

# The External Finance Premium and the Macroeconomy: US post-WWII Evidence

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## Abstract

This paper embeds the financial accelerator into a medium-scale DSGE model and estimates it using Bayesian methods. Incorporation of financial frictions enhances the model's description of the main macroeconomic aggregates. The financial accelerator accounts for approximately ten percent of monetary policy transmission. The model-consistent premium for external finance compares well to observable proxies of the premium, such as the high-yield spread. Fluctuations in the external finance premium are primarily driven by investment supply and monetary policy shocks. In terms of recession prediction, false signals of the premium can be given an economic interpretation.

Keywords: financial accelerator, external finance premium, DSGE model, Bayesian estimation

JEL: E4, E5, G32

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# 1 Introduction

This paper incorporates the agency cost framework of Bernanke, Gertler and Gilchrist (1999) into a DSGE model of the type analysed by Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2003, 2005, 2006). We estimate the model on post-war US data using Bayesian techniques. We first assess whether financial frictions help in describing the main macroeconomic aggregates. We then quantify the contribution of the broad credit channel to the transmission of monetary policy (and other) shocks. From the model, we extract a time series of the external finance premium. We discuss its relation to observable proxies of the premium and to shocks driving business cycles.

The reference model of contemporary business cycle research is the so-called New Keynesian or New Neoclassical Synthesis Dynamic Stochastic General Equilibrium (DSGE) model. Christiano, Eichenbaum and Evans (2005) show that a medium-scale version of this model is able to replicate the dynamic response of US macroeconomic aggregates to a monetary policy shock. Smets and Wouters (2003, 2005, 2006) extend the model to a wider set of shocks and frictions, and -following Schorfheide (2000)- estimate it using Bayesian methods. Their results indicate that the current strand of DSGE models is able to compete on empirical grounds with purely data driven approaches, such as (Bayesian) VAR's. The present paper combines two observations related to the New Keynesian model. First, empirically, there is room for improvement in the standard model. In particular, Smets and Wouters (2006) document a relatively poor forecasting performance of the DSGE model with respect to investment. Second, theoretically, one maintained assumption in the prototypical New Keynesian model is that of frictionless capital markets. The seminal contribution of Bernanke and Gertler (1989) and a number of subsequent calibration studies, most notably Carlstrom and Fuerst (1997) and Bernanke, Gertler and Gilchrist (1999), document how relaxing the perfect capital market assumption can generate additional features

observed in macroeconomic data.

Additionally, an enormous amount of microeconomic studies aims to quantify the extent of financial frictions to firm investment (see, e.g., Hubbard 1998 and the references cited therein). We complement this research from a macroeconomic point of view. Specifically, we estimate the external finance premium, which is essentially unobservable, on the basis of macroeconomic data. Our approach provides a number of contributions relative to the microeconomic one. First, the model allows an interpretation of fluctuations in the external finance premium in terms of shocks driving the economy. Second, from a historical perspective, data availability enables an investigation of the premium for the entire post-WWII period. Third, from a cross-sectional perspective, the model-consistent premium is exhaustive in coverage. By contrast, both micro estimates and readily available proxies of the external finance premium typically focus on limited time periods or subsets of firms, or both.

A number of related papers also take financial friction models to the data. Levin, Natalucci and Zakrajšek (2004), on the one hand, exploit the microeconomic framework of Bernanke, Gertler and Gilchrist (1999). They estimate the underlying structural parameters using a sample of US firms over the most recent business cycle. Subsequently, they analyse variations in the external finance premium over time and firms. On the other hand, a couple of papers subsume that a coherent macroeconomic framework can aid in the estimation of the magnitude of financial frictions. Meier and Müller (2006) estimate the elasticity of the premium in response to a monetary policy shock using minimum distance estimation. Christensen and Dib (2005) conduct a similar exercise using maximum likelihood techniques. The Bayesian approach enables a variety of model comparison exercises, as in Neri (2004) and Queijo (2005), who measure the relative contribution of a number of frictions, including financial imperfections. The present paper also addresses credit market frictions from the macroeconomic point of view. However, we take the variety of real and nominal frictions for granted. Their importance has been established elsewhere,

notably in Christiano, Eichenbaum and Evans (2005), Smets and Wouters (2003, 2005, 2006), and the references cited therein. We do test the contribution of the financial accelerator relative to the standard model, but take the model one step further and analyse its implications for monetary policy transmission and the external finance premium. These implications are interesting on their own, irrespective of whether the model delivers a better description of macroeconomic aggregates. In this respect, the interest of the paper is closer to the analysis of Levin, Natalucci and Zakrajšek (2004) than the aforementioned macroeconomic studies.

To anticipate our results, we find a substantial role for financial market imperfections. Incorporation of the financial accelerator further improves the prototypical New Keynesian model's ability to mimic the dynamics of the main macroeconomic aggregates. Furthermore, we perform a quantitative assessment of the strength of the financial accelerator. Our findings suggest that, in terms of GDP, 10% of monetary policy transmission is due to the existence of financial market imperfections. The estimated steady state premium for external finance in the US is 150 basis points. The premium exhibits a significant negative reaction to changes in entrepreneurial net worth. We provide a model-consistent time series of the external finance premium over the post-war period. Our estimate of the external finance premium bears close resemblance to some observable indicators of financial distress. Moreover, historical fluctuations in the premium are driven primarily by investment supply shocks, and secondly, by monetary policy shocks. Finally, although the external finance premium is generally a good predictor of recessions, supply shocks occasionally have induced false predictions.

The paper is structured as follows. In Section 2 we present the log-linearized version of the model. Section 3 discusses the estimation procedure and results. The paper then focuses on the implications for the financial accelerator (Section 4) and the external finance premium (Section 5). Section 6 concludes.

## 2 Theoretical framework

The model we propose is a version of the standard New Keynesian / New Neoclassical Synthesis model, analysed in detail in Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2003, 2006). The economy consists of households, final and intermediate goods producers, and a monetary authority. Moreover, as in Bernanke, Gertler and Gilchrist (1999) and Christiano, Motto and Rostagno (2003), we introduce a financial intermediary, capital goods producers and entrepreneurs. Since these models are quite well-known, we refrain from a full-blown exposition of their first principles. To make the paper self-contained, this section presents the log-linearized version of the model that we estimate. For details, we refer the reader to the original papers.

Households maximize utility by trading off current consumption with future consumption and current labor effort. Aggregate consumption  $\hat{C}_t$  evolves according to:

$$\begin{aligned}\hat{C}_t = & \frac{h}{1+h}\hat{C}_{t-1} + \frac{1}{1+h}E_t\hat{C}_{t+1} + \frac{\sigma_c - 1}{(1+\lambda_w)(1+h)\sigma_c}(\hat{L}_t - E_t\hat{L}_{t+1}) \\ & - \frac{1-h}{(1+h)\sigma_c}\hat{R}_t + \frac{1-h}{(1+h)\sigma_c}(\hat{\varepsilon}_t^B - E_t\hat{\varepsilon}_{t+1}^B)\end{aligned}$$

Apart from the standard terms in future consumption and the real interest rate  $\hat{R}_t$  ( $= \hat{R}_t^n - E_t\hat{\pi}_{t+1}$ ), this particular consumption process derives from habit persistence (of the "catching-up with the Joneses" form) and non-separable utility in labor ( $\hat{L}_t$ ) and consumption. Consumption is more persistent for larger values of the habit parameter  $h$ . Moreover, for  $\sigma_c > 1$ , there exists some complementarity between labor and consumption. The final term involving  $\hat{\varepsilon}_t^B$  represents a shock to the discount factor  $\beta$ , affecting intertemporal substitution decisions.

