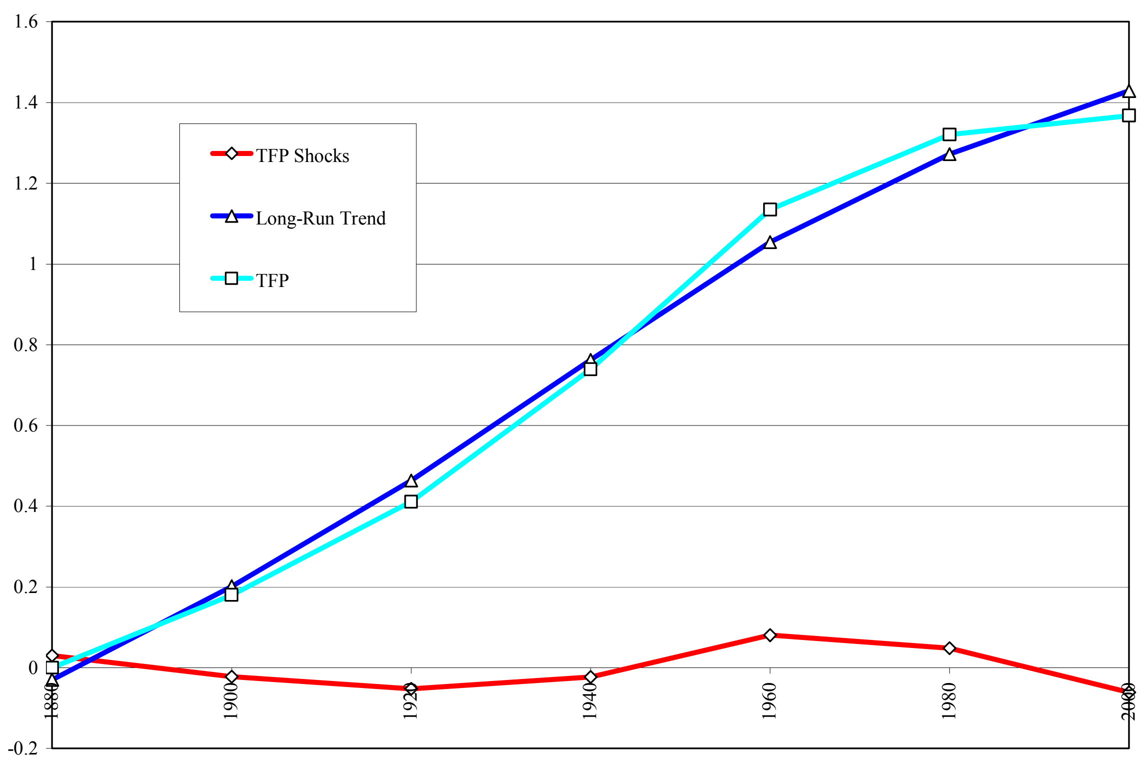

Figure 3. Child Cohort over the Simulated Baby Boom

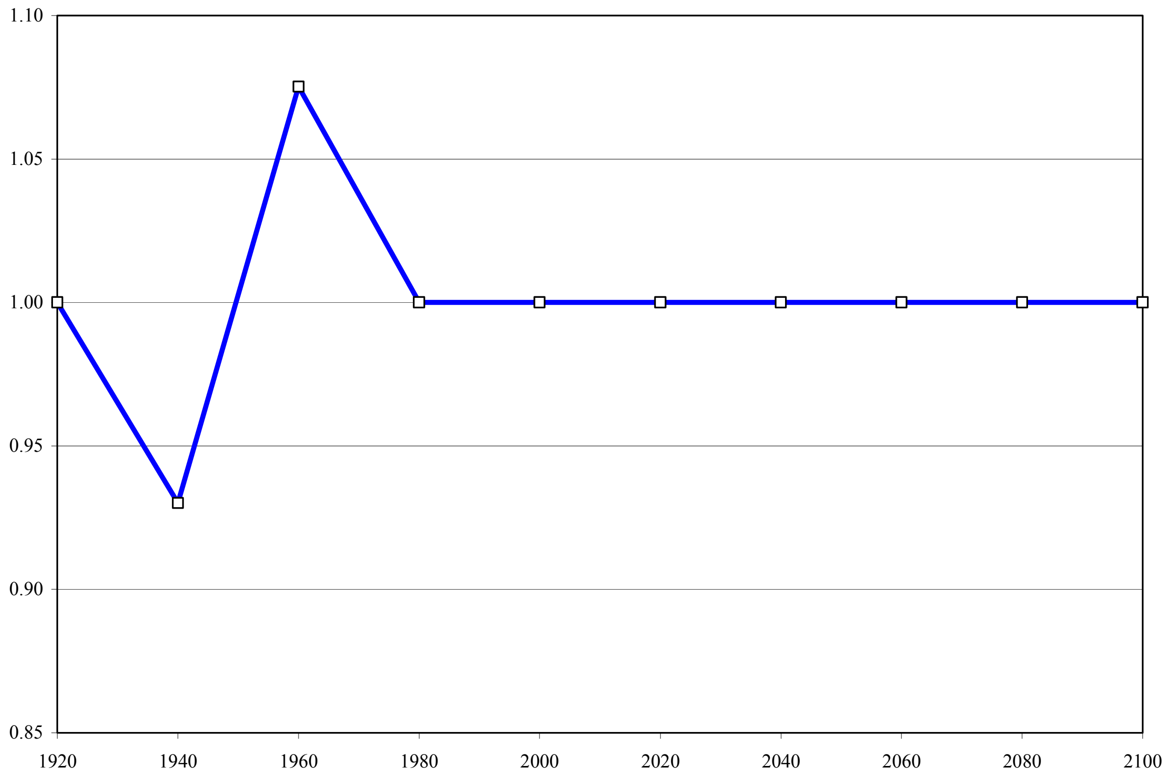

Figure 4. The Age Distribution over the Simulated Baby Boom

