ICAPM with time-varying risk aversion

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Abstract

A derivation of the ICAPM in a very general framework and previous theoretical work, argue for the relative risk aversion (RRA) coefficient to be both time-varying and countercyclical. The variables that represent proxies for the cyclical component of RRA are the market dividend yield, default spread, smoothed earnings yield and industrial production growth, all being highly correlated with the business cycle. In addition, the value spread - a proxy for the relative valuation of value stocks versus growth stocks - is included as a determinant of risk aversion. The results show that risk aversion is countercyclical, and the ICAPM with time-varying RRA performs better than the Bad beta good beta model (BBGB) from Campbell and Vuolteenaho (2004). The results from an augmented scaled ICAPM show that the market return has a negative effect on risk aversion, thus risk aversion seems to be affected by both business conditions and financial wealth. The estimates of the average RRA coefficient seem reasonable and plausible, and the model is able to capture a significant decline in risk-aversion in the 90's, in line with the mounting evidence from academics and practioneers. When compared against alternative factor models - CAPM, Fama-French 3 factor and Fama-French 4 factor models - the scaled ICAPM performs much better than the CAPM, and compares reasonably well against the Fama-French models. A crucial result relies on the fact that the scaled ICAPM models do a good job in pricing both the "extreme" small-growth portfolio and all the book-to-market quintiles, which is mainly due to the presence of the factor related with time-varying risk-aversion. Overall, the results of this paper offer a fundamental explanation - time-varying risk aversion - for the value premium. Preliminary results suggest that the ad-doc HML and UMD factors, at least partially, measure the same types of risks as the ICAPM with time-varying risk aversion.

Keywords: Asset pricing; Conditional pricing models; ICAPM; Linear multifactor models; Predictability; Time-varying risk aversion; JEL classification: G11; G12; G14; E32; E44

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I. Introduction

According to the Merton (1973) ICAPM, state variables that predict market returns, should act as risk factors that price the cross-section of ex-post average returns. Despite this prediction - and the existence of a vast literature showing that the market equity premium is time-varying and predictable at several horizons by a set of state variables linked to short term interest rates, bond yields and financial ratios - there has been not many attempts to test the ICAPM, even in the presence of the CAPM failure to explain the cross section of average returns. Among the papers that implemented empirically testable versions of the original ICAPM, are Campbell (1993, 1996), and more recently Chen (2003), Brennan et al (2004) and Campbell and Vuolteenaho (2004). Common to these papers is the assumption that the coefficient of relative risk aversion associated with the original utility function, is constant through time. Nevertheless as demonstrated in Cochrane (2001) and the next section, a derivation of the ICAPM in a very general framework produces a time varying relative risk aversion coefficient. In fact, evidence from Campbell and Cochrane (1999) show that the relative risk aversion parameter - as well as the local curvature of the utility function - should vary with the business cycle, being countercyclical. This is not just an analytical fact arising from the models, since it is economically sensible to assume that risk aversion is negatively correlated with the business cycle: In recessions or times of sustained declining prices (bear stock market), the investors' risk tolerance should be low, and the converse should happen in economic expansions, or bull market. Hence, time-varying risk aversion can be interpreted as a recession risk factor that causes marginal utility and required returns to be high in recessions and low in economic expansions.

An augmented version of the Bad beta, good beta ICAPM (BBGB) from Campbell and Vuolteenaho (2004) (CV), is specified and estimated by incorporating time-varying relative risk aversion (RRA), using a number of state variables related with the business cycle and financial wealth. The results show that, in general, all the ICAPM models with time-varying RRA perform better than the corresponding BBGB models which assume constant RRA. The estimates of the average RRA coefficient seem reasonable and plausible - in most cases under 20 - which go along with previous evidence that argue for low values of RRA. In general, the coefficient estimates in the RRA equation, have the expected sign, and in many cases are statistically significant, thus confirming that there is a negative correlation between business conditions and risk aversion. When compared against alternative factor models - CAPM, Fama-French 3 factor and Fama-French 4 factor models - the several versions of the augmented ICAPM perform reasonably well. In addition, the time-varying ICAPM models do a good job in pricing both the "extreme" small-growth portfolio and the book-to-market quintiles. The good fit of the ICAPM with time-varying RRA to both value and growth portfolios, is mainly due to the time-variation in RRA factors, thus presenting a fundamental explanation for the value premium.

II. Theoretical background

A. *A "general" ICAPM model*

The ICAPM can be derived in a very general setting with unspecified preferences. Here I adopt the structure used in Cochrane (2001) - chapter 9 - where the value function associated with the investor's optimization problem in a continuous-time setting is given by $V(W_t, z_t)$, whith W_t denoting total wealth and z_t representing a vector of state variables which forecasts future expected returns or changes in the investment opportunity set. In this context, the continuous-time stochastic discount factor (SDF)[1] is given by

$$\Lambda_t = e^{-\delta t} V_W(W_t, z_t) \qquad (1)$$

where δ is a subjective time-discount rate, and V_W denotes the marginal utility of wealth. By applying Ito's lemma to equation (1) we have

$$\frac{d\Lambda_t}{\Lambda_t} = -\delta t + \frac{W_t V_{WW}(W_t,z_t)}{V_W(W_t,z_t)}\frac{dW_t}{W_t} + \frac{V_{Wz}(W_t,z_t)}{V_W(W_t,z_t)}dz_t \qquad (2)$$

where the second derivative terms have been ignored since they will cancel out in the pricing equation. In this framework, the relative risk aversion (RRA thereafter) coefficient is given by

$$\gamma_t \equiv -\frac{W_t V_{WW}(W_t,z_t)}{V_W(W_t,z_t)} \qquad (3)$$

which can be rewritten as

$$\gamma_t \equiv -\frac{C_t u_{cc}(C_t,z_t)}{u_c(C_t,z_t)} \qquad (4)$$

since at the optimum the envelope theorem ensures that the marginal utility of wealth and the marginal utility of consumption are the same, $u_c(.) = V_W(.)$, i.e., the incremental value of a dollar consumed or a dollar invested are coincident.

Substituting (2) in the pricing model

$$E_t(\frac{dp_t^i}{p_t^i}) + \frac{D_t^i}{p_t^i}dt - r_t^f dt = -E_t(\frac{d\Lambda_t}{\Lambda_t}\frac{dp_t^i}{p_t^i}) \qquad (5)$$

we have

$$E_t(\frac{dp_t^i}{p_t^i}) + \frac{D_t^i}{p_t^i}dt - r_t^f dt = \gamma_t E_t(\frac{dW_t}{W_t}\frac{dp_t^i}{p_t^i}) - \frac{z_t V_{Wz}(W_t,z_t)}{V_W(W_t,z_t)}E_t(\frac{dz_t}{z_t}\frac{dp_t^i}{p_t^i}) \qquad (6)$$

where the left hand side represents the expected return of asset i (price appreciation plus the dividend yield) in excess of the risk-free rate. Since there is no difference between second moments and covariances in continuous time, this equation can be restated as

$$E_t(\frac{dp_t^i}{p_t^i}) + \frac{D_t^i}{p_t^i}dt - r_t^f dt = \gamma_t cov_t(\frac{dW_t}{W_t}, \frac{dp_t^i}{p_t^i}) - \frac{z_t V_{Wz}(W_t,z_t)}{V_W(W_t,z_t)}cov_t(\frac{dz_t}{z_t}, \frac{dp_t^i}{p_t^i}) \qquad (7)$$

which can be approximated to discrete time as

$$E_t(R_{t+1}^i) - R_t^f \approx \gamma_t cov_t(R_{t+1}^i, \frac{\Delta W_{t+1}}{W_t}) + \lambda_{zt} cov_t(R_{t+1}^i, \frac{\Delta z_{t+1}}{z_t}) \qquad (8)$$

with $\lambda_{zt} \equiv -\frac{z_t V_{Wz}(W_t, z_t)}{V_W(W_t, z_t)}$ being the risk price associated with the state variables z_t, $\Delta W_{t+1} \equiv W_{t+1} - W_t$ and $\Delta z_{t+1} \equiv z_{t+1} - z_t$. The growth in total wealth, $\frac{\Delta W_{t+1}}{W_t}$, can be approximated by the market return, and by the same reasoning the change in the factors that predict returns $\frac{\Delta z_{t+1}}{z_t}$, can have as proxy the returns on the corresponding factor-mimicking portfolios. Equation (7) can be alternatively derived exactly in a discrete-time framework assuming joint-normality for R^i_{t+1}, W_{t+1} and z_{t+1}, and then applying Stein's lemma, a procedure which is pursued in the Appendix A.

B. *Time-varying risk aversion*

From equation (3) it is clear that the RRA coefficient is time-varying: it is related with current wealth (realized market returns), the marginal utility of wealth (consumption), and the second derivative which measures the local curvature of the value (utility) function. These two quantities are functions of time-varying variables, (W_t, z_t), and hence should be time-varying.

The fact that γ_t is related with the marginal utility of consumption $u_c(.)$ might suggest that risk factors which are a proxy for the marginal utility growth, $\frac{u_c(C_{t+1}, z_{t+1})}{u_c(C_t, z_t)} \approx a + b' f_{t+1}$, are potential candidates for explaining time-varying RRA. In addition, γ_t should be related with the curvature of the utility function,

$$\gamma_t \equiv -\frac{W_t V_{WW}(.)}{V_W(.)} = -\frac{\partial \ln(V_W(.))}{\partial \ln(W_t)} = -\frac{\partial \ln(u_c(.))}{\partial \ln(C_t)} \frac{\partial \ln(C_t)}{\partial \ln(W_t)} = \eta_t \frac{\partial \ln(C_t)}{\partial \ln(W_t)} \qquad (9)$$

where $\eta_t \equiv -\frac{C_t u_{cc}(.)}{u_c(.)}$ denotes the local curvature of the utility function and $\frac{\partial \ln(C_t)}{\partial \ln(W_t)}$ represents the elasticity of consumption with respect to wealth. Equation (9) states that RRA moves with η_t, since the elasticity term is always positive. Variables that proxy for $u_{cc}(.)$ should be related with the business cycle or overall stance of the stock market: In periods of economic recessions or declining stock returns ("bear" market) investors should be more sensitive to additional negative shocks in returns - and hence negative shocks in wealth and consumption - i.e., the change in marginal utility measured by η_t, is higher in those periods. The converse is true for periods of economic expansion or rising stock prices ("bull" market), where investors are not so sensitive to adverse shocks. Hence γ_t should be related with "recession risk" state variables that cause RRA to be countercyclical: high in recessions ("bear markets") and low in expansions ("bull markets").

Campbell and Cochrane (1999) present a model in which $\eta_t = \gamma/S_t$, where γ is the power utility function coefficient and $S_t = \frac{C_t - X_t}{C_t}$, denoted as the "surplus consumption ratio" which measures how higher is current consumption relative to past consumption, designed by habit X_t. In this model the "recession state" variable is S_t: In recessions, consumption decreases relative to past consumption (S is low), and therefore η_t and RRA are both high. During expansions, we have the converse effect, in which consumption is high relative to the habit (S is high) and this leads to a low RRA. Additionally, in this model consumption moves more than proportionally with wealth, meaning that $\frac{\partial \ln(C_t)}{\partial \ln(W_t)} > 1$, and this causes the RRA coefficient to be always higher than η_t. Nevertheless, in what concerns the goal of this paper, the relevant feature of Campbell and Cochrane (1999) paper is that RRA - as well as the curvature of the utility function - are both countercyclical, and therefore should be explained by state variables which are negatively correlated with the business cycle or overall stance of the stock market.

Therefore γ_t will be related with state variables known in time t, and which are correlated with the business cycle, z_t. In the following analysis, let's assume that z_t is a scalar in order to simplify the algebra, but the analysis could be extended in a straightforward way for the case of z_t being a vector of state variables explaining the dynamics of γ_t. Thus the specification for γ_t is given by

$$\gamma_t = \gamma_0 + \gamma_1 z_t \qquad (10)$$

In the following pricing equations the RRA coefficient in the current period pricing equation $(t+1)$ is denoted by γ_t, since it is linearly related with state variables known in last period (time t), as specified by equation (10). In fact it seems reasonable to assume that the investor's attitude toward risk should change as a reaction to last available information - last period information set - and not on unknown information of the current period. The fact that γ depends on lagged variables also enables to condition down the model by taking the law of iterated expectations, and therefore obtain an unconditional version of the asset pricing model which can be empirically testable.