Households' labor supply is differentiated which, in combination with partial indexation of non-reoptimized wages, gives rise to the following linearized wage equation:

$$\begin{aligned}\hat{w}_t = & \frac{\beta}{1+\beta}E_t\hat{w}_{t+1} + \frac{1}{1+\beta}\hat{w}_{t-1} + \frac{\beta}{1+\beta}(E_t\hat{\pi}_{t+1} - \bar{\pi}_t) - \frac{1+\beta\gamma_w}{1+\beta}(\hat{\pi}_t - \bar{\pi}_t) + \frac{\gamma_w}{1+\beta}(\hat{\pi}_{t-1} - \bar{\pi}_t) \\ & - \frac{1}{1+\beta}\frac{(1-\beta\xi_w)(1-\xi_w)}{(1+\frac{(1+\lambda_w)\sigma_l}{\lambda_w})\xi_w}\left[\hat{w}_t - \sigma_l\hat{L}_t - \frac{\sigma_c}{1-h}(\hat{C}_t - h\hat{C}_{t-1}) - \hat{\varepsilon}_t^L\right] + \eta_t^W\end{aligned}$$

where  $\hat{w}_t$  and  $\hat{\pi}_t$  denote wage and price inflation, respectively.  $\bar{\pi}_t$  is the central bank's inflation objective. With (Calvo) probability  $1 - \xi_w$  a household gets to reoptimize its wage in period  $t$ . It does so taking into account both current and future marginal costs. The term in square brackets bears some resemblance to an error-correction term, in which the actual wage is drawn towards its flexible price counterpart. The intratemporal trade-off between consumption and work is subject to a labor supply shock  $\hat{\varepsilon}_t^L$ . The lagging terms in the wage equation result from the partial indexation assumption, parametrized through  $\gamma_w$ . Finally, this specification also allows for temporary deviations from the equilibrium wage mark-up  $\lambda_w$ , as captured by the shock  $\eta_t^W$ .

The firm sector consists of a continuum of monopolistically competitive intermediate goods firms. Their output is combined to produce final goods, which are sold in a perfectly competitive market. The aggregate conditions resulting from these agents' optimization are standard. Aggregate supply stems from the typical Cobb-Douglas production function augmented with fixed costs and variable capital utilization:

$$\hat{Y}_t = \phi \hat{\varepsilon}_t^A + \phi \alpha \hat{K}_{t-1} + \frac{\phi \alpha}{\psi} \hat{r}_t^k + \phi(1 - \alpha) \hat{L}_t$$

where  $\phi$  is one plus the share of fixed costs in production,  $\alpha$  the capital share in the production function, and  $\psi$  represents the elasticity of the capital utilization cost function.  $\hat{K}_t$  denotes capital and  $\hat{r}_t^k$  its rental rate. Variation in total factor productivity is captured by  $\hat{\varepsilon}_t^A$ .

Labor demand increases with the rental rate of capital and decreases with that of labor:

$$\hat{L}_t = -\hat{w}_t + \left(1 + \frac{1}{\psi}\right) \hat{r}_t^k + \hat{K}_{t-1}$$

Similar to wages, non-reoptimized prices are partially ( $\gamma_p$ ) indexed to past inflation. Due to Calvo-signals, each period only a fraction  $1 - \xi_p$  of firms gets to reoptimize. The resulting

inflation dynamics are captured by the following process:

$$\begin{aligned}\hat{\pi}_t - \bar{\pi}_t &= \frac{\beta}{1 + \beta\gamma_p}(E_t\hat{\pi}_{t+1} - \bar{\pi}_t) + \frac{\gamma_p}{1 + \beta\gamma_p}(\hat{\pi}_{t-1} - \bar{\pi}_t) \\ &+ \frac{1}{1 + \beta\gamma_p} \frac{(1 - \beta\xi_p)(1 - \xi_p)}{\xi_p} \left[ \alpha\hat{r}_t^k + (1 - \alpha)\hat{w}_t - \hat{\varepsilon}_t^A \right] + \eta_t^P\end{aligned}$$

In an environment of price rigidity firms will, in addition to current marginal costs (in square brackets), take into account expected future marginal costs, giving rise to the forward looking inflation term. The backward looking part follows from partial indexation. The term  $\eta_t^P$  represents a price mark-up shock.

As in Christiano, Motto and Rostagno (2003), capital goods producers work in a perfectly competitive environment and face costs to changing the flow of investment. The capital stock evolves according to:

$$\hat{K}_{t+1} = (1 - \tau)\hat{K}_t + \tau\hat{I}_t + \tau\hat{\varepsilon}_t^I$$

where  $\tau$  is the depreciation rate,  $\hat{I}_t$  stands for investment and  $\hat{\varepsilon}_t^I$  represents a shock to the investment technology. Investment dynamics are governed by:

$$\hat{I}_t = \frac{1}{1 + \beta}\hat{I}_{t-1} + \frac{\beta}{1 + \beta}E_t\hat{I}_{t+1} + \frac{1/\varphi}{1 + \beta}(\hat{Q}_t + \hat{\varepsilon}_t^I)$$

where  $\hat{Q}_t$  is the real value of installed capital and  $\varphi$  is the investment adjustment cost parameter.

Entrepreneurs buy the capital stock  $K_{t+1}$  from capital goods producers at a given price  $Q_t$ , using both internal funds (net worth,  $N_{t+1}$ ) and loans from the bank. Subsequently, they transform it using their technology, decide on capital utilization and rent out capital services to intermediate goods firms at a rate  $\hat{r}_t^k$ . The expected real return to capital is given by:

$$E_t\hat{R}_{t+1}^K = \frac{1 - \tau}{\bar{R}^K}E_t\hat{Q}_{t+1} + \frac{\bar{r}^k}{\bar{R}^K}E_t\hat{r}_{t+1}^k - \hat{Q}_t$$

where  $\bar{R}^K$  denotes the steady state return to capital and similarly,  $\bar{r}^k$  the steady state rental rate.

Following the costly state verification framework of Bernanke, Gertler and Gilchrist (1999), however, entrepreneurs cannot borrow at the riskless rate. The cost of external finance differs from the risk-free rate because entrepreneurial output is unobservable from the point of view of the financial intermediary. In order to infer the realized return of the entrepreneur, the bank has to pay a (state verification) cost. The bank monitors those entrepreneurs that default, pays the cost and seizes the remaining funds. In equilibrium, entrepreneurs borrow up to the point where the expected return to capital equals the cost of external finance:

$$E_t \hat{R}_{t+1}^K = -\epsilon E_t [\hat{N}_{t+1} - \hat{Q}_t - \hat{K}_{t+1}] + \hat{R}_t$$

The parameter  $\epsilon$  measures the elasticity of the external finance premium to variations in entrepreneurial financial health. As shown explicitly in Bernanke, Gertler and Gilchrist (1999), the premium over the risk-free rate the financial intermediary demands is a negative function of the amount of collateralized net worth. The higher the entrepreneur's stake in the project, the lower the associated moral hazard. In case entrepreneurs have sufficient net worth to finance the entire capital stock, agency problems vanish, the risk-free rate and the return to capital coincide, and the model reduces to the model of Smets and Wouters (2006)<sup>1</sup>.

Aggregate entrepreneurial net worth accumulates according to:

$$\hat{N}_{t+1} = \gamma \bar{R}^K \left[ \frac{\bar{K}}{\bar{N}} (\hat{R}_t^K - E_{t-1} \hat{R}_t^K) + E_{t-1} \hat{R}_t^K + \hat{N}_t \right]$$

where  $\gamma$  is the entrepreneurial survival rate and  $\frac{\bar{K}}{\bar{N}}$  is the steady state ratio of capital to net worth (or the inverse leverage ratio)<sup>2</sup>.

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<sup>1</sup>One difference with Smets and Wouters (2006) is the absence of an "equity premium shock" in our model. They include this shock as a non-structural proxy for fluctuations in the external finance premium. When we incorporate such a shock in the model with the financial accelerator, its variability is drawn to zero.

<sup>2</sup>We rewrite the model without the bankruptcy cost ( $\mu$ ) and default threshold ( $\bar{\omega}$ ) parameters of Bernanke et al. (1999). There are a couple advantages related to conducting such a substitution. First, it allows one to refrain from assumptions about the distribution of idiosyncratic productivity shocks, as well as its parameters. Second,



The standard goods market equilibrium condition is augmented with terms capturing the costs of variable capital utilization and bankruptcy:

$$\hat{Y}_t = c_y \hat{C}_t + \tau k_y \hat{I}_t + \varepsilon_t^G + \frac{(\bar{R}^K - 1 + \tau)}{\psi k_y} \hat{r}_t^K + k_y (\bar{R}^K - \bar{R}) \left(1 - \frac{\bar{N}}{\bar{K}}\right) (\hat{R}_t^K + \hat{Q}_{t-1} + \hat{K}_t)$$

where  $c_y$  and  $k_y$  denote the steady state ratio of consumption and capital to output, and  $\varepsilon_t^G$  can loosely be interpreted as a government spending shock.

