Networks of Relations*

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Abstract

In this paper, we model networks of relational contracts. We explore sanctioning power within these networks under different information technologies depending on the shape of the network. The value of the relational network lies in the enforcement of cooperative agreements which would not be enforceable for the agents without access to the punishment power of other network members. We identify conditions for stability of such networks, conditions for transmission of information about past actions, and conditions under which self-sustainable subnetworks may actually inhibit a stable network.

JEL Codes: L13, L29, D23, D43, O17

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1 Introduction

Increasing evidence shows that relational arrangements are an important governance mechanism over interactions of economic agents. This is not only the case in developing economies but also in well developed economic environments, most prominently in the fast changing one of high-tech industries. Especially in R&D-intensive industries, many firms enter collaborations in order to trade-off risk and return from their high-risk activities. But formal arrangements often merely represent the tip of the iceberg, "beneath which lies a sea of informal relations" (Powell et al. 1996). On the one hand, lacking contractibility over the

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main ingredients – investments into human capital and knowledge transfers – excludes market relations, the need for flexibility on the other hand excludes vertical integration. Annalee Saxenian (1994) reports a highly specialized, network-like vertical organization within the computer-industry in Silicon Valley within which informal relations play a crucial role for the success of the district in comparison with Route 128, a competing district close to Boston: "While they competed fiercely, Silicon Valley's producers were embedded in, and inseparable from, these social and technical networks." It is noteworthy that the informal relations reported by Saxenian are not only of value on their own, they are of special value due to their being part of a network of such relations between engineers. Examining the biotechnology industry, Powell et al. (1996) point out, that the "development of cooperative routines goes beyond simply learning how to maintain a large number of ties. Firms must learn how to transfer knowledge across alliances and locate themselves in those network positions that enable them to keep pace with the most promising scientific or technological developments." The networks themselves form when individuals establish relations. Using their position within the network, and therefore using the network itself for their interests, thus becomes a central issue for those firms. This paper is an attempt to model these networks of relational contracts.

In recent economic research, both, the emergence and stability of networks and relational governance mechanisms, have aroused the interest of many theoretical as well as experimental scholars. Being well connected, at the best with themselves well connected partners, is valuable. When agents set up costly links, thereby forming a network, a conflict between efficient and stable networks may arise. This line of research has been surveyed in an excellent article by Matthew Jackson (2003). Most prominent contributions to this literature are Jackson and Wolinsky (1996), who model the emergence and stability of a social information and communication network when agents choose to set up and maintain or destroy costly links, using the notion of pairwise stability, Bala and Goyal (2000a) who consider the setup of a link by one agent only, Johnson and Gilles (2000), who introduce a spatial cost structure leading to equilibria of locally complete networks, or Bala and Goyal (2000b), who explored communication reliability. The strategic aspect in these models lies in the question of whether to build and maintain a link or not. The commonly asked question in these models is: Given a value of a network, a sharing rule and the cost of maintaining a link, which networks will emerge in equilibrium and are they efficient. The underlying game and enforceability problems are thereby the left out of consideration. We depart from that literature in two ways: First, we explicitly model an underlying game, which allows us to study consequences of its features for stability of *fixed network structures*. Second, we *do not examine network formation*. We examine enforceability problems within *fixed* network structures with a *specific* game underlying the links and thereby the stability thereof.

To our best knowledge, work that explicitly models the underlying game in the past has largely focused on random matching games. Kandori (1992) and Groh (2002) consider such a random matching repeated prisoners' dilemma situation. Both show how much cooperation is possible, Kandori without information processing and Groh with reliable and unreliable communication in a network. Groh introduces the endogenous decision of players to pass on information. In contrast to Kandori and Groh, we do not consider games between changing partners, but fixed neighbors. This introduces a forward induction element into strategic behavior when defecting. We keep Groh's endogenous decision of players to pass on information on past games' actions. We introduce the possibility to pass on informations received by partners in the underlying game.

The closest theoretical literature to ours is not from the network formation literature. Our work relates the closest to the literature of multimarket contact à la Bernheim and Whinston (1990) and Maggi (1999). In Bernheim and Whinston's paper, collusion between two agents is fostered by tying the actions from one relation to the ones in the other relation. Asymmetries of payoffs drive the result. Maggi models international trade cooperation with multilateral punishment mechanisms. We depart from and extend the work of Bernheim and Whinston by allowing our agents to exploit *indirect* multimarket contact. They maintain relations by using the network that not only consists of their own but also of other agents' relations and thereby pool asymmetries in payoffs. We extend Maggis work by introducing different information transmission mechanisms that play a role in other applications than international trade cooperation.

In this paper, we model networks of relational contracts as described by Saxenian or Powell et al. We describe equilibrium conditions for different simple architectures of such networks, paying special attention to differences in these conditions for circular and non-circular architectures, and different informational regimes. The basic framework is that of repeated games between fixed partners à la Maggi (1999). We consider three information transmission structures. We study first perfect information transmission, that is each agent observes the histories of the games for all agents. We consider secondly the case where no information can be transmitted at all. Here each player only observes the histories of his own games with his direct partners. And thirdly we consider the case where, while agents meet to transact, they exchange information on the game. In this environment, in addition to observing his own history, in each period each agent transmits or receives a verified message to/from each of his partners about the histories of their games and about messages they received. We assume that it takes one period or a smaller number of periods for such an information to travel from one agent to the other, therefore with a delay, an agent may be informed about all other players' actions to whom he is connected in the network. However, we always require agents to be *willing* to pass on information, that is shouting – informing one's neighbor's neighbors is not allowed for, and we explicitely assume that exchange of information only takes place while meeting for the transactions underlying the relations.

We begin with sustainable network where agents can only have relations with two neighbors. We show that if agents cannot discipline themselves within a certain relation, a circular pooling of asymmetric payoffs may sustain the relation. In contrast to Groh, the possibility to transmit information about the cheating of someone through the links in the network will not be an equilibrium action if enforcement relies on unrelenting punishment. Once an agent deviates, a contagious process eliminates cooperation in the network. With more complex punishment strategies agents may use information transmission and thereby keep on cooperating in the rest of the network while punishing the deviator. We show that, under the complete information assumption, bilaterally unsustainable relations in a network without "redundant" links, may can be sustained by having self-sustaining relations at the ends of the network while this does not work for the other informational assumptions. Thirdly we show that having self-sustaining relations in the network may actually hurt cooperation in the case without full information because agents might not be willing to perform the punishment if this is unrelenting. In this case a network may be sustainable if we use relenting punishments. As opposed to standard results in the literature, in our model, improved outside options, possibly by more efficient spot markets, for one player may under certain conditions actually foster cooperation by making the breakup of a relation in the case of a deviation a credible threat. The results are finally generalized to more complex network architectures where players may have more than two neighbors.

The paper starts with the definition of a network of relations in section 2. In section 3, we derive results for sustainable networks with the restriction of at most two neighbors when the punishment mechanism does not provide for a re-closure of the network in a punishment period. Section 4 provides an analysis of punishment mechanisms that do allow for re-closure of the network in punishment periods. We extend the results from section 3 to situations with more neighbors in section 5. Section 6 concludes.

2 The model

2.1 Interaction

There is a set of infinitely lived agents $N = \{1, ..., n\}$, with $i \in N$. In each period t, each agent i chooses an action with respect to every other agent $j \in N$ from the set of actions $A_t^{ij} = \{C^{ij}, D^{ij}\}$. The actions taken by each individual may, but do not have to be verifiable. Denote $a_t^{ij} \in A_t^{ij}$. An action profile is then $a_t \equiv \times_{i \in N} \times_{j \in N \backslash i} a_t^{ij}$. Per period payoffs are a real valued function $U_t^i : A_t \mapsto \Re$, $i \in N$. For each pair agents (i, j) we assume the stage game to be a prisoners' dilemma with the following payoffmatrix:

		agent j	
		C^{ji}	D^{ji}
agent i	C^{ij}	$c^{i,j}, c^{j,i}$	$l^{i,j}, w^{j,i}$
	D^{ij}	$w^{i,j}, l^{j,i}$	$d^{i,j}, d^{j,i}$

with $l^{i,j} < d^{i,j} < c^{i,j} < w^{i,j}$ and $l^{i,j} + w^{i,j} < 2c^{i,j}$, $\forall i, j \in N$, $i \neq j$. Note that the stage game is constant over time. Note also that the assumptions on the payoffs imply the static Nash equilibrium characterized by (D^{ij}, D^{ji}).

Agents are assumed to interact repeatedly. Time is discrete, and all agents are assumed to share a discount factor δ, meant to capture the time preferences. For simplicity, we assume separability of agents' payoffs across interactions, that is

$$U_t^i (A_t) = \sum_{j \in N \backslash i} U_t^i \left(a_t^{ij}, a_t^{ji}\right)$$

and across time. Agents are assumed to choose actions which maximize their average discounted utility

$$V^i = \sum_{t=0}^{\infty} \delta^t U_t^i (A_t) .$$

For this stage game, we have learned from the folk theorem that for agents patient enough (that is for a high enough δ), by repeatedly interacting in this game ad infinitum, it is possible to sustain the action profile (C^{ij}, C^{ji}) as a Nash equlibrium (see e.g. Friedman, 1971).

2.2 Relations and networks

In this section, we define what mean by a relation and by a network of relations and give some definitions useful for analysing these networks. We start by defining a relation according to the usual definition in the literature.

Definition 1 (Relation) *Agents i and j are connected by a relation if and only if they repeatedly choose to play C^{ij}, C^{ji} in the stage game.*

For notational convenience let us define the difference between the payoff of player i of playing (C^{ij}, C^{ji}) forever and of defecting and then play the static Nash equilibrium (D^{ij}, D^{ji}) by

$$g^{ij} \equiv c^{i,j} - \delta d^{i,j} - (1-\delta)\, w^{i,j}.$$

A standard interpretation of g^{ij} is the net gains for i from cooperating with j considering grim trigger strategies according to Friedman (1971). Therefore, if $g^{ij} > 0$, i does not have an incentive to deviate in an infinitely repeated prisoners' dilemma with trigger strategies. However, a $g^{ij} < 0$ does *not* mean that there is no gain for agent i from cooperation with agent j. It just means that agent i would like to deviate and bilateral cooperation is, therefore, unfeasible. We call a relation of player i with player j deficient for player i if $g^{ij} < 0$ and non-deficient for player i if $g^{ij} \geq 0$.

Definition 2 (mutual, unilateral, bilaterally deficient relation) *The relation ij is called mutual iff $g^{ij} \geq 0$ and $g^{ji} \geq 0$, it is called unilateral iff either $g^{ij} < 0$ and $g^{ji} \geq 0$ or $g^{ij} \geq 0$ and $g^{ji} < 0$, it is called bilaterally deficient iff $g^{ij} < 0$ and $g^{ji} < 0$.*

We are now in the state to define a network. We interpret a collection of the agents and their relations as a network.

Definition 3 (Network) *A network $\mathcal{N}^S = (\mathcal{N}, R)$ is a graph*[1] *consisting of a finite nonempty subset $\mathcal{N}$ of the set of agents N together with a set R of two element ordered subsets of $\mathcal{N}$, where $(i,j) \in R$ iff i and j are connected by a relation.*

For simplicity, we assume $\mathcal{N} = N$.[2]

Definition 4 (Sustainability) *A relational network $\mathcal{N}^S = (\mathcal{N}, R)$ is sustainable iff all relations between the agents in $\mathcal{N}$ are simultaneously supportable in sequential equilibrium.*

Definition 5 (Stability) *A sustainable relational network $\mathcal{N}^S = (\mathcal{N}, R)$ is strategically stable if it fulfills Kohlberg and Mertens' stability criteria.*

[1] A directed **graph** $G = (V, E)$ is a finite nonempty set V of elements called **vertices**, together with a set E of two element ordered subsets of V called **edges or arcs**.

[2] We can just as well define the Network as $\mathcal{N}^S = (N, R)$.