C. *ICAPM with time-varying risk aversion*

Campbell (1993) uses an Epstein and Zin (1989, 1991) utility function and a decomposition for innovations on consumption growth based on the investor's intertemporal budget constraint, combined with joint conditional log-normality and homoskedasticity of asset returns and consumption growth, to derive a version of the ICAPM represented in unconditional form as,

$$E(r_{i,t+1} - r_{f,t+1}) + \frac{\sigma_i^2}{2} = \gamma_0 \sigma_{im} + (\gamma_0 - 1)\sigma_{ih} \qquad (11)$$

where $r_{i,t+1}$ and $r_{f,t+1}$ denote the log return on stock i and log risk-free rate respectively, γ_0 is the RRA coefficient, $\frac{\sigma_i^2}{2}$ is a Jensen's Inequality adjustment arising from the log-normal model, and

$\sigma_{im} \equiv E[Cov_t(r_{i,t+1}, r_{m,t+1})] = E[Cov_t(r_{i,t+1}, r_{m,t+1} - E_t(r_{m,t+1})]) = Cov[r_{i,t+1}, r_{m,t+1} - E_t(r_{m,t+1})]$ and $\sigma_{ih} \equiv Cov(r_{i,t+1}, r_{t+1}^h)$ represent the unconditional covariances of stock i's return with the current market return and news about future market returns, respectively. News about future market returns is given by

$$r_{t+1}^h \equiv (E_{t+1} - E_t)\sum_{j=1}^{\infty} \rho^j r_{m,t+1+j} \qquad (12)$$

In a recent paper, Campbell and Vuolteenaho (2004) - CV thereafter - using the same framework as Campbell (1993,1996), rely on the decomposition of current unexpected market returns into revisions in future expected returns (discount-rate news) and the residual which they interpret as cash-flow news,

$$r_{m,t+1} - E_t(r_{m,t+1}) = (E_{t+1} - E_t)\sum_{j=0}^{\infty} \rho^j \Delta d_{t+1+j} - (E_{t+1} - E_t)\sum_{j=1}^{\infty} \rho^j r_{m,t+1+j}$$

$$\equiv r_{t+1}^{CF} - r_{t+1}^{h} \qquad (13)$$

with $r_{t+1}^{CF} \equiv (E_{t+1} - E_t)\sum_{j=0}^{\infty} \rho^j \Delta d_{t+1+j} = r_{m,t+1} - E_t(r_{m,t+1}) + r_{t+1}^{h}$ representing "news" about future cash-flows. They come up with a version of the ICAPM based also on only two factors: the covariance (beta) with discount-rate news (good beta) and the covariance with cash-flow news (bad beta),

$$E(r_{i,t+1} - r_{f,t+1}) + \frac{\sigma_i^2}{2} = \gamma_0 \sigma_{iCF} - \sigma_{ih} \qquad (14)$$

where σ_{ih} is defined as before, and $\sigma_{iCF} \equiv Cov(r_{i,t+1}, r_{t+1}^{CF})$ is the covariance of asset i's return with cash-flow news. In (14) the difference to CV is that σ_{ih} appear with a minus sign in the pricing equation, since in their paper they define the covariance (beta) with respect to the negative (favorable change) of discount-rate news. The covariance risk with cash-flow news receives a risk price of γ_0 whereas the covariance risk with discount-rate news has a risk price of -1, thus, with $\gamma_0 > 1$, the covariance with upward revisions in future cash-flows have a higher risk price than downward revisions in future market returns. I denote equation (14) as the bad beta good beta ICAPM (BBGB).

Whereas in CV model, the RRA coefficient is assumed to be constant, one can extend it to be time-varying, making it related with state variables known in period t, and related with the business cycle, as suggested in the previous sub-section. To accomplish that, I use a "generalized" version of the Epstein and Zin utility function which accounts for time-varying RRA,

$$U_t = \{(1-\delta)C_t^{\frac{1-\gamma_t}{\theta_t}} + \delta[E_t(U_{t+1}^{1-\gamma_t})]^{\frac{1}{\theta_t}}\}^{\frac{\theta_t}{1-\gamma_t}} \qquad (15)$$

where $\theta_t \equiv \frac{1-\gamma_t}{1-\frac{1}{\psi}}$, with ψ being the elasticity of intertemporal substitution, which is assumed to be constant through time.

The objective function (15) has an associated pricing equation in simple returns given by

$$1 = E_t\{[\delta(\frac{C_{t+1}}{C_t})^{-\frac{1}{\psi}}]^{\theta_t}(\frac{1}{R_{m,t+1}})^{1-\theta_t}R_{i,t+1}\} \qquad (16)$$

which is the same as the Euler equation with constant RRA - with θ_t in place of θ due to the time variation in γ_t - since θ_t belongs to time t information set, and therefore can be put inside the expectation. Thus the stochastic discount factor (SDF) of this asset pricing model is equal to

$$M_{t+1} = \delta^{\theta_t}(\frac{C_{t+1}}{C_t})^{-\frac{\theta_t}{\psi}}(\frac{1}{R_{m,t+1}})^{1-\theta_t} \qquad (17)$$

with a corresponding log SDF,

$$m_{t+1} = \theta_t \ln(\delta) - \frac{\theta_t}{\psi}\Delta c_{t+1} - (1-\theta_t)r_{m,t+1} \qquad (18)$$

summing and subtracting both $\frac{\theta_t}{\psi}E_t(\Delta c_{t+1})$ and $(1-\theta_t)E_t(r_{m,t+1})$ yields,

$$\begin{aligned} m_{t+1} &= \theta_t \ln(\delta) - \frac{\theta_t}{\psi}E_t(\Delta c_{t+1}) - (1-\theta_t)E_t(r_{m,t+1}) - \frac{\theta_t}{\psi}(\Delta c_{t+1} - E_t(\Delta c_{t+1})) \\ &\quad -(1-\theta_t)(r_{m,t+1} - E_t(r_{m,t+1})) \\ &= E_t(m_{t+1}) - \frac{\theta_t}{\psi}(c_{t+1} - E_t(c_{t+1})) - (1-\theta_t)(r_{m,t+1} - E_t(r_{m,t+1})) \\ &= -E_t(r_{m,t+1}) - \frac{\theta_t}{\psi}(c_{t+1} - E_t(c_{t+1})) - (1-\theta_t)(r_{m,t+1} - E_t(r_{m,t+1})) \qquad (19) \end{aligned}$$

where the second equality makes use of the fact that $\Delta c_{t+1} - E_t(\Delta c_{t+1}) = c_{t+1} - E_t(c_{t+1})$, and the last equality takes into account the conditional expected log SDF $E_t(m_{t+1}) = -E_t(r_{m,t+1})$ derived in Appendix B.3. Substituting $c_{t+1} - E_t(c_{t+1})$ by its expression derived in Appendix B.2, it follows

$$\begin{aligned} m_{t+1} &= -E_t(r_{m,t+1}) - \frac{\theta_t}{\psi}[r_{m,t+1} - E_t(r_{m,t+1}) + (1-\psi)r^h_{t+1}] - (1-\theta_t)(r_{m,t+1} - E_t(r_{m,t+1})) \\ &= -E_t(r_{m,t+1}) - \gamma_t(r_{m,t+1} - E_t(r_{m,t+1})) + (1-\gamma_t)r^h_{t+1} \qquad (20) \end{aligned}$$

where the last equality follows from substituting the expression for θ_t. If we employ the decomposition of current unexpected market returns in equation (13), we have,

$$m_{t+1} = -E_t(r_{m,t+1}) - \gamma_t r^{CF}_{t+1} + r^h_{t+1} \qquad (21)$$

Finally, substituting γ_t by its expression in equation (10), yields

$$m_{t+1} = -E_t(r_{m,t+1}) - \gamma_0 r^{CF}_{t+1} - \gamma_1 z_t r^{CF}_{t+1} + r^h_{t+1} \qquad (22).$$

Making $\mathbf{f}_{t+1} \equiv (r^{CF}_{t+1}, z_t r^{CF}_{t+1}, r^h_{t+1})$ and $\mathbf{b} \equiv (b_1, b_2, b_3) = (-\gamma_0, -\gamma_1, 1)$, and using Theorem 1 in appendix B.1., one has,

$$E(r_{i,t+1} - r_{f,t+1}) + \frac{\sigma_i^2}{2} = \gamma_0 \sigma_{i,CF} + \gamma_1 \sigma_{i,CFz} - \sigma_{i,h} \qquad (23)$$

where $\sigma_{i,CFz} \equiv Cov(r_{i,t+1}, r^{CF}_{t+1} z_t)$. Equation (23) will be the benchmark model in this paper, and by imposing $\gamma_1 = 0$, one obtains the BBGB model (14) as a special case of the ICAPM with time-varying risk aversion. The innovation in (23) with respect to the BBGB model, is the $\gamma_1 \sigma_{i,CFz}$ term, resulting from the new factor, $z_t r^{CF}_{t+1}$, which represents the product of cash-flow news and the risk aversion scaled variable z_t, and has a price of risk given by γ_1, which is the time-varying component of risk aversion. Thus, this new factor is a measured of time-varying risk aversion, or a recession risk factor as argued in last sub-section.

The model in covariances (23), can be represented in expected return-beta form, as also shown in Theorem 1, appendix B.1,

$$E(r_{i,t+1} - r_{f,t+1}) + 0.5\sigma_i^2 = \boldsymbol{\lambda}' \boldsymbol{\beta}_i = \lambda_{CF} \beta_{i,CF} + \lambda_{CFz} \beta_{i,CFz} + \lambda_h \beta_{i,h} \qquad (24)$$

where $\boldsymbol{\lambda} \equiv (\lambda_{CF}, \lambda_{CFz}, \lambda_h)' = -Var(\mathbf{f}_{t+1})\mathbf{b}$ denote the vector of factor risk prices, and $\boldsymbol{\beta}_i \equiv Var(\mathbf{f}_{t+1})^{-1} Cov(r_{i,t+1}, \mathbf{f}_{t+1})$ represents the $(3x1)$ vector of multiple-regression betas for asset i. The λ's represent the risk prices of beta risk for each of the factors. As shown in appendix B.4., for the case of a single determinant of γ_t, the risk price vector is given by,

$$(\lambda_{CF}, \lambda_{CFz}, \lambda_h)' = \begin{bmatrix} \gamma_0 \sigma^2_{CF} + \gamma_1 \sigma_{CF,CFz} - \sigma_{CF,h} \\ \gamma_0 \sigma_{CF,CFz} + \gamma_1 \sigma^2_{CFz} - \sigma_{CFz,h} \\ \gamma_0 \sigma_{CF,h} + \gamma_1 \sigma_{CFz,h} - \sigma^2_h \end{bmatrix} \qquad (25)$$

where $\sigma^2_{CF} \equiv Var(r^{CF}_{t+1})$, $\sigma^2_h \equiv Var(r^h_{t+1})$, $\sigma^2_{CFz} \equiv Var(r^{CF}_{t+1} z_t)$, $\sigma_{CF,CFz} \equiv Cov(r^{CF}_{t+1}, r^{CF}_{t+1} z_t)$, $\sigma_{CF,h} \equiv Cov(r^{CF}_{t+1}, r^h_{t+1})$ and $\sigma_{CFz,h} \equiv Cov(r^{CF}_{t+1} z_t, r^h_{t+1})$. The risk prices depend on the SDF coefficients γ_0 and γ_1 - as in the case of risk prices of covariances - but also on the variances and covariances between the risk prices, since we are working with multiple-regression betas. Given $\boldsymbol{\lambda} \equiv -\boldsymbol{\Sigma}_f \mathbf{b}$, $\boldsymbol{\Sigma}_f \equiv Var(\mathbf{f}_{t+1})$, standard errors for the factor risk price estimates can be calculated as,

$$Var(\boldsymbol{\lambda}) = \boldsymbol{\Sigma}_f Var(\mathbf{b}) \boldsymbol{\Sigma}_f \qquad (26)$$

since $\boldsymbol{\Sigma}_f = \boldsymbol{\Sigma}_f'$, and given

$$Var(\mathbf{b}) = \begin{bmatrix} Var(\mathbf{b}^*) & \mathbf{0}_{(2X1)} \\ \mathbf{0}_{(1X2)} & 0 \end{bmatrix} \qquad (27)$$

with $\mathbf{b}^* \equiv (-\gamma_0, -\gamma_1)$ representing the SDF parameters to be estimated in the cross-section.