As in Smets and Wouters (2003) the model is closed with the following empirical monetary policy reaction function:

$$\begin{aligned} \hat{R}_t^n &= \rho \hat{R}_{t-1}^n + (1 - \rho) \left\{ \bar{\pi}_t + r_\pi (\hat{\pi}_{t-1} - \bar{\pi}_t) + r_Y (\hat{Y}_t - \hat{Y}_t^p) \right\} \\ &\quad + r_{\Delta\pi} (\hat{\pi}_t - \hat{\pi}_{t-1}) + r_{\Delta Y} (\hat{Y}_t - \hat{Y}_t^p - (\hat{Y}_{t-1} - \hat{Y}_{t-1}^p)) + \eta_t^R \end{aligned}$$

where the central bank output objective  $\hat{Y}_t^p$  is the flexible price, flexible wage, frictionless credit market, equilibrium. The first two terms capture the standard Taylor rule. The terms involving first differences can be seen as the allowance for "speed limit policies", as in Walsh (2003). The reaction function also contains two monetary policy shocks. The first is a temporary interest rate shock  $\eta_t^R$ . The second policy shock,  $\eta_t^\pi$ , captures changes in the authority's inflation target  $\bar{\pi}_t$  ( $= \bar{\pi}_{t-1} + \eta_t^\pi$ ).

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this approach avoids a number of computational difficulties, as in Meier and Müller (2005). Third and more important, it enables us to directly estimate  $\bar{R}^K$ , and thus the external finance premium. Finally, the remaining parameters can be thought of to arise in related frameworks. One particular strand of models we have in mind is that of costly enforcement (e.g. Kiyotaki and Moore, 1997). Although the underlying microeconomic assumptions are entirely different, these models give rise to similar acceleration phenomena.

## 3 Estimation results

### 3.1 Estimation strategy

The log-linearized version of the model is estimated using Bayesian methods. These methods use information from existing microeconomic and calibration evidence on behavioural parameters and update it with new information as captured by the likelihood. While estimation serves to increase the degree of dynamic fit of DSGE models it is not guaranteed to provide insight in the structural parameters of the underlying models. By contrast, purely calibration based approaches are unlikely to provide a good time-series characterization of the data relative to likelihood-based approaches. As stressed by Lubik and Schorfheide (2005), the combination of prior and sample information into a posterior distribution provides a meaningful compromise between calibration and (likelihood-based) estimation.

We use the priors of Smets and Wouters (2006) for the parameters we share with their model<sup>3</sup>. The last three columns of Table 1 present the prior distributions. For a thorough discussion of prior elicitation, identification and estimation methodology, we refer the reader to Smets and Wouters (2003). We discuss the priors on the financial accelerator parameters in more detail. For the steady state premium on external finance ( $\bar{R}^K - R$ ) we use a normal distribution with mean equal to 200 basis points, a value commonly used in calibration exercises (e.g. Bernanke, Gertler and Gilchrist 1999). Its prior standard deviation is set at 80 basis points. In terms of the (quarterly) model, we assume  $\bar{R}^K \sim Normal(1.0149, 0.002)$ <sup>4</sup>. The steady state inverse

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<sup>3</sup>With respect to the shock variances, we divert from the priors of Smets and Wouters (2005). They employ Inverse-Gamma prior distributions. When we estimate the model using their priors, the posterior distribution of one of the shocks' variance is bimodal, with one mode purely driven by the prior. Since most of the shock variances do not have clear economic interpretations, we set truly uninformative priors, by means of the Uniform distribution.

<sup>4</sup>The steady state level of the risk-free interest rate is undisputed throughout current macroeconomic research. Here too, it is calibrated (or given a very strict prior) such that  $R = 4\%$  annually.

leverage ratio  $\frac{\bar{K}}{\bar{N}}$  has a Normal prior with mean 2, the value used in most calibration exercises, and a standard deviation of 0.2. Based on Bernanke, Gertler and Gilchrist (1999), Carlstrom and Fuerst (1997) and others, the prior for the entrepreneurial death rate is  $\gamma \sim \text{Beta}(0.97, 0.02)$ . The elasticity of the external finance premium  $\epsilon$  has a  $\text{Normal}(0.05, 0.02)$  prior distribution. We set its mean at the value commonly used in calibrations, while its standard deviation is such that it encompasses the estimates of Meier and Müller (2006) and Christensen and Dib (2005). We set fairly diffuse priors on the financial accelerator parameters, since we hope the data are very informative in this respect.

We estimate the model on quarterly US data from 1947:1 to 2004:4. The set of observable variables consists of real GDP, consumption, investment, real wages, hours worked, prices and the short-term interest rate ( $Y, C, I, W, L, P, R$ ). These variables constitute the set of observables in Smets and Wouters (2006). Nominal variables are deflated by the GDP-deflator. Aggregate real variables are expressed in per capita terms. All variables (except hours) are linearly detrended.

Posterior simulation is done via a random walk Metropolis-Hastings algorithm on a chain of 250000 draws. We monitor convergence in a variety of ways. In particular, following Bauwens, Lubrano and Richard (2003), we track the standardized CUMSUM statistic and perform an equality in means test between the first and last 30% of posterior draws for each parameter.

### 3.2 Parameter estimates

We present the financial accelerator parameter estimates in Table 1. The estimated steady state rate of return to capital is 1.0139 on a quarterly basis. The posterior simulations reveal that, even though the estimate is not very precise, it differs significantly from the risk-free interest rate (1.0101 quarterly). Converted to a yearly basis, we find a premium for external finance of approximately 150 basis points. Moreover, we estimate  $\epsilon$  to be 6% and significantly different from zero. Starting from steady state, and holding all else equal, a one standard deviation increase in

entrepreneurial net worth results in a reduction of the external finance premium of approximately 70 basis points. The estimated value of the elasticity is very close to that of Meier and Müller (2006). The highest posterior density region on this parameter rejects the point estimate of 9% in Christensen and Dib (2005)<sup>5</sup>.

The estimates of the non-financial parameters are reported in the lower part of Table 1 and in Table 2. Overall, parameters that we share with Smets and Wouters (2006) are fairly similar. The differences between the estimates arise because of differences in sample period and detrending procedure, as well as the inclusion of the financial accelerator. Among the similarities, we find a considerable amount of rigidity in both wages and prices and a significant elasticity of the capital utilization cost function. Although consumption habits are significant, our point estimate is low relative to the one in Smets and Wouters (2006).

Several diagnostics suggest the chain of posterior draws converges. In particular, after a sufficiently long burn-in period, the standardized CUMSUM statistic for all parameters fluctuates around the final estimate with a relative error of below 10%. Moreover, for each parameter, a test between the mean of the first 30% (after burn-in) and last 30% of draws never rejects the hypothesis of equality. This reinforces the evidence in favor of stability of the draws<sup>6</sup>. The algorithm attains an acceptance rate of 28%.

## 4 The Financial Accelerator

Starting in the early nineties, a vast body of research focuses attention to an examination of the relevance of the credit channel in monetary policy transmission. Most of the existing evidence investigates cross-sectional differences in firm investment and financing conditions (see,

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<sup>5</sup>As a robustness check, we change the prior mean of the elasticity to 0.07, in view of Christensen and Dib's (2005) estimate of 9%. In this case too, our point estimate for the elasticity is drawn towards 6%.

<sup>6</sup>Moreover, different initializations of the chain converge to the same stationary distribution.

e.g., Gertler and Gilchrist 1994). While there is an awareness that credit frictions (can) affect firm investment, its economy-wide impact is largely unknown. This void follows from the microeconomic nature of these studies, which precludes a quantitative evaluation of the macroeconomic importance of the broad credit channel. We assess the contribution of financial frictions in two ways. We first measure the model's statistical performance relative to the standard New Keynesian DSGE model. Second, we document the contribution of the financial accelerator to the transmission of shocks.

