Bankruptcy in Credit Chains*

Frederic Boissay Cyril Monnet

European Central Bank, D.G. Research

March 7, 2003

Abstract

When firms use bank loans and trade credit, bankruptcy rules can magnify aggregate fluctuations. A priori, a rule where banks are senior is not appropriate to dampen fluctuations. It might force trade creditors into bankruptcy by triggering a 'domino effect' - when firms go bust because their clients default. Yet, banks are often senior. In this paper, we characterize the conditions under which such a rule limits the likelihood of bankruptcies. We model a credit chain where in equilibrium firms use trade credit and bank loans. Due to the credit chain, bank seniority minimizes the overall risk premium charged by trade creditors and banks. Although bank seniority magnifies the domino effect, we find it is optimal whenever there is a relatively high proportion of bad risks.

1 Introduction

In this theoretical paper we analyze the optimal bankruptcy procedure when an insolvent firm has bank loans and trade credit on the liabilities side of its balance sheet. Trade credit, or account receivable, is used extensively by cash constrained firms as a source of short term finance in most industrialized countries. For instance in France, it represents a third of all balance sheet liabilities (Source: INSEE). In the United States, trade credit is 1.5 times larger

*We benefited from discussion with Mike Burkart, Jean-Pierre Danthine and Chrissa Panteleou. The views expressed in this paper do not necessarily reflect those of the European Central Bank or the Eurosystem.

than bank loans regardless of the term and represents 42% of short term liabilities.[1] Despite the importance of trade credit for the undertakings of projects, its use is relatively penalized to that of cash. Indeed, there are important discrepancies between the treatment of trade credit claims and bank loans whenever a bankruptcy procedure is engaged.

There is ample evidence that when a firm fails trade credit claims owners are generally not senior claimants. Certainly trade credit claims owners are not *always* senior. In fact, in the UK most bank loans are secured with fixed charge, and therefore are paid off first, while trade credit is generally unsecured and therefore paid off last.[2] The same is true in the US, see Carey (1995) and Welch (1997). In some other cases, while the outcome of a bankruptcy procedure is uncertain, banks end up being senior.

An intuitive reasoning goes against this institutional choice. As Kiyotaki and Moore (1997) show in a framework where firms borrow from and lend to each other,“if [an entrepreneur's] customers experience a negative liquidity shock and default, he himself may run into financial difficulties and he may have to default against his suppliers, thus causing further difficulties along the credit chain”. Therefore with the objective of minimizing bankruptcies and aggregate fluctuations, one might envisage to give full seniority to trade creditors.

To provide a benchmark, we formalize this intuitive reasoning in an environment where firms that grant trade credit do not have account payable on their liability side. There, we show that banks seniority cannot be rationalized on the ground of minimizing bankruptcies. In such an environment it is always optimal to grant seniority to trade creditors because they have more resources to pay off bank loans in bad contingencies. As a consequence the bank will offer them better conditions ex-ante, thus reducing their probability of going bankrupt. Therefore a social planner seeking to minimize bankruptcies will prefer supplier's seniority.

Such a simple environment overlooks the *duality* of trade credit. In facts, almost all firms grant trade credit while they also hold trade credit on the liability side of their balance sheet. Hence we build a model of trade credit and bank loans where in equilibrium trade credit claims are on both sides of the balance sheet for some firms. With this model we are able to rationalize the fact that suppliers should not be senior in the case an entrepreneur file for

[1]Source: Flow of Funds Accounts 1995-2001, Board of Governors of the Federal Reserve System.
[2]See Directors' Briefing on Insolvency, Briefing FI 12, Business Hotline Publications Ltd.

bankruptcy.

The model has two distinctive features. The first is the chain of trade credit. The second is the uncertainty underlying the outcome of a bankruptcy procedure. With some probability banks will be senior. We show that a social planner seeking to minimize the number of bankruptcies will choose under certain conditions to make banks senior. The intuition for this result is simple.

Increasing the probability of banks' seniority induces two effects. The first obvious effect is to decrease the likelihood of suppliers seniority ex-post ('probability effect'). A second effect is to decrease the overall premium charged by creditors, and thus the threshold output level below which a firm fails *when suppliers are senior* ('threshold effect'). A planner trades off the higher probability of success when suppliers are senior ex-post with the reduction in the likelihood of this event ex-ante. In other words, while the effect on the threshold favors the adoption of banks seniority, the effect on the probability goes against it. In our framework, only the threshold effect can explain why banks are offered seniority. Interestingly, the threshold effect is due to the trade credit chain and is therefore absent of the benchmark case.

To understand the origin of the threshold effect, we use a simple example showing that the trade credit chain is most active when banks are senior ex-post and entrepreneurs fail. In this case the presence of the trade credit chain induces additional reimbursement to suppliers. Once banks have been reimbursed, any additional unit of cash transferred to a supplier can be used to reimburse his own supplier. However, when suppliers are senior ex-post and entrepreneurs fail, the multiplier does not favor banks repayment as much. In this case, once suppliers have been reimbursed, any additional unit of cash transferred to banks is drawn out of the system and cannot be used by entrepreneurs any more.

Increasing banks' seniority decreases the risk premium of banks and increases the risk premium of suppliers. But the chain effect guarantees that the increase in suppliers' risk premium does not totally offset the decrease in the bank's risk premium. As a result, the overall risk premium always decrease when banks are more likely to be senior. So does the threshold above which firms succeed.

Banks seniority is optimal ex-ante if the threshold effect is stronger than the probability effect. This obviously depends on the distribution of firms' output. If firms are distributed such that bankruptcies are important when suppliers are senior ex-post, then there is a lot to gain from reducing the threshold. This is the case when there is a relatively high proportion of bad risks in the economy.

Thus we are able to rationalize why suppliers are usually not senior claimants in most bankruptcy procedure, despite the fact that this might generate further bankruptcies in a domino effect way, as described in Kiyotaki and Moore (1997). In turn this offers a new theory of why trade credit is much more costly than bank loans.

In this paper, we do not seek to explain why there is trade credit. However the use of trade credit in equilibrium is rationalized using the recent theory proposed by Burkart and Ellingsen (2003).[3] There, cash has the property that it can easily be diverted from its intended use. This explains why firms are credit constrained by banks and need to resort to trade credit to finance their production.

The remainder of this paper is organized as follows. In Section 2 we describe the benchmark model where firms that grant trade credit do not have account payable on their liability side. We show that this model cannot explain why banks are senior. In Section 3 we develop a simple extension of the benchmark model where trade credit appears on both side of the balance sheet for some firms. We show that this model is more consistent with the facts. In Section 4 we discuss the result and conclude.

2 The benchmark model

In this section we describe a model where firms that grant trade credit do not have account payable on their liability side. We first describe the environment and the problem of the firms. Then, we consider the social planner problem whose aim is to minimize the number of failures. In this context we show that optimally suppliers are always senior.

