Heterogeneity, Asymmetries and Learning in Inflation Expectation Formation: An Empirical Assessment*

Damjan Pfajfar† University of Cambridge

Emiliano Santoro‡ University of Cambridge

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Abstract

Relying on Michigan Survey' monthly micro data on inflation expectations we try to determine the main features – in terms of sources and degree of heterogeneity - of inflation expectation formation over different phases of the business cycle and for different demographic subgroups. We identify three regions of the overall distribution corresponding to different expectation formation processes, which display a heterogeneous response to main macroeconomic indicators: a static or highly autoregressive (LHS) group, a "nearly" rational group (middle), and a group of "pessimistic" agents (RHS), who overreact to macroeconomic fluctuations. Different learning rules have been applied to the data, in order to test whether agents' are learning and whether their expectations are converging towards rational expectations (perfect foresight). The results obtained by applying conventional and recursive methods confirm our initial conjecture that behaviour of agents in the RHS of distribution is more associated with learning dynamics. We also regard the overall distribution as a mixture of normal distributions. This strategy allows us to get a deeper understanding of the existence and the main features of convergence and learning in the data, as well as to identify the demographic participation in each subcomponent.

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†Email: dp316@cam.ac.uk. Address: Trinity Hall, Trinity Lane, CB2 1TJ Cambridge, UK

‡Email: es356@cam.ac.uk. Address: Clare Hall, Herschell Road, CB3 9AL Cambridge, UK

Introduction

Throughout the history of economic thought expectations formation process has attracted much attention, although few studies have focused on household data evidence from surveys of expectations. Several different models have been proposed in the theoretical literature on expectations starting from models of where the expected future values of variables are set to the level of the last observation. The first explicit analysis of this expectation rule (usually termed naive or static expectations) is due to Ezekiel (1938). The idea of adaptive expectations originates in the work of Fisher (1930) and was formally introduced in the 1950s by several authors, e.g. Nerlove (1958). Nerlove, Grether and Carvalho (1979) first modelled expectations as a time series model of the corresponding variable and termed them as quasi-rational expectations. The concept of rational expectations was first discussed in Muth (1961) and in the 1970s it has been popularised by the work of Lucas and Sargent. Lately, a new view of expectations has emerged, postulating that agents act as econometricians when forecasting. This adaptive learning approach is discussed in Evans and Honkapohja (2001).

As far as the empirical literature is concerned the only contributions have come from the introduction of rationality tests (Pesaran, 1981), and only recently, by an empirical investigation of the degree of heterogeneity (Branch, 2004, 2005) and information stickyness (Mankiw, Reis and Wolfers, 2003). There have been a few studies that support the introduction of heterogeneous expectations in economic models, e.g. in a standard animal economy model (Baak, 1999 and Chavas, 2000) and in New Keynesian macroeconomic model (Pfajfar, 2005). Milani (2005a, b) has advanced some empirical support for macroeconomic models with learning dynamics.[1]

Economists know very little about how agents form their expectations in reality. Recently, it can be said that a consensus has been reached on the view that agents form their expectations heterogeneously, however, little has been done to investigate expectations formation in the empirical literature. The studies by Branch (2004, 2005) and Carroll (2003a,b) are noteworthy exceptions. As not all agents are economists we are focusing our research on household survey of inflation expectations. Using monthly micro data on inflation expectations provided by the University of Michigan Survey Research Center, between January 1978 and February 2005, we are trying to fill this gap in the literature by attempting to determine the sources of heterogeneity and asymmetries of households' inflation expectations.

There are three main sources of heterogeneity that have been proposed in the literature. Agents might make heterogeneous forecasts because they are using different models, they have different information sets and they have different capacities to process information. Branch (2004, 2005) assesses the importance of the first two sources of heterogeneity and finds that data are consistent with both sources of heterogeneity, as the respective models are able to replicate some characteristics of the

[1] Orphanides and Williams (2003, 2004a,b) have first advanced some empirical support for learning dynamics.

empirical distribution. Nevertheless, he concludes that model uncertainty is the more prevailing feature in the data. However, he does not consider models, which combine both sources of heterogeneity. Carroll (2003a,b) focuses on information constraints as a source of heterogeneity and proposes an epidemiological framework to study how households model inflation expectations. He finds that the diffusion process is rather slow, although the gap between household and professional forecasters narrows when inflation matters (lots of media coverage) and households become more attentive. In comparison with previous studies, this paper focuses on all the three possible roots of heterogeneity, including different capacities to process information, as we are able to analyse different responses across demographic groups.

A few studies have pointed out the difference between expectations across different demographic subgroups, but these have not stimulated further studies on the determinants of such heterogeneity. One of the most well known stylised fact in the literature is that female inflation forecasts are less accurate than those of their male counterparts (Jonung, 1981; Byran and Venkatu, 2001a, 2001b). As women are usually responsible for shopping, Jonung (1981) suggests that the difference is due to relatively larger rises in food prices compared to the aggregate CPI. We investigate further on this possible explanation in Section 3. To the best of our knowledge, only two other studies have briefly analysed inflation expectations across different demographic groups. Palmqvist and Strömberg (2004) presented some evidence that inflation opinions in Sweden are lower among male, more-educated and high income respondents. The same conclusion was also pointed out in Lindén (2005) when comparing perceived and expected inflation in the Euro area. Granato, Lo and Wong (2003) found boomerang effect[2] in the inflation diffusion process across different educational groups.[3]

We provide evidence that higher moments are important (contrary to Jonung, 1981) for studying expectation formation and also convergence. We find that the variance of inflation expectations is counter-cyclical, i.e. it increases during recessions and decreases during booms. However, skewness and kurtosis are pro-cyclical, both decreasing in recessions and increasing in expansionary periods. Also in the period of stable inflation the variance is less volatile, while skewness and kurtosis are more volatile. This implies that we can observe some form of convergence lately, while the number of outliers increases.

We report some descriptive statistics and show that expectations significantly differ across demographic groups.[4] Income, sex and education seem to be particularly

[2]If (when) there is misinterpretation in the information aquisition process, less informed agents' forecasts confound those of more informed agents (Granato, Lo, and Wong, 2003).

[3]Dominitz and Manski (2005) have analysed heterogeneity of expectations about equity prices. They present also some evidence about heterogeneity across demographic groups as they found that young agents, males and more educated tend to be more optimistic as their counterparts.

[4]We are neglecting the issue raised in e.g. McGranahan and Paulson (2005) that different demographic groups experience different inflation. In their case especially households with elderly agents experience highest deviation from overall inflation. To consider different inflation rates across de-

important characteristics when forecasting inflation. High income, male and, highly educated agents produce lower mean squared errors when forecasting inflation.

As the panel we employ is highly unbalanced, we compute the percentiles of the empirical distribution, obtaining monthly time series which entail information on the individuals comprised in different parts of the distribution. We perform several tests of rationality, learning, and convergence on the computed percentiles and find that we cannot reject the hypothesis of rationality just for a few percentiles around or slightly above the median. Tests for learning suggest that agents on the right-hand side of the distribution tend to behave in an adaptive manner, whereas agents on the left-hand side of distribution do not exhibit such behaviour. To further investigate on this issue, we estimate several additional time series models of expectation formation. These models confirm a marked degree of heterogeneity and asymmetry in the expectation formation process. The basic result is that agents positioned around the centre of the distribution behave roughly in line with the rational expectations hypothesis. However, our results suggest that agents on the left-hand side of the distribution behave in an autoregressive way. Furthermore, it can be argued that the inflation expectations of these left of centre agents are stable around some focal points and that they simply do not observe movements in any of the relevant macroeconomic variables. In contrast, on the other side of the distribution, agents are generally too pessimistic and usually produce higher inflation expectations than actual inflation. As noted above these right of centre agents' inflation expectations are more consistent with adaptive behaviour (learning), although they vary significantly in the speed of learning. Furthermore, we argue that they exhibit some features pointed out by recent advances in the macroeconomic and financial literature on inattentiveness and rationaly heterogeneous expectations models[5]. We must bear in mind that the cost of being inattentive increases as inflation increases given that agents have greater incentives to inflation forecasts which entail lower systematic errors. Thus, we carefully study the behaviour of agents over different phases of business cycle. We argue that some combination of both models is likely to be the best way of modelling agents in this area of the distribution. More precisely, right of centre agents' inflation expectations could be best explained by two or more autoregressive models (or learning) with different degrees of inattentiveness where agents would switch between different updating speeds according to their relative performance and costs.

To further investigate the degree of heterogeneity and the actual expectations formation processes, we decompose the distribution into a mixture of normal distri-

mographic groups is beyond the scope of the paper as it would cause problems in the second part of the paper.

[5]Inattentiveness - agents upadate their information sets only occasionally - was advanced by Sims (XXXX) and first implemented in macroeconomic model by Mankiw and Reis (2003). The theory of Rationaly Heterogeneous Expectations was put forward by Brock and Hommes (1997). Their basic argument is that it might not always be optimal from utility maximisation point of view to use costly-sophisticated predictor that produce lower mean squared error, thus some agents might be better of with slightly worse predictor which is less costly to use.

butions. We find that the distribution can be decomposed into a number of subcomponents which ranges between three and five Gaussians. For most of the time window observed, the mean of one of these distributions tracks actual inflation fairly well and could be characterised as the mean of the rational subgroup, whereas the others are biased and require a careful study of the underlying expectation formation processes. In most of the periods, we identify one subcomponent at each tail of the overall distribution and often at least one centered around some focal point, such as 0 or 5 percent. Since the "classification" strategy of the subdistributions matters, we analyse several different options.

We also analyse how well different models of expectation formation fit the estimated data by decomposing the distribution by ML. We conduct different exercises á la Branch (2004, 2005). The novelty is that we try different combinations of standard models of expectation formation, such as naive, AR(1), AR(2), VAR and learning, combining them with different information sets. As Branch (2005) has pointed out, the choice between different "degrees" of inattentiveness might produce the best fit. The other novelty would be also to assume different evolutionary process, replicator dynamics instead of Brock and Hommes (1997) multinomial rule. Observed data from preliminary analysis exhibit dynamics that could be explained by considering replicator dynamics style switching between groups. Groups on the left-hand side of the distribution seem to exhibit switching between two focal points.

Our strategy allows to determine the relative shares of each demographic subgroup in each of the estimated subcomponents of the overall distribution, allowing to check how well our estimated data perform in the standard New Keynesian macroeconomic model.

The remainder of this paper is organised as follows: Section 1 reports in more detail the dataset employed, while in Section 2 we deliver some preliminary descriptive statistics. In Section 3 we focus on the percentile time series analysis, with a special attention for learning dynamics. Section 4 is centered around the decomposition of the overall distribution into a mixture of normal subcomponents, while Section 5 is devoted to the analysis of the estimated parameters. In Section 6 we further deepen our knowledge on the expectation formation process and in Section 7 we implement the estimated data into the New Keynesian model. Section 8 concludes and gives some suggestions for further research.

1 The Survey of Consumer Attitudes and Behavior

The Survey of Consumer Attitudes and Behavior, conducted by the Survey Research Center (SRC) at the University of Michigan, is available at a monthly frequency from 1978.01. The survey regards an average of 591 households, with a peak of 1479 in 1978.11 and a minimum of 492 in 1992.11, with an average of approximately 500 respondents after 1987.01. Each respondent is interviewed once and then reinterviewed after six months. The sampling method is designed in a way that in any given month they interview approximately 45% of prior respondents, while the remaining 55% is composed by new households. Two relevant questions concerning inflation expectations are whether households expect prices in general to go up, down or to stay the same in the next 12 months, and to quantify the previous answer. If the answer is that prices will not change the interviewer must make sure that the interviewees does not have in mind increase with the same rate as at the time of the interview.

Although we are aware of the existence of precise quantitative data regarding all of the responds and demographic characteristics of the participants in the survey, the publicly available version of the survey reports data accumulated in groups ("go down", "stay the same or down", go up by 1-2%, 3-4%, 5%, 6-9%, 10-14%, 15+%). There might be some confusion about the category "stay the same or down". Here we follow Curtin (1996) suggestion to treat them as 0%. A word of caution is also in order when describing the coding method when households expect prices to go up: in this case data are available in such a way that 6 ranges of growth are considered, as well as the possibility that the respondent expects prices to increase but it does not know to which extent. In the last case we redistribute the respondents across the six discrete ranges (which predict the price increase), depending on the relative share on the total population which expects prices to increase and that states by which percentage. Furthermore, in every month there is an extremely small proportion of respondents who state that they "don't know" whether prices will increase, decrease or stay the same during the following 12 months or that do not give an answer: Following previous statistical analyses and the technical note provided by the SRC (see Curtin, 1996), we exclude these cases from our sample.

The remaining issue was to determine the points at both ends of the distribution beyond which observations should be truncated. It is important to note that only estimates of the mean and variance of the response distribution are influenced by the exact specification of the truncation rule. Estimates of the median of the distribution are unaffected by how the tails of the distribution are truncated. It is also important to take into account that the upper tail of the distribution is not only long but also sparse, frequently with large gaps between observations: this point will become extremely important in the decomposition of the overall distribution into a mixture of normal distributions, which are aimed at capturing the degree of heterogeneity in the formation of inflation expectations. Technical considerations regarding the cut-off procedure are outlined in Curtin (1996). As a matter of fact, in order to assess the impact of fixed truncation rules, Curtin (1996) suggests two alternatives: truncation at -10% and +50%, and truncation at -5% and +30%. The two alternatives yield nearly identical trend information, as they were correlated at 0.999. Over the entire time period, the difference in the mean estimates was just about 0.2 percentage points, while the variance appears to be constantly higher in the higher truncation range: however, as Curtin (1996) points out, for most analytic uses of these data, the level of observed variance is not as important as the change in the estimated variances over time. Using this criteria, the two alternative estimates yield nearly identical trend information. Neither the adoption of the interquantile range, which appears to be a more appropriate measure of dispersion for highly skewed distributions and to cope with problems associated with the relatively small sample sizes of the monthly surveys, points out major differences. Overall, there seems to be poor evidence supporting the choice of a truncation rule over the other. The means differ only marginally, and neither truncation rule yields desirable estimates of dispersion. Thus in the following analysis we have adopted the smaller truncation range.

An important feature of the data available is their classification for demographic subgroups: respondents in the sample are classified depending on their sex, income level (low 33%, middle 33% and top 33%), educational level (high school or less, some college degree and college degree), age (between 18-34, 35-54, and 55+) and territorial location (east, south, north west and north centre).

2 A preliminary look at the data

In this section we preliminary analyse the available data of the Consumer Survey on Inflation Expectations (CSIE hereafter). For the sake of convenience, in the first subsection we describe the main stylised facts characterising the overall sample, and subsequently we dig deeper in order to highlight major differences within demographic subgroups.

The second subsection will be devoted to the analysis of the cyclical pattern of the moments of the distribution of inflation expectations. In order to take into consideration different inflation regimes, we will pursue a parallel investigation by truncating the period between 1978.01 and 2005.02 in pre and post 1988:12. This choice allows us to take in adequate account the high inflationary period characterising the first part of the sample.

2.1 Descriptive statistics

Table 1 reports the time average of the empirical moments of the CSIE distribution, together with the time average of the actual inflation for the whole period and for different demographic groups.

Insert Table 1 about here

At this stage we can already draw some useful observations that will serve a guidelines in the subsequent analyses. We can immediately notice that the mean of the male, top level income, highly educated and elderly individuals is smaller with respect to the one of their counterparts in the other subgroups as pointed out by Palmqvist and Strömberg (2004) and Lindén (2005) for Sweden and Euro area. As regards another measure of central tendency of the population, the median, we observe that the lowest values corresponds to elderly population and top income level, while the highest value corresponds to young people and to female respondents. The time average of the second empirical moment points out how well educated and high income respondents provide less volatile predictions, especially with respect to bottom level income, less educated and female individuals who form most volatile predictions. As it will be possible to observe in the graphic analysis, skewness and kurtosis are characterised by an high level of covariance: the evidence in this cases is the same as in the case of the variance. However we will have the chance to observe in the next section how their cyclical behavior diverges. Table 2 focuses on comparison between the mean and the median, both in terms of prediction accuracy and as measures of central tendency of the distribution. We report the sum of squared errors (SSE), which is given by the difference between the mean (median) and the actual inflation.

Insert Table 2 about here

It is important to point out that the classification for income levels has started in 1979.10. To increase comparability of the data we have normalised the index in order to take into account the time difference, but we have to point out that in the early period the inflation was higher and so were the errors. Thus the estimates of these groups are still biased downwards. On average, it would seem that the mean is a better predictor than the median, although this contradicts the evidence described in Table 1, where the median is always closer to the average inflation: we argue that the role of outliers is crucial in the explanation of this situation. In order to shed light on this puzzle we argue that it is useful to conduct a separate analysis by splitting the sample in pre and post 1988. This threshold constitutes the end of the cycle and of a marked process of disinflation (see Figure 9), characterised by highly volatile inflation. Figure 6 reports the plot of skewness and kurtosis against the cycle and the actual inflation. It is possible to observe that their value is fairly stable and low during the high inflation period, while it increases and becomes more volatile in the second part of the sample, when inflation is more stable. Higher kurtosis and higher positive skewness suggests an higher number of outliers in the right tail of the overall distribution. It would seem at odds that an higher number of outliers arises when inflation is under control: however, following the argument underlying the mechanism developed by Brock and Hommes (1997), it is possible that the opportunistic cost of being inattentive or relying on a simple forecasting rule (characterised by a lower degree accuracy) is higher when inflation is high and highly volatile than in periods in which inflation is kept under control.