III. Asset pricing tests

A. *Data*

The test assets used in the asset pricing tests are the Fama-French 25 portfolios sorted on size and book-to-market (SBV25), and 38 industry sorted portfolios (IND38), all obtained from Prof. Kenneth French's website. Due to missing observations, the returns associated with five industries - Garbage, Government, Steam, Water and Other - are excluded from the sample, leading to a total of 33 industry portfolios. The 1 month Treasury bill rate used to calculate excess returns, is also obtained from Prof. French's website. Return data on the value-weighted market index is from CRSP, while monthly data on prices and earnings associated with the Standard & Poor's (S&P) Composite Index is obtained from Professor Robert Shiller's website. Macroeconomic and interest rate data, including the Federal funds rate (FFR), 10 year and 1 year Treasury bond yields, Moody's seasoned AAA and BAA corporate bond yields, and the 3 month treasury bill rate (TB3M), are all obtained from the FRED II database, available from the St. Louis FED's website.

B. *Estimating the "shifts in the investment opportunity set": a VAR approach*

Following Campbell (1991) and CV, I rely on a first-order VAR in order to estimate r_{t+1}^{h} and r_{t+1}^{CF}, the discount rate news (or shifts in the investment opportunity set) and cash-flow news components of unexpected market returns, respectively. The VAR[2] equation assumed to govern the behavior of a state vector $\mathbf{X}_t$, which includes the market return, and other variables known in time t which help to forecast changes in expected market returns, is given by

$$\mathbf{X}_{t+1} = \mathbf{A}\mathbf{X}_t + \boldsymbol{\epsilon}_{t+1} \qquad (28)$$

In this framework the news components are estimated in the following way,

$$r^h_{t+1} \equiv (E_{t+1} - E_t)\sum_{j=1}^{\infty} \rho^j r_{m,t+1+j} = \mathbf{e1}'\rho\mathbf{A}(I-\rho\mathbf{A})^{-1}\boldsymbol{\epsilon}_{t+1} = \boldsymbol{\varphi}'\boldsymbol{\epsilon}_{t+1} \qquad (29)$$

$$r^{CF}_{t+1} \equiv (E_{t+1} - E_t)\sum_{j=0}^{\infty} \rho^j \Delta d_{t+1+j} = r_{m,t+1} - E_t(r_{m,t+1}) + r^h_{t+1}$$
$$= [\mathbf{e1}' + \mathbf{e1}'\rho\mathbf{A}(I-\rho\mathbf{A})^{-1}]\boldsymbol{\epsilon}_{t+1} = (\mathbf{e1}+\boldsymbol{\varphi})'\boldsymbol{\epsilon}_{t+1} \qquad (30)$$

Here ρ is a discount coefficient linked to the average log consumption to wealth ratio $\rho \equiv 1 - \exp(\bar{c} - \bar{w})$, or average dividend yield, **e1** is an indicator vector that take a value of one in the cell corresponding to the position of the market return in the VAR, **A** is the VAR coefficient matrix, and $\boldsymbol{\varphi}' \equiv \mathbf{e1}'\rho\mathbf{A}(I-\rho\mathbf{A})^{-1}$ is the function that relates the VAR shocks with revisions in expected future market returns.

Hence, I estimate a first-order VAR with $X_t \equiv [FFPREM_t, TERM_t, EY_t, r_{mt}]'$, which represents a parsimonious representation for the variables that forecast market returns. In order to be consistent, with previous work (CV), I assume $\rho = 0.95^{\frac{1}{12}}$. FFPREM represents the spread between the Federal Funds rate and the 3 month Treasury bill rate, and thus it is a measure of both monetary policy and short-term interest rates. Its inclusion in the VAR is justified by previous evidence that both monetary policy (Patelis (1997), Goto and Valkanov (2002)) and short-term interest rates (Ang and Bekaert (2003)) do forecast future expected market returns, at least for short term forecasting horizons. TERM refers to the term structure spread - measured here as the difference between the 10 year and 1 year Treasury bond yields - which represents a proxy for the yield curve slope, and has been widely used in the predictability of returns literature, since Fama and French (1989) have found that TERM tracks the business cycle. EY denotes the earnings yield (calculated as the log of the earnings to price ratio associated with the S&P Composite index), used instead of the dividend yield, in light of recent evidence that the forecasting power of the dividend yield has decreased since the 90's, which might be related to a possible structural break in the firms' dividend policy, causing more firms to paying less dividends (Fama and French (2001)). The fourth variable used in the VAR is the log excess market return, which uses the value-weighted market index return from CRSP. CV use in their VAR specification an additional variable, the value spread, which they define as the difference between the log book-to-market ratios of small value and small growth stocks. I performed both Fama-French long-horizon regressions and first-order VAR estimation by including the value spread, and it revealed not significant at forecasting returns for the sample in analysis. Therefore I have opted to leave it outside the VAR vector. The sample used in estimating the VAR is 1954.07-2003.09.

Descriptive statistics for the VAR state variables are presented in table I, panel A. From the first-order autocorrelation coefficients, we can conclude that both TERM and especially EY are very persistent, while the VAR state variables are not highly correlated.

The VAR estimates corresponding to the market return equation on the VAR are presented at table II, panel A. FFPREM predicts negative market excess returns 1 month ahead, consistent with previous evidence (Patelis (1997)), and both TERM and EY predict positive market returns, also consistent with previous evidence, with all three regressors being statistically significant. EY is highly significant (1% level) which is remarkable, given previous evidence that the forecasting power of financial ratios is greater for long-horizon returns (beyond 1 year). In addition, the small degree of 1 month momentum in market returns, as captured by the estimate of $r_{m,t}$, is not statistically significant. The adjusted R^2 of 0.03 is in line with the values for monthly predictive regressions in other papers.

The results for the estimated "news" components are presented in Table II, Panel B, which is similar to Table 3 in CV. The discount-rate news variance represents 0.72% of the unexpected market return variance, compared to a weight of 0.28% for the cash-flow news component. This result goes in line with previous evidence (Campbell (1991), CV) that discount-rate news is the main determinant of unexpected market return's volatility. Additionally, r_{t+1}^{h} and r_{t+1}^{CF} are almost uncorrelated, as shown by the respective correlation coefficient (-0.003), a result also obtained in CV. Thus, this VAR specification seems to model the two news components as different and almost independent forces that drive unexpected market returns.

By analyzing the correlations of shocks in the individual VAR state variables with both r_{t+1}^{h} and r_{t+1}^{CF}, we can verify that the innovations on FFPREM are weakly negatively correlated with cash-flow news and almost uncorrelated with discount-rate news, suggesting that a unexpected rise in the FED Funds rate is associated with a minor negative impact on cash-flow news, i.e., negative revisions on future cash-flows or earnings. The magnitude of this correlation is nevertheless small, given that monetary policy has a short-term effect on stock prices (Patelis (1997), Maio (2005)). Shocks in TERM are almost uncorrelated with both news components, whereas Innovations on EY are strongly positively correlated with r_{t+1}^{h}, confirming that EY forecasts positive returns, in part due to the mean reversion in stock prices. Innovations in market returns are strongly negatively correlated with discount-rate news, reflecting the existence of long-term reversion in prices, and weakly positively correlated with r_{t+1}^{CF} suggesting that, at least partially, the rise in current prices is linked to future growth in cash-flows (earnings). The correlations between shocks in both EY and market return and the news components are in line with those obtained in CV.

C. *Econometric framework*

A natural econometric framework to estimate and test the asset pricing models presented in the previous section, is GMM, where the N sample moments correspond to the pricing errors for each of the N test assets at hand,

$$g_T(\mathbf{b}^*) \equiv \frac{1}{T}\sum_{t=1}^{T}(r_{i,t+1} - r_{f,t+1}) + \frac{\sigma_i^2}{2} - \gamma_0\sigma_{i,CF} - \gamma_1\sigma_{i,CFz} + \sigma_{i,h} = 0, \quad i = 1,\ldots,N \qquad (31)$$

where the covariances and variances were previously estimated. In the ICAPM with time-varying RRA there are two parameters to estimate, γ_0 and γ_1, so there will be $N-2$ overidentifying conditions, whereas the BBGB model will have $N-1$ overidentifying conditions in the associated GMM system. The standard errors for the parameter estimates and moments are presented in Appendix C, and the test that the pricing errors are jointly zero, with $\hat{\boldsymbol{\alpha}} \equiv g_T(\hat{\mathbf{b}}^*)$, is given by

$$\hat{\boldsymbol{\alpha}}' var(\hat{\boldsymbol{\alpha}})^{-1}\hat{\boldsymbol{\alpha}} \sim \boldsymbol{\chi}^2(N-K) \qquad (32)$$

Following Cochrane (1996), and given the fact that $var(\hat{\boldsymbol{\alpha}})$ is singular in most of the cases, I perform a eigenvalue decomposition of the moments' variance-covariance matrix, $var(\hat{\boldsymbol{\alpha}})=\mathbf{Q\Lambda Q}'$, where $\mathbf{Q}$ is a matrix containing the eigenvectors of $var(\hat{\boldsymbol{\alpha}})$ on its columns, and $\boldsymbol{\Lambda}$ is a diagonal matrix of eigenvalues, and then I invert only the non-zero eigenvalues of $\boldsymbol{\Lambda}$.

D. *Time variation in the risk-aversion coefficient*

In order to have a first impression of time variation in the RRA coefficient, the BBGB model is estimated with rolling samples. Thus a 5-year rolling sample window is used to produce estimates of covariances between the asset returns with the factors, which are then used in the GMM estimation of the RRA parameter, according to the pricing equation (14), therefore producing a time-series of RRA estimates. The values for RRA obtained from tests with both SBV25 and IND38, are presented in Figure 1. The graph shows, for both sets of test assets, that while there is no apparent time trend in RRA, there is important variation through time in the estimates: the RRA coefficient achieves values as high as 120 and on the other extreme, it assumes negative values (although not statistically significant). The main range is between 0 and 50, which represents a broad interval. The figure also gives some evidence in favor of a sharp decline in risk aversion, in late 90's. Although these estimates should be interpreted with caution, given the small sample size employed in each estimation, it represents nevertheless preliminary evidence that the RRA parameter is time-varying.

E. *Cyclical risk-aversion: estimating the risk premia*

Some of the variables that qualify to explain the time-variation in RRA should be correlated with business conditions, in order to make RRA countercyclical as argued above. I have opted for a specification for γ_t where it is explained contemporaneously by the market dividend yield (DY), smoothed earnings yield (EY*), default spread (DEF), industrial production growth (IPG) and the value spread (VS) .

E.1. *The BBGB model*

As a benchmark that enables comparison with the time-varying RRA ICAPM models, I estimate the BBGB model for three classes of test assets - the 25 size and book-to-market portfolios (SBV25), the 38 industry sorted portfolios (IND38), and the combination of SBV25 with IND38 (SBV25+IND38). Along with the first stage GMM estimates, I present the second stage GMM estimates, where the weighting matrix is S^{-1} with S being the spectral density matrix. The results for BBGB are presented in Table III, Panel A, in lines 1, 3 and 5. The first-stage GMM estimates of γ_0 are statistically significant at the 5% level, for the 3 classes of test assets, with the estimates for SBV25 being higher than the corresponding estimates for IND38 (10.771 versus 8.683). In the case with the combined portfolios, one gets an intermediate estimate for the RRA coefficient (9.607). The statistical significance of γ_0 confirms that the cash-flow news factor r_{t+1}^{CF} is a valid determinant of the SDF. The estimated risk price associated with the cash-flow news factor, λ_{CF}, is much higher than the symmetric of the discount-rate news risk price, λ_H, in line with the results of CV, and as predicted by the BBGB pricing equation, where the risk price of covariance with cash-flow news (the RRA coefficient) is higher than the symmetric of risk price for covariance with discount-rate news (1), if γ_0 is greater than 1. Notice that CV define the beta(covariance) with the negative of discount-rate news (good news), thus their estimate for λ_H and λ_{CF} are both positive. The estimates for λ_{CF} are significant at the 5% level, whereas λ_H is significant at the 1% level. For the 3 classes of portfolios, the model is not rejected by the asymptotic χ^2 test that the pricing errors are jointly equal to zero. The results from the efficient GMM estimation, show that the estimates for both γ_0 and λ_{CF}, are lower than the corresponding estimates in the first stage GMM.