There are two periods. In the first period, 5 types of agents are considered: a measure 1

[3]See Nadiri (1969), Ferris (1981), Emery (1987), Brennan, Maksimovic and Zechner (1988), Mian and Smith (1992), Biais and Gollier (1997), Frank and Maksimovic (1998) or Cunat (2000) for other explanations for the existence of trade credit.

of entrepreneurs of type 1 (1 for short), a measure 1 of entrepreneurs of type 2 (2 for short), a measure 2 of deep pockets, a measure 2 of banks and a measure 1 of consumers. In a second period, only consumers and successful entrepreneurs subsist.[4]

Banks are endowed with one unit of a good that will be referred to as cash. Deep-pockets are endowed with one unit of a generic investment good. Entrepreneurs 2 are endowed with one unit of a specific investment good and with a technology (technology 2). In contrast, entrepreneurs 1 are only endowed with a technology (technology 1). Investment goods can be invested in a safe technology which return R units of a good as valuable as cash per unit invested.

Technology 1 requires $\gamma \in (0, 1)$ units of the generic investment good and $(1 - \gamma)$ units of the specific investment good to produce one unit of final good. Therefore, entrepreneurs 1 will buy γ goods from deep pockets and $(1 - \gamma)$ goods from entrepreneurs 2. All investment goods are sold at a unit price. Technology 2 requires one unit of cash to produce one unit of final good. A single unit of final good can be produced by entrepreneurs 1 and 2.

Final goods 1 and 2 are sold to consumers in a specific market. The price for final goods is distributed according to a probability density function $g_i(.)$ for $i = 1, 2$ with support $[0, \infty)$. We let y_i be the nominal output of entrepreneurs i. Therefore, y_i is distributed according to $g_i(.)$ with support $[0, \infty)$. y_1 and y_2 are independently drawn. As a shortcut, we write the joint distribution as $g(y_1, y_2)$.

In both periods, consumers are endowed with cash and have preferences represented by a simple utility function $u(x) = x$. Banks, entrepreneurs and deep-pockets maximize their profit. There is no discounting.

2.1 First period

Entrepreneurs 1 can finance the purchase of 1 unit of investment good using either bank loans and/or trade credit. We focus our attention exclusively on debt contracts. A debt contract $\{I_j, R_j\}$ specifies an amount of cash or trade credit I_j granted and a gross interest rate R_j on this amount. We denote $\{I_b, R_b\}$ a bank loan signed by 1s, and $\{I_s, R_s\}$ a trade credit contract signed between 1s and their suppliers. We impose further symmetry and let $(1 - \gamma)I_s$ be the

[4] We will precise at a later stage the meaning of 'successful'.

trade credit granted by suppliers 2 and γI_s be the trade credit granted by deep pockets. As entrepreneurs 2 and deep pockets face the same outside option and given perfect competition on the loan side, they will charge the same break even interest rate R_s. Finally, we denote by $\{\tilde{I}_b, \tilde{R}_b\}$ a bank contract signed by 2s. One type of contract can only be signed with one agent. Hence it is not possible to get cash from several banks for instance. Agreed upon contracts are perfectly enforceable.

The timing is as follows. First 1s contract with a bank. Then they contract with a suppliers and a deep-pocket. Once 1 contracted with 2, the latter contracts with a bank. Production takes place and if possible payments are made. If the contract cannot be honored, entrepreneurs file for bankruptcy and their assets are split according to a given bankruptcy rule - specified at a later stage - between creditors. We say that an entrepreneur is successful whenever her debts are payed off.

2.2 Second period

In the second period, those entrepreneurs who did not fail have access to a risk free production technology at no cost which requires no input.

2.3 Cash

Cash has the property that it is not traceable. This implies that cash holders can - up to a certain proportion δ - divert it from its primarily intended use with no chance of being caught. In other words, whenever an entrepreneur receives a unit of cash, it can divert a share δ for its private use, be it a first class private trip or gas on the company account. For diversion to have a bite, we assume that the utility $A > R$ derived from diverting one unit of cash is higher than the safe return from investing it.[5] This will ultimately motivate the fact that entrepreneurs are cash constrained and resort to trade credit. Entrepreneurs can also decide to contract with the bank, receive cash but divert it all.

[5] An alternative would be to introduce discounting and assume that it takes time to produce.

2.4 Bankruptcy procedure

Whenever an entrepreneur fails, a creditor seizes all assets only if he is the only creditor. If the creditor is not the only one, two procedures are envisaged. The bank has seniority over the claims. There, the bank is reimbursed first and the residual claimants share pro-rata the rest of the assets, if any. The suppliers have seniority over the claims. There suppliers are reimbursed pro-rata first and the banks is the residual claimant.

Depending on the priority structure p ($p = b$ if the bank has seniority, or $p = s$ if suppliers have seniority) we will denote the reimbursement function to entrepreneurs 2 in case 1s fail as $\rho_p(y_1)$. It is straightforward to show that $\rho_b(y_1) \leq \rho_s(y_1)$ for any y_1.

The distinctive feature of this environment is the uncertainty that surrounds the decision concerning creditors seniority. With a probability α banks are senior, while with a probability $(1-\alpha)$ suppliers are senior. This formulation has some advantage as it embeds the two most important cases - full priority to banks or suppliers - in a very tractable way. Also it fits well with the idea that it is difficult for economic agents to know with certainty the outcome of a bankruptcy procedure.[6]

2.5 Entrepreneurs 1 problem

Given cash diversion, entrepreneurs 1 will choose I_b, I_s such that $(1-\delta)I_b + I_s = 1$. We let $R^* = R_b I_b + R_s I_s$ be the minimum nominal output above which 1s are successful.

Interest rates have to satisfy break-even conditions taking into account the uncertainty on the seniority structure of claims. When creditors $i \in \{b, s\}$ are senior, their expected return is

$$\Pi_i^i = \int^{R_i I_i} y_1 G(dy_1) + \int_{R_i I_i} R_i I_i G(dy_1)$$

Otherwise,their expected return is, for $j \in \{b, s\} \neq i$,

$$\Pi_i^j = \int_{R_j I_j}^{R^*} (y_1 - R_j I_j) G(dy_1) + \int_{R^*} R_i I_i g(y_1) dy_1.$$

[6] Another possible structure is to split assets in fixed proportions among creditors. However this introduces many unnecessary complications.

Hence, the break even rates R_s, R_b have to satisfy:

$$\alpha\Pi_b^b + (1-\alpha)\Pi_b^s = RI_b \tag{2.1}$$

$$\alpha\Pi_s^b + (1-\alpha)\Pi_s^s = RI_s \tag{2.2}$$

The summation of equations (2.1) and (2.2) together with the use of $(1-\delta)I_b + I_s = 1$ returns,

$$\int_0^{R^*} y_1 G(dy_1) + R^*[1 - G(R^*)] = R(1 + \delta I_b) \tag{2.3}$$

This gives R^* as a function of I_b which is not explicitly dependent on the seniority structure. Hence entrepreneurs 1 face the following problem.

$$\begin{aligned} \underset{I_b}{Max} \quad & \int_{R^*(I_b)} [y_1 - R^*(I_b)]G(dy_1) + A\delta I_b \\ & \text{subject to equation} \\ & \int_{R^*(I_b)} [y_1 - R^*(I_b)]G(dy_1) + A\delta I_b \geq AI_b \end{aligned} \tag{2.4}$$

Inequality (2.4) guarantees that 1 uses technology 1, after having diverted a share of the cash δ rather than opting out of the economy. We then have a simple result.

