

# Fiscal Policy Effects in the European Union\*

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## Abstract

This paper analyzes empirically the impact of fiscal policy on the price level for the cases of Germany and Spain. We investigate whether the fiscal theory of the price level (FTPL) is able to deliver a reasonable explanation for the different performances of the price level in these two countries during recent years. We apply two different approaches. The first is a Bayesian VAR model using sign restrictions to assess the relation between surpluses and public debt. Afterwards, we use a Bayesian regime-switching model to uncover changes in monetary and fiscal policy behavior. The analysis basically shows that in each of the two countries fiscal shocks have a significant impact on the price level. Nonetheless, the FTPL does not deliver a reasonable explanation for the differences in the pattern of inflation between the two countries.

*Keywords:* Fiscal theory, policy interaction, monetary policy, public debt, price level, Euro area

*JEL classification:* E30, E31, E42, E62, E63

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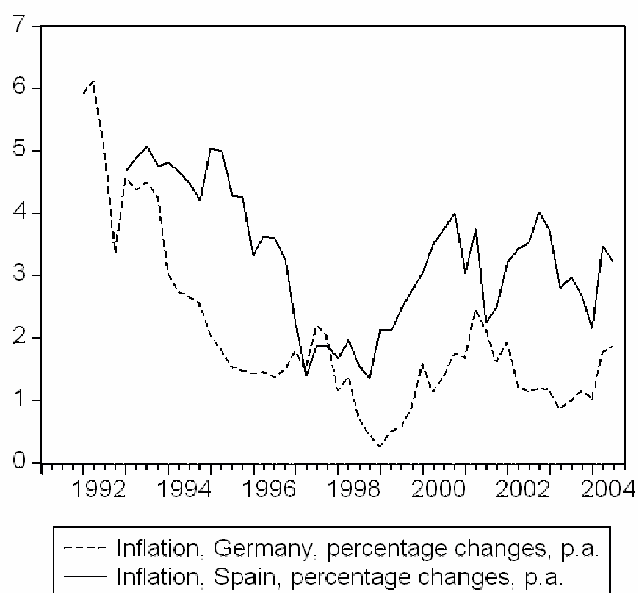


Figure 1: *Inflation in Germany and Spain*

# 1 Introduction

## 1.1 Motivation

Since the introduction of the Euro as the common currency in twelve member states of the European Union (EU) in 2002, there has been a steady debate about the effectiveness of the Stability and Growth Pact (SGP), which requires all countries in the Euro Zone to have a deficit of less than 3% of its GDP. The aim is to prevent excessive government deficits from occurring.

Theory predicts that excessive government deficits may lead to substantial increases in the overall price level, even if the central bank is independent and hardly fighting inflation.

Government deficits have grown in almost all member states of the European Monetary Union (EMU) during recent years. In 2003 Germany, Greece, France and the Netherlands had a deficit ratio of above 3%, while the deficits of Italy and Portugal remained slightly below it. At the same time the rate of inflation in the EMU was with 2.1% considerably modest<sup>1</sup>. This raises the question, whether control of public debt is really a requirement for price stability.

This paper aims to answer this question by investigating German and Spanish data as an example of two countries which performed very differently in terms of inflation during recent years, although both countries were subject to the same monetary policy. Figure 1 provides the run of inflation in both countries for the period 1991-2004. Since 1998, the rate of inflation in Germany is considerably lower than in Spain. We examine if fiscal policy behavior may deliver a possible explanation for this development.

<sup>1</sup>This corresponds to current ECB statistics.

We will base the analysis on two different approaches. The first goes back to Canzoneri, Cumby and Diba (2000). We extend and modify this approach by using Bayesian techniques and sign restrictions for the variables. The second part of the analysis is based on the work of Davig and Leeper (2005) using a Markov-switching model. Again, we use a Bayesian approach. To our knowledge, this is the first attempt to test the relevance of the so-called fiscal theory of the price level (FTPL) for Germany and Spain.

## 1.2 Literature Review

During the 1990s there has been a considerable amount of literature devoted on the impact of fiscal policy on inflation. Cornerstones of this theory are the works of Leeper (1991), Sims (1994), Woodford (1994, 1995, 1996 and 2001) and Cochrane (1998, 2000). While traditional theory regards the stock of money as the sole determinant of the price level, the FTPL argues that if fiscal policy is free to set primary surpluses independently of government debt, fiscal shocks may well have an impact on the price level. Whereas traditional theory assumes that fiscal authorities adjust primary surpluses to guarantee solvency of the government for any price level<sup>2</sup>, the FTPL considers the possibility that fiscal policy is able to set primary surpluses independently of government debt accumulated. As a result the price level will adjust to make the government's intertemporal budget constraint hold at any point of time. Woodford (1995) refers with the terms "non-Ricardian" and "Ricardian" to these two cases of fiscal policy behavior. While "Ricardian" fiscal policy describes the case in which primary surpluses may *not* be set independently of government debt, "non-Ricardian" refers to the latter case.

In both cases the intertemporal budget constraint holds in equilibrium. The crucial difference between the two scenarios is the causal link between prices and surpluses.

Woodford (1996, 1998) argues that fiscal shocks affect aggregate demand in non-Ricardian environments. Households regard government debt as net wealth affecting their future path of consumption due to the exogeneity of government deficits.

Sims (1997) argues that government commitments to stable prices can easily turn out to be unsustainable. Furthermore, there are practical bounds for governments on primary surpluses and unpredictable disturbances to fiscal balance. For a monetary union Sims concludes that generally an interest-rate-pegging policy, which is what a monetary union finally is about, can only work, if each country with an initial level of public debt larger than zero commits itself to some positive level of primary surpluses in the future. From a game theoretic perspective each government has an incentive to deviate from this strategy to increase welfare of its own citizens leading to an upward jump in the price level. The costs of this policy have to be paid by members of the monetary union. This implies that a monetary union can only survive, if national governments have to commit themselves to a deficit or surplus rule, i.e. a limit on borrowing as done by the existing SGP or as Sims argues to a path of some positive primary surpluses.

Cochrane (1998) argues that the "FTPL per se has no testable implications for the time series of debt, surplus and price level". The budget constraint of the government written in nominal terms holds in both Ricardian and non-Ricardian regimes. If this equilibrium is restored by

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<sup>2</sup>Barro (1974).

price or surplus adjustments remains unclear<sup>3</sup>. Hence, all we observe are equilibrium points. Woodford (1995) supports this view saying that it does not make much sense to test the FTPL in empirical terms. Heading in the same direction Buiter (1999) states that “[...] the government’s intertemporal budget constraint is a constraint on the government’s instruments that must be satisfied for all admissible values of the economy-wide endogenous variables.” So what really matters for the characterization of fiscal policy behavior is the question, whether prices or future surpluses of the government adjust to make the government budget constraint hold.

Despite the rather pessimistic view of Woodford there have been some attempts to measure empirically the effect of fiscal policy on the price level. One of the earliest works dealing with this question is the paper of Shim (1984). Shim uses a vector autoregressive (VAR) model to analyze the interdependencies between the rate of inflation and the government budget constraint for the U.S. and 12 other industrialized countries. He finds little evidence for a strong comovement between government debt and the price level for most of the countries investigated.

A recent paper by Canzoneri, Cumby and Diba (2000) investigates U.S. data for the period 1951-1995 with a bivariate VAR model in Surplus/GDP and Liabilities/GDP, both quoted in nominal terms. This VAR specification allows to identify, whether prices or surpluses adjust in order to make the intertemporal government constraint hold. The paper comes to the conclusion that fiscal policy in the U.S. may rather considered to be Ricardian than non-Ricardian.

Bohn (1998) finds out that U.S. fiscal surpluses have responded positively to debt. He argues that this provides evidence that U.S. fiscal policy has been sustainable, and although he does not directly comment on the FTPL, his results are consistent with those of Canzoneri, Cumby and Diba.

Janssen, Nolan and Thomas (2002) analyze the impacts of monetary and fiscal policy on the path of inflation in the UK. This paper is especially remarkable as it is built on almost 300 years of data starting in 1705. They also conclude that there is little econometric evidence that fiscal policy has significantly affected the price level or the overall money supply.

For the EMU, Afonso (2002) concludes, applying a panel data approach, that the FTPL is not supported for the EU-15 countries during the period 1970-2001. The member states of the EMU tend to react with larger future surpluses to increases in the government liabilities. Therefore, fiscal policy may *not* considered to be exogenous implying a rejection of the “non-Ricardian” hypothesis.

So far, there seems to be empirical evidence that Ricardian fiscal policies are possible and likely.

Very recent papers by Davig, Leeper and Chung (2004) as well as Davig and Leeper (2005) analyze regime switches in both fiscal and monetary policy for the U.S. They distinguish between active and passive behavior for monetary and fiscal authorities<sup>4</sup>. Their work shows that tax cuts always generate wealth effects and non-Ricardian outcomes, as long as there is a positive probability for an active fiscal policy in the next period. Therefore, their work may be interpreted as a support of the FTPL mechanism.

Another attempt to examine fiscal policy regimes in the light of Markov switching processes is carried out by Favero and Monacelli (2005). They investigate U.S. data for the period 1960-

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<sup>3</sup>We will comment on this issue in greater detail within the next section of this paper.

<sup>4</sup>The terms active and non-Ricardian fiscal policy are equivalent as well as passive and Ricardian.

2002 and come to similar conclusions, namely that fiscal policy has switched between active and passive regimes.