E.2. *Dividend yield and default spread*

The Default spread (DEF), which represents the difference between BAA and AAA corporate bond yields, and DY - dividend yield on the value-weighted market index - are both employed by Fama and French (1989) to forecast market returns at several horizons, being interpreted as variables related with the longer-term components of business conditions. Here we're not concerned about the predictive power of DEF and DY over market returns. In fact, results from long-horizon regressions suggest that the forecasting ability of these two variables in recent samples has either erased (DEF) or declined substantially (DY), which in this latter case can be attributable to a potential shift in the dividend payout policy of firms, as suggested above. In response to that, these two variables were not included in the VAR vector - used to measure the changes in the investment opportunity set - in this paper, as well as in CV. What concern us here, is that both DEF and DY are negatively correlated with the business cycle, and therefore are candidates to explain the cyclical component of RRA. By using a business cycle dummy (CYCLE) - which takes the value 1 in an economic expansion as defined by the NBER, and takes value 0 in recessions - and performing monthly regression of either DEF and DY on CYCLE, one gets the following results (OLS t-statistics in parenthesis),

$$\begin{aligned} DY_t &= \underset{(37.978)}{0.039} - \underset{(-7.572)}{0.008}CYCLE_t \qquad Adj.R^2 = 0.087 \\ DEF_t &= \underset{(29.215)}{0.013} - \underset{(-8.133)}{0.004}CYCLE_t \qquad Adj.R^2 = 0.100 \qquad (33) \end{aligned}$$

These results confirm that both DEF and DY are countercyclical. Hence our specification for RRA is given by $\gamma_t = \gamma_0 + \gamma_1 z_t$, with $z_t = DY_t, DEF_t$, and we expect γ_1 to be positive for both DY and DEF, i.e., worsening business conditions lead to rising risk aversion.

The risk-price estimates associated with the model having RRA scaled by DY are presented in Table III, Panel A, lines 2, 4 and 6. The first stage estimates for γ_1 are positive in all 3 classes of portfolios, being highly significant in the case of SBV25 (1% level) although not significant for the industry portfolios. In the case of the combined portfolios, γ_1 is significant at the 10% level. The estimates for γ_0 are negative, and there is statistical significance only in the case of SBV25. This is a signal that the point estimates for RRA in the BBGB model, hidden the dependence of RRA from other variables. If we calculate the average time-varying RRA coefficient as

$$E(\gamma_t) = \gamma_0 + \gamma_1 E(DY_t) \qquad (34)$$

there is an increase on the average RRA values relative to the constant RRA coefficient, γ_0 in the BBGB model, for SBV25 (15.746 versus 10.771), while for IND38 there is no significant change. Thus by incorporating time-variation in RRA related with DY, one has an increase in risk aversion for the case of SBV25, and no significant impact in RRA in the case of the industry portfolios. In terms of the factor risk prices, λ_{CF} is higher than the symmetric of the discount-rate news risk price $-\lambda_H$, similarly to BBGB, with all 3 risk prices being statistically significant at the 5% level, for all 3 sets of portfolios. Comparing the scaled ICAPM with BBGB, λ_{CF} increases slightly (becoming significant at the 1% level) while λ_H decreases in magnitude, in the case of SBV25, while for IND38, both λ_{CF} and λ_H register a decline in magnitude. As expected, it seems, that by incorporating the additional risk factor, λ_{CFDY} -

which prices the time variation in RRA - some of the pricing ability of the two other factors is lost and transferred to the new factor. The estimates for λ_{CFDY} are positive and significant for the 3 classes of portfolios, although its magnitudes are much lower than the corresponding values for λ_{CF}. The second stage GMM estimates for the risk aversion parameters, have lower magnitude when compared to the first stage estimates, although the statistical significance is not altered. In addition, this causes the magnitude of both λ_{CF} and λ_{CFDY} to also decline relative to the first stage estimates. In both first and second stage estimates, the ICAPM scaled by DY is not rejected by test (32).

The results for the ICAPM scaled with DEF are presented in Table III, Panel B. For SBV25, γ_1 is negative, and marginally significant at 10%, thus it seems that RRA decreases with DEF, as opposed to the expected relation. For the IND38, γ_1 has the expected sign, but it is nevertheless not significant. Combining the 2 sets of portfolios, γ_1 is negative but highly insignificant (t-statistic of -0.762). The sign of γ_1 contributes to λ_{CF} being smaller than $-\lambda_H$ and λ_{CFDEF} to be negative for SBV25, while it is positive and significant for IND38. The statistical significance of the risk aversion parameters and risk prices increases in the second stage GMM, and in addition the average RRA declines, relative to the first stage estimates. Compared to the ICAPM scaled by DY, the first stage average RRA is lower for SBV25 (10.502 versus 15.746), being similar for IND38. In sum, in opposition with the model scaled with DY, DEF is not very satisfactory in explaining time-varying risk-aversion.

E.3. Smoothed earnings *yield*

The earnings yield like the dividend yield, should be a countercyclical state variable which can be used to explain time-varying risk aversion. Instead of the earnings yield, which was used in the VAR, I use a smoothed log earnings yield, which uses a 10 year moving-average of S&P 500 earnings (EY*). EY* is countercyclical as illustrated in the following regression,

$$EY_t^* = -2.555 - 0.322 CYCLE_t \qquad Adj.R^2 = 0.072$$
$$(-58.242) \qquad (-6.815) \qquad (35)$$

Hence, similarly to the model scaled with DY, we expect γ_1 to be positive. The results presented in Table III, Panel C, indicate that both γ_0 and γ_1 are positive for the three classes of portfolios, being highly significant for SBV25 (1% level), although not significant for IND38, as it was the case in the DY model. For SBV25+IND38, γ_0 and γ_1 are significant at the 5% and 10%, respectively. The average RRA estimates are similar to the corresponding estimates in the DY model. In terms of factor risk prices, λ_{CF} is higher than $-\lambda_H$ in the 3 sets of portfolios, whereas, the factor related with time-varying RRA, λ_{CFEY^*}, is significant for both IND38 and SBV25+IND38, although not significant for SBV25. Similar to the DY ICAPM, the second stage GMM produces lower magnitude estimates for the RRA coefficients and beta risk prices, and the average RRA estimates are similar to the second stage corresponding values for the DY model. Thus, as expected, the ICAPM models scaled with DY and EY* are very approximate.

E.4. *Industrial production growth*

So far, the state variables used to explain time-varying RRA are directly linked to asset prices, whether they are financial ratios (DY and EY*) or interest rates spreads (DEF). As noticed above, although all those 3 variables are related with business conditions, they have been used for some time in the predictability of returns literature, as predictors of expected market returns. Thus their role as determinants of γ_t might be to some extent, mixed with their role as forecasters of future market returns. To overcome this issue, and as a robustness check, I use the industrial production monthly growth as an alternative determinant of γ_t, a variable which is not directly linked to asset prices, being in addition highly procyclical, as the following regression confirms,

$$IPG_t = -0.008 + 0.012 CYCLE_t \qquad Adj.R^2 = 0.213$$
$$(-8.644) \qquad (12.670) \qquad (36)$$

The IPG measure used in equation (10) is the cyclical component of IPG calculated as $IPG_t^* = 0.012CYCLE_t$. Being positively correlated with the business cycle, we expect γ_1 to be negative, i.e., a rise in IPG, corresponding to increasing business conditions, leads to declining risk aversion. The results for the model scaled by IPG are presented in Table III, Panel D. The SDF parameter estimates, show that γ_1 has the expected sign, being negative for the 3 sets of portfolios, and in addition it is highly significant for SBV25 (1% level), and not significant for the industry portfolios, which in this latter case, goes along the results of the previous scaled models. On the other hand, γ_0 is positive and significant for SBV25. Unfortunately, λ_{CFIPG} has very small values and it is not significant for the 3 classes of portfolios. Comparing with the previous scaled models, the average RRA is much lower in the case of SBV25 (2.550 versus 15.746 for DY) while for IND38 the difference in magnitudes is not as significant (7.631 versus 8.973 for DY). In fact, contrary to the other ICAPM scaled models, the average RRA associated with IND38 is higher than the corresponding from SBV25. The second stage average RRA is higher than the first stage estimates (4.224 for SBV25 and 5.899 for IND38), but still lower when compared with the other scaled models.

E.5. *Value spread*

CV use the value spread (VS) defined above, in their VAR specification as a state variable that predicts expected market returns. A rise in VS signals an increase in the valuations/prices of growth stocks relative to value stocks, which might be a result of a funds flow from value to growth. As we'll see in section V, growth stocks have higher magnitudes in both discount-rate and cash-flow betas, relative to value stocks, hence, growth stocks are riskier than value stocks during the sample in analysis. Thus a rise in VS is associated with a decrease in risk aversion.

The results for the model scaled by VS are presented in Table III, Panel E. As expected γ_1

is negative for all 3 sets of test assets, being significant for SBV25 (1% level) and SBV25+IND38 (5%). γ_0 is positive and has similar statistical significance relative to γ_1. The factor related with time-varying RRA λ_{CFVS}, is positive and marginally significant (10%) for IND38, being negative and non-significant for SBV25. For SBV25+IND38, λ_{CFVS} is positive but not significant. By incorporating RRA scaled by VS, it causes λ_{CFVS} to have higher magnitudes than the cash-flow news factor, although its higher standard errors make it less significant than λ_{CF}. Since λ_{CFVS} is negative for SBV25, we have that λ_{CF} is lower than $-\lambda_H$, whereas for both IND38 and SBV25+IND38, the relation $\lambda_{CF} > -\lambda_H$ holds. The average RRA estimates are slightly lower when compared with the models scaled with DY and EY, for SBV25, whereas for IND38, it achieves similar values relative to those values. Overall, these results confirm that a rise in the value spread, and hence a higher demand for growth stocks, in disfavor of value stocks, is linked with a decline in risk aversion.

E.6. *Market return*

From equation (3) above, γ_t is related with wealth or equivalently, market returns. In Appendix B.5, it is shown that under certain conditions, γ_t is negatively correlated with wealth or market returns. Nevertheless, it seems reasonable to assume that declining prices/negative returns, for some period, originates an increase in risk aversion, leading investors to be more reluctant to invest in stocks. Thus, time-varying risk aversion can be determined of two types of forces. On one hand, we have cyclical risk-aversion, which includes the state variables used so far, which is related with the business-cycle fluctuations that causes changes in non-financial wealth (labor income), leading the investors to require higher expected returns to invest in stocks. The other component of risk-aversion, which can be related with the first one, is related to direct losses in market returns and financial wealth, i.e. overall market conditions. By choosing DY as the variable that explains the cyclical risk-aversion, the specification for RRA is given by $\gamma_t = \gamma_0 + \gamma_1 DY_t + \gamma_2 r_{mt}$, where r_{mt} is the log excess market return. In the preceding equation, γ_2 is expected to be negative, i.e., rising market returns lead to lower risk aversion.

The results for the model scaled by the market return are presented in Table IV. In panel A, I present the results for the specification with only market return explaining time-varying RRA. γ_1 is positive for SBV25, and negative for IND38, being significant in both cases. For SBV25+IND38, γ_1 is negative, although not significant. The average RRA for SBV25 is much higher than in the previous models (37.422), while for IND38 is slightly negative, due to the strong negative effect of r_{mt} on γ_t. The positive correlation between r_{mt} and γ_t for SBV25 might be a consequence of a small degree of short-term momentum in market returns, as indicated by the VAR estimated in Table II above.

