Ambiguous Contracting: Natural Language and Judicial Interpretation

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Abstract

We study the relationship between ambiguity (which comes into the picture since contracts have to be written in natural language), and contractual incompleteness. The contracting process is modelled as a signalling game between the parties and the judge, with the contract as the signal. The judge is assumed to be bound by the content of the contract (in as far as it can be ascertained unambiguously). Two kind of examples are presented: The first set of examples shows how ambiguity can lead to incompleteness. Here incompleteness is a way of hedging against adverse judgements on the part of an imperfectly informed judge. The remaining example illustrates a sort of converse intuition: It shows how incompleteness might lead the contracting parties to write ambiguous contracts in order to afford a relatively well-informed judge freedom to enforce the parties'will.

1 Introduction

The first-generation literature on contracts (for an overview, see Salanié 1996, or Laffont and Martimort 2002) took it for granted that the parties to a contract could communicate any agreement they reached to the instance in charge of enforcing it. In fact, besides the occasional reference to verifiability, the process of enforcement was rarely mentioned. Lately, a growing literature (for references, see Tirole 1999) has focused on explaining why contracts are or seem 'incomplete' - i.e., 'insufficiently state-contingent'(Ayres and Gertner 1989), certainly compared to what first-generation contract theory would have predicted. This literature has shifted the emphasis towards the enforcement process in attempts to link 'incompleteness' to costs of drafting contracts, describing contingencies, and the like (see, for example, Anderlini, Felli and Postlewaite 2001, and Maskin and Tirole 1999, plus references therein). The present paper is in this line, offering yet another explanation of incompleteness in terms of the constraints imposed by the enforcement process. We innovate in that our explanation relies on the constraints that natural language imposes on drafting contracts.

Natural language is ambiguous, imprecise, vague, etc.(and has to be, for deeper linguistic reasons that we do not explore, see Pinker 1995, Jackendof 2002, Solan 1993). This makes communication via a written contract very difficult, often leading to 'obligational incompleteness' (Ayres and Gertner 1989), i.e., a situation in which the enforcing instance (the judge) is uncertain about the nature of the agreement entered by the parties.'Obligational incompleteness', in turn, feeds back into the written form of the contract and, of course, into the sort of substantial commitments the parties aim for.

In this paper, we show, via examples, that when imperfectly informed judges are bound by a contract written in ambiguous language, well informed parties might prefer to write incomplete contracts.

The intuition behind these examples is simple: Contracts are left incomplete as a way to hedge against uncertain enforcement, which results both from the ambiguity of the language, and from the limited information available to judges .Note that it does not make sense to write contracts which condition on enforcement decisions, for who would enforce such a contract? Attempting to do this would only push the issue one level back, without actually resolving the key problem: The limited communication possibilities between the parties and an ill-informed judge.

We offer a second set of examples illustrating a sort of converse logic. We argue that when language is imprecise (thus directly limiting the commitments the parties can enter into), and the judge is well-informed, the parties might prefer to choose ambiguous or vague formulations. The intuition behind this is that ambiguity affords well-informed judges the discretion to enforce the will of the parties (which the judges, being well-informed, can reconstruct without relying on the written contract).

In developing these examples, it is crucial to distinguish between the contract as written object and the commitments entered via that document. To do this, we work with a game in which the parties signal their agreement to a judge via a written, binding contract. The judge then chooses a continuation game (compatible with the contract) to be played amongst the parties.

This framework is considerably more explicit than others along two dimensions: The enforcement instance is an active player; and we describe explicitly not only the space of possible agreements, but also the space of possible written contracts. This allows us to differentiate clearly between the commitments entered by the parties, and the means to achieve those commitments -the written contract (thus, incompleteness here is a feature of the commitments entered, while ambiguity and imprecision are features of the written contract). Drawing this distinction clearly is, we think, crucial; after all, contract theory is nothing but an exploration of the interaction between the written contract and the substantial commitments that derive from it[1].

While the examples we present do not amount to a theory, and we certainly are too ignorant of linguistics to be able to connect in a serious way with the insights of that discipline[2], we do think that these examples help in delimiting the issues involved, and, at the very least, suggest the outline of a more thorough exploration into the role of communication via natural language for the design of contracts.

Relation to the Literature The work nearest to ours is Bernheim and Whinston 1998. Our arguments share with their paper a basic second-best

[1] How important it is to draw this distinction clearly can be gleaned from the inconsistent definitions of incompleteness that permeate the literature: Sometimes incompleteness is taken to describe the written contract, sometimes the substantial commitments. So, for example, some authors describe as incomplete contracts that do not directly specify what the parties should do in each contingency, but instead only describe a procedure to decide what should be done after the uncertainty is resolved (e.g., those studied in by Maskin and Tirole 1999) Note, though, that these contracts commit the parties fully, if only in an indirect way. Similarly, contracts which optimally specify one and the same action for a whole range of contingencies, are often considered incomplete ('endogenously incomplete' -see, for example, Segal ?). In fact, such a contract is specifying an action for each and every contingency (and, by the way, doing this directly) . In contrast, other authors consider a contract incomplete if and only it does not commit the parties to a particular course of action in each contingency, (for example, Bernheim and Whinston 1998).

[2] It would seem though, that conventional linguistics focuses on describing the structure of language, rather than on studying the relationship between language and the real world. The latter being the primary concern in this work. See Jackendof 2002.

flavor: While they show how incompleteness along certain dimensions might lead to incompleteness along others, we show how ambiguity might lead to incompleteness and viceversa. Moreover, our story of how ambiguity leads to incompleteness fundamentally resembles their story of how incompleteness leads to more incompleteness. In both papers the parties aim is to secure additional flexibility. On the other hand, there are clear differences: In our work, the judge can verify all actions by the parties, and has full enforcement powers. The judge's problem lies in interpreting the contract accurately, i.e., in establishing intent. In their work, the judge is not an explicit player, and establishing intent is not an issue. These differences explain why we work with a signalling game, while they work with conventional perfect information games.

Two papers that model the enforcement process explicitly are Anderlini, Felli and Postlewaite 2001, and Baliga, Corchon and Sjostrom 1997. The first studies optimal judicial enforcement (void or uphold a contract) when the parties face a trade-off between insurance and incentives to make specific investments. This trade-off arises because contracts are assumed to be (exogenously) incomplete (in the sense of insufficiently state contingent). The latter paper takes an implementation approach, and looks at what happens when the planner herself has to act as enforcer. Following the implementation tradition, the enforcer is assumed to have limited information. The standard implementation problem is thus transformed into a signalling game, quite similar to the one we work with. Besides the fact that these authors are not concerned with natural language and its peculiarities, two basic differences between their work and ours are, one, that they look at cheap talk communication (while talk is costly in our setup as contracts are binding), and, two, that their enforcer pursues an independent objectives (over and beyond enforcing the most preferred outcome for the participants in the mechanism).

Maskin and Tirole 1999, whose main line of argument, however, is not directly concerned with the issues that occupy us here, draws attention to the importance of how contracts are written (the indescribability of certain contingencies requires that the procedure to determine the actions to be implemented ex-post be written in a way which does not refer directly to indescribable contingencies -this leads these authors to introduce a language formalism of sorts).

Finally, Ayres and Gertner 1989 and Hart and Moore 1999 clarify substantially some conceptual issues. Particularly, the distinction between 'obligational complete' and 'insufficiently state contingent'. As said, the present paper is very much about how obligational incompleteness interacts with incompleteness in the sense of 'insufficiently state contingent'.

2 The Model

We consider a bargaining situation between two parties. As a result of this bargaining, the parties will enter into agreements which will be 'coded' into written contracts. The exact informational conditions under which this bargaining will take place will vary, and will be described more closely further down. We will assume that there is an entity in charge of enforcing agreements between the parties ('the judge'). Though this entity will rely only on 'written contracts' in order to ascertain the will of the parties (i.e., will not observe the agreement directly), it is nevertheless necessary to keep track of both agreements and written contracts (the reasons for this will be explained in section ?).

2.1 Reference Game and Agreements

In order to describe agreements and, later on, written contracts, we introduce a so-called 'reference game-form' (henceforth, **RG**). As will become apparent, this is not the game-form of the game the parties will actually play (we refer to this latter game as the 'contracting game', **G** - to be described further down), but just a means to characterize it.

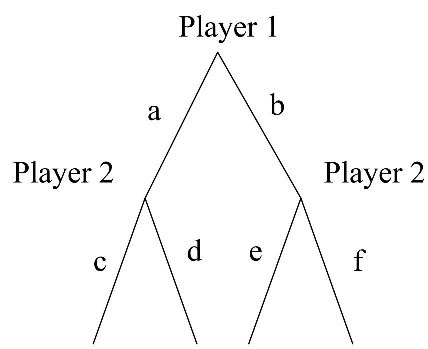
The reference game-form will be one of perfect information. It is convenient to describe the reference game-form in the following terms: For each party $i \in \{1, 2\}$, specify a set of actions A_i. It is assumed that these suffice to unambiguously describe the decisions in a given reference game-form (more precisely, that every action is described by a distinct symbol). If the reference game-form is simultaneous, the aforesaid amounts to a full description of the game-form. If the game-form is sequential, one has to specify a set T of admissible sequences of actions corresponding to paths of play in **RG** (for the details of such a formulation, consult, for example, Rubinstein and Osborne 1994).

Definition 1 *Given a* **RG***, an* ***agreement*** $\boldsymbol{\alpha}$ *is a set of terminal histories of* **RG**[3]*.*

[3]Note that agreements are 'positive', meaning that only those actions mentioned in

We adopt the notational convention of expressing $\boldsymbol{\alpha}$ as a pair of allowable actions' sets, one for each party, $[\alpha_1; \alpha_2]$, with $\alpha_i \subset A$. Each such set will be a string of actions, with individual actions separated by forward slashes (a logically redundant practice when applied to agreements; not so when applied to 'written contracts', as we shall see). Of course, the actions for both parties, when combined, must correspond exactly to a set of terminal histories. We will denote the agreement as a collection of terminal histories by α^T.

Example: Say **RG** is given by the following tree,



An agreement might be $[a; c]$. The actions in this agreement add up to exactly one terminal history, namely, ac.

An agreement that consists of more than one terminal history allows at least one of the parties discretion to decide his or her course of action, in the sense that the game-form constructed using the terminal histories in the agreement will at some stage afford this party a choice of actions. We will refer to such an agreement as **incomplete**. Agreements that are made up of only one terminal history, will be referred to as **complete**.

Example (cont.): The agreement $[a; c]$ is complete. An incomplete agreement would be $[a/b; c/f]$. This last agreement is consistent with two terminal histories of the given reference game-form, namely, ac and bf. The game-form constructed using these histories looks as follows,

Here, player 1 can choose between actions a and b.

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the agreement are allowed. Furthermore, note that while agreements can be specified in terms of outcomes of **RG** (terminal histories of **RG**), written contracts will be described in terms of actions. Outcome-based contracts are probably not equivalent to action-based ones, even when outcomes are described via actions sequences. Of course, if outcomes can be described directly, these two types of contracts need not be equivalent.