The remainder of this paper is structured as follows. In section 2 we assess data for Germany and Spain using a Bayesian version of the approach developed by Canzoneri, Cumby and Diba (2000). Following the method of Uhlig (1999) we impose sign restrictions on the impulse responses of the two variables included, i.e. Surplus/GDP and Liabilities/GDP. The rationale for this is outlined throughout the next section. After the interpretation of the results obtained we check in section 3 if these conclusions coincide with those we may draw from the procedure of Davig and Leeper (2005) using a Markov-switching model. Finally, in section 4 we briefly summarize the findings and comment on the policy implications arising from the analysis, i.e. if fiscal policy is a determinant of the price level in Spain or in Germany. Furthermore, we attempt to answer the question whether the FTPL is able to explain the different processes of inflation in these two countries.

## 2 Deficit-Debt Approach

In the following we will basically apply the method of Canzoneri, Cumby and Diba (2000) to find out, whether fiscal policy in Germany and Spain may be regarded as being Ricardian or non-Ricardian. The VAR will be estimated using a Bayesian approach, which imposes sign restrictions on the impulse responses. The rationale for this procedure as well as the details of this empirical approach will be outlined later on.

### 2.1 The Theoretical Foundation of the Model

The government's budget constraint written in nominal terms for period  $t$  is naturally given by

$$B_t = (T_t - G_t) + (M_{t+1} - M_t) + \frac{B_{t+1}}{1 + i_t}, \quad (2.1)$$

where  $M_t$  denotes the stock of base money and  $B_t$  the stock of government debt outstanding at the beginning of period  $t$ . At this point it is important to notice that  $B_t$  and  $M_t$  are quoted in nominal terms and their values are fixed at the beginning of each period. The real value of these two variables is determined by the price level. The difference between taxes  $T_t$  and government expenditures  $G_t$  in period  $t$  yields the primary surplus.  $i_t$  is the nominal interest rate at time  $t$ .

(2.1) states that government liabilities outstanding in period  $t$  have to be repaid by either running a surplus in the same period, monetized by increasing the stock of base money, or financed by issuing new debt at the beginning of the next period.

We divide (2.1) by nominal GDP  $P_t y_t$ . After some rearrangements using simple algebra we obtain

$$\frac{M_t + B_t}{P_t y_t} = \frac{T_t - G_t}{P_t y_t} + \frac{M_{t+1}}{P_t y_t} \frac{i_t}{1 + i_t} + \frac{y_{t+1}/y_t}{(1 + i_t)P_t/P_{t+1}} \frac{M_{t+1} + B_{t+1}}{P_{t+1} y_{t+1}}. \quad (2.2)$$

On the left-hand side of (2.2) we find the ratio of total government liabilities and GDP. As a short form of writing total government liabilities in period  $t$  we use  $L_t$  in the following.

At first glance the right-hand side seems to be somewhat more complicated.  $\frac{T_t - G_t}{P_t y_t}$  is the primary surplus of the government in period  $t$  set in relation to nominal GDP. When we think of the government as renting the money supply to the private sector<sup>5</sup> charging  $\frac{i_t}{1 + i_t}$ , then the second term represents the central bank transfers also set in relation to current nominal GDP. Thus, the first two terms on the right-hand side of (2.2) add up to the total surplus-GDP ratio of the government, which we will denote in the following by  $S_t/Y_t$ . In the notation introduced above  $\frac{M_{t+1} + B_{t+1}}{P_{t+1} y_{t+1}}$  boils down to  $L_{t+1}/Y_{t+1}$ , where  $Y_t = P_t y_t$ . Finally,  $\frac{y_{t+1}/y_t}{(1 + i_t)P_t/P_{t+1}}$  has as numerator real growth of GDP and the denominator gives the real interest rate using the well-known Fisher equation. Thus, we may interpret this term as a discount factor of next period's total government liabilities. In the following we will refer to this discount factor as  $\beta_t$ .

This enables us to simplify (2.2) so that we obtain

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<sup>5</sup>Obstfeld and Rogoff (1996), p. 537.

$$\frac{L_t}{Y_t} = \frac{S_t}{Y_t} + \beta_t \frac{L_{t+1}}{Y_{t+1}}. \quad (2.3)$$

Iterating this equation forward and recursively substituting  $\frac{L_{t+1}}{Y_{t+1}}$  we end up with

$$\frac{L_t}{Y_t} = \frac{S_t}{Y_t} + E_t \sum_{j=t+1}^{\infty} \left( \prod_{k=t}^{j-1} \beta_k \right) \frac{S_j}{Y_j}, \quad (2.4)$$

with  $E_t$  being the expectations operator conditional on information available at time  $t$ . This flow budget constraint has to be fulfilled at any point of time, which can be achieved in two ways<sup>6</sup>:

1. Consider the case in which the surpluses follow an endogenous process so that (2.4) is fulfilled by adjustments in the sequence of  $S_t$ , whereby the values of the discount factor  $\beta_t$  and nominal GDP  $Y_t$  are determined outside the system. We refer to this type of fiscal policy behavior as *Ricardian* as both real GDP and inflation remain unaffected by changes of the fiscal variables.
2. Let the sequence of primary surpluses be determined by an arbitrary exogenous process. Now, to make (2.4) hold, either the discount factor or the liabilities-GDP ratio have to move. As mentioned before we assume nominal government liabilities to be fixed at the beginning of each period. That means that the numerator in  $\frac{L_t}{Y_t}$  remains unchanged as it was the case before in the Ricardian scenario. Hence, equality of (2.4) can only be restored through  $Y_t$  in the denominator which also implies an impact on the discount factor  $\beta_t$ . Fiscal policy is said to be *non-Ricardian*.

That means, whenever surpluses are set independently of the stock of government debt accumulated, nominal income is determined by fiscal policy actions. By definition, nominal GDP is the product of real GDP  $y_t$  and the price level  $P_t$ . Thus, an increase in nominal GDP will generally affect both real GDP as well as the price level<sup>7</sup>.

Using these basic insights in the FTPL we now try to figure out which of the variables considered above responds to changes in the fiscal variables using German and Spanish data.

## 2.2 The Model

In the following we investigate how total government liabilities divided by nominal GDP react to changes in the surplus-GDP ratio.

Assume that  $\frac{S_t}{Y_t}$  increases in period  $t$ . Then, if fiscal policy is Ricardian we should either expect future surpluses to decrease or to use the surplus to repay the debt, if possible. Thus,

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<sup>6</sup>Note that the stock of total nominal government liabilities  $B_t + M_t$  is fixed at the beginning of each period.

<sup>7</sup>A theoretical quantification of the impact fiscal policy has on both real GDP and inflation can be found for instance in Woodford (1996).

an indicator for a Ricardian policy behavior would be a negative or zero-response of  $L_t/Y_t$  to a positive shock in  $S_t/Y_t$ . This Ricardian interpretation would only be reasonable, if the surplus shock is persistent in a sense that it does not immediately changes in sign to a deficit shock so that the impact is immediately diminished. For this reason it will be important to regard the pattern of  $S_t/Y_t$  for conclusions about the character of the shock.

The non-Ricardian case is somewhat easier to describe in terms of the results we should expect. A non-Ricardian fiscal policy is definitively at work if the reaction of  $L_t/Y_t$  is positive to a positive shock in  $S_t/Y_t$  for reasons which should be obvious from equation (2.4). Furthermore, a negative response of  $L_t/Y_t$  should also be considered to be non-Ricardian, if  $S_t/Y_t$  is significantly negatively autocorrelated, i.e. the shock is not persistent and quickly changes in sign.

Formally we analyze a VAR of the form

$$\begin{bmatrix} S_t/Y_t \\ L_t/Y_t \end{bmatrix} = const + \sum_{s=1}^p \underbrace{\begin{bmatrix} B_{11}(s) & B_{12}(s) \\ B_{21}(s) & B_{22}(s) \end{bmatrix}}_{B(s)} \begin{bmatrix} S_{t-s}/Y_{t-s} \\ L_{t-s}/Y_{t-s} \end{bmatrix} + \underbrace{\begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}}_{u_t}, \quad (2.5)$$

where the  $B(s)$  is a set of  $p$  ( $m \times m$ ) coefficient matrices with  $m$  denoting the number of dependent variables included ( $m = 2$ ).  $u_t$  is Gaussian with zero mean and

$$E[u_t u_t' | (S_{t-s}/Y_{t-s}; L_{t-s}/Y_{t-s})] = \Omega \quad (2.6)$$

with  $\Omega$  being the positive definite symmetric and time-invariant covariance matrix of size ( $m \times m$ ). It is often referred to the inverse of  $\Omega$  as the precision matrix  $H$ . Following Canzoneri, Cumby and Diba (2000) we set the lag length  $p$  to two.

In opposite to Canzoneri, Cumby and Diba we choose a Bayesian instead of a classical approach to avoid any problems arising from potential non-stationarity of the data. Bayesian inference has the advantage that it does generally not raise specific difficulties like classical inference when the data analyzed is non-stationary. The application of Bayes' theorem does not require the data to be stationary. The same is true for the conditional log likelihood function which is one pillar of the Bayesian approach in the determination of the parameters' joint posterior probability density function (pdf). Furthermore, the Bayesian approach allows us to express the information we have about the parameters under consideration by specifying the prior density of the parameters, which includes the specification of our beliefs about the presence of a unit root in the data analyzed.

When examining the data for both Germany and Spain it should be quite obvious that government liabilities as well as government liabilities divided by GDP is steadily increasing for almost the entire observation period. That means that the sample data is not mean reverting and hence not stationary in a common sense. When we perform a formal ADF test for the series of Liabilities/GDP for each of the two countries the null of a unit root may not be rejected. This result is interesting. Of course, the result, i.e. the non-stationarity of Liabilities/GDP, is statistically reasonable, but from an economic point of view it implies that any point between plus and minus infinity is equally likely. If this were really true for the series of Liabilities/GDP, fiscal policy would definitively not be sustainable as government debt may grow without bound. A classical approach would require an estimation of the VAR in differences in order to obtain



statistically correct results. But this procedure would be problematic from an economic point of view for the same reasons as stated above, i.e. we would implicitly assume a random walk behavior for the pattern of government debt, which leads to a non-sustainability of fiscal policy. By applying a Bayesian approach we still allow for these scenarios to occur, when choosing an appropriate prior, but we do not impose any restrictions of stationarity or non-stationarity on the data during the analysis. Thus, we do not assume that the series of Liabilities/GDP will actually tend towards infinity as we would do, when accepting the null of the ADF test and consequently estimating the VAR in differences.