If we include DY in the specification of γ_t, as specified above, the RRA coefficients have the desired signal. γ_1 is positive whereas γ_2 is negative, for all 3 classes of portfolios. In terms of statistical significance, γ_1 is strongly significant for SBV25 and marginally for SBV25+IND38, while, γ_2 is significant only for IND38. Hence, the dominant determinant of risk aversion is DY (business conditions) in the case of SBV25, while for IND38 the dominant force is the market return (financial wealth). λ_{CFDY} is positive and significant for SBV25 and SBV25+IND38, whereas λ_{CFRM} is negative and significant for IND38. Comparing with the ICAPM scaled with DY, the average RRA is slightly lower for SBV25, but the biggest decline is for IND38 with a value lower than 1, being even negative in the second stage GMM. This a consequence of γ_t being negatively determined by the market return hat in the case of IND38. Overall, these results suggest evidence in favor of 2 types of forces influencing time-varying risk aversion, recession risk factors, related with the business cycle, and shocks in financial wealth.

F. *Individual pricing errors*

F.1. *Size-book to market portfolios*

Both BBGB and scaled ICAPM models were not rejected using the test of joint nullity of the pricing errors (32). As emphasized before (Cochrane (1996, 2001), Hodrick and Zhang (2001)), inference using this test can be misleading due to the singularity of $var(\hat{\boldsymbol{\alpha}})$, and the inherent problems in inverting it. As a consequence I have opted for a generalized inverse as described above. Nevertheless, it could be that the low test values, are not so much the result of individual low pricing errors - what we want - but rather the economic uninteresting result of low values for $var(\hat{\boldsymbol{\alpha}})^{-1}$. To address this issue, we have to pursue an analysis of the individual pricing errors. Figure 2 presents pictures of the pricing errors for SBV25, from BBGB and the ICAPM scaled by DY, DEF, EY*, IPG, VS and DY+RM. We can see that, with the exception of DEF, all the scaled ICAPM models compare favorably with BBGB. In particular the ICAPM scaled with DY, EY*, VS and DY+RM, have lower magnitude pricing errors than BBGB, for most portfolios. The BBGB individual errors have a robust pattern across size quintiles: within each size quintile, the growth portfolio has large negative pricing errors and the value portfolio has large positive errors. This has been referred as the value premium, and has been originally documented for the CAPM (Fama and French (1992, 1993)). This pattern is strongly attenuated, and in some cases non-existent for the 4 scaled ICAPM models mentioned above. We can confirm these results in Table V, with the scaled ICAPM having much lower pricing errors than BBGB, across the book-to-market quintiles, with the sole exception of the DEF model. Regarding the size quintiles, with the exception of the smallest size quaitile, the scaled models perform favorably against BBGB, in terms of average pricing errors. Although many of the pricing errors are individually significant, as indicated by the respective t-statistics presented in panel C, the magnitudes and economic significance are small. For example, in the case of DY+RM model, the largest errors across book-to-market (size) quintiles are 0.141% (0.263%), corresponding to annualized errors of 1.693% (3.153%), while for the VS model, the same quantities are 0.095% (0.137%) and 1.135% (1.648%), on a monthly and annual basis, respectively.

F.2. Industry *portfolios*

The pricing errors and respective t-statistics, for the industry portfolios, are presented in table VI. The magnitudes are in general low, and only 3 industries, FOOD, METAL and SMOKE have significant pricing errors. Contrary to the case of SBV25 portfolios, the errors magnitudes between BBGB and the scaled models is not very different, although most of the scaled models have slightly lower pricing errors.

G. *Robustness checks*

G.1. *Standard errors with estimation error*

The standard errors associated with the GMM system (31), don't take into account the fact that the covariances are generated regressors and thus estimated with error. Instead, they are assumed to be fixed parameters estimated outside the system. In order to take into account the estimation error associated with the covariances, in the spirit of Shanken (1992), I conduct a generalized GMM system, where the first set of moments, for N test assets and K factors, is given by

$$g_{1T}(\mathbf{b}^*) \equiv \frac{1}{T}\sum_{t=1}^{T}[r_{i,t+1} - E(r_{i,t+1})][\mathbf{f}_{t+1} - E(\mathbf{f}_{t+1})] - \boldsymbol{\sigma}_{i,f} = 0, \quad i = 1,\ldots,N \qquad (37)$$

which identifies the covariances between the log individual excess returns and the factors, $\boldsymbol{\sigma}_{if} \equiv Cov(r_{i,t+1}, \mathbf{f}_{t+1})$, and corresponds to NK orthogonality conditions, thus this set of moments is exactly identified. System (37) corresponds in this framework to the time-series regressions of the time series/cross sectional regression framework, which estimate the betas, and that will be conducted in the next section. The second set of moments correspond to the N pricing errors (31), hence the number of overidentifying conditions is $N - K$, as previously.

The results, presented in Table VII, show that, the GMM coefficient estimates of γ_0, γ_1 and γ_2 are equal to the estimates produced from the system (31), as we would expect. In terms of statistical significance, the parameter that measures time-varying RRA γ_1, is highly significant (1% level) for the models scaled with DY, DEF, EY*, IPG and VS, for all portfolios. In particular for the case of IND38, γ_1 is now significant at the 1% level. For the DY+RM model, γ_1 is strongly significant for both SBV25 and SBV25+IND38, whereas γ_2 is marginally significant for IND38. Overall, the results indicate that taking into account the covariances estimation error, it strengths the significance of the time-varying component of RRA in the pricing equation.

G.2. *The Hansen-Jagannathan distance*

As a robustness check, I estimate and evaluate the models above, using the Hansen and Jagannathan (1997) (HJ-distance), which was employed by Hodrick and Zhang (2001), to evaluate a set of alternative asset pricing models. The HJ-distance is defined as,

$$\delta = [g_T(\mathbf{b}^*)'\mathbf{W}_{HJ}g_T(\mathbf{b}^*)]^{\frac{1}{2}} \qquad (38)$$

with $\mathbf{W}_{HJ} = E(\mathbf{r}_t\mathbf{r}_t')^{-1}$ being the inverse of the second moment of log excess returns for the test assets in analysis, and $g_T(\mathbf{b}^*)$ is the vector of average pricing errors defined above. δ is interpreted as the minimal distance between a SDF proxy and the set of true pricing Kernels. The parameters from our model $\mathbf{b}^*$, can be estimated within a GMM system like system (31) defined above, with the moments weighting matrix given by $\mathbf{W}_{HJ}$,

$$\hat{\mathbf{b}}^* = \arg\min \delta^2 = \arg\min g_T(\mathbf{b}^*)'\mathbf{W}_{HJ}g_T(\mathbf{b}^*) \qquad (39)$$

This approach has the advantage of allowing the comparison across different models (similarly to the First stage GMM), since $\mathbf{W}_{HJ}$ is model invariant, which is not the case for $\mathbf{S}^{-1}$.

The standard errors formulas for the parameter estimates and moments are given in the appendix. Hodrick and Zhang (2001) have found that even if a 3-factor model is true, the HJ-distance test has size greater than the theoretical size, for the case of SBV25 portfolios. Since, in addition, $E(\mathbf{r}_t\mathbf{r}_t')^{-1}$ is near singular for SBV25, I report results only for the industry portfolios, which are presented in Table VIII. The estimates for the risk aversion parameters and average RRA have lower magnitudes, when compared to the first stage and second stage estimates given above. The models are not rejected by the asymptotic test that the pricing errors are jointly zero, and in addition, the test $\delta = 0$, is not rejected for all models. Nevertheless, these results should be interpreted with caution, given the near-singularity of $E(\mathbf{r}_t\mathbf{r}_t')^{-1}$ also in the case of industry portfolios.

IV. Time-series cross-sectional regressions

As an additional robustness check, and as an alternative technique to GMM, I use the time-series/cross-sectional regressions approach (TSCS), presented in Cochrane (2001), chapter 12. In this approach, on a first step, individual time-series regressions are performed to obtain estimates of the factor loadings for each test asset, and in a second step, the average individual returns are regressed on the individual factor betas within a cross-sectional regression, in order to estimate the common factor risk premiums.

This approach enables to obtain direct estimates of betas and the risk prices of betas, allowing us to assess its economic plausibility. In addition, this method allows the comparison with other ICAPM models estimated by this approach (CV, Brennan et al (2004)), and alternative factor models, which are often estimated in terms of beta risk, rather than covariance risk.

As in CV, I use simple excess returns, $(R_{i,t+1} - R_{f,t+1})$ instead of log returns on the left hand side of the expected return-beta equations - with the assumption of a second-order Taylor assumption, the two quantities are the same - in order to facilitate the comparison with other multifactor models, followed in the next section.

Since our model, is a theoretically derived asset pricing model, the cross-sectional regression will estimate the risk prices of covariances, **b**, rather than the beta's risk prices. In vector form, the pricing errors can be represented by,

$$E(\mathbf{R}_t - R_{f,t}\mathbf{1}_N) = -\boldsymbol{\beta}\boldsymbol{\Sigma}_\mathbf{f}\mathbf{b} \qquad (40)$$

and the betas' risk prices are recovered by $\boldsymbol{\lambda} \equiv -\boldsymbol{\Sigma}_f\mathbf{b}$.

The results are reported in Table IX. In order to save space, I report results only for BBGB (Panel A), and ICAPM scaled by DY (Panel B), and VS (Panel C). For each model, I estimate both OLS and GLS cross-sectional regression estimates of risk premia, with two types of standard errors associated. The second row of t-statistics associated with factor risk prices, and the second column of pricing errors tests, use standard errors calculated with the Shanken (1992) adjustment in order to take into account the estimation error in betas. I will denote these standard errors as Shanken or type II standard errors. The first row of t-statistics and first column of pricing errors tests, are based on "robust" standard errors, calculated within a GMM system that relaxes the assumptions implicit in the Shanken correction of i.i.d. errors (from the time-series regressions) and factors, and independence between the time-series residuals and the factors. These standard errors are derived in the appendix, and will be denoted thereafter by robust or type I standard errors.

In all three models, the risk-aversion parameters and beta risk prices estimates from OLS, are very similar to those arising from first stage GMM in Table III, as we would expect, since the two methods are equivalent. On the other hand, the GLS estimates have higher magnitude compared to the second stage GMM estimates in Table III. Since the GLS cross-sectional regression is not equivalent to the second stage GMM, we should expect the difference in results. In general, the χ^2 tests have higher values, when the type II standard errors are used instead of type I standard errors, and in some cases, the model is rejected if we use the Shanken standard errors.

Thus these results suggest that the restrictive and implausible assumptions - of i.i.d. and conditionally homoskedastic time series residuals - do play an important role in the statistical inference about the pricing errors, in the ICAPM scaled models, especially for SBV25 and SBV25+IND38 portfolios.

V. Discussion

A. *Risk aversion estimates*

The average RRA estimates, $E(\gamma_t)$ associated with the scaled ICAPM models, and presented in table III above, seem reasonable: For SBV25 we have average RRA below 16 for all the models, whereas for IND38, most of the estimates are around 8. The second stage estimates are even lower. Since these are averages, it is important to have an idea of the behavior of the time-series of γ_t, and its associated dispersion. In figure 3, I present estimates of $\gamma_t = \gamma_0 + \gamma_1 z_t$, for $z_t = DY_t, EY_t^*, VS_t$, and the augmented model $\gamma_t = \gamma_0 + \gamma_1 DY_t + \gamma_2 r_{mt}$, for SBV25+IND38. The parameter estimates are the first stage estimates obtained in tables III and IV. When γ_t is negative, I truncate it at zero, since negative γ_t makes no economic sense. It is remarkable that the time-series of RRA estimates from the 4 models, have a similar pattern, and there is no apparent time trend, although there is lots of dispersion in γ_t. The estimates from the VS model are the most volatile ones. Until the 90's most of the estimates are in a range 5-20, whereas, in the 90's there is a sharp decline in risk aversion, in line with existent evidence.

B. *Pricing the market return*

Most of the equity premium puzzle literature, has focused on the (in)consistency between plausible risk-aversion parameter estimates and the ability to price consumption or the market return. For this reason, I evaluate whether the ICAPM with time-varying risk aversion can accurately price the market excess return, $r_{m,t+1} - r_{f,t+1}$.

