The Interaction of Inflation and Financial Development with Endogenous Growth*

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Abstract

A cash-in-advance, endogenous growth, economy defines financial development within a banking sector production function as the degree of scale economies for normalized capital and labor. Less financially developed economies have smaller such returns to scale, and can be credit constrained endogenously by a steeply sloping marginal cost of credit supply. The degree of scale economies uniquely determines the marginal cost curvature and the unit cost of financial intermedition, which is expressed in terms of an interest differential. The interest differential result allows for calibration of the finance production function using industry data. A hypothesis of how financial development interacts with inflation and growth is tested, using fixed effects panel estimation with endogeneity tests, dynamic panel estimation, and an extended use of multiple inflation rate splines in estimation of the growth rate.

JEL: C23, E44, O16, O42

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1. Introduction

Recent work emphasizes that financial development can lead to more investment (for example, Ndikumana (2005)), while examining whether financial development itself actually causes growth (Manning (2003), Shan (2005), Rousseau and Sylla (2006)). The traditional finding that the investment-output ratio can positively affect growth, put forth in Kormendi and Meguire (1985), is updated by Gillman, Harris, and Mátyás (2004) on the basis that the investment to output ratio proxies the real rate of interest that largely determines the growth rate in the theoretic Euler equation. Including investment in growth regressions along with financial development has been found to leave the latter with no direct role per se, as in Dawson (2003) and Rousseau and Vuthipadadorn (2005). Aghion, Howitt, and Mayer-Foulkes (2005) include the level of output, instead of investment, and find no effect of financial development per se on growth. However they also include an interaction term between financial development and output which they find significant. Gillman et al. include investment in growth regressions amongst regions differing in their level of development, and find possible interaction between inflation and development in the way that they affect the growth rate. While the Aghion et al. interaction term suggests the investment role that financial development can play, the Gillman et al. interaction results suggest the exchange role that financial development can facilitate.

The possible growth interaction of inflation with financial development has had limited focus.[1] Part of the difficulty here is in providing a standard definition of financial development, within a standard monetary growth framework. This can be viewed as the need to define financial development within a decentralized financial sector in which there exists a robust mixed exchange equilibrium of both money and credit, or money and interest earning demand deposits. The problem of finding a mixed equilibrium goes back, for example, to Wallace's (1980) overlapping generations model, in which there is no unique equilibrium between money and interest-earning assets that are a substitute for money. Specifically within the exchange framework, many approaches have been used to establish a mixed equilibrium, from putting money and credit, or money and demand deposits, in the utility function [Lucas and Stokey (1983), Hartley (1988), Englund and Svensson (1988), Einarsson and Marquis (2001), Christiano, Motto, and Rostagno (2003)], to generalized transaction cost functions including both shopping time [Bansal and Coleman (1996), Goodfriend (1997), Lucas (2000), Gavin and Kydland (1999),Canzoneri and Diba (2005)], and costly isoquant-like combina-

[1] Exceptions include for example Paal and Smith (2000) and Rousseau and Wachtel (2001).

tions of money and demand deposits (Einarsson and Marquis (2002)).

Unlike this previous work, we achieve a mixed money-credit equilibrium at the point where the marginal cost of money use equals the marginal cost of the intermediary-supplied credit. This may be closest in spirit to Marquis's approach, except that our credit is produced in a decentralized sector, rather than having a generalized transaction cost; and this results in a banking time instead of a shopping time economy. In using a decentralized intermediary to solve the mixed equilibrium problem, the paper contributes a novel but parsimonious micro-based model that has a standard definition of financial development, within a cash-in-advance economy. It does this by borrowing from the banking literature a specification for the production function. Set also within Lucas (1988) endogenous growth, this enables extension of the theory of how inflation and financial development interact and affect the growth rate, resulting in a testible hypothesis that is set out and examined empirically.

The intuition of the equilibrium problem that we solve was well put forth by King and Plosser (1984) as being based in the standard assumption of constant returns to scale (CRS): "The constant returns to scale structure implies that at given factor prices the finance industry supply curve is horizontal." When applied to supplying exchange credit backed by interest-earning-demand-deposits, this flat marginal cost schedule competes against a similarly flat price of money, being the nominal interest rate, and there is no equilibrium (Proposition 1).[2] The paper here solves this, following Sealey and Lindley (1977), Clark (1984) and Hancock (1985), by including the intermediary's funds as a third factor in its CRS production function. A normalization of output by the quantity of deposit funds leads to a per unit production function that is less than CRS in the normalized inputs of labor and capital, per unit of funds, giving a rising, convex, marginal cost curve per unit of funds.[3]

The upward-sloping marginal cost not only allows for a unique equilibrium between money and credit. It suggests an immediate sense of how financial development affects the equilibrium. With a greater degree of diminishing returns to labor and capital factors in producing the credit, and a lessor returns to scale

[2] Aiyagari, Braun, and Eckstein (1998) and Li (2000) specify a production approach with CRS in labor and capital for the production of the financial services. Aiyagari, Braun, and Eckstein (1998) assumes an exogenous money demand function to pin down a money-credit allocation, while Li (2000) drops the CRS assumption during the calibration of the model and instead uses less than constant returns to scale.

[3] Hansen and Prescott (2005) specify a third factor of production as the number of production plants, with a normalization by plants. Lucas (2000) and Canzoneri and Diba (2005) normalize their transaction cost function arguments by the quantity of exchange funds (equal to output).

in the normalized labor and capital, the convex marginal cost curve has more curvature. High curvature suggests a "credit-constrained" economy: in the limit of highest curvature, it is similar to a Keynesian full-output schedule that is horizontal until it reaches capacity and then vertical [see for example Clark, Laxton, and Rose (2001)]. But here the curvature is an endogenous function that results given the production function parameter specification. A high curvature economy is our candidate to describe a less financially developed economy. In contrast, with a lessor degree of diminishing returns to factors, and higher returns to scale in the normalized labor and capital, the marginal cost is more smoothly rising as output rises. This is the candidate for the more financially developed economy. Thus, in the normalized production function, the degree of economies of scale in the normalized labor and capital is put forth here as the description of the degree of development, consistent with other fields.[4]

One result is that the interest differential between the government nominal bond interest rate and the yield received on total deposits exactly equals the degree of the economies of scale (Proposition 2). By approximating the interest differential using industry data, this gives a calibration for the degree of the normalized economies of scale. It is then demonstrated that with the advantage of greater economies of scale comes the disadvantage of possibly higher marginal costs at low levels of output, because of a lessor curvature of the marginal cost curve (Proposition 3). This implies a limited but testible proposition that, given a non-hyperinflation rate of inflation, an increase in financial development within the exchange credit sector causes a decrease in the growth rate (Proposition 4).

The empirics of the paper test the proposition with panel data, and endogenous inflation-growth splines, instrumental variables and dynamic panel estimation. These results extend the threshold literature as for example in Drukker, Gomis-Porqueras, and Hernandez-Verme (2004). Standard financial development measures are included, with a focus on the liquidity measure, along with the inflation rate and the investment ratio, in explaining growth. Results appear robust in their support for the model's explanation of financial development in terms of economies of scale, and in terms of the different standard of financial development, but show sensitivity to the spline division points under instrumental variables.

[4] R E Lucas (2002)Lucas (2001) similarly uses the degree of diminishing returns to indicate aggregate development in a growth context; such scale economies are defined as in Julien and Sanz (2005); and economies of scale have been used to characterize development, for example as in Boldrin and Levine (2005), who considers this in terms of the parameters of a production technology for new ideas.

2. Representative Agent Model

The model extends Gillman and Kejak (2005b) by decentralizing the financial intermediation sector. This results in the similar equilibrium conditions as when the agent acts in part as a credit supplier, in a Robinson Crusoe fashion (Hicks 1935), except that now the price of the credit services and the profit of the credit provider is derived, and the consumer supplies labor and capital to the financial intermediary and receives explicit wages and rents. Two other differences are that both consumption and investment require exchange, while only consumption requires exchange in Gillman and Kejak; and both capital and labor are inputs into all three sectors, while capital was not included in the credit production function in Gillman and Kejak. The main propositions of this paper are robust to excluding these later extensions, but these add realism for the calibration and are still consistent with the previous inflation, Tobin (1965) and growth effects studied in Gillman and Kejak. Unlike Stockman (1981), with its inverse Tobin effect and no leisure or human capital, here inflation causes substitution from goods to leisure, a rise in the real-wage-to-real-interest-rate ratio, and substitution from effective labor to capital in all sectors, resulting in a positive Tobin effect, except at very high rates of inflation. This is despite the fact that inflation directly taxes a fraction of investment that is bought with money, as in the Stockman channel, because the factor reallocation across sectors dominates this other effect.[5]

Although deterministic, it clarifies the model's structure by setting out its timing into four sequential sub-sets within each period, called 1) asset trading, 2) working and producing, 3) goods trading, and 4) account settling, with only 2) takes real time. The consumer initially opens with the financial intermediary a depository account, both non-interest bearing and interest bearing, is issued a credit card from the intermediary, and is given a credit limit for purchases made during good trading. The consumer agrees to have the intermediary pay for the consumer's credit purchases from the goods producer's store using the consumer's deposit account, at the end of each period during account settling. The consumer receives one share of ownership in the intermediary, for every real dollar deposited in the intermediary, and receives dividends accordingly, during account settling. The consumer works for, and rents capital to, both the goods producer and the financial intermediary, and receives the corresponding wages and rental incomes

[5] A similar model with both consumption and investment requiring exchange, and with capital and labor used in all three sectors, but without the decentralization of the credit sector, can be found in Gillman and Kejak (2005a) A related decentralization can be found in Gillman and Kejak (2004), but without the microfoundations of the credit production function as presented here.

through direct deposits into the intermediary account, during account settling. Also during account settling, the consumer is informed of next period's credit limit by the intermediary, and then transfers funds from the non-interest bearing account to the interest-bearing account, just leaving the amount of money needed for the next period's money purchases in the non-interest bearing account. The consumer makes purchases using money and the credit card, up to its limit.

The financial intermediary receives a charter from the government to operate depositor accounts, buy government bonds, supply credit, and to distribute money through money machines that can be located at the goods producer's store. The charter requires solvency and liquidity each period, and provides ownership equity to the consumer for each unit deposited. Solvency requires that liabilities in the form of deposits do not exceed assets in the form of money and government bonds. And liquidity requires that non-interest bearing deposits on call during goods trading must be 100% backed by money holdings. Each period, the intermediary manages assets and liabilities, and produces exchange credit with labor and capital.

