Endogenous Cycles and Liquidity Risk

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Using an overlapping generations model with liquidity risk, we show that equilibrium aggregate investment and asset prices are cyclical. In an economy with neither a beginning nor an ending date, a stationary equilibrium can be obtained. In a *startable equilibrium* however, economic activity is highly cyclical. The first generation and consecutive odd ones invest most of their wealth in new long lived technologies, while *even* generations flock to seasoned claims that are sold by liquidity challenged older cohorts. We find that this liquidity driven cyclicality is driven by the optimal length of the investment horizon, not by agent live spans.

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1. Introduction

The phenomenon of business cycles is widely documented and studied. Despite a rich literature on the subject, there still exists significant controversy on the subject […]. In this paper we suggest a novel explanation for the existence of business cycles. We show that in world of identical overlapping generations that are subject to liquidity risk, a pattern of cyclical consumption constitutes a stable equilibrium.

Rational agents invest in a mix of new, long term production technologies and seasoned short term existing claims thereon. Agents who become 'impatient' sell their claims to younger generations. We find that even though returns on production technologies are stationary and certain, prices and returns in the secondary market for financial claims are not. Consider the case where a generation invest large amounts in long term technologies. When more members of this generation become impatient, supply in the secondary market increases, depressing security prices. This entices a second generation to buy more existing claims, and invest less in new technologies. As these short term claims expire, their prices increase because old agents demand them, but middle aged can no longer supply them. This leads newborns to shy away from the secondary market and instead invest more in new technologies. Then the cycle repeats itself.

This story is congruent with several observations regarding business cycles. [….]. When stock prices are low, investment in new technologies is depressed, leading to lower future consumption and production.

2. The model

We consider a two-periodic, three-date overlapping generations model à la Samuelson (1958). On each date of an infinite horizon economy a new generation of price taking agents is born, each with an endowment of one unit of consumption good, denoted dollars. Agents live either one period, with probability λ, or two periods, and consume before they die. They derive increasing utility, U, from consumption on the last date of their life.[1]

[1] In Samuelson's model, two dates were for production, one for consumption. Because we aim to study the affect of agents' liquidity needs, we split their *post*-productive life up in two periods. This simplified assumption and the assumption that patient agents do not consume at date 1, are innocuous and do not change the main findings of our model.

Agents can invest their dollars in either one- or two-period bonds. Two-period bonds with facevalue \$*R* are issued in the primary market by the competitive productive sector of the economy, at a price of \$1. One-period bonds are traded in the secondary market between patient and impatient agents. This model was earlier studied by Qi (1994), Bhattacharya and Padilla (1996), Fulghieri and Rovelli (1998), and Bhattacharya, Fulghieri and Rovelli (1998).[2]

In the next section we derive the set of *stationary* equilibriums which meet the Nash-condition in an economy where time is infinite both backward and forward. In section 4 we derive the equilibrium for the economy that has a beginning date, but no end. We will see that the allocation in such an economy is uniquely determined. We refer to this latter equilibrium it as the *startable* equilibrium.

3. Stationary Equilibriums

If, in a game with certain payoffs but uncertain preferences, time is represented by $t \in \mathbb{Z}$, we can find the equilibrium from the maximization problem of an agent who is born on date *t*. Because agents are atomistic, their objective function is:

$$\max_{y_t} \; \lambda U\left((1-y_t)\frac{R}{p_t} + y_t p_{t+1} \right) + (1-\lambda) U\left((1-y_t)\frac{R^2}{p_t \, p_{t+1}} + y_t R \right) \tag{1}$$

Where y_t is the investment, at *t*, in two-period bonds and p_t the market price of one-period bonds. The maximand has as first order condition:

$$\lambda U'(\cdot)\left(p_{i+1} - \frac{R}{p_i} \right) + (1-\lambda) U'(\bullet)\left(R - \frac{R^2}{p_i \, p_{i+1}} \right) = 0 \tag{2}$$

Because borrowing and lending are not ruled out, this first order condition must hold to avoid infinite demand for borrowing or lending. Therefore we must have:

Proposition 1 (stationary equilibrium)

In the stationary economy there exist infinitely many two-periodic equilibriums. For any pair of consecutive prices we have $p_i \, p_{i+1} = R$, $p_s, p_t \in [1,R]$.

Proof: follows immediately from the first order condition.