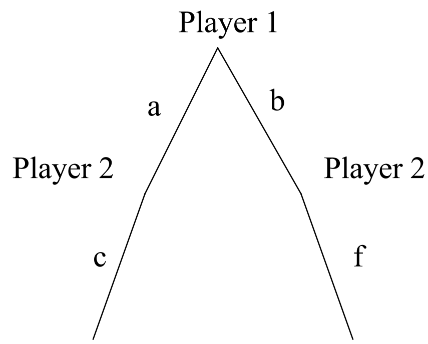


Figure 1:

2.2 Written Contracts and Readings

Two crutial aspects of any language are its syntaxis and its semantics. Roughly speaking, the semantics is concerned with the mapping from language to meaning, while syntaxis deals with the rules governing how the basic units of meaning, 'words', are combined into sentences. Ambiguity being the focus of this paper, it is natural for us to concentrate on the meaning aspect. In the present framework, the issue of meaning turns around the transition from agreements to written contracts. To achieve this transtion, we introduce a so called *Encoding Device,* a pair (C, E) , where C is a set of codes (a 'lexicon'), elements of which will be used to write contracts, and a mapping $E : A \rightarrow C$, specifying a code (or set of codes) for each action in A $(= A_1 \cup A_2)$, onto but not necessarily one to one. Note that the Encoding Device is exogenously given, a primitive of the model. It is meant to summarize the ('semantic') constraints imposed by Natural Language, and will serve as the driving force in the analysis of the paper.

Note further that we are not requiring language to be exact, i.e., one to one. In particular, different actions can share the same code. When this happens, we say that the encoding is *polysemic* (rather than *ambiguous*, a term we will use to designate a feature of written contracts).

We now describe how the lexicon will be used for writing contracts. Generally, let $S(B)$ denote the set of separated strings (i.e., strings with individual elements separated by forward slashes), generated from elements of the set B. Given a specific string $s \in S(B)$, let s^k denote its $kth.$ element, $k = 1, ..., n$. Abusing notation, let $E(A_i)$ denote all codes associated with elements of A_i by the encoding map E. Then $S(E(A_i))$ stands for the set of all separated strings generated from $E(A_i)$. Intuitively, $S(E(A_i))$ describes the universe of written sentences that might be used to describe behavior by player i. A written contract will then be an ordered pair of such sentences, where the first refers to player 1's behavior.

Definition 2 *A* ***written contract*** *is an ordered pair* $w = [w_1; w_2]$, $w_i \in S(E(A_i))$.

In linguistic terms, this definition of written contract embodies a form of syntax (syntax broadly understood as a prescribed, meaningful ordering of words). This order precludes certain misunderstandings. For example, there can be no confusion regarding whose actions the contract is referring to. Also, given the separation between individual entries, no confusion can arise regarding the number of allowed actions. For the same reason, it is not possible for confusion to result from grouping the symbols making up a sentences in differing ways (see appendix ? for a relaxation of this feature).

Even though the sorts of confusion that can obtain in going back from written contracts to actual behavior is limited by such syntactic devices, the exercise is still far from straightforward. Not surprisingly, given the posited inexact encoding, it will often be possible to 'trace back' the same sentence to quite different behavior. In other words, a written contract will often admit many 'readings'.

The following definition makes this precise,

Definition 3 *Given a written contract* w, *a* ***reading*** *of* w, $r(w)$, *is an ordered pair of separated action strings,* $[r(w_1); r(w_2)]$, $r(w_i) \in S(A_i)$, *each of the same length as the corresponding written component, such that for each entry* w_i^k *in that component, the kth. entry of* $r(w_i)$, $r^k(w_i)$, *belongs to the counterimage of* w_i^k *under* E, *i.e.,*

$$r^k(w_i) \in E^{-1}\left(w_i^k\right)$$

Let $\left\{r^k(w_i)\right\}_{k=1}^n$, where n is the length of w_i, denote the set of actions appearing in the string $r(w_i)$. Then we denote by $T\left(\left\{r^k(w_1)\right\}_{k=1}^n \cup \left\{r^k(w_2)\right\}_{k=1}^m\right)$ the subset of T (the set of terminal histories in **RG**) made up of actions in $\left\{r^k(w_1)\right\}_{k=1}^n \cup \left\{r^k(w_2)\right\}_{k=1}^m$.

Definition 4 *Given a contract w, a reading $r(w)$ is admissible iff*

$$T\left(\left\{r^k(w_1)\right\}_{k=1}^{n} \cup \left\{r^k(w_2)\right\}_{k=1}^{m}\right) \neq \emptyset.$$

In words, this reading of the contract allows one to construct at least one terminal history of the reference game-form.

Definition 5 *A contract is **ambiguous** iff it has more than one admissible reading.*

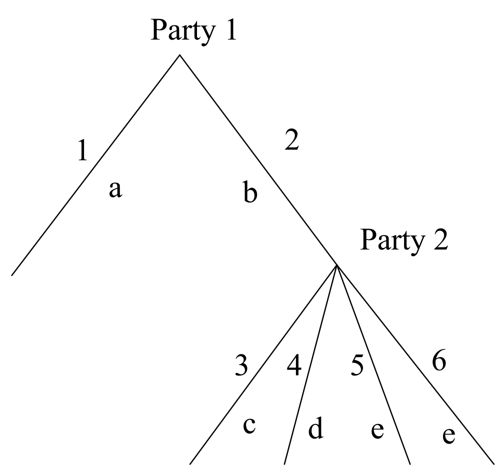
Definition 6 *An admissible reading is **complete** iff $T\left(\left\{r^k(w_1)\right\}_{k=1}^{n} \cup \left\{r^k(w_2)\right\}_{k=1}^{m}\right)$ is singleton. A contract such that all its admissible readings are complete will be called a complete contract.*

We feel is important to bear in mind that a contract might have both complete and incomplete readings. Even an incomplete contract in the above sense might suffice to fully commit the parties, as in equilibrium a complete reading might get implemented. We come back to this point further below.

2.2.1 Example

To illustrate the possibilities implicit in this formalization, we work through a somewhat more elaborate example. We adopt the convention of using numbers to describe actions, and letters to codify them.

Take the following **RG**,



Here, $A_1 = \{1, 2\}$, $A_2 = \{3, 4, 5, 6\}$, while

$$T = \{\{1\}, \{2, 3\}, \{2, 4\}, \{2, 5\}, \{2, 6\}\}$$

Say, the encoding function is given by

$$E(1) = a; E(2) = b; E(3) = c; E(4) = d; E(5) = E(6) = e$$

Note that the encoding function is polysemic, since actions 5 and 6 are encoded into the same letter. An agreement might be

$$[(1/2); (4/5)]$$

This agreement would allow for three outcomes, $\{1\}$, $\{2, 4\}$ and $\{2, 5\}$. In other words, it is incomplete. Now, using the encoding above, such an agreement has to be written

$$[(a/b); (c/e)]$$

This written contract is clearly both incomplete and ambiguous. It admits the readings $[(1/2); (3/5)]$ and $[(1/2); (3/6)]$,hence, it is ambiguous. On the other hand, both readings are consistent with multiple terminal histories, i.e., both $T(\{1, 2\} \cup \{3, 5\})$ and $T(\{1, 2\} \cup \{3, 6\})$ contain more than one element, hence, both are incomplete.

Note, though, that, generally, a written contract can be ambiguous without being incomplete, and viceversa. Consider the written contract $[(a); (d/e)]$. This written contract is ambiguous, since it has more than one reading, yet each such reading is complete, and, hence, so is the contract.

Also, there are ambiguous contracts which might admit both complete and incomplete readings (which, under our definition, would be described simply as incomplete). To illustrate, we need to modify the encoding, letting now$E(1) = E(2) = a$. Under this modified encoding, the written contract $[(a); (d/e)]$ has the reading $[(1); (4/5)]$, which is complete; and the reading $[(2); (4/5)]$, which is incomplete.

2.3 The Contracting Game

We are now in a position to describe the game the contracting parties will actually play. This will be an extensive form game between 3 differentially informed players: Two contracting parties and a judge.

The general structure will be that of a sender-receiver game (with the written contracts playing the role of signals, the parties the role of senders, and the judge the role of receiver). As usual, we model the initial uncertainty by moves of nature ('states of the world'), chosen from a set Ω, with generic element ω. Nature moves will be observed directly by the contracting parties, though not necessarily by the judge. In so far, the resolution of ex-ante uncertainty describes the knowledge of the world the contracting parties have at the moment of contracting. This knowledge will determine the terms of agreement between them.

Importantly, we are assuming that ex-ante nature moves are not describable. This is reflected in two features of the setup: One, the reference game-form is assumed to be the same after every nature move, and, two, the language being used to describe this game-form is also the same regardless of nature moves. A caveat: As will become apparent, this undescribability will not drive the results of this paper in any substantial way.

In describing the contracting game further, we distinguish between two stages: The proposing stage, and the enforcement stage.

2.3.1 Proposing Stage

In what follows, we will reduce the **ex-ante uncertainty** to two dimensions which we will think of as 'summarizing' the manifold considerations that might determine the outcome of contractual bargaining : One dimension refers to the assignment of 'bargaining power'. Nature will assign 'bargaining power' to either one or the other party, and the party with the bargaining power will then make a take-it-or-leave-it 'contractual proposal' to the other. The other dimension concerns the payoffs in the 'reference game'. Payoffs in $\mathbf{G}$ will be given by a mapping from the product of 'states of the world',Ω, and terminal histories in the reference game-form, T, to the positive reals,

$$u_i : \Omega \times T \to R^+$$

Given a state of the world, $\omega' \in \Omega$, we can define its corresponding **reference game**, $\mathbf{RG}(\omega')$, as being characterized by the reference game-form $\mathbf{RG}$, and payoffs $u_i(\omega', .)$, $i = 1, 2$.

We will assume that the states of the world display the following structure: Each state $\omega \in \Omega$ will be representable as a pair (b, π) where $b \in \{1, 2\}$ denotes whether player 1 has the bargaining power $(b = 1)$, or two does

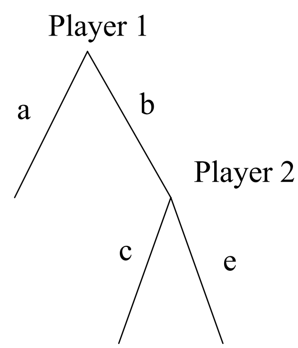
$(b = 2)$; while $\pi \in \Pi$, denotes the payoff structure of the given reference-game form.

The 'contractual proposal' will consist of a pair made up of an agreement and a written contract (the reasons for including an agreement in this proposal will be discussed when we discuss the judge's objective). If this proposal is rejected, the game is over and the parties receive a default payoff (which will be normalized to zero). If accepted, an enforcement stage follows.

We will require that the agreement and the written contract making up a proposal are 'compatible' with each other. In order to make this precise, we define 'continuation game',

Definition 7 *Given a written contract w, and a reading of this contract, $r(w)$, the* ***continuation game-form*** *under this contract reading,* $\mathbf{CG}(r(w))$, *corresponds to the game-form constructed using* $T\left(\left\{r^k(w_1)\right\}_{k=1}^{n} \cup \left\{r^k(w_2)\right\}_{k=1}^{m}\right)$.