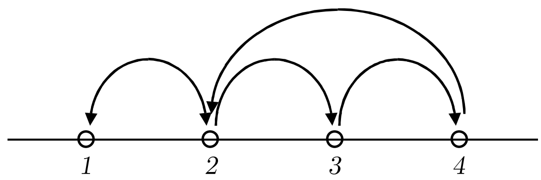


Figure 1: Graphical representation of a network of relations

A way to represent such a network is of course graphical, where a line is drawn from agent j to agent i if $(i, j) \in R$. In our graphical representation, we depart from our original undirected network definition by adding information on deficiency of relations. We depict a non-deficient relation for player i by an incoming arc to player i. A unilateral relation, thus, is depicted by an arc originating from the agent for whom the relation in deficient. A mutual relation is depicted by an incoming arc to both players. A bilaterally deficient relation is just a line. Refer to figure 1: Agents 1 and 2 share a mutual relation and the relation between 2 and 3 is unilateral, it is deficient for player 2 and non-deficient for player 3.

Definition 6 (mutual, non-mutual, mixed network) *A relational network is called a mutual network if it only consists of mutual relations. A network is called a non-mutual network if it does not contain mutual relations. A network is called a mixed network if it consists of both, mutual and other relations.*

Agent i is called **adjacent** from agent j and agent j adjacent to agent i if $(i, j) \in R$ and $g^{ij} \geq 0$. Two agents are called **directly connected** in the social network (or adjacent) if $(i, j) \in R$. **The set of agents adjacent to or from** i are the neighborhood of i, denoted by N_i, and $j \in N_i \Leftrightarrow i \in N_j$.

Given $\mathcal{N}^S = (\mathcal{N}, R)$, the number of agents in $\mathcal{N}$ is called the **order** of $\mathcal{N}^S$ and the number of relations in R the **size** of $\mathcal{N}^S$. The number of arcs directed away from agent i is called the **outdegree** of agent i and denoted by $\operatorname{od} i$, and the number of arcs directed into agent i is called the **indegree** of agent i, denoted by $\operatorname{id} i$. The **degree** of vertex i is the number of arcs directed away or into agent i, denoted $\deg i = \operatorname{od} i + \operatorname{id} i$. An agent of degree 1 is called end vertex. In figure 1, 1 is an end vertex, $\deg 2 = 3$, $\operatorname{id} 2 = 2$, $\operatorname{od} 2 = 1$.

Let i and j be two agents of $\mathcal{N}^S$. A $i - j$ **walk** in $\mathcal{N}^S$ is a finite alternating sequence of agents and links that begins with agent i and ends with agent j and in which each link in the sequence joins the agent that precedes it in the sequence to the agent that follows in the sequence. The number of links in an $i - j$ walk is **length** $l(i, j)$ of the $i - j$ walk. A **path** connecting i_1 and i_k is an $i_1 - i_k$ walk in which no agent is repeated. An $i-j$ **walk** is **closed** if $i = j$ and **open** otherwise. A closed path is a **cycle**. A network of order c that consists only of a cycle is called the c**-cycle**. If a network contains no cycles, it is called acyclic. The network $\mathcal{N}^S$ is called **circular** if there exists a path $\{i_1, i_2, ..., i_k\}$ with $k = 1$.

2.3 Information structures

We will consider the following mechanisms for the transmission of information between agents. Let H^{ij} be the set of histories in the relation between agents i and j with $\left(a_t^{ij}, a_t^{ji}\right)_{t=1,...,T} \in H^{ij}$.

(**I1**) Complete Information: At time τ, each agent $i \in \mathcal{N}^S$ observes

- $(a_t^{mn})_{t=1,...,\tau} \in H^{mn}$ $\forall m, n \in \mathcal{N}^S$.

Each agent observes the histories of the games for all agents.

(**I2**) No Information Transmission: At time τ, each agent $i \in \mathcal{N}^S$ observes

- $\left(a_t^{ij}, a_t^{ji}\right)_{t=1,...,\tau} \in H^{ij}$ $\forall j \in N_i$.

Each agent only observes the histories of his own games with his direct opponents.

(**I3**) Network Information Transmission: At time τ, each agent $i \in \mathcal{N}^S$ observes

- $\left(a_t^{ij}, a_t^{ji}\right)_{t=1,...,\tau} \in H^{ij}$ $\forall j \in N_i$ and
- $(a_t^{mn}, a_t^{nm})_{t=1,...,int\left[\tau - \frac{l}{v}\right]} \in H^{mn}$, $m \in N_n$, where $\min\left[l\left(i,m\right), l\left(i,n\right)\right] = l$ if there exists an $i - m$ path.

In information structure (**I3**), in addition to observing his own history, in each period each agent i transmits or receives verifiable messages to/from each agent $j \in N_i$ about the histories of their relations or about messages they received. A message on past behavior can, thus, travel over v links per period. However, since agents only meet when they cooperate, information can only be transmitted in that case.

For an illustration of the three informational assumptions, suppose a non-circular connected network with 7 agents, call them agent 1 through 7, as in figure 2. Suppose first (**I1**). Each agent immediately knows everything that happened between every other two players, that is for example between agents 6 and 7 or between agents 2 and 3. Next suppose (**I2**).

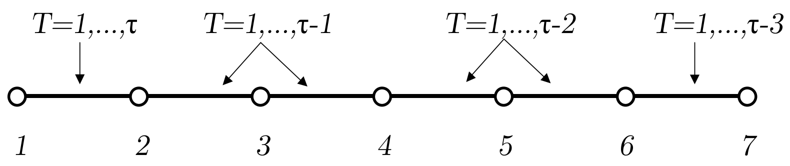


Figure 2: Agent 1's "observations"

Here each agent only knows the history of his own play, that is agent 1 only knows what happend between agents 1 and 2.

At last, suppose (**I3**) and suppose $v = 2$. Then in $t = \tau$, agent 1 observes the full history of his own play starting at $t = 1$ through $t = \tau$. Furthermore, he will receive messages from agent 2 about the play between 2 and 3 and thus "observe" actions $\left(a_t^{2,3}, a_t^{3,2}\right)_{t=1,\dots,\tau-1}$. The messages from 2 will also contain his received messages and thus agent 1 will "observe" actions $\left(a_t^{3,4}, a_t^{4,3}\right)_{t=1,\dots,\tau-1}, \left(a_t^{4,5}, a_t^{5,4}\right)_{t=1,\dots,\tau-2}$, and so on.

There are many situations, in which it seems natural to assume that there does not exist an institution that gathers and disseminates truthfully any information concerning the behavior of network members. In (**I3**), we thus suppose instead that information can only be transmitted through personal contact of members of the network and that each transmission takes time, e.g. one period. Information transmission is being delayed and therefore punishment will set in at a later point in time. Therefore, a higher discount factor δ, that is more patience of agents, will be necessary to sustain the network. An essential feature of this information structure is that agents have to have an incentive to actually transmit information to their neighbor. Thus, even with high speeds of information transmission, if agents do not want to transmit information but rather deviate and reap deviation profits, this potential higher speed of information transmission will not show an effect as to the networks sustainable.

We assume that in networks of relations communicating besides interacting is not costly. This, we think, is a reasonable assumption since very often chatting next to everyday business – if anything – gives pleasure to agents.

3 Sustainable networks

Most insights can be gained by examining networks with restricted number of neighbors and identical payoffs across individuals. Therefore, we focus on networks with nodes of a maximal degree of two, that is each agent can have at most two neighbors. We will later discuss how the results generalize in larger networks. Throughout this section, we suppose agents are not able to close the network by creating new links. We, therefore focus on mechansisms that do not involve a re-closure of the network in a punishment period. A justification for such a restriction may be that there is a geography underlying the network, i.e. that not all members of the network can have a relation with each other. Often there are very specific transactions underlying the relations and it is not possible to substitute one relation with another one. A second reason for such a focus may be that networks using re-closure are either not sustainable (they are not an equilibrium) or not strategically stable (they are unlikely to be chosen as an equilibrium). A relaxation of this assumption will be discussed in section 4.

3.1 Unilateral networks

In the theory of repeated games it is stated that in two-player repeated prisoners' dilemmas, in order to sustain a cooperative outcome as a Nash equilibrium, it is necessary that the gain from deviating net of the loss from punishment must be outweighed by the gain agents incur from cooperating for ever. Translated into the language we used above, bilateral relations are sustainable, if and only if they are mutual or in other words unilateral relations would not be sustainable. Agents would not cooperate with each other since the most severe bilateral punishment available is not strong enough and there are no other agents to discipline them.

However, once agents are aware of the network structure of their potential relations and form a punishment coalition, where deviations from agreed on behavior will be punished multilaterally, it may well be possible to pool these asymmetric payoffs in a way that also networks containing unilateral or even bilaterally deficient relations become sustainable. In this section, we explore how this pooling has to take place. We start with a negative result in Lemma 1.

Lemma 1 *There does not exist a sustainable non-mutual non-circular network, independent of the discount factor and the information structure.*

Proof. A network has been defined non-circular if for no agent $i_1 \in \mathcal{N}^S$ there exists a path $\{i_1, i_2, ..., i_k\}$ with $i_1 = i_k$. It has been defined non-mutual if $g^{ij} > 0 \Leftrightarrow g^{ji} \leq 0$. In such a network, there would have to be either an agent e at the end vertex with $\operatorname{od} e = 1$ or an agent m in the middle with $\operatorname{od} m = 2$. Since we assumed $\deg i \leq 2$, there will not be any punishment from *other* neighbors and agent $e's$ or agent $m's$ dominant strategy is to defect from the relation. *Q.E.D.* ■

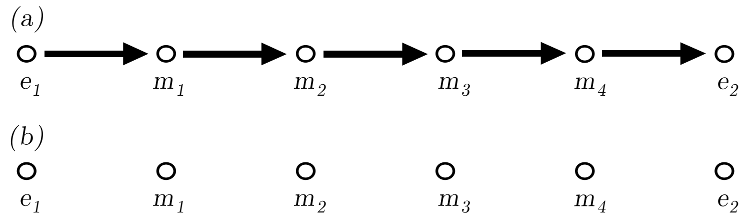


Figure 3: Only the empty network (b) is sustainable

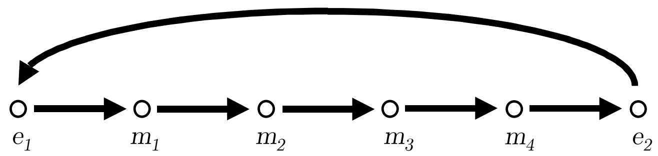


Figure 4: Circular unilateral network

Lemma 1 says that as long as relations are not mutual, they are not sustainable by a multilateral mechanism within a non-circular network. Figure 3 illustrates this: Part (a) shows a network that is not sustainable. In that situation, agent e_1 always has an incentive to deviate and the only sustainable network is empty, as shown in (b).

Leaving non-circular networks, imagine agents e_1 and e_2 share a unilateral relation that is non-deficient for e_1, thus consider a circular network as in figure 4. In this case, each agent in the network has an incoming and an outgoing arc which suggests that we may exploit payoff asymmetries with a multilateral mechanism. The network will be sustainable if the punishment coalition agrees on strategies that – given the information structure – makes the losses from a deviation big enough for the agents to prefer to cooperate. Under (**I1**) for example, such strategies may require every agent to play the cooperative action in every period and in the case of a deviation that the deviator gets punished by both his neighbors.

Let us formally define some strategy profiles for non-mutual relational networks under the three information structures. Strategies (**S1**) will serve for the full information case (**I1**), while (**S2**) serves for (**I2**) and for (**I3**).

Strategy profile (S1)

1. Each agent $i \in \mathcal{N}^S$ starts by playing the agreed upon action vector C^{ij} $\forall i \in \mathcal{N}^{\mathcal{S}}, \forall j \in N_i$.

2. Each player i goes on playing C^{ij} $\forall j \in N_i$ as long as no deviation by any player in the network is observed.

3. Every agent i reverts to D^{ij} $\forall j \in N_i$ for ever if a deviation by any player in the network occurred.

Strategy and belief profile (S2)

1. Each agent $i \in \mathcal{N}^S$ starts by playing the agreed upon action vector C^{ij} $\forall i \in \mathcal{N}^{\mathcal{S}}, \forall j \in N_i$.

2. If player i observes every of his neighbors $j \in N_i$ play C^{ji} she goes on playing C^{ij} $\forall j \in N_j$.

3. If player i observes a neighbor j play D^{ji} in $t = \tau$ she reverts to D^{ij} $\forall j \in N_i$ $\forall t \geq \tau + 1$, that is in all his future interactions with all neighbors.