Lemma 1. *Entrepreneurs 1 are always cash constrained.*

Proof. Taking the first derivative of the objective function for entrepreneurs with respect to I_b we have:

$$-\frac{\partial R^*}{\partial I_b}[1 - G(R^*)] + A\delta$$

Equation (2.3) gives us $\partial R^*/\partial I_b = \delta R/(1 - G(R^*))$ Which implies that as long as $A > R$ the payoff of entrepreneur 1 is strictly increasing in I_b. Hence 1 will always borrow cash $\bar{I}_b$ where $\bar{I}_b$ solve (2.4) with equality. □

In the Appendix, we show that under certain assumptions $\bar{I}_b$ exists, is unique and is less than 1. Hence entrepreneurs 1 need trade credit. The unique solution to entrepreneurs 1' problem is given by the two constraints above. One property of this solution is that the amount of cash borrowed from the bank does not depend on the seniority structure. This will prove very useful in characterizing the solution to an entrepreneur 2's problem.

2.6 Entrepreneurs 2's problem

The problem of entrepreneur 2s is a little more simple as we only have to take into consideration one source of finance. Indeed, a bank loan is the only financing source of 2s as they do not have access to trade credit. In equilibrium the bank knows that 1 buys $(1-\gamma)I_b < 1$ investment goods cash from 2s, we then restrict our attention to equilibrium where banks only lend $\tilde{I}_b = 1-(1-\gamma)I_b$. In this case, 2s cannot combine cash diversion and production, since one unit of cash is necessary to produce. Hence if 2 invests, his payoff is given by $\alpha\Pi_2^b + (1-\alpha)\Pi_2^s$, where:

$$\Pi_2^p = \int_{\{(y_1,y_2):y_2+\rho_p(y_1)>\tilde{R}_b\tilde{I}_b\}} [y_2 + \rho_p(y_1) - \tilde{R}_b\tilde{I}_b] G(dy_1, dy_2)$$

Otherwise, entrepreneur 2 consumes all the cash and gets $A(\tilde{I}_b + (1-\gamma)I_b) = A$. Therefore production from entrepreneur 2 is only incentive compatible if $\alpha\Pi_2^b + (1-\alpha)\Pi_2^s > A$.

Since the bank is the only creditor, it is always senior. Therefore, the interest rate charged by the banks will satisfy $\alpha\tilde{\Pi}_b^b + (1-\alpha)\tilde{\Pi}_b^s = R\tilde{I}_b$ where

$$\tilde{\Pi}_b^p = \int_{\{(y_1,y_2):y_2+\rho_p(y_1)<\tilde{R}_b\tilde{I}_b\}} [y_2 + \rho_p(y_1)] G(dy_1, dy_2) + \tilde{R}_b\tilde{I}_b \int_{\{(y_1,y_2):y_2+\rho_p(y_1)\geq\tilde{R}_b\tilde{I}_b\}} G(dy_1, dy_2).$$

Given $\tilde{I}_b$ is fixed by the bank, the equations above give the break even interest rate charged by banks, $\tilde{R}$. A simple assumption allows for the production of 2s to take place:

$$\int y_2 G(dy_2) > A + R$$

2.7 Social planner problem

The problem of the social planner is to choose the ex-ante seniority structure that will maximize the welfare of consumers. It is easy to see that this implies that the social planner will try to minimize the measure of firms that fail.

We found that the total debt R^* of entrepreneurs 1 is not affected by the seniority structure, so that the measure of successful entrepreneurs 1 is independent of the seniority structure. However the total debt $\tilde{R}_b \tilde{I}_b$ of entrepreneurs 2 is. Indeed, the seniority structure affects the asset *level* of entrepreneurs 2 as well as its *distribution*. We then have the following result.

Proposition 1. *Suppliers seniority maximizes the measure of successful entrepreneurs.*

Proof. It is easy to show (see the Appendix) that, for a given interest rate $\tilde{R}_b$, $\tilde{\Pi}_b^s > \tilde{\Pi}_b^b$. Since $\alpha \tilde{\Pi}_b^b + (1-\alpha)\tilde{\Pi}_b^s = R\tilde{I}_b$ always holds with equality, it must be that $\tilde{R}_b$ is lower when suppliers have seniority. Given $\tilde{I}_b$ is fixed, the total debt of entrepreneurs 2 is minimized when suppliers have seniority. □

This result is simple. Given a financing structure for entrepreneurs 1 - which can be summarized by I_b - their overall debt is not affected by the seniority structure. The reason is that, since lenders are risk neutral, the change in interest rates charged by suppliers resulting from a different seniority structure perfectly offsets the change in interest rates charged by banks. Furthermore, entrepreneurs 1 choose their financing structure only as a function of overall debt. Therefore the seniority structure does not affect 1s choice. As a consequence, 1s face the same ex-ante probability of bankruptcy independently of the seniority structure.

Given 1s do not modify their bank loans entrepreneurs 2 have the same cash holdings when they go for a bank loan, independently of the seniority structure. However when the seniority shifts from banks to suppliers, 2 receives more in the case 1 fails. In this case, the bank takes into account that 2 has a higher level of assets in bad contingencies and as a consequence of perfect competition charge a lower interest. In turn this lowers the likelihood of failure of 2. Hence suppliers seniority is optimal from the viewpoint of minimizing bankruptcies.

However as underlined in the introduction there are evidences that in many occasions banks are senior claimants whenever a firm is bankrupt. The benchmark model cannot explain this simple fact. We feel that it overlooks an important aspect of trade credit, its duality. In facts, many firms grant trade credit while they also hold trade credit on the liability side of their balance sheet. Considering this aspect will affect the optimal degree of seniority as equilibrium assets and liabilities now go pairwise. Hence in the next section we construct from the benchmark model an environment with trade credit chains.

3 A model of trade credit chains

3.1 Structure and equilibrium definition

There are two differences with the previous environment. First, in addition to technology 1, type 1 entrepreneurs are endowed with a specific investment good 1. Second, type 2 entrepreneurs are now endowed with a technology similar to type 1 technology. This technology requires $(1-\gamma)$ units of the specific investment good 1 combined with γ units of the generic good to produce one unit of final good 2.

In the first period, there are three stages. In the first stage, entrepreneurs are randomly matched with a bank. They propose a loan $\{I_b, R_b\}$ to the bank. If the latter rejects the offer, the entrepreneur is matched randomly with another bank.