Example (cont.): Using the original encoding in the preceding example, again consider the contract $[(a/b);(c/e)]$. Take the reading $[(1/2);(3/5)]$. The continuation game-form for this reading is



Since the reading is incomplete, this continuation game-form is non-trivial. Note that the continuation game-form corresponding to a complete reading allows no discretion to the parties.

Definition 8 *An agreement/contract pair is said to be* ***compatible*** *iff the game-form made up of the terminal histories of* **RG** *that are consistent with the agreement is a continuation game-form under some admissible reading of the contract.*

Note that in permitting only compatible proposals, we are implicitly excluding non-sensical contracts (i.e., contracts which have no reading such that $T\left(\left\{r^k(w_1)\right\}_{k=1}^{n} \cup \left\{r^k(w_2)\right\}_{k=1}^{m}\right) \neq \emptyset$, i.e., no admissible reading).

2.3.2 Enforcement Stage

In this stage, the only decision the judge makes is to choose a continuation game. Once such a game is chosen, parties will be free to play it as they see fit.

We will assume that the judge, in choosing a continuation game, is bound by a 'plain language rule':

Assumption (**Plain Language Rule**) Given a contract w, the judge can choose only amongst continuation game-forms corresponding to admissible readings.

Note that this rule just captures the obvious fact that a written contract not only binds the parties to a contract, but also the judge enforcing it.

Choosing a continuation game is then equivalent to choosing a reading amongst all admissible ones. It follows that if w is not ambiguous, i.e., admits only one reading, then the judge has nothing to decide. On the other hand, if the contract is complete, then the continuation game-form chosen by the judge will not allow the parties any choices. Note that the clear language rule makes it directly costly to choose (written) contracts; in other words, (written) contracts are not 'cheap'.

Example (continued): Looking only at the written contract $[(a/b)\,;(c/e)]$, the judge cannot determine whether e stands for action 5 or action 6. Note, though, that he or she does know that only one of these is meant. Hence, the judge has to decide what e stands for, i.e., choose an admissible reading. Depending on this decision, the allowable terminal histories will be either $\{1\}$ and $\{2,5\}$, or $\{1\}$ and $\{2,6\}$. In either case, party 1 has a choice of actions, i.e., the continuation game-forms are non-trivial.

Finally, even though it is implicit in the above, it is worth stating explicitly that the judge here can, in principle, observe all actions undertaken by the parties, i.e., that there is **full verifiability** -using the terminology of Bernheim and Whinston ?. Throughout this paper, this assumption will be maintained. Note that what prevents the judge from forcing the parties to take certain actions is not his or her inability to monitor compliance, but the plain language rule.

What does the judge maximize? In choosing amongst continuation games corresponding to admissible readings, the judge's objective will be to try and implement the will of the parties as accurately as possible[4]. It is in order to capture this formally, that we introduced a dual proposal above. The judge observes only one component of the proposal, the written contract, not so the agreement. Yet his or her aim will be to 'match the agreement' between the contracting parties. Specifically, the judge obtains an infinite payoff if the resulting terminal history of the chosen continuation game-form corresponds to one allowable under the agreement, and a payoff of minus infinity should the outcome differ from one allowable under the agreement. Note that both the agreement and the continuation game-form are made up of terminal histories of **RG**. The requirement of compatibility, on the other hand, guarantees that there will be at least one continuation game-form which incorporates the terminal histories in the agreement.

This payoffs' specification implies that the judge will be indifferent between choosing continuation games that, in equilibrium, do not lead for sure to a terminal history (of **RG**) in the agreement, as well as between continuation games that lead to such terminal histories for sure. In such cases, we will assume that the judge has recourse to a 'default rule',

Default Rule In any equilibrium in which the judge cannot be sure that choosing a continuation game-form corresponding to an admissible reading will result in a terminal history of **RG** consistent with the parties agreement, he or she will randomize evenly in choosing amongst continuation game-forms whose outcomes might correspond with the agreement.

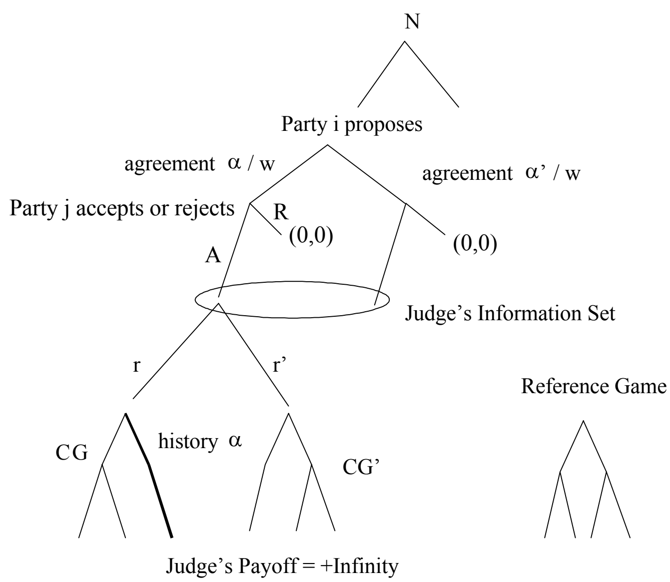
This dual procedure in implementing (private) contracts does seem common in practice: The judge sticks to the contracts when no interpretative problems arise. If the latter arise, then judges often have recourse to 'default rules' ('substantive canons of construction', 'rule of leniety', etc., see Solan 1993, p.66).

The Judge's Information We will consider situations in which the judge may not be fully informed along two dimensions: The judge might ignore who has the bargaining power, i.e., who is proposing. Secondly, the judge

[4] On the question of defining a judge's objective, see, for example, Posner 1993. That author takes the view that judge's maximize 'the same thing as everyone else'. We feel that defining the judge's objective is not quite as straightforward. The text hints at the issues.

might ignore the exact payoffs of the reference game being played. Of course, the judge will not be able to directly observe the agreement reached between the parties, only the written contract 'embodying' that agreement. Towards the end, we will consider briefly situations in which the judge has other information.

The diagram below illustrates schematically the structure of the contracting game:



2.4 Legal Equilibrium

As above, let Ω stand for the set of states of the world (i.e., initial nature moves), with Ω_i denoting the states in which contracting party i proposes. Further, let P denote the set of all agreements, and let W denote the set of all written contracts. Given a contract $w \in W$, we defined $\mathbf{CG}(r(w))$ as the (continuation) game-form constructed using terminal histories in

$$T\left(\left\{r^k(w_1)\right\}_{k=1}^{n} \cup \left\{r^k(w_2)\right\}_{k=1}^{m}\right)$$

Let $\mathbf{CG}(w)$ stand for the set of all continuation game-forms associated with some admissible reading of w.

Denote by $\sigma_i(r(w))$ a strategy of contracting party i $(i = 1, 2)$ in continuation game-form $\mathbf{CG}(r(w))$, and let $\Sigma_i(r(w))$ stand for the set of all such strategies. A **strategy for contracting party** i in the contracting game $\mathbf{G}$, s_i, consists of two mappings, s_i^k, $k = 1, 2$, such that

$$s_i^1 \;:\; \Omega_i \rightarrow W \times P$$

$$s_i^1 \;:\; \Omega_{-i} \rightarrow \{accept, reject\}$$

$$s_i^2 \;:\; W \times P \times \Omega \rightarrow \Sigma_i \left(r\left(w\right)\right)$$

A **judge's strategy**, r, is simply a mapping from written contracts to probability distributions over continuation games,

$$r : W \rightarrow \triangle \mathbf{CG}\left(w\right)$$

The **judge's beliefs** are given by a mappingΨ from written contracts to probability distributions over Ω,

$$\Psi : W \rightarrow \triangle\Omega$$

The judge's beliefs, will, of course, be required to be consistent with the equilibrium, in the usual manner.

In the light of the posited objective for the judge, the use of a default rule, and the assumption of compatibility, it is more economical to describe the judge's behavior in the following way: Given beliefs Ψ and a pair of strategies $s_i, i = 1, 2$, the distribution over states induces a distribution over agreements. Whenever an agreement is assigned probability one, an optimizing judge must choose the continuation game corresponding to this agreement with probability one (such a continuation game exists by compatibility). Else, the judge has to mix evenly amongst all admissible continuation games -by the default rule. Note that this behavior does not conflict with maximization of payoffs on the part of the judge -at least in as far as this maximization is well defined.

Define **modified beliefs** as

$$\widetilde{\Psi} : W \rightarrow \triangle P$$

and let the judge follow the behavioral rule

$$\delta : W \rightarrow \triangle \mathbf{CG}\left(w\right)$$

satisfying

$$\widetilde{\Psi}(w)(p) = 1 \rightarrow \delta(w)(p) = 1$$

$$\delta(w)(.) = \tfrac{1}{\#\mathbf{CG}(w)} \quad else$$

Of course, $\widetilde{\Psi}$ will again be required to be consistent with the contracting parties' strategies.

Definition 9 *A **legal equilibrium** is a behavioral rule for the judge δ, a profile of strategies for the contracting parties, s, and beliefs $\widetilde{\Psi}$, such that the latter are consistent with δ and s, while the strategies s are optimal under δ. In particular, the projection of s to any continuation game admissible under s forms a subgame perfect equilibrium in that game.*

3 How Polysemy Leads to Incompleteness

We start out by presenting three examples of how ambiguity might lead to incompleteness. As it turns out, the examples are less special than might appear at first, and seem to represent the main ways in which ambiguity can lead to incompleteness.

In all instances we deal with situations in which, roughly speaking, the parties have too many things to say, but too few words to express them (explicitly in the first example; implicitly in the others). This circumstance forces pooling in equilibrium (i.e., the same written contract obtains in at least two distinct states).

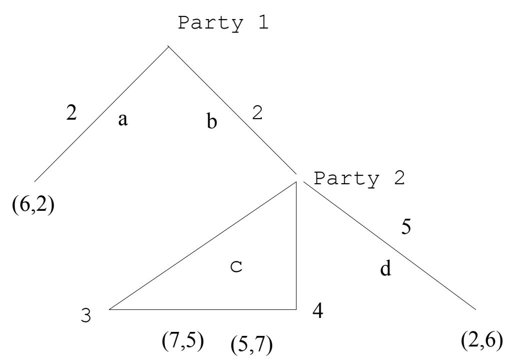
In the first example, pooling creates incentives for the party with the bargaining power (i.e., the party proposing the contract/agreement pair), to include additional actions in the written contract in order to be able to 'escape' an eventually erroneous judgment. It illustrates a 'hedging' motive for incompleteness, which we shall argue is specific to contracting under ambiguity.

In the second example we illustrate a self-enforcement motive for incompleteness. The idea being that parties allow themselves some discretion in order to exploit the informational advantage they enjoy. This form of incompleteness is not specific to the type of situations studied here.

In essence, Bernheim and Whinston ? also obtain this sort of incompleteness. However, there are differences. A third example aims to make these plain -besides illustrating a special form of self-enforcement based on coordination in reference games with simultaneuos moves.