With attention to the demographic analysis, it is evident that respondents in the top income range are more efficient. However, as we have already noted, the income codification has started with a year lag: there are no data available for the period featured by extremely high inflation. From the comparison of the forecast accuracy between and within groups, it is possible to put forward a conjecture on the role of the income level (followed by sex and education) as one of the most relevant demographic characteristics for the inflation forecasting process[6]. The basic lesson we can learn is that for the more biased subgroups, the mean is a more accurate forecast than the median: opposite evidence holds for the less biased groups. This evidence is in line with what we have preliminary observed in terms of dynamic behaviour of skewness and kurtosis.

Insert Table 3 and 4 about here

Tables 3 and 4 report the moments of the empirical moments of the CSIE distribution for the truncated sample. The data confirms a lower level of skewness and kurtosis in the first part of the sample (opposite evidence holds for the second moment). Furthermore, the prediction accuracy, both in terms of the mean and the median, is higher in the second part of the sample. The difference between and within demographic groups pointed out before maintains the same features even by splitting the sample.[7]

We now perform a brief graphic analysis of the variables of interest. It is worth pointing out at this stage that all the series of expectational variables are reported at the realised date. Figure 1 plots the mean and the median against the actual inflation.

Insert Figure 1 about here

It is evident how both constantly underestimate the rise in inflation at the beginning of the sample, although the forecasting performance improves remarkably for the subsequent deflation: this improvements is probably due to the credibility that the FED acquired in lowering inflationary pressures and also as noted before the opportunistic cost of being inattentive in this period is comparatively higher.

As regards the post-1988 subsample, expectations are quite stable, although they almost systematically fail to forecast periods of low inflation. Furthermore, we can observe how expectation fail to account for the marked rise in the price level during the first Gulf War, reacting with a consistent delay. This over reaction is also present after the 9/11, but in this case in the opposite direction.

Insert Figure 2, 3, 4 about here

[6]We do not dig deeper into the analysis of the statistical evidence available in the case of the regional partition, even because it would require to take in adequate account the presence of asymmetric shocks within the country.

[7]To properly asses the importance of inattention we would probably have to compare the relative error. We investigate this further in the next section.

Figure 2 and Figure 3 plot the mean, variance and skewness and kurtosis, respectively: It is trivial to observe how higher inflationary expectations are associated to high volatility. Opposite evidence, as we noted in the previous subsection, holds with respect to skewness and kurtosis.

Figure 4 reports the 25_{th}, the 50_{th} and the 75_{th} percentiles. This graph helps to understand the different variability characterising different parts of the distribution. A further, more analytical study will be performed in the next section, in which we analyse the macroeconomic determinants of the dynamics of each percentile, in order to detect sources of asymmetry in the response of the distribution over the business cycle. At this stage we limit ourselves to a mere graphic analysis. Firstly, the 75_{th} percentile appears to be remarkably stable after 1988 with respect to the two remaining series, although the 50_{th} percentile appears to react less, although with delay, to the inflationary pressures brought by the first Gulf War, probably because the respondents comprised in this range have partially internalised in their expectations that the rise in inflation is not due to last. The 25_{th} percentile, on the other way, reacts less to the 9/11. Interestingly, the 50_{th} percentile reacts most to the 9/11.

Insert Figure 5 about here

Figure 5 reports the plot of mean of the distribution against the actual level of inflation and the mean of the Survey of Professional Forecasters (SPF) on inflation expectations: it is striking how the former, generally more accurate in the second part of the sample, is more biased than the one of the consumers in the highly inflationary period. The two predictions are remarkably similar from 1984 to 1990 and from this point onwards the SPF seems to provide a more accurate prediction.

2.2 Cyclical behavior of the CSIE distribution

In this section we will outline the cyclical features of the empirical moments of the CSIE distribution. As there are no major differences in the dynamic pattern of the moments of the overall distribution with respect to the subgroups' distributions that can be observed from the graphical analysis,[8] the first part of this section will be devoted to the study of the general features. The second part will focus on the description of the major differences within groups. Differences within groups are limited to the level and to the variability of the moments.

Insert Figure 6, 7 about here

Figures 6 and 7 report the higher moments of the distribution against the output gap series and an indicator of the cycle (the HP detrended Industrial Production Index (IPI) and interpolated estimates of Kuttner' (1994) model of multivariate Kalman

[8]This is studied more in depth later when we do the percentile time series analysis.

filtering). It is clear how variance has a counter cyclical behaviour, while skewness and kurtosis are highly procyclical. As pointed out before, the third and the fourth moments display higher variability in the period post-1988. Furthermore, kurtosis exhibits increasing variability in correspondence with peaks in the cycle, which probably reflect uncertainty about the future and hence more unstable tails of the overall distribution.

In appendix A we report a number of graphs analogous to the ones presented in this section for each demographic subgroup. The cyclical behaviour of the empirical moments outlined in this section is confirmed by moving at a higher level of disaggregation. Generally speaking, the visual impression we get from the observation of the plots confirms the results reported in tables 1-4: in terms of mean and median, we can state that male, highly educated, top income level respondents are usually provide more accurate predictions. In terms of variance, their counterparts appear to be much more volatile (see, e.g., the difference between male and female). The analysis of the skewness and the within group comparison point out an interesting situation: the subgroups of male, highly educated, top income level respondents have generally an higher level of skewness, which appears to be much more volatile. These groups are associated with lower mean square errors when forecasting inflation, so we would expect that there are more outliers on the right-hand side of the distribution and thus higher skewness.

As regards the kurtosis, the same observations as drawn in the case of skewness can be pointed out and for the same groups. The dynamics in skewness is strikingly similar to the one chracterising kurtosis: their level points out the existence of a long right tail chracterised by high variability. From this evidence we might conjecture the existence of an highly volatile component on the right end of a hypothetical mixture of distributions, and we will actually investigate further on this. However, bare eye can lead to conclusion that high peaks in variability are not associated to any cyclical phase or any change in the cycle.[9] Thus, we might conclude that the rise of outliers in the subgroups with higher kurtosis and positive skewness could at this stage be considered as erratic and non systematic, while their subgroup counterparts have a lower but more stable level of outliers in the right tail. We investigate this further in the next section.

2.3 Time series analysis of the empirical moments

To further investigate the properties of empirical moments we estimate the following two models:

$$em_{t|t-12} = \theta_0 + \theta_1 \pi_{t-12} + \theta_2 y_{t-12} + \theta_3 i_{t-12} + \theta_4 r_{t-12} + \theta_5 t + \theta_6 em_{t-1|t-13} \quad (1)$$

$$\Delta em_{t|t-12} = \theta_{10} \Delta\pi_{t-12} + \theta_{11} \Delta y_{t-12} + \theta_{12} \Delta i_{t-12} + \theta_{13} \Delta r_{t-12} + \theta_{14} \Delta em_{t-1|t-13} \quad (2)$$

[9] This is studied more in depth later when we do the percentile time series analysis.

y_t denotes a cycle indicator (detrended industrial production index (IPI)), i_t is the real short term interest rate (3 months t-bill), r_t is the long term interest rate (10 years t-bond yield). We estimate the above equations (em) for the inerquantile range, variance, skewness and kurtosis.

Insert Table 5, 6 about here

In this section we briefly discuss the results of the analysis on the macroeconomic drivers of the empirical moments of the overall distribution of inflation expectations. The models employed have been reported by equations (1-2). The set of regressors comprises a constant, a deterministic trend, a cycle indicator, actual inflation and real short and long term interest rates. We immediately confirm the evidence of a negative trend in the variance and of a positive one in skewness and kurtosis, which was clear from a graphical inspection. The interpretation of the estimated coefficients for cycle and inflation at the time of the forecast do not pose any particular problem. In accordance with what observed graphically, we can point out a clear counter-cyclicity of variance, while skewness and kurtosis display a marked pro-cyclical behaviour. On the other side, as we would expect upon theoretical grounds, the sign of the estimated coefficients is inverted in the case of inflation. This evidence is particularly important under the perspective provided by the literature on rational inattention, whereas higher and more volatile inflationary pressures should lead agents to raise the level of attention and accuracy in forecasting, while periods of relatively stable inflation, such as the post-1988 period, imply a lower level of attention. A decreasing number of outliers (i.e. lower kurtosis) as inflation increases might fit within this framework. The evidence on kurtosis, moreover, will be confirmed by the percentile regressions models that will be presented in the next section, as units in the upper end of the distribution seem to be more reactive with respect to inflation dynamics. In order to compare the different impacts of the exogenous regressors under different inflationary regimes we have splitted the sample in pre- and post- December 1988.[10] The empirical exercise shows that coefficients attached to cycle and inflation keep the same sign, although they both decrease in absolute value in the second part of the sample. The interpretation of this evidence can be enriched by adding to the picture the effect brought by interest rate regressors. After 1988 agents probably understand the relevance of the informational content of interest rates, being the short term interest rate the main intermediate target adopted by the FED to fight inflation, while the t-bond yield incorporates a premium for inflation, hence providing an important benchmark for inflation forecasting.

The regressions carried out on the full sample deliver interesting results as regards the informational content of short and long term interest rates. The sign of the estimated coefficients is consistent with the considerations outlined in the previous paragraph. For the variance, t-bill assumes a positive coefficient while t-bond has a

[10] Results are reported in Appendix C.

negative estimated impact, meaning that if the long term yield increases, it probably reflects increased inflation expectations that cause more volatility at a cross sectional dimension, while if the t-bill rate increases, it reflects the will of the central bank to fight inflation strenuously. As regards skewness and kurtosis, the impact of the estimated coefficients is now inverted: higher t-bill rate (which is likely to reflect commitment to fight inflation) leads to an increased number of outliers, especially on the RHS of the distribution, probably because agents, relying on the CB's commitment, have a lower degree of attention. Opposite arguments hold for the estimated coefficient of t-bond yield . In order to check whether a term structure effect is at work, we have also estimated the two models adopting as a regressor the spread between long and short term interest rates. As expected, in the first model (variables in levels), the coefficient is highly significant and negative for the third and the fourth moment, while it positive for the variance. However, the contribution of the change in the horizontal spread to changes in the moments is null. The evidence on the role played by interest rates and the term structure gains more relevance if we consider the post-1988 period, when the new route undertaken by monetary policy has been designed by taking more in consideration the detrimental effects of inflation. In the first part of the sample the estimated coefficients attached to the interest rate variables are always insignificant in the model expressed in first differences for all the moments under scrutiny. On the other hand for the first model they are significant, although lower in absolute value with respect to their counterparts estimated in the first part of the sample.

2.3.1 Evidence on Demographic Subgroups

The evidence carried out for the overall sample is generally confirmed by considering different demographic subgroups.[11] In this case we have considered the effect of interest rate variable just by introducing the horizontal spread in the set of regressors. Both for the first and the second model, the sign of the estimated coefficients and their significance denote the same features outlined in the general case. The most interesting result comes from the analysis of the degree of heterogeneity characterising the response of the moments of the various sub-distributions to macroeconomic determinants. The first result is that both models have a better fit (in terms of R^2) in the case of the variance, and a lower one for skewness and kurtosis, if we look at the outcome for male, highest income level, highest education and highest age range, with respect to their counterparts. As regards the first model (in levels) applied to the variance, we can notice how for these classes constant and trend have a lower impact, while the estimated coefficients are higher in absolute value for the remaining regressors. As regards skewness and kurtosis, all regressors seem to have a higher impact, with exception for the trend and the horizontal spread for the skewness.

In the second model (in differences), these results are confirmed for skewness

[11] Results are reported in Appendix C.

and kurtosis. For the variance, apart from the autoregressive component, all the remaining explanatory variables have lower impact for male, highest income level, highest education and highest age range with respect to their counterparts. In all the cases the term structure seems not to have any importance in this model.

3 Percentile Time Series Analysis

3.1 Rational Expectations, Learning Dynamics and Convergence

3.1.1 Asymmetries in the Expectation Formation Process

In this section we perform a quantile time series analysis. The aim is to move a first step towards the detection of heterogeneity in the response of different regions of the CSIE distribution with respect to macroeconomic variables which are relevant to the rational process of expectation formation (such as output gap, actual inflation, short and long term interest rates and the horizontal spread between them, which is aimed at capturing some term structure effect). Furthermore, we introduce in the set of regressors the mean of the distribution determined by the Survey of Professional Forecasters (SPF hereafter), currently conducted by the FED of Philadelphia[12]. This choice is motivated by the need to observe whether a diffusion process is at work: such a mechanism is likely to have an asymmetric effect on different households. Carroll (2003a,b) designs an epidemiological framework to model how respondents to the Michigan Survey actually form their expectations. For this purpose, he models the evolution of inflationary expectations relying on the assumption that households update their information set from news reports, which at the same time are strongly influenced by the expectations of professional forecasters. As Pesaran and Weale (2005) point out, the diffusion process is, however, slow due to inattentiveness of the households.[13]

The choice of the percentile time series, apart from being a useful device to capture asymmetric responses in the distribution of inflationary expectations, is also driven by a more practical consideration: the panel retrievable from the survey is highly unbalanced, as the households interviewed change over time. Thus we extract a set of time series that can be used to capture the evolution of the cross-sectional distribution

[12] From 1968 to 1990 NBER and ASA were responsible for the conduction of the survey. Before the 1981 the data exists only for GDP deflator forecasts. So we use that data for the first few years of our sample.

[13] Thus as Pesaran and Weale (2005) point out, even if the expectations of professional forecasters are rational the expectations of households will only slowly adapt. Carroll (2003a,b) finds that the Michigan Survey has a mean square error on average almost twice that of the SPF. He finds that the SPF inflation expectations Granger-cause household inflation expectations but household expectations do not seem to Granger-cause professional forecasts.

over the cycle. Here is a brief explanation of the technique employed. Regard the expected change in price level during the following 12 months as a random variable, denoted by π^e, which is distributed with respect to some continuous distribution, $F(\cdot)$. The p^{th} quantile of the distribution, denoted by π^e_p, is the value below which $(100 * p)\%$ of the distribution lies, hence $F(\pi^e_p) = p$. Thus, we can compute a set of ordered statistics for each month, obtaining a $99(= p)$ time series of percentiles. Of course, the number of observation in the cross-section varies over time. This method is a convenient way to build up a balanced panel of quantiles, after fixing p.

Given our sample sizes, at each cut-off, we can be confident that the estimated quantiles are good estimates of population quantiles: for any two sample ordered statistics $\pi^e_{t,k}$ and $\pi^e_{t,k+h}$, the amount of probability in the population distribution contained in the interval $(\pi^e_{t,k}\ ,\ \pi^e_{t,k+h})$ is a random variable, which does not depend on $F(\cdot)$. Relying on these considerations, for each time period the cross sectional sample is classified in percentiles, thus obtaining 99 time series of percentiles. Percentiles have been obtained by interpolating the distribution obtained after applying the redistribution and the truncation methods outlined in the previous section. Interpolation is a convenient way for obtaining the percentiles at this stage, as the survey reports the percentage of respondents in each range of price movement, hence constituting already a sort of ordered statistic. In section 4 we recover a population of respondents through a different interpolation method: the resulting percentiles display a 0.999 correlation with the counterparts used in this section. Furthermore, in order to perform some robustness analysis, different interpolation methods have been applied (such as linear and cubic), which do not yield to major differences.

Given these premises, we specify the following percentile regression:

$$\begin{aligned} \pi^k_{t|t-12} &= \alpha_0 + \alpha_1\pi_{t-12} + \alpha_2\pi_{t-14} + \alpha_3\pi_{t-24} + \alpha_4\pi_{t-30} + \alpha_5 y_{t-12} + \alpha_6 y_{t-14} + \\ &\quad \alpha_7 y_{t-24} + \alpha_8 y_{t-30} + \alpha_9 i_{t-24} + \alpha_{10} r_{t-24} + \alpha_{11}\pi^k_{t-1|t-13} + \alpha_{12}\pi^F_{t|t-12} + \varepsilon_t \\ k &= 1, ..., 99 \end{aligned} \tag{3}$$

We denote with $\pi^k_{t|t-12}$ the k^{th} percentile of the 12 months ahead expected change in prices, while $\pi^F_{t|t-12}$ denotes the mean of the 12 months ahead expected change in prices derived from the Survey of Professional Forecasters.

In order to capture the determinants of monthly changes in inflation expectations, the following percentile time series regression have been specified:

$$\begin{aligned} \pi^k_{t|t-12} - \pi^k_{t-13|t-25} &= \beta_0 + \beta_1\left(\pi_{t-13} - \pi^k_{t-13|t-25}\right) + \beta_2\left(\pi_{t-14} - \pi^k_{t-14|t-26}\right) + \\ &\quad \beta_4\left(\pi^k_{t-1|t-13} - \pi^k_{t-13|t-25}\right) + \beta_5\left(\pi^k_{t-2|t-14} - \pi^k_{t-13|t-25}\right) + \\ &\quad \beta_6\left(y_t - y_{t-13}\right) + \beta_7\left(i_t - i_{t-13}\right) + \beta_8\left(r_t - r_{t-13}\right) + \\ &\quad \beta_9\left[(i_{t-1} - r_{t-1}) - (i_{t-13} - r_{t-13})\right] + \beta_{10}(\pi^F_{t|t-12} - \pi^F_{t-13|t-25}) + \varepsilon_t \\ k &= 1, ..., 99 \end{aligned} \tag{4}$$

where coefficients β_1 and β_2 describe the effect determined by lagged forecast errors for the k^{th} percentile, β_4 and β_5 capture the degree of persistence, β_6, β_7 (β_8) capture the effect of changes in the output gap and in the short (long) term interest rate respectively, while β_9 is aimed at capturing the effect of flips in the yield curve. Finally, β_{10} captures the extent to which non-professional forecasters revise their expectation, when also professional forecasters revise their expectation.