2.1. Consumer Problem

The representative agent's discounted utility stream depends on the consumption goods c_t and leisure x_t in a constant elasticity fashion:

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+\rho} \right)^t \frac{c_t^{1-\theta} x_t^{\alpha(1-\theta)}}{(1-\theta)}. \tag{2.1}$$

Exchange is required for both consumption and investment goods, by using either nominal money, M_t, or credit from the credit card; d_t denote the real quantity of credit, and P_t denoting the nominal goods price. This makes the exchange constraint:

$$M_t + P_t d_t = P_t(c_t + i_t). \tag{2.2}$$

Money comes from bank machines linked to nominal non-interest bearing deposit accounts, denoted by D_t^n, with the constraint that

$$D_t^n = M_t. \tag{2.3}$$

The exchange constraint (2.2) can be rewritten with this substitution in for the money stock, but this implies a change in the purchasing process: exchange would be a cashless purchase using only the debit card and no actual money. While the debit card world is equivalent in most aspects, instead, the assumption will continue to be made that the consumer gets the money from a bank machine before purchasing at the store. Equation (2.3) is a linear production function of money from the bank machine using deposits; costly fees could be built in with a more complicated technology.

Its assumed that all expenditures are sourced from the deposit accounts, since all income is deposited in the accounts. With q_t denoting total real deposits, this means that

$$P_t q_t = P_t \left(c_t + i_t\right). \tag{2.4}$$

These deposits are held in either interest-bearing or non-interest bearing accounts, with the nominal quantity of each denoted by D_t^i and D_t^n, this give the total as

$$P_t q_t = D_t^i + D_t^n.$$

The consumer's allocation of capital and time constraints are that the shares of capital and labor across the credit (D), goods (G) and human capital (H) sectors add up as follows: $1 = s_{Dt} + s_{Gt} + s_{Ht}$ and $1 - x_t = l_{Dt} + l_{Gt} + l_{Ht}$. The nominal income received from capital and labor, with P_t denoting the price of goods, and r_t and w_t denoting the real rental and wage rates, is $P_t r_t (s_{Dt} + s_{Gt}) k_t$ and $P_t w_t (l_{Dt} + l_{Gt}) h_t$. Also there is a lump sum government transfer V_t, and a dividend distribution from the intermediary per share, denoted by Π_{Dt}^*, which is multiplied by the total number of shares, q_t, for a total of $\Pi_{Dt}^* q_t$.

Expenditures on consumption and investment, denoted by $P_t \left(c_t + i_t\right)$, and the payment of the fee for credit services, denoted by $P_{dt} d_t$, subtract from income, as do the net addition to interest-bearing and non-interest-bearing deposits, $-D_{t+1}^i + (1+R_t^i) D_t^i - D_{t+1}^n + D_t^n$, where R_t^i denotes the interest rate paid on interest-bearing deposits. Substituting in for q_t from equation (2.4), together these items make the income constraint:

$$\begin{aligned} 0 \;=\; & P_t r_t \left(s_{Gt} + s_{Dt}\right) k_t + P_t w_t \left(l_{Gt} + l_{Dt}\right) h_t + V_t + \Pi_{Dt}^* (P_t c_t + P_t i_t) \\ & - P_t c_t - P_t i_t - P_{dt} d_t - D_{t+1}^i + (1 + R_t^i) D_t^i - D_{t+1}^n + D_t^n. \end{aligned} \tag{2.5}$$

Human capital is accumulated through a CRS production function using effective labor and capital. With $A_H > 0$,and $\epsilon \in [0,1]$, $h_{t+1} = A_H \left(s_{Ht} k_t\right)^{1-\epsilon} \left(l_{Ht} h_t\right)^{\epsilon} + (1 - \delta_h) h_t$; using the allocation of time and goods constraints to substitute in for l_{Ht} and s_{Ht},

$$h_{t+1} = A_H [(1 - s_{Gt} - s_{Dt}) k_t]^{1-\epsilon} [(1 - l_{Gt} - l_{Dt} - x_t) h_t]^{\epsilon} + (1 - \delta_h) h_t. \tag{2.6}$$

Physical capital changes according to

$$k_{t+1} = i_t + \left(1 - \delta_k\right) k_t. \tag{2.7}$$

2.2. Financial Intermediary Problem

The intermediary jointly produces credit and manages assets and liabilities. The production function for credit services is CRS in effective labor, capital and the total real deposit funds q_t. With $A_D > 0$, and $\gamma_1 \in [0,1]$, $\gamma_2 \in [0,1]$, and assuming that $\gamma_1 + \gamma_2 < 1$, the production is given by

$$d_t = A_D \left(l_{Dt} h_t\right)^{\gamma_1} \left(s_{Dt} k_t\right)^{\gamma 2} q_t^{1-\gamma_1-\gamma_2}, \tag{2.8}$$

Normalizing by funds, the function is

$$d_t / q_t = A_D \left(\frac{l_{Dt} h_t}{q_t}\right)^{\gamma_1} \left(\frac{s_{Dt} k_t}{q_t}\right)^{\gamma 2}. \tag{2.9}$$

The third factor in the production function results because the financial intermediary is supplying jointly two outputs, in which "there is a joint supply of things...which cannot easily be produced separately; but are joined in a common origin" [Marshall (1923), p. 222]. Here the credit services and the deposits are supplied given the supply of funds to the intermediary from the consumer's income deposits, which are also the purchases of shares in the intermediary at the preset unit price. As a result, the total deposits are taken as a given to the intermediary in the production of the credit.

The management of funds is assumed to involve no cost. Deposits are used to purchase government bonds, denoted by B_t, or are kept as money holdings. The solvency restriction that assets equal liabilities is given by

$$B_t + M_t = D_t^i + D_t^n. \tag{2.10}$$

And with R_t denoting the nominal rate earned on the bonds, the change in the assets net of liabilities provides additional income to the intermediary, this being equal to $+B_{t+1} - (1+R_t)B_t + M_{t+1} - M_t - D_{t+1}^i + (1+R_t^i)D_t^i - D_{t+1}^n + D_t^n$.

The intermediary's competitive profit maximization problem, for joint credit production and asset management, with p_t the discount factor,[6] can be written as

[6] Here $p_t = \left(\frac{1}{1+\rho}\right)^{t+1} \left(\frac{u_{c,t+1}}{P_{t+1}}\right)$ is the discounted value of the adjusted marginal utility of income. (Dotsey and Ireland 1995).

the discounted sum of per period nominal profits, denoted by Π_{Dt} :

$$\underset{d_t, l_{Dt}, s_{Dt}, B_{t+1}, D^i_{t+1}, D^n_{t+1}, M_{t+1}}{Max} \sum_{t=0}^{\infty} p_t \Pi_{Dt} \tag{2.11}$$

$$\begin{aligned} = \quad & \sum_{t=0}^{\infty} p_t [P_{dt} d_t - w_t l_{Dt} h_t P_t - r_t s_{Dt} k_t P_t \\ & -B_{t+1} + (1+R_t) B_t - M_{t+1} + M_t \\ & +D^i_{t+1} - (1+R^i_t) D^i_t + D^n_{t+1} - D^n_t], \end{aligned} \tag{2.12}$$

subject to the solvency restriction that assets equal liabilities, the liquidity restriction that

$$D^n_t = M_t, \tag{2.13}$$

and the production function for d_t in equation (2.8).

Maximizing equation (2.11) subject to (2.8), the first order conditions can be written as in terms of average and marginal products, AP_t and MP_t, and marginal cost per unit MC_t :

$$P^d_t / P_t = \frac{w_t}{\gamma_1 \left(\frac{d_t}{l_{Dt} h_t} \right)} \equiv \frac{w_t}{\gamma_1 AP_{l_{Dt} h_t}} = \frac{w_t}{MP_{l_{Dt} h_t}} = MC_t; \tag{2.14}$$

$$P^d_t / P_t = \frac{r_t}{\gamma_2 \left(\frac{d_t}{s_{Dt} k_t} \right)} \equiv \frac{r_t}{\gamma_2 AP_{s_{Dt} k_t}} = \frac{r_t}{MP_{s_{Dt} k_t}} = MC_t. \tag{2.15}$$

These set the marginal cost of credit funds equal to the value of the marginal products of effective labor and capital in producing the credit, the standard price theoretic conditions for factor markets; and the marginal products are fractions, γ_1 and γ_2, of the average products.

Zero profit for the intermediary means that the intermediary pays out all profit as dividends to the consumer, given by $\Pi_{Dt} = \Pi^*_{Dt} q_t$. Now consider defining the interest yield of the dividends as the total dividends per unit of credit produced, or $\Pi_{Dt}/d_t \equiv R^c_t$. With this normalization, and substituting in $P^d_t/P_t = R_t$ from the consumer equilibrium conditions, zero profits can be written as

$$0 \equiv (R_t - R^c_t) - \left(\frac{w_t l_{Dt} h_t}{d_t} + \frac{r_{dt} s_{Dt} k_t}{d_t} \right). \tag{2.16}$$

Now, the other intermediary first-order conditions equate the interest paid on interest-bearing deposits to the government bond interest rate,

$$R^i_t = R_t \tag{2.17}$$

(and equate the shadow value of the liquidity constraint to R_t, thereby equating non-interest-bearing deposits to money holdings). Substituting in for R_t from equation (2.17) and solving for R_t^c, zero profit implies that R_t^c equals the return on the interest-bearing deposits net of the per unit credit production costs, or $R_t^c \equiv R_t^i - \left(\frac{w_t l_{Dt} h_t}{d_t} + \frac{r_{dt} s_{Dt} k_t}{d_t} \right)$.

2.3. Goods Producer Problem

The goods producer competitively hires labor and capital for use in its Cobb-Douglas production function. Given $A_G > 0$, $\beta \in [0,1]$,

$$y_t = A_G (l_{Gt} h_t)^{\beta} (s_{Gt} k_t)^{1-\beta}, \tag{2.18}$$

with the first-order conditions of

$$\begin{aligned} w_t &= \beta A_G (l_{Gt} h_t)^{\beta-1} (s_{Gt} k_t)^{1-\beta}, && (2.19) \\ r_t &= (1-\beta) A_G (l_{Gt} h_t)^{\beta} (s_{Gt} k_t)^{-\beta}. && (2.20) \end{aligned}$$

Wage and rental payments are contractually owed to the labor and capital owner (the consumer) only during account settling. The goods producer sells the goods at its store, and uses the receipts, both money and credit, to pay the wage and rents the consumer.

2.4. Government Financing Problem

The government uses money seignorage and lump sum taxation V_t to pay interest on its debt, and returns the rest by a lump sum transfer back to the consumer, whereby its total liabilities next period are given by

$$M_{t+1} + B_{t+1} = M_t + V_t + (1 + R_t) B_t. \tag{2.21}$$

And it is assumed that the government lends to the intermediary during each period the amount that the intermediary demands at the given R_t, using the funds for oversight of the property rights that define the markets. The rate of growth of the money supply is assumed constant at σ.