3.1 Example 1: Hedging

Assume the judge knows only that the reference game is one out of a continuous set. That is, we will assume that initial nature moves are drawn from a continuum, and that the judge cannot directly observe those moves (though, of course, the parties to the contract can). The uncertainty will concern only the payoffs that obtain in the reference game. The diagram below illustrates: After player 1 takes action 2, party 2 chooses between a single action (5) and an interval of actions, all encoded c. Hence, the encoding is polysemic. In the interval of actions, there is always exactly one action leading to the outcome $(7,5)$, another to the action $(5,7)$. All other actions lead to the payoffs $(0,0)$.The exact location of the two 'good' actions along the continuum is determined by nature, and will not be directly observed by the judge.



Finally, assume that party 1 has the bargaining power (the judge is aware of this assignment).

Trivially, there must pooling in equilibrium, as the cardinality of the set of possible reference games vastly exceeds that of the set of written contracts (which are also the potential signals). By the default rule adopted here (see section ?), the judge, when confronted with a written contract which includes c, picks an action from the interval randomly. Under these circumstances, it will be the case that the written contract

$$[(a \backslash b)\,;(c)]$$

dominates all others for player 1; while the contract

$$[(b)\,;(c\backslash d)]$$

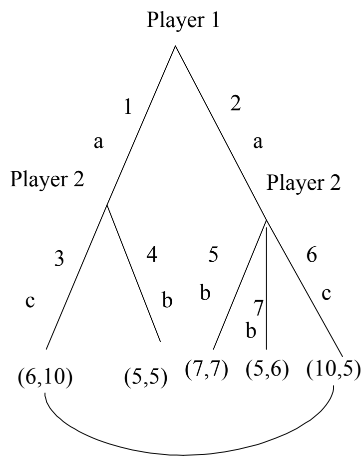
is dominant for player 2.

These incomplete contracts dominate because they allow the players to **hedge** against adverse judgments. Should the judge choose the 'wrong' action, the party proposing the contracting will always be able to 'opt out' by taking his or her 'hedging' action. Note that incompleteness is the only meaningful way to do this: It would not make sense to make a contract contingent on the judge's ruling, for, who would enforce it? Allowing for appeals would only push the central issue one level up each time one of the parties has recourse to a higher court. Eventually, a final enforcing instance (supreme court) has to be reached, and we would be back at the problem as posed here.

This example brings forth what we think is an interesting linguistic problem: The issue of **pragmatic signalling**. We will attempt to define this notion more precisely further on (though we will not go beyond this in the main text). For now, let us just say loosely that pragmatic signalling amounts to 'cheap-talk' signalling via 'redundant statements'. We feel that, in practice, the scope for such pragmatic communication is extremely limited, and, hence, in much of what follows we will simply assume it away (for some examples of why such pragmatic signalling might not be possible even when the cardinality of the signal space exceeds that of the state space, see Appendix ?). By the way, note this hedging equilibrium is separating (and must be, see Section ?).

3.2 Example 2: Self-Enforcement

Nature has two moves: One move leads to the payoffs shown in the diagram below, while the other interchanges them (leaving the rest of the reference game unchanged):



As before, the parties to the contract observe nature moves, but the judge does not. Assume that party 2 has the bargaining power, and that this is known by the judge. Thus, the problem of the judge is just to figure out which actions to enforce in order to induce the best outcome for player 2. Note that the encoding is polysemic. If it were not, despite the judge's ignorance, the parties would agree to a complete contract, which would provide the judge with precise instructions. This shows, by the way, that the posited undescribability of nature moves does not by itself lead to incompleteness in this sort of setup (the key feature in this respect is the ex-ante nature of the uncertainty).

With a polysemic encoding, however, player 2 has to figure out whether there is contract that would allow him or her to achieve the best outcome in all states. Since the polysemic encoding does not allow player 2 to directly tell the judge which route to follow (the desired terminal history in either state can only be described by $[(a)\,;(c)]$), such a contract will have to allow player 1 some discretion, and, hence, be incomplete. However, if player 1 is allowed discretion, player 2 must have discretion too. For, given that the judge does not know which history leads to the desired outcome, and since the polysemic encoding does not allow player 2 to directly tell the judge which route to follow, the only option for the proposing player is to, in effect, take over the judge's role and enforce the right action by player 1 him- or herself by means of state contingent threats. This can, in fact, be achieved here, through the following incomplete contract:

$$[(a/a)\,;(b/b/c/c)]$$

The judge will then choose the reading

$$[(1/2)\,;(4/7/3/6)]$$

The continuation game corresponding to this reading is non-trivial, and has in each state the 'right' subgame perfect equilibrium. Note that the equilibrium is fully pooling.

Again, the issue of **pragmatic signalling** comes up here. For example, one could, quite trivially, obtain a fully separating equilibrium by having the proposing party send the contract

$$[(a)\,;(c/c)]$$

in the first state, and the contract

$$[(a)\,;(c)]$$

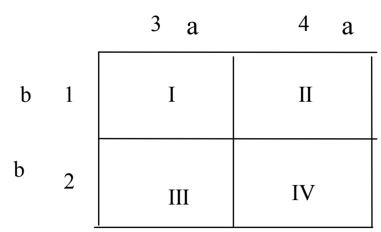
in the second. Note that both contracts are complete. The second c in the first contract just serves the purpose of signalling the state to the judge. As we said in the introductory comments to this section, we feel this sort of communication is rare in the real world. Formally, it would be easy to modify this example along the lines of example 1 so that the cardinality of the state space exceeds that of the signals space (the space of written contracts), and thus exclude full separation.

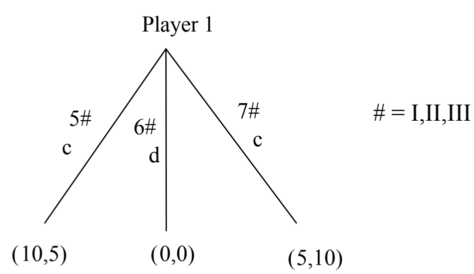
This example also illustrates some of the subtleties of incompleteness in this framework. The pooling contract proposed has, besides incomplete **readings**, complete ones (unlike the hedging contract in example 1, which only had incomplete readings). It is the judge who chooses the incomplete reading, precisely in order to induce self-enforcement. In other words, substantive incompleteness (i.e., partial commitment) is really a property of the legal equilibrium, not of the contract itself.

As already pointed out, Bernheim and Whinston ? also offer a self-enforcement explanation for incompleteness (in a model with limited verifiability though). While essentially the same logic as in their work operates here, there is a substantial difference: In their framework, one party is granted additional flexibility because the flexibility of the other cannot be constrained. Here, additional flexibility is granted to one party in order to grant additional flexibility to the other. The next example illustrates this difference.

3.3 Example 3: Coordination

The reference game is as follows: To start with, the two parties choose between 2 actions simultaneously. Afterwards, they proceed to play a sequential game (one and the same game-form, regardless of the outcome of the simultaneouous moves). Assume that uncertainty concerns here only who has the bargaining power. The diagram below illustrates:





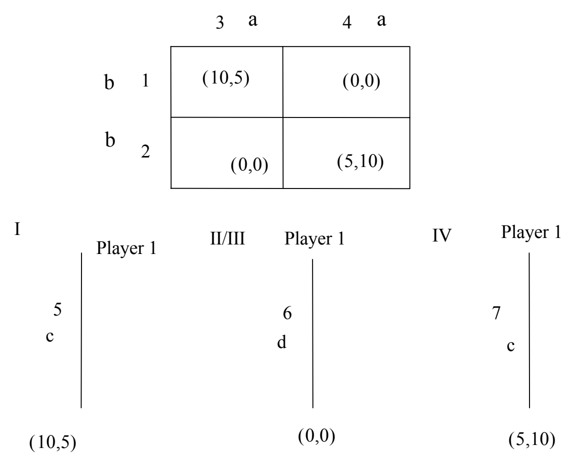
The following contract implements the will of the parties in each state,

$$[(b/b/c/c/d/d)\,;(a/a)]$$

This contract admits the reading

$$[(1/2/5I/6II/7IV/6III)\,;(3/4)]$$

The corresponding continuation game is illustrated below



Note that the 'reduced form' simultaneous moves game that results has two Nash equilibria (note, by the way, that these are 'self-enforcing' in the sense of Aumann ?). One equilibrium outcome corresponds the most desirable outcome in **RG** for player 1, while the other corresponds to the most desirable outcome for player 2. Hence, this contract can implement the will of the parties in every state.

This example illustrates well the differences with the Bernheim and Whinston approach: The key to incompleteness here is the **multiplicity of equilibria**. If it were not possible to contractually induce a simultaneous game with multiple equilibria, it would not make sense to allow the parties discretion. In Bernheim and Whinston, in contrast, the parties would not want to induce multiple equilibria in this fashion. In fact, in their framework incompleteness (appropriately redefined to take into account limited verifiability) cannot result in static setups, except in cases in which a complete contract would induce multiple equilibria, and then only as a means to eliminate such multiplicity, not as a means to exploit it (see Bernheim and Whinston ?, p.907).

In order to clarify the relationship of the present work to Bernheim and Whinston ?, we attempt to reinterpret the example just presented in terms of limited verifiability. Start by assuming that the judge is fully informed but cannot distinguish between actions 1 and 2, on the one hand, and actions 3 and 4, on the other. This reinterpretation requires further modifications: It would not seem reasonable to allow the judge to condition 'downstream' enforcement decisions on the outcomes of the simultaneous move game, given that he or she is not in a position to distinguish amongst them (Bernheim and Whinston follow this modelling convention, see the discussion on p.908). Hence, the same 'downstream' actions will have to be enforced regardless of the simultaneous moves outcome. This represents no problem for the party proposing though: He or she will simply have the judge enforce the actions leading to his or her best outcome. By doing this, the simultaneous game becomes trivial. Any outcome will correspond to a Nash equilibrium, and all will yield the desired outcome for the player proposing. Note that the resulting contract would be complete, as completeness can here only reasonably mean 'as complete as possible' given the limits of verifiability[5].

We could modify the example further in order to make the reinterpretation less trivial, say by letting the terminal histories leading to the best outcome for the party proposing vary with the outcome of the simultaneous moves stage. Since the judge has to enforce the same sequence of actions regardless of the outcome of the simultaneous moves game, the payoffs will now vary across such outcomes. Under these conditions, the party proposing will aim ideally to select payoffs yielding a unique Nash equilibrium whose outcome corresponds to the best possible for this party. Inducing multiple equilibria would only make it more difficult to force coordination on the desired outcome. By the same logic, adding additional verifiable actions (making incompleteness feasible) would only lead to incompleteness if allowing these actions helps eliminate undesired equilibria.