To investigate more in depth the nature of the forecast error we estimate the following relations: evidence of serial correlation in the forecast error process indicates that there is an inefficient exploitation of information from last year's forecast in generating current year's forecast, hence violating the rationality hypothesis. Furthermore, in order to capture the possibility of an efficient exploitation of relevant information, we include in the set of regressors the forecast error of the professional forecasters:

$$\begin{aligned} \pi_t - \pi^k_{t|t-12} &= \gamma_0 + \gamma_1 \left(\pi_{t-13} - \pi^k_{t-13|t-25}\right) + \gamma_2 \left[(i_t - r_t) - (i_{t-13} - r_{t-13})\right] + \\ &\quad \gamma_3 \left(y_t - y_{t-13}\right) + \gamma_4 (\pi_t - \pi^F_{t|t-12}) + \gamma_5 \left(\pi_t - \pi_{t-13}\right) + \varepsilon_t \\ k &= 1, ..., 99 \end{aligned} \tag{5}$$

3.1.2 Testing for Rationality

We now apply some tests of rationality commonly employed in the literature[14]. The rational expectations hypothesis can be interestingly applied to survey expectations data, as these allow to determine different degrees of forecast efficiency: the latter has to be intended as the result of a forecasting procedure that does not yield to predictable errors. The simplest test of efficiency is a test of bias[15]. It is possible, by regressing the expectation error on a constant, to verify whether inflation expectations are centred around the right value

$$\pi_t - \pi^k_{t|t-12} = \alpha \tag{6}$$

where π_t is inflation at time t and $\pi^k_{t|t-12}$ is k^{th} percentile from the survey inflation expectations. The following regression represents a convenient test for rationality

$$\pi_t = a + b\pi^k_{t|t-12} \tag{7}$$

where rationality implies that $a = 0$ and $b = 1$, jointly. The last expression can be simply augmented to test whether information in a forecast is fully exploited

$$\pi_t - \pi^k_{t|t-12} = a + (b-1)\,\pi^k_{t|t-12} \tag{8}$$

[14]See Pesaran (1989), Mankiw, Reis and Wolfers (2004) and Bakhshi and Yates (1998) for a review of these tests.

[15]See, for an application, Jonung and Laidler (1988) and Mankiw, Reis, and Wolfers (2003).

Testing remains the same as in the previous regression: under the null of rationality these regressions are meant to have no predictive power[16].

3.1.3 Estimating Simple Learning Rules

Different learning rules will be implemented for the Michigan Survey data, in order to test whether agents' expectations are converging towards rational expectations (perfect foresight). For a discussion on different learning rules and convergence to rational expectations see Evans and Honkapohja (2001). Learning will be first tested in a model with constant gain learning, where convergence to rational expectations is not generally observed. The model below is equivalent to the adaptive expectations formula

$$\pi^k_{t|t-12} = \pi^k_{t-13|t-25} + \vartheta \left(\pi_{t-13} - \pi^k_{t-13|t-25} \right) \tag{9}$$

where ϑ is the constant gain parameter. Under this learning rule agents revise their expectations according to the error of the last realised forecast. Since in the survey of inflation expectations agents are asked to forecast inflation in the next year time (hence they make their forecast at time $t-12$), the revision will regard the previous period's forecast (at time $t-13$), which was made at time $t-25$.

Below we represent a learning mechanism with decreasing gain parameter

$$\pi^k_{t|t-12} = \pi^k_{t-13|t-25} + \frac{\iota}{\varsigma t^{\varkappa}} \left(\pi_{t-13} - \pi^k_{t-13|t-25} \right) \tag{10}$$

The empirical approach will consist in estimating ϑ and ι. ς is arbitrary small number and $\varkappa$ is the coefficient that controls the dampening of the learning gain. If we want that the learning always converges to the rational expectations $\varkappa \leq 1$. If the estimated parameters will be significantly different from 0, then we could conclude that agents are actually learning from their past mistakes.

3.1.4 Recursive Representation of Simple Learning Rules

The above specification is mainly devoted to test whether data support the adaptive behaviour of agents. Since in the adaptive learning literature we are assuming that agents behave like econometricians, and they use all the available information at the time of the forecast, we have to specify a recursive model for the two different learning rules mentioned above. We will assume that agents' perceived law of motion (PLM) will be a simple AR(1) process

$$\pi^s_{t|t-12} = \phi_{0,t} + \phi_{1,t} \pi_{t-13} + \varepsilon_t \tag{11}$$

[16] Alternative test for rationality could take into account that inflation and inflation expectations data are I(1): in this situation the rational expectations hypothesis suggests that they cointegrate, i.e. that expectations errors are stationary, and that the cointegrating vector has no constant terms, as well as coefficients on expected and actual inflation which are equal in absolute value (Bakhshi and Yates, 1998).

When agents are estimating their PLM they exploit all the available information up to period t. As new data become available they update their estimates according to a constant gain learning or a decreasing gain learning rule. First we specify stochastic gradient learning with constant or decreasing gain and then we focus on least square learning. Let X_t and $\widehat{\phi}_t$ be the following vectors: $X_t = \begin{pmatrix} 1 & \pi_{t-13} \end{pmatrix}$ and $\widehat{\phi}_t = \begin{pmatrix} \phi_{0,t} & \phi_{1,t} \end{pmatrix}'$. In stochastic gradient learning agents update coefficients according to the following rule:

$$\widehat{\phi}_t = \widehat{\phi}_{t-1} + \vartheta X_t' \left(\pi_t - X_t \widehat{\phi}_{t-1} \right) \tag{12}$$

In the updating algorithm for decreasing gain learning we replace ϑ with $\frac{\iota}{\varsigma t^{\varkappa}}$.

$$\widehat{\phi}_t = \widehat{\phi}_{t-1} + \frac{\iota}{\varsigma t^{\varkappa}} X_t' \left(\pi_t - X_t \widehat{\phi}_{t-1} \right) \tag{13}$$

In least square learning agents take into account also the matrix of second moments of X_t, R_t. In the case of constant gain they update their coefficients in the following way:

$$\widehat{\phi}_t = \widehat{\phi}_{t-1} + \vartheta R_{t-1}^{-1} X_t' \left(\pi_t - X_t \widehat{\phi}_{t-1} \right) \tag{14}$$

$$R_t = R_{t-1} + \vartheta \left(X_{t-1} X_{t-1}' - R_{t-1} \right) \tag{15}$$

As before, in the updating algorithm for decreasing gain learning we replace ϑ with $\frac{\iota}{\varsigma t^{\varkappa}}$.

$$\widehat{\phi}_t = \widehat{\phi}_{t-1} + \frac{\iota}{\varsigma t^{\varkappa}} R_{t-1}^{-1} X_t' \left(\pi_t - X_t \widehat{\phi}_{t-1} \right) \tag{16}$$

$$R_t = R_{t-1} + \frac{\iota}{t^{\varkappa}} \left(X_{t-1} X_{t-1}' - R_{t-1} \right) \tag{17}$$

The empirical approach will consist in finding ϑ and ι and $\varkappa$ that minimise the sum of squared errors (SSE), i.e. $\left(\pi^s_{t|t-12} - \pi^k_{t|t-12} \right)^2$.

3.1.5 Testing for convergence: Weighted least squares learning and the Kalman filter

In this section the coefficients in the PLM are updated through the following algorithm

$$\widehat{\phi}_t = \widehat{\phi}_{t-1} + \frac{\alpha_t}{t} R_{t-1}^{-1} X_t' \left(\pi_t - X_t \widehat{\phi}_{t-1} \right) \tag{18}$$

where $R_t = \frac{1}{t} \sum_{\tau=1}^{t} \alpha_\tau X_\tau X_\tau'$ and α_τ is a sequence of positive numbers. This formula is a version of weighted least squares, which also corresponds to recursive least squares if $\alpha_\tau = 1$. This updating procedure can be implemented into the Kalman filter formulae. Let $P_t = \frac{1}{t}R_t^{-1}$ and $f_t = X_t P_{t-1} X_t' + \frac{1}{\alpha_t}$.

We end up with the following equation

$$\widehat{\phi}_t = \widehat{\phi}_{t-1} + P_{t-1} X_t' f_t^{-1} \left(\pi_t - X_t \widehat{\phi}_{t-1} \right)$$

$$P_t = P_{t-1} - P_{t-1} X_t' X_\tau P_{t-1} f_t^{-1}$$

which corresponds to the state-space model

$$\pi^s_{t|t-12} = \phi_{0,t} + \phi_{1,t} \pi_{t-13} + e_t$$

$$\forall i \quad \phi_{i,t} = \phi_{i,t-1} + \eta_{i,t}$$

with hyper-parameters given by

$$Var\left(e_t\right) = \frac{1}{\alpha_t} \tag{19}$$

$$Var\left(\eta_t\right) = 0 \tag{20}$$

As the least square estimation assumes that the coefficients are stable, while their estimated counterparts are time varying, the learning process is not optimal. The results in Marcet and Sargent (1989) on the convergence of the learning process towards rational expectations hold only when the law of motions of parameters are viewed as invariant. Hence, if $Var\left(\eta_t\right) \neq 0$ then P_t does not converge towards 0, and consequently the learning does not converge to rational expectations. The coefficients in a more general state-space setting would be derived as follows

$$\widehat{\phi}_t = \widehat{\phi}_{t-1} + P_{t-1} X_t' f_t^{-1} \left(\pi_t - X_t \widehat{\phi}_{t-1} \right) \tag{21}$$

$$P_t = P_{t-1} + Q_t - P_{t-1} X_t' X_\tau P_{t-1} f_t^{-1} \tag{22}$$

where $f_t = X_t P_{t-1} X_t' + H_t$, $Var\left(e_t\right) = H_t$ and $Var\left(\eta_t\right) = Q_t$. Therefore, the expectations of bounded rational agents are computed as the prediction of π_t^s

$$\pi^s_{t|t-12} = \phi_{1,t|t-1} + \phi_{2,t|t-1} \pi_{t-13} \tag{23}$$

Note that Kalman filter delivers the optimal gain that agents apply when updating their parameters. It also allows to test whether the learning is perpetual or whether is converging to rational expectations. Practically, the procedure implies a test of significance of the variance of the state variables.

3.2 Results: Overall Sample

As already mentioned, we perform the estimation of the models outlined in the previous section for each percentile. Our goal is to establish some stylised facts about sources of asymmetries in the response to movements in macroeconomic variables useful for the process of expectation formation, degree of biasness, and the learning dynamics. In this section we present the results obtained. We have estimated several of the model outlined above also for the subsamples of our period of analysis but we did not observe mayor differences, so we are reporting the results mainly for the whole sample and just outlining some interesting results for the subsamples.

3.2.1 Rationality tests

We start percentile time series analysis with rationality tests. When running regressions on equation (6) we can observe that only agents between 51-55th (52-54th) percentile are not biased at 1% significance (5% significance). When splitting the sample to pre-1988 and post-1988, we found that for pre-1988 sample agents between 55-63th (56-62th) percentile are not biased at 1% significance (5% significance). For 1999-2005 period we found that agent between 47-50th (48-50th) percentile are not biased at 1% significance (5% significance). Estimating equation (8) and computing the F-test we find that it is always possible to reject the null hypothesis (rationality) that the first coefficient (a) is 0 and the second (b) is 1 for the whole sample and two subsamples.

3.2.2 Learning

The next test are the tests for simple learning rules or in the first case the degree of adaptiveness of inflation expectations. First we estimate equation (9) and then (10). In the latter we set $\varsigma = \varkappa = 1$.[17]

Insert Figures 8-11 about here

The estimates suggest that agents in the upper part of the distribution are behaving at least partly in an adaptive way while for the agents comprised in the poor hand of the distribution the past error has little or no explanatory power. As regards the estimated constant gain and the overall R^2, we can observe the hump-shaped response between 40th and 99th percentile which peaks around 75th percentile, i.e. in the range in which that percentile past errors have the highest explanatory power. Below we will generalise this regression by including also other possible explanatory variables.

The decreasing gain learning estimates are confirming that agents between 40th and 95th percentile are behaving partly in an adaptive way. Indeed, the decreasing

[17] Results are oulined below in the Table 8.

gain estimates are suggesting that this method of learning is more in line with the bahaviour of agents in the upper part of distribution. As noticed before, also this method of learning has no explanatory power for agents comprised in the left-hand side of the distribution. Also in this case we observe a hump-shaped response, although the adjusted R^2 peaks around 0.75, compared to a value of about 0.35 obtained in the case of constant gain learning. A higher explanatory power of the decreasing gain learning might be due to high inflationary period at the beginning of our sample.

We have also estimated the constant gain learning and decreasing gain learning by means of recursive techniques, determining the optimal gains with the minimum SSE. We set $\iota = 1$. The problem when estimating recursively learning is how to set initial values. This problem is extensively discussed in Carceles-Poveda and Giannitsarou (2005). Stricktly speaking this problem should not occur in our case since we are just trying to replicate our time-series as close as possible. Thus in the case of gradient learning we designed this exercise so that we are searching for the best combinations of gain and initial values to match as closely as possible each percentile.[18] Preliminary results are suggesting that agents starts learning above the 55^{th} percentile, where the gain almost immediately jumps to the highest value. Afterwards the gain starts slowly decaying and converging to zero.[this results are very preliminary]

Next we focus on least squares learning. To simplify the exercise (for this preliminary version) we decided to approximate initial conditions by $\begin{bmatrix} 0.01 + \frac{k}{100} & 0.5 + \frac{k}{100} \end{bmatrix}$ and set variance-covariance matrix to be constant across the percentiles.

Insert Figures 12-15 about here

The results confirm our initial conjecture that behaviour of agents in the RHS of distribution is more associated with learning dynamics as specified above. The "optimal" gain in CGL was estimated between 0 and 0.045. Overall we can say that decreasing gain learning slightly better replicates the bahaviour of agents. It would be interesting to proceed with analysing learning behaviour where agents could endogenously switch between both algorithms (especially after structural breaks). This way of learning would probably result in even better fit of the data.

[To be added more]

3.2.3 Perpetual Learning vs. Convergence

[To be added]

3.2.4 Percentile Regressions

The evidence arising from the percentile time series regressions generally confirm the presence of a marked degree of heterogeneity in the process of expectation formation. Since we get very similar results by splitting the sample we report just the results for

[18] This approach has however an obvious practical drawback as it is computationally very intensive.

the whole sample. Relying on a visual impression obtained from the models (3)−(5), we can identify (at least) three intervals of marginal response of the dependent variable to the regressors introduced in the estimation. This evidence might be due to the existence of different models of expectation formation for the individuals comprised in the overall distribution. In the next section we will investigate further on this issue, regarding the CSIE distribution as a mixture of Gaussian distributions, each of these corresponding to a different sub-group characterised by a peculiar model of expectation formation.

On empirical grounds, we can roughly consider the first interval, the one at the poor hand of the distribution, as the one characterised by agents that do not observe (or do not take into account) the relevant variables for producing one-year-ahead inflation expectations. On the other way, individuals comprised in the interval corresponding to the upper tail, although observing the relevant information, seem to overreact to movements in the regressors, denoting a high degree of "pessimism". Intuitively, the middle range of response should comprise rational individuals.

Insert Table 7 about here

We now describe the main stylised facts arising from the estimation of each model. As already mentioned the model (3) aims at characterising the relevance of the determinants of the one-year-ahead inflation expectations. We introduce in the set of exogenous regressors the following contemporaneous (hence at the time in which expectations are formed) and lagged variables: rate of inflation, cycle (measured as the HP detrended IPI), short term interest rate (3m treasury bill), long term interest rate (10y bond yield) and the one-year-ahead expectation taken from the Survey of Professional Forecasters. It turns out that just some of the mentioned regressors can actually account for the movements in the dependent variable and contextually have a clear cut interpretation. Thus we are setting in our model $\alpha_2 = ... = \alpha_4 = 0$ and $\alpha_6 = ... = \alpha_9 = 0$ as they are almost always insignificant. We can mention, among these, the contemporaneous rate of inflation and the autoregressive term and to some degree as well the forecasts of the SPF. As far as the remaining regressors are concerned, on empirical grounds we can argue that these variables are generally either not observed or not taken into account for the determination of the expectation at each range of the CSIE distribution. Only the contemporaneous cycle is slightly significant. The resulting response functions have been plotted in figures XX in the Appendix B. Furthermore, figure 8 reports the total R^2 for each regression as well as the contribution of each regressor to the explanation of the variation of a dependent variable (Sherrer (1984))[19]. This statistic provides important information on the dif-

[19] As it is well known, the coefficient of multiple determination measures the proportion of the variance of a dependent variable y explained by a set of explanatory variables. It can be computed as $R^2 = \sum_{j=1}^{k} a_j r_{yx_j}$, where a_j is the standardized regression coefficient of the j^{th} explanatory variable and r_{yx_j} is the simple correlation coefficient (Pearson's r) between y and x_j. Scherrer defines $a_j r_{yx_j}$ as the contribution of the j^{th} variable to the explanation of the variance of y.

ferent information structure underlying the mechanism of expectation formation for the individuals comprised in different ranges of the distribution.

Insert Figure 16 about here

As it is clear from figures in the Appendix B, in the upper tail of the distribution the constant (from the 85^{th} percentile onwards) as well as the estimated coefficient associated to the actual inflation (from the 70^{th} percentile onwards) assume high values. This element corroborates the evidence arising from the observation of the descriptive statistics, confirming a marked degree of pessimism for the upper tail of the distribution. On the other way, looking at the response function for the actual inflation in the middle range (in the interval $[25^{th}, 70^{th}]$), we can notice an evident hump-shaped pattern. On the other way, within the same interval, the autoregressive term determines a U-shaped response. These results are in line with what we would expect on a theoretical ground, as more rational individuals should rely less on past expectations, displaying a lower degree of stickyness, and rely more on actual inflation, which is likely to have a higher informational content.

An interesting situation can be outlined from the observation of the graph reporting the overall R^2 and the partial "contribution" coefficients. It is clear that up to the 70^{th} percentile most of the variance in the dependent variable can actually be explained by taking into consideration the autoregressive term, while the second highest contribution comes from the introduction of the contemporaneous rate of inflation, which becomes more important for the upper tail.