2.5. Social Resource Constraint

Consolidating the economy's accounts, by substituting into the consumer budget constraint, in equation (2.5), for the total intermediary nominal profit per period,

$\Pi_{Dt} = \Pi^*_{Dt} q_t$, given by equation (2.11), and for the lump sum transfer V_t, given by the government's budget constraint in equation (2.21), results in the social resource constraint of

$$P_t r_t s_{Gt} k_t + P_t w_t l_{Gt} h_t = P_t c_t + P_t i_t.$$

2.6. Equilibrium

2.6.1. Definition

The consumer maximizes utility in equation (2.1) subject to the exchange, income, and human capital investment constraints, in equations (2.2), (2.3), (2.5), (2.6), and (2.7) with respect to c_t, x_t, l_{Gt}, l_{Dt}, s_{Gt}, s_{Dt}, d_t, k_{t+1}, h_{t+1}, D^i_{t+1}, D^n_{t+1} and M_{t+1}. The financial intermediary maximizes discounted profit in equation (2.11) subject to the CRS production function in (2.8), the solvency constraint (2.10), and the liquidity constraint (2.13), with respect to d_t, l_{Dt}, s_{Dt}, B_{t+1}, D^i_{t+1}, D^n_{t+1}, and M_{t+1}, giving the equilibrium conditions (2.14), (2.15), and (2.17). The goods producer maximizes profit subject to the CRS production function constraint (2.18), giving the marginal product conditions (2.19) and (2.20). And the government's budget constraint (2.21) provides the market clearing condition for the money market, while the deposit condition (2.4) provides market clearing for the intermediary's deposit market.

2.6.2. The Effect of Inflation on the Balanced Growth Path

Key balanced-growth-path conditions are given here for log-utility, and used to describe how inflation affects the equilibrium.

$$\frac{x}{\alpha c} = \frac{1+\tilde{R}}{wh}, \tag{2.22}$$

$$\begin{aligned} 1+g &= \frac{1+r_H-\delta_H}{1+\rho} = \frac{1+\frac{r_K}{1+\tilde{R}}-\delta_K}{1+\rho}, & (2.23)\\ \tilde{R} &= \left(1-\frac{d}{q}\right) R + (\gamma_1+\gamma_2) R\left(\frac{d}{q}\right), & (2.24)\end{aligned}$$

$$r_H = \varepsilon A_H \left(\frac{l_H h}{s_H k}\right)^{-(1-\varepsilon)} (1-x), \tag{2.25}$$

$$R_t = P_{dt}/P_t = w l_D h / (\gamma_1 d) = r s_D k / (\gamma_2 d), \tag{2.26}$$

$$R_t = \sigma + \rho + \rho\sigma, \tag{2.27}$$

$$\left(\frac{d}{q}\right) = A_D \left(\frac{l_{Dt}h_t}{q_t}\right)^{\gamma_1} \left(\frac{s_{Dt}k_t}{q_t}\right)^{\gamma 2} = A_D^{\frac{1}{(1-\gamma_1-\gamma_2)}} \left(\frac{R\gamma_1}{w}\right)^{\frac{\gamma_1}{(1-\gamma_1-\gamma_2)}} \left(\frac{R\gamma_2}{r}\right)^{\frac{\gamma_2}{(1-\gamma_1-\gamma_2)}}. \tag{2.28}$$

At the Friedman optimum, the nominal interest R, given in equation (2.27), equals zero and no credit is used. But as inflation rises, the agent substitutes from goods towards leisure while equalizing the margin of the ratio of the shadow price of goods to leisure, $x/(\alpha c) = \left[1 + \tilde{R}\right]/wh$, in equation (2.22). Here $\tilde{R}$ in equation (2.24) is the average exchange cost per unit of output, a weighted average of the average cost of using cash, R, with the weight $\left(1 - \frac{d}{q}\right)$, and the average cost of using credit, $(\gamma_1 + \gamma_2)R$, with the weight $\frac{d}{q}$. That $(\gamma_1 + \gamma_2)R$ is the average cost can be computed by dividing the total cost of credit production by the total output of credit production. The equalization of marginal exchange costs in equation (2.26), between money and credit, determine by how much the exchange cost of consumption rises. And this determines how much substitution there is from money to credit, as given in equation (2.28), and from goods to leisure. Substitution towards leisure causes a fall in the human capital return of $r_H \equiv \varepsilon A_H (s_H k/l_H h)^{(1-\varepsilon)}(1-x)$, given in equation (2.25). The marginal product of physical capital r_K, in equation (2.23), also then falls, as a result of a Tobin-type substitution from labor to capital across all sectors in response to the higher real wage rate; the Tobin like rise in $s_H k/l_H h$ mitigates but does not reverse the fall in the return to human capital caused by the increase in leisure. The growth rate, in equation (2.23), falls as R rises since both r_H and r_K fall. As the inflation rate continues to rise, the credit substition channel allows the growth rate to decline at a decreasing rate, as more credit and less leisure is used as the substitute for the inflation-taxed good (Gillman and Kejak 2005b).

2.6.3. Non-Existence of Equilibrium with $\gamma_1 + \gamma_2 = 1$

Proposition 1: Assume that $\gamma_1 + \gamma_2 = 1$, $A_D = A_G$, $\gamma_1 = \beta$, and log-utility. For all $R < 1$, there exists no equilibrium.

Proof: From equation (2.9), $d_t/q_t = A_d \left(\frac{l_{Dt}h_t}{q_t}\right)^{\gamma_1} \left(\frac{s_{Dt}k_t}{q_t}\right)^{\gamma 2}$, and with $\gamma_1 + \gamma_2 = 1$, then $1 = A_d \left(l_{Dt}h_t/d_t\right)^{\gamma_1} \left(s_{Dt}k_t/d_t\right)^{\gamma 2}$. Equation (2.26) indicates that $l_{Dt}h_t/d_t = \gamma_1 R_t/w_t$, and $s_{Dt}k_t/d_t = \gamma_2 R_t/r_t$. Substituting these into the previous equation, $1 = A_D \left(\gamma_1 R_t/w_t\right)^{\gamma_1} \left(\gamma_2 R_t/r_t\right)^{\gamma 2}$, or $R_t = A_D^{-1} \left(\gamma_1/w_t\right)^{-\gamma_1} \left(\gamma_2/r_t\right)^{-\gamma 2}$.

Now substituting in for w_t and r_t from the equations (2.19) and (2.20), $R_t = A_D^{-1} (\gamma_1)^{-\gamma_1} (\gamma_2)^{-\gamma 2} \left[\beta A_G (l_{Gt} h_t)^{\beta-1} (s_{Gt} k_t)^{1-\beta}\right]^{\gamma_1} \left[(1-\beta) A_G (l_{Gt} h_t)^{\beta} (s_{Gt} k_t)^{-\beta}\right]^{\gamma_2}$. With $\gamma_1 = \beta$, and $A_D = A_G$, the last expression becomes $R = 1$. But this gives a contradiction since from equation (2.27), with log-utility, $R = \sigma + \rho + \sigma\rho < 1$.[7]

Even more simply, if there is no physical capital, but still human capital, and all production functions are CRS (linear, with $\gamma_1 = 1, w_t = A_G$) while utility is the log form, then it is always true that $R_t = A_G / A_D$ and that $R = \sigma + \rho + \sigma\rho$, a contradiction in general.

2.6.4. The interest differential

From equation (2.16), the interest differential equals the average cost of producing the credit; this can be further simplified.

Proposition 2: The percentage interest differential, $(R_t - R_t^c) / R_t$, equals the sum of the production parameters, $\gamma_1 + \gamma_2$.

Proof: Dividing Π_{Dt} in equation (2.11) by $P_t d_t$, substituting in $l_{Dt} h_t / d_t = \gamma_1 R_t / w_t$, $s_{Dt} k_t / d_t = \gamma_2 R_t / r_t$, and $P_{dt} / P_t = R_t$, from equation (2.26) into the current profit $\Pi_{Dt} / P_t d_t$, and substituting in $M_t = D_t^n$, and $B_t = D_t^i$, from equations (**??**), (2.10) and (2.13), it results that $\Pi_{Dt} / P_t d_t \equiv R_t^c = (1 - \gamma_1 - \gamma_2) R_t$. Thus $R_t - R_t^c = (\gamma_1 + \gamma_2) R_t$, and the normalized interest differential is $(R_t - R_t^c) / R_t = \gamma_1 + \gamma_2$.

This analytic result facilitates calibration of the credit production function parameters. For example, if there is a 6% nominal interest rate in short term treasury bills, and $\gamma_1 + \gamma_2$=0.25, then R_t^c=(0.75)(6.0)% = 4.5%. And the interest differential is the 1.5%. This illustrative calibration can be refined by using data of the mutual fund industry, as is done in Section 4.[8]

3. Financial Development

Changes in the scale parameters γ_1 and γ_2 cause changes in the curvature of the production function, and in the economies of scale. Consider $d_t / q_t = A_D \left(\frac{l_{Dt} h_t}{q_t}\right)^{\gamma_1} \left(\frac{s_{Dt} k_t}{q_t}\right)^{\gamma 2}$. For $\gamma_1 = \gamma_2 = 0.1$, and $A_D = 0.5$, Figure 1 graphs the production function:

[7] We thank Toni Braun for a first version of a related proof.

[8] "Although many investors try to pick the mutual fund with the best return, they really should be looking for the mutual fund with the lowest expenses. Studies show that most mutual-fund managers fail over the long term to beat the indexes. Their shortfall, in fact, tends to resemble their expense levels." (Browning 2006) . See also Berk and Green (2004).