4 Characterizing Incompleteness

4.1 Polysemic Structure

In trying to characterize more generally the conditions under which incompleteness might result, and the form it might take, we will restrict attention to **sequential games** (and thus we will not consider coordination incompleteness of the sort illustrated by Example 3). Moreover, we will restrict attention to sequential games with **generic payoffs** , i.e., with no payoff-ties. An implication of this last assumption is that, for each state $\omega \in \Omega$, and for each player i, $i = 1, 2$, there will be one and only one **best terminal**

[5] Of course, incompleteness in the broader sense of Bernheim and Whinston ? is unfeasible in this example. However, it is obvious that even if we were to add an additional verifiable move for each player at the initial stage (making incompleteness in this broader sense feasible), things would not change.

history, denoted by $t^*(i,\omega) \in T$, i.e., one and only one history such that

$$u_i(t^*(i,\omega),\omega) \geq u_i(t,\omega) \quad \forall\, t \in T$$

We do not think any of these restrictions bears on the substance of our conclusions, and they certainly make the characterizations much less notation intensive.

Define a 'contractual situation', $\mathbf{G}_{E,C}$, made up of a contracting game, $\mathbf{G}$, and an encoding device, (E,C).We proceed to define what we will call, for lack of a better name, the 'polysemic structure' of such a 'contractual situation', denoted by $\mathcal{P}_{\mathbf{G},(E,C)}$. This concept will play a crucial role in the analysis that follows. Informally, a 'polysemic structure' will be a partition of all terminal histories in **RG** such that histories in a cell cannot be distinguished linguistically, i.e., are all described by the same sentences. To make this notion precise, define the *minimal encoding mapping*

$$E_{\min} : T \to C^2$$

associating with any terminal history the smallest cardinality set of codes that can be used to write contracts that can be read as such a history. To simplify, we will be assuming that the encoding does not allow for synonyms, thus guaranteeing that this mapping is a function.

Definition 10 *Given a contractual situation* $\mathbf{G}_{E,C}$*, its* ***polysemic structure*** $\mathcal{P}_{\mathbf{G},(E,C)}$ *will be a partition of all terminal histories in* **RG***, such that, for any cell* $p' \in \mathcal{P}_{\mathbf{G},(E,C)}$*,*

$$E_{\min}(t) = E_{\min}(t') \quad \forall t,t' \in p'$$

This implies that if a history in a cell can be described by the written statement $[w_1; w_2]$, all histories in that cell can be described in those terms as well.

Example: Refer to the reference game and encoding in the example of section 2.2.1. Assume that uncertainty concerns only who has the bargaining power, and that terminal history $\{2,5\}$ is best for player 1, while terminal history $\{2,6\}$ is best for player 2. These two histories cannot be distinguished by means of written descriptions, since

$$E_{\min}(\{2,5\}) = E_{\min}(\{2,6\}) = \{e,b\}$$

Hence the polysemic structure of this contractual situation has four cells only.

Define the polysemic structure of the best terminal histories, $\mathcal{P}^*_{\mathbf{G},(E,C)}$, analogously to polysemic structure, but referred only to the set of best histories as defined above.

The following proposition is immediate,

Proposition 11 *Incompleteness cannot arise in contractual situations* $\mathbf{G}_{E,C}$ *whose polysemic structure* $\mathcal{P}^*_{\mathbf{G},(E,C)}$ *consists only of singleton cells.*

4.2 No Incomplete Agreements

The next proposition shows that incomplete agreements are in a sense weakly dominated in this framework.

Proposition 12 *Given a legal equilibrium specifying an incomplete agreement in some state, there always exists a legal equilibrium specifying complete agreements in all states that generates an equivalent or better outcome.*

Proof. Say the agreement $\alpha^T(\omega)$ is incomplete. If the judge chooses a reading of $w(\omega)$ with probability one, then the incompleteness of the agreement is clearly redundant. If the judge mixes, and $w(\omega)$ is proposed only in this state, then by dropping those histories in the original agreement that yield lower payoffs, while sticking to the original written contract, one obtains a legal equilibrium specifying only complete agreements whose outcome is superior (note that in such an equilibrium, the judge will not mix but will choose the reading that leads to the realization of the agreement). If $w(\omega)$ is proposed in more than one state, then dropping dominated histories from the agreement (while sticking to the original written contract), results in an outcome-equivalent legal equilibrium (the judge will continue to mix amongst exactly the same readings as before). ■

4.3 Pragmatic Signalling

We will define more precisely what we mean by 'pragmatic signalling'. As said, our notion of 'pragmatic signalling' coincides in essence with the conventional notion of 'cheap signalling' in sender-receiver games. The only complication is that, in the present framework, whether a code is cheap or not depends on the context in which that code appears.

The most obvious case of 'cheap' communication arises with **repeated codes,** either non-polysemic or polysemic (provided the latter are repeated often enough). The inclusion of such repetitions in any written contract will leave the readings of that contract as well as the set of terminal histories associated with any such reading, unchanged. On the other hand, there could be non-repeated codes in a written contract that, when deleted, leave the set of continuation games associated with the contract unchanged, though the readings associated with the resulting contract will change. Such codes we will call **isolated**, in as far as the actions associated with them do not enter into the construction of any terminal history under any reading.

These considerations motivate the following definition,

Definition 13 *Given a written contract,* ***redundant codes*** *are those that, if deleted, leave the set of continuation games associated with any reading of the original contract unchanged.*

We want to strengthen this definition somewhat to

Definition 14 *Given a legal equilibrium,* ***equilibrium redundant codes*** *are those that, if deleted, leave the set of continuation games associated with any reading chosen with positive probability under the original contract unchanged.*

There is another form of redundancy which we want to exclude as well: It concerns codes or combinations of codes that only introduce dominated choices into the continuation games associated with the contract that results from their deletion.

Definition 15 *Given a written contract w, we say that a code item w_i^k in the contract is* ***dominated*** *if there is no reading of this contract such that an action $r_i^k(w_i)$ would have been chosen in the subgame perfect equilibrium of the subgame starting at the point where this action is available, in some state $\omega \in \Omega$.*

The following definition is a strengthening of the preceding one,

Definition 16 *Given a legal equilibrium, a code item w_i^k in w_i is* ***equilibrium dominated*** *if there is no reading $r(w)$ and associated continuation game in which the action $r_i^k(w_i)$ would have been chosen in the subgame perfect equilibrium of the subgame starting at the point where this action is available, in some state $\omega \in \Omega$ in which this contract is proposed.*

This can be strengthened further to

Definition 17 *Given a legal equilibrium, a code item w_i^k in contract w is* ***strongly equilibrium dominated*** *if there is no reading $r(w)$ and associated continuation game chosen with positive probability in some state $\omega \in \Omega$ in which this contract is proposed, and such that action $r_i^k(w)$would have been chosen in the subgame perfect equilibrium of the subgame starting at the point where this action is available.*

We can now define 'cheap signalling' as follows,

Definition 18 ***(Pragmatic Signalling)*** *Given a legal equilibrium, we say that equilibrium contracts signal pragmatically if they are differentiated across states only by redundant or dominated codes.*

Definition 19 ***(Pragmatic Signalling in Equilibrium)*** *Given a legal equilibrium, we say that equilibrium contracts signal pragmatically if they are differentiated across states only by equilibrium redundant or strongly equilibrium dominated codes.*

Clearly, communication via redundant and/or isolated codes represents an instance of what linguists might call 'pragmatic communication'. That is, communication via items that only adquire meaning from knowledge of the situation in which the communicating parties find themselves. Or to put it differently, communication that does not rely on intrinsic meaning. We feel the scope for this sort of communication in the sort of situations we are considering is very limited, and we are quite comfortable in excluding it a priori. On the other hand, it is equally clear that by excluding any form of equilibrium domination we exclude forms of communication that do rely on intrinsic meaning (for example, passive hedging). These meaningful communication is characterized by having the judge rely on irrelevant statements in order to infer the state. The irrelevance of these statements is of course relative: Here they are irrelevant in so far as the actions they describe could be deleted without altering the outcome of the equilibrium in the continuation games being chosen with positive probability. Of course, deleting the codes describing those actions would alter the overall outcome as the judge is making inferences based on those codes. In what follows, we exclude this form of communication as well, even though it is much less clear to us how much people rely on such devices in practice.

Once one excludes strong equilibrium domination, the following proposition is immediate,

Proposition 20 *In a fully separating legal equilibrium, in which no contracts include strongly equilibrium dominated codes, all contracts are complete.*

Proof. Assume the contract admits an incomplete reading. Take an action in the corresponding continuation game which is not being played in its equilibrium. Since the equilibrium is separating, the judge will know exactly which history he or she should enforce in each state. Consequently, the judge will choose only continuation games which yields this outcome and no other. Hence this action (or one corresponding to the same code item in the contract) will not be played. In other words, the code item corresponding to this action will be strongly equilibrium dominated. ■

4.4 Hedging

In this section, we try to characterize formally the idea of 'hedging' introduced in Example 1 above.

Denote by $CG(\omega)$ the continuation game made up of the game-form CG and payoffs corresponding to state ω. Similarly, let w_ω be the contract proposed in that state, and $\alpha^T(\omega)$ be the agreement proposed in that state. We will denote by $t_{CG(\omega)}$ the terminal history which obtains in equilibrium in game $CG(\omega)$. Define the 'joint' of a set of historiesT' as their highest common decision node, i.e.,as the node that actions shared by all histories in T'. We denote it $\mathbf{n}(T')$. Finally, we denote the set of codes describing a set of histories T' in a given contract w_ω, by $w_\omega(T')$.

Definition 21 *A contract w_ω is hedging if it admits two readings with corresponding continuation games $CG_1(\omega)$ and $CG_2(\omega)$ such that*

$$CG_1 = CG_{1a} \cup CG_1 \backslash CG_{1a}$$

$$CG_2 = CG_{2a} \cup HCG_2$$

with

$$w_\omega(CG_{1a}) = w_\omega(CG_{2a})$$

and such that

$$t_{CG_1(\omega)} = t_{CG_{1a}(\omega)} \in \alpha^T(\omega)$$

$$t_{CG_{2a}(\omega)} \neq t_{HCG_2(\omega)} = t_{CG_2(\omega)} \notin \alpha^T(\omega)$$

Also, it must be that

$$t_{CG_{2a}(\omega)} \prec t_{HCG_2(\omega)}$$

If these conditions are satisfied, we say that $HCG_2(\omega)$ hedges against $CG_{2a}(\omega)$.

Building on this definition, we can define more precisely 'active' and 'passive' hedging.