In a second stage, entrepreneurs of type i are randomly allocated a supplier and a client of type j as well as a deep-pocket. In meetings with suppliers, clients offer a contract $\{I_s, R_s\}$.[7] If suppliers reject, clients are matched randomly with another supplier. A meeting between a client and an entrepreneur-supplier is separated from all other meetings. Similarly for meetings between a client and a deep-pocket. Hence an entrepreneur of type 1 is assigned a deep-pocket as well as a client and a supplier of type 2. Since we consider a continuum of agents in each type, the client and supplier of type 2 will typically be different entrepreneurs. The pairs (client, entrepreneur) and (client, deep-pocket) are fixed throughout period 1. In this second stage, suppliers and deep-pockets provide their client with intermediate goods and possibly trade credit.[8]

In a third stage, production of the final good takes place. Within types, entrepreneurs are subject to the same production shock. Contracts are honored if possible. Otherwise entrepreneurs file for bankruptcy.

[7] With the understanding that if the supplier is an entrepreneur only $(1-\gamma)I_s$ trade credit will be granted, while if the supplier is a deep pocket, γI_s trade credit will be granted.

[8] The separation of the role of supplier/client rules out possible arrangements between suppliers and clients. More elegantly but at the price of simplicity, we could have assumed 3 types of entrepreneurs, where type 1 needs the good of type 2 who needs the good of type 3 who needs the good of type 1. However while more intuitive when dealing with meetings, this formulation significantly complicates the analysis without changing the main message of the paper.

We concentrate our analysis on symmetric Nash equilibria. A symmetric Nash equilibrium is pair of contract $(\{I_b, R_b\}, \{I_s, R_s\})$, such that given this choice of contract by all other entrepreneurs, an entrepreneur maximizes his payoff by choosing $(\{I_b, R_b\}, \{I_s, R_s\})$.

3.2 Entrepreneurs' problem

Given the strategy adopted by his client, an entrepreneur has an asset level a with a cumulative distribution function G_p that depends on the priority structure, where $p \in \{b, s\}$. Note that this distribution is given to the entrepreneur. Also, as $\underline{y}$ is the minimum realization of nominal output, note that for all entrepreneurs $a \geq \underline{y}$ unconditionally on the seniority structure. Then we can proceed as for the problem of entrepreneur 1 of the benchmark model. When creditors $i \in \{b, s\}$ are senior, their expected return is

$$\Pi_i^i = \int^{R_i I_i} a G_i(da) + \int_{R_i I_i} R_i I_i G_i(da)$$

Otherwise, their expected return is, for $j \in \{b, s\} \neq i$,

$$\Pi_i^j = \int_{R_j I_j}^{R^*} (a - R_j I_j) G_j(da) + \int_{R^*} R_i I_i G_j(da).$$

Using the new definition of Π_i^i and Π_i^j, the break even rates R_s, R_b also have to satisfy equations (2.1) and (2.2). In turn, equations (2.1) and (2.2) sum up to

$$\alpha \left[\int^{R^*} a G_b(da) + R^*[1 - G_b(R^*)] \right] + (1 - \alpha) \left[\int^{R^*} a G_s(da) + R^*[1 - G_s(R^*)] \right] = R(I_s + I_b) \tag{3.1}$$

Now, given the strategy of all other entrepreneurs, an entrepreneur expect to receive a certain amount of cash, ω from his client. From this, an amount $(1 - \delta)\omega$ will be diverted. Therefore, an entrepreneur will choose I_b, I_s such that $(1 - \delta)I_b + I_s = 1 - (1 - \delta)\omega$. Therefore the left hand side of equation (3.1) becomes $R(1 - (1 - \delta)\omega + \delta I_b)$. Deriving equation (3.1) with respect to I_b, we obtain:

$$\frac{\partial R^*}{\partial I_b} = \frac{\delta R I_b}{\alpha(1 - G_b(R^*)) + (1 - \alpha)((1 - G_b(R^*)))}$$

Since R^* satisfies (3.1), entrepreneurs 1 face the following problem.

$$\begin{aligned} &\underset{I_b}{Max} \quad \alpha \int_{R^*} (a - R^*) G_b(da) + (1-\alpha) \int_{R^*} (a - R^*) G_s(da) + A\delta I_b \\ &\text{subject to} \\ &\alpha \int_{R^*} (a - R^*) G_b(da) + (1-\alpha) \int_{R^*} (a - R^*) G_s(da) + A\delta I_b \geq A I_b \qquad (3.2) \end{aligned}$$

Then using the same procedure as in the benchmark model, we obtain a similar result.

Lemma 2. *Entrepreneurs are always cash constrained.*

However note that we are not able to say at this stage that the maximum amount borrowed, $\bar{I}_b$ does not depend on the seniority structure. Now we just know that equation (3.2) is binding. This will prove very useful in the sequel.

3.3 Assets derivation in equilibrium

Since we concentrate our analysis on symmetric equilibrium, we can derive the equilibrium asset levels fairly easily. In this section we derive the assets of each entrepreneur in the general case where y_1 and y_2 are distributed according to a distribution with support $[\underline{y}, \bar{y}]$.
We also assume for simplicity that $\underline{y} \geq max\{R_s I_s, R_b I_b\}$, with the understanding that one should give conditions such that this is satisfied in equilibrium. We have to distinguish 2 cases.

Case 1: Banks are senior.

Subcase 1.1: The two types of entrepreneur fail.
Then we have for $i \neq j$ and $i, j \in \{1, 2\}$, $a_i = y_i + (1-\gamma)(a_j - R_b I_b)$. Solving for a_1 and a_2, we obtain

$$a_i = \frac{y_i + (1-\gamma) y_j - R_b I_b [(1-\gamma)^2 + (1-\gamma)]}{1 - (1-\gamma)^2}.$$

Since we are in the case where both firms fails, we have to impose $a_i < R_s I_s + R_b I_b = R^*$ for $i = 1, 2$. Hence we obtain the following condition on y_1 and y_2 (where $\bar{R} = R_b I_b + \gamma R_s I_s$):

$$\begin{aligned} y_1 + (1-\gamma) y_2 &< (2-\gamma)\bar{R} \\ y_2 + (1-\gamma) y_1 &< (2-\gamma)\bar{R} \end{aligned}$$

Subcase 1.2: Entrepreneurs 1 fail and entrepreneurs 2 succeed.

Since 2s succeed they are able to pay back their supplier with an amount R_sI_s. Therefore, assets of 1s are $a_1 = y_1 + (1-\gamma)R_sI_s$. However since 1s fail and banks are senior, 2s only get the residual claims that they share pro-rata with deep pockets. Hence 2s' assets are $a_2 = y_2 + (1-\gamma)(a_1 - R_bI_b)$. Since 1s fail and 2a succeed, $a_1 < R_sI_s + R_bI_b$ while $a_2 > R_sI_s + R_bI_b$. We then obtain the following condition on (y_1, y_2).

$$\begin{aligned} y_1 &< \bar{R} \\ y_2 + (1-\gamma)y_1 &> (2-\gamma)\bar{R} \end{aligned}$$

Subcase 1.3: Entrepreneurs 2 fail and entrepreneurs 1 succeed.

This case is symmetrical to the one where 1s fail and 2s succeed.

Subcase 1.4: Both types of entrepreneur succeed.

In this case, entrepreneurs are able to pay back both banks and suppliers. Hence, asset levels are $a_i = y_i + (1-\gamma)R_sI_s$. As both entrepreneur succeed $a_i > R_sI_s + R_bI_b$ for $i = 1, 2$ and we obtain

$$y_i > \bar{R},$$

All these cases are summarized in Figure 1.