If we apply the pricing equation (23) to $r_{m,t+1} - r_{f,t+1}$, as shown in Appendix B.6., it follows,

$$E(r_{m,t+1} - r_{f,t+1}) = \gamma_0(\sigma^2_{CF} - \sigma_{CF,h}) + \gamma_1(\sigma_{CF,CFz} - \sigma_{h,CFz}) + 0.5(\sigma^2_h - \sigma^2_{CF}) \qquad (41)$$

and the associated pricing error is given by,

$$\hat{\alpha}_m = \frac{1}{T}\sum_{t=1}^{T}(r_{m,t+1} - r_{f,t+1}) - \gamma_0(\sigma^2_{CF} - \sigma_{CF,h}) - \gamma_1(\sigma_{CF,CFz} - \sigma_{h,CFz}) - 0.5(\sigma^2_h - \sigma^2_{CF}) \qquad (42)$$

Estimates of $\hat{\alpha}_m$ are given in Table X, where the RRA parameter estimates are from tables III and IV. The average pricing errors associated with the scaled models have low magnitudes: For SBV25, DY has an annualized error of -1.82%, whereas for IND38, the largest absolute error is for IPG model (-0.473%). In addition, within each of the 3 classes of portfolios, the scaled models have lower magnitude errors compared with the BBGB model. Hence, the time-varying RRA ICAPM also outperforms the static ICAPM, in pricing the market return, in addition to the size/book-to-market and industry portfolios. Overall, these results suggest that the risk-aversion estimates from the scaled ICAPM are consistent with the market return.

C. *Comparison with other asset pricing models*

Given the good performance of the ICAPM with time-varying RRA in last two sections, and given its theoretical derivation, it is important to compare it with other models, namely the traditional CAPM, and the Fama-French model, which has less theoretical background, being nevertheless a empirically successful factor model.

Figure 4 plots the average pricing errors associated with SBV25 portfolios, for the ICAPM scaled with DY, VS and DY+RM, against 3 alternative factor models: the CAPM, the Fama and French 3 factor model (FF3F) and the Fama and French 4 factor model (FF4F), which incorporates a momentum related factor (UMD). The specification for FF4F is

$$E(R_{i,t+1} - R_{f,t+1}) = \lambda_M \beta_{i,M} + \lambda_{SMB} \beta_{i,SMB} + \lambda_{HML} \beta_{i,HML} + \lambda_{UMD} \beta_{i,UMD} \qquad (43)$$

where λ_M, λ_{SMB}, λ_{HML} and λ_{UMD} denote the risk premia related with the market, size, book-to-market and momentum factors, respectively, and where the factors SMB, HML and UMD have been orthogonalized relative to the market factor, before conducting the time-series regressions to estimate the factor loadings. The FF3F model is obtained by making $\lambda_{UMD} = 0$, whereas the CAPM is derived by imposing $\lambda_{SMB} = \lambda_{HML} = \lambda_{UMD} = 0$.

Figure 4 shows that the 3 scaled ICAPM models have lower pricing errors than the CAPM, and approximate pricing errors relative to both FF3F and FF4F models. On the other hand, the BBGB model has approximate pricing errors relative to CAPM. These results are confirmed in Table XI. In Panel C, containing the average pricing errors for book-to-market quintiles, we can observe that the 3 scaled ICAPM models have lower average errors than the CAPM, across all quintiles. Relative to FF3F and FF4F, the scaled ICAPM have slightly higher average errors, but this difference is not economically significant. On the other hand the BBGB has similar average errors compared to the CAPM, across all quintiles. Regarding the size quintiles, the scaled ICAPM have approximate average errors relative to the already low pricing errors associated with the CAPM. The only exception is for the lowest quintile (S1), where some of the scaled models - DY and DY+RM - have higher average errors than the CAPM, although they are not economically large. These results show that the BBGB model can not price the value premium, remaining an anomaly, as it was the case for the CAPM.

Overall, these results confirm that the various versions of the scaled ICAPM perform very well compared with the alternative factor models, and on the other hand, the static ICAPM does not improve the CAPM in pricing both growth and value stocks.

D. *The Value Premium*

One of the biggest difficulties of the existing pricing models is in being able to price the extreme small-growth portfolio (11) within the class of SBV25 portfolios. In fact, the CAPM, FF3F and the BBGB ICAPM from CV fail to price this portfolio accurately. For example, CV obtained an annualized pricing error of -8.8%, in one of the samples used in the paper. The results displayed in table XI (panel A, first line), indicate that the pricing error for portfolio 11 is economically non significant, when estimated from the ICAPM scaled models - the biggest annualized error is -1.335% for DY, a value difficult to sustain after transactions costs. Furthermore, the SBV11 pricing error is lower in the ICAPM scaled models, compared with both BBGB and the alternative factor models CAPM, FF3F and FF4F. It seems paradoxical that the scaled ICAPM performs so well in pricing the book-to-market quintiles, and in special, the risky small-growth portfolio, given the previous evidence from CV, that their static ICAPM, does not price SBV11 properly. The difference in results should arise from time-variation in risk aversion. In addition, the pricing ability of the ICAPM is sustained across all the book-to-market quintiles: the model fits equally well to both growth and value stocks. What drives the ability of the ICAPM to price equally well both growth and value stocks? Which of the fundamental components referred earlier - cash-flow news, discount rate news or time-varying RRA - contributes the most to the low pricing errors in book-to-market quintiles? To address these questions, we need to measure the individual contribution, of each of the factors, r_{t+1}^{CF}, $r_{t+1}^{CF}z_t$, and r_{t+1}^{h}, in explaining the average returns across book-to-market quintiles.

Tables XII, XIII and XIV present factor loading estimates associated with the SBV25 portfolios, for BBGB and the ICAPM scaled with DY and VS, respectively. In panels B and C of each table, are presented the average factor loadings associated with book-to-market and size quintiles, respectively. In table XII, the betas associated with r_{t+1}^{CF} and r_{t+1}^{h} are respectively positive and negative, as expected. From panel B, growth stocks (BV1) have higher magnitudes betas for both cash-flow and discount-rate news factors, relative to value stocks (BV5). Furthermore, the decline in the betas magnitude is monotonic as we move from low to high book-to-market quintiles. Hence, these results seem to suggest that growth stocks are riskier than value stocks. The ICAPM scaled with DY has the same pattern regarding the cash-flow and discount-rate news betas, with growth stocks exhibiting higher magnitudes in both types of betas, relative to value stocks. On the other hand, the sensitivity to the factor related with time-varying RRA, $r_{t+1}^{CF}DY_t$ differs both in magnitude and sign for growth and value stocks, with BV1 having a beta of -9.998 versus 9.360 for BV5. In addition, the betas associated with $r_{t+1}^{CF}DY_t$ rise in a monotonic way, being highly negative for BV1, around zero and slightly positive for intermediate quintiles (BV2 and BV3), and highly positive for BV5. In the case of the ICAPM scaled by VS, value stocks (BV5) have much higher cash-flow betas than growth stocks (BV1), whereas, the factor related with time-varying RRA, $r_{t+1}^{CF}VS_t$ has factor loadings which differ in sign between growth and value stocks, similarly to the DY model, with BV1 having a beta of 0.216 versus -0.844 for BV5. In addition, the betas decrease monotonically as book-to-market increases.

Do these factor loading estimates make economic sense? Growth stocks have higher duration risk than value stocks, i.e. their prices are more sensible to upward revisions in future expected returns or future cash-flow growth. In periods characterized by economic boom, rapid growth in earnings and cash-flows, financial stability and lower and stable interest rates, growth stocks benefit the most, since they enable to profit from the benign side of higher duration risk, and higher "elasticity" to the business cycle. These are also periods of lower risk aversion, and hence investors require a lower premium to invest in growth stocks, relative to value stocks. In periods of economic downturn, low or negative cash-flow growth, financial instability, increasing and volatile interest rates and in addition higher risk aversion, we have the "negative" side of higher duration risk and higher "elasticity" to the business cycle:

investors require a higher return to invest in growth stocks relative to the safer, lower duration risk and lower elasticity (utilities are a good example here) value stocks. Thus a decline in risk aversion (measured here by either a decrease in $r_{t+1}^{CF}DY_t$ or a increase in $r_{t+1}^{CF}VS_t$) leads to rise in prices of growth stocks, producing lower expected returns. We should expect the rise in value stocks prices to be less pronounced than for growth stocks, but actually we see a decrease in value stocks prices in response to a decrease in risk aversion, thus explaining the signs of the betas associated with time-varying RRA risk factors: A decline in $r_{t+1}^{CF}DY_t$ is equivalent to a decrease in risk aversion, thus growth stocks have negative betas while value stocks have positive betas. Similarly, a rise in $r_{t+1}^{CF}VS_t$, represents lower risk aversion, leading to positive (negative) betas for growth (value) stocks. Hence, these results suggest that the changes in RRA lead to a investment flow between growth and value stocks, explaining the overall pricing ability of the model concerning these two categories of stocks. To address this issue in more detail, we need to analyze the risk premium (beta times risk price) for each of the 3 factors, and its relative contribution at explaining the average returns for each book-to-market quintile. In Table XV, we have the factor risk premium associated with book-to-market quintiles, for BBGB and the scaled ICAPM with DY and VS. For comparison, I present the average pricing errors associated with the CAPM, and the average returns and average pricing error, for each quintile. In respect to the BBGB model, the risk-premium associated with r_{t+1}^{CF}, contributes the most to average returns for all the quintiles, especially for BV1, where r_{t+1}^{CF} has a premium of 0.713 versus 0.175 for r_{t+1}^{h}. It turns out that the model has book-to-market pricing errors similar to those arising from the CAPM, as noticed above, and therefore the model is not able to explain the value premium. In the case of the ICAPM scaled by DY, the risk premium is still higher for r_{t+1}^{CF} relative to r_{t+1}^{h}, whereas $r_{t+1}^{CF}DY_t$ has a negative (positive) risk premium for growth (value) stocks (-0.458 for BV1 versus 0.429 for BV5). In addition the $r_{t+1}^{CF}DY_t$ risk premium rises monotonically with book-to-market, similarly with the betas, and $r_{t+1}^{CF}DY_t$ is the main responsible for the low pricing errors across all quintiles, i.e., small negative pricing errors for growth stocks and small positive errors for value stocks. In the ICAPM scaled by VS, the risk premium associated with r_{t+1}^{CF} is lower relative to r_{t+1}^{h}, in opposition with the former two models (0.058 versus 0.522 for BV1, and 0.139 versus 0.405 for BV5.) The factor $r_{t+1}^{CF}VS_t$ has a negative risk price for growth stocks and positive for value stocks (-0.085 for BV1 versus 0.332 for BV5), and similarly to the DY model, it is the responsible for the low pricing errors in both low and high book-to-market quintiles. These results show that in both scaled ICAPM models, it is the factor related with time-varying risk aversion, that drives the low pricing errors in all book-to-market quintiles, a feature that is not present in the BBGB model.

Given that the post-war period is characterized mainly by economic expansions and financial stability, and hence low risk aversion, this explains the low (high) premia required by investors to invest in growth (value) stocks. It is important to emphasize that it is low risk aversion, rather than high risk aversion, that explains the high excess return required to invest in value stocks: in periods of low risk aversion, growth stocks tend to outperform value stocks, and hence investors require low prices - and hence higher expected returns - to invest in those stocks, instead of investing in the more "attractive" growth stocks.

E. *Comparison to Campbell and Vuolteenaho (2004)*

CV have found that their BBGB had some success in pricing the SBV25 plus 20 risk-sorted portfolios, but they pointed out that the pricing ability depends crucially on the inclusion of VS on the VAR state vector. Given that the BBGB in this paper has not improved significantly the pricing ability of the CAPM, I estimated the BBGB, by including VS in the state vector, $X_t \equiv [FFPREM_t, TERM_t, VS_t, EY_t, r_{mt}]'$, and the results are reported in table XVI. Compared to the former BBGB, the difference in magnitude for discount-rate betas between BV1 and BV5 is now greater. On the other hand, regarding the cash-flow betas, the difference between growth and value stocks is much smaller (1.130 for BV1 versus 1.022 for BV5). Thus, we have almost flat cash-flow betas across the book-to-market quintiles, more in accordance with CV, which have found higher cash-flow betas for value stocks relative to growth stocks. Regarding the model's performance, we have lower pricing errors compared with the former BBGB, but still higher when compared to the scaled ICAPM, for all quintiles, and in special, growth stocks have large negative pricing errors. These results seem to suggest that by incorporating VS in the calculation of the cash-flow and discount -rate news components, we are able to capture some part of time-varying risk-aversion. Nevertheless, it is the theoretical derived factor $r_{t+1}^{CF} z_t$ that properly captures time-varying risk aversion, and improve the pricing ability for book-to-market quintiles, thus explaining the value premium. Overall, these results suggest that the value premium probably is not an anomaly, rather being rationalized by fundamental factors related to the business cycle and time-varying risk aversion.