Definition 22 *Given an ambiguous legal equilibrium, a contract w_ω* ***actively hedges*** *if it is hedging, and, moreover, both readings involved in the hedging are chosen with positive probability.*

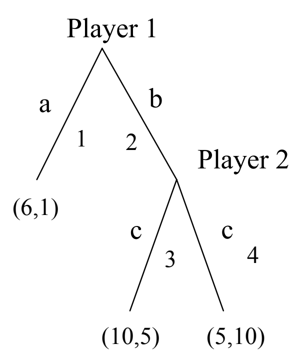
Definition 23 *Given an ambiguous legal equilibrium, a contract w_ω* ***passively hedges*** *if it is hedging, but $CG_2(\omega)$ is not chosen in equilibrium.*

We can define further 'direct' hedging versus 'indirect' hedging

Definition 24 *A contract w_ω is* ***directly hedging*** *if in addition to being hedging, it is the case that the proposer has the move at the joint between $t_{CG_1(\omega)}$ and $t_{CG_{2a}(\omega)}$. Else, we say that the contract hedges indirectly.*

Example of Separating, Passive, Direct Hedging:

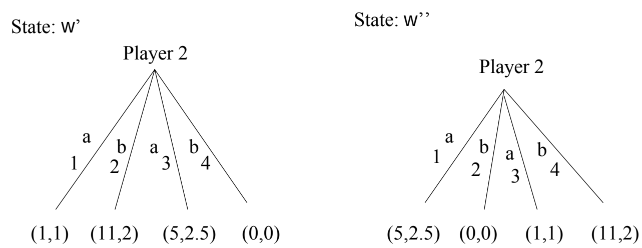
There will only be uncertainty regarding who has the bargaining power. The reference game is as illustrated below,



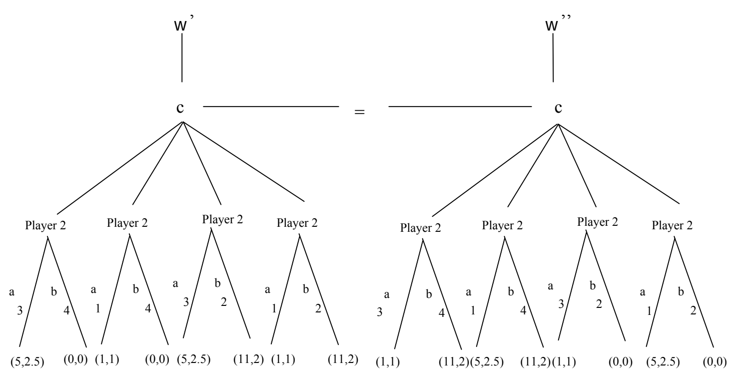
The following is a legal equilibrium: Player 1 proposes the contract $w(1) = [(a/b);(c)]$ and the agreement $\alpha(1) = [2;3]$, while player 2 proposes $w(2) = [b;c]$ and the agreement $\alpha(2) = [2;4]$. This equilibrium is hedging with $CG_1(1) = \{1;(2,3)\}$, $HCG_2(1) = \{1\}$, $CG_{1a}(1) = \{(2,3)\}$, $CG_2(1) = \{1;(2,4)\}$ and $CG_{2a}(1) = \{(2,4)\}$. Here player 1 hedges. Evidently, this equilibrium is separating, and so the judge will enforce the agreement with probability one. Note that code a in $w(1)$ is strongly equilibrium dominated.

Example of Pooling, Active Indirect Hedging:

There will be only uncertainty regarding payoffs. For concreteness, say party 1 proposes. Two reference games are possible:



We claim that proposing the contract $[a/b]$ in both states, with agreements $\alpha^T(\omega') = \{2\}$ and $\alpha^T(\omega'') = \{4\}$, and the judge mixing evenly between the readings $[1/2]$, $[3/4]$, $[3/2]$ and $[1/4]$, is a legal equilibrium in which there is active indirect hedging. The diagram below illustrates the equilibrium:



This equilibrium is best understood by considering what would happen if party 1 were to propose a complete contract, say $[b]$. The judge would still resort to the default rule. However, conditional on either of the two states, the expected payment to the proposing party would only be $\frac{1}{2}11 + \frac{1}{2}0 = 5.5$, instead of $\frac{1}{4}11 + \frac{1}{4}5 + \frac{1}{4}1 + \frac{1}{4}11 = 7$, as under the proposed equilibrium.

Note that the contract and the equilibrium are incomplete (i.e., in equilibrium an incomplete reading is chosen for sure). It is not self-enforcing, as the outcome does not always correspond with the agreement. It is hedging. To see this, take, for example, the contract $w(\omega') = [a; b]$, and the corresponding agreement $\alpha^T(\omega') = \{2\}$. We have $CG_1 = \{1, 2\}$ and $CG_2 = \{3, 4\}$, with $CG_{1a} = \{2\}$ and $CG_{2a} = \{4\}$. Clearly,

$$w_\omega(CG_{1a}) = w_\omega(CG_{2a}) = b$$

Moreover, we have

$$t_{CG_1(\omega')} = t_{CG_{1a}(\omega')} = \{2\} = \alpha^T(\omega')$$

$$\{4\} = t_{CG_{2a}(\omega')} \neq t_{HCG_2(\omega')} = t_{CG_2(\omega')} = \{3\} \neq \alpha^T(\omega')$$

This is a case of active hedging, as the judge mixes, and thus the proposer gets to exercise the hedging option.

4.5 Self-Enforcement

Definition 25 *A pooling ambiguous legal equilibrium, is said to be* ***self-enforcing*** *in the pooled states if in each of those states the subgame perfect equilibria of the continuation games selected with positive probability correspond to the agreement in that state, i.e., if*

$$w(\omega') = w(\omega'')$$

then

$$t_{CG(\omega)} \in \alpha^T(\omega)$$

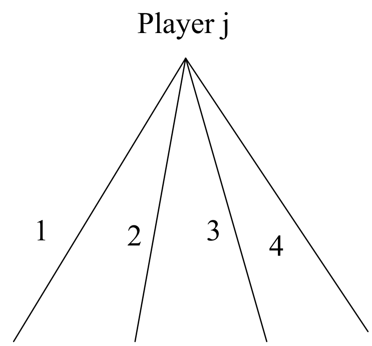
for $CG(\omega) \in \mathbf{CG}(\omega), \omega \in \{\omega', \omega''\}$, *chosen with positive probability, and such that at least one continuation game-form is non-singleton.*

The main difference between hedging and self-enforcing lies in the fact that (active) hedging is a response to uncertainty regarding the judge's choice of readings, while self-enforcement can obtain even if the judge is certain to pick a particular reading.

For an illustration, the reader is referred to section ?.

5 Characterizing Incompleteness in Simple Reference Games

In this section, we specialize our setup even further. The following propositions are proved for reference games in which only one of the parties chooses an action, i.e., reference games of the form illustrated below,



Call these '**simple reference games**'. Situations in which the party proposing is also the party moving in the reference game can be understood as 'right to play' scenarios: The party proposing needs the permission of the other party to move -hence the need for a contract.

We conjecture that close counterparts of the propositions we present below can be obtained in the more general setup sektched above, though at the cost of a lot more notation.

We will argue that incompleteness in this environment essentially takes two forms, (indirect) hedging and self-enforcement.

Proposition 26 *With two states, in a legal equilibrium with simple reference games and complete agreements, in which no contract includes strongly equilibrium dominated codes, and in which at least one contract reading chosen with positive probability is incomplete, incompleteness only takes two forms, hedging and self-enforcement.*

Proof. Let the states be ω and ω'. Proposition ? implies that the equilibrium must be pooling (fully, since there only two states). Take an incomplete contract reading, say $r(\omega)$, and take an action $r_i^k(\omega)$ which is not played in the corresponding continuation game $CG(\omega)$. Since the code corresponding to that action is not strongly equilibrium dominated, it must be that there is some other continuation game, say $CG'(\omega)$ chosen with positive probability in some state such that an action encoded by w_i^k is actually played.

If this happens in the same state as the one we started with, then two conditions must be satisfied: First, the judge must be mixing. Second, either this action is not in the agreement, or, if it is, it must be that the equilibrium action of the original continuation game is not (since we take the agreement to be complete). Without loss of generality, assume that it is not in the agreement (hence the equilibrium outcome of the original continuation game. $CG(\omega)$, is in the agreement).

Say the new action should not be preferred by the proposer in that state to the action that would be played in the corresponding continuation game should this action not be available. Then assuming that dropping of codes does not change judges beliefs regarding the agreement in each state, it would be better for the proposer to drop the corresponding code in this state.

Finally, by definition of strong equilibrium domination, it must be that this other action is either the same unplayed action or an action encoded by the same code in the contract. Thus the condition

$$w_\omega(CG_{1a}) = w_\omega(CG_{2a})$$

is fulfilled.

Hence, we either have hedging in ω.

If the state in which this happens is ω', **and only** ω', then it must be that in that state this action corresponds to the agreement. Hence, we would have self-enforcement. ■

In fact, in this scenario, it is possible to characterize even more precisely the form incompleteness might take,

Proposition 27 *With only two states, a legal equilibrium with incomplete contracts and complete agreements can only take the following forms: 1) If hedging, then there can be at most two continuation games chosen with positive probability, and each can include at most two actions. 2) If self-enforcing, only one continuation game will be chosen, and it will include two actions, with each action being played in the state in which it corresponds to the agreement.*

6 Implications of the Dual Structure of Uncertainty

The dual structure of uncertainty turns out to be important in explaining the form legal equilibria might take here. This is particularly interesting since in our opinion bargaining power uncertainty can be reasonably interpreted as the primary source of obligational incompleteness in the sense of Ayres and Gertner ?. If the judge does not know who has the bargaining power, he or she cannot know what the parties actually agreed to. Uncertainty about the reference game, in contrast, does not prevent the judge from establishing the will of the parties, but rather makes it difficult for him or her to know how to implement that will. Still, one can argue that this also amounts to a form of obligational incompleteness, if of a lower level variety. In any case, these two forms of uncertainty turn out to have distinct implictions, which we explore in the next sections.

6.1 Active Hedging versus Passive Hedging

Note that in Example 1 above, the hedging option was exercised with positive probability in equilibrium (active hedging). This need not always be the case: The hedging history can serve only to separate states (passive hedging). The following proposition establishes a connection between bargaining power uncertainty, separation and hedging.

Proposition 28 *In an ambiguous legal equilibrium (i.e., a legal equilibrium in which in at least one state an ambiguous contract is proposed), in those states of the world in which at least one party hedges, the contracts will signal who has the bargaining power, i.e., will at least partially separate.*

Proof. Fix an ambiguous legal equilibrium. Take two states, ω' and ω'' such that in each the reference game is the same but the party proposing varies. Assume that the party with the bargaining power in state ω' has a hedging strategy. Then, it won't be in the interest of the other party in state ω'' to use that same hedging strategy. To see this, note that these hedging histories must be associated with a payoff for the first party higher than the one that party would obtain in any other outcome in that state, except the outcomes desired by this player. Any other outcome in that state must correspond to a desired outcome for the remaining party. If the hedging action of the first player would yield a higher payoff for the other player as well, it would be yielding more than that player's desired outcomes, contradicting the fact that it was not included amongst the desirable outcomes to start with (even though that was feasible, since hedging histories can be described unambiguously). ■

Corollary 29 *In an ambiguous legal equilibrium, if uncertainty concerns only the assignment of bargaining power, then the equilibrium must be fully separating with the contracts differentiated by the hedging action. The hedging option will not, however, be used in equilibrium.*

Proof. From the previous proposition, the contracts must vary according to whom has the bargaining power. Since the uncertainty concerns only this aspect, this implies that the equilibrium must be fully separating. In such an equilibrium, the judge will enforce the desired outcome with probability 1, and there won't be a need to exercise the hedging option ■

Note that passive hedging represents a form of equilibrium pragmatic signalling. The hedging statement is equilibrium dominated, since in the reading selected in equilibrium the hedging decision is dominated.