Case 2 : Suppliers are senior.

For any of the subcases, since $\underline{y} > max\{R_sI_s, R_bI_b\}$ by assumption, clients are always able to pay back at least their suppliers. Hence for all $i = 1, 2$ we have $a_i = y_i + (1-\gamma)R_sI_s$. Therefore $a_i > R_sI_s + R_bI_b$ will be equivalent to $y_i > \bar{R}$ and ibidem for the reverse inequality. As a consequence we obtain that entrepreneurs $i \in \{1, 2\}$ fail if $y_i < \bar{R}$ and succeed otherwise. This is represented in Figure 2.

The main difference between the case where banks and suppliers have seniority is the presence of regions A' and A'' in Figure 1. Within these regions, suppliers fail because their clients were not successful enough.

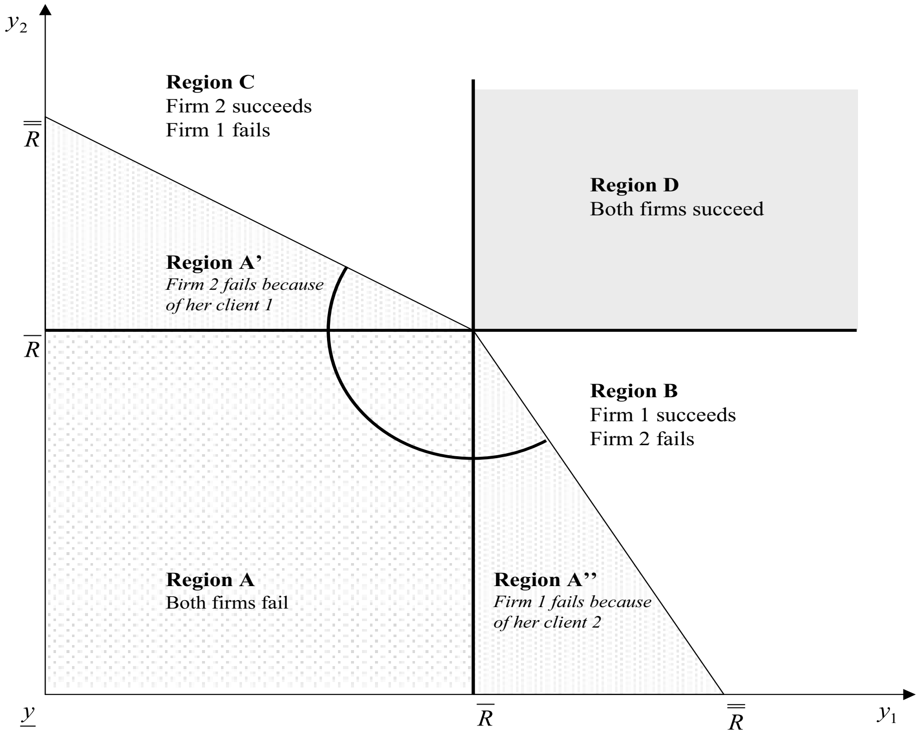

Figure 1: Banks are senior, where $\bar{\bar{R}} = \bar{R} + (1-\gamma)(\bar{R} - \underline{y})$

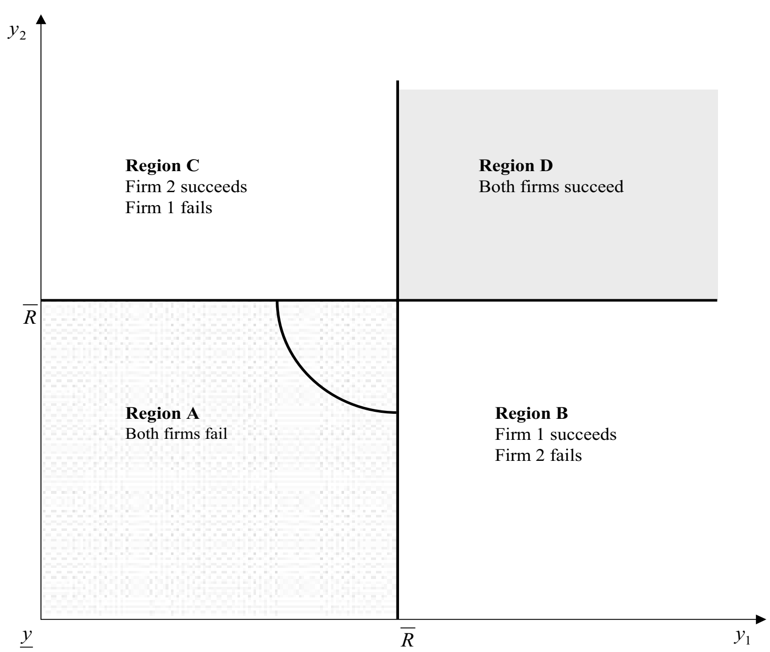

Figure 2: Suppliers are senior

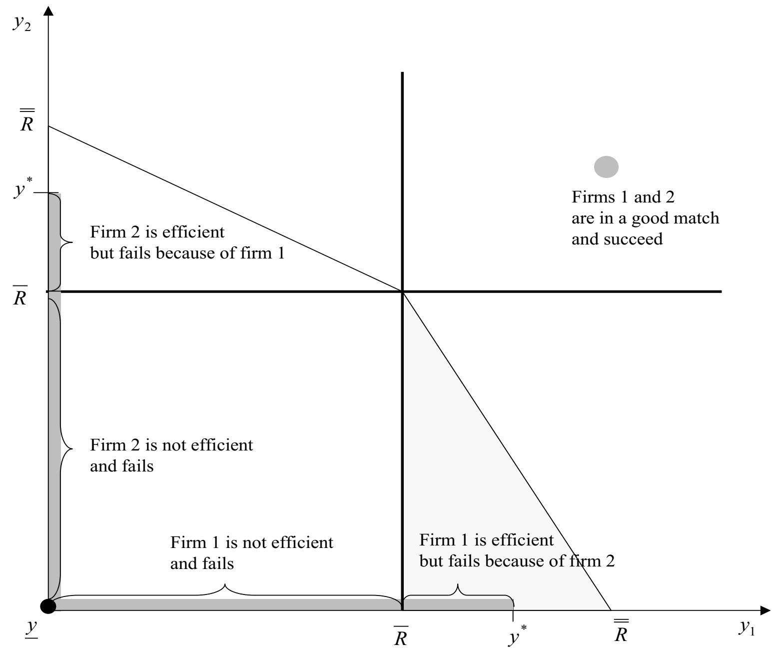

Figure 3: A simple example of bad and good match

3.4 Social planner problem: a simple case

It is fairly difficult to have a closed form solution for the equilibrium debt contracts when we use a general formulation for the distribution of nominal output. Therefore, we choose to present here a simple case that summarizes the forces at work. We assume that with probability $(1-\theta)$, entrepreneurs 1 and 2 get $\bar{y}$ so that in equilibrium $\bar{y} > \bar{R}$. Hence with a probability $(1-\theta)$, entrepreneurs are in a good match since they always succeed. Then with a probability θ, entrepreneurs are in a bad match where one of the type will always fail. With a probability $\theta/2$, 1s receive $\underline{y}$ while 2s receive y distributed according to a probability density function f with support $[\underline{y}, y^*]$, where in equilibrium $y^* \in [\bar{R}, \bar{\bar{R}}]$ and $\bar{\bar{R}} = \bar{R} + (1-\gamma)(\bar{R} - \underline{y})$. We let $\mu = \int y f(y) dy$.[9] With probability $\theta/2$, the outcome for 1s and 2s is reversed. This is illustrated in Figure 3.