Insert Table 8 and Figure 17 about here

The second model aims at explaining what determines change in the forecasts. In order to do that we assume as regressors the first differences of the variables introduced in the previous model. It turns out that the explanatory power of the regressors is quite poor, apart from the first autoregressive term. Thus we are setting $\beta_7 = ... = \beta_9 = 0$. which has a high partial contribution coefficient along the whole distribution as can be seen in Figure 9 (see also figures in the Appendix B). Its contribution starts decreasing only from the 70^{th} percentile, leaving room for the last observed error and the second autoregressive term. The overall coefficient of determination still displays a hump-shaped pattern in the middle range. This model can actually be treated as extended model of the estimated simple learning rules. As in that model here we can also observe the coefficient on the observed past forecast error to be significant on the right-hand side of the distribution.

Insert Table 9 about here

The third model (5) proposed aims at providing a deeper understanding of the determinants of the forecast error, which is assumed as the dependent variable ($\pi_t - \pi^k_{t|t-12}$). It is worth mentioning again that evidence of serial correlation in the forecast error process indicates that there is an inefficient exploitation of information from last year's forecast in generating current year's forecast, which violates the rationality hypothesis. We have introduced in the set of exogenous regressors the last observed forecast errors, the horizontal spread between long and short term interest rate, the change in the cycle, the change in inflation and the contemporaneous forecast error from the SPF. The resulting response functions are reported in various figures in the Appendix B. It turns out that the coefficient associated to horizontal spread is never found to be significantly different from zero, at any percentile. The same evidence holds for the coefficient of the cyclical component, but just from the 45^{th} percentile onwards, while in the previous range it has a negative sign. It is also worth noting that the function built up with constant is downward sloping and crosses the zero line in correspondence of the 51^{th} percentile, which is classically associated with the "rational" group. The response function associated to the last observed forecast error is fairly constant up to the 30^{th} percentile and it assumes a marked U-shaped pattern afterwards. As regards the average error of the professional forecasters, which on theoretical grounds is actually expected to get a significant and positive coefficient, we can actually notice that the response is first constant and then hump shaped around 55^{th} percentile, while it decreases in the last deciles.

Insert Figure 18 about here

The most important inference probably comes from the observation of the coefficient of the determination and from the partial "contribution" coefficients associated to each regressor. The first one declines as we move towards the upper end of the distribution, but not monotonically, displaying a quite marked hump-shaped pattern in the first two ranges and assuming a U-shaped pattern from the 70^{th} percentile. This evidence has important implications for the informational structure underlying each group. The interpretation will appear more clear cut after observing the partial contribution coefficients. It appears that the last observed error has a great importance for the first range, which has classically displayed a backward looking, "adaptive" behaviour. This might be also due to that they are not observing current inflation as their error could be explained by the past errors and they are just making inflation expectations around focal points such as 0 or 5 percent. The variance of forecast errors of the third group, located in the upper end is almost exclusively explained by the variance of the change in the actual inflation. Total R^2 decreases thus they are observing all the variables, although their error could not be explained by these variables, but just with the constant. That again points to the "pessimism" of that agents. But there is another possible interpretation that arises from the results as the change in the inflation is the most important variable and the rise in inflation decreases forecast errors. This leads together with the fact that autoregressive component has almost no explanatory power to the conclusion that agents comprising this part of the distribution might be behaving in lines with recent literature on inattentiveness and rationally heterogeneous expectations. They are more attentive when inflation rises since the opportunity cost of being inattentive in this period rises. As regards the middle range, we can actually notice that the contribution of the past error decreases, while the one of the error of the professional forecasters gains importance. Considering the professional forecasters as a "general" stereotype of rational agents, we can actually infer that the middle range, especially around the $50^{th} - 55^{th}$ percentile, is the less biased, as the evidence arising from the test of unbiasedness in the pervious section. In that region the error of professional forecasters is actually almost the only important variable for determination of the forecast error. This equation could be considered as a test of rationality. The test could be that the $\gamma_0 = ... = \gamma_3 = \gamma_5 = 0$. The only significant could be γ_4. We tried to add several lags of the SPF to the equation to assess the Carroll's (2003a,b) finding that the transmission effect from professional forecasters to households is quite slow, but in our case the additional lags tend to be insignificant.

3.3 Results: Different Demographic Groups

Below we briefly outline the main results for different demographic groups. Most of the results are quite similar than the results for the overall sample. Especially responses from regressions (3)-(5) are quite similar. The most interesting results are for different income groups. Thus we will focus on explaining them while for the other groups we will just point out the main features. Main features of the rationality and learning tests are reported in the Table 8. In the first two columns are result of the test for bias (6) in the third there are results for the second test of the rationality (8). The next three columns are reporting results for constant gain learning and the last three for decreasing gain learning. In these results there are reported the range of percentiles for which the variance of the explanatory variable (past forecast error) explains more than 5% of the variance of dependent variable. The percentile for which R^2 has the highest value (and the value of R^2) is reported in the middle column and in the last column is reported the percentile with the highest estimated coefficient for respective learning and its corresponding value.

Insert Table 10 about here

This subsection also outlines some of the main differences in the responsiveness of the percentiles of the distributions describing every demographic subgroup, along the lines indicated by the models (3)-(5).

The comparison will be based on the observation of the partial correlation coefficients regarding only the regressors that produce estimated coefficients significantly different from zero. In this way we are able to discriminate which variables have a higher importance for some classes of agents and whether their effect has a heterogeneous impact on different parts of the distribution. It is important to say that the general features outlined for the overall sample still survive at a more disaggregate level, with quantitative differences among classes within the same subgroup. As already mentioned, the model expressed in equation (3) aims at characterising the relevance of the determinants of the one-year-ahead inflation expectations. The second model aims at explaining what determines that change in the forecasts. It is worth mentioning that, as in the general case, the explanatory power of the model is quite poor. In particular, major differences between classes within the same subgroup are confined to the extreme upper end of the distribution and do not deliver a clear picture.

The third model (5) proposed aims at providing a deeper understanding of the determinants of the forecast error, which is assumed as dependent variable. With regard to this model, evidence of serial correlation in the forecast error process indicates that there is an inefficient exploitation of information from last year's forecast in generating current year's forecast, which violates a rationality criterion.

3.3.1 By gender

The first regression points out a greater importance of the inflation level at the time of forecast for men, which has a monotonic increasing impact up to the 95^{th} percentile. Furthermore, the autoregressive term has a greater importance for men compared to women, even at higher percentiles. For women, this component loses importance in favour of the forecast produced by the SPF around the 65^{th} percentile. The evidence can point out how women in the upper end of the distribution may rely less on their own past forecasts.

In the third model men's forecast error at the upper end of the distribution is better described by contemporaneous changes of actual inflation, while the forecast error of the SPF acquires for them a greater importance around the median of the distribution. The general fit of the model is better for men, especially for the right half of the distribution.

3.3.2 By age group

The first regression points out a homogeneous impact of inflation for younger people in the middle range, while it gains more importance for older people at higher percentiles. Younger people also rely less on their autoregressive term and more on the SPF forecast.

There is substantial equivalence in the impact of the regressors for the first two classes (18-34, 35-54). Elderly people's forecast errors seem to depend less on changes in actual inflation and to be more correlated with the contemporaneous forecast error produced by the SPF.

3.3.3 By region

There are not many differences related to a different location: the only thing we can point out for the first model is just that forecasts produced by people living in the NE are more influenced by the SPF component.

In the third model, forecast errors of people from West and South in the upper part of the distribution are accounted by changes in actual inflation, while there are not significant differences otherwise.

3.3.4 By income

Again first we have to point out that grouping interviewees by their income has started at the end of the 1979. Thus the results are not completely comparable to the others, but as we checked the responses to different percentile regressions by splitting the sample we can argue that the responses do not significantly change if we analyse subsample of our time span. Percentile regression on different income groups are representing probably the most interesting results, especially due to the case of high degree of rationality that arises from running regression (5) on the highest income group. They are almost better forecasters that the professional forecasters.

The autoregressive component for higher percentiles gradually loses importance as we move from the bottom income level to the top one: the last one has the general best fit for the first model.

From the estimation of the third model it is clear how more upper level income people's forecast error is just described by changes in actual inflation after the 70^{th} percentile, while the middle range is mainly described by the SPF forecast error. Its area has a clear hump-shaped behaviour. We have to point out that the results for (5) are significantly different that the results for other groups or overall sample.

[more to be written more in this part]

3.3.5 By education

There are not major differences between the two upper levels of scholarisation. As regards individuals comprised within the category "High School or Less", we can observe how the autoregressive component maintains a high importance at higher percentiles.

From the estimation of the third model it is clear how upper level educated people's forecast error is just described by changes in actual inflation after the 70th percentile, while the middle range is mainly described by the SPF forecast error.

4 Decomposition of Normal Mixture by Maximum Likelihood

The graphic analysis of the distribution of inflationary expatiations clearly points out the existence of multimodality. This characteristic constitutes one of the main stylised facts at the root of the literature on heterogeneous expectations and on learning mechanisms. In this section we estimate a mixture of normal distributions, in order to characterise the heterogeneity in the forecasting process of different groups. This exclusively statistical approach constitutes a novelty in the literature. Branch (2004) develops a model of expectation formation where agents form their forecasts by rationally selecting a predictor function from a set of three costly alternatives. The framework he employs is constituted by a mixture of three normal distributions. However, as Pesaran and Weale (2005) point out, instead of finding the parameters of each distribution and the weight attached to each of them in every period, he imposes strong assumptions on the choice of the models used to generate the means of each distribution from three relatively simple specifications. Our statistical approach is aimed at estimating the mean, the variance and the proportion of each component in the mixture. Furthermore, we do not restrict the number of components, but we consider a sequence of nested hypotheses, testing them one by one on the basis of the information criteria and goodness of fit tests[20] retrievable through maximum likelihood estimation.

4.1 Model Description and Interpretation of the Parameters

The likelihood function describing the model under scrutiny has the following form

$$
\begin{aligned}
f(x \quad | \quad \theta) &= \sum_{p=1}^{P} n_p \frac{1}{\sqrt{2\pi\sigma_p^2}} \exp\left[-\frac{(\pi_{t|t-12} - \mu_p)^2}{2\sigma_p^2}\right] \\
\theta &= (P, \mu_1, \sigma_1^2, n_1, \mu_2, \sigma_2^2, n_2, ..., \mu_P, \sigma_P^2, n_P) \qquad \sum_{p=1}^{P} n_p = 1
\end{aligned}
$$

where θ is the set of model parameters and P is the total number of components in the mixture. We denote with n_p the proportion of the p^{th} Gaussian, while μ_1 and σ_1^2 are its mean and variance respectively. As we have anticipated, the estimation of the number of components is one of the most important questions in the maximisation

[20] A Stata module, *Denormix*, developed by Stanislav Kolenikov, performs a numeric maximisation of the likelihood function, as well as some diagnostic tests that are useful to determine the number of components. The package is freely available at the author's webpage, http://www.komkon.org/∼tacik/stata/.

problem[21]. Kolenikov (2005) asserts that the likelihood ratio test has not a convenient χ^2 distribution (see McLachlan, 1987 and Feng and McCulloch, 1996). Thus we resort to the response of goodness of fit tests, such as the χ^2 Pearson fit test. The model yielding the highest p-value of the χ^2 test is chosen. Another remedy can be found in the use of information criteria (Bearse et. al. (1997) and Schwarz (1978)). In this case we choose the model that minimizes the criterion.

5 Analysis of mixed distribution estimation

5.1 Properties of the estimated distributions

In this section we analyse the cyclical properties of the parameters obtained from the ML estimation of the mixture of normals outlined in the previous paragraph. The aim of this analysis is to detect whether the degree of heterogeneity has some characteristic and identifiable temporal patterns. The decomposition of the overall distribution in mixture of Gaussians provides us with statistical data that are relevant for the identification of the number of subcomponents, the description of their intrinsic characteristics and the dispersion among them.

Once we identify the number of components that determines the highest goodness of fit, the first step is to observe whether this is constant over time, and in particular over the cycle and in correspondence of periods characterised by different levels of uncertainty. The rise of a different number of components, shifts in their mean or modifications in their variances with respect to phases of expansion or contraction, point out the existence of an asymmetric response of the individuals comprised in the sample in the process of expectation formation. For example, it might be the case that during periods of marked uncertainty, dispersion in the sample will increase, giving rise to additional subcomponents or to an higher variance of the existing ones. Furthermore, different individuals might resort to different models of inflation forecasting depending on the cyclical phase.

We design a number of indices which are useful to provide preliminary evidence on (i) the degree of dispersion, in terms of distance between the subcomponents, (ii) concentration, measured by a Herfindahl-style index, (iii) **convergence**, in terms of the net flow of people comprised in the distribution characterised by the lowest forecast error, evaluated as the spread between its mean and the realised inflation. We briefly present the indices employed in this part of the analysis

- **An Herfindahl-type Index**

  The index is aimed at determining the concentration of the subcomponents of the overall distribution, and it is give by

[21] On this point see, for instance, Day (1969), Hathaway (1985), Basford and McLachlan (1985), Kiefer and Wolfowitz (1956) and Fowlkes (1979).

$$H_t = \sum_{p=1}^{P_t} n_{p,t}^2 \tag{24}$$

The value of H varies between zero and one: the first case corresponds to minimum concentration, while the second case corresponds to maximum concentration. For a given finite number of components, the lowest value occurs in the case of homogeneous shares.

- **An Average Distance Index**

  The index is aimed at determining the average distance between the component whose mean is a more accurate predictor of future price inflation and the remaining components. It is represented by the following expression

$$D_t = \sum_{p=1}^{P_t} \frac{\Delta m_{p,t}^2}{\sigma_{p,t}} \tag{25}$$

  where Δm_p is the difference between the mean of the p^{th} component and the mean of the "minium bias" component, normalised by the standard deviation of the former.

- **A further Dispersion Index**

  The following index captures the change in the sum of the dispersion of each component of the distribution, weighted by the relative share.

$$\Psi_t = \sum_{p=1}^{P_t} \Delta n_{p,t} \sigma_{p,t} \tag{26}$$

- **A** $\sigma - type$ **Convergence Index**

  The index, similar to the previous one, is the sum of the dispersion of each component weighted by its own share. A negative trend in the dynamics of the index might point out the validity of a $\sigma - type$ convergence hypothesis

$$C_t = \sum_{p=1}^{P_t} n_{p,t} \sigma_{p,t} \tag{27}$$

5.1.1 Properties of overall sample

Before proceeding with the discussion we have to point out that the data retrieved is not "ordered". It is very important how we order our results into different time series, since it produces significantly different results as with ordering we can to certain degree control properties such are variability of the mean, variance, and/or the share of the subdistribution. For the below figures we employ the following ordering: first we determine the distributions with the highest and the lowest mean. Then we order the remaining distributions according how accurate are their forecasts. Thus we determine the group that potentially could be rational. But there might arise a situation where the group with the lowest mean is closer to the actual inflation.

From the results we can argue that usually it is optimal to decompose the distribution into 5 components, although especially in the periods of stable inflation 4 components or even 3 components might be optimal. In 231 periods we could decompose our distribution into 5 components, in 88 periods into 4 components and in 7 periods into 3 components. Figure 19 reports the results of decomposition of empirical distribution.

Figure 20 reports the movement of the means of different components and Figure 21 reports their respective shares. The data here is ordered as discussed in Appendix D.

Insert Figures 20 and 21 about here

It can be seen that the mean of the rational group is tracking the actual inflation quite well. In most periods we have one component at each tail of the distribution. Their respective shares are usually quite low as can be observed from figure 16. In the period of stable inflation the share of the "rational" group increases.

We have analysed the macroeconomics determinants that are likely to drive the dynamics of the indices presented above, by implementing the model expressed by equation (...) and inserting the sigma convergence index and the average distance index as dependent variables. The empirical exercise is useful to get some insight on the existence of some phenomenon of between/within group convergence.

As mentioned, the first index is the sum of the dispersion of each component weighted by its own share. A negative trend in the dynamics of the index might point out the validity of a dynamics towards a common level of dispersion, pointing out the existence of a within group convergence. On the other way, the average distance index is aimed at determining the average distance between the component whose mean is a more accurate predictor of future price inflation and the remaining components. Evidence of a negative trend would be a signal of a between group convergence. It has to be pointed out that calculated indices are invariant to any ordering.

The analysis on the first index actually points out the possibility of a within groups convergence process at work, as the trend has a negative coefficient and, more importantly, the cycle variable coefficient is not significantly different from zero. Evidence of a negative trend arises in the case of the second index as well, although in this case the cyclical component gains some importance in explaining its variation.

[To be added more]

5.1.2 Properties of different groups

[To be added]

5.2 Learning and convergence

5.2.1 Determination of the Demographic Structure of Each Mixture Component

In this section we refine the estimation of the proportion of each component in the mixture of normals, by determining their demographic structure. The empirical exercise we are going to outline, inspired by Branch (2004, 2005), constitutes a convenient way to impose more structure on the data, and to obtain estimates of the demographic composition of the shares determined in the previous step, through the unrestricted decomposition of the mixture. We design a likelihood function for the response of each individual, under the assumption that the forecast error, v_{it}, with respect to the mean of each subcomponent, $H_p(\pi^t)$, is normally distributed

$$\pi_{it|t-12} - H_p(\pi^t) = v_{it} \sim N(0, \sigma_{it}), \qquad p = 1, ..., P \qquad t = 1, ..., T$$

Given the stochastic properties of v_{it}, the maximisation problem is consists in

$$\begin{aligned} \max_{n_p, \sigma_{p_{p\in(1,...,P)}}} \quad P(\pi_{it|t-12},\ i &= 1, ..., N \mid H_p(\pi^t),\ p = 1, ..., P) \\ &= \sum_i \ln \sum_p n_p \frac{1}{\sqrt{2\pi\sigma_p^2}} \exp\left[-\frac{(\pi_{it|t-12} - H_p(\pi^t))^2}{2\sigma_p^2}\right] \\ \sum_{p=1}^{P} n_p &= 1 \end{aligned}$$

The estimation will provide us with valid information on accumulation of the population of each demographic subgroup (e.g. male and female, income level, educational level) around the mean of the components of the mixture estimated in the first step for the overall sample, together with the standard deviation.