From a Bayesian perspective the data is seen as deterministic, whereas the model's parameters are stochastic. Within a Bayesian analysis we aim at finding the posterior distribution of the parameters. This posterior distribution is obtained in two steps. First, we choose a prior pdf, which expresses our prior beliefs about the coefficients in  $B(s)$  and the covariance matrix  $\Omega$ . Afterwards, we may compute the likelihood function, i.e. the joint pdf of the data conditional on the unknown parameters.

As Uhlig (1994) suggests it is reasonable to assume a Normal Wishard distribution for the prior and the posterior pdf,  $\phi_{NW}(B, H|\bar{B}, N, S, v)$ , with  $\bar{B}$  being the mean coefficient matrix of size  $(p \times m)$ ,  $S$  the positive definite mean covariance matrix of size  $(m \times m)$ ,  $N$  a positive definite matrix of size  $(p \times p)$  and finally  $v \geq 0$  denotes the degrees of freedom to describe the uncertainty about  $B$  and  $\Omega$  around  $(\bar{B}, S)$ . The precision matrix of the prior distribution  $H$  follows a Wishard distribution of the form  $W_m(S^{-1}/v, v)$ . For the specification of the prior we have to choose values for  $\bar{B}, S, N, v$ .

We assume that our prior information is diffuse so that basically the parameters in  $B(s)$  may take any value in the interval  $-\infty$  to  $\infty$  with equal probability. This implies that our prior beliefs are best represented by a flat prior. We obtain a flat prior by setting  $N_0 = v_0 = 0$  and  $\bar{B}_0$  as well as  $S_0$  arbitrarily.

Thus, the analysis applies to both explosive and nonexplosive cases. If the process is actually explosive or not, will be determined by the sample information we have<sup>8</sup>. That means that inferences are unaffected by information external to the current data<sup>9</sup>.

As stated above we aim at examining the reaction of  $L_t/Y_t$  to a positive shock in  $S_t/Y_t$ . Generally, this reaction or impulse response may be both positive and negative in sign. When we do not differentiate between these two scenarios in the analysis we may possibly obtain an "average" response, which may be misleading in measuring and evaluating the impacts of a shock. Furthermore, we know from the theoretical considerations given above that a positive response of  $L_t/Y_t$  leads to a non-Ricardian interpretation of the data in the corresponding period.

For the case differentiation between responses which are positive and negative in sign we have chosen the pure-sign-restriction approach by Uhlig (2004). Using this approach we only consider those cases in which the orthogonalized impulse responses head for the desired direction in the period the shock takes place<sup>10</sup>. We divide the sample in those impulse responses which are

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<sup>8</sup>For a general discussion on choosing the appropriate prior pdf, the interested reader may have a look at Zellner (1971).

<sup>9</sup>See Gelman et al. (1995).

<sup>10</sup>The sign restriction is binding for only one period.

candidates for a Ricardian interpretation and those which are candidates for a non-Ricardian interpretation, i.e.

1. A positive (negative) shock in  $S_t/Y_t$  immediately leads to a zero or negative (positive) impact on  $L_t/Y_t$ .
2. A positive (negative) shock in  $S_t/Y_t$  immediately leads to a positive (negative) impact on  $L_t/Y_t$ .

Our focus will then lie on the question, if the further process, i.e. the process of the two variables after the shock has occurred also matches a Ricardian pattern in case of scenario 1 and how many of the draws generally match scenario 1 and 2. We made 1,000 draws from the reduced-form posterior density and for each reduced-form draw 50 draws of the  $\alpha$ -vector<sup>11</sup>.

## 2.3 The Data

The data we use corresponds to statistics of the International Monetary Fund. All data is denoted in nominal terms. For monetary liabilities  $M_t$  we take money in circulation. Government debt  $B_t$  is represented by the total government debt which includes in the case of Germany both debt of federal and federal state authorities.  $L_t$  is then defined as the sum of total government debt  $B_t$  and money in circulation  $M_t$ . For  $S_t$  we correspondingly choose to take the difference of total government revenues and expenditures. Nominal GDP  $Y_t$  is seasonally adjusted. The data has a quarterly frequency and starts for Germany with the 1<sup>st</sup> quarter 1970 and ends in the 4<sup>th</sup> quarter 1998. To take the German reunification into account we split the German data in two periods, that means we first analyze the time span 1970-1990 and afterwards 1991-1998. Unfortunately, the Spanish data covers only the period from 1986-1998. Data before 1986 is only partially available.

## 2.4 The Results

In the following we give the results for both countries in form of the impulse responses of the two variables to a one-standard-deviation shock in  $S_t/Y_t$ . All impulse responses show the median response as well as the 18% and 84% quantiles corresponding to a one standard deviation band if the distribution was normal.

### 2.4.1 Germany

Figure 2 shows the results obtained for Germany based on the sample 1970-1990. We can see that a positive shock in  $S_t/Y_t$  leads to a significant and persistent negative impact on  $L_t/Y_t$  in the first period. This should not be surprising as we used a sign restriction on the impulse responses to exclude all those cases in which a positive Surplus-GDP shock leads to a positive

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<sup>11</sup>Details about the meaning of the  $\alpha$ -vector may be found in the appendix.

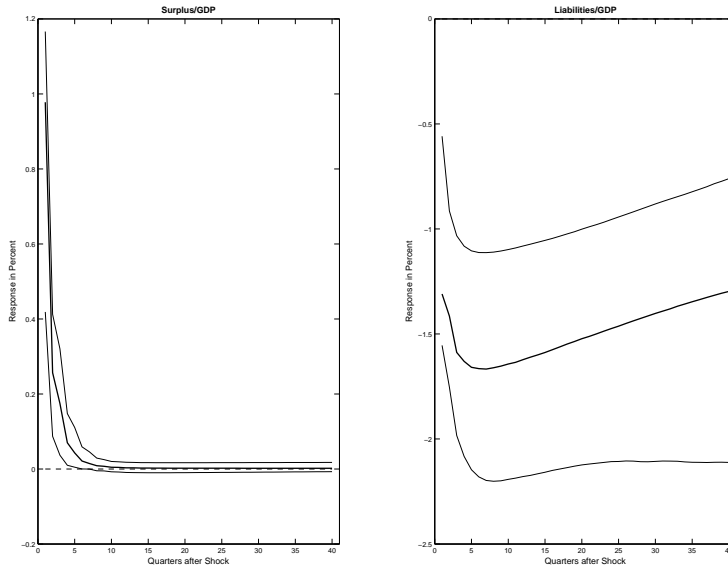


Figure 2: *Germany, Response to a Surplus/GDP Shock for the Period 1970-1990 in % with 68% Error Bands.*

impact on  $L_t/Y_t$ . The fact that the impact of the shock seems to be persistent and significant in the process of  $L_t/Y_t$  for a horizon of 10 years suggests a fairly Ricardian interpretation of figure 2. The initial shock vanishes quite fast after 2.5 years, but it does not change in sign which gives further evidence for a Ricardian fiscal policy behavior<sup>12</sup>. That means that with the sign restriction imposed, we obtained those results which allow for a Ricardian interpretation. More important now is to consider the fraction of draws that match the priors for the sign restriction. This number was computed as 72.43%. That means that about 30% of all impulse responses drawn from the posterior distribution does not follow the sign restriction imposed which may be seen as a necessary condition for a fiscal policy to be Ricardian. Thus, about one-third of the draws seem to follow a non-Ricardian policy behavior.

Figure 3 shows the behavior of the two variables when hit by a one-standard-deviation shock in  $S_t/Y_t$  for Germany during the period 1991-1998. We can see that both processes are somewhat different to the ones we obtained before. Again, as induced by the sign restriction the shock in  $S_t/Y_t$  is positive for the first period. In opposite to the earlier sample the Surplus-GDP ratio changes in sign after about two quarters. The lower error band has its minimal value at about -0.25%. This seems to suggest a rather non-Ricardian policy behavior since the initial shock itself is hardly persistent.  $L_t/Y_t$  reacts negatively as indicated by the the upper error band which remains strictly below zero. Although the impact on  $L_t/Y_t$  is significant and persistent, it is questionable if figure 3 shows Ricardian results. Furthermore, for about 35% of the draws

<sup>12</sup>We also estimated the VAR with 4-6 lags. The results obtained were similar.

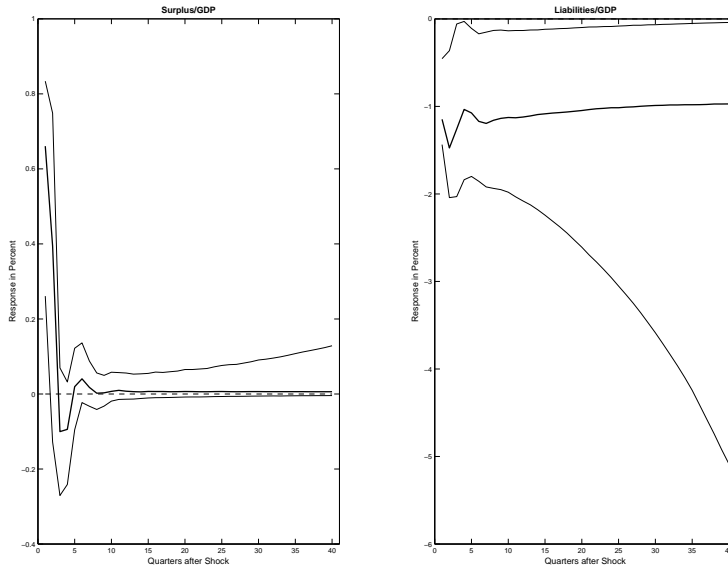


Figure 3: *Germany, Response to a Surplus/GDP Shock for the Period 1991-1998 in % with 68% Error Bands.*

positive Surplus-GDP shocks lead to a positive impact on the Liabilities-GDP ratio and hence to a clearly non-Ricardian pattern.