The problem of the social planner is to choose a seniority structure α that maximizes the welfare of consumers. It is easy to see that this implies that the social planner will try to

[9]We prove in the Appendix that the set of economies we are considering is non-empty.

minimize the probability of firms bankruptcy. This probability is given by

$$P = \frac{\theta}{2}[1 + \alpha + (1-\alpha)F(\bar{R})].$$

Hence we have the following lemma.

Lemma 3. *It is optimal to give seniority to banks whenever*

$$\left[(1-\alpha)\frac{f(\bar{R})}{1-F(\bar{R})} \times \frac{\partial \bar{R}}{\partial \alpha}\right]_{\alpha=0} < -1 \tag{3.3}$$

Proof. Banks will sometimes be senior if $\alpha > 0$. But $\alpha > 0$ is optimal whenever $[\partial P/\partial \alpha]|_{\alpha=0} < 0$. Now, $\partial P/\partial \alpha = 1/2(1-\theta)[(1-\alpha)f(\bar{R})(\partial \bar{R}/\partial \alpha) + (1 - F(\bar{R}))]$, from which the result follows. □

This result is of limited usefulness as we know from the benchmark model that it is well possible that $\frac{\partial \bar{R}}{\partial \alpha} = 0$ for all α. However, we will devote the remainder of this section to showing the following proposition.

Proposition 2. *There exist $\theta \in (0,1)$ and a distribution $f(.)$ such that (3.3) holds in equilibrium.*

We will first show that $\partial \bar{R}/\partial \alpha < 0$ due to the trade credit multiplier. Then we will show that there exists a distribution $f(.)$ such that (3.3) holds.

The main difference between the social planner problem and the individual entrepreneur problem is that the planner takes into consideration the effect of additional borrowing of an entrepreneur on the asset of his supplier. Hence we have to consider the pattern of circulation of cash in equilibrium. We will refer to this pattern as the 'cash-chain'.

With a bank loan I_b, entrepreneurs i transfer $(1-\delta)(1-\gamma)I_b$ to their supplier j. In turn a supplier can use this cash stream in order to finance the purchase of intermediate goods, after having diverted a share δ. Given unit price, a supplier j can then buy $(1-\delta)^2(1-\gamma)I_b$ goods from his own supplier and deep-pocket. Hence, entrepreneurs j buys for $(1-\delta)^2(1-\gamma)^2 I_b$ goods from their suppliers i and $\gamma(1-\delta)^2(1-\gamma)I_b$ goods from deep pockets. Therefore this cash-chain implies that an entrepreneur i will spend

$$\sum_{s=0}^{\infty}(1-\delta)^{s+1}(1-\gamma)^s I_b = I_b \frac{1-\delta}{\gamma(1-\delta)+\delta}$$

on the purchase of intermediate goods. It is convenient to define $\beta = \frac{\gamma(1-\delta)+\delta}{1-\delta}$, so that whenever a bank lends one unit more of cash, there are $1/\beta$ intermediate goods purchased. We will refer to $1/\beta$ as the cash multiplier. Note that as $\delta \geq 0$, we always have $\beta \geq \gamma$. In other words, cash diversion implies that the cash multiplier is always weakly lower than the trade credit multiplier (defined as $1/\gamma$).

The total amount of trade credit requested by an entrepreneur is then

$$I_s = 1 - \frac{1}{\beta} I_b. \tag{3.4}$$

This is split between the amount of trade credit requested from the supplier is $(1-\gamma)(\beta - I_b)/\beta$ and the amount of trade credit requested from deep pockets is $\gamma(\beta - I_b)/\beta$. Hence, $I_b + I_s = [\beta + (\beta - 1)I_b]/\beta$.

Now the equilibrium break even rates are easily determined using the derivation of asset levels in equilibrium. We find that the suppliers' expected return when banks are senior is[10] $R_s I_s + \frac{\theta}{2\gamma}(\underline{y} + \mu - 2\bar{R})$. While when suppliers are senior, they always get fully payed back $R_s I_s$. Hence the ex-ante expected return to suppliers is $(1-\alpha)R_s I_s + \alpha[R_s I_s + \frac{\theta}{2\gamma}(\underline{y} + \mu - 2\bar{R})]$, which determines the break even rate R_s as a solution to

$$R_s I_s = R I_s + \alpha \frac{\theta}{2\gamma}(2\bar{R} - \underline{y} - \mu). \tag{3.5}$$

The last term in (3.5) is the risk premium. The risk for suppliers only occur when they are in a bad match, which occurs with a probability θ and when banks are senior which occurs with probability α. With a probability $1/2$ a supplier's client is a bad type whose nominal output is $\underline{y}$ and with probability $1/2$ it is a relatively good type whose nominal output on average is μ. However, both type of client fails in equilibrium. Hence, the expected net loss on trade

[10] Again, this is multiplied by $(1-\gamma)$ if the supplier is an entrepreneur and by γ if it is a deep pocket.

credit in these events is

$$\frac{1}{2}\left[R_sI_s - \frac{1}{\gamma}(\underline{y} - R_bI_b)\right] + \frac{1}{2}\left[R_sI_s - \frac{1}{\gamma}(\mu - R_bI_b)\right]$$

where $1/\gamma$ is the trade credit multiplier. Hence, in this instance the existence of a trade credit chain decreases the net loss on trade credit. In turn this decreases the risk premium imposed by suppliers. Arranging this term, we get the bracketed expression in equation (3.5). The break even rate for banks is determined similarly and using $\psi(x) = \int_{\underline{y}}^{x}(y-x)F(dy)$ we obtain the following expression

$$R_bI_b = RI_b + (1-\alpha)\frac{\theta}{2}(\bar{R} - \underline{y} - \psi(\bar{R})). \tag{3.6}$$

Again, the last term in (3.6) is the risk premium and has the same interpretation as the risk premium imposed by suppliers. When suppliers are senior, the trade credit chain only plays an implicit role for banks since it allows a reduction in the threshold level of nominal output $\bar{R}$ below which entrepreneurs fail. However, when entrepreneurs fail, there is no explicit channel through which trade credit has a role. In our view, this is the fundamental difference between banks and suppliers seniority. When entrepreneurs fail, the trade credit chain becomes active only in the case where banks are senior.