[2] The mentioned papers used the model, an overlapping generations variant of the Diamond Dybvig (1983)model where risksharing agents face the possibility of having future unverifiable liquidity needs, to investigate the bank versus exchange question.

Remark 1: Every equilibrium price-process $\{p_i,p_{i+1}\}$, can be supported by one pure investment strategy, and many mixing strategies. In both cases agents born on date *i* invest an expected amount of y_i = $(1-\lambda)(R-p_i)/(R-1)$ in the long term production technology, and use the remainder of their endowment to purchase existing claims.

Remark 2: An agent born on date *i* expects to consume is $\lambda p_{i+1}+(1-\lambda)R$. In general then agents of consecutive generations consume different amounts. If agents are risk averse, the equilibrium which maximizes intergenerational welfare would then have $p_i = \sqrt{R}$ for all *i*. We assume that it is the social attractiveness of this one-periodic equilibrium's that lead Qi (1994), Bhattacharya and Padilla (1996) and Fulghieri and Rovelli (1998) to disregard the cyclical equilibria.[3] From a pure game theoretic point of view however, there is no reason to believe that the welfare maximizing equilibrium is obtained. In fact, in the following we find that if the economy has a starting date, the unique equilibrium is the Pareto inferior two-periodic business cycle.

4. Startable Equilibrium

If, in a game with certain payoffs but uncertain preferences, time is represented by $t \in \mathbb{Z}^+$, the first generation decides which of the stationary equilibria is played. Since there is no secondary market at date zero, and the only alternative to investing is storing, the first generation solves:

$$\max_{y_0} \ \lambda U\left((1-y_0)+y_0 p_1\right)+(1-\lambda)U\left((1-y_0)\frac{R}{p_1}+y_0 R\right) \tag{3}$$

Because the first derivative with respect to y_0, $\lambda U'(\cdot)\left(p_1-1\right)+(1-\lambda)U'(\bullet)R\left(1-\frac{1}{p_1}\right)$, is positive for all $p_1 > 1$, the first generation invests it entire endowment in the two-period bonds. We then have:

Proposition 1 (startable equilibrium)

In the $\mathbb{Z}^+$ *economy we have* $p_{odd} = 1$, $p_{even} = R$, $y_0 = y_{even} = 1$, $y_{odd} = 1-\lambda$.

Proof: follows immediately from the f.o.c.

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[3] Bhattacharya, Fulghieri and Rovelli (1998) mention that there exist only *two* equilibriums, $p_i = \sqrt{R}\ \forall i$ and $\boldsymbol{p} = ..,1,R,1,R,....$, and focus on the former.

The cyclical equilibrium is extraordinarily robust. Although the agents' aversion to risk makes the one-periodic stationary equilibriums the most desirable one, risk preferences do *not* determine the equilibrium. This is simply because there is no uncertainty. In the following we discuss several extensions of our model.

5. Extensions

To demonstrate the main trust of our story, we have resorted to the simplest possible model. In the following we relax some of these abstracting assumptions.

5.1. Short term production technologies

The base case assumes that the only alternative to the long-lived production technology is costless storage. In the following we show that also if storage is costly or if there exists a productive short term investment, cyclicality is retained, unless the per period return on the short term technology is higher than that on the long term technology. In particular:

Proposition 3: *If, apart from the long term technology, agents have access to a one-period technology that offers a return* $0 < r < \sqrt{R}$ *, the startable equilibrium will have:*

$$p_{\text{odd}} = \max(1,r),\ \ p_{\text{even}} = \min(R, R/r),$$

and $$y_0 = 1-\lambda+\lambda^2\frac{R-r}{rR-r},\ \ y_{\text{odd}} = 1-\lambda\frac{R-r}{R-1},\ \ y_{\text{even}} = 1-\lambda\frac{rR-R}{rR-r}$$

<u>proof</u>: We only need consider the first generation's problem:

$$\max_{y_0}\ \lambda U\left((1-y_0)r + y_0 p_1\right) + (1-\lambda)U\left((1-y_0)\frac{rR}{p_1} + y_0 R\right)$$

Which has as f.o.c. $r = p_1$. The accompanying y_0 follow from the market clearing conditions.

Not surprisingly, the existence of a productive alternative to the storage technology reduces the cyclicality. If $r \geq \sqrt{R}$, no cyclicality will survive as the long term technology is superfluous. If on the other hand storage is costly, $r < 1$, the equilibrium will not be affected, as in the base case there is no storage.