Up to here we may summarize for the case of Germany that for the period 1970-1990 fiscal policy may be considered to be mainly Ricardian. Nonetheless, there is a substantial part of draws which does not match the prerequisites constituting a Ricardian policy. From 1991 onwards it is hard to find sufficient evidence for Ricardian fiscal policy behavior as the surplus shock vanishes quite rapidly.

### 2.4.2 Spain

The results for Spain are given in figure 4. Basically they show the same pattern as those for Germany during the period 1970-1990. After the shock has occurred,  $S_t/Y_t$  moves quite rapidly towards zero. The error bands are almost symmetric around zero within a range of less than 0.1%. Thus, although the lower error band becomes smaller than zero, we should not regard this as a violation of the Ricardian requirements since the extent of the change in sign is considerably small. Also  $L_t/Y_t$  delivers a purely Ricardian picture under the sign restriction as the initial negative response is persistent and significant. With 59.8% of the draws matching the sign restriction of scenario 1 the share of impulse responses that are Ricardian candidates is lower than in Germany during 1970-1990. Non-Ricardian outcomes seem to be more likely.

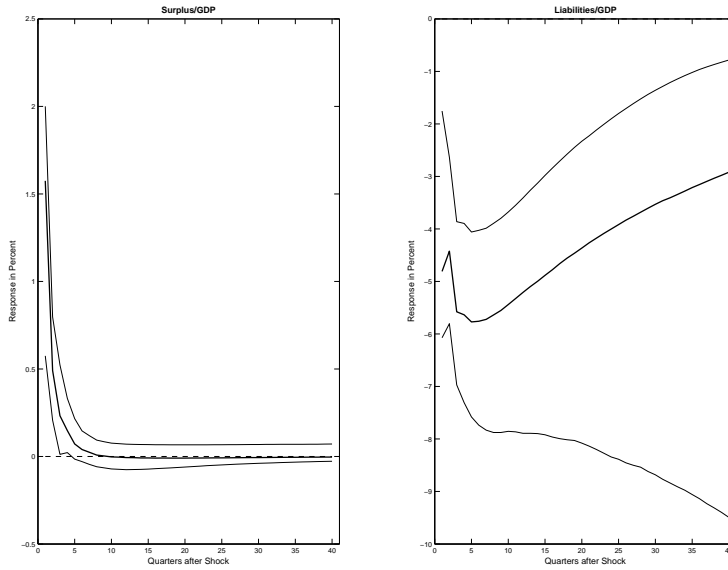


Figure 4: *Spain, Response to a Surplus/GDP Shock for the Period 1986-1998 in % with 68% Error Bands.*

## 2.5 Conclusions and Outlook

So far, the analysis shows Ricardian and non-Ricardian equilibria in both countries. Ricardian outcomes seem to be slightly more frequent as non-Ricardian. During the period 1991-1998 non-Ricardian characteristics seem to be predominating in Germany's fiscal policy. Also Spain shows a large share of non-Ricardian equilibria.

The approach chosen to test for the occurrence of non-Ricardian equilibria is fairly simple, but allows to figure out whether the general requirements for a Ricardian fiscal policy are fulfilled. While common VAR approaches yield some kind of average responses to the shock in question, the sign restriction used here allows us to differentiate between those impulse responses that are potential candidates for Ricardian equilibria and those which are not. Unfortunately, it does not tell us anything about the temporal distribution of the two policy states. Besides this drawback the previous analysis exhibits another problem, which is less obvious. Davig, Leeper and Chung (2004) state that the approach applied above considers equilibria that “are potentially ones with an unbounded debt-output ratio”. Davig, Leeper and Chung argue that the deficit-debt approach does not require the fiscal response to be strong enough to make the evolution of government debt stable. Suppose there is a positive shock in the Surplus-GDP ratio in period  $t$  and  $L_t/Y_t$  does - as we imposed by the sign restriction - respond negatively. Furthermore, let us assume that the responses of both variables are persistent and significant in the desired direction as outlined above so that we may conclude that fiscal policy seems to be Ricardian. For this case Davig, Leeper and Chung say that a significant negative impact on  $L_t/Y_t$  does not necessarily

lead to a Ricardian fiscal policy behavior, as there may be an unobserved impact on  $Y_t$  in the denominator so that not the entire surplus is used to repay the outstanding debt. This basically allows for unbounded debt-output ratios.

The preceding analysis should be seen as a first indicator whether Ricardian or non-Ricardian equilibria have occurred. Thus, our analysis for Spain and Germany has shown that there have not only been potential Ricardian equilibria throughout the sample but more important that there have definitively been non-Ricardian outcomes. This is a quite powerful result, which holds regardless of the arguments made by Davig, Leeper and Chung.

In the following section we proceed with the approach of Davig and Leeper (2005) based on a Markov-switching model.

### 3 Regime-Switching Approach

Throughout this section we consider a Bayesian analysis of the Markov-switching approach chosen by Davig and Leeper. We aim at uncovering changes in monetary and fiscal policy behavior with the help of simple policy rules.

Before we start with the actual analysis, we first provide the reader with the definitions used in the following and the theoretical considerations underlying the empirical model. We then give a brief introduction to Bayesian analysis of linear Markov-switching regression models. More mathematical details may be found in the appendix.

#### 3.1 Theoretical Background and Definitions

The FTPL implies that both the specification of monetary and fiscal policy is important in determining the price level. Whenever fiscal policy becomes active, i.e. deficits are exogenous and set regardless of public debt outstanding, fiscal shocks necessarily have an impact on the price level. The size of this impact is among other things determined by the characteristics of monetary policy.

We model the behavior of monetary and fiscal authorities by assuming the following policy rules:

1. For monetary policy we estimate a Taylor rule specification, which expresses the nominal interest rate,  $i_t$ , as a function of inflation,  $\pi_t$ , and output,  $Y_t$ , at time  $t$ ,

$$i_t = \alpha_0(S_t^M) + \alpha_\pi(S_t^M)\pi_t + \alpha_Y(S_t^M)Y_t + \sigma_i\varepsilon_t^i. \quad (3.1)$$

$S_t^M$  denotes the state of monetary policy at time  $t$ . We assume that monetary regimes evolve according to a Markov chain with corresponding transition matrix  $P^M$ . We allow for two different states of the parameters.

2. Following Davig and Leeper we choose a policy rule that links current government revenues to output, government expenditures and last period's government debt, i.e.

$$\tau_t = \gamma_G(S_t^F)G_t + \gamma_Y(S_t^F)Y_t + \gamma_B(S_t^F)B_t + \sigma_\Delta \varepsilon_t^\Delta, \quad (3.2)$$

where  $S_t^F$  is the fiscal policy regime at time  $t$ , which likewise the monetary policy regime follows a Markov chain with a transition matrix  $P^F$ . The remaining variables correspond to the conventional notation:  $\tau_t$  denotes the government's tax yields in period  $t$ ,  $B_t$  stands for public debt outstanding and finally  $Y_t$  is output in period  $t$ .  $\varepsilon_t^\Delta$  is an *i.i.d.* random shock and  $\sigma_\Delta$  the corresponding time-invariant standard error.

By estimating the policy rules as given above we try to figure out, if and when monetary and fiscal policy have been active or passive. Following Taylor (1993) we consider monetary policy to be active if  $\alpha_\pi > 1$  and passive if  $\alpha_\pi \leq 1$ . For fiscal policy to be passive it requires that  $\gamma_b > 0$  so that a larger stock of public debt outstanding significantly decreases government deficits. Correspondingly, the case in which  $\gamma_B \leq 0$  is referred to as an active fiscal policy behavior.

Theory has shown that only a combination of active monetary policy and passive fiscal policy leaves the price level unaffected by fiscal policy shocks. Following the literature we refer to this scenario as a *Ricardian regime*.

### 3.2 Bayesian Analysis of Markov-Switching Models

The Bayesian analysis of Markov-switching models goes back to McCulloch and Tsay (1993). They show that Bayesian estimation of Markov-switching models is kept relatively simple when using the Gibbs sampler. Gibbs sampling belongs to the class of iterative Monte Carlo methods. It solves the problem of drawing samples from a multivariate density by drawing successive samples from corresponding univariate densities. The exposition given in the following is based on Harris (1999) and Krolzig (1997).

Consider a VAR<sup>13</sup> of order  $p$ , where the parameters can take on  $k$  different states  $S$ ,

$$x_t = \mu(S_t) + \sum_{h=1}^p B_{S(t)}^h x_{t-h} + \varepsilon_t(S_t), \quad (3.3)$$

where  $\varepsilon_t(S_t)$  is an normally distributed *i.i.d.* error term with mean zero and regime-dependent covariance matrix  $\Omega(S_t)$ .  $\mu(S_t)$  denotes a constant and  $B_{S(t)}^h$  is the matrix of coefficients for the  $h^{th}$  lag included in state  $S_t$ . Furthermore, we define the transition probabilities for a switch from regime  $i$  to regime  $j$  as  $p_{ij} = p(S_t = j | S_{t-1} = i)$ . We summarize these probabilities in the transition matrix  $P$  with size  $(k \times k)$ .