For agents j with $\text{id}(j) = 1$, beliefs are such that

(i) if they observe cooperation on both sides, they believe that all agents in the network cooperated so far,

(ii) if they observe a deviation on both sides, they believe that the neighbor with whom they share their deficient relation was the first to deviate, and

(iii) if they observe a deviation only from the agent with whom they share their non-deficient relation, they give an equal probability to the event that any of the other players was the first to deviate.

For agents[3] j with $\text{id}(j) = 2$, beliefs are such that

(iv) if they observe cooperation on both sides, they believe that all agents in the network cooperated so far,

(v) if they observe a deviation on both sides, they believe whatever.

(vi) if they observe a deviation on only one side, they give an equal probability to the event that any of the other players was the first to deviate.

[3] We will need this part of the belief structure only when we consider mixed networks. In unilateral networks, by definition there are no agents with an indegree of two.

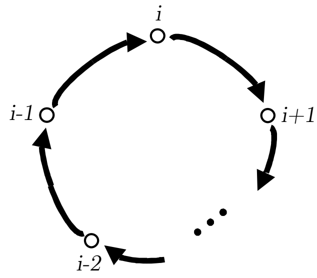


Figure 5: Circular unilateral network

Proposition 1 *Suppose the network is a c-cycle. Then*

1. *Under information structure* (**I1**)*, a non-mutual relational network is sustainable if and only if* $\forall i \in \mathcal{N}^S \; g^{i,i-1} + g^{i,i+1} > 0$*;*

2. *Under information structures* (**I2**) *, a non-mutual relational network is sustainable if and only if* $\forall i \in \mathcal{N}^S \; \delta^{c-2} g^{i,i-1} + g^{i,i+1} > 0$*, where, w.o.l.o.g.,* $g^{i,i+1} < 0$.

3. *Under information structure* (**I3**)*, a strategy profile using unforgiving punishment constitutes a sustainable non-mutual relational network if and only if* $\forall i \in \mathcal{N}^S \; \delta^{c-2} g^{i,i-1} + g^{i,i+1} > 0$*, where, w.o.l.o.g.,* $g^{i,i+1} < 0$*, regardless of the speed of information transmission.*

For the proof of proposition 1, refer to figure 5. It visualizes a non-mutual circular network. Note that in a non-mutual network, if a deviation is ever profitable, it is optimal for an agent i to immediately deviate from a relation that is deficient and to deviate from a relation that is non-deficient in the period before a punishment from that respective neighbor is expected. This follows directly from definition 2.

Proof. *Part 1* of proposition 1: *Sufficiency*: Consider strategies (**S1**) Since a deviator faces immediate Nash-reversion from both his neighbors, no matter whether she deviates towards one partner or both, she can just as well deviate from both her relations. Therefore, the network is a Nash-Equilibrium in a circular network if $\forall i \; g^{i,i-1} + g^{i,i+1} > 0$. It is subgame perfect since in the punishment phase, the stage Nash equilibrium is played.

Necessity: Since during the punishment phase the agents play their minimax strategy and the punishment phase is infinitely long, this is the strongest punishment available to the agents. If cooperation is not possible with these strategies, it will not be possible with other

– less strong – punishments. Therefore $g^{i,i-1} + g^{i,i+1} > 0 \; \forall i \in \mathcal{N}^S$ is also necessary for the relational network to be supportable.

Part 2 of proposition 1: *Sufficiency*: Consider strategies (**S2**). An agent might want to deviate only towards one neighbor in the first period and continue cooperating with the other neighbor until the period in which this other neighbor is being communicated the deviation of i in his interaction with the first neighbor. If at all, the agent would sensibly first deviate from his deficient relation, that is from his relation with $i+1$, and – as late as possible, since deviating from a bilaterally non-deficient relation is a cost – from his other relation. This would be after $c-2$ periods. Therefore deviation will not be profitable if

$$\delta^{c-2} g^{i,i-1} + g^{i,i+1} \geq 0 \qquad \forall i \in \mathcal{N}^S \text{ and } \{i-1, i+1\} = N_i.$$

Since every agent i in the network would want to deviate bilaterally from his relation with $i+1$, was it not for the threat of the loss of cooperation in her other relation, after losing this other relation for ever, infecting is rational and the equilibrium is subgame perfect. This is true for *any* belief about the history of the game.

Necessity: Since during the punishment phase the agents play their minimax strategy and the punishment phase is infinitely long, this is the strongest punishment available to the agents. If cooperation is not possible with these strategies, it will not be possible with other – less strong – punishments. Therefore $\delta^{c-2} g^{i,i-1} + g^{i,i+1} > 0 \; \forall i \in \mathcal{N}^S$ is also necessary for the relational network to be supportable.

Part 3 of proposition 1: Assume information structure (**I3**) and non-forgiving strategies. Suppose agent i observes a deviation of his neighbor $i-1$ in his ($i-1$'s) deficient relation. Then, since, due to the unforgiving strategies, there will never be a return to cooperation with $i-1$, the best response of i in his (i's) remaining deficient relation would be to deviate from that relation. Therefore agent i will not make use of her ability to transmit information, leaving only room for the same strategies as under (**I2**). *Q.E.D.* ■

As we will see in section 4, the only if part of part 3 of proposition 1 may depend on the agents' ability or rather inability to re-close the network.

In the next paragraph we will see that if one does not assume a non-mutual but a mixed network instead, agents that share a mutual relation may be reluctant to exercise punishments if strategies are unforgiving. This means, we will see that the equilibrium described is inrobust with respect to the inclusion of mutual relations into the network.

3.2 Mixed networks

A network that consists only of mutual relations is sustainable (by definition). Thus, replacing a unilateral relation in a network with a mutual one, one might think, should, if it changes something all, not reduce the set of sustainable networks. After all, we replace someone who has an incentive to deviate from on of her relations with someone who does not. However, this reasoning only refers to the cooperation phase of the strategies. Once one examines incentives in the punishment phase, it will be required that an agent has an incentive to deviate from cooperative behavior. Since a mutual is sustainable on a stand-alone basis and the strategies described above involve the loss of cooperation if some agent in the network deviates, segunda facie it does not seem to be that clear anymore, that the agents have an incentive to exercise the agreed multilateral punishment. In examining the effects of a replacement on a unilateral relation with a mutual one, we will again first study non-circular networks. We will see that there is a week cooperation-enhancing effect. We will then turn to circular networks.

3.2.1 Non-circular networks with unforgiving punishments

In the previous section, we have seen that there does not exist a non-circular non-mutual network. This was the case because there would be an agent at the end vertex of the non-circular network who would have a deficient relation and since he only faces punishment from one side, he has an incentive to deviate from that relation. Once one takes an agent as an end vertex who has a mutual relation, as in figure 6 (a), this incentive to deviate in the cooperation phase should vanish. Under full information, then, agent e_1 would know everything agent m_1 does in her interactions and so a multilateral punishment like (**S1**) could be agreed on. Part 1 of proposition 2 states that. Part 2, on the other hand, retains the result of lemma 1. Part 3 of that proposition argues that the equilibrium of Part 1 does not satisfy reasonable stability criteria put forward by Kohlberg and Mertens (1986). In particular, if their *Iterated Dominance* criterion is applied, there exists a profitable deviation from a strategy like (**S1**) for the agent adjacent to the end vertex.

Proposition 2 *Suppose* $\deg i \leq 2$. *Then*

1. *under information structure* (**I1**), *a non-circular network* $\mathcal{N}^{\mathcal{S}}$ *is sustainable if*

   (a) $\mathrm{id}\, i\,|_{\deg i=1} = 1$ *and*

   (b) *for all other agents in the network* $g^{i,i-1} + g^{i,i+1} > 0$ *holds, and*

2. *under information structures* (**I2**) *and* (**I3**), *there exists no sustainable non-circular mixed network.*

3. *If the network under* (**I1**) *relies on unforgiving punishments, it is not strategically stable.*

Proof. Part 1 of proposition 2: Consider again (**S1**) . Assumption (b) rules out the possibility that an agent has $\text{od}\, i > 1$. Therefore, all agents with $\deg i = 2$ face immediate punishment after deviating from both sides and have no incentive to deviate if $g^{i,i-1}+g^{i,i+1} > 0$. The only agents that might have an incentive to deviate then are the ones with $\deg i = 1$, the end vertices of the network, which have no incentive to deviate if their indegree is 1.

Part 2 of proposition 2: Under (**I2**) or (**I3**), enforcement relies on contagion or transmission of information about past actions through the agents. In a non-mutual subnetwork of a non-circular network, no agent i would get punished by another agent j than the one from whose relation she is deviating. Agent j will not be infected or be informed about the deviation by anyone, respectively. This is because i is the only one who could infect or inform j, respectively. Therefore it is a dominant strategy of any agent $i \in \mathcal{N}^{\mathcal{S}}$ to defect to any neighbor $k \in N_i$ if $g^{ik} \leq 0$.

Part 3 of proposition 2: *Unforgiving* punishment in our framework means to play according to (**S1**), i.e. to play D on both sides *forever* if a deviation occured in the network.

As argued above, ruling out the play of strictly dominated strategies gives rise to a profitable deviation for each agent i of the mutual subnetwork who is also part of a non-mutual subnetwork. Let agent m_1 in figure 6 (a) defect only from her relation with m_2. Then sticking to the multilateral punishment mechanism ($S1$), is part of a strictly dominated strategy for m_1. It is strictly dominated by the strategy "defect from both relations and then stick to the multilateral punishment mechanism". Thus, if agent e_1 observes m_1 deviate only from her relation with m_2, he knows that he does not want to stick to the punishment. Given that m_1 played $C^{m1,e1}$, there exists a focal equilibrium. This focal equilibrium is to switch to a bilateral punishment mechanism, the normal grim trigger strategy. Since going on to cooperate is in e_1's own interest, he should go on playing grim trigger. The resulting – stable – equilibrium is the same as the one under ($I2$) and ($I3$), scetched in figure 6 (b). This gives rise to a profitable deviation for agent m_1.*Q.E.D.* ■

Figure 6 illustrates proposition 2. If agent e_2 has the possibility to tell m_3 about m_4 having deviated and deviating to both e_2 and m_3 is not profitable for m_4, this network is supportable. This is the case under (**I1**), thus part 1 of proposition 2 says given (**I1**), figure

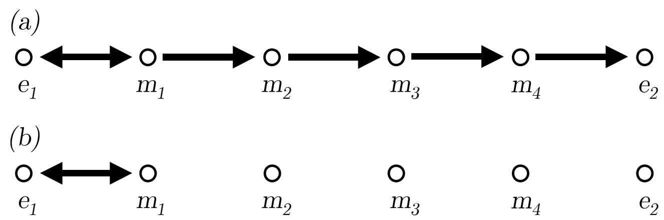


Figure 6: Sustainable networks under (a) info structure (I1), (b) info structure (I2) and (I3)

6 (a) is an equilibrium. It is not the case under (**I2**) and (**I3**), thus part 2 of proposition 2 says given (**I2**) or (**I3**), figure 6 (a) is not an equilibrium. The equilibrium network in that case would be figure 6 (b).

However, there is a caveat. The resulting network under (**I1**) is not strategically stable. The mutual interest in cooperation, which made cooperation of all agents in the non-circular network an equilibrium, puts it on weak feet as it makes it unlikely to be selected as the equilibrium played.

3.2.2 Circular networks with unforgiving punishments

We now turn to circular networks. We will start with a sustainable non-mutual network and replace one of the unilateral relations by a mutual one. We will see that the agent who net-gains from both sides thereby is being given an incentive to deviate from the punishment if punishment involves playing the stage Nash-equilibrium with both neighbors forever. We will also see that rewarding punishments may heal this.

Under full information, all members of the network observe a deviation and can therefore enter a punishment phase immediately. Since punishment involves playing the static Nash equilibrium forever, in expectation of the punishment, a deviator will play play according to the punishment no matter whether the relation is a mutual or non-mutual one. This leads to proposition 3, part 1.

Under the other two information regimes however, it is not possible to identify the initial deviator. The contagious equilibrium, given by strategies (**S2**), in the case of a non-mutual circular network, thus relied on the fact that, each agent that has been cheated on by a neighbor, had an incentive to carry out the punishment on the deficient side. If we introduce a mutual subnetwork, there exist agents who do not have a deficient relation. These agents may be reluctant to enter into an punishment phase immediately if they observe a deviation on only one side. This leads to proposition 3, part 2.