As a measure of statistical comparative model performance, we compute the marginal density of our model  $p(Y^T|M_1)$ , where  $Y^T$  and  $M_1$  denote the set of observables and the model including the accelerator, respectively. We then compare it with the predictive performance of the model without credit frictions,  $p(Y^T|M_0)$ . The resulting Bayes factor is  $\frac{p(Y^T|M_1)}{p(Y^T|M_0)} = e^{17}$ . This suggests (placing equal prior probability on each model) the model with the financial accelerator performs substantially better in matching the dynamic behaviour of  $(Y, C, I, W, L, P, R)$  relative to the model without the financial accelerator. Neri (2004), Christensen and Dib (2005) and Queijo (2005) also favor model specifications that incorporate financial frictions. Meier and Müller (2006), by contrast, find the financial accelerator to contribute only marginally to describing the effects of monetary policy shocks.

One of the reasons underlying the improved empirical performance of our model relative to that of e.g. Smets and Wouters (2006) is the following. The latter model generates crowding out effects between consumption and investment following preference and investment supply shocks. Greenwood, Hercowitz and Krusell (2000) show that the consumption response to an investment supply shock is a priori uncertain<sup>7</sup>. In the present model, the estimated parameters generate a

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<sup>7</sup>This is due to two opposing effects. On the one hand, following a positive investment supply shock, there is a shift away from consumption to investment, in response to the latter's increased rate of return. On the other hand, increased investment serves to increase production, demand and thus consumption.

positive consumption impulse response function following a positive realisation of  $\varepsilon^I$ . Peersman and Straub (2005) too find, using sign restrictions on VAR's, that such crowding out effects are not evident in the data. The data seem to contain many episodes of comovement between consumption and investment, rather than crowding out effects between the two. Another case in which this is evident is the preference shock,  $\varepsilon^B$ . As in Smets and Wouters (2006) we find that this shock crowds out investment. However, we find a substantially smaller variance of the shock in comparison to their estimate.

Figure 1 plots the response of output to one standard deviation impulses to all shocks in the model, both with and without the financial accelerator. To compute the response barring capital market frictions, we simulate the model under  $\epsilon = 0$  and  $R^K = \frac{1}{\beta}$ .<sup>8</sup> The figures also contain the 90% confidence interval of the difference in responses.

The response of output to both monetary policy shocks exhibits the prototypical acceleration effect, as in the calibration of Bernanke, Gertler and Gilchrist (1999). Qualitatively, the additional effect generated by the financial accelerator implies a significant increase in the potency of monetary policy during the first ten to fifteen quarters following the shock. Quantitatively, the contribution of the financial accelerator to monetary policy transmission amounts to approximately 10% of the total output response<sup>9</sup>.

For the other shocks, the picture is somewhat more complicated. With respect to the three supply shocks, the financial accelerator amplifies the immediate impact of a shock, yet reduces their medium term responses. After a number of periods, the output response to an investment supply shock becomes negative. The reason is that the substantial fall in the price of capital (or

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<sup>8</sup>Conditional on credit frictions being absent, the values of  $\gamma$  and  $\frac{\bar{K}}{N}$  are irrelevant. In this case, they only contribute to the evolution of net worth, which is then immaterial. Moreover, the latter ratio is, by the Modigliani-Miller theorem, indeterminate.

<sup>9</sup>More precisely, the average difference in impulse responses over the first 20 quarters is 9% for inflation objective and 11% for interest rate shocks.

rise in relative efficiency of investment) advances the optimal timing of investment. Moreover, for this shock there is hardly any amplification. This follows from the ensuing rise in the external finance premium, mitigating the investment response. The increase in the premium is due to the reduction in net worth which, in turn, is caused by the fall in  $Q$ . The mild response of investment relative to the model without financial frictions also rationalizes its comovement with consumption. A stronger response of investment to  $\varepsilon^I$  would aggravate substitution effects between investment and consumption.

We observe the mirror effect of the financial accelerator on the output reaction following a government spending shock. Here, the impact effect on output is small relative to the credit-frictionless response, albeit more persistent. Thus, depending on the particular shock under consideration, amplification and persistence can both rise and fall due to the inclusion of the financial accelerator. In the estimated model, however, the presence of financial frictions does not generate additional propagation relative to the nominal and real frictions of Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2006). In terms of Figure 1, the peak response of the model without frictions never predates that of the overall model. Finally, with respect to preference and mark-up shocks the differences in transmission between the two models are negligible. The differences in impulse responses are either statistically insignificant ( $\varepsilon^B$  and  $\eta^P$ ) or economically very small ( $\eta^W$ ).

## 5 The external finance premium

The previous section aimed to provide evidence of financial accelerator effects in macroeconomic data. One of the reasons why macroeconomic evidence on financial frictions is scarce is because one of the central variables of these theories, *viz.* the external finance premium, is unobservable. In the present section, we first estimate the model-consistent premium. As a means of external

validation, we then compare our estimate with a number of observable proxies of the premium. Finally, we interpret movements in the premium in relation to shocks driving the business cycle.

## 5.1 A time series of the premium

Figure 2 plots the external finance premium implied by the model. Shaded areas denote NBER recessions. From the figure, it is evident that almost all of the post-war recessions are preceded by substantial (relative) increases in the premium. The leading character of the premium relative to the business cycle arises naturally in the model: While output responds relatively slow due to real (and nominal) frictions, the premium reacts instantaneously. The premium is low relative to its steady state level during most of the sixties, seventies and eighties. Following this prolonged period of relatively low external financing costs, the premium experiences a steady rise peaking prior to the early nineties recession. After this recession the external finance premium returns towards its steady state level. Starting in the middle nineties, another surge initiates, ending with the early millenium slowdown.

## 5.2 An external validation exercise

It is of interest to know to what extent our estimate relates to other indicators of the external finance premium suggested in the literature. On the one hand, there are a number of readily available series that bear on the premium for external finance. Among these, the most widely used are the prime spread (prime loan rate-federal funds rate) and the corporate bond spread (Baa-Aaa). Gertler and Lown (1999) argue that in the last two decennia, the high-yield bond spread (<Bbb-Aaa) emerges as particularly useful indicator of the external finance premium and financial conditions more generally. On the other hand, using microeconomic data on a sample of US firms Levin, Natalucci and Zakrajšek (2004) provide an estimate of the premium over the most recent business cycle. Figure 3 plots these indicators joint with our estimate of the



premium.

Only the prime and corporate bond spread are available over the entire sample period. Overall, the relation between our estimate and the former two series is rather weak. The correlations amount to  $-37\%$  (corporate) and  $20\%$  (prime). Nevertheless, they share a number of important characteristics. For one, they all rise around the time of a recession. There is, however, a difference in timing, especially with respect to the prime spread, which lags a couple of quarters<sup>10</sup>. Second, the hike in the mid-sixties that was not followed by a recession is observable in all three indicators. Similarly, the substantial decrease in the premium following the 1973-75 recession is also apparent. In the late eighties, with the emergence of a market for below investment grade corporate bonds, an additional indicator comes to the fore. Gertler and Lown (1999) show that the high-yield spread is strongly associated with both general financial conditions and the business cycle (as predicted by the financial accelerator). Along the lines of their arguments, we believe this spread to be a more thorough indicator of the external finance premium, relative to the two proxies discussed above. In particular, the prime loan spread is a poor indication for financing conditions of firms typically deemed vulnerable to financial frictions. It focuses on firms of the highest credit quality, upon which financial constraints impinge the least. The (Baa-Aaa) corporate bond spread accounts for this discrepancy too some extent, by isolating developments specific to firms that have a less solid financial status. Evidently, this argument holds *a fortiori* for the high-yield spread. As shown in Figure 3, our estimate of the external finance premium is closely related to this high-yield spread. Although our estimate misses most of the high frequency movements in the high-yield spread, the longer frequencies have more aligned patterns.

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<sup>10</sup>The lagging character of the prime spread is noticeable over the entire sample. The sluggish response of retail bank interest rates has spurred a vast amount of independent research. Due to the interest rate hikes in the early 70's, the rigidity of loan rates occasionally resulted in negative spreads. Moreover, starting in 1994, the prime spread ceases to be a useful indicator of fluctuations in the external finance premium. From then onwards the prime loan rate is set as the federal funds rate plus 3 percent.