Let  $\lambda$  denote the set of all unknown parameters, i.e.

$$\lambda = [\mu(1), \dots, \mu(k), B(1), \dots, B(k), \Omega(1), \dots, \Omega(k), P].$$

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<sup>13</sup>We consider the more general case of a multivariate process. The results similarly apply to univariate processes.

In partitioned notation this boils down to  $\lambda = [\Theta, P]$ . Inference on  $\lambda$  depends on the posterior distribution

$$p(\lambda|X) \propto \pi(\lambda)p(X|\lambda), \quad (3.4)$$

where  $X' = (x'_1, \dots, x'_T)$  is the vector of observations and  $\pi(\lambda)$  the prior for the parameter vector. Since we are in Markov-regime switching environment, we have additional unknown parameters given by the unobservable states. Therefore, the posterior density (3.4) is obtained by the integration of the joint probability distribution with respect to the state vector  $S$ , i.e.

$$p(\lambda|X, \lambda) = \int p(\lambda, S|X, \lambda)dS. \quad (3.5)$$

Using the Gibbs sampler we may draw successive samples from univariate distributions for  $\lambda$  and  $S$ , namely  $Pr(S|X, \lambda)$  and  $p(\lambda|X, S)$  instead of the multivariate distribution  $p(\lambda, S|X)$ . The Gibbs sampler constructs a Markov chain on  $(\lambda, S)$  such that the limiting distribution of the chain is the joint distribution of  $p(\lambda, S|X)$ . There are two types of Gibbs sampler, single-move and multi-move, which differ in the way the states  $S$  are generated. We apply multi-move sampling as it - according to Liu, Wong and Kong (1994) - will lead to a faster convergence than single-move sampling.

The idea of multi-move Gibbs sampling is to draw all states in  $S$  at once conditional on the observations. The starting point is to make use of the structure of the underlying Markov chain, i.e.

$$Pr(S|X, \lambda) = Pr(S_T|X, \lambda) \prod_{t=1}^{T-1} Pr(S_t|S_{t+1}, x_t, \lambda). \quad (3.6)$$

The probabilities  $Pr(S_T|X, \lambda)$  can be calculated using the filter introduced by Hamilton (1989) after having chosen initial values for  $Pr(S_0|X)$ . As we are not able to say anything about  $S_t$  for  $t < 1$ , we assume that the economy was in a steady state in  $t = 0$ . This enables us to choose steady-state probabilities for  $Pr(S_0|X)$ , which are easy to compute. We then may generate  $Pr(S_T|X, \lambda)$ , which allows us to compute  $Pr(S_t|S_{t+1}, x_t, \lambda)$  by

$$Pr(S_t|S_{t+1}, x_t, \lambda) = \frac{Pr(S_t, S_{t+1}|x_t, \lambda)}{Pr(S_{t+1}|x_t, \lambda)} = \frac{Pr(S_{t+1}|S_t)Pr(S_t|x_t, \lambda)}{Pr(S_{t+1}|x_t, \lambda)}. \quad (3.7)$$

The regimes can now be jointly generated according to (3.6). It is then possible to draw the unknown parameters from the conditional densities

$$p(\Theta_j|S, \Theta_{-j}, X) \propto L(X|S, \lambda) \cdot p(\Theta_j) \quad (3.8)$$

$$p(P|S, \Theta, X) \propto p(S_q|P) \prod_{t=q+1}^T p(S_t|S_{t-1}, P) \cdot p(P). \quad (3.9)$$

In the following section we give a brief overview of the data we used for the analysis and then provide the results obtained. Some further details on the mathematical backgrounds of Bayesian analysis of Markov-switching models may be found in the appendix.



### 3.3 The Data

For government debt we take the data as described in section 2.3. In contrast to the deficit-debt approach, GDP is provided in real terms. Government expenditures are related to total expenditures. German nominal interest rates are represented by the 3-month interbank deposit rate. For Spain we take the so-called Bank of Spain rate. The rate of inflation is for both countries given by annual changes in the CPI. Except for the interest rates we computed annual percentage changes for all variables. The data set corresponds to IMF statistics.

### 3.4 The Results

In the following we provide the estimated parameters of the fiscal and monetary policy rule as well as the temporal distribution of the regimes. For the estimation we use a Matlab-based code which makes 30,000 draws for each scenario from the corresponding posterior distribution. The prior pdf of the transition probabilities  $p_{ij}$  are assumed to follow a  $\beta$ -distribution.

#### 3.4.1 Germany

Figure 5 shows the temporal distribution of the regimes underlying the monetary policy rule during period the 1970-1990. We observe significant switches from regime 1 to regime 2 and vice-versa. Regime 2 was predominating German monetary policy during the observation period, particularly the 1980s. The parameter estimates given in the appendix clearly show that regime 2 describes an active policy rule as the parameter for inflation  $\alpha_\pi$  is significant and larger than one<sup>14</sup>. Also GDP turns out to be significant and enters with a positive sign.

Regime 1 dominating most of the 1970s is characterized by a significant and positive influence of inflation on interest rates, while GDP turns out to be not significant at all. As  $\alpha_\pi$  is smaller than one we may say that regime 1 describes a passive monetary policy behavior.

These results are plausible. The Bundesbank responded to the oil price shocks at the beginning of the 1970s with an active monetary policy fighting strongly inflation. During the 1980s when the German economy experienced slow growth, interest rates were kept at a low level as predicted by regime 2. Both active and passive monetary policy regimes have occurred during the 1970s and 1980s in Germany. The 1970s were dominated by a passive monetary policy rule except for the two years after the oil price shocks took place, while the 1980s were characterized by an active policy rule.

In figure 6 the temporal distribution of fiscal regimes are similarly reported. Except for the early 1970s and the late 1980s German fiscal policy was almost exclusively described by regime 2. At first glance, the estimated coefficients for the fiscal policy rule suggest a passive interpretation of regime 2 as annual changes in debt turn out to be significant and positive in modeling revenues. This suggests that fiscal authorities will generally try to respond to positive changes in government debt with larger tax yields. Nonetheless, we propose that regime 2 is only weakly passive as the reaction of revenues to positive changes in government debt is not strong enough

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<sup>14</sup>Taylor (1993).

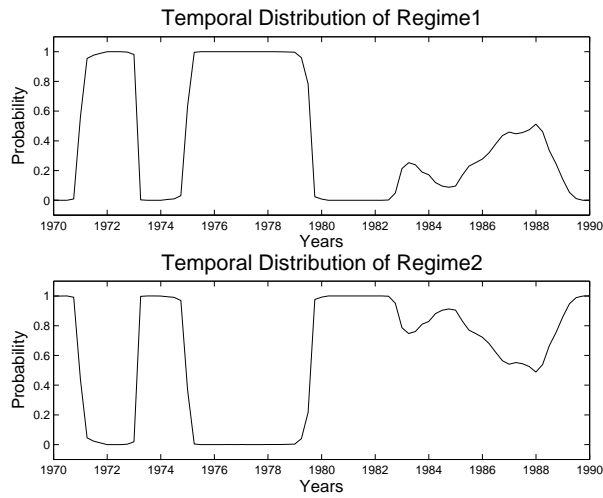


Figure 5: *Germany, Monetary Policy Rule, Temporal Distribution of Regime Probabilities for the Period 1970-1990.*

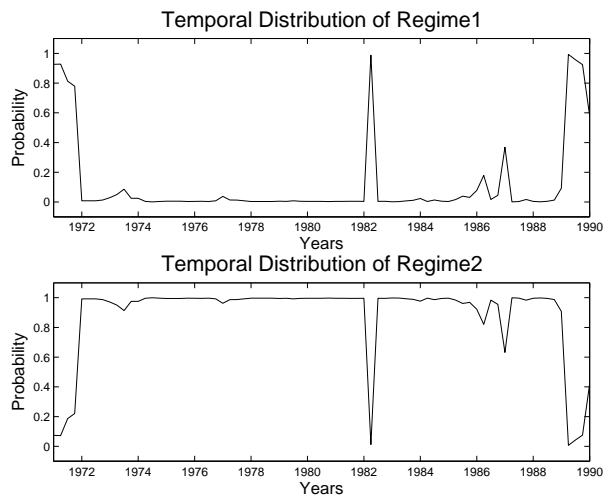


Figure 6: *Germany, Fiscal Policy Rule, Temporal Distribution of Regime Probabilities for the Period 1970-1990.*

to reduce deficits persistently and stabilize the path of public debt. The major aim of running a fiscal policy as the one described by regime 2 is to stimulate growth and not to return to a stable path of debt as  $\gamma_Y$  is significant and highly positive such that fiscal policy follows a countercyclical pattern. Therefore, we think that it is reasonable to call regime 2 *weakly passive*. Government expenditures do not seem to affect revenues at all as indicated by the 95% confidence band. This implies that increased expenditures in the current period will leave the tax yields unaffected so that we should expect a rise of the debt level in the short run.

In contrast to that, regime 1 exhibits not only a significant debt coefficient but also a highly positive one. Therefore, an interpretation towards a passive fiscal policy regime seems to be reasonable. Furthermore, government expenditures show at least some significant influence while GDP is not significant at all.

These results are plausible as they cover a lot of issues in actual German fiscal policy behavior during the observation period. Till the end of the 1960s government debt was hardly changing and almost negligible. In response to a drastic jump in the stock of public debt combined with growing government expenditures in the year 1967 fiscal authorities increased tax revenues to return to a balanced budget in the following years. This behavior is well described by regime 1 as both debt and expenditures are significant. In 1972 and even more strongly in 1974 deficits started to increase again while expenditures steadily grew till the end of the 1980s. Regime 2 exhibits these characteristics as debt is still significant but with a smaller coefficient than in regime 1. At the same time the government tried to stimulate growth as indicated by  $\gamma_Y$  while allowing for a larger stock of debt. Finally, towards the end of the 1980s the growth rate of government expenditures was reduced and tax revenues increased particularly in 1989 to stabilize the debt process. The peak in 1982 in favor for the active regime 1 can be related to freezing of expenditures.