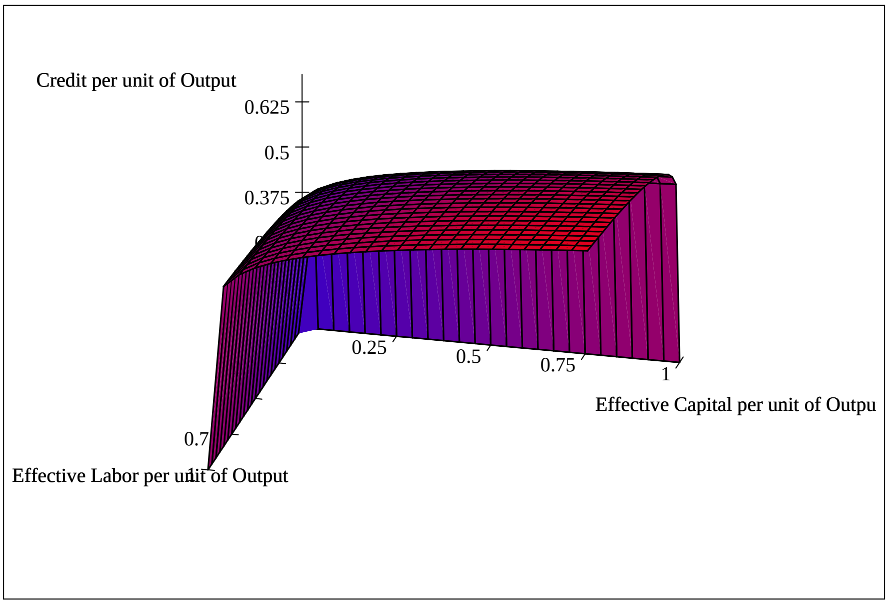


Figure 3.1. Production of Credit

Increases in γ_1 and γ_2 cause a less sharp curvature. The changes in curvature can also be seen in the marginal cost curves.

Proposition 3. The curvature of the marginal cost of credit (MC) is inversely related to the normalized economies of scale, in the following way: Let $\gamma_1 = \gamma_2$, and $\gamma \equiv \gamma_1 + \gamma_2$, then $\eta \equiv \left[\frac{\partial MC}{\partial (d/q)}\right] / \left[\frac{d/q}{MC}\right] = (1-\gamma)/\gamma$.

Proof: Define the MC per unit of deposits, using equation (2.26) that equates the marginal cost of using money to the marginal cost of using credit; dropping time subscripts $MC \equiv R_t = wl_D h/(\gamma_1 d) = rs_D k/(\gamma_2 d)$. Then $MC = \left(\frac{w}{\gamma_1}\right)\left(\frac{l_d h}{q}\right)\frac{q}{d}$. From the production function in equation (2.9), $\frac{l_{Dt}h_t}{q_t} = A_D^{-\gamma_1}\left(\frac{s_D k}{q}\right)^{-\gamma 2/\gamma_1}\left(\frac{d}{q}\right)^{1/\gamma_1}$. Substituting this into the MC expression $MC = \left(\frac{w}{\gamma_1}\right)\left[A_D^{-\gamma_1}\left(\frac{s_{Dt}k_t}{q_t}\right)^{-\gamma 2/\gamma_1}\left(\frac{d}{q}\right)^{1/\gamma_1}\right]\left(\frac{d}{q}\right)^{-1}$. Finally, substituting in for $\left(\frac{s_D k}{q}\right)$ from equation (2.26), in which $\frac{s_D k}{d} = \frac{\gamma_2 R}{r}$, and simplifying $MC = \left[w^{\left(\frac{\gamma_1}{\gamma_1+\gamma_2}\right)} r^{\left(\frac{\gamma_s}{\gamma_1+\gamma_2}\right)} \gamma_1^{\left(\frac{-\gamma_1}{\gamma_1+\gamma_2}\right)} \gamma_2^{\left(\frac{-\gamma_2}{\gamma_1+\gamma_2}\right)} A_D^{\left(\frac{-1}{\gamma_1+\gamma_2}\right)}\right]\left(\frac{d}{q}\right)^{\left(\frac{1-\gamma_1-\gamma_2}{\gamma_1+\gamma_2}\right)}$. Then it follows immediately, with $\gamma_1 = \gamma_2$, and $\gamma \equiv \gamma_1 + \gamma_2$, that the curvature is $\eta = (1-\gamma)/\gamma$.

The proposition indicates that a more financially developed economy has a less curved marginal cost, and one with increasingly high marginal costs (convex) as long as $\gamma < 0.5$. A less financially developed economy can be credit constrained with a high curvature MC curve, approaching a reverse-L shape.

Figure 2 shows that the MC curvature increases monotonically as $\gamma = \gamma_1 + \gamma_2$, where $\gamma_1 = \gamma_2$, increases from its greatest value at 0.06 to its least value at 0.40. Here the variable values are inserted using the baseline calibration below for a fixed w, r, and A_d; this gives only an approximation of the MC curve for illustrative purposes, in that w and r are endogenous to the γ_1 and γ_2 values, and change as these values change. An interest rate of $R = 0.15$ is also inserted, implying a decrease in equilibrium credit production as γ rises.

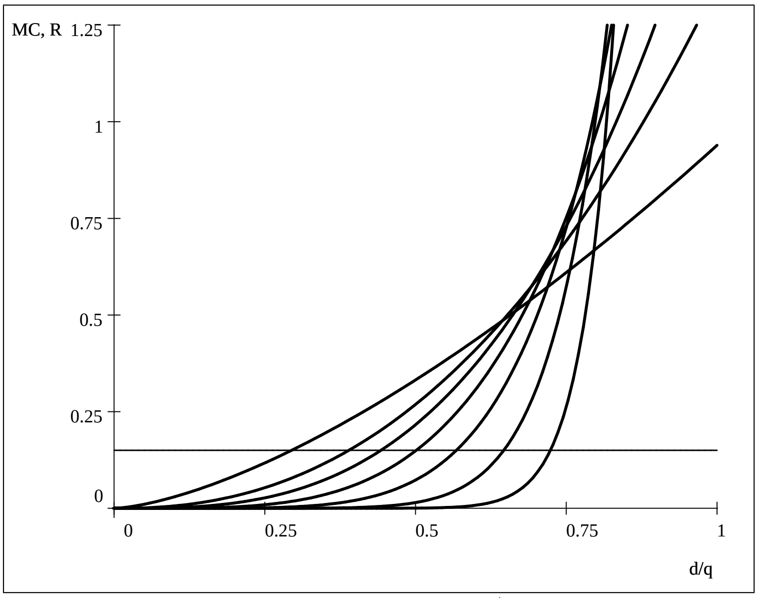


Figure 3.2. Marginal Cost of Credit d/q, with Changes in γ

3.1. Effect of Financial Development on Growth

The role of financial development can be shown analytically within a no-physical capital case of the economy.

Proposition 4. In the case with no physical capital, linear production functions for goods and human capital, and log-utility ($\beta = \varepsilon = \theta = 1$, $\gamma_2 = 0$), and with a bounded interest rate of $R = \sigma + \rho < \widehat{R}(\gamma_1, A_D, A_G) = \frac{A_G^{1+\gamma_1}}{A_D}\frac{1}{\gamma_1}e^{-(1-\gamma_1)}$, an increase in parameter γ_1 causes an increase in leisure x and a decrease in the growth rate g; i.e. $\partial x/\partial\gamma_1 > 0$ and $\partial g/\partial\gamma_1 < 0$.

Proof: With no physical capital and CRS production of goods and human capital, $c = A_G l_G h$, $\Delta h = A_H l_H h - \delta_h h$, and $\frac{d}{q} = A_D(l_D h/c)^{\gamma_1}$. Deriving the closed form solution from the first order conditions along the balanced-path growth rate, the growth rate is given by $g = A_H(1-x) - \rho - \delta_h$, leisure is $x = \frac{\alpha\rho}{A_H}\left[1+R-(1-\gamma_1)\left(\frac{d}{q}\right)R\right] / \left[1+\left(\frac{d}{q}\right)R\right]$, and the credit ratio is $\frac{d}{q} = \left[A_D\left(\frac{\gamma_1 R}{A_G}\right)^{\gamma_1}\right]^{1/(1-\gamma_1)}$. It follows that $\frac{\partial g}{\partial \gamma_1} = -A_H \frac{\partial x}{\partial \gamma_1}$, and that $\frac{\partial x}{\partial \gamma} = \frac{\alpha\rho}{A_H} \frac{\left(\frac{d}{q}\right)R[1+\left(\frac{d}{q}\right)R]-R[2-\gamma_1+R]\frac{\partial\left(\frac{d}{q}\right)}{\partial\gamma}}{[1+\left(\frac{d}{q}\right)R]^2}$, which depends negatively on the sign of $\frac{\partial\left(\frac{d}{q}\right)}{\partial\gamma_1}$, for all γ_1. Now

$$\frac{\partial\left(\frac{d}{q}\right)}{\partial\gamma_1} = \left[A_D\left(\frac{\gamma_1 R}{A_G}\right)^{\gamma_1}\right]^{\frac{1}{1-\gamma_1}} \frac{1}{1-\gamma_1}\left[1+\frac{1}{1-\gamma_1}\ln(\gamma_1 R)+\frac{1}{1-\gamma_1}\ln A_D - \frac{1+\gamma_1}{1-\gamma_1}\ln A_G\right],$$

and so $\frac{\partial\left(\frac{d}{q}\right)}{\partial\gamma_1} < 0$ for $R < \widehat{R}(\gamma_1, A_D, A_G) = \frac{A_G^{1+\gamma_1}}{A_D}\frac{1}{\gamma_1}e^{-(1-\gamma_1)}$. Then, for all $R < \widehat{R}(\gamma_1, A_D, A_G)$, it results that $\frac{\partial x}{\partial\gamma_1} > 0$, and $\frac{\partial g}{\partial \gamma_1} < 0$.

Consider how binding is the restriction that $R < \widehat{R}$. With the additional assumption that $A_G = 1$ and $A_D = 0.91$, as in the calibration below, the figure below shows that the resulting $\widehat{R}$ ranges above 100% (straight line) for all γ_1, and so is well-above non-hyperinflation rates to which the model applies. For example, for $\gamma_1 = 0.15$, as in the calibration below, $\widehat{R} = 3.13$, and so R needs to be below 313%. For $\gamma_1 \in (0.5, 1)$, the marginal cost is increasing at a decreasing rate (concave) contrary to more normal convex marginal cost curves. In the linear case of $\gamma_1 = 0.5$, then $\widehat{R} = 1.33$, or 133%, still well above a hyperinflation rate. And the $\widehat{R}$ remains well above 1.00 for all $\gamma_1 \in (0.5, 1)$, for which the marginal cost curve is concave instead of convex as is the case for $\gamma_1 \in (0, 0.5)$.

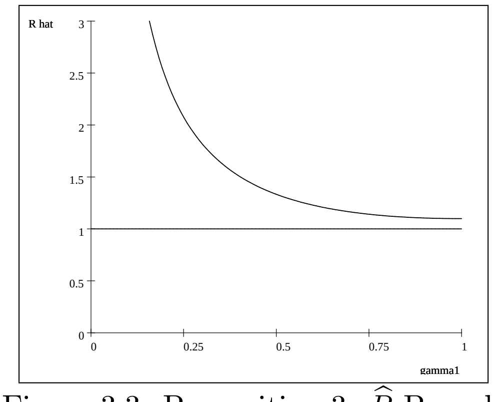


Figure 3.3. Proposition 3: $\widehat{R}$ Bounds

Intuitively, an increase in the normalized returns to scale to labor and capital in credit production, while maintaining a given nominal interest rate, causes a more flexible but more costly credit production, less total credit production, a more inelastic money demand, and more leisure because of a greater use of the money to leisure channel for avoiding inflation, instead of the money to credit channel for avoiding inflation. More leisure causes less growth.