As a rough approximation, our estimate almost envelopes the high-yield spread. The correlation between the two series is 68%. Finally, the graph also contains the premium as estimated by Levin, Natalucci and Zakrajšek (2004). They estimate the premium on the basis of micro data by exploiting the microeconomic friction underlying the model of Bernanke, Gertler and Gilchrist (1999). As in the case of the high-yield spread, its behaviour and relation to our estimate of the premium are very similar. Given the enormous difference in empirical approach this similarity is somewhat surprising, yet comforting.

In conclusion, our estimate of the premium for external finance seems to have a substantial realistic content. It is closely related to readily available proxies of the premium. Using macroeconomic data we establish roughly the same behaviour of the premium as Levin, Natalucci and Zakrajšek (2004), who estimate firm-level premia. Due to the span of the data in the present analysis, however, we are able to generalize these properties over a more comprehensive set of economic cycles. One advantage of our estimate relative to the indicators suggested in the literature is its coverage. By estimating the premium on the basis of macroeconomic data, it should cover the entirety of US firms. By contrast, other indicators typically pertain to a specific subset of firms<sup>11</sup>. Another advantage follows from distilling the premium out of a full-fledged DSGE model. Hence, one can interpret movements in the premium in relation to structural shocks driving the economy, as the next section illustrates.

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<sup>11</sup>Although we do not push this issue any further, this economy-wide coverage might explain a number of observations related to the model. First, by means of the law of large numbers, it is consistent with our estimate of the premium not sharing high-frequency movements observed in indicators for subsets of firms. Second, this wide coverage possibly generates the wide range of the highest posterior density region of the steady state cost of external finance,  $\bar{R}^K$ .

### 5.3 Decomposing the premium

Table 3 and Figures 4 and 5 provide variance and historical decompositions of the external finance premium and GDP. Such decompositions provide insight into the manner in which the model interprets movements of the premium and the business cycle.

First, it seems that investment supply shocks are the primary source of fluctuations in the premium. In the short run investment supply shocks account for about half to two-thirds of the forecast error variance of the premium. At longer horizons, this percentage increases to around 90%. The historical decomposition of the premium in Figure 4 confirms that investment supply shocks are responsible for the bulk of variations in the external finance premium. The graph traces the low frequency component of the premium very closely. Not only for the premium, but also for the business cycle the role of investment supply shocks is substantial. We find that the contribution of these shocks to GDP ranges from a lower bound of 13% (at long horizon) to an upper bound of 34% (immediate). This is somewhat higher than in Smets and Wouters (2006) and is more in line with the findings of Greenwood, Hercowitz and Krusell (2000). They attribute up to 30% of business cycle fluctuations to these shocks. Moreover, the substantial increases in the premium due to  $\varepsilon^I$  in the second half of the sample are consistent with the increased role of technological investment since the mid-seventies (Greenwood and Yorukoglu, 1997).

Second, monetary policy shocks also cause a great deal of movements in the premium. Table 3 shows that the inflation objective and monetary policy shock jointly account for 10 to 35% of the short run fluctuations of the premium. Importantly, the model interprets the early eighties surge in the premium as being largely driven by the Fed's disinflationary policy. The corresponding recession is also attributed to the stance of monetary policy, as is evident from the historical decomposition of GDP. Following the 2001 recession, favorable monetary policy shocks have contributed to the reduction of the external finance premium.

Third, we also find a small, yet significant contribution of preference shocks (4 – 12%) to the short horizon variance decomposition of the premium. Another minor portion (6% on average) of the high frequency movements in the premium is generated by labor supply shocks. Government spending as well as both mark-up shocks have only minor effects on the premium. The price and wage mark-up shocks also have a small effect on output fluctuations. The government spending shock, by contrast, generates most of the short horizon and a substantial part of the long horizon variance of GDP.

Finally, historical contributions can also shed light on the leading indicator properties of the external finance premium. In particular, consider the peaks in the external finance premium during the early fifties and mid-sixties in Figure 2. These peaks did not signal a recession. Historical decompositions can provide insight into these episodes, which would be labelled "false signals" from a forecasting perspective. The surge in the premium in 1950 is driven almost entirely by positive investment supply shocks, as shown by the second peak in its contribution in Figure 4. The increase in the second half of the sixties is mainly the result of increases in total factor productivity. Both these shocks induce a positive correlation between GDP and the external finance premium. The reason is that the borrowing needs of firms ultimately rise. After a productivity shock, for instance, the increase in investment opportunities surmounts the rise in private net worth. While the premium rises consequently, this does not prevail the substantial positive output response, thus creating the false signal. In addition, the favorable business cycle stance during these episodes was supported by positive contributions of government spending and -to a lesser extent- price mark-up shocks. Both these shocks have limited effects on the premium.

## 6 Conclusion

This paper incorporates the financial accelerator of Bernanke, Gertler and Gilchrist (1999) into a medium-scale, empirically able DSGE model of the type described by Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2003). This combined model allows us to address a number of important issues. We first measure the contribution of the financial accelerator to the standard model, in both statistical and economic terms. We find that the marginal likelihood of the standard New Keynesian model increases substantially when a financial accelerator is accounted for. Moreover, the model is consistent with a number of independent observations. These include, the relatively high importance of investment supply shocks in generating business cycles (as in Greenwood, Hercowitz and Krusell 2000), and the comovement of investment and consumption in response to such shocks (as in Peersman and Straub 2005). The model also allows an assessment of the significance and the quantitative importance of the financial accelerator as a transmission mechanism of monetary policy. In particular, posterior simulations of the model suggest that the proportion of monetary transmission due to financial frictions is 10%. While this is a significant contribution, it also makes clear that these effects should not be overstated. The financial accelerator really is a complement to the traditional channels of policy transmission.

A second line of inquiry focuses on the external finance premium. In essence, this premium is unobservable. While there exist a number of observable indicators, each of them is, in a sense, imperfect. We provide a model-consistent estimate of the premium and its fluctuations over the post-WWII era. While this estimate too, has its limitations, a number of interesting implications can be derived from it. Recessions are typically preceded by surges in the premium. From a leading indicator perspective, however, the reverse is not true. The estimated premium, as well as existing indicators, exhibit a number of peaks that were not followed by a recession. One advantage of the present model is that it allows an economic interpretation of such episodes, by

means of historical decompositions.