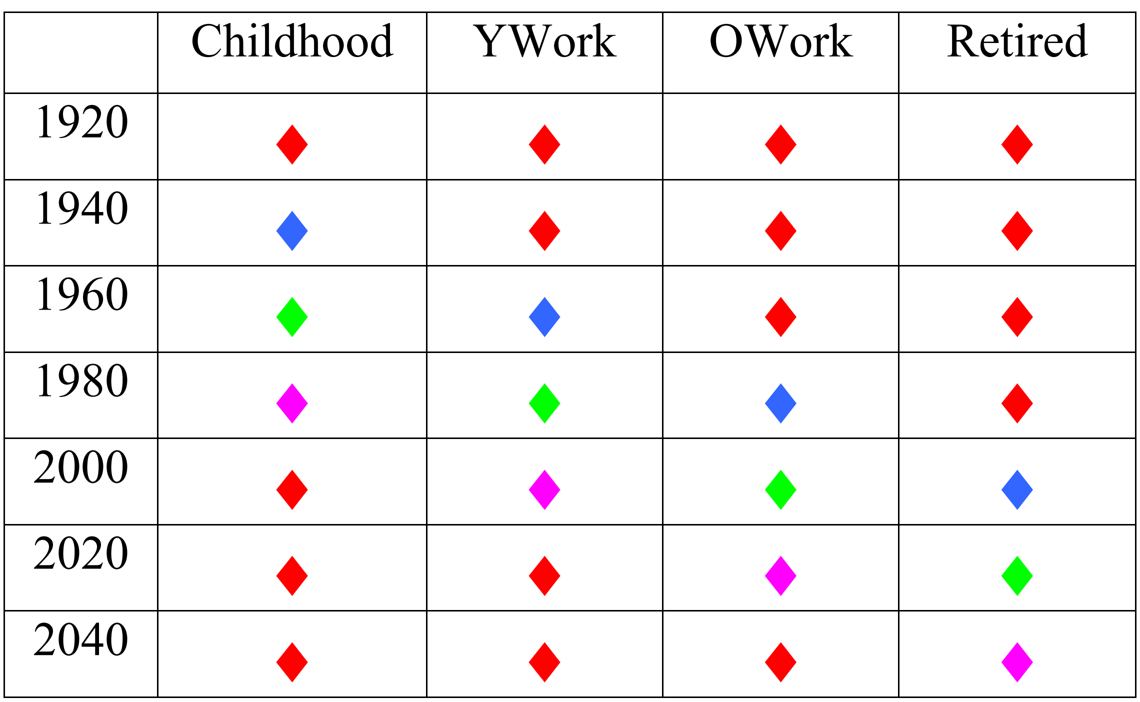

Figure 5. The Unconstrained Model: Expected Real Returns on Risky Capital and Safe Bonds (Simulation: 1940 pre-war bust; 1960 post-war boom; 1980 post-war bust)