F. *Comparison to Fama and French (1993)*

In the last sub-section, we concluded that the ICAPM with time-varying risk aversion can take into account the value premium anomaly. The Fama and French (1993) 3 factor model earned great acceptance, by being able to price the value premium. The model which can be rationalized in an APT context, uses the HML factor in order to explain the CAPM negative (positive) pricing errors for growth (value) stocks. Since the scaled ICAPM is derived in a theoretical context, it is important to compare the 2 models, and more speciphically to infer if the less theoretical based HML factor is still significant in explaining the cross-section of returns, in the presence of the time-varying RRA factors. This has the advantage of offering a fundamental coherent story for the value premium, instead of the ad-hoc HML portfolio, justified by absence of arbitrage opportunities (APT). Hence, I estimate the scaled ICAPM by adding the HML factor, and test whether it has explanatory power over the cross-section of returns, using the specification,

$$E(r_{i,t+1} - r_{f,t+1}) + \frac{\sigma_i^2}{2} = \gamma_0\sigma_{i,CF} + \gamma_1\sigma_{i,CFz} - \sigma_{i,h} + \gamma_{HML}\sigma_{i,HML} \qquad (44)$$

with $\sigma_{i,HML} \equiv Cov(r_{i,t+1}, HML_t)$.

The results are displayed in Table XVII, including the DY ICAPM (Panel A) and VS ICAPM (Panel B). HML is not significant, both in terms of covariance (γ_{HML}) or beta risk price (λ_{HML}). The only exception is for SBV25 in the DY ICAPM, where both γ_{HML} and λ_{HML} are marginally significant (1% level). These preliminary results indicate that to some degree, both the recession risk factor from the scaled ICAPM and the HML factor measure the same risks, which are related to time-varying risk aversion.

H. *Momentum revisited*

It is possible that the recession risk factor accounts for some of the momentum observed in stock prices. In fact there are potential valid economic arguments for the momentum to be related with macroeconomic risk factors: The observed positive autocorrelation in some macro variables, the progressive response of the stock market to news in macro variables, and most important, it is possible that some part of the momentum observed in prices or realized returns, might be a consequence of cyclical variation in RRA: After a period of declining stock prices, investors' risk tolerance should decrease, originating further declines in stock prices - and hence in realized returns - which represents momentum. In this way, risk aversion which is related with previous realized returns, should amplify the observed positive autocorrelation in stock prices, producing therefore a explanation for momentum. Hence, a possible test to be made is to analyze whether the momentum factor, UMD is still significant after accounting for time-varying risk aversion in the ICAPM model. In order to do that, I specify the following model,

$$E(r_{i,t+1} - r_{f,t+1}) + \frac{\sigma_i^2}{2} = \gamma_0\sigma_{i,CF} + \gamma_1\sigma_{i,CFz} - \sigma_{i,h} + \gamma_{UMD}\sigma_{i,UMD} \qquad (45)$$

with $\sigma_{i,UMD} \equiv Cov(r_{i,t+1}, UMD_t)$.

The results are displayed in Table XVIII, again, for the DY ICAPM (Panel A) and VS ICAPM (Panel B). In the model scaled by DY, UMD is significant both in terms of covariance (γ_{UMD}) or beta risk price (λ_{UMD}) for SBV25 (1%) and SBV25+IND38 (5%), whereas it is not significant for the industry portfolios. In the ICAPM scaled by VS, UMD is not significant, both in terms of covariance and beta risk, for all 3 classes of portfolios. While these results are not entirely conclusive, they do suggest that some part of the momentum observed in stock returns, might be explained by time-varying risk aversion in the context of a fundamentally derived ICAPM. Nevertheless, this explanation for momentum should be analyzed in more detail, and it is beyond the scope of this paper.

VI. Conclusion

Using a derivation of the ICAPM in a very general framework - with general unspecified preferences - the RRA coefficient can be made time-varying. In addition, there are reasons to consider the RRA - or local curvature of the utility function - both time-varying and countercyclical (Campbell and Cochrane (1999)). By extending the RRA coefficient to be time-varying, it can be decomposed in two components: The constant coefficient which can be interpreted as the long term RRA, and a time-varying component, that is negatively correlated with the business cycle, and thus is interpreted as the cyclical component of risk aversion. The variables that represent proxies for the cyclical component of RRA are the market dividend yield, default spread, smoothed earnings yield and industrial production growth, all being highly correlated with the business cycle. In addition, the value spread - a proxy for the relative valuation of value stocks versus growth stocks - is included as a determinant of risk aversion. I specify and estimate an extended version of the bad beta-good beta model (BBGB) from Campbell and Vuolteenaho (2004), by incorporating time-varying RRA, and the results show that risk aversion is countercyclical. In addition, the ICAPM with time-varying RRA perform better than the BBGB model, when tested on portfolios sorted on both size and book-to-market and also industry portfolios. The results from an augmented scaled ICAPM show that the market return has a negative effect on risk aversion, thus risk aversion seems to be affected by both business conditions and financial wealth. The estimates of the average RRA coefficient seem reasonable and plausible - in most cases under 16 - which is line with previous micro evidence that argue for low values of RRA. The model is able to capture a significant decline in risk-aversion in the 90's, in line with the mounting evidence from academics and practioneers.

When compared against alternative factor models - CAPM, Fama-French 3 factor and Fama-French 4 factor models - the scaled ICAPM performs much better than the CAPM, and compares reasonably well against the Fama-French models. A crucial result relies on the fact that the ICAPM models do a good job in pricing both the "extreme" small-growth portfolio and all the book-to-market quintiles. The very good fit of the ICAPM with time-varying RRA to both value and growth portfolios, is mainly due to the presence of the factor related with time-varying risk-aversion, which is not present in the static ICAPM. This suggests that the value premium probably is not an anomaly, rather being rationalized by fundamental factors related to the business cycle and time-varying risk aversion. Preliminary results suggest that the ad-doc HML and UMD factors, at least partially, measure the same types of risks as the ICAPM with time-varying risk aversion.

Given these results some interesting extensions and robustness checks for the current paper are in place for future research. First, the assumption of homoskedasticity can be relaxed, and it should be interesting to investigate whether time-varying risk aversion is still important after accounting for time-varying covariances or betas. Second, one should investigate, with more detail, whether business conditions and changes in risk aversion are associated with an investment flow between growth and value stocks, as suggested by the results. A third possible extension for this model is to analyze in more detail, whether the ICAPM with time-varying risk aversion can explain momentum.

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Notes

1 This is not the marginal rate of substitution (MRS), as in discrete time, but rather the level of marginal utility, since the MRS is not well behaved in continuous-time.

2 Any P order VAR, with $P > 1$, can be restated as a first order VAR, if the state vector is expanded by including lagged state variables, with A denoting the VAR companion matrix.

3 In alternative we can assume joint log-normality for the SDF and asset return.

Appendices

A. *Derivation of a general ICAPM specification in discrete time*

Let $V(W_{t+1}, z_{t+1})$ denote the value function of the investor's optimization problem, with the SDF given by

$$m_{t+1} = \beta \frac{V_W(W_{t+1}, z_{t+1})}{V_W(W_t, z_t)} \equiv g(W_{t+1}, z_{t+1}) \qquad (A.1)$$

where β is a subjective time discount factor and $V_i(.)$ denotes the partial derivative of the value function with respect to argument i. By applying Stein's lemma, the conditional covariance between stock i's return and the SDF is given by,

$$\begin{aligned} &Cov_t[R_{i,t+1}, g(W_{t+1}, z_{t+1})] \\ &= E_t[\frac{\partial g(W_{t+1}, z_{t+1})}{\partial W_{t+1}}]Cov_t(R_{i,t+1}, W_{t+1}) + E_t[\frac{\partial g(W_{t+1}, z_{t+1})}{\partial z_{t+1}}]Cov_t(R_{i,t+1}, z_{t+1}) \\ &= \frac{\beta W_t E_t[V_{WW}(W_{t+1}, z_{t+1})]}{V_W(W_t, z_t)} Cov_t(R_{i,t+1}, \frac{W_{t+1}}{W_t}) + \frac{\beta z_t E_t[V_{Wz}(W_{t+1}, z_{t+1})]}{V_W(W_t, z_t)} Cov_t(R_{i,t+1}, \frac{z_{t+1}}{z_t}) \qquad (A.2) \end{aligned}$$

Using (A.2) in the general pricing equation in discrete time, we have,

$$\begin{aligned} &E_t(R_{i,t+1}) - R_{f,t+1} = -\frac{Cov_t(R_{i,t+1}, m_{t+1})}{E_t(m_{t+1})} \\ &= \frac{V_W(W_t, z_t)}{\beta E_t[V_W(W_{t+1}, z_{t+1})]}\{-\frac{\beta W_t E_t[V_{WW}(W_{t+1}, z_{t+1})]}{V_W(W_t, z_t)} Cov_t(R_{i,t+1}, \frac{W_{t+1}}{W_t}) - \\ &\frac{\beta z_t E_t[V_{Wz}(W_{t+1}, z_{t+1})]}{V_W(W_t, z_t)} Cov_t(R_{i,t+1}, \frac{z_{t+1}}{z_t})\} \\ &= \gamma_t \frac{V_W(W_t, z_t)}{E_t[V_W(W_{t+1}, z_{t+1})]} Cov_t(R_{i,t+1}, \frac{W_{t+1}}{W_t}) - \frac{z_t E_t[V_{Wz}(W_{t+1}, z_{t+1})]}{V_W(W_t, z_t)} Cov_t(R_{i,t+1}, \frac{z_{t+1}}{z_t}) \\ &= \gamma_t Cov_t(R_{i,t+1}, \frac{W_{t+1}}{W_t}) + \lambda_{zt} Cov_t(R_{i,t+1}, \frac{z_{t+1}}{z_t}) \qquad (A.3) \end{aligned}$$

Where I have made use of the fact that $E_t[V_W(W_{t+1}, z_{t+1})] = V_W(W_t, z_t)$ and $\lambda_{zt} \equiv -\frac{z_t E_t[V_{Wz}(W_{t+1}, z_{t+1})]}{V_W(W_t, z_t)}$. Equation (A.3) is the discrete-time analogous to equation (8).

B. *ICAPM with time-varying risk aversion*

B.1. *Theorem 1*

Given the asset pricing model

$$1 = E_t(M_{t+1} R_{i,t+1}) \qquad (B.1)$$

and with the assumption that the log SDF, $m_{t+1} \equiv \ln(M_{t+1})$, is a linear function of K risk factors $\mathbf{f}_{t+1}$,

$$m_{t+1} = a + \mathbf{b}'\mathbf{f}_{t+1} \qquad (B.2)$$

the unconditional model in discount factor form for log returns, $r_{i,t+1} \equiv \ln(R_{i,t+1})$, can be represented as,

$$E(r_{i,t+1} - r_{f,t+1}) + 0.5\sigma_i^2 = -\mathbf{b}'Cov(r_{i,t+1}, \mathbf{f}_{t+1}) \qquad (B.3)$$

which corresponds to the following expected return-beta representation

$$E(r_{i,t+1} - r_{f,t+1}) + 0.5\sigma_i^2 = \boldsymbol{\lambda}'\boldsymbol{\beta}_i \qquad (B.4)$$

where $\boldsymbol{\lambda} \equiv -Var(\mathbf{f}_{t+1})\mathbf{b}$ and $\boldsymbol{\beta}_i \equiv Var(\mathbf{f}_{t+1})^{-1}Cov(r_{i,t+1}, \mathbf{f}_{t+1})$.