Corollary 30 *The existence of payoff uncertainty is a necessary condition for active hedging.*

Proof. From the previous corollary, if there is no payoff uncertainty a hedging equilibrium will be fully separating. If the equilibrium is fully separating, the hedging option will be superfluous. Generally, full separation voids the need to exercise the hedging option. ■

6.2 Obligational Incompleteness and Self-Enforcement

Proposition 31 *In an ambiguous legal equilibrium, a contract cannot be self-enforcing across states which only differ by who has the bargaining power.*

Proof. In each pooled state the desired outcome of the reference game will vary but the reference game itself will not, since uncertainty refers only to bargaining power assignment. As the reading chosen will be the same in all pooled states, the associated continuation game will be the same as well. The continuation game played being one of perfect information with sequential moves and generic payoffs, must have a unique subgame perfect equilibrium. Hence, the outcome must be the same in all pooled states ■

Corollary 32 *If the uncertainty concerns only the assignment of bargaining power, a legal equilibrium cannot be self-enforcing.*

This result is interesting, in our view, because, broadly speaking, 'self-enforcement' incompleteness is the only form of incompleteness that can arise in Bernheim and Whinston ?. If one accepts the interpretation identifying obligational incompleteness with bargaining power uncertainty, this proposition supports the view that obligational uncertainty generates a distinct form of incompleteness, namely, hedging incompleteness (which clearly cannot obtain in Bernheim and Whinston ?).

The preceding results have a stronger implication still,

Corollary 33 *In an ambiguous legal equilibrium, with only bargaining power uncertainty, incompleteness can only take the form of (passive) hedging.*

Proof. If incompleteness takes the form of hedging, by proposition ?, the hedging must be passive and the equilibrium must be fully separating. If the equilibrium is fully separating then the judge will know which history to enforce, and there will be no need for discretion ■

In other words, there might exist equilibria which display incompleteness beyond hedging, but they will have the same outcomes as the pure hedging arrangement (i.e., they will involve some form of pragmatic signalling).

7 Incompleteness Leading to Ambiguity

We illustrate a sort of converse to the logic underlying the previous examples: When complete contracts cannot be written, or only so at enormous expense (the reasons for this will be discussed below), parties might prefer to use ambiguous formulations, even should unambiguous ones be available. This in order to afford a well-informed judge the freedom to enforce their preferred actions (which, due to the exogenous incompleteness, they cannot enforce directly).

7.1 Imprecision

One reason it might be impossible to fully specify the appropriate actions under every contingency is if written language is inherently imprecise. In order to capture this notion, it is necessary to generalize the description of contracts. As before, we define an *Encoding Device* as a pair $\left(C, \widetilde{E}\right)$, where C is a set of codes. The mapping $\widetilde{E}$ though goes now from the power set of A, rather than just A, into C,

$$\widetilde{E} : A^2 \to C$$

Definition 34 ***(Imprecision)*** *A code $c \in C$ is imprecise iff not all $a \in \widetilde{E}^{-1}(c)$ are singletons.*

Definition 35 *An Encoding Device $\left(C, \widetilde{E}\right)$ is imprecise if there exist actions in A that can only be described using imprecise code.*

Note that code can be precise but ambiguous ($\widetilde{E}^{-1}(c)$ is made up of singleton sets); imprecise but not ambiguous ($\widetilde{E}^{-1}(c)$ consists of only one non-singleton set); and, of course, imprecise and ambiguous ($\widetilde{E}^{-1}(c)$ consists of more than one set, some non-singleton).

In fact, this extension allows us to draw even subtler distinctions. In particular, it allows us to define a notion of vagueness (for discussions of this concept, see Keefe and Smith 1999).

Definition 36 ***(Vagueness)*** *A code c is vague if $\widetilde{E}^{-1}(c)$ consists of more than one set, some non-singleton, and the intersection of at least two of those is non-empty.*

7.2 Example of Imprecision Leading to Ambiguity

The reference game is as follows: The two parties to a contract must each choose an action. Each such action is represented by a point on the unit interval. Player 1 moves first; player 2 second. Party 1 preferred action pair is $\{x, x\}$. When player 1 plays x, it is best for player 2 to play the action furthest away from x. Assume that player 1 has the bargaining power.

Language imprecision is modelled by assuming that a contract can only specify intervals of actions, i.e., that it can only partition the unit interval finitely. In terms of the formalism we work with here, for all $c \in C/d$,

$$\widetilde{E}^{-1}(c) = [a(c), b(c)]$$

where for no c, $a(c) = b(c)$.

One convenient way to represent such contracts is to specify that written descriptions of actions can at most include a finite number of decimals, zeros included (to describe an action accurately one would need an infinite number of decimals).

Note how the 'clear language' requirement applies: The judge will read such imprecise statements as incomplete, not as ambiguous. That is, he or she would read them as descriptions of intervals, not as one word which refers to any one but only one action in the interval.

On the other hand, assume there is a fully ambiguous expression which might refer to any action of player 2 (something like 'the appropriate action' or the 'expected action'). Formally, there exists a $d \in C$ such that

$$\widetilde{E}^{-1}(d) = A_2$$

Finally assume the judge knows player 1 has the bargaining power. The judge can moreover enforce any action pair he or she chooses subject to two conditions: The first, a rather obvious one, is the existence of a contract[6]. The second, a clear language requirement.

It is immediate that player 1 would prefer to write a contract which requires 2 to take an 'appropriate action', rather than one which is unambiguous but incomplete. The judge, being fully informed, will then be entitled to force 2 to play 1's favorite choice.

One can easily endogeneize imprecision by introducing costs of writing, so that even when a precise and unambiguous language is available, parties prefer to resort to ambiguous expressions. This could be done, for example, by introducing a post-contracting move by nature that results in one of a continuum of states, with the preferred actions varying with the state (as is commonly done in the indescribability literature, see, for example, Anderlini and Felli 1998).

Note finally that the standard justification for incompleteness, non-verifiability, cannot be invoked here. It is essential in the example that the judge is well informed and can enforce fully.

[6] In practice, it is clear that without a written contract, or some evidence of the existence of a contractual relationship, a court will not intervene. Theoretically, one could capture this by assuming that contractual parties meet randomly, and the judge has no way of telling whether the parties met, other than the written contract.

7.3 An Example of Imprecision Leading to Vagueness

The reference game is as follows: There are two contracting parties, plus nature. The state of nature (nature's move) is realized after contracting, and corresponds to a point in an interval $[1,2]$. The judge knows who has the bargaining power (say, party 1), but cannot observe nature moves fully. He can only tell whether nature's move is in $[1,1.5]$ or in $(1.5,2]$. Party 1 has no moves, besides proposing, while party 2's moves correspond to points in the unit interval $[0,3]$. The action of party 2 that maximizes party 1's payoff varies continuously in $[1,1.75]$ with states of nature in $[1,1.5]$, and in $[1.25,2]$ with states of nature in $(1.5,2]$. The maximum payoff for party 2 is located at 2 for states of nature in $[1,1.5]$, and at 1 for states of nature in $(1.5,2]$. Party 2's payoffs attain a local maximum at the maximum payoff for party 1. The encoding device ('language') available to the parties is the following: There is an imprecise but unambiguous code for 2's actions, c_2, such that

$$\widetilde{E}^{-1}(c_2) = A_2 = [0,3]$$

There is another code, c_{2a}, which is vague: $\widetilde{E}^{-1}(c_{2a}) = \{[1,1.75],[1.25,2]\}$. It is not difficult to see that party 1 would prefer using the vague code. In this case, the judge could rule that party 2 has to take an action in $[1,1.75]$, whenever the state of nature is in $[1,1.5]$, and one in $[1.25,2]$ whenever the state is in $(1.5,2]$. This would lead party 2 to take the action that maximizes her utility locally, thus maximizing party 1's payoff.

The vaguely formulated contract allows party 1 to exploit both party 2's and the judge's information optimally, while still committing the other party.

7.4 An Incomplete Language?

In all the previous examples, the party proposing the contract has been fully informed. Clearly, this party has an incentive to convey all its information to the judge, yet we have implicitly assumed that he or she is unable to do so, i.e., he or she cannot simply 'state' the state of the world in the contract. The issue of pragmatic signalling, at least in part, boils down to this question as well; pragmatic signalling being just an indirect way of stating the state of the world.

The question is then if our examples are just an artifact of language being incomplete; of a too narrow view of language. The languages we have used focused on describing the contracting parties actions, and lacked codes to describe nature's moves. We feel that this is just an apparent problem: Presumably such language would also be imperfect (i.e., ambiguous, imprecise, etc.), and we conjecture one could construct examples similar to the ones presented here even when such a wider but imperfect language is available[7].

8 Conclusions

We have presented a series of examples illustrating how ambiguity might lead to incompleteness, and how incompleteness (more precisely, imprecision) might lead to ambiguity. The first link obtains as hedging against uninformed enforcement; the second, as a way of relying on a well-informed judge to fine tune the parties' commitments.

Though it is, of course, risky to draw broad conclusions from a few examples aiming at illustrating possibilities, the ones presented here do suggest a pattern: When judges are well informed, contracts will tend to be more ambiguous, the parties retaining much less discretion vis-á-vis the judge; while contracts that are to be adjudicated by relatively uninformed judges will tend to be more flexible, i.e., incomplete, as the parties try to keep their options open[8].

References

[1] Anderlini L., L. Felli and A. Postlewaite (2001) Courts of Law and Unforeseen Contingencies, Unpublished Manuscript.

[2] Anderlini L. and L. Felli (1998) Describability and agency problems, *European Economic Review* **42**, 35-59.

[3] Ayres I. and Robert Gertner (1989) Filling Gaps in Incomplete Contracts: An Economic Theory of Default Rules, *Yale Law Journal* **99**, 87-130.

[7] In fact, attempts to describe the general setup in which a contractual transaction takes place are often made. So-called 'statements of intention' can be understood in this way.

[8] In the light of this, one wonders whether contracts that are adjudicated by specialized judges tend to be less precise in the above sense, compared to contracts that are to be adjudicated by non-specialized courts?

[4] Baliga S., Luis C. Corchón and T. Sjöstrom (1997) The Theory of Implementation When the Planner is a Player, *Journal of Economic Theory* **77**, 15-33.

[5] Bernheim D. and M. D. Whinston (1998) Incomplete Contracts and Strategic Ambiguity, *American Economic Review* **88**, 902-932.

[6] Farell, S. (1993) Meaning and Credibility in Cheap-Talk Games, *Games and Economic Behavior* **5**, 514-531.

[7] Hart O. and J. Moore (1999) Foundations of Incomplete Contracts, *Review of Economic Studies* **66**, 115-118.

[8] Jackendof R., *Foundations of Language: Brain, Meaning, Grammar, Evolution,* Oxford University Press 2002.

[9] Keefe R. and P. Smith, *Vagueness: A Reader*, The MIT Press 1999.

[10] Laffont J.J. and D. Martimort, *The Theory of Incentives,* Princeton University Press 2002.

[11] Maskin E. and J. Tirole (1999) Unforeseen Contingencies and Incomplete Contracts, *Review of Economic Studies* **66**, 83-114.

[12] S. Pinker, *The Language Instinct,* Harper Perennial 1995.