4. Calibration and Simulation

Simulation of a calibrated model illustrates Proposition 4 within the full general equilibrium model of Section 2. The calibration matches stylized facts of the US economy, following Jones, Manuelli, and Siu (2005). Values of the shares of human capital in the goods and human capital sectors, β and ε, are both set equal to 0.64; the rates of human and physical capital depreciation, δ_H and δ_K, are both set to 0.1; and the values of the sectoral productivity parameters are set to $A_G = 1$, $A_H = 0.577$, and $A_D = 0.909$. The average rate of growth of real GDP, g, and average inflation rate, π, are set at 2% and 5%, respectively; and the discount rate ρ is set to 0.04. This implies a steady state of money growth, σ, of 7%. To set working time l_G to 0.17, as in Jones, Rodolfo, and Rossi (1993), the weight for leisure in the utility function, α, is 4.36. The intertemporal elasticity of substitution, θ^{-1}, which controls the steepness of the growth-inflation relationship, is set at $(1.5)^{-1}$, within the usual range.

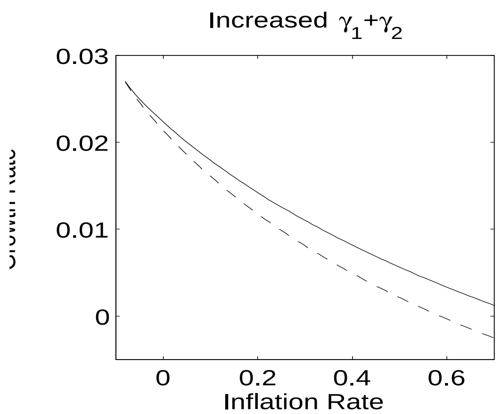
4.1. Credit Sector

The credit production γ_1 and γ_2 are set using US monetary and banking sector data. The nominal output per capita, Py, equals $41,632$ million USD in June 2005. Using M2 data for June 2005, for the money stock, the velocity of money, $y/(M/P)$, is set equal to 2.27. To approximate $\gamma_1 + \gamma_2$, the interest differential formula of Proposition 2 is used, whereby $\gamma_1 + \gamma_2 = (R - R^c)/R$. For a rough estimate, we use the average annual fee for the American Express credit cards, of 109 USD, for how much interest is paid on average. This implies that the total interest cost is $(R - R^c) \cdot d = 109$ USD. Then the ratio of this interest cost to the interest earnings on bonds, Rd, is computed, by estimating the interest earning on nominal bonds. The average interest rate of 0.0351 during 2005 is multiplied by the total credit used; the total credit is calculated using income minus the money, which with a given income of $41,632$ and a velocity of 2.27, is equal to $23,292$. This makes the total interest estimate equal to $R \cdot d = (0.0351) \cdot 23,292 = 817.5$ USD. The ratio of $(R - R^c) \cdot d$ to $R \cdot d$, is 109/817.5, giving that $\gamma_1 + \gamma_2 = 0.133$. This is what might be considered a lower bound estimate for such credit production costs, in that it ignores all other costs like late fees; and other credit cards issued to consumers with higher risks for repayment may be more expensive on average. On the side of this estimate being an overestimate is the fact that with prompt payment some credit cards are fee free. Also note that using a higher velocity of money makes the estimate lower.

With these qualifications, the simulation presented below assumes that $\gamma_1 + \gamma_2 = 0.1$, which is then increased to $\gamma_1 + \gamma_2 = 0.15$. The relative proportions between the physical and human capital shares is the same as in other sectors of the economy, with γ_1 and γ_2 being set to 0.064 and 0.036, respectively.

4.2. Simulation

The simulation shows a negative relation between inflation and growth. Figure 4.1 shows that this relation becomes more negative with 50% proportional increases in both γ_1 (0.064 to 0.096) and γ_2 (0.036 to 0.054) to $\gamma_1 + \gamma_2 = 0.15$. Similar graphs result when only γ_1 is increased, or when only γ_2 is increased.



Baseline; and 50% increase in $\gamma_1+\gamma_2$.

5. Testing the Theoretical Model with Panel Evidence

The theory indicates that, for a given inflation rate, an increase in financial development, as defined in terms of the returns to scale of $\gamma_1 + \gamma_2$, causes a decrease in the growth rate. This can be tested using standard growth rate estimations by extending them to include an interaction term between the inflation rate and financial development. The two primary variables for such a regression, on the basis of the endogenous growth perspective (discussed in GHM), is to include variables that capture the returns to the two capitals, physical and human. The real interest rate is the physical capital return and this is proxied by the investment to output ratio. The inflation rate acts as a systemic tax on human capital, acting through leisure, and so the inflation rate is included. These variables give the initial model, which is then extended with an interaction term between inflation and financial development in the spirit of Propostion 4.

The measures of financial development used are the standard ones from Levine, Loayza, and Beck (2000). However note that with respect to the optimization model, the measure that may fit best is the Liquid Liabilities measure that is M2/GDP. This most closely fits the type of exchange credit that appears in the model: a means of exchange provided by the banks that allows for interest to be earned on the funds during the period. This observation is based on the fact that while M1 is comprised of non-interest bearing instruments, and some low interest-bearing instruments, M2 instruments also can be means of exchange, like checks written against money market accounts to pay off American Express cards, while earning more significant interest rates as does the credit in the model.

5.1. The Econometric Model

The initial econometric model is specified as

$$g_{it} = \mu_i + \lambda_t + \beta_1 \dot{p}_{it} + \beta_2 I_{it} + \alpha_{it}^k F_{it}^k + [Other]_{it}' \boldsymbol{\eta} + \varepsilon_{it}, \tag{5.1}$$

where: g_{it} is the real per capita annual rate of growth of GDP of country i in period t; μ_i an unobservable effect (or "individual effect") for country i; λ_t an unobservable effect for time period t; and F_{it}^k is the level of financial intermediary development as proxied by the $k = 3$ variables, with unknown weights α_{it}^k. However since the ratio of commercial assets to total banking assets is found to be generally insignificant, results for this variable are not reported, and only *private* and *lly* results are presented. The variables in the vector $Other_{it}$ with unknown weights $\boldsymbol{\eta}$ include the initial income, trade, and black market variables; the disturbance term is ε_{it}.

The model further proposes that the financial intermediary effect, α_{it}^k, can be a function of the inflation rate:

$$\alpha_{it}^k = \beta_3^k + \beta_4^k \dot{p}_{it}, \tag{5.2}$$

where β_3^k and β_4^k are parameters for the k number of financial development variables. Substituting in for α_{it}^k gives

$$g_{it} = \mu_i + \lambda_t + \beta_1 \dot{p}_{it} + \beta_2 I_{it} + \beta_3^k F_{it}^k + \beta_4^k (\dot{p}_{it} F_{it}^k) + [Other]'_{it} \boldsymbol{\eta} + \varepsilon_{it}, \tag{5.3}$$

where $\dot{p}_{it} F_{it}^k$ represents the interaction between inflation and financial development.

5.2. The Data

The financial development data is that of Levine, Loayza, and Beck (2000), as is the data for output, prices, government expenditure, trade and black market premiums.[9] While the original sample consists of 74 countries over the period 1961-1995, supplementing this data with the investment to output ratio (*EconData*) and the money supply (*IFS*), as used in Gillman, Harris, and Mátyás (2004), results in reducing the sample to 27 countries with full information on all required.[10]

Five-yearly, non-overlapping, data averages are used such that are seven observations per country. The variables are defined as g, the real per capita growth in GDP; $\dot{p}$, the natural log of one plus the CPI rate of inflation; I, the ratio of gross domestic investment to GDP; y_0, the natural log of the real per capita GDP in the initial period; *gov*, the natural log of the share of government expenditure in GDP; *trade*, the natural log of the share of total international trade in GDP; and *bmp*, the natural log of one plus a black market premium.

For the three financial development variables that are used, the notation is *private*, which is the natural log of the ratio of the value of credits by financial intermediaries to the private sector relative to GDP; *lly*, which is the natural log of the ratio of liquid liabilities of the financial system to GDP; $\dot{p}private$, which is the product of $\dot{p}$ and *private* (an "interaction term"); $\dot{p}lly$,which is the product of $\dot{p}$ and *lly* (an alternative interaction term). The third financial development variable is the ratio of commercial assets to total banking assets, but these results are not reported as they generally are dominated by the other two variables.

[9] We are very grateful to those authors for kindly supplying their data.

[10] These countries are Australia, Austria, Belgium, Canada, Chile, Denmark, Finland, France, Greece, Ireland, Italy, Japan, the Republic of Korea, Malaysia, Mexico, Netherlands, New Zealand, Norway, Peru, Philippines, Portugal, Spain, Sweden, Switzerland, Thailand, the United Kingdom, and the United States. Note that for the inflation rate data, 4 data points of the 186 are above 50%, three for Peru and one for Mexico, and there are no negative rates of inflation.

5.3. Estimation Results

A panel data estimation with two-way fixed effects techniques is used since Hausman tests indicate the presence of correlations between the observed and unobserved effects. With such correlations both fixed and random effects specifications can yield consistent parameter estimates. In the fixed effects approach the effects are treated as constants. In a random effects approach, instrumental variable (IV) techniques can be applied. Table 5.1 presents first the fixed effects approach. Inflation and the investment ratio are highly significant with the expected negative and positive signs respectively. The level of financial development is consistently statistically insignificant; results are presented with it removed from the model except as it enters through the interaction term. The interaction term of financial development and inflation for both the $\grave{p}private$ and $\grave{p}lly$ variables is significant and negative. Endogeneity tests indicate that the inflation rate enters exogenously in models 1 to 3.

Because the Hausman specification tests suggest possible correlation between the observed and unobserved effects, as a further robustness check, this correlation is accounted for by using a random effects framework as in Hausman and Taylor (1981) and Amemiya and MaCurdy (1986). These results indicate a significant and negative effect of the level of financial intermediation. The interaction term between inflation and financial intermediation generally remains significant and negative. However a problem is that there is a rejection of the null hypothesis of valid instruments using the Sargan criteria, and so the results are not reported here [see Gillman and Harris (2004) for these details]. Thus the baseline model apparently finds only negative effects of financial development, assuming a linear relation between inflation and growth.