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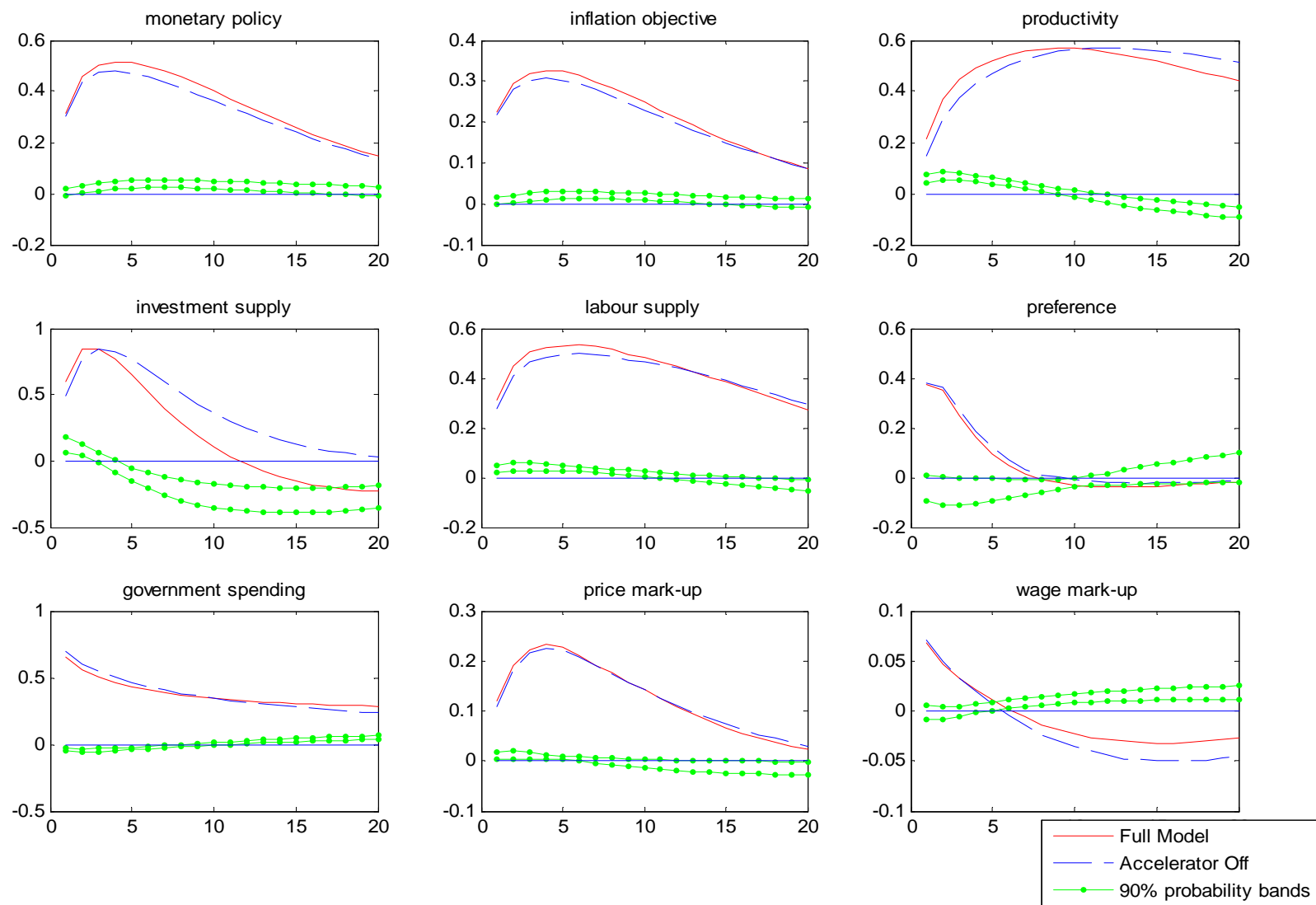
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**Figure 1: Impulse Response Functions for GDP**



Note: Probability bands denote 5 and 95% pointwise draws for the difference in impulse response function between the two models.

Figure 2: The External Finance Premium

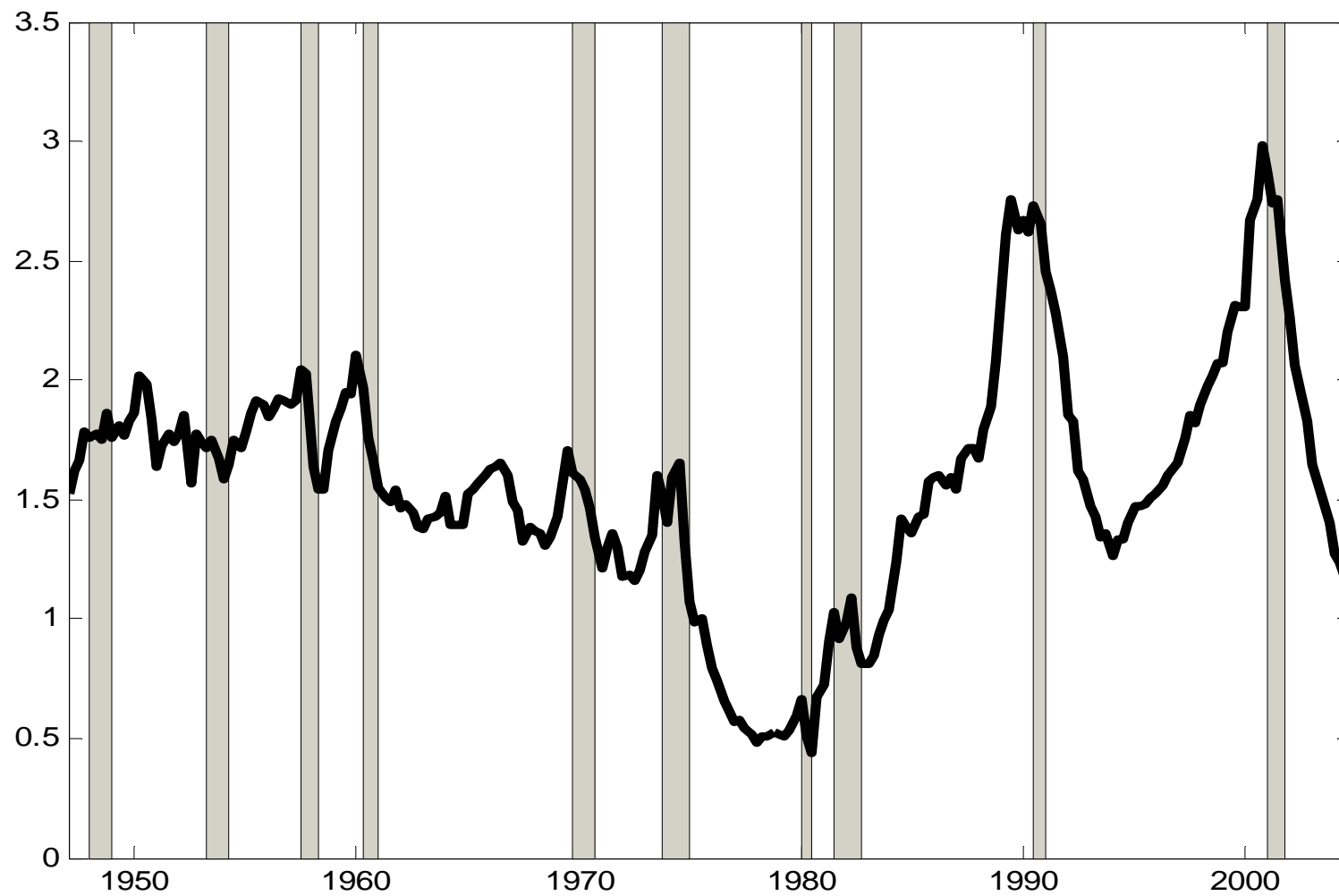
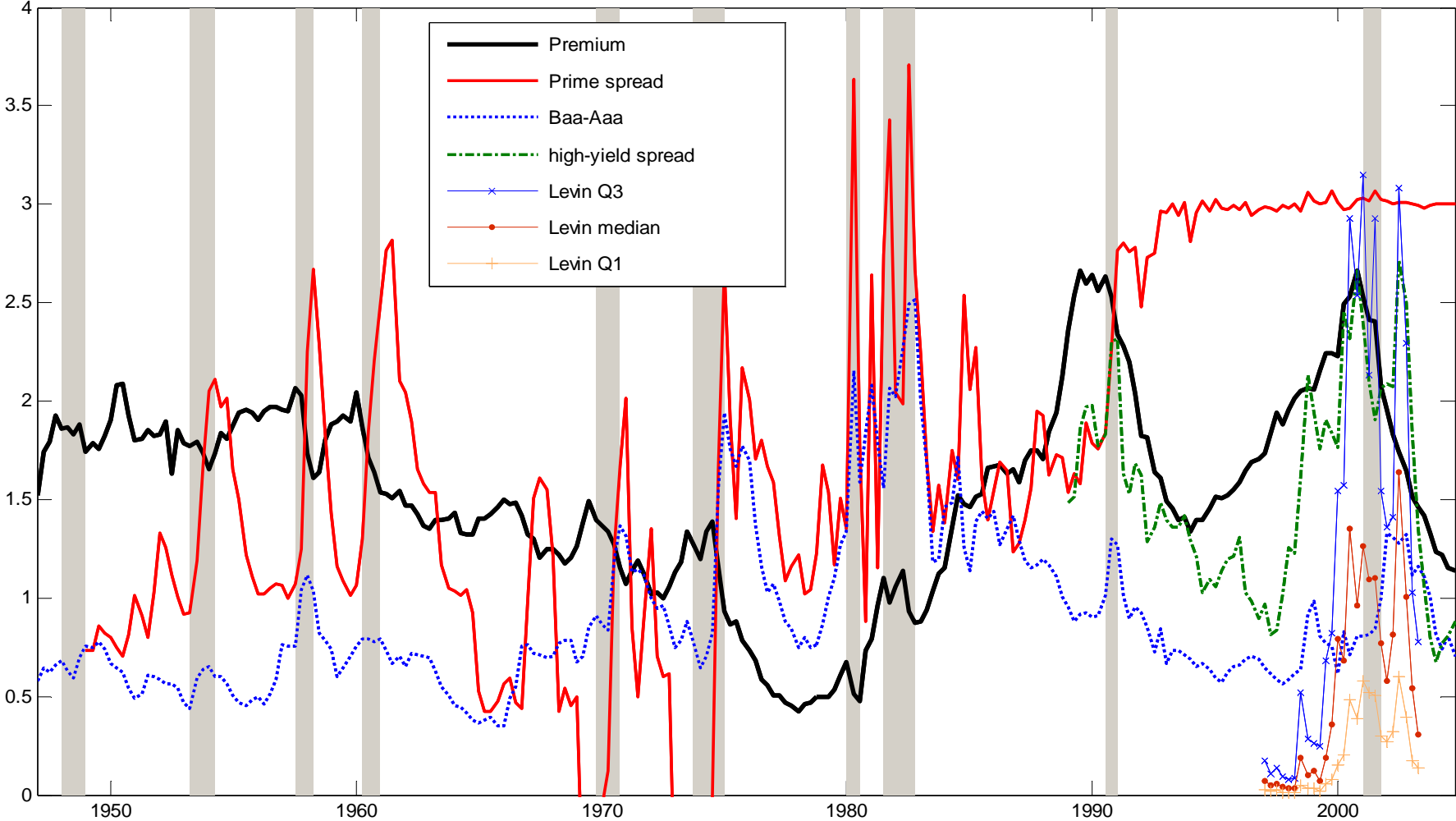