Adding both break even equations (3.5) and (3.6) and using equation (3.4) to replace for I_s we obtain an expression that uniquely determines $\bar{R}$,

$$R(\gamma + \frac{\beta-\gamma}{\beta}I_b) = \bar{R} + \frac{\theta}{2}(\underline{y} - \bar{R}) + \frac{\theta}{2}(\alpha(\mu - \bar{R}) + (1-\alpha)\psi(\bar{R})). \tag{3.7}$$

We can now state an interesting result which follows in a straightforward manner from this equation.

Lemma 4. *In equilibrium, I_b does not depend on the seniority structure.*

Proof. This easily follows from the binding incentive compatibility constraint. See the Appendix for some steps of the derivation. ☐

Given I_b does not depend on α it is straightforward to obtain an expression for $\partial\bar{R}/\partial\alpha$.

Indeed we have

$$\frac{\partial \bar{R}}{\partial \alpha} = -\frac{\theta}{2} \frac{\int_{\bar{R}}^{y^*} (y - \bar{R}) F(dy)}{1 - P} < 0$$

Our analysis of the constituents of $\bar{R}$ allows a simple interpretation for this expression. $\partial \bar{R}/\partial \alpha$ is the change in the sum of the risk premium charged by banks and suppliers due to a modification in the seniority structure. Given entrepreneurs fail, this is the change in the loss on trade credits and bank loans.

Here, the role of the trade credit chain is crucial. When banks are senior ex-post and entrepreneurs fail, the trade credit multiplier is active and induces additional reimbursement to suppliers. However, when suppliers are senior ex-post and entrepreneurs fail, the multiplier does not favor banks repayment as it is inactive.

The direct effect of increasing banks' seniority is to decrease the risk premium of banks and to increase the risk premium of suppliers. However, due to the trade credit multiplier, the increase in suppliers' risk premium does not totally offset the decrease in the bank's risk premium. As a result, the overall risk premium always decrease when banks become more senior.

As $\partial \bar{R}/\partial \alpha < 0$ it is well possible that inequality (3.3) is satisfied. Furthermore we can now give an intuitive interpretation of the trade-offs which come into play in the decision of the optimal seniority structure. When more seniority is given ex-ante to banks the threshold $\bar{R}$ diminishes thanks to the trade credit multiplier. This favors entrepreneurs as the success threshold decreases. However, since entrepreneurs in a bad match fail whenever banks are senior, the benefits of a decrease in $\bar{R}$ only materialize when suppliers are senior ex-post (see Figures 2 and 3). But giving more seniority to banks decreases the likelihood of such an event. Hence the planner trades off the decreases in the likelihood of having suppliers senior with the decrease in the success threshold $\bar{R}$.

This implies that, in our simple case, it is never optimal to give ex-ante full seniority to banks, i.e. to set $\alpha = 1$, since the benefits from having the lowest $\bar{R}$ would never be realized in this case. Hence, if (3.3) holds, we know there exists an interior optimal α. Of course the example is the worst case scenario for giving seniority to banks as when confronted with banks seniority, all entrepreneurs in a bad match fail. It is therefore surprising that banks get some ex-ante seniority.

We are now in a position to determine a distribution function $f(.)$ under which it is optimal to give some seniority to banks. This is the case whenever

$$\left[(1-\alpha)\frac{f(\bar{R})}{1-F(\bar{R})} \times \frac{\theta \int_{\bar{R}}^{y^*}(y-\bar{R})F(dy)}{2-\theta(1+\alpha+(1-\alpha)F(\bar{R}))}\right]_{\alpha=0} > 1 \tag{3.8}$$

which is equivalent to

$$\left[\frac{f(\bar{R}_0)}{1-F(\bar{R}_0)} \times \frac{\theta \int_{\bar{R}_0}^{y^*}(y-\bar{R}_0)F(dy)}{(1-\theta)+(1-F(\bar{R}_0))}\right] > 1$$

where $\bar{R}_0$ solves $\tilde{R} = R(\gamma I_s + I_b) = \bar{R}_0 + \frac{\theta}{2}(\underline{y} - \bar{R}_0) + \frac{\theta}{2}\psi(\bar{R}_0)$. Hence $f(.)$ has to satisfy,

$$\left[\frac{f(\bar{R}_0)}{1-F(\bar{R}_0)} \times \frac{\theta(\mu - \tilde{R} + \frac{\theta}{2}(\underline{y} - \bar{R}_0))}{(1-\theta)+(1-F(\bar{R}_0))}\right] > 1$$

In words, $f(\bar{R}_0)$ has to be sufficiently large that many entrepreneurs would gain from having a decrease in $\bar{R}_0$.

4 Discussion

To be written.

A Proof that $\bar{I}_b < 1$

Let $\mu_1 = \int y_1 G(dy1)$.

Lemma 5. *If $\mu_1 > R$ and $\mu_1 < R + A - \delta(A - R)$ then $\bar{I}_b$ exists, $\bar{I}_b \in (0, 1)$ and is unique.*

Proof. From equation (2.3), we obtain

$$R^*(1 - G(R^*)) = R(1 + \delta I_b) - \int^{R^*} y_1 G(dy_1)$$

so that the inequality (2.4) can be written as

$$\mu_1 - R + (A - R)\delta I_b \geq A I_b$$

When $I_b = 0$, the inequality is satisfied as $\mu_1 > R$. However when $I_b = 1$, the inequality is not satisfied as $\mu_1 < R + A - \delta(A - R)$. Moreover, since the right and left hand sides of (2.4) are strictly increasing, $\bar{I}_b$ exists, is less than 1 and is unique. □

B Proof of Proposition 1

We want to know the sign of

$$\int_{\{(y_1,y_2):y_2+\rho_s(y_1)<\tilde{R}_b\tilde{I}_b\}} [y_2 + \rho_s(y_1)]G(dy_1, dy_2) + \tilde{R}_b\tilde{I}_b \int_{\{(y_1,y_2):y_2+\rho_s(y_1)\geq\tilde{R}_b\tilde{I}_b\}} G(dy_1, dy_2) -$$

$$\int_{\{(y_1,y_2):y_2+\rho_b(y_1)<\tilde{R}_b\tilde{I}_b\}} [y_2 + \rho_b(y_1)]G(dy_1, dy_2) - \tilde{R}_b\tilde{I}_b \int_{\{(y_1,y_2):y_2+\rho_b(y_1)\geq\tilde{R}_b\tilde{I}_b\}} G(dy_1, dy_2)$$

Now, for any y_1, we have $\rho_s(y_1) \geq \rho_b(y_1)$. Therefore we obtain the following inclusion:

$$\{(y_1, y_2) : y_2 + \rho_s(y_1) < \tilde{R}_b\tilde{I}_b\} \subseteq \{(y_1, y_2) : y_2 + \rho_b(y_1) < \tilde{R}_b\tilde{I}_b\}$$