5.4. Threshold Effect Extensions

Non-linear effects can be estimated by breaking the regression into segments, or splines, and then looking for threshold levels of inflation with differentiated marginal effects for either side of the threshold value. Some advances in such estimated thresholds include Hansen (1999), who provides procedures for estimating multiple unknown breakpoints within the context of a one-way fixed effects panel model. Hansen (2000) presents distribution theory for the estimation of multiple threshold effects for either cross-section or time series data; Gonzalo and Pitarakis (2002) introduce a model selection based procedure which simultaneously estimates the unknown threshold parameters and their optimal number; and Drukker, Gomis-Porqueras, and Hernandez-Verme (2004) combine the model selection procedures

Table 5.1: Growth Regression Results

	Model 1	Model 2	Model 3	Model 4
Constant	0.425	0.458	0.388	0.446
	(0.05)**	(0.06)**	(0.06)**	(0.06)**
$\dot{p}$	-0.250	-0.187	-0.189	-0.206
	(0.05)**	(0.04)**	(0.04)**	(0.04)**
I	0.003	0.002	0.002	0.002
	(0.00)**	(0.00)**	(0.00)**	(0.00)**
y_0	-0.052	-0.056	-0.053	-0.054
	(0.01)**	(0.01)**	(0.01)**	(0.01)**
gov	-	-	-0.020	-
			(0.01)**	
$trade$	-	-	-0.016	-
			(0.01)**	
bmp	-	-	-	-0.028
				(0.02)*
$\dot{p}private$	-	-0.043	-0.043	-0.058
		(0.02)**	(0.01)**	(0.02)**
$\dot{p}lly$	-0.088	-	-	-
	(0.02)**			
$\overline{R}^2$	0.735	0.722	0.737	0.727
$LR \sim \chi^2_{33}$	169.697	163.91	169.244	167.578
$Hausman \sim \chi^2_{df}$	47.34 (4)	50.19 (4)	56.83 (7)	49.03 (5)
$Endogeneity \sim N(0,1)$	0.224	1.723	1.056	2.173
NT	186	186	186	186

**Significant at 5% size (2-sided). *Significant at 10% size (1-sided). LR refers to Likelihood Ratio tests of $\alpha_i = \lambda_t = 0, \forall i, t$. *Hausman* tests are of fixed *versus* random specifications. *Endogeneity* tests the null-hypothesis that the inflation variable is exogenous; the critical value is 1.96.

of Gonzalo and Pitarakis (2002) with the panel data aspects of Hansen (1999) in estimating one-way fixed effects for the non-linear effect of inflation on growth.

Here a multiple threshold approach is developed and applied to panel data for the inflation-growth relation as an extension of the model in equation (4.3). The threshold results come from several novel econometric extensions. These are that two-way country (individual) and time fixed effects are used in the panel's endogenously determined splines, as compared to one-way (individual) fixed effects in Drukker, Gomis-Porqueras, and Hernandez-Verme (2004); the use of instrumental variables where the splined variable is potentially endogenous; the application of the model selections criteria to choose simultaneously across both the model type (OLS, one-way panel, two-way panel) and the number of breakpoints (thresholds); and the estimation of the multiple endogenous splines when they are forced to be piecewise continuous. The methodology of these extensions is presented in the Appendix A.3.

The econometric model is estimated without any unobserved effects (labelled OLS), with fixed unobserved country effects ($1 - Way$), and with fixed unobserved country and time effects ($2 - Way$). Table 5.2 presents the results for the estimated breakpoints in terms of the inflation rate using each method, and for the optimal number of thresholds for each method and overall as based on the Information Criterion (IC) procedure detailed in Appendix A.3.1-A.3.2. The procedure is undertaken for both *tied* and *untied* spline functions; see Appendix A.3.3. Although IC methods are used to ascertain the optimal number of breakpoints, the estimation procedure works sequentially, implying in a sense that the first breakpoint reported in Table 4.2, which is the first found in the estimation, is the "strongest" one, the second reported in the table is less strong, and the third the least strong.

When lly is used, the optimal combination of number of *untied* breakpoints and estimation method is $2 - Way$ with two splines; with the breakpoints occurring at inflation rates of 16% and 23% (the optimal number is zero for OLS and two for $1 - Way$). Note that both the $1 - Way$ and $2 - Way$ select the same number and value of breakpoints. If *private* is used as the proxy for financial development, again $2 - Way$ is preferred. Here there are three thresholds at 4, 16 and 23% rates of inflation, similar to the lly results in that the 16% and 23% breakpoints coincide. Forcing the spline function to be piecewise continuous, the optimal model for both lly and *private* is $2 - Way$. For both there is now only one threshold effect at a 3% rate of inflation.

Table 5.3 contains the estimation results corresponding to the estimated threshold values of Table 5.2. All of the estimations presented are undertaken using

Table 5.2: Estimated Threshold Effects

Breaks	OLS	$1-Way$	$2-Way$	OLS	$1-Way$	$2-Way$
	$lly : untied$			$private : untied$		
0	*	-	-			
1	0.07	0.16	0.23	0.07*	0.23	0.23
2	0.23	0.23*	0.16**	0.23	0.16*	0.16
3	0.16	0.03	0.03	0.16	0.03	0.04**
	$lly : tied$			$private : tied$		
0	*	-	-			
1	0.23	0.03	0.03**	0.23*	0.23	0.03**
2	0.07	0.05*	0.04	0.07	0.05*	0.04
3	0.10	0.04	0.03	0.10	0.02	0.23

**Preferred model overall (based on minimum IC). *Preferred model for each estimation procedure (based on minimum IC).

$2-Way$ fixed effects and correspond to the optimally chosen model and number of breakpoints. With regard to the variables I, y_0, and the interaction term between inflation and financial development, results vary little across specifications and in comparison to Table 5.1. Investment has a positive and significant effect on growth; initial GDP has a significantly negative effect, as does the interaction term, while the level of financial development has a insignificant negative effect. For the inflation rate, the lly proxy shows a significant negative effect at all levels; the $private$ proxy shows an insignificant positive effect at low levels and a significant negative effect at all other levels. Forcing the inflation-growth splines to be piecewise continuous, the effect of inflation at low levels up to 3% is significant and positive for both proxies. However these later results with a positive effect at low levels are not robust to using instrumental variables, as the next subsection indicates.

Another way to compare Models 1 to 4 in Table 4.3 is using the Information Criterion numbers. These are -3.1646 for lly not tied, -3.1362 for lly tied, -3.1287 for $private$ not tied, and -3.0831 for $private$ tied. With a lower IC value being a better one, this indicates that the models using lly are the preferred ones, and that the results not forcing the continuity of the splines are preferred over the tied spline results for both financial development proxies. Model 1 is the preferred model.

Table 5.3: Threshold Growth Results: Standard Errors in Paranetheses.

	lly		*private*	
	untied	*tied*	*untied*	*tied*
Constant	0.511	0.513	0.542	0.559
	(0.06)**	(0.06)**	(0.06)**	(0.06)**
I	0.002	0.002	0.002	0.002
	(0.00)**	(0.00)**	(0.00)**	(0.00)**
y_0	-0.061	-0.062	-0.064	-0.066
	(0.01)**	(0.01)**	(0.01)**	(0.01)**
$\dot{p}FD$	-0.049	-0.074	-0.011	-0.034
	(0.02)**	(0.02)**	(0.02)	(0.02)**
FD	-0.005	-0.005	-0.002	-0.002
	(0.00)	(0.00)	(0.00)	(0.00)
$\dot{p}_{low}$	-0.214	0.222	0.093	0.252
	(0.05)**	(0.10)**	(0.13)	(0.11)**
$\dot{p}_{medium}$	-0.278	-	-0.131	
	(0.05)**	-	(0.05)**	
$\dot{p}_{medium-high}$	-0.180		-0.215	
	(0.05)**		(0.04)**	
$\dot{p}_{high}$		-0.228	-0.110	-0.169**
		(0.05)**	(0.04)**	(0.04)
$\overline{R}^2$	0.736	0.723	0.733	0.708
NT	175	175	175	175

**Significant at 5% size (2-sided). *Significant at 10% size (1-sided). FD refers to the appropriate measure of financial development.

5.5. Simultaneity Bias

Using splines it is difficult to test for endogeneity in the splined variable, in this case the inflation rate. However an instrumental variables (IV) estimation can be made and the results can then be compared to the Table 4.3. For the estimation with IVs, a procedure similar to two-stage least squares is used. First fitted values of inflation are constructed by regressing it against all of the exogenous variables in the model plus a money supply instruments. The observed inflation rate is then replaced by its fitted value $\widehat{\dot{p}}_{it}$ and the spline procedure as described above is then implemented on $\widehat{\dot{p}}_{it}$ as opposed to $\dot{p}_{it}$. To take into account the issue of generated regressors, coefficient standard errors are estimated by bootstrap methods. Note that the sample loses one time period and one country due to missing observations on the money supply and this means that the procedure searches over somewhat different ranges of inflation.

Table 5.4 presents the IV results for the optimal number and position of the breakpoints. Using *lly* and a piecewise discontinuous function, the optimal model now is OLS with two breakpoints at 8% and 17% rates of inflation, as compared to the optimal $2 - Way$ with two breakpoints at 3% and 16% in Table 5.2. However the test statistic that choses OLS as optimal is very close to the test statistics for the $1 - Way$ and $2 - Way$. For *private untied*, the optimal choice is OLS with 3 breakpoints as compared to $2 - way$ with 3 breakpoints in Table 5.2.

Table 5.4: Estimated Threshold Effects using IVs

Breaks	OLS	$1 - Way$	$2 - Way$	OLS	$1 - Way$	$2 - Way$
	$lly : untied$			$private : untied$		
0	-	-	-			
1	0.17	0.28*	0.18	0.10	0.29	0.05
2	0.08**	0.06	0.05	0.14	0.13	0.09
3	0.11	0.04	0.15*	0.11**	0.11*	0.03*
	$lly : tied$			$private : tied$		
0	-	-	-			
1	0.08	0.04*	0.05	0.11**	0.11	0.15
2	0.29**	0.05	0.04*	0.10	0.13	0.05
3	0.07	0.05	0.18	0.06	0.15*	0.09*

**Preferred model overall (based on minimum IC). *Preferred model for each estimation procedure (based on minimum IC).

Table 5.5: IV Threshold Growth Results: Standard Errors in Paranetheses.

	lly		*private*	
	untied	*tied*	*untied*	*tied*
Constant	0.045	0.015	0.095	0.024
	(0.03)*	(0.02)	(0.10)	(0.02)
I	0.003	0.002	0.002	0.002
	(0.00)***	(0.00)	(0.00)**	(0.00)***
y_0	-0.007	-0.006	-0.010	-0.005
	(0.00)***	(0.00)	(0.01)*	(0.00)***
$\dot{p}FD$	-0.142	0.053	-0.209	0.217
	(0.08)**	(0.05)	(0.40)	(0.05)***
FD	-0.008	-0.015	-0.001	-0.014
	(0.01)	(0.01)	(0.01)	(0.00)***
$\dot{p}_{low}$	-0.397	0.170	-0.647	0.163
	(0.26)*	(0.07)	(1.18)	(0.07)***
$\dot{p}_{low/medium}$	-	-	-0.748	-
	-	-	(1.15)	-
$\dot{p}_{medium/high}$	-0.535	-0.148	-0.900	-
	(0.20)***	(0.88)	(1.13)	-
$\dot{p}_{high}$	-0.379	0.046	-0.640	0.532
	(0.16)***	(0.11)	(1.08)	(0.14)***
$\overline{R}^2$	0.457	0.434	0.456	0.457
Sargan	0.430	0.078	0.022	0.072
NT	144	144	144	144

***Significant at 5% size (2-sided). **Significant at 5% size (1-sided);*Significant at 10% size (1-sided); *Sargan* refers to the bootstrapped empirical $p-$value of the Sargan statistic for instrument validity, accept H_0 of valid instruments for $p > 0.05$.