Figure 3: The External Finance Premium: Other indicators



**Figure 4: Historical Contributions to External Finance Premium (90% probability bands)**

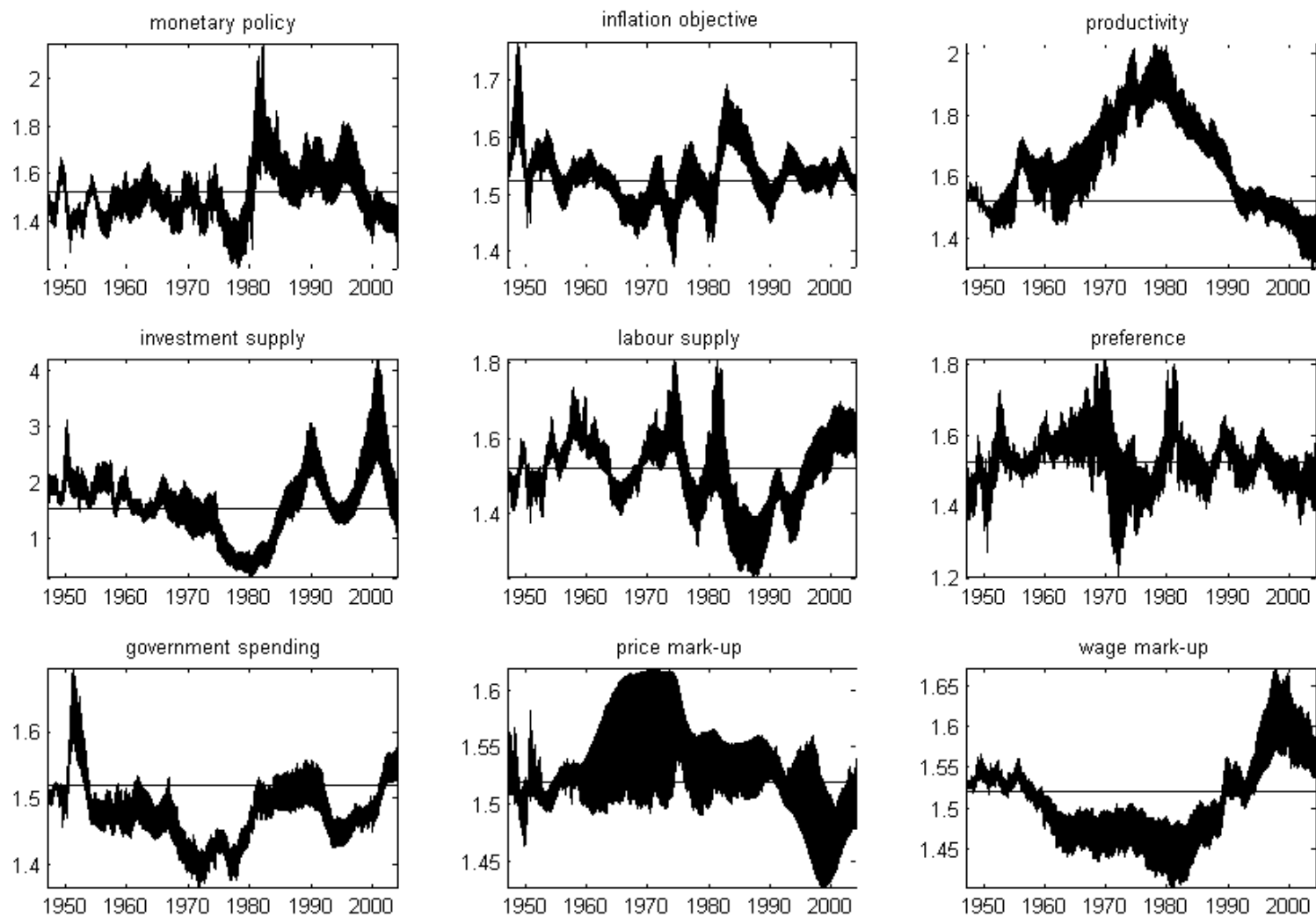
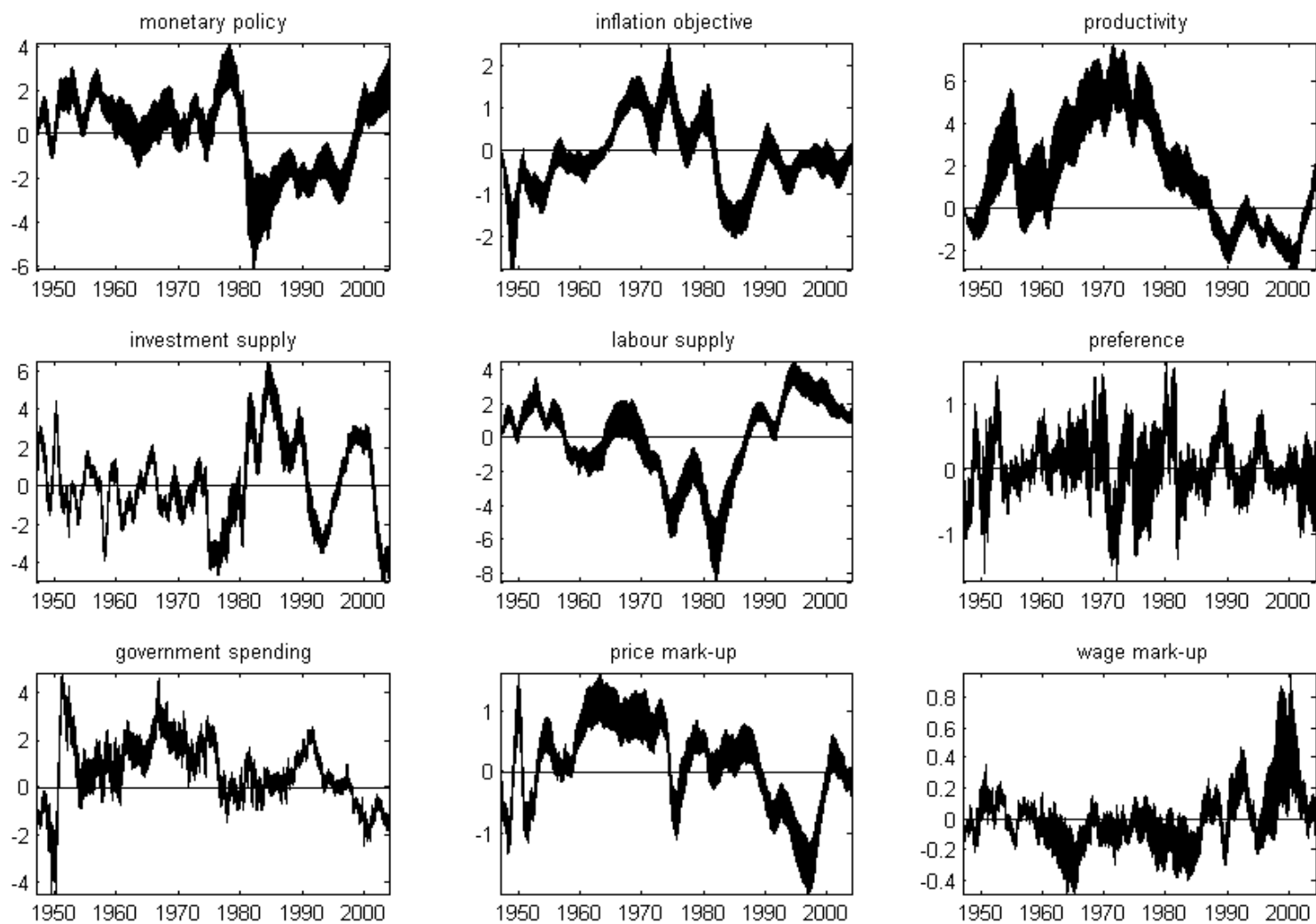