This empirical exercise should help to identify the demographic composition of each share for the total population of respondents. The resulting information is useful to perform both static and dynamic descriptive statistics analysis, as well as more accurate investigation of the expectation formation process of different demographic subgroups over the cycle and under different inflation regimes.

[To be added more]

5.2.2 Learning and convergence within different groups

6 Analysis of the expectation formation process

[To be added]

6.1 Digit preference?

[To be added]

7 Survey Inflation Expectations in the New Keynesian Macro Model

In this section we design two different exercises to test the performance of the New Keynesian (NK) model with survey inflation expectations[22]. Several papers have tried to estimate the NK model with survey expectations, e.g., Roberts (1997), Rudebusch (2002), Basdevant (2003) and Adam and Padula (2004).[23] They generally conclude that the model cannot explain the inflation persistence just by including survey expectations, while finding that additional lags of inflation in the Phillips curve (PC) are still significant. We reexamine this by using the results in the previous section.

The framework we are going to outline is an augmented standard version of the NK model with heterogeneous expectations. In the first exercise we replace the forward looking inflation expectations with the means of subdistributions obtained from the estimation of the mixture. We then estimate the shares of each subdistribution and study the fit of the model, especially in terms of description of the inflation persistence. In the second exercise we also replace the shares of each subdistribution with the shares estimated in the previous section.

The econometric approach consists of estimating a system of equations through Hansen's (1982) Generalised Method of Moments (GMM) estimator. For the sake of comparison, we also report the results of ML and OLS estimators. We use lagged values of the dependant variables as instruments[24]. The model employed is derived in Flamini and Pfajfar (2005). The main features of the model are the aggregate demand, which is predetermined for 2 quarters, and the Phillips curve, which is predetermined for 4 quarters. Furthermore, the demand function is derived under the assumption that part of the consumption goods are employed in the production process

[22]Before proceeding in the analysis, we transform the frequency of the data estimated in the previous section from monthly to quarterly (as average of the quarter).

[23]These authors use survey inflation expectations for US, Euro area, and New Zealand. Paloviita (2002) pursues a different approach by using OECD forecasts of inflation and output for the Euro area.

[24]Data definitions are in the appendix A2.

$$y_{t+2} = \beta y_{t+1} + (1-\beta) y_{t+3|t} - \lambda (1-\beta) \sigma r_{t+2|t} + \tau y^n_{t+1} + \eta_{t+2} \tag{28}$$

$$\pi_{t+4} = \frac{1}{1+\zeta} \left[\zeta \pi_t + \pi_{t+5|t} + \kappa y_{t+4|t}\right] + \varepsilon_{t+4}, \tag{29}$$

$$i_t = a_1 \pi_t + a_2 \pi_{t+1|t} + a_3 y_t + a_4 y^n_t + a_5 i_{t-1} \tag{30}$$

where $\tau = \left[(1-\beta)\gamma_y^{n2} - \gamma_y^n (1+\beta)\right]$. We augment this model for heterogeneous expectations

$$\begin{aligned} y_{t+2} \quad &= \quad \beta y_{t+1} + (1-\beta) \sum_p n_p H^y_p \left(y_{t+3|t}\right) - \lambda (1-\beta) \sigma \sum_p n_p H_p \left(r_{t+2|t}\right) \\ &\quad + \left[(1-\beta)\gamma_y^{n2} - \gamma_y^n (1+\beta)\right] y^n_{t+1} + \eta_{t+2} \end{aligned} \tag{31}$$

$$\pi_{t+4} = \frac{1}{1+\zeta} \left[\zeta \pi_t + \sum_p n_p H_p \left(\pi_{t+5|t}\right) + \kappa \sum_p n_p H^y_p \left(y_{t+4|t}\right)\right] + \varepsilon_{t+4}, \tag{32}$$

$$i_t = a_1 \pi_t + a_2 \sum_p n_p H_p \left(\pi_{t+1|t}\right) + a_3 y_t + a_4 y^n_t + a_5 i_{t-1} \tag{33}$$

where $H^y_p \left(y_{t+4|t}\right)$ are corresponding output gap expectations of the p-th subgroup.

7.1 Results

[To be added]

Concluding Remarks

[To be added]

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8 Tables and figures

Table 1:

Demographic Group	Mean	Median	Variance	Int. Range	Skew	Kurt	Inflation
Male	4.28	3.8	20.6	4.44	1.79	8.69	
Female	5.37	4.16	34.9	5.55	1.54	5.6	
18-34	5.14	4.16	29.5	4.93	1.64	6.52	
35-54	4.95	4.09	27.6	4.81	1.72	7.12	
55+	4.48	3.46	28.1	4.9	1.69	6.71	
West	4.91	4.09	27.1	4.89	1.61	6.69	
North/Centre	4.77	3.9	27.6	4.79	1.73	7.1	
North/East	4.82	3.92	28.9	5.09	1.61	6.54	**4.19**
South	4.95	3.94	30.2	4.99	1.66	6.41	
Bottom Income Level	5.28	3.95	36.7	5.83	1.44	5.08	
Middle Income Level	4.59	3.71	26.8	4.71	1.79	7.36	
Top Income Level	4.01	3.57	19.2	4.29	1.9	9.41	
HS or less	5.23	3.97	34.8	5.43	1.53	5.47	
Some college	4.78	3.96	26.4	4.77	1.66	6.97	
College degree	4.51	4.11	20	4.27	1.79	8.95	
Overall	**4.87**	**4.16**	**28.7**	**5.55**	**1.73**	**6.98**	

Table 2:

Demographic Group	Mean	Mean SSE	Median	Median SSE	Inflation
Male	4.28	741	3.8	849	
Female	5.37	1474	4.16	1089	
18-34	5.14	1143	4.16	900	
35-54	4.95	1035	4.09	810	
55+	4.48	1253	3.46	1560	
West	4.91	1021	4.09	834	
North/Centre	4.77	1021	3.9	1030	
North/East	4.82	1115	3.92	1106	**4.19**
South	4.95	1174	3.94	1033	
Bottom Income Level	5.28	1610	3.95	772	
Middle Income Level	4.59	834	3.71	507	
Top Income Level	4.01	431	3.57	392	
HS or less	5.23	1420	3.97	1183	
Some college	4.78	1012	3.96	980	
College degree	4.51	759	4.11	745	
Overall	**4.87**	**1015**	**4.16**	**1089**	

Table 3:

Demographic Group	Mean	Median	Variance	Int. Range	Skew	Kurt	Inflation
Male	5.64	5.05	29.7	5.77	1.41	6.38	
Female	6.65	5.15	46.6	7.58	1.18	4.02	
18-34	6.78	5.56	40.5	6.48	1.25	4.57	
35-54	6.41	5.37	37.8	6.36	1.32	5.19	
55+	5.27	4	37.2	6.37	1.4	5.32	
West	6.33	5.4	37.2	6.43	1.26	5.15	
North/Centre	6	4.92	37.9	6.39	1.36	5.19	
North/East	6.27	5.13	40.3	6.75	1.27	4.69	
South	6.18	4.99	40.7	6.58	1.32	4.81	**6.2**
Bottom Income Level	6.12	4.46	48.6	7.8	1.22	4.02	
Middle Income Level	5.7	4.6	37.2	6.27	1.46	5.49	
Top Income Level	5.28	4.73	28.9	5.7	1.48	6.6	
HS or less	6.29	4.77	45.4	7.19	1.25	4.24	
Some college	6.08	5.11	35.7	6.18	1.34	5.34	
College degree	6.12	5.71	28.2	5.38	1.3	6.19	
Overall	**6.18**	**5.15**	**39.3**	**7.58**	**1.33**	**4.96**	

Table 4:

Demographic Group	Mean	Median	Variance	Int. Range	Skew	Kurt	Inflation
Male	3.36	2.94	14.5	3.54	2.05	10.26	
Female	4.5	3.49	27	4.17	1.78	6.68	
18-34	4.03	3.21	22	3.89	1.91	7.85	
35-54	3.95	3.22	20.7	3.76	1.99	8.42	
55+	3.94	3.09	22	3.9	1.88	7.65	
West	3.94	3.19	20.3	3.83	1.85	7.74	
North/Centre	3.93	3.21	20.6	3.7	1.98	8.39	
North/East	3.84	3.09	21.2	3.96	1.84	7.79	
South	4.11	3.22	23.1	3.91	1.89	7.5	**2.98**
Bottom Income Level	4.81	3.65	29.9	4.7	1.57	5.69	
Middle Income Level	3.95	3.2	20.8	3.82	1.98	8.43	
Top Income Level	3.27	2.9	13.6	3.48	2.13	11.02	
HS or less	4.51	3.42	27.6	4.24	1.72	6.31	
Some college	3.89	3.18	20.1	3.81	1.88	8.08	
College degree	3.42	3.03	14.5	3.51	2.12	10.82	
Overall	**3.98**	**3.49**	**21.6**	**4.17**	**2.01**	**8.35**	

Table 7:

Percentile	α_0	α_1	α_5	α_{11}	α_{12}	adj R^2	DW	LM
5	-0.09473	-0.01145	0.10487	0.661447	-0.005838	0.590783	1.86518	3.419019
	-1.125544	-0.446112	3.864332	14.81215	-0.141948			
	0	0.000622	0.093356	0.501669	0.000592			
20	0.126709	0.007963	0.038741	0.825245	-0.011359	0.783729	2.108067	1.241417
	1.953621	0.382037	2.238046	25.62935	-0.3611			
	0	0.012736	0.023533	0.758311	-0.007968			
35	0.414404	0.111881	0.078971	0.692024	-0.054613	0.811333	2.085508	10.78214
	4.118989	3.463982	3.067221	16.27749	-1.223784			
	0	0.207533	0.029997	0.631416	-0.055098			
50	0.643896	0.142411	0.052998	0.641576	0.028623	0.883729	2.090109	1.599532
	5.453873	4.041255	2.100157	14.24378	0.623681			
	0	0.234476	0.009356	0.614024	0.027424			
65	0.597332	0.140803	0.013555	0.767223	-0.00221	0.959869	2.20107	5.629957
	5.67896	4.616663	0.724973	20.20503	-0.064008			
	0	0.194938	0.001289	0.765969	-0.001791			
80	0.829583	0.221726	-0.018074	0.574501	0.266797	0.926136	2.170334	20.81655
	5.35557	4.668941	-0.494793	12.41056	3.710707			
	0	0.21286	-0.000767	0.557264	0.157764			
95	4.922529	0.30972	-0.112973	0.378513	0.727935	0.884826	2.073057	5.272365
	10.51711	4.295885	-1.833474	7.054447	5.674966			
	0	0.215654	-0.002423	0.351977	0.321154			

Table 8:

Percentile	β_0	β_1	β_4	β_5	β_6	β_{10}	adj R^2	DW	LM
5	0.021859	-0.005743	0.025232	0.049844	0.817661	-0.013736	0.744347	2.002395	1.546538
	0.371232	-0.558999	0.423693	2.688042	13.81945	-0.335997			
	0	-0.000336	0.01775	0.055185	0.678658	-0.002705			
20	0.07958	-0.031382	0.082998	0.033491	0.831623	-0.030514	0.85678	1.95855	5.297139
	2.075881	-3.376631	1.418898	2.818029	14.71095	-1.080296			
	0	0.019337	0.066222	0.027191	0.748616	-0.002231			
35	0.043559	-0.029567	0.176302	0.039439	0.720604	0.074932	0.786758	1.987575	0.297668
	0.934461	-1.791201	2.973088	2.301914	12.51742	1.528925			
	0	-0.005047	0.130957	0.028763	0.609267	0.026326			
50	-0.009211	-0.016224	0.210276	0.03232	0.733404	0.072917	0.819273	2.037169	5.649271
	-0.25824	-0.712788	3.508346	1.813141	12.99337	1.40703			
	0	-0.007853	0.159673	0.015974	0.633335	0.021116			
65	-0.032842	-0.020933	0.222331	0.010812	0.748923	0.111278	0.892344	1.996421	4.886389
	-1.001481	-0.929733	3.676228	0.848683	13.37323	2.325793			
	0	-0.014367	0.17827	0.005332	0.673895	0.050984			
80	0.434589	0.180954	0.211123	0.029217	0.552788	0.087704	0.793901	1.906255	9.317524
	3.305049	3.655734	3.532268	1.192772	9.566334	1.050179			
	0	0.154989	0.154948	0.006915	0.456531	0.023909			
95	2.592649	0.247461	0.203516	0.015123	0.434981	0.20605	0.728849	2.018644	3.853632
	4.738284	4.835443	3.604924	0.370143	7.71224	1.665398			
	0	0.199175	0.15013	0.001599	0.349341	0.033063			

Table 9:

Percentile	γ_0	γ_1	γ_2	γ_3	γ_4	γ_5	adj R^2	DW	LM
5	0.724896	0.830685	-0.212333	-0.011714	0.410927	0.568801	0.913086	0.876929	96.00554
	5.767167	31.09996	-9.282281	-0.350267	7.493298	9.842804			
	0	0.739137	0.001195	0.000887	0.160839	0.012453			
20	0.377044	0.88218	-0.109709	0.039082	0.291533	0.544748	0.877859	0.484068	177.1469
	3.598453	28.69231	-5.047493	1.231526	6.057569	10.19863			
	0	0.749382	-0.005299	-0.00375	0.131293	0.008235			
35	0.53637	0.713859	-0.130196	0.05539	0.234986	0.530016	0.736719	0.661733	141.0765
	5.702844	15.31129	-4.846561	1.431296	3.631449	7.897191			
	0	0.484395	-0.004798	-0.00693	0.14764	0.120729			
50	0.098423	0.212925	-0.033953	0.060264	0.493078	0.173569	0.620452	0.526092	168.8809
	1.984261	3.633577	-1.33278	1.652009	6.883843	2.502988			
	0	0.09868	-0.004153	-0.014114	0.448939	0.097322			
65	-0.888195	0.219273	-0.005963	0.055606	0.253756	0.427515	0.751401	0.534202	167.7825
	-14.1763	5.283664	-0.326761	2.103352	5.49416	10.61973			
	0	0.070305	-0.001293	-0.018546	0.268237	0.436772			
80	-1.957707	0.235571	-8.12E-05	0.010553	0.05723	0.815214	0.702703	0.88426	100.8469
	-16.61696	6.522812	-0.002851	0.252221	1.076451	17.48691			
	0	0.047348	-1.15E-05	-0.002567	0.032803	0.630004			
95	-7.059817	0.32647	0.046956	0.107927	-0.321297	1.240188	0.618541	1.112078	67.12759
	-16.67411	8.625145	1.006713	1.582705	-3.971579	17.93305			
	0	0.11527	0.004719	-0.011237	-0.086966	0.603009			

Table 10:

group	α=0 (1%)	α=0 (5%)	a=0,b=1	CGL*	CGL peak	mCGL coef.	DGL*	DGL peak	mDGL coef.
male	54-58	55-58	never	44-99	75 (37%)	62 (0.50)	42-94	76 (77%)	77 (37)
female	50-53	50-53	never	42-97	71 (34%)	58 (0.37)	43-87	68 (69%)	70 (30)
18-34	50-54	51-54	never	36-97	66 (39%)	55 (0.53)	36-87	68 (72%)	70 (32)
35-54	51-54	51-54	never	40-98	72 (38%)	57 (0.47)	40-92	70 (72%)	72 (33)
55-97	56-60	57-60	never	54-98	78 (27%)	78 (0.35)	52-94	76 (57%)	79 (36)
West	51-54	51-54	never	39-98	62 (39%)	55 (0.56)	39-92	70 (74%)	71 (35)
Nort-centr.	52-56	53-56	never	44-98	76 (30%)	61 (0.40)	44-90	72 (60%)	73 (34)
Northeast	52-56	53-56	never	42-98	64 (34%)	60 (0.50)	43-91	70 (59%)	75 (33)
South	52-56	53-55	never	42-98	73 (35%)	59 (0.40)	43-90	72 (65%)	73 (32)
Bottom	48-53	49-52	never	60-97	71 (26%)	72 (0.29)	17-31, 58-95	67 (50%)	64 (39)
Middle	51-55	52-55	never	48-98	75 (38%)	77 (0.33)	14-33, 57-97	71 (65%)	62 (47)
Top	53-58	54-57	never	50-99	81 (43%)	58 (0.51)	12-34, 54-98	76 (71%)	61 (51)
HS or less	51-55	52-55	never	48-96	73 (30%)	73 (0.31)	47-89	71 (66%)	72 (32)
Some coll.	52-56	53-55	never	40-98	72 (35%)	57 (0.53)	40-91	73 (67%)	74 (33)
Coll. degree	51-55	52-54	never	35-99	64 (43%)	56 (0.60)	35-94	73 (74%)	75 (36)
Overall sample	51-55	52-53	never	44-98	74 (35%)	59 (0.42)	42-90	72 (75%)	73 (33)

* R^2 above 5%

Table 11:

	Constant	Cycle	Inflation	AR(1)	SX	abs error	lag abs error	R^2	DW
Model for 0%	9.433	-0.349	-0.170	0.761	-0.283	-	-	0.740	2.143
	6.214	-2.303	-2.426	22.593	-4.568				
Model for 5%	19.848	-	-	0.131	-0.139	-1.058	-0.153	0.640	1.967
	15.686			2.410	-6.669	-10.598	-1.915		

Figure 1:

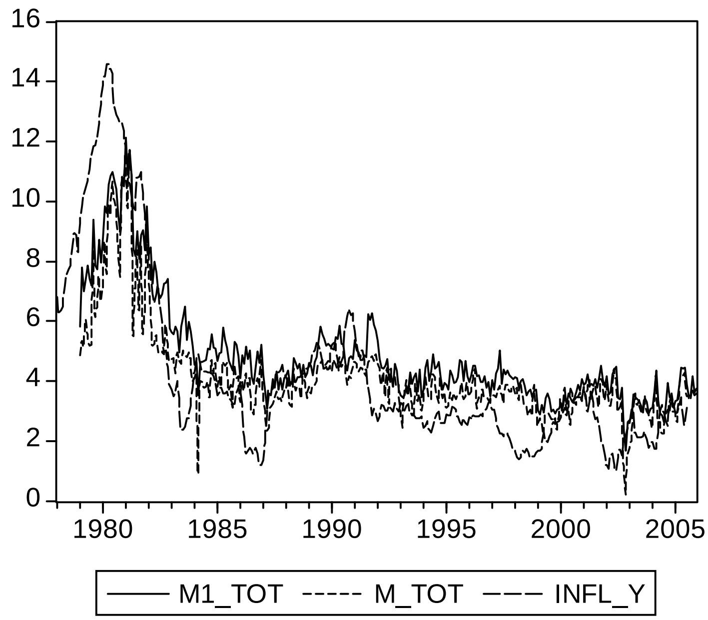


Figure 2:

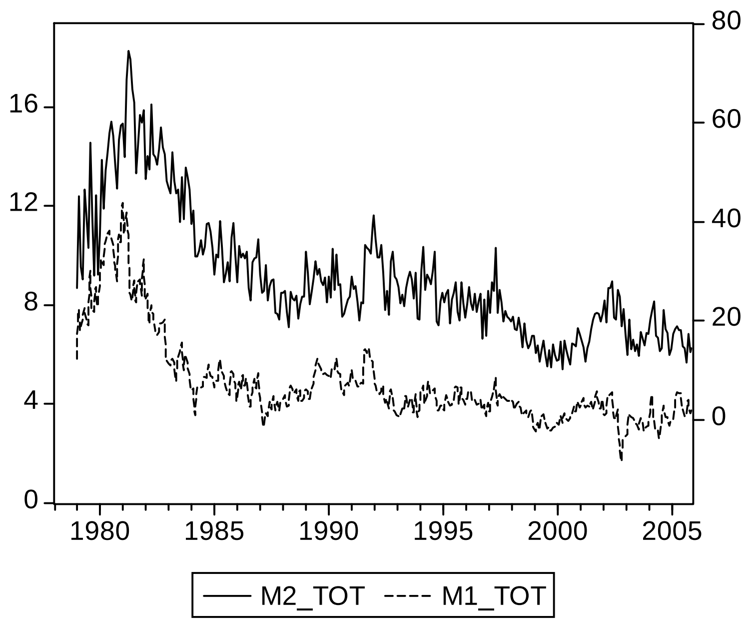


Figure 3:

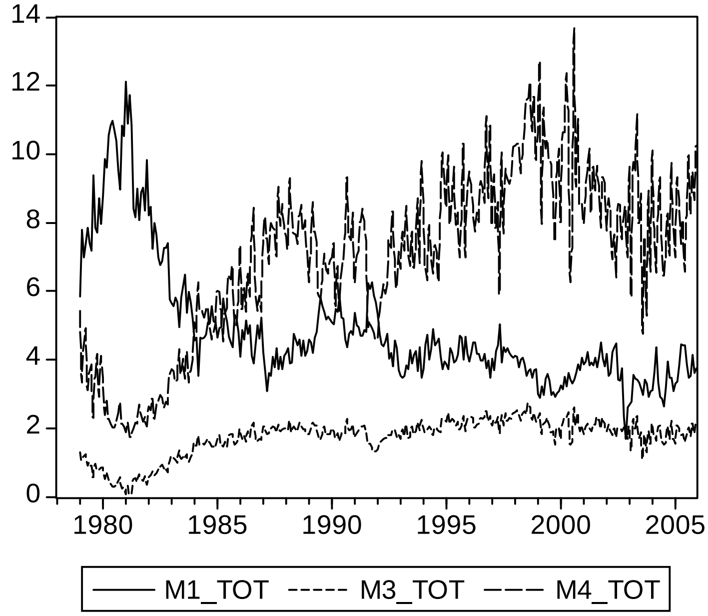


Figure 4:

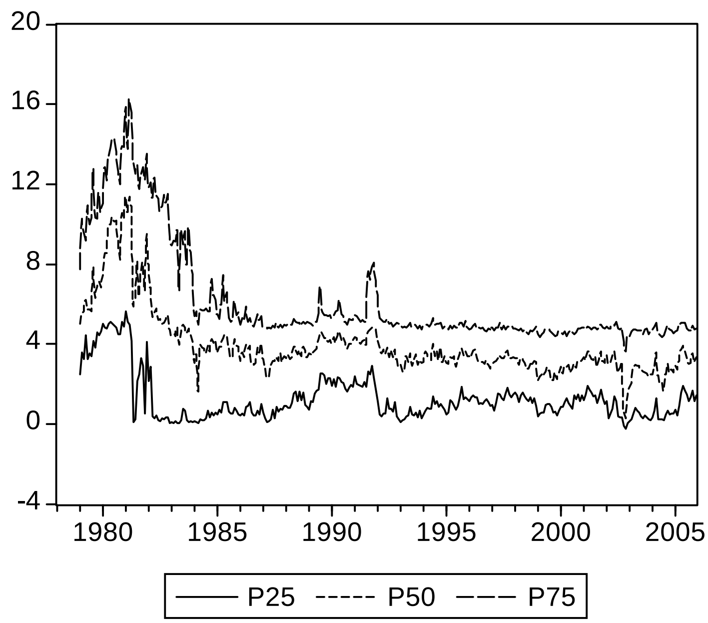


Figure 5:

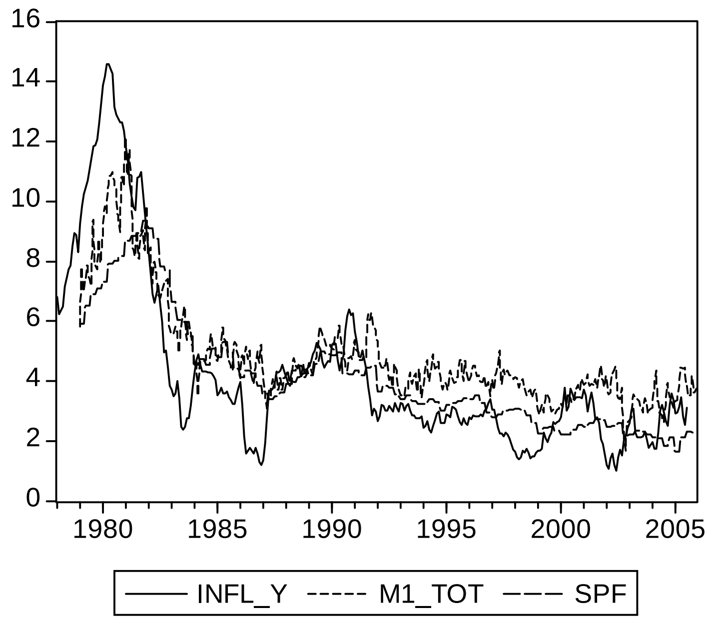
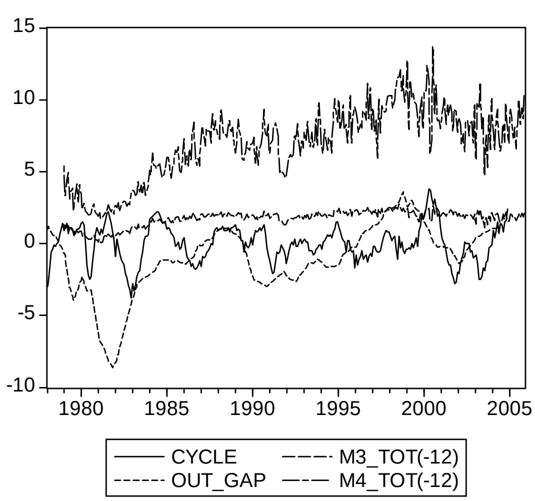


Figure 6, 7:



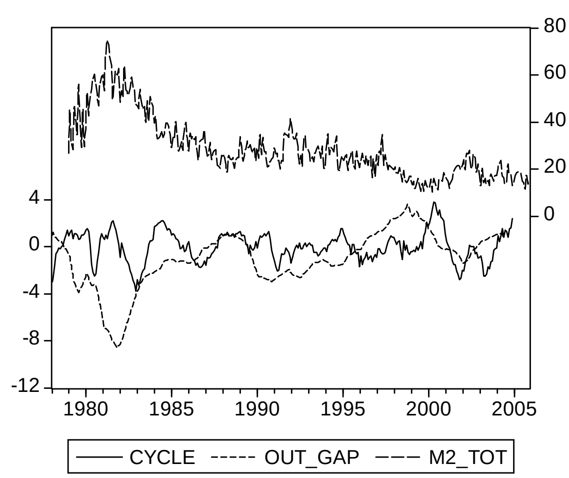


Figure 8:

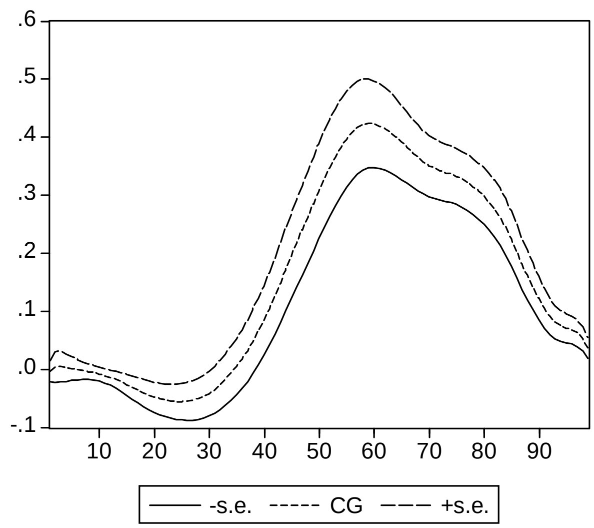


Figure 9:

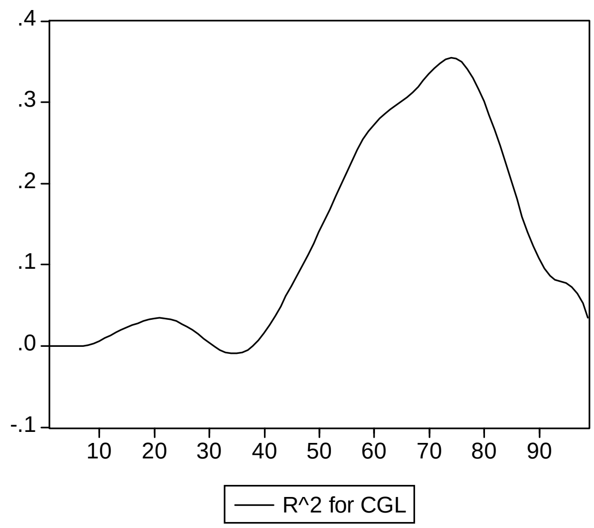


Figure 10:

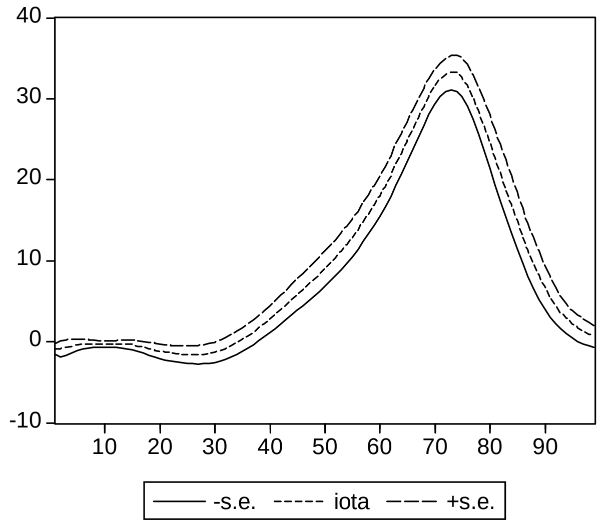


Figure 11:

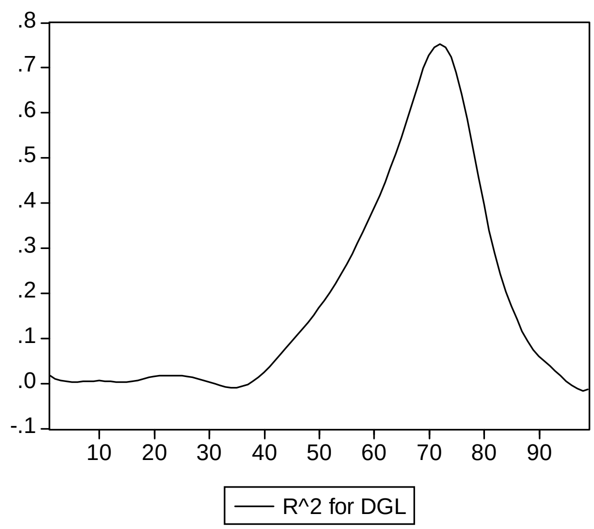


Figure 12:

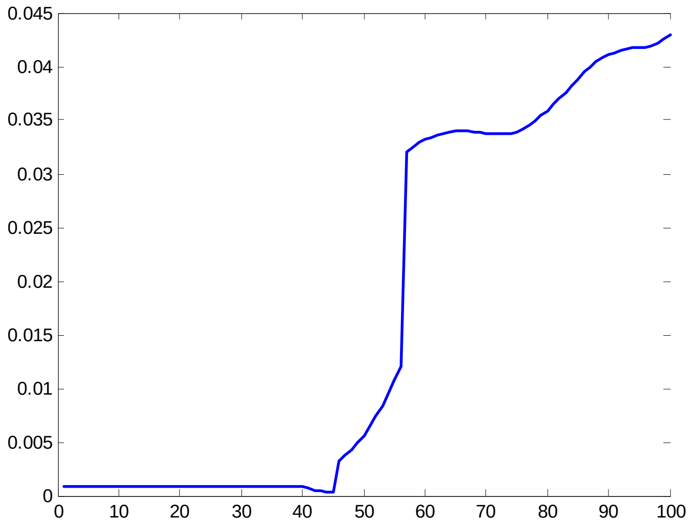


Figure 13:

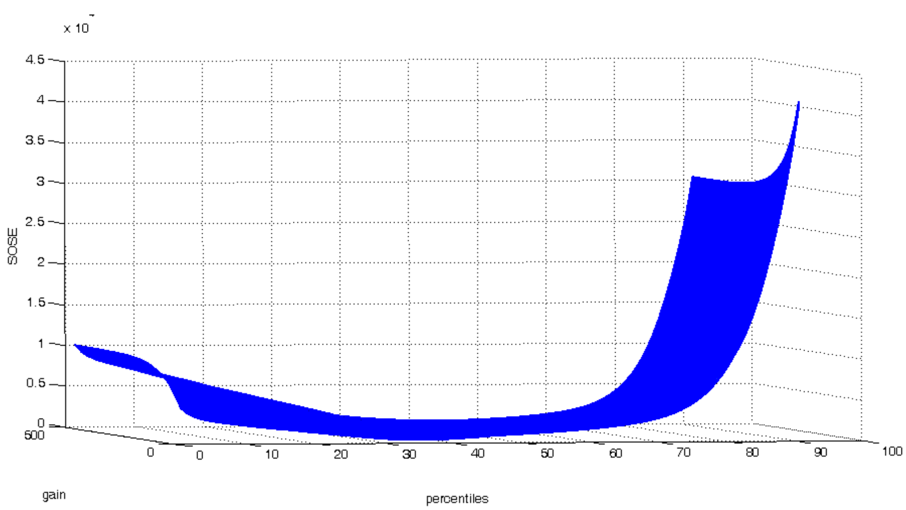


Figure 14:

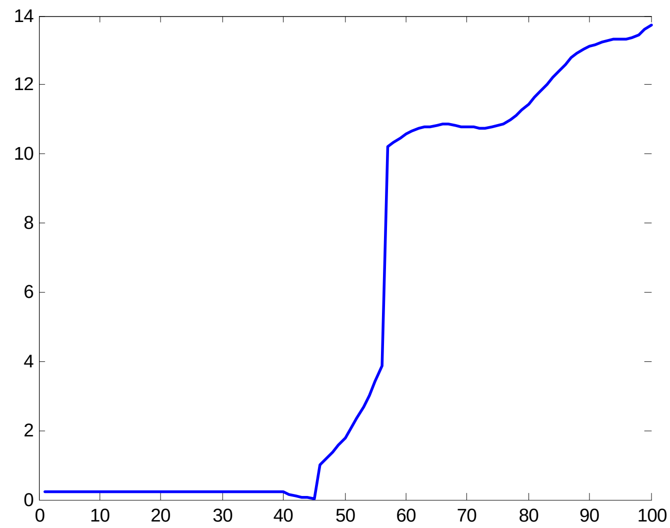


Figure 15:

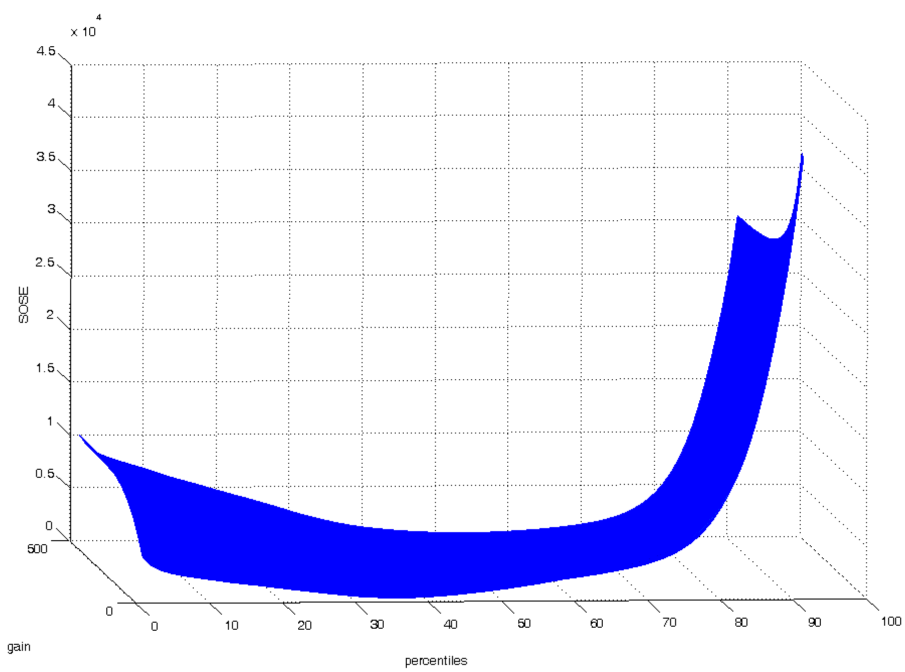


Figure 16:

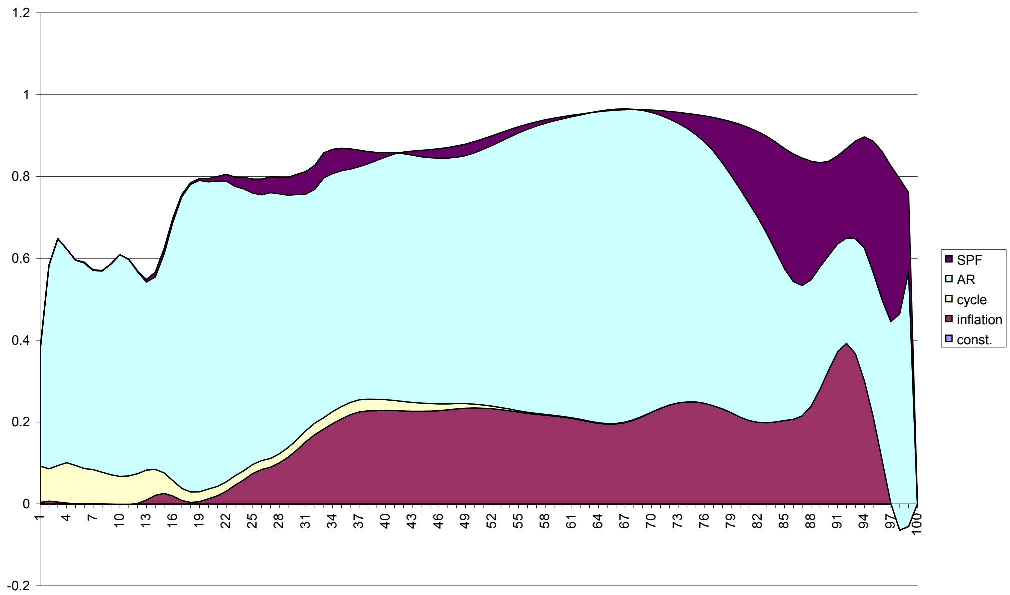


Figure 17:

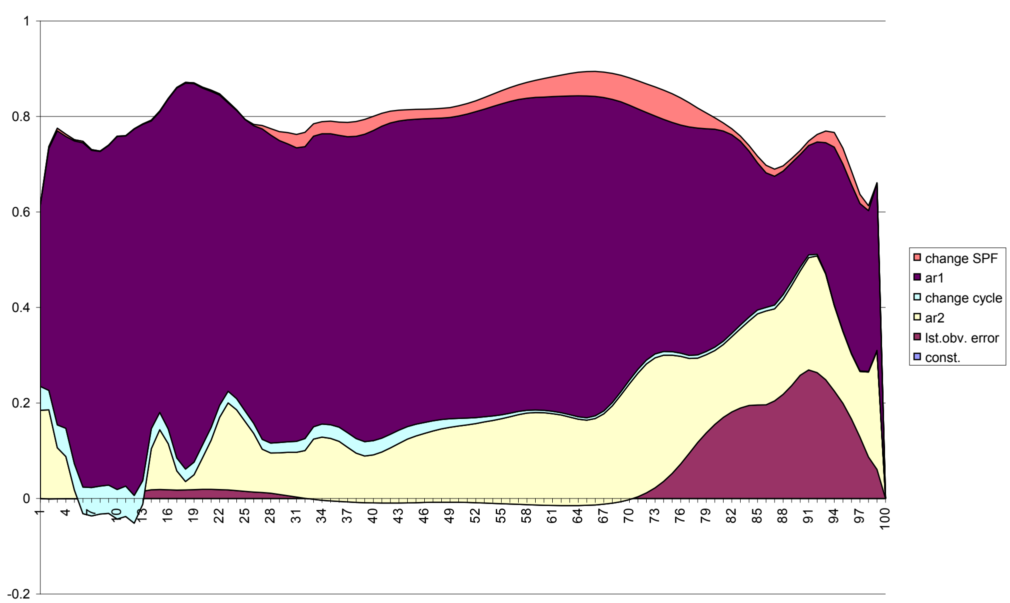


Figure 18:

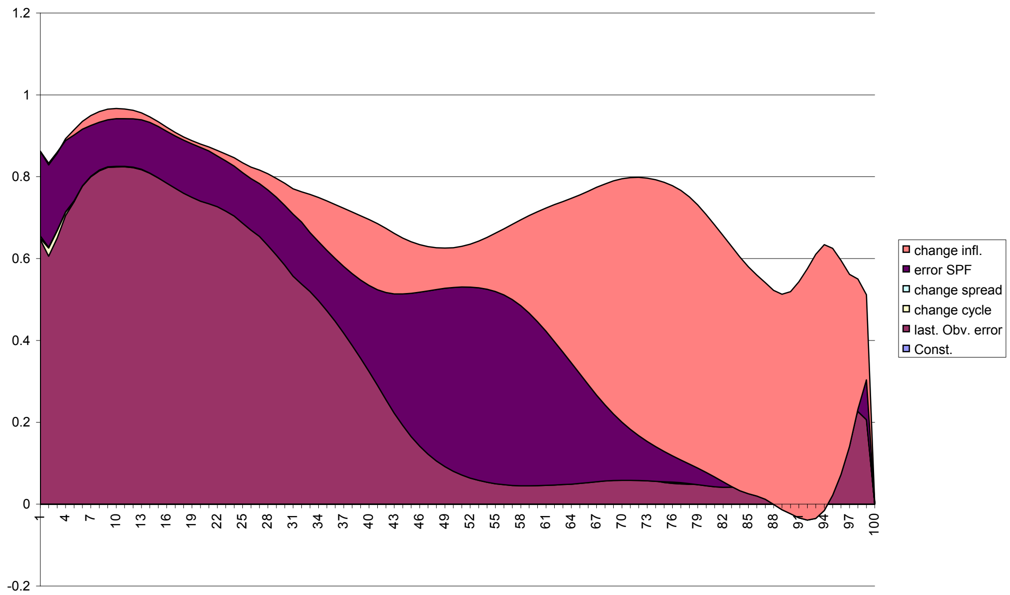


Figure 19:

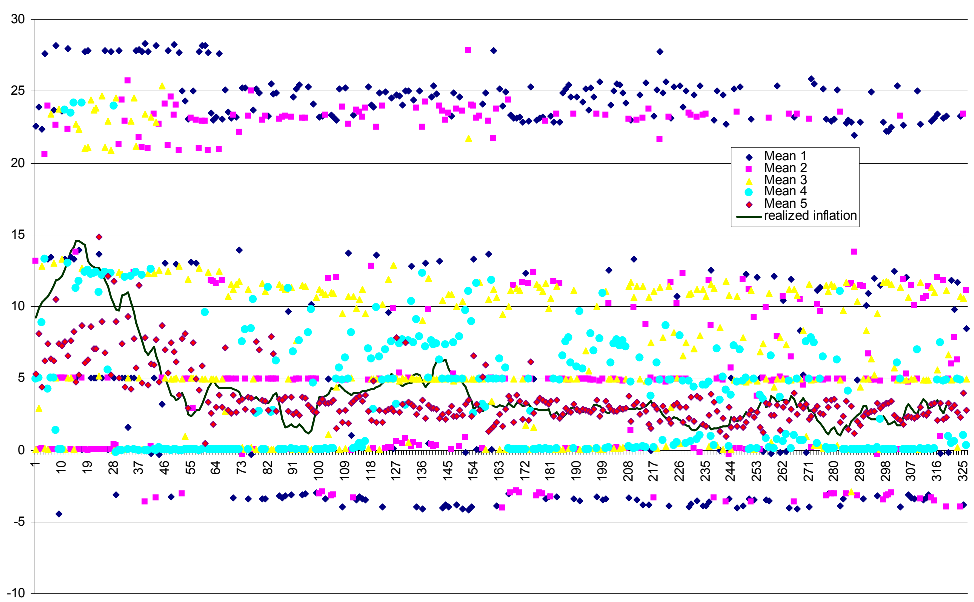


Figure 20:

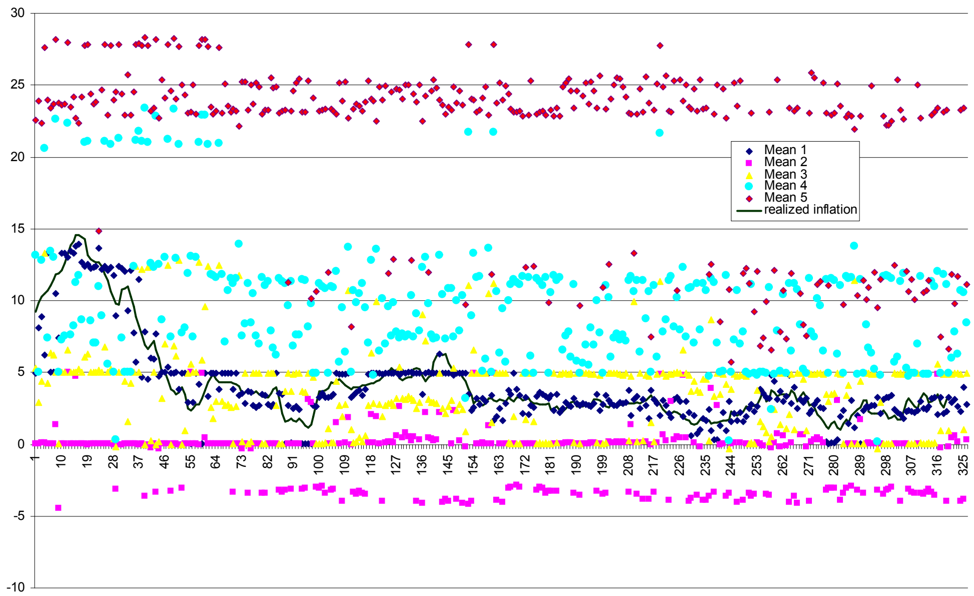
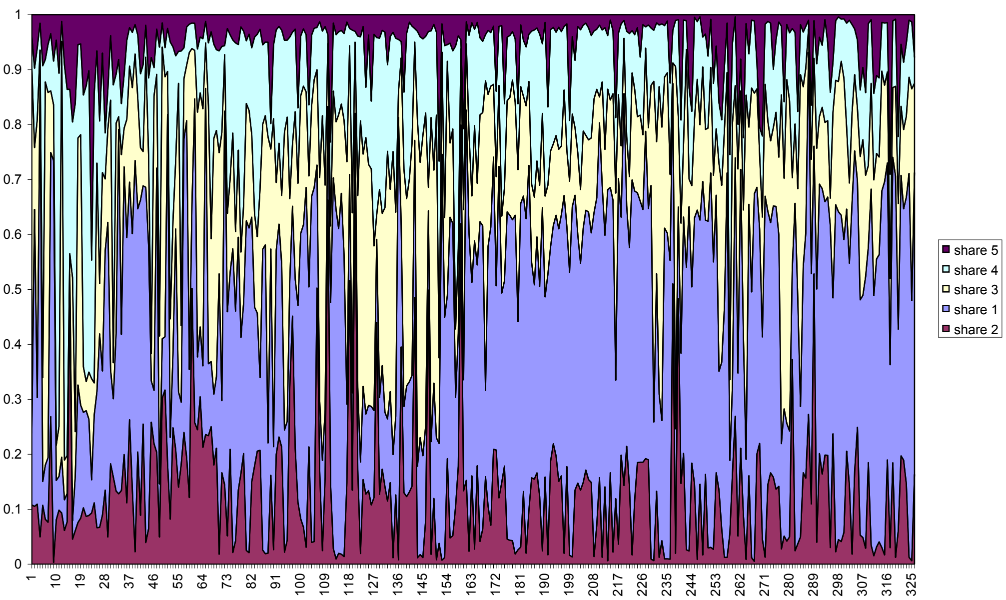


Figure 21:



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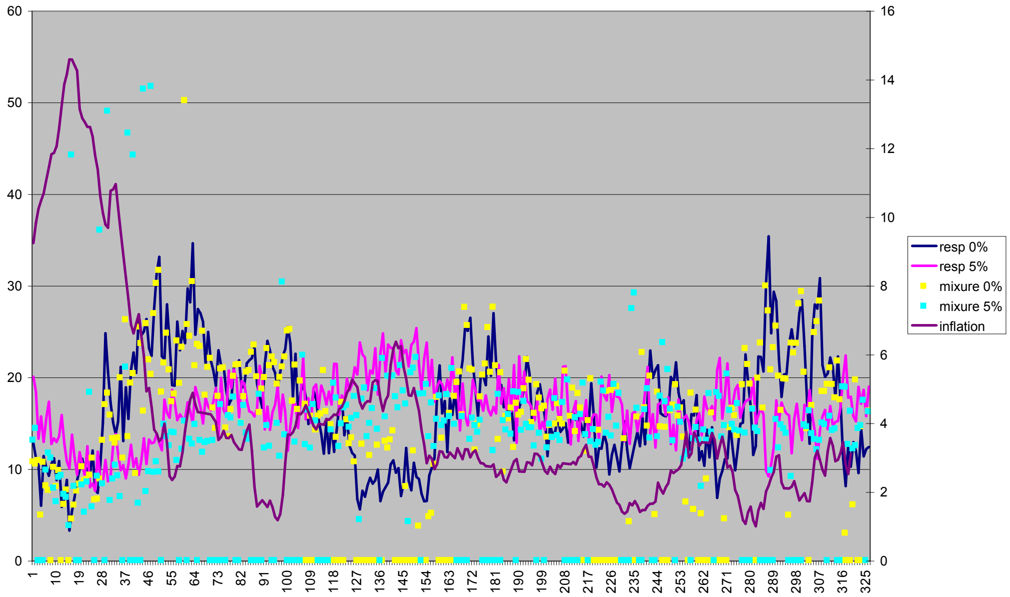
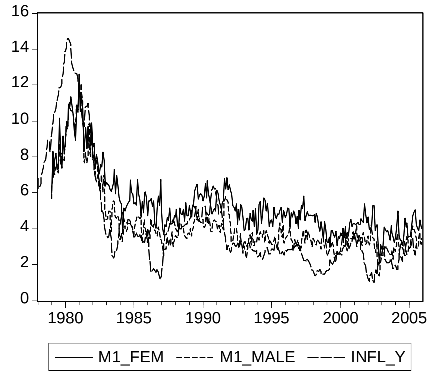
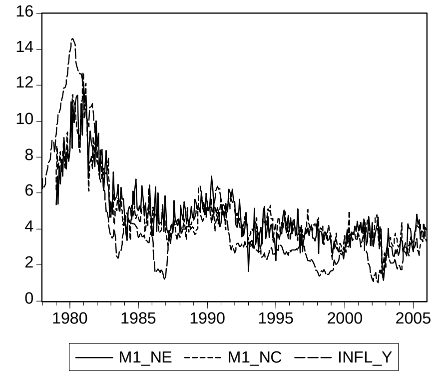
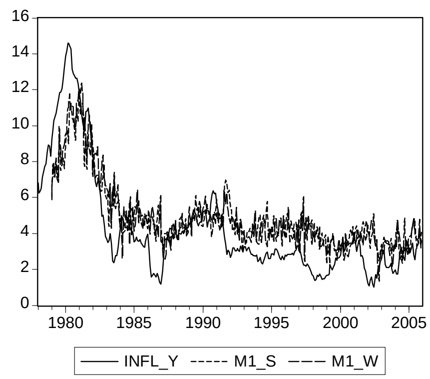
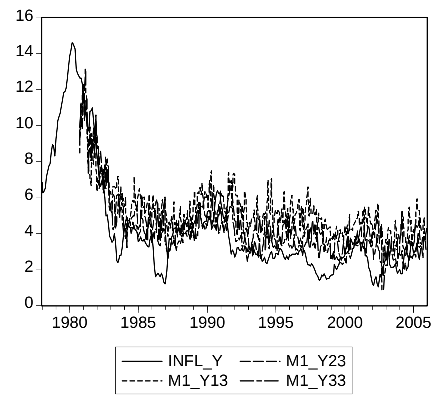


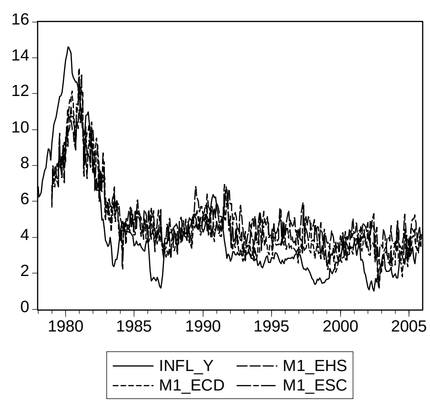
9 Appendix A: Figures across demographic subgroups (figures to be changed)