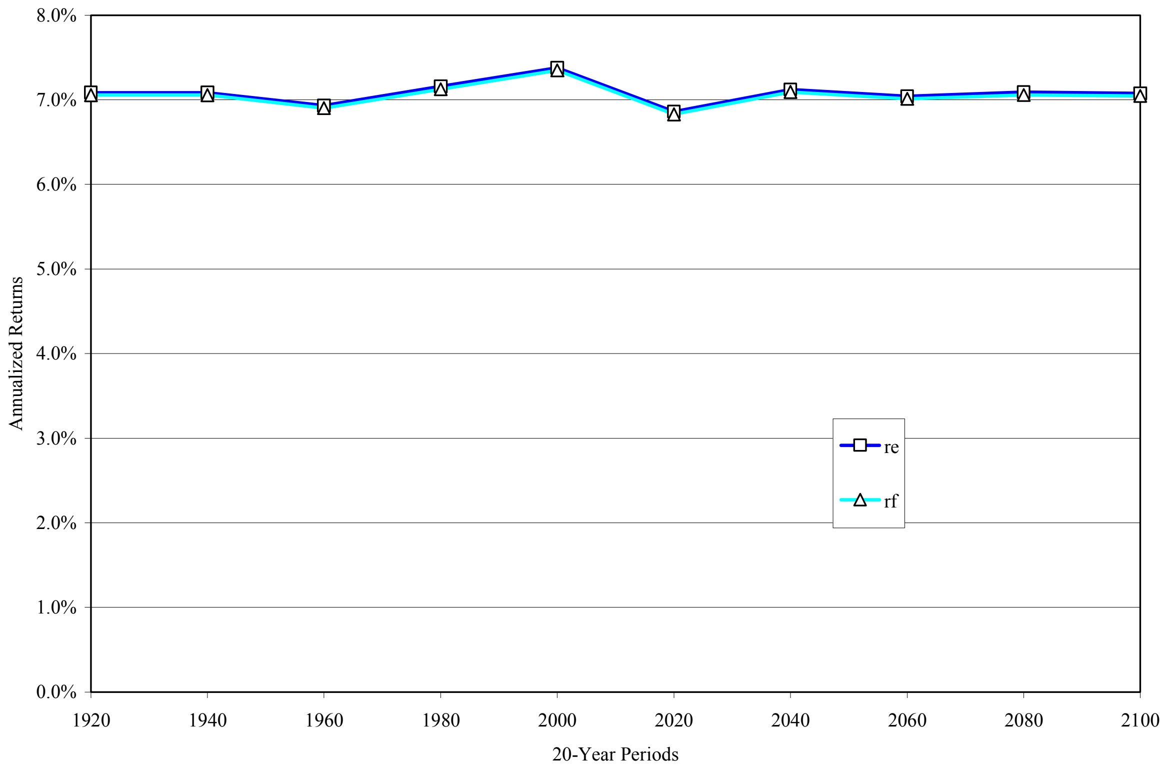

Figure 6. The Unconstrained Model: Lifetime Consumption in Percent Deviation from Steady State by Generation (Deviations by year of birth: 1940 pre-war bust; 1960 post-war boom; 1980 post-war bust)