Therefore the expression we are interested in can be rewritten as:

$$\int_{\left\{\begin{array}{rl}(y_1,y_2): y_2+\rho_s(y_1) < & \tilde{R}_b\tilde{I}_b \\ \text{and } y_2+\rho_b(y_1) < & \tilde{R}_b\tilde{I}_b\end{array}\right\}} \rho_s(y_1)-\rho_b(y_1)G(dy_1,dy_2) \tag{B.1}$$

$$- \quad \int_{\left\{\begin{array}{rl}(y_1,y_2): y_2+\rho_s(y_1) > & \tilde{R}_b\tilde{I}_b \\ \text{and } y_2+\rho_b(y_1) < & \tilde{R}_b\tilde{I}_b\end{array}\right\}} y_2+\rho_b(y_1)G(dy_1,dy_2) \tag{B.2}$$

$$+ \quad \tilde{R}\tilde{I}_b \int_{\left\{\begin{array}{rl}(y_1,y_2): y_2+\rho_s(y_1) > & \tilde{R}_b\tilde{I}_b \\ \text{and } y_2+\rho_b(y_1) < & \tilde{R}_b\tilde{I}_b\end{array}\right\}} G(dy_1,dy_2) \tag{B.3}$$

which is clearly positive. Therefore, for a given interest rate $\tilde{R}_b$, the left hand side of (2.4) is higher when suppliers have seniority rather than when banks do.

C Proof of Lemma 4

Since the incentive compatibility constraint of entrepreneurs is binding, I_b is given by

$$(1-\theta)(\bar{y}-\bar{R})+\frac{\theta}{2}(1-\alpha)(\mu-\bar{R}-\psi(\bar{R})) = AI_b(1-\delta)$$

But it is easy to show that this is equivalent to

$$(1-\theta)\bar{y}+\frac{\theta}{2}(\mu+\underline{y})-\left[\bar{R}+\frac{\theta}{2}(\underline{y}-\bar{R})+\frac{\theta}{2}(\alpha(\mu-\bar{R})+(1-\alpha)\psi(\bar{R}))\right] = AI_b(1-\delta)$$

and using the expression for $\bar{R}$ given by (3.7) we find

$$AI_b(1-\delta) = (1-\theta)\bar{y}+\frac{\theta}{2}(\mu+\underline{y})-R(\gamma+\frac{\beta-\gamma}{\beta}I_b)$$

which uniquely determines I_b independently of the seniority structure.

D Set of economy is non-empty

In this section we verify that the set of economies $\{\underline{y}, \bar{y}, y^*, \theta, \gamma, \alpha, R, F\}$ we consider in section 3.4 is non-empty. For a given α, an economy needs to verify 10 equalities or inequalities.

$$\underline{y} > R_b I_b \tag{D.1}$$

$$\underline{y} > R_s I_s \tag{D.2}$$

$$\underline{y} < \bar{R} \tag{D.3}$$

$$y^* > \bar{R} \tag{D.4}$$

$$y^* < \bar{\bar{R}} \tag{D.5}$$

$$\bar{R} = R_b I_b + \gamma R_s I_s \tag{D.6}$$

$$\bar{\bar{R}} = \bar{R} + (1-\gamma)(\bar{R} - \underline{y}) \tag{D.7}$$

$$R_b I_b = R I_b + (1-\alpha)\frac{\theta}{2}(\bar{R} - \underline{y} - \psi(\bar{R})) \tag{D.8}$$

$$R_s I_s = R I_s + \alpha\frac{\theta}{2\gamma}(2\bar{R} - \underline{y} - \mu) \tag{D.9}$$

$$I_b = \frac{(1-\theta)\bar{y} + \frac{\theta}{2}(\mu + \underline{y})}{A(1-\delta) + R(\gamma + 1 - \frac{\gamma}{\beta})} \tag{D.10}$$

where $I_s = 1 - I_b/\beta$.

Let $\underline{y} = \bar{R} - \varepsilon$, $y^* = \bar{R} + \varepsilon$ and F be the uniform distribution on $[\bar{R} - \varepsilon, \bar{R} + \varepsilon]$, such that $\mu = \bar{R}$. In addition we have $\psi(\bar{R}) = -\varepsilon/4$. Multiplying both sides of equation D.9 by γ and adding the result to equation D.8, we obtain a simple expression for $\bar{R}$,

$$\bar{R} = R(I_b + \gamma I_s) + (1-\alpha)\frac{\theta}{2}\frac{5}{4}\varepsilon + \alpha\frac{\theta}{2}\varepsilon \tag{D.11}$$

Now, using the expression of I_b and I_s, we get

$$\bar{R} = \frac{\gamma R\tilde{A} - \varepsilon[R(1-\theta+\frac{\theta}{2})(1-\frac{1}{\beta}) + (1-\alpha)\tilde{A}\frac{5}{4}\frac{\theta}{2} + \alpha\tilde{A}\frac{\theta}{2}]}{\tilde{A} - R(1-\frac{1}{\beta})} \tag{D.12}$$

where $\tilde{A} = A(1-\delta) + R(\gamma + 1 - \gamma/\beta)$. This economy exists only if $\bar{R} > 0$. Sufficient conditions are

$$A > \frac{(1-\gamma)(1-\frac{1}{\beta})R}{(1-\delta)}$$

and ε is small enough. Then by construction, an economy $\{\underline{y}, \bar{y}y^*, \theta, \gamma, \alpha, R, F\}$ satisfying equations (D.1)-(D.10) exists.

References

[1] Ambrus-Lakatos, L. and U. Hege [2000] "Trade Credit Chains and Liquidity Supply", Mimeo, Tillburg University.

[2] Brennan, M-J., Maksimovoc, V. & Zechner, J. [1988]: "Vendor financing", *Journal of Finance*, pp. 1127-1141.

[3] Biais, B. and Gollier, C. [1997]: "Trade credit and credit rationing", *The Review of Financial Studies*, No. 4.

[4] Burkart, M. and T. Ellingsen [2002] "In-Kind Finance", Financial Markets Group, LSE, DP No 421.

[5] Carey, M. [1995] "Are Bank Loans Mispriced?", Discussion paper, Board of Governors of the Federal Reserve System.

[6] Cunat, V. [2000] "Trade Credit: Suppliers as Debt Collectors and Issurance Providers", Financial Market Group, LSE DP No 365.

[7] Emery,G. [1987]: "An optimal financial response to variable demand", *Journal of Financial and Quantitative Analysis*, No. 22.

[8] Ferris, J. [1981]: "A transaction theory of trade credit use", *Quarterly Journal of Economics*, No. 94.

[9] Frank, M. and V. Maksimovic [1998] "Trade Credit, Collateral and Adverse Selection", mimeo, University of Maryland.

[10] Kiyotaki, N. and J. Moore [1997] "Credit Chains", Mimeo, London School of Economics.

[11] Mian, S. and C. Smith [1992]: "Extending trade credit and financing receivables", *Journal of Applied Corporate Finance*, pp. 74-84.

[12] Petersen, M.A. and R.G. Rajan [1997] "Trade Credit: Theories and Evidence", *The Review of Financial Studies*, Vol. 10, No. 3, pp. 661-691.

[13] Welch, I. [1997] "Why is Bank Debt Senior? A Theory of Priority Based on Influence Costs", *The Review of Financial Studies*, Vol.10, No.4, Winter, pp. 1203-1236.