Hence, for the period 1970-1990 there have been switches between active and passive monetary policy and passive and weakly passive fiscal policy. From the beginning of the 1970s till the mid of the 1980s fiscal policy has been weakly passive, while the early 1970s and the late 1980s show a purely passive pattern. Due to the general passiveness of fiscal policy for most of the observation period we may expect rather Ricardian outcomes in which the price level is not determined by fiscal policy actions.

An analysis of monetary policy after the German reunification shows that there have been no changes in the underlying parameters as the distribution of the regime parameters in figure 7 demonstrates. As  $\alpha_\pi(S_t^M = 1)$  is larger than one the underlying regime may be considered as active, while regime 2 constitutes a passive monetary policy regime. This implies that for the entire period 1992-1998 monetary policy was active responding positively to both changes in inflation and GDP.

Turning to the characteristics of fiscal policy we see that except for 1992 regime 1 is slightly more likely than regime 2. While in the period 1970-1990 the fiscal policy rule specification was able to explain the process of revenues quite good as the standard error of regression suggested,  $\sigma$  has now become very large. The same is true for the standard errors of the coefficients and their confidence bands. As the confidence bands for  $\gamma_G$  and  $\gamma_Y$  make up an interval around zero, we assume that government expenditures and GDP are not significant. The only remaining variable is debt. The estimate for  $\gamma_B$  is characterized by large standard errors so that the policy rule is

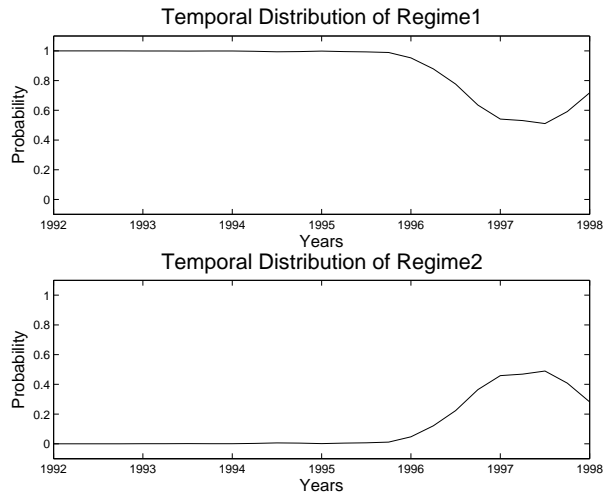


Figure 7: *Germany, Monetary Policy Rule, Temporal Distribution of Regime Probabilities for the Period 1991-1998.*

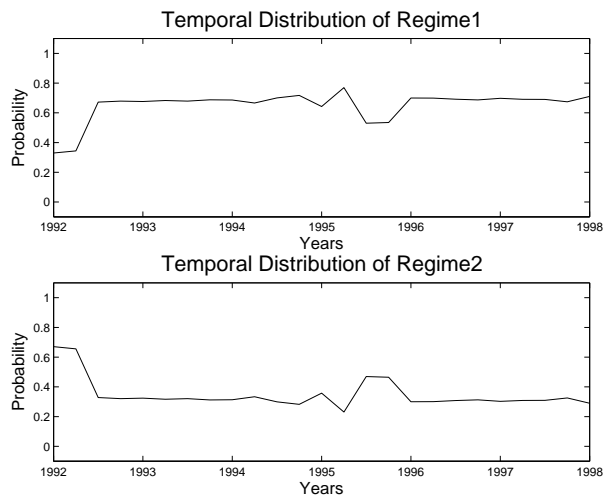


Figure 8: *Germany, Fiscal Policy Rule, Temporal Distribution of Regime Probabilities for the Period 1991-1998.*

not able to capture satisfyingly movements in tax revenues. Thus, we conclude that tax revenues during 1992-1998 follow a rather exogenous process.

Again, we should ask ourselves whether these results are economically plausible. The German economy was exposed to inflationary pressures right after the unification. The Bundesbank reacted to this upward movement in the price level with an active monetary policy. As inflation returned to lower levels towards the end of the observation period we observe a declining probability for the active monetary regime 1 in figure 7. At the same time, fiscal policy was affected by one-time effects related to the reunification. Till 1994 debt and expenditures grew without much response of tax yields. This explains the activeness of fiscal policy as described by regime 1.

The analysis has shown so far that there have been switches between passive and active monetary policy behavior particularly before 1991. Even more important is the result that fiscal policy has been at least weakly passive during the period 1970-1998, which constitutes Ricardian outcomes. The later sample shows significant evidence for non-Ricardian equilibria.

### 3.4.2 Spain

Figure 9 shows the estimated temporal distributions of monetary policy regimes in Spain. We can see that it is hard to define significant regime switches for the specified monetary policy rule. This raises the question if there have been regime changes in Spanish monetary policy at all or if we may regard the two regimes as basically identical. The estimates for  $\alpha_\pi$  show no significant difference as the confidence bands exhibit. Regime 1 and 2 are overlapping each other widely. Therefore, it is reasonable to presume that the character of monetary policy in Spain has been stable in a sense that the reaction of monetary authorities to changes in the price level has been similar at any point of time. That means there have not occurred any regime switches. The categorization in active and passive policy behavior is somewhat more difficult than for Germany as the parameter estimates are provided with quite large confidence bands. The mean as well as the median of  $\alpha_\pi$  is greater than one. Furthermore, the confidence bands are rather located in the region of  $\alpha_\pi > 1$  than below it. Therefore, it should be reasonable to consider Spanish monetary policy as active. GDP seems to contribute little to the explanation of interest rates.

The results for the fiscal policy rule given in figure 10 are also less precise than the ones we obtained for Germany. We can see that except for the years 1991 and 1997 regime 1 was slightly more probable than regime 2. The estimates for  $\gamma_B$  combined with the corresponding confidence bands imply a non-significance of public debt in regime 1, while the opposite is true for regime 2. Therefore, we may say that regime 1 constitutes an active, regime 2 a passive fiscal policy behavior. Besides the noncyclical characteristics of any of the two fiscal regimes and the significance of government expenditures in regime 1, it is important to notice that the fit is very poor as indicated by the large standard error of regression  $\sigma$ . This should not be surprising as we could prove that regime 1, which was in place for most of the time, describes active behavior of fiscal authorities implying an exogeneity of tax revenues. Hence, if tax revenues follow an exogenous process and are consequently not explained by the variables included in the fiscal policy rule, the standard error of regression should indeed become quite large.

Recapitulating we may say that for the period 1986-1998 a combination of active fiscal and

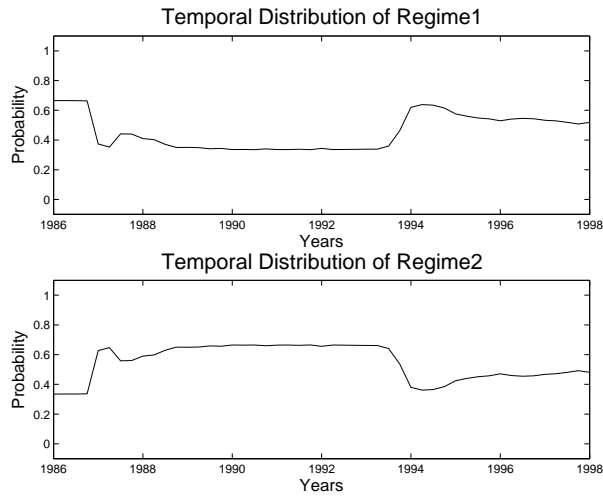


Figure 9: *Spain, Monetary Policy Rule, Temporal Distribution of Regime Probabilities for the Period 1986-1998.*

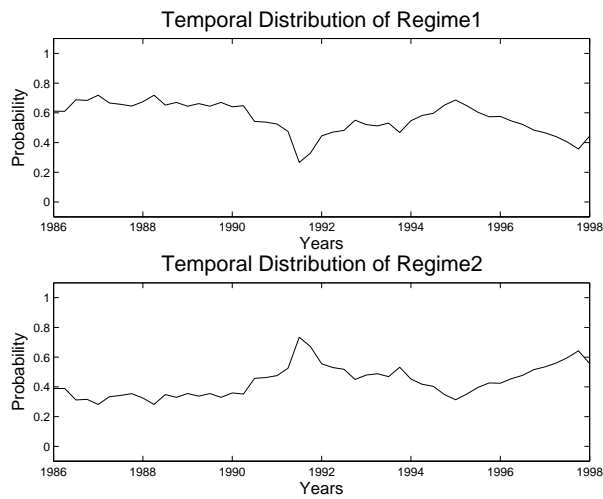


Figure 10: *Spain, Fiscal Policy Rule, Temporal Distribution of Regime Probabilities for the Period 1986-1998.*

monetary policy behavior was predominant in the Spanish economy. Therefore, fiscal policy shocks will necessarily have impacts on the price level.

These results are supported by the works of De Castro (2003) and Diaz-Roldan and Montero-Soler (2005). De Castro shows that on the one hand growing government expenditures have no significant impact on tax yields in Spain and on the other hand that increases in taxes have no persistent positive influence on budget deficits. Diaz-Roldan and Montero-Soler find that Spanish monetary policy was characterized by a strong impact of inflation on interest rates.

## 4 Conclusions and Policy Implications

The analysis has shown that fiscal policy plays a significant role in the determination of the price level in Germany as well as in Spain. We could show that the FTPL is a relevant mechanism in explaining the data. These results hold regardless of the approach chosen for the analysis. Both deficit-debt and regime-switching approach offer a consistent picture as they suggest an entirely non-Ricardian interpretation for Germany in the period 1991-1998, a mainly Ricardian policy behavior for Germany during 1970-1990, and a mostly non-Ricardian behavior for Spain. Hence, differences in the rate of inflation between the two countries during recent years cannot be traced back to the non-relevance of the FTPL in Spain or Germany.