Proposition 3 *In a non-mutual circular network of size c with* $g^{i,i+1} \leq 0$ *and* $g^{i,i-1} \geq 0$ $\forall i \in \mathcal{N}^S$*, let* $\underline{\delta} \equiv \left\{ \delta | \, g^{i,i+1} + \delta^{c-2} g^{i,i-1} = 0 \right\}$*. Replace the unilateral relation between* i *and* $i+1$ *with a mutual one.*

1. *Then, under information structure* (**I1**)*,*

   (a) the resulting unidirected network is still sustainable

   (b) but not strategically stable.

2. *Denote with* $\underline{\underline{\delta}}$ *the minimum discount factor necessary to sustain the resulting network under* (**I2**) *and* (**I3**) *with strategy and belief profiles* (**S2**)*. Then*

   (a) for sufficiently low $l^{i,i+1}$ ***or*** *sufficiently high* $w^{i,i+1}$*,* $\underline{\underline{\delta}} = \underline{\delta}$*.*

   (b) for insufficiently low $l^{i,i+1}$ *and insufficiently high* $w^{i,i+1}$*,* (**S2**) *does not result in a sustainable network.*

   (c) if we require strategic stability, a low $w^{i,i+1}$ *is sufficient for the breakdown of the network.*

Proof. Part 1 (a): The optimality of the actions during a punishment phase proposed in part 1 of the proof of proposition 1 only depended on the fact that the strategies played by the deviator and his neighbors were in fact a stage Nash equilibrium. Since we have full information, everybody knows everybody elses history and expecting the other to stick to the prescribed strategy (**S1**), would lead to playing D^{ij} whenever a deviation is observed.

Part 1 (b): The proof parallels the one for proposition 2 part 3.

Part 2 (a) through (c) we relegate to the appendix. *Q.E.D.* ■

The intuition for parts 2 (a) and (b) is the following (refer to figure 7): With beliefs specified in the appendix, if agent i in figure 7 observes $D^{i-1,i}$ and $C^{i+1,i}$ in $t = \tau$, he assigns probability $\frac{1}{c-1}$ to the event that any of the other agents in the network started to deviate. Then, the bigger the network becomes, the more likely it is a priori that the agent that started the contagious process is an agent other than $i+1$ and $i+2$. Since in this case, $i+1$ will not play $D^{i+1,i}$ until $t = \tau + 2$, and since the net gain from cooperating with $i+1$ is positive for i, for a big size of the network, it is not a best response to play $D^{i,i+1}$ in $t = \tau + 1$. However, for agent i, with probability $\frac{1}{c-1}$ agent $i+1$ started. Because of that, if the loss from playing $C^{i,i+1}$ if $i+1$ plays $D^{i,i+1}$, $l^{i,i+1}$ is high enough, the expected payoff from carrying out the punishment may be higher than the one from going on cooperating for one more period. Furthermore, for agent i, with probability $\frac{1}{c-1}$ agent $i+1$ started. In that case, agent i expects $D^{i+1,i}$ from $t = \tau + 2$ on. Then, if the payoff from playing $D^{i,i+1}$ in $t = \tau + 1$, i.e. $w^{i,i+1}$, is very high in comparison to the payoff from playing $C^{i,i+1}$, agent i might also prefer to punish immediately.

The intuition for part 2 (c) is the following: Strategic stability rules out the belief that agent $i+1$ started and then sticks to the multilateral punishment since this is strictly dominated by having played $D^{i+1,i}$ in $t = \tau$. This only leaves a high $w^{i,i+1}$ as a reason to carry out punishments immediatly.

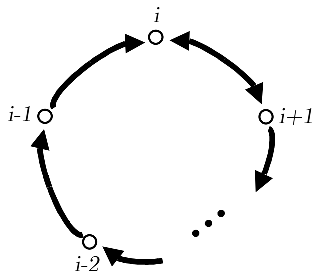


Figure 7: Circular network with a mutual relation

Proposition 3 resembles an everyday intuition: An agent, who benefits from everybody, hurts cooperation because he might be unwilling to punish. But we can say even more if we restrict our attention to equilibria fulfilling the strategic stability criteria put forward by Kohlberg and Mertens.

This brings us to a discussion about stability and self-enforcement of the equilibria described so far in this section.

3.2.3 Circular networks with forgiving punishments

Network information transmission Note that (**S2**) does not make use of the possibility to transmit information on observed behavior and on transmitted information offered by ($I3$). Due to that, the results in both informational regimes do not differ. Note also again, that not to transmit information is an equilibrium choice of an agent if, as in (**S2**), the punishment phase lasts forever and thus a deviation from a would-be prescribed transmission of information is not costly for an agent. One result of that is a complete breakdown of the network during the punishment phase. That holds also if one considers a change in (**S2**) such that the reversion to the stage Nash equilibrium does not last forever but only for T periods. Agents will chose to infect instead of sending information and keeping up cooperation. Another result is that even if information could be transmitted with a high speed and therefore induce an earlier punishment, relaxing the incentive constraint of the agents in the network, this potential is left unused.

There are two different avenues to follow if a punishment phase should not comprise of a complete breakdown of the network. One is to close the network without the agent that deviated. A second one is to reward an agent for transmitting information instead of infecting her neighbor and to punish harshly if infection occurs. That second avenue makes use of transmitting information and, thereby, relaxes the agents' incentive constraint, allowing a sustainable network for a lower δ than (**S2**).

For that end, let us define the following strategy profile:

Strategy profile S3

1. Agents start by playing C^{ij} $\forall i \in \mathcal{N}^{\mathcal{S}}, \forall j \in N_i$.

2. As long as any agent i observes C^{ji} $\forall j \in N_i$, and as long as no message containing D^{mn} for any $m \in \mathcal{N}^S$, agent i goes on playing C^{ij} $\forall j \in N_i$.

3. If agent i observes D^{ji} for any $j \in N_i$ and she received no message about an earlier defection of j, agent i then sends a message about the deviation to her other neighbor and plays D^{ij} until j and i played D^{ij}, C^{ji} for T periods. After that i sends her other neighbor a message about the end of the punishment phase for player j and they go back to 2. thereafter. Each agent truthfully passes on the messages.

4. If a neighbor k of j receives a message about j's initial deviation, she plays D^{kj} until both, she receives the message that D^{ij}, C^{ji} has been played for T periods and D^{kj}, C^{jk} has been played for T periods. She returns to 2. thereafter.

5. If agent j played D^{ji}, she plays C^{ji} for the next T periods, D^{jk} in the period when k receives the information on her initial deviation and C^{jk} for the next T periods. She returns to 2. thereafter.

6. If some agent deviates from the actions in 3. – 6., the punishment starts against this agent.

For notational convenience the following definition will be useful.

Definition 7 *We define a function*

$$\theta(c,v) \equiv \begin{cases} \max\left\{\frac{c-2}{v},1\right\} & if\ int\left(\frac{c-2}{v}\right)=\frac{c-2}{v} \\ \max\left\{int\left(\frac{c-2}{v}+1\right),1\right\} & if\ int\left(\frac{c-2}{v}\right)\neq\frac{c-2}{v} \end{cases}.$$

This function maps the order of the cycle c and the speed of information transmission v into the strictly positive natural numbers and indicates the period in which an information about play between agents i and $i+1$ in period 0 reaches agent $i-1$.

Proposition 4 *In a non-mutual circular network of size c with $g^{i,i+1} \leq 0$ and $g^{i,i-1} \geq 0$ $\forall i \in \mathcal{N}^S$, let $\underline{\delta} \equiv \left\{\delta | \, g^{i,i+1} + \delta^{c-2} g^{i,i-1} = 0\right\}$. Let $\widetilde{\Delta}$ be the set of δ for which – together with an appropriate T – $(S3)$ constitutes a sustainable non-mutual network with $g^{i,i+1} \leq 0$ and $g^{i,i-1} \geq 0$ under* (**I3**) *and $\widetilde{\delta} = \min\left\{\widetilde{\Delta}\right\}$. Then*

(i) $\widetilde{\delta} \leq \underline{\delta}$ with a strict inequality for high speeds of information transmission, i.e. for $v > 1$.

(ii) the network is still sustainable and strategically stable $\forall \delta \in \widetilde{\Delta}$ for any l if one substitutes non-mutual subnetworks for mutual ones.

For the proof, which we relegate to appendix , there are four incentive constraints to consider:

1. Every agent has to have an incentive to stick to C^{ij} $\forall j \in N_i$ as long as neither he observes D^{ji} for a $j \in N_i$ nor he receives a message containing D^{mn} for an $m \in \mathcal{N}^S$. $\left(IC^{CI}\right)$
2. Given one neighbor m of i played $D^{m,i}$, each agent j has to have an incentive to send a message containing $D^{m,i}$ her other neighbor n and stick to $C^{i,n}$. $\left(IC^{CII}\right)$
3. Every neighbor of an original cheater has to have an incentive to carry out the punishment. $\left(IC^{P}\right)$
4. Every original cheater has to agree to be punished. $\left(IC^{LP}\right)$

We first show that $\left(IC^{CII}\right)$ and $\left(IC^{P}\right)$ are never binding. Using $\left(IC^{LP}\right)$ and $\left(IC^{CI}\right)$, we then show that, for a speed of $v = 1$, by choosing an appropriate lenght T of the punishment, the conditions for cooperation can be made equivalent to the ones for (**S2**). Increasing the speed of information transmission reduces the delay $\theta(c,\nu)$ and, thus, relaxes $\left(IC^{LP}\right)$ which in turn gives room to make punishment more severe. This establishes *(i)*. Since agents are being rewarded for punishing their neighbor, they always have an incentive to do so during a punishment phase even if they want to cooperate bilaterally, which establishes *(ii)*.

It is worth pointing out that again a pooling of asymmetries across agents will under some parameter constellations lead to a sustainable network and, thus, to cooperation where it would be impossible with bilateral implicit contracts.

Proposition 4 also shows that it is not necessary to have a complete breakdown of cooperation in the network in case of a deviation if information about past actions can be transmitted. The equilibrium is, thus, also more robust against mistakes of players and increases welfare during punishment periods.

Perfect information transmission Since under the perfect information transmission regime (**I1**) the initial cheater is known, the complete breakdown of the network in a punishment phase can be avoided by similar punishments as in (**S3**): All neighbors $j \in N_i$ of an initial cheater i start playing $D^{j,i}$ until i has played $C^{i,j} \forall j \in N_i$ for T periods and then they go back to plaing $C^{i,j}, C^{j,i}$. In all other games in the network, the players go on playing the cooperative action during the punishment phase for player i. As the initial cheater can always get his minimax payoff forever, which is the payoff from the punishment in (**S1**), the biggest T, for which this strategy profile is an equilibrium, gives him exactly this payoff. Therefore, these strategies result in the same set of equilibria as (**S1**).

No information transmission While strategy profile (**S3**) avoids the breakdown of the network due to mutual subnetworks for (**I3**), it can not be used for (**I2**) since it makes use of the transmission of information. Without the transmission of information, it is impossible to know, who deviated from the equilibrium path first. Without this, a targeted punishment of only the original deviator becomes impossible.

4 Circular networks with exclusion and re-closure

In the strategy profiles used so far, the members of possible networks and therefore the size of such networks were fixed. Strategies ($S1$) and ($S2$) result in the complete breakdown of the network in case of a single deviation. ($S3$) on the other hand, features a hard, shorter punishment period where only the initial deviator and his neighbors are required to stop cooperating with each other during the punishment phase in their games. All original members of the network, including the cheater, formed a network again after the punishment phase. The results obtained, thus, apply both, to situations without an exogenous geography and to situations where there is an exogenously given natural neighborship relation for each member of the network, i.e. a geography.

In this section, we consider a commonly used punishment both, in reality and in the networks literature: The permanent exclusion of a network member from the network together with the assumption that the remaining members close the gap in the network. By definition, the results in this section will therefore *not apply* to situations with an *exogenously given geography*. This is the case because punishments involve a change in the shape of the network.

Strategies will involve a recursive element because defections are deterred by the creation of a new network. If this new network is not sustainable, there is no deterrence. Therefore, also a deviation from this new network - if the same punishment is applied - has to be deterred by the existence of a sustainable network.

As before, we also examine exclusion for the three information transmission regimes. Throughout the section we assume unilateral networks. We will first define exclusion, we will then show that pure exclusion equilibria do not exist in this environment, and finally we show that almost pure exclusion equilibria do not sustain networks with lower discount factors than the mechanisms examined before.