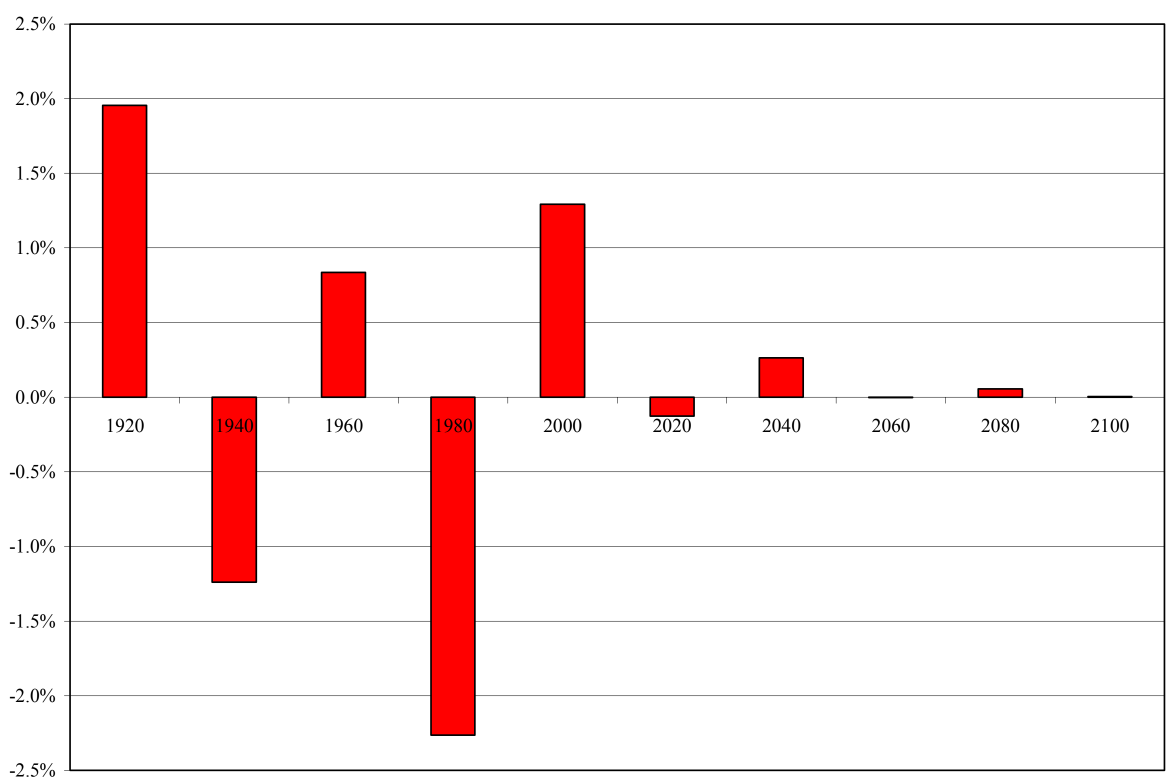

Figure 7. The Constrained Model: Expected Real Returns on Risky Capital and Safe Bonds (Simulation: 1940 pre-war bust; 1960 post-war boom; 1980 post-war bust)

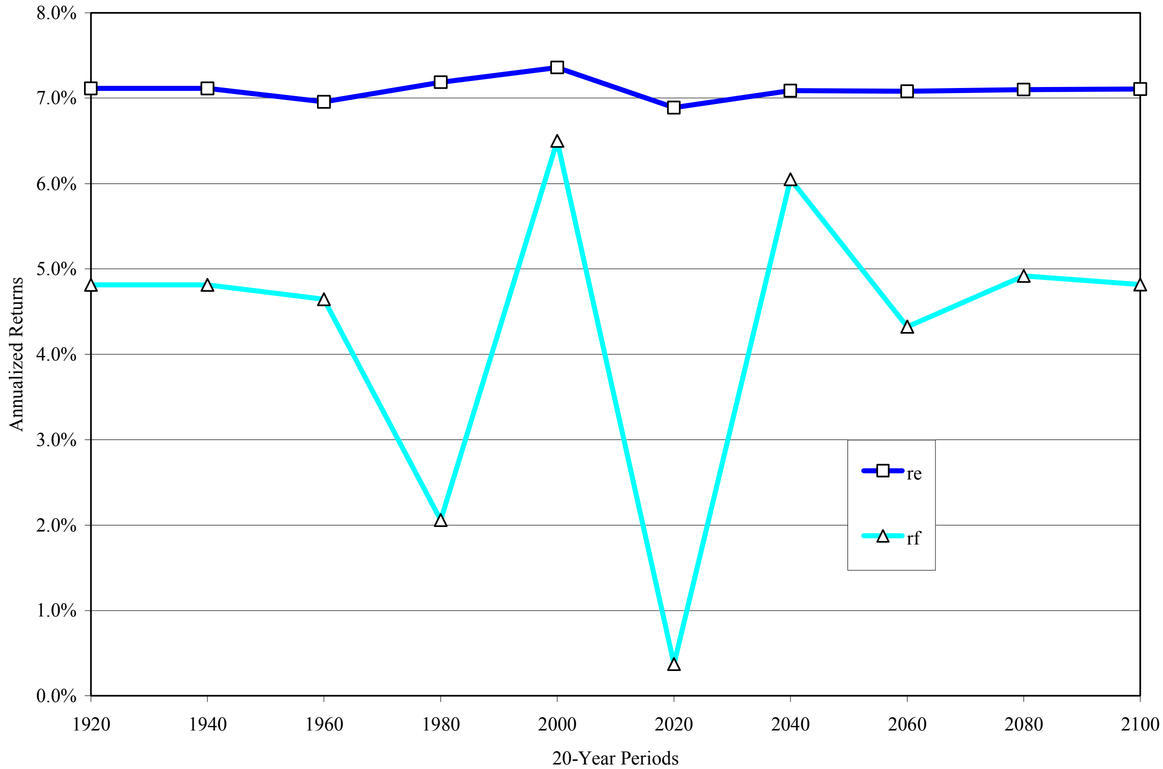

Figure 8. The Constrained Model: Lifetime Consumption in Percent Deviation from Steady State by Generation (Deviations by year of birth: 1940 pre-war bust; 1960 post-war boom; 1980 post-war bust)