The deficit-debt approach with sign restrictions turned out to be useful as a general indicator for the existence of non-Ricardian equilibria. In direct comparison of the two approaches we can say that it requires at least 70% of draws fitting the Ricardian sign-restriction to allow for a mainly Ricardian interpretation of the data. The sign-restriction approach should be especially valuable in small samples and used as a supplementary instrument to check for the existence of non-Ricardian equilibria. Furthermore, it gives a simple way to test for the plausibility of the results obtained with the help of the regime analysis.

The significance of the FTPL has some powerful policy implications. A limit on borrowing is needed and appropriate to guarantee the success of a monetary union like the EMU. That means that the SGP is important and rather needs strengthening than weakening. Furthermore, demand management by national governments is generally effective as the FTPL induces impacts on real economic activity due to fiscal shocks.



## 5 Appendix

### 5.1 Pure-Sign-Restriction Approach

The basic idea of the pure-sign-restriction approach is to consider only those impulse responses heading in the desired direction for at least  $Z$  periods. Let  $a \in \mathbf{R}^m$  be an impulse vector, if there exists a matrix  $A$  such that  $\Omega = AA'$  with  $a$  being a column of  $A$ . Following the notation used throughout the paper  $\Omega$  denotes the covariance matrix,  $m$  denotes the number of variables in the vector of dependent variables  $X_t$ , and  $p$  the lag length. Furthermore, let  $e_i$  for  $i = 1, \dots, m$  be the eigenvectors of  $\Omega$ , normalized to form an orthonormal basis of  $\mathbf{R}^m$ , and  $v_i$  the corresponding eigenvalues. Then, if there are coefficients  $\alpha_i$  for  $i = 1, \dots, m$  such that  $\sum_{i=1}^m \alpha_i^2 = 1$ , the impulse vector  $a$  is given by

$$a = \sum_{i=1}^m (\alpha_i \sqrt{v_i}) e_i. \quad (5.1)$$

To obtain the corresponding impulse responses we define  $\mathbf{a} = [a', 0_{1,m(p-1)}]'$ . Given the impulse vector  $a$ , the impulse response of variable  $j$  with  $j = 1, \dots, m$  at horizon  $z$  may be computed as

$$r_{z,j} = (\Gamma^z \mathbf{a}), \quad (5.2)$$

where  $\Gamma = \begin{bmatrix} & B & \\ I_{m(p-1)} & & 0_{m(p-1),m} \end{bmatrix}$ .

For the application of the sign-restriction approach we make joint draws from both the posterior distribution of the VAR parameters and a uniform distribution over the  $(m-1)$ -dimensional sphere  $(\alpha_1, \dots, \alpha_{m-1})$ . It is then possible to obtain the impulse vector  $a$  according to (5.1), which then may be used to calculate the impulse responses. Then, if the impulse response fulfills the sign restrictions imposed, we keep the draw. Otherwise we drop it from the further analysis.

### 5.2 Bayesian Analysis of Markov-Regime Switching Models

We consider a VAR of order  $p$  given by

$$x_t = \mu(S_t) + \sum_{h=1}^p B_{S(t)}^h x_{t-h} + \varepsilon_t(S_t), \quad (5.3)$$

where  $\varepsilon_t(S_t)$  is an normally distributed *i.i.d.* error term with mean zero and regime-dependent covariance matrix  $\Omega(S_t)$ .  $\mu(S_t)$  denotes a constant and  $B_{S(t)}^h$  is the matrix of coefficients for the  $h^{\text{th}}$  lag included. As indicated by  $S_t$  we assume that both the parameters included in  $B$  as well as the covariance  $\Omega$  can adopt  $k$  different states. In any period parameters and covariance matrix may switch to a new state with a probability  $p_{ij} \geq 0$ . We define the transition probabilities for a switch from regime  $i$  to regime  $j$  as  $p_{ij} = p(S_t = j | S_{t-1} = i)$ . We then summarize these probabilities in the transition matrix  $P$  with size  $(k \times k)$ .

We aim at estimating the set of unknown parameters given by

$$\lambda \equiv \{\mu_1, \dots, \mu_k, B_1, \dots, B_k, \Omega_1, \dots, \Omega_k, P\}.$$

In partitioned notation this expression reduces to  $\lambda \equiv \{\Theta, P\}$ . Furthermore, it is convenient to rewrite (5.3) in stacked form as a VAR(1) model, i.e.

$$X_t = \bar{\mu}(S_t) + B(S_t)X_{t-1} + \bar{\varepsilon}_t(S_t), \quad (5.4)$$

where

$$X_t = \begin{bmatrix} x_t \\ x_{t-1} \\ \vdots \\ x_{t-p+1} \end{bmatrix}, \bar{\mu}(S_t) = \begin{bmatrix} \mu(S_t) \\ 0 \\ \vdots \\ 0 \end{bmatrix}, B(S_t) = \begin{bmatrix} B_{S(t)}^1 & B_{S(t)}^2 & \cdots & B_{S(t)}^p \\ I_m & 0 & \cdots & 0 \\ 0 & I_m & \ddots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 0 \end{bmatrix}, \bar{\varepsilon}(S_t) = \begin{bmatrix} \varepsilon_t(S_t) \\ 0 \\ \vdots \\ 0 \end{bmatrix}.$$

Furthermore, let  $X = (x_1, \dots, x_T)$  be the vector of all observations.

### 5.2.1 The Likelihood Function

The contribution of the  $t^{\text{th}}$  vector of observations  $x_t$  to the likelihood conditional on the regime  $S_t$  is given by

$$l(x_t|S_t, X_{-1}, \lambda) = (2\pi)^{-m/2} |\Omega^{-1}(S_t)|^{1/2} \cdot \exp \left\{ -\frac{1}{2} \varepsilon_t'(S_t) \Omega^{-1}(S_t) \varepsilon_t(S_t) \right\} \quad (5.5)$$

with  $\varepsilon_t(S_t) = x_t - \mu(S_t) - \sum_{h=1}^p B_{S(t)}^h x_{t-h}$ .

$m$  denotes the number of variables in  $x_t$  and  $X_{-1} = \{x_1, \dots, x_{T-1}\}$ , i.e. all observations up to period  $T-1$ . Next, we exploit the recursiveness of (5.4) for the first  $p$  observations by substituting for  $X_{t-1}$ . This yields

$$\begin{aligned} X_p &= \bar{\mu} + BX_{p-1} + \bar{\varepsilon}_p \\ &= \bar{\mu} + B(\bar{\mu} + BX_{p-2} + \bar{\varepsilon}_{p-1}) + \bar{\varepsilon}_p \\ &= \bar{\mu} + B\bar{\mu} + B\bar{\varepsilon}_{p-1} + \bar{\varepsilon}_p + B^2X_{p-2} \\ &= \dots \\ &= \sum_{\tau=0}^{\infty} B^\tau \bar{\mu} + \sum_{\tau=0}^{\infty} B^\tau \bar{\varepsilon}_{p-\tau} \end{aligned} \quad (5.6)$$

under the assumption that there is no regime shift prior to  $p$ . This enables us to write the unconditional mean of  $X_p$  as

$$E[X_p] = \sum_{\tau=0}^{\infty} B^\tau \bar{\mu}.$$

For the existence of  $E[X_p]$  it requires that all eigenvalues of  $B$  have absolute value less than one. For the variance of  $X_p$  it follows

$$\begin{aligned} Var[X_p] &= E(X_p - E(X_p))(X_p - E(X_p))' \\ &= E \left( \sum_{\tau=0}^{\infty} B^\tau \bar{\varepsilon}_{t-\tau} \right) \left( \sum_{\tau=0}^{\infty} B^\tau \bar{\varepsilon}_{t-\tau} \right)' \\ &= E \left( \sum_{\tau=0}^{\infty} B^\tau \bar{\varepsilon}_{t-\tau} \bar{\varepsilon}'_{t-\tau} (B^\tau)' \right) \\ &= E \left( \sum_{\tau=0}^{\infty} B^\tau \bar{\Omega} (B^\tau)' \right) \\ &= V(\Omega, B). \end{aligned} \tag{5.7}$$

We are now able to approximate  $l(X_p|S_p, \lambda)$ , which is the contribution of the first  $p$  data vectors to the likelihood, by

$$l(X_p|S_p, \lambda) = (2\pi)^{-(mp)/2} |V(\Omega, B)^{-1}(S_p)|^{1/2} \cdot \exp \left\{ -\frac{1}{2} X_p'(S_t) V(\Omega, B)^{-1}(S_p) X_p \right\}. \tag{5.8}$$

For the full likelihood conditional on the regimes we obtain, using (5.5) and (5.8)

$$L(X|S_T, \lambda) = l(X_p|S_p, \lambda) \prod_{t=p+1}^N l(x_t|S_t, X_{-1}, \lambda). \tag{5.9}$$

Integrating over all possible states we end up with the unconditional likelihood of the parameter set  $\lambda$  given by

$$L(X|\lambda) = l(X_p|\lambda) \prod_{t=p+1}^N l(x_t|X_{-1}, \lambda). \tag{5.10}$$

### 5.2.2 Generating the Regimes $S$ using Gibbs-Sampling

We generate the regimes  $S$  with the help of multi-move Gibbs sampling. The idea is to obtain the  $T$  elements in  $S$  within one draw conditional on  $\lambda$  and the observed data  $X$ . The starting point is to make use of the structure of the underlying Markov chain. The density of the regimes  $Pr(S|X, \lambda)$  can easily be rearranged in a multiplicative relationship as

$$\begin{aligned}
Pr(S|X, \lambda) &= Pr(S_1, \dots, S_T|X, \lambda) \\
&= Pr(S_T|X, \lambda)Pr(S_{T-1}, \dots, S_1|S_T, X_{-1}, \lambda) \\
&= Pr(S_T|X, \lambda)Pr(S_{T-1}|S_T, X_{-1}, \lambda)Pr(S_{T-2}, \dots, S_1|S_{T-1}, X_{-2}, \lambda) \\
&= Pr(S_T|X, \lambda) \prod_{t=1}^{T-1} Pr(S_t|S_{t+1}, x_t, \lambda).
\end{aligned} \tag{5.11}$$

Knowing  $Pr(S_T|X, \lambda)$  and  $Pr(S_t|S_{t+1}, x_t, \lambda)$  we could first draw  $S_T$ . Conditional on  $S_T$  it would then be possible to obtain  $S_{T-1}$ , and again conditional on  $S_{T-1}$  we could draw  $S_{T-2}$  etc. Finally,  $Pr(S_t|S_{t+1}, x_t, \lambda)$  can be determined using the filter proposed by Hamilton (1989). This procedure demands initial values for  $S_0$ . We will briefly outline in the following how these density may reasonably be chosen.