[13] Posner R. (1993), What do Judges and Justices Maximize? (The Same as Everybody Else Does), *Supreme Court Economic Review* **3,** 1-41.

[14] Salanié B., *The Economics of Contracts,* The MIT Press 1997.

[15] Segal I. (1999) *A Theory of Incomplete Contracts,* Review of Economic Studies **66**, 57-82.

[16] Solan L. M. *The Language of Judges,* The University of Chicago Press 1993.

[17] Tirole J. (2000) Incomplete Contracts: Where Do We Stand?, *Econometrica* **67**, 741-781.

A Pragmatic Signalling

In this appendix, we present two additional examples, which show how ambiguity might lead to incompleteness. They offer increasingly elaborate stories to justify the existence of pooling equilibria. In linguistic terms, the issue is one of pragmatic versus 'verbal' communication: Why is language combinatorial? Why is there not a word or code for each meaning? In game theoretic terms, this deeper question translates into one of pooling versus separation. Why can't the parties to a contract signal the state of the world to the judge via the contract? The basic answer we offer (example in section ?) is, from a game-theoretic point of view, trivial: The cardinality of the signal space is assumed smaller than that of the set of states of the world. Yet, linguistically, it is not absurd. As Humboldt put it, "language makes infinite use of finite media"[9].

In these additional two examples, we move away somewhat from this basic scenario, and provide various ('pseudo-linguistic') stories which could explain why, even when the cardinality of the signal space exceeds that of the set of states, it might yet prove impossible to signal pragmatically.

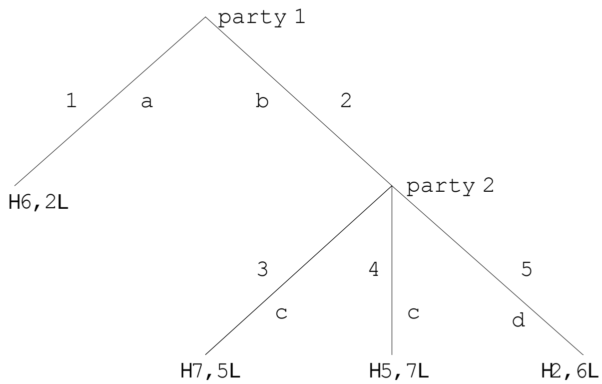
A.1 Example A: Enough Words, but Compromising to Say Them

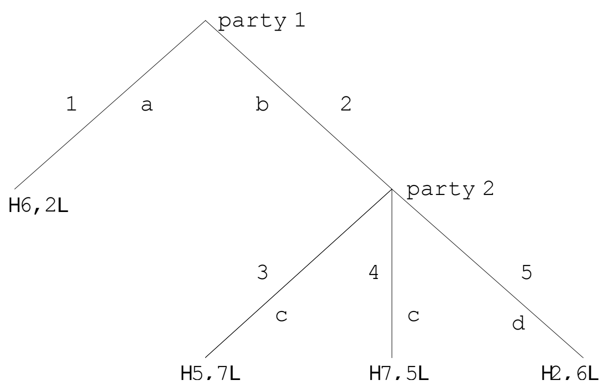
In the previous example, the point of making the cardinality of the set of possible reference games exceed that of the set of written contracts, was to prevent what we call 'cheap-talk' signalling. 'Cheap-talk' signalling amounts to meaningless communication, or 'pragmatic' communication: The judge infers the state of the world from knowledge of the equilibrium via contracts whose meaning does not differ in any substantial way, but which he or she can nevertheless differentiate (for example, through redundant statements). From daily experience, it is obvious that the reach of such pragmatic communication is limited (certainly between a judge and the parties to a contract), though the reasons for this are not quite as obvious. One possible answer is the one suggested above, namely, that the uncertainty is just too large relative to the communication possibilities. Here, we present an alternative hypothesis: Even when the cardinality of the set of contracts exceeds that of the possible states of the world, the fact that talk is not cheap, i.e., the

[9] Quoted in Pinker 1995, p.84.

fact that the judge is bound by what is said in the contract, might suffice to induce pooling (and, hence, incompleteness). A caveat: The example is rather ad-hoc, and we present it to illustrate an underlying logic which we think will apply in more realistic cases as well.

Assume there are two possible reference games, and that the judge cannot directly distinguish between them. These are illustrated below,





The encoding is given by

$$e(1) = a; e(2) = b; e(3) = e(4) = c; e(5) = d$$

In addition, the judge does not know ex-ante which party has the bargaining power. Hence, there are 4 states of the world, which we will denote (j/i), $j, i \in \{1, 2\}$, with the first number denoting the reference game, while the second denotes the party who proposes the contract.

Finally, add a 'syntactic restriction':

Definition 37 *In a well written contract, code cannot be repeated.*

Despite this last restriction, the number of written contracts remains larger than the number of states, so that, in principle, the parties could pragmatically signal the state to the judge. If contracts were 'cheap-talk', the party proposing the contract would not have incentives to mislead the judge (and, under Farell 1993's neologism proof refinement, the only equilibrium would be separating). The example below shows that, in contrast, when the judge is bound by the content of the contract (the 'clear language' requirement), pooling can nevertheless result. To see this, note that in order to be able to reach their most preferred outcomes both parties would want to include the actions b and c. Of course, if both proposed this same contract, pooling would result, and assuming that the 'default rule' assigns positive probability to all admissible actions in such eventuality, incentives to hedge would lead the parties to want to make the contract incomplete by adding their respective hedging options. Note though that no party would want to add the hedging option of the other: This would prevent them from getting their most preferred outcome even should the judge rule in their favor. In conclusion, only three contracts are really 'good' for pragmatic signalling, namely,

$$[(a/b)\,;(c)]\,,[(b)\,;(c)]\,,[(b)\,;(c/d)]$$

But there are four states, so that pooling is inevitable. The conclusion is that party 1 would strictly prefer to propose the contract

$$[(a/b)\,;(c)]$$

while party 2 would strictly prefer to propose the contract

$$[(b)\,;(c/d)]$$

The role of the 'syntactic restriction' should be clear: It is introduced to prevent the parties from using redundancies to communicate pragmatically with the judge. We believe that some such syntactic restrictions operate in practice. We doubt that a judge confronted with a totally nonsensical contract would ever enforce it, even should he or she feel she can reconstruct the will of the parties from his or her knowledge of the contracting environment. The identification of realistic syntactical restrictions that might force this sort of outcome is, evidently, outstanding.

A.2 Example B: Trade-Off Between Syntactical and Lexical Ambiguity

The intuition behind the example presented here is simple: Often adding redundant statements only serves to 'muddle' the original message. In order to capture this intuition, we weaken the assumption that the judge is free to interpret the contract. We assume that the judge can only interpret the lexically ambiguous portions of the contract, i.e., ambiguous words. On the other hand, in interpreting syntactically ambiguous parts of the contract, i.e., statements in the contract which can be read in different ways, and, hence, lead to different interpretations, the judge cannot rule out any of the continuation games that might result from different readings. We will refer to this as a 'strict letter-of-the-contract' requirement. While, again, this is rather ad-hoc, it does not seem completely unreasonable. After all, there is quite a lot of evidence that judges do not interpret contracts consistently (see, for example, Solan 1993).

Now, syntactical ambiguity cannot result under the syntaxis implicit in the previous examples. That syntaxis distinguished clearly between the actions of both parties (by means of a dot-colon), as well as between the individual actions of each party (by means of a forward-slash). These are the only dimensions along which syntactical ambiguity might result. In the present example, we will drop both these features, and assume that the contract consists simply of string of letters, following an alphabetic order -regardless of whose actions we are referring to. For example, the contract

$$[(a/b)\,;(c)]$$

will now read

$$[abc]$$

If there is an action of party 1 (say) denoted ab as well as one denoted a, and another denoted b,(but there is only one action denoted c, and this action corresponds to party 2), the judge, when confronted with the contract $[abc]$, under the 'strict letter-of-the-contract' requirement, will have to allow two readings,

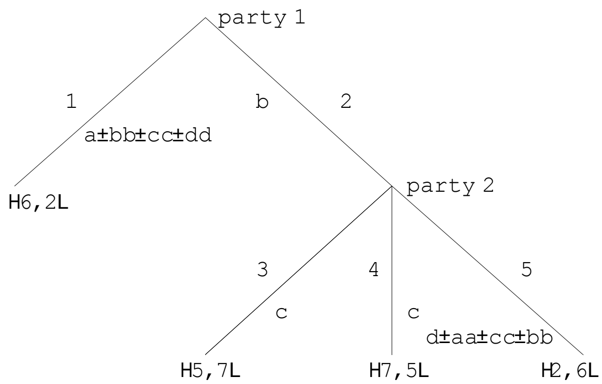
$$[(a/b)\,;(c)]$$

and also

$$[(ab)\,;(c)]$$

This sort of syntactic ambiguity is of no consequence for party 1, as it just gives him or her extra freedom. Assume now, though, that ab is an alternative designation for action d of party 2. In such case, the syntactic ambiguity becomes a real problem for party 1: It means that this party will never be able to enforce his preferred outcome.

Now, it is easy to modify the previous example along the lines just sketched so as to exclude signalling through redundant statements: Specify various designations for the hedging action of each player as shown below,



The encoding is

$$e(1) = \{bb, cc, dd, a\}\,; e(2) = b; e(3) = e(4) = c; e(5) = \{d, aa, cc, bb\}$$

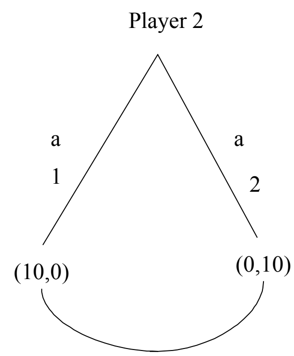
Note that we are allowing here for synonyms -i.e., the encoding is a correspondence.

The contracts $[abc]$ and $[bcd]$, while lexically ambiguous, are not syntactically ambiguous. Hence, the judge is free to interpret them fully (i.e., restrict the parties to play one of the admissible continuation games). On the other hand, any contract containing repeated codes is syntactically ambiguous as well. Since any such contract can be read so as to allow the hedging action of the opposing party, the parties will strictly prefer to propose a syntactically unambiguous contract. Since there are only 3 such contracts ($[abc]$, $[bcd]$ and $[bc]$), this will prevent fully pragmatic communication, and, hence, separation. Pooling, in turn, will lead the parties to strictly prefer the incomplete contract that includes their hedging action ($[abc]$ or $[bcd]$) to the complete contract $[bc]$.

A.3 But all this complete?!

Here is an example that shows that other forms of incompleteness can result:

Assume that there is uncertainty about payoffs only. Say party 1 has the bargaining power. Let the reference game be given by



In a legal equilibrium, party 1 might propose the contract $[a]$ in both states. The judge will not be able to infer the state, and so will choose each of the two possible readings randomly. Note that an equilibrium in which party 1 proposes $[(a/a)]$ in one of the states, and $[a]$ in the other, would entail equilibrium pragmatic signalling. Of course, the outcome of such an equilibrium would be superior, since the judge would choose the right action in each state. Either equilibrium is neither self-enforcing nor hedging.