Figure 5: Historical Contributions to GDP (90% probability bands)



**Table 1: Prior and posterior distribution for structural parameters**

		Posterior mode		Posterior sample			Prior distribution		
		Mode	st. err.	5%	50%	95%	Type	Mean	st. err.
$\frac{\bar{K}}{\bar{N}}$	inv. leverage	1.7681	0.1712	1.5707	1.8359	2.0912	Normal	2	0.2
$\gamma$	survival rate	0.9850	0.0086	0.9698	0.9816	0.9942	Beta	0.97	0.02
$\epsilon$	elasticity	0.0567	0.0140	0.0374	0.0604	0.0821	Normal	0.05	0.02
$\bar{R}^K$	capital return	1.0137	0.0019	1.0106	1.0139	1.0171	Normal	1.015	0.002
$\varphi$	investment adj cost	5.24	1.01	3.82	5.54	7.22	Normal	4	1.5
$\sigma$	consumption utility	2.26	0.23	1.85	2.22	2.61	Normal	1	0.375
$h$	consumption habit	0.25	0.04	0.19	0.28	0.36	Beta	0.7	0.1
$\sigma$	labour utility	2.50	0.57	1.74	2.66	3.57	Normal	2	0.75
$\phi$	fixed cost	1.67	0.07	1.56	1.68	1.79	Normal	1.25	0.125
$\psi$	capital util adj cost	0.36	0.10	0.24	0.40	0.57	Gamma	0.2	0.075
$\xi_w$	calvo wages	0.81	0.03	0.77	0.82	0.87	Beta	0.75	0.05
$\xi_p$	calvo prices	0.91	0.01	0.89	0.91	0.93	Beta	0.75	0.05
$\gamma_w$	indexation wages	0.37	0.09	0.21	0.36	0.50	Beta	0.5	0.15
$\gamma_p$	indexation prices	0.39	0.08	0.27	0.41	0.54	Beta	0.5	0.15
$r_\pi$	inflation	1.52	0.10	1.36	1.52	1.68	Normal	1.5	0.1
$r_{\Delta\pi}$	d(inflation)	0.10	0.03	0.06	0.11	0.15	Gamma	0.3	0.1
$r_Y$	output	0.14	0.04	0.09	0.17	0.25	Gamma	0.125	0.05
$r_{\Delta Y}$	d(output)	0.24	0.03	0.19	0.23	0.28	Gamma	0.063	0.05
$\rho$	lagged interest rate	0.91	0.02	0.88	0.91	0.94	Beta	0.75	0.1

**Table 2: Prior and posterior distribution for parameters of shock processes**

Standard deviation		Posterior mode		Posterior sample			Prior distribution		
		Mode	st. err.	5%	50%	95%	Type	LB	UB
$\sigma(\hat{\varepsilon}_t^A)$	$\sigma$ productivity shock	0.49	0.02	0.45	0.49	0.54	Uniform	0	5
$\sigma(\hat{\varepsilon}_t^B)$	$\sigma$ preference shock	0.29	0.11	0.16	0.39	0.62	Uniform	0	5
$\sigma(\varepsilon_t^G)$	$\sigma$ govt. spending shock	0.73	0.03	0.68	0.74	0.79	Uniform	0	5
$\sigma(\hat{\varepsilon}_t^L)$	$\sigma$ labor supply shock	3.14	0.64	1.98	3.17	4.39	Uniform	0	5
$\sigma(\hat{\varepsilon}_t^I)$	$\sigma$ investment shock	0.76	0.06	0.68	0.78	0.88	Uniform	0	5
$\sigma(\eta_t^R)$	$\sigma$ interest rate shock	0.21	0.02	0.17	0.21	0.24	Uniform	0	5
$\sigma(\eta_t^\pi)$	$\sigma$ inflation objective shock	0.11	0.03	0.07	0.12	0.16	Uniform	0	5
$\sigma(\eta_t^p)$	$\sigma$ price mark-up shock	0.29	0.02	0.27	0.30	0.33	Uniform	0	5
$\sigma(\eta_t^w)$	$\sigma$ wage mark-up shock	0.29	0.02	0.27	0.29	0.32	Uniform	0	5
Persistence								Mean	st. err.
$\rho_A$	$\rho$ productivity shock	0.98	0.01	0.97	0.98	0.99	Beta	0.85	0.1
$\rho_B$	$\rho$ preference shock	0.84	0.07	0.66	0.80	0.93	Beta	0.85	0.1
$\rho_G$	$\rho$ govt. spending shock	0.98	0.01	0.96	0.97	0.99	Beta	0.85	0.1
$\rho_L$	$\rho$ labor supply shock	0.94	0.02	0.91	0.94	0.97	Beta	0.85	0.1
$\rho_I$	$\rho$ investment shock	0.63	0.06	0.54	0.63	0.73	Beta	0.85	0.1



**Table 3: Variance decompositions: 5% – 95% bounds**

Shock	Output			Premium		
	$t = 1$	$t = 10$	$t = 20$	$t = 1$	$t = 10$	$t = 20$
$\hat{\varepsilon}_t^A$	0.02 – 0.06	0.11 – 0.24	0.15 – 0.34	0.00 – 0.03	0.00 – 0.01	0.01 – 0.02
$\hat{\varepsilon}_t^B$	0.05 – 0.12	0.01 – 0.04	0.01 – 0.03	0.04 – 0.12	0.01 – 0.02	0.01 – 0.03
$\varepsilon_t^G$	0.31 – 0.42	0.12 – 0.19	0.11 – 0.19	0.00 – 0.01	0.00 – 0.00	0.00 – 0.01
$\hat{\varepsilon}_t^I$	0.23 – 0.34	0.17 – 0.34	0.13 – 0.26	0.47 – 0.72	0.80 – 0.92	0.85 – 0.94
$\hat{\varepsilon}_t^L$	0.05 – 0.15	0.11 – 0.30	0.11 – 0.31	0.02 – 0.11	0.01 – 0.04	0.01 – 0.03
$\eta_t^\pi$	0.02 – 0.06	0.03 – 0.09	0.02 – 0.08	0.02 – 0.09	0.01 – 0.03	0.00 – 0.02
$\eta_t^R$	0.05 – 0.12	0.08 – 0.23	0.07 – 0.22	0.09 – 0.27	0.03 – 0.10	0.02 – 0.07
$\eta_t^P$	0.01 – 0.02	0.01 – 0.05	0.01 – 0.04	0.00 – 0.01	0.00 – 0.00	0.00 – 0.00
$\eta_t^W$	0.00 – 0.01	0.00 – 0.00	0.00 – 0.00	0.00 – 0.01	0.00 – 0.01	0.00 – 0.01