### 5.2.3 Deriving the Initial Probabilities

Using the filter of Hamilton (1989) to compute  $Pr(S_T|X, \lambda)$  requires initial values for  $Pr(S_0|X)$ . By assuming that the economy was in a steady state in  $t = 0$ , we may use steady-state probabilities for  $Pr(S_0|X)$ . The general condition for a steady-state probability is given by

$$P \cdot Pr(S_0|X) = Pr(S_0|X), \tag{5.12}$$

where  $P$  denotes the matrix of transition probabilities. This equation can be rearranged to

$$(I - P)Pr(S_0|X) = 0, \tag{5.13}$$

with  $I$  being a  $(k \times k)$  identity matrix. We know that by construction the  $k$  probabilities in the vector of  $Pr(S_0|X)$  add up to one. Thus, with  $\iota = (1, \dots, 1)'$  we may express this fact in vector notation as

$$\iota Pr(S_0|X) = 1. \tag{5.14}$$

In matrix notation (5.13) and (5.14) can be rewritten as

$$\underbrace{\begin{bmatrix} I - P \\ \iota \end{bmatrix}}_{\equiv M} Pr(S_0|X) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{5.15}$$

We premultiply this expression by  $(M'M)^{-1}M'$  and obtain for the initial probabilities

$$Pr(S_0|X) = (M'M)^{-1}M' \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{5.16}$$

## 5.2.4 Generating the Parameters

After having generated  $S$ , we are now able to formulate the conditional density of the parameters, which is generally given by

$$Pr(\lambda_j|S, \lambda_{-j}, X) \propto L(X|S, \lambda) \cdot p(S|\lambda) \cdot p(\lambda_j), \quad (5.17)$$

where  $\lambda_{-j}$  denotes the set of all parameters except for  $\lambda_j$ .

## 5.3 Numerical Results of the Regime-Switching Approach

### 5.3.1 Germany

- Monetary Policy Rule: 1970-1990

	mean	Std. Dev.	2.5%	Median	97.5%
Constant	2.7646	0.4729	2.1936	2.7281	3.3897
$\alpha_Y(S_t^M = 1)$	0.1732	0.1158	0.0318	0.1686	0.3236
$\alpha_Y(S_t^M = 2)$	0.3219	0.0817	0.2193	0.3218	0.4274
$\alpha_\pi(S_t^M = 1)$	0.4376	0.1125	0.2952	0.4354	0.5814
$\alpha_\pi(S_t^M = 2)$	1.1880	0.0821	1.0802	1.1883	1.2916
$\sigma$	1.5332	0.2624	1.2194	1.5043	1.8825
$P_{11}$	0.8875	0.0556	0.8125	0.8957	0.9514
$P_{12}$	0.1125	0.0556	0.0486	0.1043	0.1875
$P_{21}$	0.0835	0.0485	0.0287	0.0745	0.1497
$P_{22}$	0.9165	0.0485	0.8503	0.9255	0.9713

- Fiscal Policy Rule: 1970-1990

	mean	Std. Dev.	2.5%	Median	97.5%
$\gamma_G(S_t^F = 1)$	0.5183	0.3220	0.2427	0.5072	0.7801
$\gamma_G(S_t^F = 2)$	0.1080	0.1064	-0.0191	0.1020	0.2383
$\gamma_Y(S_t^F = 1)$	-0.7624	0.7936	-1.8426	-0.6094	0.1438
$\gamma_Y(S_t^F = 2)$	2.2330	0.4228	1.7168	2.2445	2.7549
$\gamma_B(S_t^F = 1)$	1.8741	0.4434	1.3428	1.9087	2.3784
$\gamma_B(S_t^F = 2)$	0.2997	0.0573	0.2243	0.3030	0.3693
$\sigma$	20.3249	4.0922	15.6124	19.8016	25.9082
$P_{11}$	0.6076	0.1505	0.4079	0.6145	0.8036
$P_{12}$	0.3924	0.1505	0.1964	0.3855	0.5921
$P_{21}$	0.0546	0.0360	0.0198	0.0474	0.0972
$P_{22}$	0.9454	0.0360	0.9028	0.9526	0.9802

- Monetary Policy Rule: 1992-1998

	mean	Std. Dev.	2.5%	Median	97.5%
Constant	1.6230	0.2720	1.2864	1.6144	1.9695
$\alpha_Y(S_t^M = 1)$	0.1523	0.0880	0.0410	0.1543	0.2634
$\alpha_Y(S_t^M = 2)$	0.1170	0.4006	-0.4583	0.2063	0.5569
$\alpha_\pi(S_t^M = 1)$	1.3231	0.0757	1.2258	1.3214	1.4195
$\alpha_\pi(S_t^M = 2)$	0.3943	0.2059	0.1632	0.3628	0.6709
$\sigma$	0.5099	0.1128	0.3792	0.4958	0.6582
$P_{11}$	0.9346	0.0474	0.8705	0.9465	0.9827
$P_{12}$	0.0654	0.0474	0.0173	0.0535	0.1295
$P_{21}$	0.1121	0.0899	0.0189	0.0911	0.2379
$P_{22}$	0.8879	0.0899	0.7621	0.9089	0.9811

- Fiscal Policy Rule: 1992-1998

	mean	Std. Dev.	2.5%	Median	97.5%
$\gamma_G(S_t^F = 1)$	0.3086	0.6310	-0.1593	0.1969	0.9793
$\gamma_G(S_t^F = 2)$	0.4255	0.9010	-0.3309	0.2310	1.7786
$\gamma_Y(S_t^F = 1)$	0.5137	1.1461	-0.9007	0.4374	2.0513
$\gamma_Y(S_t^F = 2)$	1.7293	1.3828	-0.0993	1.7706	3.4856
$\gamma_B(S_t^F = 1)$	0.6230	0.7248	0.1766	0.5055	1.2901
$\gamma_B(S_t^F = 2)$	1.6827	1.0731	0.4424	1.6816	3.0818
$\sigma$	26.3741	11.5776	12.5658	24.8717	41.2548
$P_{11}$	0.8114	0.2029	0.4945	0.8894	0.9853
$P_{12}$	0.1886	0.2029	0.0147	0.1106	0.5055
$P_{21}$	0.3304	0.2366	0.0343	0.3052	0.6683
$P_{22}$	0.6696	0.2366	0.3317	0.6948	0.9657

### 5.3.2 Spain

- Monetary Policy Rule: 1986-1998

	mean	Std. Dev.	2.5%	Median	97.5%
Constant	1.7709	0.8930	0.6063	1.7436	2.9575
$\alpha_Y(S_t^M = 1)$	0.4791	0.5412	-0.0933	0.3151	1.3145
$\alpha_Y(S_t^M = 2)$	0.2936	0.5126	-0.1851	0.1651	1.1361
$\alpha_\pi(S_t^M = 1)$	1.2563	0.4921	0.7427	1.0540	2.0008
$\alpha_\pi(S_t^M = 2)$	1.5664	0.5037	0.8184	1.7638	2.0911
$\sigma$	2.9396	0.7330	2.1268	2.8127	3.9220
$P_{11}$	0.8550	0.1269	0.6667	0.8978	0.9733
$P_{12}$	0.1450	0.1269	0.0267	0.1022	0.3333
$P_{21}$	0.1150	0.1046	0.0235	0.0842	0.2502
$P_{22}$	0.8850	0.1046	0.7498	0.9158	0.9765

- Fiscal Policy Rule: 1986-1998

	mean	Std. Dev.	2.5%	Median	97.5%
$\gamma_G(S_t^F = 1)$	0.8612	0.6127	0.0150	1.0024	1.3603
$\gamma_G(S_t^F = 2)$	0.2772	0.7770	-0.6998	0.3066	1.1376
$\gamma_Y(S_t^F = 1)$	-0.0094	0.8735	-1.0373	-0.0283	1.0961
$\gamma_Y(S_t^F = 2)$	1.3900	1.2088	-0.1495	1.3752	2.9819
$\gamma_B(S_t^F = 1)$	0.3667	0.7124	-0.1175	0.1722	1.2231
$\gamma_B(S_t^F = 2)$	0.9781	0.9206	0.0991	0.7112	2.3303
$\sigma$	125.4221	29.9270	89.9012	122.1884	164.8915
$P_{11}$	0.7769	0.2002	0.4703	0.8479	0.9624
$P_{12}$	0.2231	0.2002	0.0376	0.1521	0.5297
$P_{21}$	0.2735	0.2178	0.0381	0.2172	0.5939
$P_{22}$	0.7265	0.2178	0.4061	0.7828	0.9619

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