Definition 8 (Punishment by exclusion) *We define punishment by exclusion as the permanent choice of a cheater's neighbors to play the non-cooperative action w.r.t. the cheater and the permanent choice of the neighbors to play the cooperative action w.r.t. each other.* ***Pure exclusion*** *strategies deter deviations in every subgame with exclusion.* ***Almost pure exclusion*** *strategies deter deviations in every subgame except those with networks of size 3 with exclusion.*

There is one obvious drawback of these strategies: In order to be able to link to the neighbor of the neighbor who cheated on a player, this player has to know who is the neighbor of that cheater. This requires information on the history of his neighbor's play, which he does not have under (**I2**). We can therefore exclude the no information case from our analyis in this subsection.

Lemma 2 *Under* ($I2$)*, no network is sustainable using exclusion.*

With (**S4**), we formalize pure exclusion strategies.

Strategy profile (S4)

1. Players $k \in \mathcal{N}^{\mathcal{S}}$ start by playing C^{kj} $\forall k \in \mathcal{N}^{\mathcal{S}}, \forall j \in N_k$.

2. Each player k goes on playing C^{kj} $\forall j \in N_k$ as long as no deviation by any player in the network is observed.

3. If an agent i played $D^{i,j}$,

   (a) her neighbors $j \in N_i = \{i+1, i-1\}$ will play $D^{j,i}$ forever and form a link $i-1, i+1$ and

   (b) all $k \in \mathcal{N}^{\mathcal{S}}_{-i} \equiv \mathcal{N}^{\mathcal{S}} - \{i, i-1; i, i+1\} + \{i-1, i+1\}$ go to point 1.

Lemma 3 *No non-mutual, circular network can be sustained by the pure exclusion strategies* (**S4**).

Proof. We give the proof for the full information case (**I1**). Consider figure 5 on page 13 and strategies (**S4**). For (**S4**) to be an equilibrium, we need the following conditions to hold:

i. For any $i \in \mathcal{N}^{\mathcal{S}}$ we must have

$$g^{i,i+1} + g^{i,i-1} \geq 0.$$

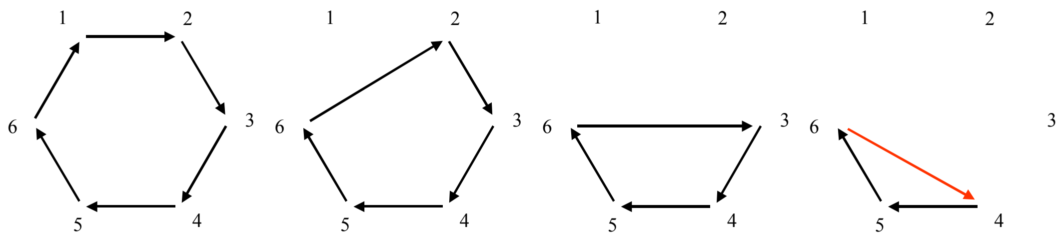
ii. For $i+1$, the link with $i-1$ has to be non-deficient as $g^{i+1,i+2} < 0$.

iii. For $i-1$ and $i+1$, we need

$$\begin{aligned} g^{i-1,i+1} + g^{i-1,i-2} &\geq 0 \\ g^{i+1,i+2} + g^{1+1,i-1} &\geq 0. \end{aligned}$$

iv. Points (i) through (iii) must hold for any member of any network $\mathcal{N}^{\mathcal{S}}_{-i}$ and any member of any reduced network thereof.

The consequence of equilibrium condition iv. is that the smallest reduced network thinkable, i.e. each bilateral relation in the original network must be sustainable. To see the implication of condition ii. together with condition iv., remember that the deviation of any member of the network has to be deterred. Also remember that the deviation of any member of the resulting shrinked network has to be deterred. This requires not only $g^{i+1,i-1} > 0$, but also $g^{i+2,i-1} > 0$, $g^{i+3,i-1} > 0$, and so on. Consider the following figure. It represents the consequences of using pure exclusion for the deterrance of a deviation of first agent 1, then agent 2, and then agent 3. For the original network to be sustainable, the second one has to be sustainable. For its sustainability, the third one has to be sustainable, and so on. Once arrived at the triangular network, the deviation of e.g. 6 has to be deterred – and thus taking the punishment literally – the relation between 4 and 5 has to be mutual, which is a violation of the assumption of a non-mutual network. The same has to hold for any other relation in the network.



Q.E.D. ■

Let us formalize strategies using almost pure exclusion. Lemma 3 implies that a strategy that involves exclusion as defined above has to include at some point in time other punishments as well. One way of doing that is to make the punishment dependent on the size of the remaining network, that is to focus on *almost pure exclusion.* Strategies $(S4')$ make the assumption that the punishment changes to defection with all neighbors if the residual network is triangular.

Strategy profile $(\mathbf{S4}')$

1. Players $k \in \mathcal{N}^{\mathcal{S}}$ start by playing C^{kj} $\forall k \in \mathcal{N}^{\mathcal{S}}, \forall j \in N_k$.

2. Each player k goes on playing C^{kj} $\forall j \in N_k$ as long as no deviation by any player in the network is observed.

3. If an agent i played $D^{i,j}$,

   (a) her neighbors $j \in N_i = \{i+1, i-1\}$ will play $D^{j,i}$ forever

   (b) if $size\left(\mathcal{N}^{\mathcal{S}}_{-i}\right) \geq 3$, $\mathcal{N}^{\mathcal{S}}_{-i} \equiv \mathcal{N}^{\mathcal{S}} - \{i, i-1; i, i+1\} + \{i-1, i+1\}$, her neighbors $j \in \{i+1, i-1\}$ will form a link $i-1, i+1$ and all agents $k \in \mathcal{N}^{\mathcal{S}}_{-i}$ go to point 1.

   (c) if $size\left(\mathcal{N}^{\mathcal{S}}_{-i}\right) < 3$, every agent k reverts to D^{kj} $\forall j \in N_k$ forever.

Proposition 5 *Let* $\widehat{\delta} \equiv \{\delta | \, g^{i,i-1} + g^{i,i+1} = 0\}$. *Let* $\widehat{\widehat{\delta}}$ *be the minimum discount factor necessary to sustain a network with* $(\mathbf{S4}')$*under* $(\mathbf{I1})$. *Then*

1. $\widehat{\widehat{\delta}} \leq \widehat{\delta}$.

2. *the network is not strategically stable.*

Proof. Part 1.: (**S1**) punishes a deviation immediately with the strongest possible punishment, i.e. the one that gives the cheater his minimax payoff forever. It is, therefore, not possible to decrease the delay until punishment takes place and the strengh of the punishment.

Part 2.: We first give the equilibrium conditions. These are the same as for (**S4**), with a slight change in iv.

i. For any $i \in \mathcal{N}^{\mathcal{S}}$ we must have

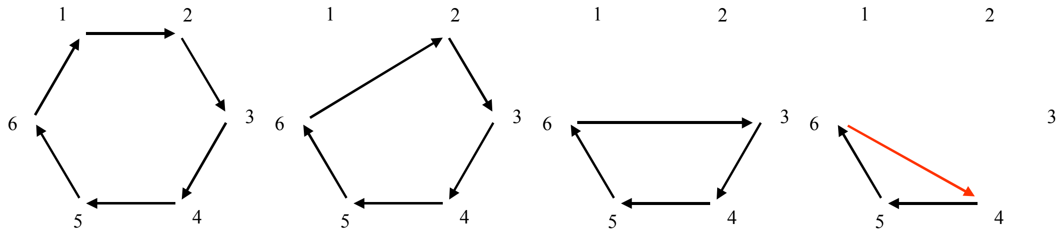
$$g^{i,i+1} + g^{i,i-1} \geq 0.$$

ii. For $i-1$ and $i+1$, we need

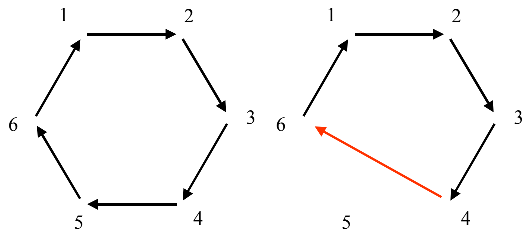
$$\begin{aligned} g^{i-1,i+1} + g^{i-1,i-2} &\geq 0 \\ g^{i+1,i+2} + g^{1+1,i-1} &\geq 0. \end{aligned}$$

iii. Points i. and ii. must hold for any member of any network $\mathcal{N}^{\mathcal{S}}_{-i}$ and any member of any reduced network thereof except the triangular networks. In the triangular one, only i. has to hold.

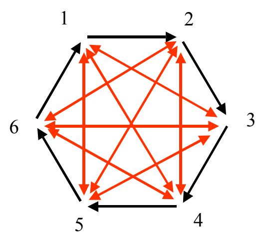
It is condition ii. together with iii. that drives part 2. To see the implication of condition ii., remember that the deviation of any member of the network has to be deterred. Also remember that the deviation of any member of the resulting shrinked network has to be deterred. This requires not only $g^{i+1,i-1} > 0$, but also $g^{i+2,i-1} > 0$, $g^{i+3,i-1} > 0$, and so on. Consider the following figures. The first row represents the consequences of the deterrance of a deviation of agent 1, then 2, and then 3. We see, that the relation of 4 with 6 has to be non-deficient for 4.



The second row represents the consequences of a deterrance of a deviation of agent 4. We see now that the relation of 4 with 6 has to be non-deficient for 6.



The consequence is that all agents inside the circle have to potentially have mutual relations.



However, if all these potential relations have to be mutual, this has consequences for the strategic stability of the equilibrium as shown in proposition 3. *Q.E.D.* ■

With (**S4′**), as with restitution punishments, agents enjoy the advantage of avoiding the breakdown of the network during a punishment phase. Thus, there is a utility gain compared with (**S1**). Note that for (**I1**), there is no increase in stability thanks to the shrinked network after a deviation. However, compared to restitution punishments (similar to (**S3**)), the neighbors of a cheater lose utility - a payback of the damages is not done. Furthermore, if the network fulfills any other function, such that the size of the network matters for overall welfare, there is a loss in welfare compared to restitution punishments due to the exclusion of the cheater.

Let us think about network information transmission (**I3**). In Lemma 2, we have stated that lack of knowledge about the players in the network prevents players under (**I2**) from closing the network in a punishment phase and therefore from using any form of exclusion defined in Definition 8. This knowledge could be created under (**I3**) if strategies prescribe to pass on info on your play in cooperation periods. A modification of the strategy profile (**S4**) accounts for this.

Strategy profile (**S4′′**)

1. Players $k \in \mathcal{N}^{\mathcal{S}}$ start by playing $C^{kj}\ \forall k \in \mathcal{N}^{\mathcal{S}}, \forall j \in N_k$ and transmitting information on his play and received messages to each neighbor $j \in N_k$.

2. Each player k goes on playing $C^{kj}\ \forall j \in N_k$ as long as he observes $C^{jk}\ \forall j \in N_k$, and as long as he does not observe $C^{ik}, i \notin N_k$.

3. If an agent k observes $D^{j,k}$ without having played $D^{k,\cdot}$ before, and if *size* $\left(\mathcal{N}^{\mathcal{S}}_{-j}\right) \geq 3$,

   (a) her neighbor k will

      i. play $D^{k,j}$ forever,

      ii. play $C^{k,i}$ w.r.t. j's other neighbor $i \in N_j$

   (b) If an agent $i \notin N_k$ observes $C^{k,i}$ he will

      i. play $D^{i,j}$ with agent $j \in N_k, j \in N_i$ forever.

      ii. play $C^{i,k}$ starting from the next period

   (c) all agents $k \in \mathcal{N}^{\mathcal{S}}_{-j}$ go to point 1.

4. If an agent k observes $D^{j,k}$ without having played $D^{k,\cdot}$ before, and if *size* $\left(\mathcal{N}^{\mathcal{S}}_{-j}\right) < 3$, then he reverts to $D^{kj}\ \forall j \in N_k$ forever.

Note again that a closure of the network – after excluding a defector – by agents formerly not connected requires that there is *no underlying geography* for the network, i.e. that agents are able to do so. Furthermore, there will be additional conditions to fulfill for these strategies to be an equilibrium.

Proposition 6 *Assume information structure* (**I3**).