To the referee: I am currently in the process of “beefing up” the paper with a more complete literature review and additional extensions as mentioned below. I am confident that by the summer the paper will be finished and polished, and will certainly make for an interesting presentation.

ii) population growth, call it g. As long as $g < \sqrt{R}$, cyclicality obtains in the startable economy. In can easily be shown that in this case prices are $p_{odd} = g$, $p_{even} = R/g$. Only in the (less intuitive) case of $g > \sqrt{R}$ are prices stationary: $p_i = \sqrt{R}\ \forall\ i$. The intuition behind this one-periodic equilibrium (proof omitted) is that the second generation will not drive the price beyond $\sqrt{R}$. In this case $y_0 = 1$, $y_i = 1 + \lambda \sum_{j=1}^{i} \left(\left(-\sqrt{R}/g \right)^i \right)$ $\forall\ i = 1$, with $y_\infty = 1 - \frac{\lambda\sqrt{R}}{g + \sqrt{R}}$.

iii) investor’s life span. Assume has an infinite life, but dies with probability λ(a) where a stands for age (the continuous variation of this, was introduced by Blanchard (1985)). we could use any function, including the mortality rate predicted by “Gomperty’s Law” (see Wetterstrand 1981)). The objective function of an agent born on date i then becomes:

$$\max_{y_t(a)}\ \lambda(1)U\left((1-y_t(1))\frac{R}{p_t} + y_t(1)p_{t+1}\right) + \lambda(2)U\left(\left(\frac{(1-y_t(1))R}{p_t} + y_t(1)p_{t+1} - y_t(2)\right)\frac{R}{p_{t+1}} + y(2)p_{t+1}\right) + \ldots$$

Clearly we have as f.o.c.s

$$\max_{y_t(a)}\ \lambda(1)U'(\cdot)\left(p_{t+1} - \frac{R}{p_t}\right) + \sum \lambda(2)U(\bullet)\left(R - \frac{y_t(1)R^2}{p_{t+1}p_t}\right) + \ldots$$

So that $p_t p_{t+1} = R$ ➔ we still have **two-periodicity**.

iv) number of dates before LT asset pays of. Assume asset pays R in *three* periods. Rest is the same. Then agents can choose between one-year bonds, two-year bonds and three-year bonds. Then it turns out that we have *three-periodicity*.

Denote the price of one-year bonds p, those of two year bonds q. The maximization problem becomes:

$$\max_{y_t, x_t}\ \lambda U\left((1-x_t-y_t)\frac{p_{t+1}}{q_t} + x_t\frac{R}{p_t} + y_t q_{t+1}\right) + (1-\lambda)U\left((1-x_t-y_t)\frac{R}{q_t} + x_t\frac{R^2}{p_t\, p_{t+1}} + y_t p_{t+2}\right)$$

With foc.’s

$$\lambda U'(\cdot)\left(\frac{R}{p_t}-\frac{p_{t+1}}{q_t}\right)+(1-\lambda)U'(\bullet)\left(\frac{R^2}{p_t p_{t+1}}-\frac{R}{q_t}\right)=0$$

$$\lambda U'(\cdot)\left(q_{t+1}-\frac{p_{t+1}}{q_t}\right)+(1-\lambda)U(\bullet)\left(p_{t+2}-\frac{R}{q_t}\right)$$

Or

$$\lambda U'(\cdot)\left(\frac{R}{p_t}-\frac{p_{t+1}}{q_t}\right)+(1-\lambda)U'(\bullet)\frac{R}{p_{t+1}}\left(\frac{R}{p_t}-\frac{p_{t+1}}{q_t}\right)=0 \text{ hence } R=\frac{p_t p_{t+1}}{q_t}=\frac{p_{t+2}p_{t+1}}{q_{t+1}}$$

$$\lambda U'(\cdot)\left(q_{t+1}-\frac{p_{t+1}}{q_t}\right)+(1-\lambda)U(\bullet)\frac{p_{t+2}}{q_{t+1}}\left(q_{t+1}-\frac{p_{t+1}}{q_t}\right) \text{ hence } p_{t+1}=q_{t+1}q_t \text{ and } p_t=q_t q_{t-1}$$

Hence we have a ***three-periodic*** equilibrium where $R=q_{t-1}q_t q_{t+1}=q_t q_{t+1}q_{t+2}$.

Hence the two-periodicity depends on the length of the payoff, not on the maximum age of the agents!!!

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