Table 5.5 presents the regression results corresponding to the optimal model as indicated in Table 5.4. The results that pass the *Sargan* test are those using *lly untied*. As in previous findings, I, y_0 and the interaction of financial development and inflation are all significant, and once more the level of financial development is insignificant. All levels of inflation have a significant negative effect, although weaker at lower levels. For *private untied*, the model fails the *Sargan* test. Forcing the continuity of the splines, the *tied* results for *lly* and *private* suggest that inflation positively affects growth in ranges, but the *Sargan* statistic that indicate instruments with borderline validity.[11]

[11] These are bootstrapped empirical values of the standard *Sargan* test.

5.6. A Dynamic Growth Approach

Finally, consider for robustness dynamic growth equations. Here the basic model is extended by including lagged growth, $g_{i,t-1}$. For the dynamic panel model the usual estimation techniques are inconsistent. To allow for growth to follow an autoregressive process while removing the unobserved effects, it is common to write the model in terms of first differences and including a lagged dependent variable

$$\Delta g_{it} = \delta \Delta g_{i,t-1} + \Delta \mathbf{x}'_{it}\boldsymbol{\beta} + \Delta \varepsilon_{it}. \tag{5.4}$$

Following Arellano and Bond (1991) it is possible to consistently estimate the model by GMM estimation based upon the moment conditions,

$$E\left(\Delta \varepsilon_{it} g_{i,t-j}\right) = 0,\ j = 2, \ldots, t-1;\ t = 3, \ldots, T. \tag{5.5}$$

The moment conditions imply that the $\Delta \varepsilon_{it}$ do not follow a second-order serial correlation process, a condition that is tested here.

Table 5.6: Dynamic Growth Results

Variable	Coefficient	Standard Error
$g_{i,t-1}$	-0.488	$(0.06)^{**}$
I	0.002	$(0.00)^{**}$
y_0	-0.046	$(0.01)^{**}$
$\dot{p}lly$	-0.058	$(0.02)^{**}$
lly	0.000	(0.01)
$\dot{p} \times 1\,(\dot{p} < 16\%)$	-0.166	$(0.04)^{**}$
$\dot{p} \times 1\,(16\% \leq \dot{p} < 23\%)$	-0.244	$(0.04)^{**}$
$\dot{p} \times 1\,(23\% \leq \dot{p})$	-0.1943	$(0.04)^{**}$
$\overline{R}^2$	0.694	
$Sargan$	0.1	
m_2	-2.4	
NT	125	

**Significant at 5% size (2-sided). $Sargan$ refers to the $p-$value of the Sargan statistic for instrument validity, accept H_0 of valid instruments for $p > 0.05$. m_2 tests for second-order serial correlation and is $\tilde{a}N\,(0,1)$ under the null hypothesis.

The results of the instrumental variables estimation of the previous subsection indicate a greater validity for the lly proxy of financial development, with discontinuous splines. Here this model is used (Model 1 in Table 4.3) and it is assumed that the true threshold model has two breakpoints as found in both Table 5.2 and Table 5.4. Re-estimating the model with the inclusion of the lagged dependent variable as in equation (5.4), Table 5.6 presents the results.

The model passes the $Sargan$ test for instrument validity and indicates results consistent with Tables 5.3 and 5.5 in terms of the significance and signs of the explanatory variables. The growth process is autoregressive with the lagged dependent variable being strongly significant. This variable's negative sign indicates a cyclical return to the equilibrium growth path following a shock. The strong significance of the lagged growth term while the remaining variables have similar effects to the static estimations suggests that the potential omitted variable bias arising from the previous exclusion of $g_{i,t-1}$ is small. Note that there is some evidence that the $\Delta\varepsilon_{it}$ follow a second-order serial correlation process.

6. Discussion

Related models of intermediation include the "limited participation" model, such as Fuerst (1994), Fuerst (1995), Christiano and Eichenbaum (1995), Dotsey and Ireland (1995), Einarsson and Marquis (2001), Canova (2002), and Christiano, Motto, and Rostagno (2003). Typically, the consumer lends the wage funds to the bank as interest-bearing deposits and the bank lends them to the firm. The interest-paying deposits equal the loans to firms needed for wages and capital expenses, so that the inputs are bought with loan credit. This gives a mixed cash/credit equilibrium with the disadvantage of a rigid structure of what is a cash good and what is a credit good. Many of these models assume no costs of intermediation, with no interest differential between the depositor and lender rates except when there is a positive reserve requirement on deposits. This is a result consistent with Fama (1980), who postulates a Modigliani-Miller theorem of finance as applied to banks in which, without intermediation costs, only the reserve requirements imposed by banks give them value. In this case only the reserve requirements create a spread between depositor and loan interest rates, which is often the basis for valuing the output of banks (Wang 2003). More generally, the value of banks, and the interest differential, would include the resource cost of the intermediation efforts [Baltensperger (1980), Wang, Basu, and Fernald (2004)].

With positive costs assumed for providing interest-bearing deposits or credit, models since Baumol-Tobin have used different devices to derive a mixed exchange equilibrium. In Baumol (1952)-Tobin (1956), the marginal cost of getting interest-bearing deposits (using banking) equals the marginal cost of money, being the nominal interest rate. Baltensperger (1980) focuses on the problem of a unique interior solution for the costly supply of intermediation services in partial equilibrium. He specifies that the production function must be of decreasing returns to scale in capital and labor, or that there needs to be a convex cost function, so that the constant marginal revenue per unit of funds equals the marginal cost per unit funds, which rises with the level of services output.[12] Berk and Green (2004) take a related approach in their partial equilibrium study of mutual funds intermediation, specifying a convex cost function. And Wang, Basu, and Fernald (2004) similarly assume an exogenous convex marginal cost for a variety of value-added bank services.

In general equilibrium, the rising-marginal-cost principle emerges in models with costly credit but without an explicit intermediation sector. Gillman (1993), Ireland (1994), Khan, King, and Wolman (2000), Erosa and Ventura (2000) use monotonically varying costs across a continuum of stores, with the marginal credit store being where the marginal cost of credit equals the nominal interest rate. But the store continuums used here do not easily integrate into the one-good neoclassical model. And the lack of a financial intermediary makes it difficult to define financial development.

In terms of the empirical results, they are robust with respect primarily to the ratio of liquid financial assets to GDP, the financial development measure that best corresponds to the model. Using instead the private credit variable, indicates for example a positive effect of inflation at low levels of the inflation rate that is not robust to considerations of endogeneity of the inflation rate, since the inflation rate is insignificant at all levels for the IV results. The IV results are important to consider since such endogeneity can be suspected *a priori* at low inflation rates because of the interaction of the business cycle with the price level. The price level has been found to co-move with output in the short run (Den Haan 2000), which is a manifestation of how low levels of the measured inflation rate and the output level could be simultaneously determined. However this can be more of a relative price change involving changes in the aggregate price level due to real output changes over the business cycle than to monetary, inflation-type, effects and so it should be controlled for with instruments. Use of the money supply as an instrument is also found in Gillman, Harris, and Mátyás (2004).

The other way in which a positive effect of inflation at low levels is replicated is through a procedure to force the multiple splines to be piecewise continuous. But this approach yields consistently worse results using the Information Criterion and so is found to involve a nontrivial assumption that can yield misleading results. The models without the tied-spline assumption perform better.

[12] Baltensperger (1980), pp. 14, 17, 31, equations (15) and 21).

For other variables in the econometric model, the investment ratio has a robust postive effect; the initial value of GDP has a robust negative effect, as consistent with transitional dynamics (see Gillman, Harris, and Mátyás (2004)). By itself the level of financial development is robustly insignificant; but note that the standard results of a positive financial development effect can be replicated with the data set. These results are not reported because they lack significant missing variables. For example, using the private credit measure of financial development and the black market variable, but excluding the investment ratio, the results show a significant financial development variable at the 1% level (two-sided). And here a significant negative inflation effect is replicated at a 5% level of significance (one-sided). Or, excluding only the investment ratio from the Model 1 in Table 5.1, while using the liquid liabilities measure of financial development, finds that the level of financial development is positive while the interaction term is negative, although both with weak levels of significance ($t-$statistics respectively of 1.296 and -1.182). Finally, in the reported results, the level of financial development robustly has a negative effect on growth through its interaction term with the inflation rate.

7. Conclusion

The optimization model has a unique equilibrium between non-interest-bearing and interest bearing means of exchange, using intermediary structural parameters instead of free utility or transaction cost parameters, and providing an intuitive definition of financial development based on the production function borrowed from the banking literature. As a result, the theory modestly extends the mainstream monetary general equilibrium model towards a fuller integration of finance. The implied money demand function is characterized by a unitary income elasticity and a plausible Cagan (1956) type interest elasticity (Mark and Sul 2003). This plausibility is important in that the money demand derives partly from the intermediary production function and provides another reflection of its specification; also the money demand, as Lucas (2000) shows in a related model, determines the welfare cost of inflation. Here such welfare cost comes from the use of both intermediary resources and leisure to avoid inflation. And financial development, in terms of greater economies of scale, here counterintuitively implies more intermediary resources used to avoid the inflation tax, a disadvantage of achieving greater flexibility in expanding output at all levels of the inflation rate without the constraint of a rapidly rising marginal cost. The implication is that developing countries, for a given rate of inflation, may incur less such costs of avoidance, using technology with less scope such as limited banking, including exchange for foreign currency as a type of bank-provided means to avoid the inflation tax.[13]

The model provides a formal basis to investigate the interaction with inflation of financial development on one type of intermediation service. Other types of financial intermediation might be included, each with its own production function and similarly derived interest differentials, such as the cost of buying a market portfolio, or of buying government bonds. Further, here there is no actual difference between short and long run securities, making difficult an explanation of the term structure puzzle as in Bansal and Coleman (1996). And the liquidity effect from open market operations is not present by construction, but might be investigated by building a fuller capital market.