1. *Let $\widetilde{\Delta}$ be the set of δ for which – together with an appropriate T – (**S3**) constitutes a sustainable non-mutual network with $g^{i,i+1} \leq 0$ and $g^{i,i-1} \geq 0$ under (**I3**) and $\widetilde{\delta} = \min\left\{\widetilde{\Delta}\right\}$. Let $\widetilde{\widetilde{\Delta}}$ be the set of δ for which (**S4**$''$) constitutes a sustainable non-mutual network with $g^{i,i+1} \leq 0$ and $g^{i,i-1} \geq 0$ under (**I3**) and $\widetilde{\widetilde{\delta}} = \min\left\{\widetilde{\widetilde{\Delta}}\right\}$. Then if $l^{i,i-2}$ is not too small, if ν is not too high, and if all potential relations between members of the network, which are not links in the network, are* mutual, $\widetilde{\widetilde{\delta}} < \widetilde{\delta}$.

2. *Sustainable networks resulting from (**S4**$''$) are not strategically stable.*

Proof. Before showing the two parts of the proposition, we give the conditions for sustainability of the network. As before, we assume optimal deviations given the punishment.

i. No agent has to have an incentive to deviate from the cooperative action:
$$g^{i,i+1} + \delta g^{i,i-1} \geq 0.$$

ii. An agent i who has been cheated on by an agent $i-1$ has to have an incentive to play $C^{i,i+1}$ and $C^{i,i-2}$:
$$g^{i,i+1} + (1-\delta)\, l^{i,i-2} + \delta c^{i,i-2} - d^{i,i-2} \geq 0,$$

iii. and to go on playing that in the next period:
$$g^{i,i+1} + \delta g^{i,i-2} \geq 0.$$

iv. Any agent $i-2$ who observes $C^{i,i-2}$ from a member of the network who is not his neighbor has to have an incentive to play $C^{i-2,i}$:
$$g^{i-2,i} + \delta g^{i-2,i-3} \geq 0.$$

v. Points i. through iv. must hold for any member of $\mathcal{N}^{\mathcal{S}}$ and of any network $\mathcal{N}^{\mathcal{S}}_{-i}$ and any member of any reduced network thereof except the triangular networks. In the triangular ones, only i. has to hold.

Part 1.: In this equilibrium, permanent Nash reversion of $i+1$ arrives immediately. Permanent Nash reversion of $i-1$ arrives after one period. With strategy profile (**S3**), a punishment of $i+1$ as strong as permanent Nash reversion arrives immediately. The punishment of $i+1$ as strong as permanent Nash reversion arrives after $\theta(c,\nu)$ periods. Since $\theta(c,\nu)$ is decreasing in ν, for low ν condition i. is less strict than the equivalent condition for (**S3**).

Conditions iii. through v. imply, similarly to conditions ii. and iii. from (**S4′**), that all potential relations between members of the network, which are not links in the network, have to be *mutual*.

Condition ii. is only less stringent than condition i. if $l^{i,i-2}$ is not too low.

Part 2.: As conditions iii. through v. imply that all potential relations between members of the network, which are not links in the network, have to be *mutual*, it is possible to deviate suboptimally in a network $\mathcal{N}_{-i}^{\mathcal{S}}$ which makes a punishment of an agent in that mutual relation by the other agent in that mutual relation a dominated action. *Q.E.D.* ■

Part 1 of proposition 6 says that under certain conditions *more* networks are possible than with the forgiving, hard punishments from strategy profile (**S3**). However, these *certain conditions* are quite restrictive: all potential relations between members of the network, which are not links in the network, have to be mutual, the loss from playing C if your partner plays D has to be not too low, and the speed of information transmission has to be not too high. The first of the restrictions causes, in addition, the network to be not strategically stable, and thus (**S4″**) unlikely to be chosen as equilibrium strategies.

5 Sustainable networks of higher degree

In this section we show that the results we obtained for the simple networks above generalize for networks in which agents have more than two neighbors. For this end, we will use a c-cycle as a basic structure and add a link such that there now exist two subnetworks that share the added relation.

The underlying structure of the stage game is a prisoners' dilemma and maintaining a relation as such is not costly. This means that the utility agents receive from having a relation, as compared to not having it, is always bigger. If it was not for the incentive problem, agents would always choose to cooperate in all their interactions. Adding a relation to the network benefits the agents who add this relation. Therefore, if we allow for a higher degree of agents, they will have an incentive to add relations, including even *bilaterally deficient* ones, as long as this results in a sustainable network, given a basis structure.

Furthermore, the lower the discount factor of agents in a network, the more difficult is it to sustain a network of relations where information travels with delay or where information

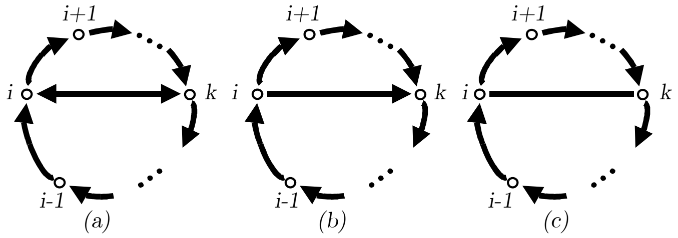


Figure 8: Adding a relation to a circular, non-mutual network

cannot "travels" via contagion. In these networks it disciplines to have cycles of smaller order and thereby shorten ways.

In the remainder of the section, we consider for each of the three informational regimes, (**I1**) – (**I3**), adding to a non-mutual circular network a bilaterally deficient, a unilateral, and a mutual relation, one at a time.

Full information (**I1**) Let us consider the full information (**I1**) paradigm. In this case, every agent immediately knows about a deviation. With strategy profile (**S1**), a deviating agent will face immediate Nash reversion from all neighbors and rationally deviate only if it pays to deviate from all relations. It is therefore straightforward to generalize proposition 1 part 1 in the following proposition stated without proof.

Proposition 7 *Assume* (**I1**) *and the strategy profile* (**S1**). *Then the network is sustainable iff*

$$\sum_{j \in N_i} g^{ij} > 0 \qquad \forall i \in \mathcal{N}^{\mathcal{S}}. \tag{1}$$

As long as (1) holds, even bilaterally deficient relations can be and will maintained in equilibrium. However, while this is an equilibrium, the same forward induction caveat that applied to proposition 1, part 1 also applies here.

Consider first figure 8, networks (b) or (c). Since ik is a deficient relation for i, $\mathcal{N}^{\mathcal{S}}$ is only sustainable with (**S1**), if $\mathcal{N}^{\mathcal{S}} \setminus ik$ is sustainable in autarky. If this is the case, then the equilibrium $\mathcal{N}^{\mathcal{S}}$ is not forward induction-proof in the sense of Lippert (2003). If e.g., agent i deviates only from her relation with agent k, but not from his other two relations, then this deviation, together with sticking to the multilateral punishment mechanism, is strictly dominated by a simultaneous deviation from all relations and sticking to the multilateral punishment. Furthermore there is an equilibrium – $\mathcal{N}^{\mathcal{S}} \setminus ik$ – which (i) Pareto-dominates the continuation equilibrium in the punishment phase of (**S1**) and which is (*ii*) a focal point after this deviation. This is a profitable deviation, given the agents indeed coordinate on $\mathcal{N}^{\mathcal{S}} \setminus ik$, since $g^{ik} < 0$.

Consider network (a) with strategy profile (**S1**), on the other hand. This network *is* forward induction proof with (**S1**) if both, $\mathcal{N}^{\mathcal{S}}$ and $\mathcal{N}^{\mathcal{S}} \setminus ik$ are sustainable in autarky. There are six equilibrium networks: The empty network, ik, $\mathcal{N}^{\mathcal{S}} \setminus ik$, $\mathcal{N}^{\mathcal{S}} \setminus \{i, i+1, ..., k\}$, $\mathcal{N}^{\mathcal{S}} \setminus \{k, k+1, ..., i\}$, and $\mathcal{N}^{\mathcal{S}}$. The empty network is the continuation equilibrium of (**S1**) if a deviation occured. Equilibria that Pareto-dominate the empty network other than $\mathcal{N}^{\mathcal{S}}$ are ik, $\mathcal{N}^{\mathcal{S}} \setminus ik$, $\mathcal{N}^{\mathcal{S}} \setminus \{i, i+1, ..., k\}$, and $\mathcal{N}^{\mathcal{S}} \setminus \{k, k+1, ..., i\}$. The network $\mathcal{N}^{\mathcal{S}} \setminus ik$ is focal after a deviation that does not involve $i, i+1$ and $i, i-1$. Deviating from ik, even if agents then play the focal equilibrium $\mathcal{N}^{\mathcal{S}} \setminus ik$, is not profitable since ik is a mutual relation. The network $\mathcal{N}^{\mathcal{S}} \setminus \{k, k+1, ..., i\}$ is focal after i deviated from her relation with $i+1$. If this were the final outcome, the deviation would be profitable since $g^{i,i+1} < 0$. However, $\mathcal{N}^{\mathcal{S}} \setminus \{k, k+1, ..., i\}$ is not forward induction proof: ($S1$) requires to play the empty network if a deviation occurs. The network ik Pareto-dominates the empty network, and it is focal after a deviation of k from her relation with $k+1$. Given that, if in $\mathcal{N}^{\mathcal{S}}$ agents that observe agent i deviate only from his relation with $i+1$, the network $\mathcal{N}^{\mathcal{S}} \setminus \{k, k+1, ..., i\}$ will not appear, since agents other than k will anticipate k's deviation, and the network ik will emerge immediately after i's initial deviation. Since we assumed $\mathcal{N}^{\mathcal{S}} \setminus ik$ to be sustainable with (**S1**), $g^{i,i+1} + g^{i,i-1} > 0$, the deviation is not profitable and network (a) is forward induction-proof.

No information transmission (**I2**) Again refer to figure 8. Consider network (a). Obviously, if both subnetworks ik and $\mathcal{N}^{\mathcal{S}} \setminus ik$ were sustainable in autarky, by treating the subnetworks separately, adding ik to $\mathcal{N}^{\mathcal{S}} \setminus ik$, will of course result in a sustainable network. However, $\mathcal{N}^{\mathcal{S}} \setminus ik$ does not have to be sustainable on its own: If $g^{i,i+1} + \delta^{c-2} g^{i,.i-1} < 0$ and $g^{i,i+1} + \delta^{m-2} g^{i,k} + \delta^{c-2} g^{i,.i-1} > 0$, where m is the size of the subnetwork $\{i, i+1, ..., k\}$, adding ik will make the network sustainable if both, i and k have, given their beliefs, an incentive to contribute to a multilateral punishment using their mutual relation.

Proposition 8 *Let a network $\mathcal{N}^{\mathcal{S}}$ consist of a non-mutual circular network of size c, $\mathcal{N}^{\mathcal{S}} \setminus ik$, with $g^{i,i+1} \leq 0$ and $g^{i,i-1} \geq 0$ $\forall i \in \mathcal{N}^{\mathcal{S}} \setminus ik$ and a mutual relation ik between two non-adjacent agents. Let $\underline{\delta} \equiv \left\{ \delta | \, g^{i,i+1} + \delta^{c-2} g^{i,i-1} = 0 \right\}$ $\forall i \in \mathcal{N}^{\mathcal{S}} \setminus ik$. Let $\widehat{\Delta}$ be the set of δ for which, with* (**S2**) *and beliefs specified in appendix A.3, $\mathcal{N}^{\mathcal{S}}$ is sustainable, and let $\widehat{\delta} = \min \left\{ \widehat{\Delta} \right\}$. Then* for *$l^{i,k}$ and $l^{k,i}$* small enough, *$\widehat{\delta} < \underline{\delta}$.*

Proof. Assume (**S2**) and the beliefs specified in appendix A.3. Similar to the proof of proposition 3, by assuming $l^{i,k}$ and $l^{k,i}$ low enough, i's (k's) expected profit from playing C^{ik} (C^{ki}) after having observed agent $i-1$ ($k-1$) deviate is smaller than if they not only play $D^{i,i+1}$ ($D^{k,k+1}$), i.e. infect agent $i+1$ (agent $k+1$), but also $D^{i,k}$ ($D^{k,i}$), i.e. infect also agent k (agent i). Therefore punishment sets in earlier and a lower discount factor is needed to sustain $\mathcal{N}^{\mathcal{S}}$. *Q.E.D.* ■

Again, if i's (k's) loss from playing C^{ik} (C^{ki}) if k (i) plays D^{ki} (D^{ik}) is high, the expected payoff from not punishing is very low and the agents sharing the mutual relation are willing to contribute to a collective punishment mechanism.