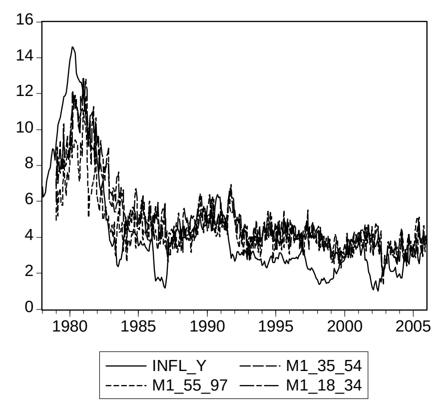


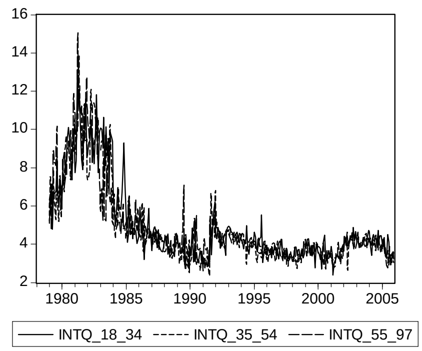








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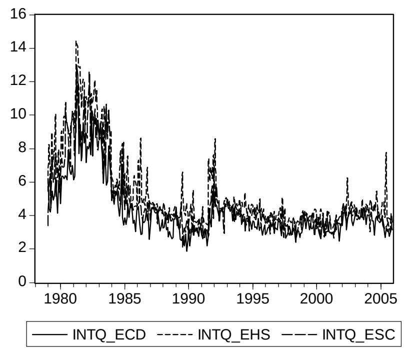
INTQ_35_54
INTQ_55_97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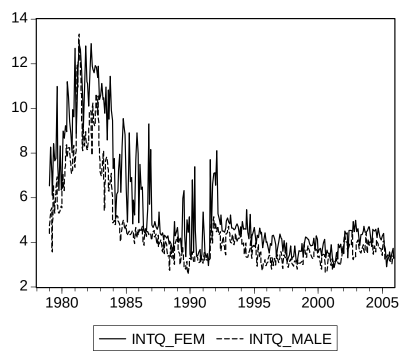
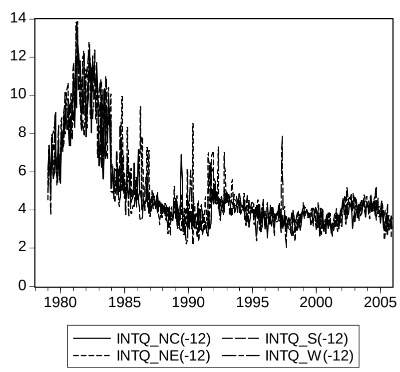
INTQ_EHS
INTQ_ESC


INTQ_FEM
INTQ_MALE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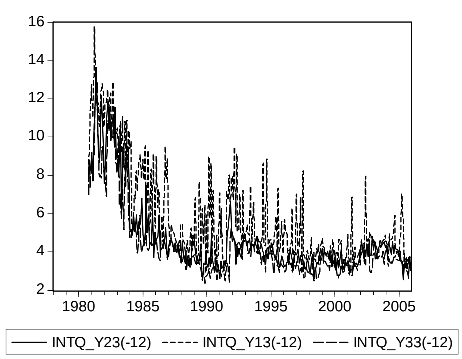
INTQ_S(-12)
INTQ_NE(-12)
INTQ_W(-12)


INTQ_Y23(-12)
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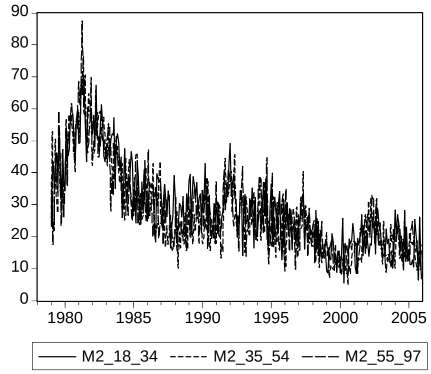
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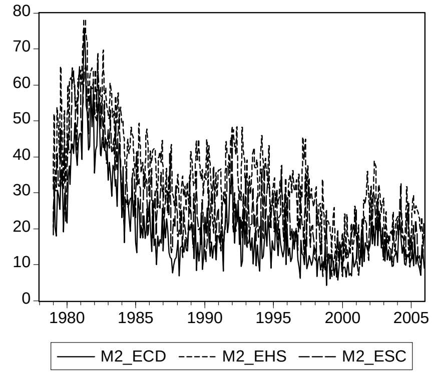
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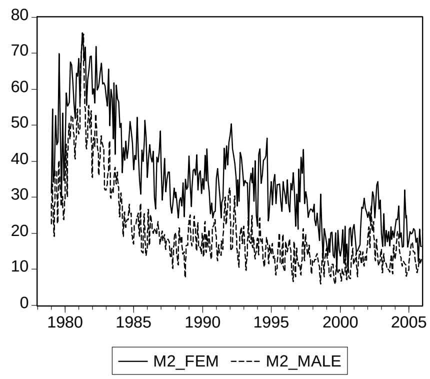
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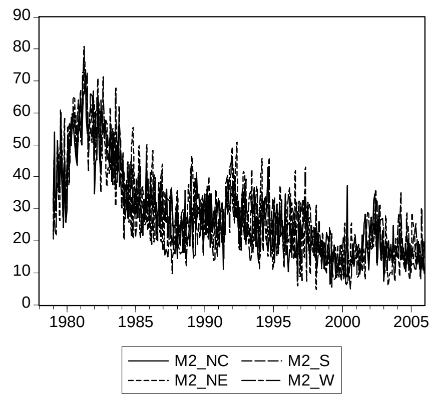
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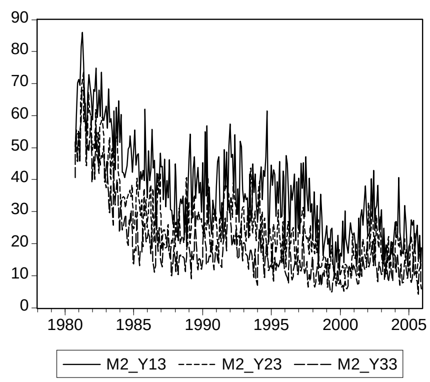
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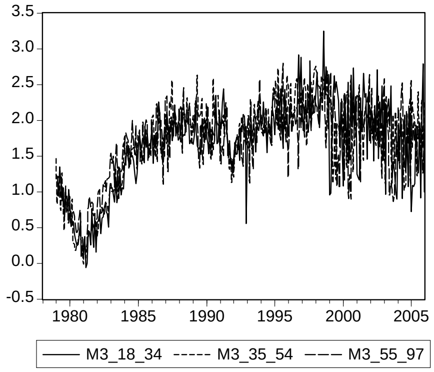
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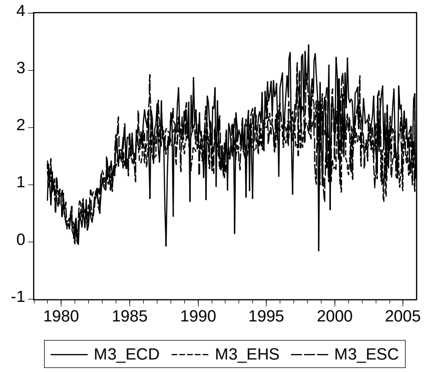
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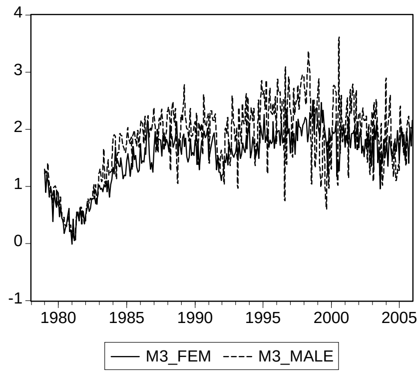
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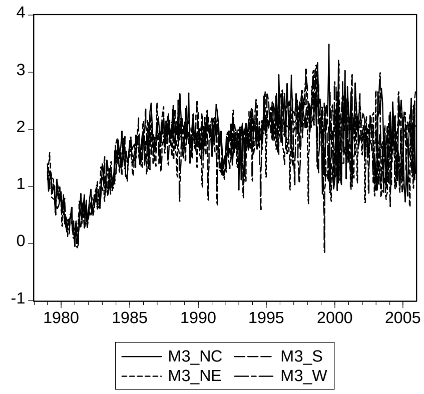
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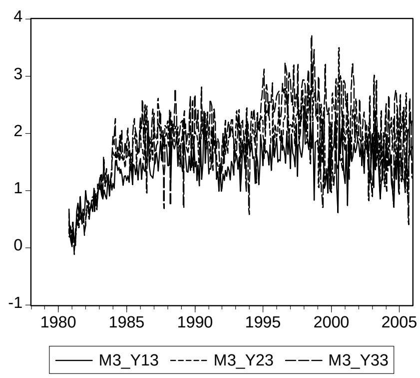
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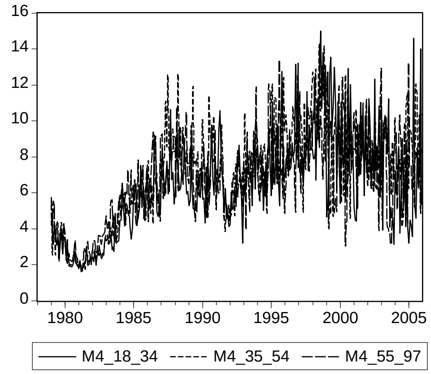
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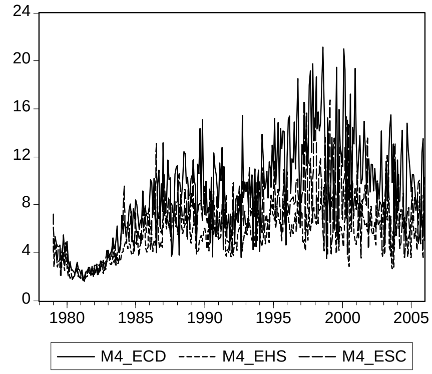
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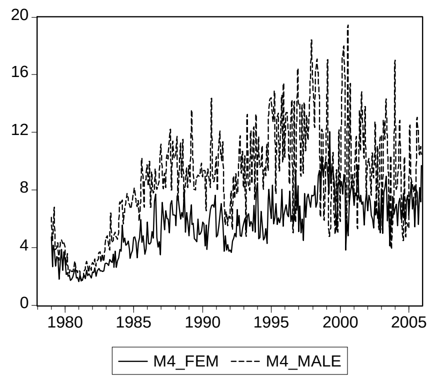
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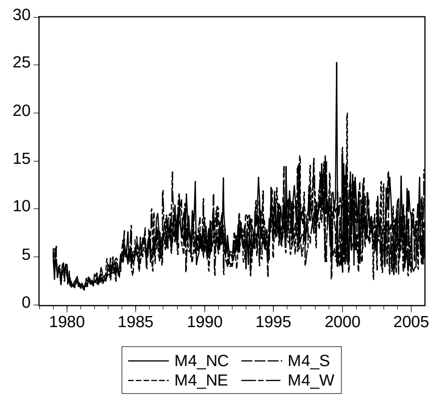
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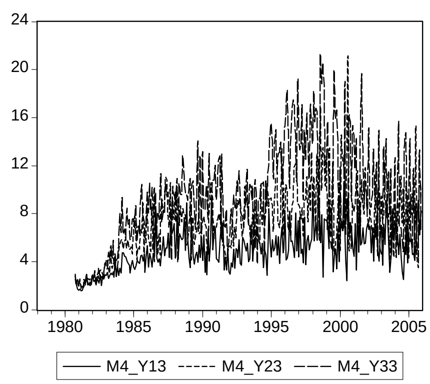
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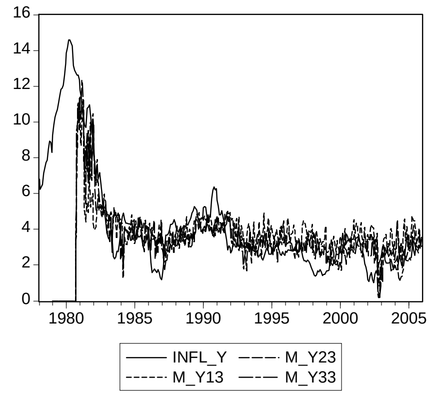
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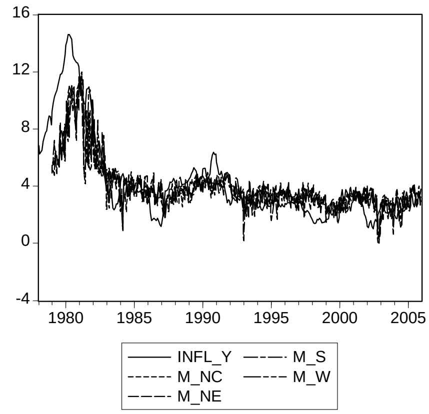
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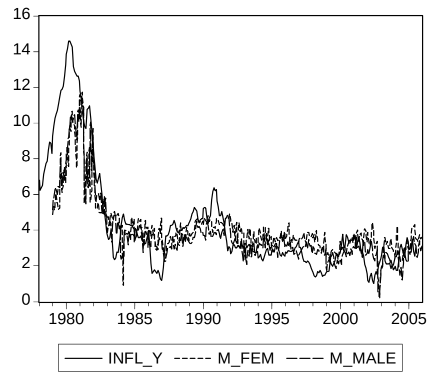
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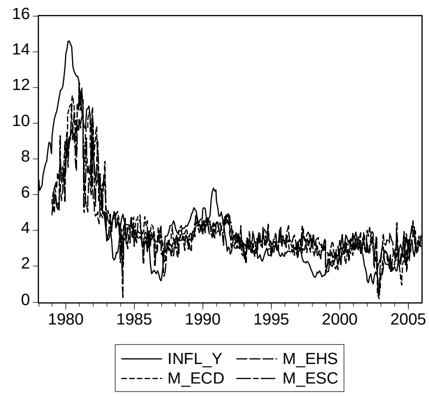
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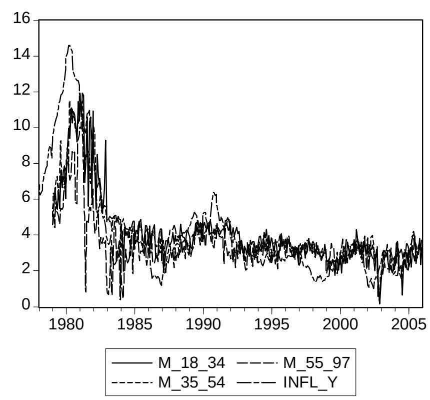
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10 Appendix B: Percentile time series analysis

11 Appendix C: Moments regressions

Table C1:

1st Model-pre1988	Constant	Trend	AR(1)	Cycle	Inflation	Term Str	R^2	DW
Interquantile Range	2.415	-0.010	0.604	-0.045	0.180	0.051	0.782	2.288
	2.994	-1.830	8.893	-0.487	2.716	0.464		
Variance	15.219	-0.035	0.415	-0.629	1.595	0.055	0.824	2.053
	4.447	-1.675	5.451	-1.558	4.732	0.120		
Skewness	0.908	0.004	0.345	0.007	-0.057	0.028	0.943	2.061
	6.367	5.942	4.358	0.727	-5.696	2.285		
Kurtosis	2.433	0.015	0.496	-0.015	-0.140	-0.032	0.895	2.079
	4.621	4.629	6.889	-0.313	-3.705	-0.555		

2nd Model-pre1988

	AR(1)	Cycle	Inflation	Term Str	R^2	DW
Interquantile Range	0.371	0.004	0.234	-0.039	0.349	2.149
	4.389	0.054	3.536	-0.433		
Variance	0.223	-0.806	2.105	-0.874	0.559	2.010
	2.552	-2.488	6.131	-2.344		
Skewness	0.323	0.012	-0.058	0.043	0.631	2.093
	3.719	1.328	-5.336	3.672		
Kurtosis	0.448	-0.019	-0.132	0.041	0.458	2.093
	5.436	-0.472	-3.425	0.840		

Table C2:

1st Model-post1988	Constant	Trend	AR(1)	Cycle	Inflation	Term Str	R^2	DW
Interquantile Range	2.201	-0.001	-0.155	0.215	0.124	0.307	0.392	2.083
	4.604	-0.655	-3.289	3.454	2.585	4.376		
Variance	12.122	-0.026	-0.610	2.122	0.894	0.350	0.717	2.042
	4.650	-3.882	-2.445	5.765	3.449	5.143		
Skewness	2.061	-0.001	0.034	-0.109	-0.044	0.313	0.335	2.152
	8.094	-2.854	2.085	-4.607	-2.594	4.494		
Kurtosis	11.454	-0.002	0.248	-0.923	-0.422	0.109	0.445	1.946
	9.332	-1.002	2.833	-6.818	-4.413	1.489		

2nd Model-post1988

	AR(1)	Cycle	Inflation	Term Str	R^2	DW
Interquantile Range	-0.121	0.223	0.106	0.313	0.224	2.065
	-2.668	2.823	1.549	4.493		
Variance	-0.700	2.893	0.808	0.278	0.416	2.006
	-3.114	6.663	2.421	4.117		
Skewness	0.025	-0.050	-0.015	0.243	0.115	2.068
	1.842	-2.071	-0.728	3.405		
Kurtosis	0.188	-0.608	-0.184	0.141	0.168	1.955
	2.489	-4.356	-1.573	1.952		