Given such qualifications, and in summary, the paper presents a general equilibrium monetary model with an exchange constraint that is determined by the production of credit and the free substitution between money and credit to buy output. The model is used to analyse how financial development affects the inflation-growth profile. Effects of changes in the economies of scale are shown graphically in terms of marginal cost, in an analytic solution of the financial intermediary problem, and also in numerical simulations of the inflation-growth effect. Extensive empirical results find support for the proposition of a negative interaction effect of financial development and inflation upon growth. These results show robustness across multiple threshold testing, instrumental variable estimation and dynamic panel estimation.

A. Appendix

A.1. Multiple Threshold Effects

The threshold model, for two regimes, considered in Hansen (1999) and Drukker, Gomis-Porqueras, and Hernandez-Verme (2004) is of the form $g_{it} = \alpha_i + \mathbf{x}'_{it}\boldsymbol{\beta} + \gamma_1 \dot{p}_{it} \times 1\left(\dot{p}_{it} \leq \gamma_1^*\right) + \gamma_2 \dot{p}_{it} \times 1\left(\dot{p}_{it} > \gamma_1^*\right) + \varepsilon_{it}$.This can be written more compactly; by defining $\dot{\mathbf{p}}_{it}\left(\gamma_1^*\right) \equiv \left[\dot{p}_{it} \times 1\left(\dot{p}_{it} \leq \gamma_1^*\right), \dot{p}_{it} \times 1\left(\dot{p}_{it} > \gamma_1^*\right)\right], \boldsymbol{\gamma} \equiv \left(\gamma_1, \gamma_2\right)'$:

$$g_{it} = \alpha_i + \mathbf{x}'_{it}\boldsymbol{\beta} + \dot{\mathbf{p}}_{it}\left(\gamma_1^*\right)\boldsymbol{\gamma} + \varepsilon_{it}. \tag{A.1}$$

Here $\mathbf{x}_{it}$ is the vector of explanatory variables net of the splined variable. Thus if inflation is less than or equal to the (unknown) threshold value γ_1^*, its marginal effect on growth is given by γ_1 and by γ_2 otherwise. For identification, $\mathbf{x}_{it}$ cannot

[13] Cziraky and Gillman (2006) find evidence of the limited use of banking for exchange credit in Croatia since its stabilization after its early 90s hyperinflation.

contain any time-invariant variables; it is also assumed that the threshold effects are time-invariant. The error term, ε_{it}, is iid with zero mean and finite variance, σ_{ε}^{2}.

The usual approach to estimating one-way panel models, is to use the $Within$ operator to transform the variables into differences from time means for each i and then to apply ordinary least squares (OLS) to the transformed model (Mátyás and Sevestre 2005). For the unsplined variables, the transformation is such that for typical element of $\mathbf{x}$ we have $x_{it}^{*} = x_{it} - \overline{x}$, with $\overline{x}_{i.} = T^{-1}\sum_{t=1}^{T} x_{it}$. For the inflation variable the relevant transformation is $\overline{\dot{\mathbf{p}}}_{it}(\gamma_1^*) = T^{-1}\sum_{t=1}^{T}\dot{\mathbf{p}}_{it}(\gamma_1^*) = \left[T^{-1}\sum_{t=1}^{T}\dot{p}_{it}\times 1(\dot{p}_{it}\leq\gamma_1^*), T^{-1}\sum_{t=1}^{T}\dot{p}_{it}\times 1(\dot{p}_{it}>\gamma_1^*)\right]$. With $\mathbf{G}^*, \mathbf{X}^*$ and $\dot{\mathbf{P}}^*$ defined as the matrix stacked versions of $g_{it}^*, \mathbf{x}_{it}^*$ and $\dot{p}_{it}^*$ respectively, the estimating equation is $\mathbf{G}^* = \mathbf{X}^*\boldsymbol{\beta} + \dot{\mathbf{P}}^*(\gamma_1^*)\boldsymbol{\gamma} + \boldsymbol{\varepsilon}^*$.With $\mathbf{Z}^*(\boldsymbol{\gamma}_1^*) = \left[\mathbf{X}^*, \dot{\mathbf{P}}^*(\gamma_1^*)\right]$ and $\boldsymbol{\phi} = (\boldsymbol{\beta}', \boldsymbol{\gamma}')'$, this rewrites as $\mathbf{G}^* = \mathbf{Z}^*(\boldsymbol{\gamma}_1^*)\boldsymbol{\phi} + \boldsymbol{\varepsilon}^*$.For any given value of γ_1^*, the matrix $\boldsymbol{\phi}$ can be estimated by $\widehat{\boldsymbol{\phi}} = \left[\mathbf{Z}^*(\boldsymbol{\gamma}_1^*)'\mathbf{Z}^*(\boldsymbol{\gamma}_1^*)\right]^{-1}\mathbf{Z}^*(\boldsymbol{\gamma}_1^*)'\mathbf{G}^*$, with covariance matrix $V\left(\widehat{\boldsymbol{\phi}}\right) = \sigma_{\varepsilon}^{2}\left[\mathbf{Z}^*(\boldsymbol{\gamma}_1^*)'\mathbf{Z}^*(\boldsymbol{\gamma}_1^*)\right]^{-1}$. However, γ_1^* is unknown. The estimation procedure (Chan 1993, Hansen 1999, Hansen 2000, Gonzalo and Pitarakis 2002) involves a grid search over all possible values of γ_1^*, while ensuring that a sufficiently large number of observations ($\eta\%$) lie in each regime (η is set equal to 5%). The optimal value of γ_1^* is obtained by minimising the concentrated sum of squared errors, which means choosing the value of γ_1^* that yields the smallest sum of squared errors (SSE) over the grid-searched possible values of γ_1^*. In practice, the sorting is on the observed $\dot{p}_{it}$ with search between the $\eta\%$ and $(1-\eta)\%$ quantile.

It is possible that there may be several such threshold effects. A convenient result is that sequential estimation of the breakpoints is consistent (see Chong 1994, Bai 1997, Bai and Perron 1998, Hansen 1999, Hansen 2000, Gonzalo and Pitarakis 2002). This suggests a procedure to estimate multiple breakpoints: estimate the single threshold point; fix the first stage estimate at $\widehat{\gamma}_1^*$; conditional on this estimate, repeat the procedure to find $\widehat{\gamma}_2^*$; with both $\widehat{\gamma}_1^*$ and $\widehat{\gamma}_2^*$ treated as fixed, repeat the procedure to find $\widehat{\gamma}_3^*$; continue for $m = 1, \ldots, M$ possible breakpoints. In subsequent grid searches, the range over which to search is reduced so as to ensure a minimum number of observations ($\eta\%$) in each regime.

A.1.1. Model Selection Criteria

Hansen (1999) and Hansen (2000) suggest using bootstrapped versions of likelihood ratio statistics to determine the optimal number of breakpoints. Gonzalo and Pitarakis (2002) alternatively offer an appealing approach of choosing the model which minimises the information criterion (IC) function: $IC(\gamma_1^*, \gamma_2^*, \ldots, \gamma_m^*) = \ln SSE(\gamma_1^*, \gamma_2^*, \ldots, \gamma_m^*) + \frac{\omega_S}{S}[k^*(m)]$, where S is the sample size, k^* is the number of freely estimated response parameters that in turn are functions of m, and ω_S is a penalty term, typically a function of the sample size.[14] Gonzalo and Pitarakis (2002) suggest that $\omega_S = \ln(S)$, which corresponds to a Bayesian Information Criteria, performs the best.

This procedure can be adapted to the panel data by letting $S = NT$, with $k^*(m)$ reflecting the reduction in degrees of freedom involved in the panel estimation. This involves a loss of $N-1$ degrees of freedom for a fixed effects one-way model and of $(N-1)(T-1)$ degrees of freedom for a fixed effects two-way model.

A.1.2. Time effects

As it currently stands, equation (A.1), or its multiple regime counterpart, is inconsistent with equation (5.3) due the former's omission of the time, or business cycle, effects of λ_t.[15] Time effects can be incorporated into the threshold procedure described above. The relevant data transformations for a typical element of $\mathbf{x}$ are $x_{it}^* = x_{it} - \overline{x}_{i.} - \overline{x}_{.t} + \overline{x}$, with the appropriate definition of the time, individual and overall mean variables. Applied to the splined variable ($\dot{p}_{it}$), it needs to be determined if there are unobserved time and/or country effects present in the data, and what are the optimal number of breakpoints in the inflation-growth profile. Devising such a testing procedure is complicated since for example a two-way fixed effects panel model can yield a different optimal value of m as compared to a simple OLS model. An alternative approach is to use the Information Criteria procedure suggested by Gonzalo and Pitarakis (2002) to choose both across m and among estimation technique (OLS, one- and two-way models), once appropriate degrees of freedom corrections have been made to $k^*(m)$. That is, fix M; estimate for $m = 0, \ldots, M$ the model by each of the three estimation procedures; and for each model estimation calculate the $IC(\gamma_1^*, \gamma_2^*, \ldots, \gamma_m^*)$. Finally, choose the optimal model with regard to the m number of breakpoints and the estimation procedure that yields the smallest value of the $IC(\gamma_1^*, \gamma_2^*, \ldots, \gamma_m^*)$.

[14] For example, if the total number of explanatory variables, including the inflation variable is denoted k, and $m = 0$, then $k^*(m) = k$, for $m = 1$, $k^*(m) = k+1$ and for $m = 2$, $k^*(m) = k+2$.

[15] In the baseline model, which is an unsplined specification, there is a clear rejection of both of the null hypotheses: $H_0 : \lambda_t = 0$ and $\alpha_i = 0$, for all t, i and $H_0 : \lambda_t = 0$, for all t.

A.1.3. Tied versus Untied Splines

The inflation-growth relation with thresholds may be assumed to be piecewise continuous or allowed to be discontinuous at the spline knot. To force the relationship to be continuous, as is made explicit in Tables 4.2-4.5, it is possible to follow Greene (2003), p.122, and re-define $\dot{\mathbf{p}}_{it}(\gamma_1^*)$ as $\dot{\mathbf{p}}_{it}(\gamma_1^*) = [\dot{p}_{it}, (\dot{p}_{it} - \gamma_1^*) \times 1(\dot{p}_{it} > \gamma_1^*)]$.

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