Consider networks (b) and (c). Here, adding the relation ik, which is unilateral (bilaterally deficient), involves a trade-off. On the one hand, punishment will be faster, which relaxes the incentive constraint for each agent in the network and makes the network sustainable for lower discount factors. On the other hand, one agent (two agents) will have to sustain one deficient relation more, which tightens the incentive constraint for this agent (these agents). It is, thus, not clear whether the set of discount factors for which the network is sustainable increases or shrinks with adding the additional relation.

The conditions for sustainability of the network, which we give together with the belief structure in appendix A.3, are a straightforward generalization of the conditions we had for the simple network with $\deg(i) \leq 2$.

Network information transmission (**I3**) Again, consider network (a) and strategies (**S3**). Since (**S3**) involves transmission of hard evidence, agents only have information sets that are singletons, and thus, beliefs are not necessary to specify. For network (a) to be sustainable, the incentive constraints for agents other than i and k, are equivalent to the ones given in appendix A.2 with one change: Since the ways are shorter, $\theta(c, \nu)$ will be substituted by $\theta(m, \nu)$ for agents $j \in \{i+1, ..., k-1\}$ and by $\theta(c-m+2, \nu)$ for agents $j \in \{k+1, ..., i-1\}$. As an example for the incentive constraints for agents i and k, we give the ones for i in appendix A.4. As we see, the sustainability conditions from appendix A.2 generalize.

Consider networks (b) and (c). As under (**I2**), adding the relation ik, which is unilateral (bilaterally deficient), involves a trade-off. On the one hand, punishment will be faster, which relaxes the incentive constraint for each agent in the network and makes the network sustainable for lower discount factors. This is true for networks large enough or information transmission slow enough – such that there is a difference to full information. On the other hand again, one agent (two agents) will have to sustain one deficient relation more, which tightens the incentive constraint for this agent (these agents). It is, thus, not clear whether the set of discount factors for which the network is sustainable increases or shrinks with adding the additional relation.

6 Conclusion

In our model, agents maintain relations by using a network that, in addition to their own relations, consists of other agents' relations. We identify equilibrium conditions for different architectures of such networks, paying special attention to differences in these conditions for circular and non-circular architectures. The basic framework is that of repeated games between fixed partners with three basic information structures: complete information, no information, and information transmission through the network's links. We distinguish equilibria which make use of the creation of new links in the punishment period from those that do not.

We show that if agents cannot discipline themselves within a certain relation, pooling asymmetries in payoffs can sustain the relation under these three informational assumptions. In contrast to previous literature, the possibility to transmit information about the cheating of someone through the links in the network has not been an equilibrium action if enforcement relied on unforgiving punishment. With unforgiving punishment, the deviation of an agent starts a contagious process that eliminats cooperation in the network. We showed that with more complex punishment strategies, agents use information transmission, and thereby keep on cooperating in the rest of the network while punishing the deviator – which increases efficiency and decreases the discount factor necessary to sustain the network. We show that, under the complete information assumption, bilaterally unsustainable relations in a non-circular network, can be supported by having self-sustaining relations at the ends of the network while this does not work for the other informational assumptions. We also showed that having self-sustaining relations in the network may actually hurt cooperation in the case without full information because agents might not be willing to perform the punishment if a suboptimal deviation occured. In this case a network may be sustainable if agents use less severe punishments than grim trigger or by rewarding the punisher. The results were finally generalized to more complex network architectures.

Possible applications of our model or of modifications thereof, include the organization of inter-firm relations in industrial districts, social capital or collusive behavior that is enforced in networks of very different players. In her much acclaimed book, Saxenian (1994) attributes a large part of Silicon Valley's success to a special culture of cooperation in that industrial district, which stems from a common background of the early workforce in that area. Our model may help explain what Saxanian calls a "culture of cooperation" with the means of economics and game theory as a network of long-term relations, each of which perhaps might not be sustainable on a bilateral basis. The main contrast of our model to Saxenian's discussion is that we do not use a "culture" but an implicit multilateral threat of retaliation to keep the members of the network cooperating so closely.

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A Proofs and beliefs

A.1 Proposition 3

First we proof that with an unforgiving punishment, cooperation may break down if we replace a unilateral relation with a mutual one. We then show that for $U^i\left(C^{ij}, D^{ji}\right)$ in the mutual relation small enough, the set of equilibria will not shrink.

Proof. Part 2 (a) and (b). Consider strategies (**S2**) and beliefs as outlined above. Suppose, we are in the situation of figure 7 with agents i and $i+1$ forming a mutual subnetwork. Consider the following defection: Agent $i+1$ plays $D^{i+1,i+2}$ and after $c-2$ periods goes on playing $C^{i+1,i}$. After $c-2$ periods, say in period $t=\tau$, agent i observes $D^{i-1,i}$ and $C^{i+1,i}$. Playing $D^{i,i+1}$ in $t=\tau+1$ is rational for agent i only if she expects $i+1$ to play $D^{i+1,i}$ in $t=\tau+1$. Whether she expects this to happen, depends on her beliefs on who started the deviation. Agent i may have three possible beliefs about who defected initially.

(a) Agent $i+1$ started and deviated only from his relation with $i+2$. If agent $i+1$ after his initial deviation sticks to the strategies prescribed, he will play $D^{i+1,i}$ in $t=\tau+1$. Then it is in i's best interest to play $D^{i,i+1}$ as well. In the expected dicounted payoff, this receives a bigger weight, the lower $l^{i,i+1}$.

(b) Agent $i+2$ started: Then $i+2$ would infect $i+1$ in $t=\tau+1$, thus, no matter what agent i plays in $t=\tau+1$, agent $i+1$ will play $D^{i+1,i}$ in $t=\tau+2$. Therefore it is better to have a deviation profit in $t=\tau+1$ and play $D^{i,i+1}$. In the expected dicounted payoff, this receives a bigger weight, the higher $w^{i,i+1}$.

(c) An agent $m\in\mathcal{N}^S\setminus\{i,i+1,i+2\}$ started: The earliest period when $i+1$ would be infected by $i+2$ would be $\tau+2$. Thus i will expect $i+1$ to play $C^{i+1,i}$ at least until $t=\tau+2$. Since we assumed $g^{i,i+1}>0$, for this belief it is *not* a best response to play $D^{i,i+1}$ in $t=\tau+1$.

Since agent i does not have any information, a consistent belief is that cases (a) and (b) have occured with probability $\frac{1}{c-1}$ and case (c) with probability $\frac{c-3}{c-1}$. If c gets large, therefore, the expected payoff for agent i from deferring the punishment phase by one period may become positive.

This in turn delays the expected punishment date of an initial deviator, which leads to a breakdown of the network if $l^{i,i+1}$ is not small and $w^{i,i+1}$ is not big.

Part 2 (c). The proof parallels the one for proposition 2 part 3. *Q.E.D.* ■

A.2 Proposition 4

In the proof we first consider the incentive constraints for agents in the network not to deviate from cooperation in phase I (IC^{CI}), from cooperation with their other neighbor in phase II that is if one neighbor cheated (IC^{CII}), from punishing the original cheater in phase II (IC^{P}), and from letting the others punish when she deviated in the first place (IC^{LP}). In a second step we show that $\widetilde{\delta} \leq \underline{\delta}$. It is shown that IC^{CII} and IC^{P} are never binding, so we can concentrate on IC^{CI} and IC^{LP}. For a speed of $v = 1$, by an appropriate choice of the lenght of the punishment, the conditions for cooperation can be made equivalent to the ones for (**S2**). Increasing the speed then relaxes IC^{LP} which gives room to make punishment more severe, which establishes (i): $\widetilde{\delta} \leq \underline{\delta}$. Since agents are being rewarded for punishing their neighbor, they always have an incentive to do so during a punishment phase even if they want to cooperate bilaterally, which establishes (ii).

Proof. The following incentive constraints are to be satisfied:

1. (IC^{CI}) For each agent i, playing $D^{i,i+1}$ in $t = 0$ and $D^{i,i-1}$ in $t = \theta(c,v)$, which is her best deviation, yields $w^{i,i+1}$ in $t = 0$, $l^{i,i+1}$ for the following T periods and $c^{i,i+1}$ thereafter, as well as $c^{i,i-1}$ until $t = \theta(c,v) - 1$, $w^{i,i-1}$ in $t = \theta(c,v)$, $l^{i,i-1}$ for the following T periods and $c^{i,i-1}$ thereafter. Playing $C^{i,i+1}$ and $C^{i,i-1}$ forever yields $\frac{1}{1-\delta}\left(c^{i,i+1} + c^{i,i-1}\right)$. Summing up leads to $\left(IC^{CI}\right)$, which is the condition for (**S3**) to be a Nash equilibrium.

$$IC^{CI} \equiv \left(c^{i,i+1} - w^{i,i+1}\right) + \sum_{t=1}^{T} \delta^t \left(c^{i,i+1} - l^{i,i+1}\right) + \delta^{\theta(c,\nu)}\left(c^{i,i-1} - w^{i,i-1}\right) + \sum_{t=\theta(c,\nu)+1}^{\theta(c,\nu)+T} \delta^t \left(c^{i,i-1} - l^{i,i-1}\right) \geq 0 \quad \forall i \in N^{\mathcal{S}},\ i+1, i-1 \in N_i.$$

2. (IC^{CII}) Suppose we are in phase II and in period $t = 0$, agent $i-1$ played $D^{i-1,i}$.

   (a) Furthermore suppose $\theta(c,v) \geq T-1$. Then nothing changes in his interactions with $i+1$ from the case where $\theta(c,v) < T-1$. However in his interactions with $i-1$, I will already have returned to phase I, which means he will give up $c^{i,i-1}$ for T periods by infecting $i+1$. Thus, i is in the same situation as if he never had been cheated on by $i-1$, which means $IC^{CII} = IC^{CI}$.

   $$IC^{CII} = IC^{CI} \qquad if\ \theta(c,v) \geq T-1,$$

   (b) Suppose now $\theta(c,v) < T-1$. After observing $D^{i-1,i}$ in $t = 0$, a deviation, that is playing $D^{i,i+1}$, yields the same payoffs from the interactions with $i+1$ as in phase I. Thus the first line of IC^{CII} coincides with the first line in IC^{CI}. If in $t = 1$, agent i plays $D^{i,i+1}$ istead of sticking to cooperation and just sending a message, this results in agent $i+1$ sending a message that reaches agent $i-1$ in $t = \theta(c,v)+1$. This yields agent i a utility of $l^{i,i-1}$ until $t = \theta(c,v)+T+2$. By sticking to cooperation, she would have had a utility of $w^{i,i-1}$ from $t = \theta(c,v)+1$ until $t = T$ and of $c^{i,i-1}$ from $t = T+1$. This difference constitutes the second and third line of IC^{CII}.

   $$IC^{CII} \equiv \left(c^{i,i+1} - w^{i,i+1}\right) + \sum_{t=1}^{T} \delta^t \left(c^{i,i+1} - l^{i,i+1}\right)$$
   $$+ \sum_{t=\theta(c,\nu)+1}^{T-1} \delta^t \left(w^{i,i-1} - l^{i,i-1}\right) + \sum_{t=T}^{\theta(c,\nu)+T} \delta^t \left(c^{i,i-1} - l^{i,i-1}\right) \geq 0$$
   $$\forall i \in N^{\mathcal{S}},\ i+1, i-1 \in N_i \qquad if\ \theta(c,v) < T-1,$$

   Since

   $$IC^{CI} - IC^{CII} = \begin{cases} \sum_{t=\theta(c,\nu)}^{T-1} \delta^t \left(c^{i,i-1} - w^{i,i-1}\right) < 0 & \forall \theta(c,v) < T-1 \\ 0 & \forall \theta(c,v) \geq T-1 \end{cases},$$

   whenever IC^{CI} holds, IC^{CII} is satisfied.

3. (IC^{P}) Suppose agent i receives the message that agent $i+1$ deviated in their relation with one of their other neighbors. Then agent i has to have an incentive to punish him. Since $w^{i,j} > c^{i,j}$ together with $\left(IC^{CI}\right)$, this is always the case.

4. (IC^{LP}) Suppose we are in phase II and in period $t = 0$, agent $i-1$ played $D^{i-1,i}$. An agent $i-1$ who has cheated on i has to agree to the punishment, i.e. agree to playing $(C^{i-1,i}, D^{i,i-1})$ for T periods instead of his minimax strategy forever. After having played $D^{i-1,i}$ in $t = 0$, for agent $i-1$ sticking to punishment strategies means incurring $l^{i-1,i}$ for T periods and $c^{i-1,i}$ thereafter. It furthermore means $w^{i-1,i-2}$ in $t = \theta(c, v)$, $l^{i-1,i-2}$ for the following T periods and $c^{i-1,i-2}$ thereafter. Deviating from punishment strategies yields $d^{i-1,i}$ forever, $w^{i-1,i-2}$ in $t = \theta(c, v)$ and $d^{i-1,i-2}$ forever thereafter. The difference between these utilities (transformed to the situation where player i deviated) is represented by $\left(IC^{LP}\right)$.

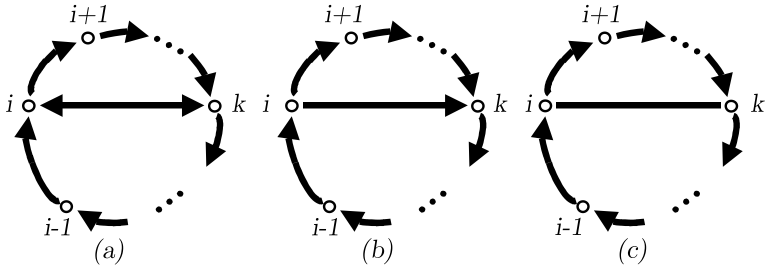
$$IC^{LP} \equiv \sum_{t=0}^{T-1} \delta^t \left(l^{i,i+1} - d^{i,i+1}\right) + \sum_{t=T}^{\infty} \delta^t \left(c^{i,i+1} - d^{i,i+1}\right)$$
$$+ \sum_{t=\theta(c,\nu)}^{\theta(c,\nu)+T} \delta^t \left(l^{i,i-1} - d^{i,i-1}\right) + \sum_{t=\theta(c,\nu)+T+1}^{\infty} \delta^t \left(c^{i,i-1} - d^{i,i-1}\right) \geq 0$$
$$\forall i \in \mathcal{N}^{\mathcal{S}},\ i+1, i-1 \in N_i.$$

Constraint $\left(IC^{CI}\right)$ consists of addends that are either strictly increasing in δ or strictly positive. Constraint $\left(IC^{LP}\right)$ is strictly increasing in δ for $\delta \in (0,1)$. Both conditions do not hold for a δ close to 0. They do hold strictly for a δ close enough to 1, thus there exists a $\widetilde{\delta}$ for which both constraints hold. Therefore under the conditions stated, strategy (**S3**) is subgame perfect for $\delta > \widetilde{\delta}$.

Since $l^{i,j} < d^{i,j}$, it is possible to fix a T such that $IC^{LP} = 0$[4]. Given that T, fix $v = 1$, such that $\theta(c, v) = c-2$. For this, IC^{CI} is satisfied for all δ that satisfy $\delta^{c-2} g^{i,i-1} + g^{i,i+1} \geq 0$. Now consider $v > 1$. Again, it is possible to fix a T such that $IC^{LP} = 0$. That ensures the same strentgh of the punishment. But now the punishment in the non-deficient relation sets in earlier which reduces the value of the deviation and therefore for $v > 1$, $\widetilde{\delta} < \underline{\delta}$. *Q.E.D.* ■

[4]That means that the punishment is as strong as if the deviator was punished with infinite reversion to the static Nash equilibium.

A.3 Belief structure and sustainability conditions for section 5, information regime (I2)



For networks (a), (b), and (c), we assume the following beliefs:
For agents $j \notin \{i, k\}$, beliefs are such that

(i) if they observe cooperation on both sides, they believe that all agents in the network cooperated so far,

(ii) if they observe a deviation on both sides, they believe that the neighbor with whom they share their deficient relation was the first to deviate, and

(iii) if they observe a deviation only from the agent with whom they share their non-deficient relation, they give an equal probability to the event that any of the other players was the first to deviate.

For agents i and k, beliefs are such that

(iv) if they observe cooperation from all neighbors, they believe that all agents in the network cooperated so far,

(v) if they observe a deviation by all neighbors, they believe that everybody in the network deviated,

(vi) if i (if k) observes agent $i-1$ (agent $k-1$) deviate, but the other neighbors cooperate, agent i (agent k) gives an equal probability to the event that any agent $j \in \{k, k+1, ..., i-1\}$ (any agent $j \in \{i, i+1, ..., k-1\}$) was the first to deviate,

(vii) if i (if k) observes agents $i-1$ and k (agents $k-1$ and i) deviate, but the other neighbor cooperate, he believes that agent k (agent i) was the first to deviate, (*viii*) if i (if k) observes agent k, agent $i+1$, or both, agents k and $i+1$, (agent i, agent $k+1$, or both, agents i and $k+1$) deviate, but the other neighbors cooperate, agent i (agent k) gives an equal probability to the event that any agent $j \in \{i+1, i+2, ..., k\}$ (any agent $j \in \{k+1, k+2, ..., i\}$) was the first to deviate, and

(*ix*) if i (if k) observes agents $i-1$ and $i+1$ (agents $k-1$ and $k+1$) deviate, but the other neighbor cooperate, agent i (agent k) gives an equal probability to the event that any agent $j \in \mathcal{N}^{\mathcal{S}} \setminus i$ (any agent $j \in \mathcal{N}^{\mathcal{S}} \setminus k$) was the first to deviate.

Let $\mathcal{N}^{\mathcal{S}} \setminus ik$ be of size c and the subnetwork $\{i, i+1, ..., k-1, k, i\}$ be of size m. Then for the beliefs given, information structure (**I2**), and $l^{i,k}$ and $l^{k,i}$ low $\mathcal{N}^{\mathcal{S}}$ is sustainable iff

$$\begin{aligned} g^{i,i+1} + \delta^{m-2}\left(g^{i,k} + \delta^{c-m} g^{i,i-1}\right) &\geq 0 \\ g^{k,k+1} + \delta^{c-m}\left(g^{k,i} + \delta^{m-2} g^{k,k-1}\right) &\geq 0 \\ g^{j,j+1} + \delta^{m-2} g^{j,j-1} &\geq 0 \ \ \forall j \in \{i+1, ..., k-1\} \\ g^{j,j+1} + \delta^{c-m} g^{j,j-1} &\geq 0 \ \ \forall j \in \{k+1, ..., i-1\} \end{aligned}$$

A.4 Sustainability conditions for agent i in section 5, information regime (I3)

1. $\left(IC_i^{CI}\right)$ During a cooperation phase, it must be profitable for i to play $C^{i,i+1}, C^{i,k}, C^{i,i-1}$ at any time, which yields $c^{i,i+1}$, $c^{i,k}$, and $c^{i,i-1}$ in each period, instead of choosing his best deviation ("static" best reply), which would be to play $D^{i,i+1}$ in $t=0$, $D^{i,k}$ in $t=\theta(m,\nu)$, and $D^{i,i-1}$ in $t=\theta(c,\nu)$ and then to face a $T-$ period punishment during which he has to endure payoffs of only $l^{i,i+1}$, $l^{i,k}$, and $l^{i,i-1}$. Such a deviation is not profitable iff

$$\begin{aligned} IC_i^{CI} \equiv \left(c^{i,i+1} - w^{i,i+1}\right) + \sum_{t=1}^{T} \delta^t \left(c^{i,i+1} - l^{i,i+1}\right) \\ + \delta^{\theta(m,\nu)}\left(c^{i,k} - w^{i,k}\right) + \sum_{t=\theta(m,\nu)+1}^{\theta(m,\nu)+T} \delta^t \left(c^{i,k} - l^{i,k}\right) \\ + \delta^{\theta(c,\nu)}\left(c^{i,i-1} - w^{i,i-1}\right) + \sum_{t=\theta(c,\nu)+1}^{\theta(c,\nu)+T} \delta^t \left(c^{i,i-1} - l^{i,i-1}\right) \geq 0. \end{aligned}$$

2. $\left(IC_i^{CII}\right)$ Suppose that agent $i-1$ deviated in $t=-1$. Agent i has to have an incentive to pass on this information in $t=0$ to both his neighbors, $i+1$ and k, instead of infecting

his neighbors $i+1$ in $t=0$ and k in $t=\theta(m,\nu)$ and then facing the punishment prescribed against himself. Again, we have to distinguish two cases depending on the speed of information transmission.

(a) If $T-1<\theta(c,\nu)$, then the information that i did not pass on the info, but cheated instead against $i+1$, reaches $i-1$ *after* i and $i-1$ have gone back to cooperation. Therefore,

$$IC^{CII} = IC^{CI} \qquad \forall\theta(c,v)\geq T-1.$$

(b) If $T-1\geq\theta(c,v)$, then the information that i did not pass on the info, but cheated instead against $i+1$, reaches $i-1$ *after* i and $i-1$ have gone back to cooperation. That means that i looses punishment profits $w^{i,i-1}$ for a number of periods equal to the difference between $T-1$ and $\theta(c,\nu)$. Therefore,

$$\begin{aligned}IC_i^{CII} \equiv& \left(c^{i,i+1}-w^{i,i+1}\right)+\sum_{t=1}^{T}\delta^t\left(c^{i,i+1}-l^{i,i+1}\right)\\ &+\delta^{\theta(m,\nu)}\left(c^{i,k}-w^{i,k}\right)+\sum_{t=\theta(m,\nu)+1}^{\theta(m,\nu)+T}\delta^t\left(c^{i,k}-l^{i,k}\right)\\ &+\sum_{t=\theta(c,\nu)+1}^{T-1}\delta^t\left(w^{i,i-1}-l^{i,i-1}\right)+\sum_{t=T}^{\theta(c,\nu)+T}\delta^t\left(c^{i,i-1}-l^{i,i-1}\right)\geq 0\\ &\forall\theta(c,v)<T-1.\end{aligned}$$

Again, we see that

$$\left(IC^I-IC^{II}\right)=\begin{cases}\sum_{t=\theta(c,\nu)}^{T-1}\delta^t\left(c^{i,i-1}-w^{i,i-1}\right)<0 & \forall\theta(c,v)\geq T-1\\ 0 & \forall\theta(c,v)<T-1\end{cases}.$$

Thus, $\left(IC^I\right)$ holds implies that $\left(IC^{II}\right)$ holds. Agent i also always has an incentive to punish a deviator immediately, thus, the equivalent to $\left(IC^P\right)$ always holds. We have to verify that $\left(IC^{LP}\right)$ holds.

3. (IC^P) Suppose agent i receives the message that agent $i+1$ (agent k) deviated in their relation with one of their other neighbors. Then agent i has to have an incentive to punish them. Since $w^{i,j}>c^{i,j}$ together with $\left(IC^{CI}\right)$, this is always the case.

4. $\left(IC^{LP}\right)$ Lastly, agent i has to have an incentive to let his neighbors carry out the punishment on him if he deviated. He can ensure himself a payoff of $d^{i,i+1}$, $d^{i,k}$, and $d^{i,i-1}$ forever by playing $D^{i,i+1}$, $D^{i,k}$, and $D^{i,i-1}$ forever. This limits the punishment available to the community.

$$IC_i^{LP} \equiv \sum_{t=0}^{T-1} \delta^t \left(l^{i,i+1} - d^{i,i+1}\right) + \sum_{t=T}^{\infty} \delta^t \left(c^{i,i+1} - d^{i,i+1}\right) + \sum_{t=\theta(m,\nu)+1}^{\theta(m,\nu)+T} \delta^t \left(l^{i,k} - d^{i,k}\right) + \sum_{t=\theta(m,\nu)+T+1}^{\infty} \delta^t \left(c^{i,k} - d^{i,k}\right) + \sum_{t=\theta(c,\nu)+1}^{\theta(c,\nu)+T} \delta^t \left(l^{i,i-1} - d^{i,i-1}\right) + \sum_{t=\theta(c,\nu)+T+1}^{\infty} \delta^t \left(c^{i,i-1} - d^{i,i-1}\right) \geq 0$$

By choosing an appropriate T_i, the punishment can again be made as hard as in the contagious equilibrium (with strategies (**S2**) and the respective beliefs). With $\nu > 1$, due to a faster punishment, the discount factor necessary to sustain the network will again be lower than with (**S2**).