Proof:

Taking logs of $(B.1)$, one gets the pricing equation in the log form,

$$0 = \ln[E_t(\exp(m_{t+1} + r_{i,t+1})] \qquad (B.5)$$

Since the log is a non-linear function, one can use a second-order Taylor expansion to the right hand side of $(B.5)$, leading to the following approximation[a1]

$$0 = E_t(m_{t+1} + r_{i,t+1}) + 0.5Var_t(m_{t+1} + r_{i,t+1}) \qquad (B.6)$$

By expanding $(B.6)$ and rearranging, one obtains,

$$E_t(r_{i,t+1}) + 0.5Var_t(r_{i,t+1}) = -E_t(m_{t+1}) - 0.5Var_t(m_{t+1}) - Cov_t(m_{t+1}, r_{i,t+1}) \qquad (B.7)$$

Applying the pricing equation $(B.7)$ to the risk-free rate, $r_{f,t+1}$, and noting that $Var_t(r_{f,t+1}) = Cov_t(m_{t+1}, r_{f,t+1}) = 0$, since $r_{f,t+1}$ is known in period t, one has,

$$r_{f,t+1} = -E_t(m_{t+1}) - 0.5Var_t(m_{t+1}) \qquad (B.8)$$

Subtracting $(B.8)$ from $(B.7)$, we obtain,

$$E_t(r_{i,t+1}) - r_{f,t+1} + 0.5Var_t(r_{i,t+1}) = -Cov_t(m_{t+1}, r_{i,t+1}) \qquad (B.9)$$

Given the assumption that the log SDF is linear in the risk factors, $m_{t+1} = a + \mathbf{b}'\mathbf{f}_{t+1}$, and substituting in $(B.9)$, we have the following conditional pricing equation for excess returns,

$$E_t(r_{i,t+1}) - r_{f,t+1} + 0.5Var_t(r_{i,t+1}) = -\mathbf{b}'Cov_t(r_{i,t+1}, \mathbf{f}_{t+1}) \qquad (B.10)$$

By applying the law of iterated expectations to equation $(B.10)$, one has the unconditional pricing model

$$E(r_{i,t+1} - r_{f,t+1}) + 0.5\sigma_i^2 = -\mathbf{b}'Cov(r_{i,t+1}, \mathbf{f}_{t+1}) = \sum_{k=1}^{K} -b_k \sigma_{i,k} \qquad (B.11)$$

where $\sigma_i^2 \equiv Var(r_{i,t+1})$, $\sigma_{i,k} \equiv Cov(r_{i,t+1}, f_{k,t+1}), k = 1,\ldots,K$ and $f_{k,t+1}$ denotes the kth factor.

The equation in the expected return-covariance form $(B.11)$ can be translated in an equivalent expected return-beta model, in the following way,

$$\begin{aligned} E(r_{i,t+1} - r_{f,t+1}) + 0.5\sigma_i^2 &= -\mathbf{b}'Cov(r_{i,t+1}, \mathbf{f}_{t+1}) \\ = -\mathbf{b}'Var(\mathbf{f}_{t+1})Var(\mathbf{f}_{t+1})^{-1}Cov(r_{i,t+1}, \mathbf{f}_{t+1}) &= \boldsymbol{\lambda}'\boldsymbol{\beta}_i \qquad (B.12) \end{aligned}$$

where $\boldsymbol{\lambda} \equiv -Var(\mathbf{f}_{t+1})\mathbf{b}$ denote the vector of factor risk prices, and $\boldsymbol{\beta}_i \equiv Var(\mathbf{f}_{t+1})^{-1}Cov(r_{i,t+1}, \mathbf{f}_{t+1})$ is a vector containing the K betas for asset i.

Equation $(B.12)$ can be restated in a vector form, for the vector of N excess returns $\mathbf{r}_{t+1}$,

$$E(\mathbf{r}_{t+1} - r_{f,t+1}\mathbf{1}_N) + 0.5 diag(Var(\mathbf{r}_{t+1})) = \boldsymbol{\beta\lambda} \qquad (B.13)$$

where $\boldsymbol{\beta} \equiv Cov(\mathbf{r}_{t+1}, \mathbf{f}_{t+1})Var(\mathbf{f}_{t+1})^{-1}$ is a (NxK) factor beta matrix with row i containing the factor loadings for asset i, and $\mathbf{1}_N$ is a N-dimension vector of ones.

Theorem 1 represents a straightforward generalization of the theorem in section 6.3 of Cochrane (2001), for the case in which the SDF is nonlinear, but the log SDF is a linear function of the factors.

B.2. *Substituting out consumption as in Campbell (1993) model*

Using an extension of the Epstein and Zin utility function which accounts for time-varying risk aversion,

$$U_t = \{(1-\delta)C_t^{\frac{1-\gamma_t}{\theta_t}} + \delta[E_t(U_{t+1}^{1-\gamma_t})]^{\frac{1}{\theta_t}}\}^{\frac{\theta_t}{1-\gamma_t}} \qquad (B.14)$$

where $\theta_t \equiv \frac{1-\gamma_t}{1-\frac{1}{\psi}}$, ψ is the elasticity of intertemporal substitution, and γ_t is the time-varying

RRA coefficient. The corresponding SDF is given by

$$M_{t+1} = \delta^{\theta_t}\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta_t}{\psi}}\left(\frac{1}{R_{m,t+1}}\right)^{1-\theta_t} \qquad (B.15)$$

and the corresponding log SDF is equal to

$$m_{t+1} = \theta_t \ln(\delta) - \frac{\theta_t}{\psi}E_t(\Delta c_{t+1}) - (1-\theta_t)E_t(r_{m,t+1})$$
$$-\frac{\theta_t}{\psi}(\Delta c_{t+1} - E_t(\Delta c_{t+1})) - (1-\theta_t)(r_{m,t+1} - E_t(r_{m,t+1})) \qquad (B.16)$$

Applying the conditional log pricing equation $(B.7)$ to the market return, $r_{m,t+1}$, leads to

$$E_t(r_{m,t+1}) + 0.5Var_t(r_{m,t+1}) = -E_t(m_{t+1}) - 0.5Var_t(m_{t+1}) - Cov_t(m_{t+1}, r_{m,t+1}) \qquad (B.17)$$

substituting the expressions for $E_t(m_{t+1})$, $Var_t(m_{t+1})$ and $Cov_t(m_{t+1}, r_{m,t+1})$, and using the fact that $Cov_t(m_{t+1}, r_{m,t+1}) = Cov_t(m_{t+1}, r_{m,t+1} - E_t(r_{m,t+1}))$ and $Var_t(r_{m,t+1}) = Var_t(r_{m,t+1} - E_t(r_{m,t+1}))$, we have

$$E_t(r_{m,t+1}) + 0.5\sigma_{mt}^2 = -\theta_t \ln(\delta) + \frac{\theta_t}{\psi}E_t(\Delta c_{t+1}) + (1-\theta_t)E_t(r_{m,t+1})$$
$$-0.5\left[\left(\frac{\theta_t}{\psi}\right)^2\sigma_{ct}^2 + (1-\theta_t)^2\sigma_{mt}^2 + 2\frac{\theta_t}{\psi}(1-\theta_t)\sigma_{c,mt}\right] + \frac{\theta_t}{\psi}\sigma_{c,mt} + (1-\theta_t)\sigma_{mt}^2 \qquad (B.18)$$

where $\sigma_{ct}^2 \equiv Var_t(\Delta c_{t+1} - E_t(\Delta c_{t+1}))$, $\sigma_{mt}^2 \equiv Var_t(r_{m,t+1} - E_t(r_{m,t+1}))$ and $\sigma_{c,mt} \equiv Cov_t(\Delta c_{t+1} - E_t(\Delta c_{t+1}), r_{m,t+1} - E_t(r_{m,t+1}))$

Solving for $E_t(\Delta c_{t+1})$, and imposing joint conditional homoskedasticity for log consumption growth and log market returns, it follows,

$$E_t(\Delta c_{t+1}) = \psi \ln(\delta) + 0.5\theta_t\left[\frac{1}{\psi}\sigma_{ct}^2 + \psi\sigma_{mt}^2 - 2\sigma_{c,mt}\right] + \psi E_t(r_{m,t+1})$$
$$= \psi \ln(\delta) + 0.5\theta_t\left[\frac{1}{\psi}\sigma_c^2 + \psi\sigma_m^2 - 2\sigma_{c,m}\right] + \psi E_t(r_{m,t+1}) \qquad (B.19)$$

where $\sigma_c^2 \equiv Var(\Delta c_{t+1} - E_t(\Delta c_{t+1}))$, $\sigma_m^2 \equiv Var(r_{m,t+1} - E_t(r_{m,t+1}))$ and $\sigma_{c,m} \equiv Cov(\Delta c_{t+1} - E_t(\Delta c_{t+1}), r_{m,t+1} - E_t(r_{m,t+1}))$.

As stated in Campbell (1993), θ_t is infinite when ψ is near one, and hence, the expression in covariances in $(B.19)$ must be zero, in order to have finite expected consumption growth. If we make the assumption that $\psi \approx 1$, then it follows that

$$E_t(\Delta c_{t+1}) = \mu_m + \psi E_t(r_{m,t+1}) \qquad (B.20)$$

with $\mu_m \equiv \psi \ln(\delta)$.

Giving a relation similar to equation $(B.20)$, Campbell (1993) shows that innovations in log consumption and log market returns are related by the following expression,

$$c_{t+1} - E_t(c_{t+1}) = (r_{m,t+1} - E_t(r_{m,t+1})) + (1-\psi)(E_{t+1} - E_t)\sum_{j=1}^{\infty} \rho^j r_{m,t+1+j} \qquad (B.21)$$

B.3. *Conditional expected SDF in Campbell (1993) model*

The log SDF in equation $(B.16)$ has a conditional first moment given by

$$E_t(m_{t+1}) = \theta_t \ln(\delta) - \frac{\theta_t}{\psi} E_t(\Delta c_{t+1}) - (1-\theta_t)E_t(r_{m,t+1}) \qquad (B.22)$$

given the fact that $E_t[\Delta c_{t+1} - E_t(\Delta c_{t+1})] = E_t[r_{m,t+1} - E_t(r_{m,t+1})] = 0$.

Replacing $E_t(\Delta c_{t+1})$ by its expression in $(B.20)$ and solving, leads to

$$E_t(m_{t+1}) = -E_t(r_{m,t+1}) \qquad (B.23)$$

thus, the conditional expected SDF is equal to minus the conditional market return, in this framework.

B.4. *Factor risk prices in the expected return-beta representation*

Given

$$\boldsymbol{\lambda} \equiv -\boldsymbol{\Sigma}_f \mathbf{b},\ \boldsymbol{\Sigma}_f \equiv Var(\mathbf{f}_{t+1}) \qquad (B.24)$$

from Theorem 1, with $\mathbf{f}_{t+1} \equiv (r_{t+1}^{CF}, z_t r_{t+1}^{CF}, r_{t+1}^{h})$, it follows that the variance-covariance matrix of the factors is given by

$$\boldsymbol{\Sigma}_f \equiv \begin{bmatrix} \sigma_{CF}^2 & \sigma_{CF,CFz} & \sigma_{CF,h} \\ & \sigma_{CFz}^2 & \sigma_{CFz,h} \\ & & \sigma_h^2 \end{bmatrix} \qquad (B.25)$$

where $\sigma_{CF}^2 \equiv Var(r_{t+1}^{CF})$, $\sigma_h^2 \equiv Var(r_{t+1}^{h})$, $\sigma_{CFz}^2 \equiv Var(r_{t+1}^{CF} z_t)$, $\sigma_{CF,CFz} \equiv Cov(r_{t+1}^{CF}, r_{t+1}^{CF} z_t)$, $\sigma_{CF,h} \equiv Cov(r_{t+1}^{CF}, r_{t+1}^{h})$ and $\sigma_{CFz,h} \equiv Cov(r_{t+1}^{CF} z_t, r_{t+1}^{h})$.

Substituting (B.25) in (B.24) leads to

$$(\lambda_{CF},\lambda_{CFz},\lambda_h)' = \begin{bmatrix} \gamma_0\sigma^2_{CF} + \gamma_1\sigma_{CF,CFz} - \sigma_{CF,h} \\ \gamma_0\sigma_{CF,CFz} + \gamma_1\sigma^2_{CFz} - \sigma_{CFz,h} \\ \gamma_0\sigma_{CF,h} + \gamma_1\sigma_{CFz,h} - \sigma^2_h \end{bmatrix} \qquad (B.26)$$

If there are two state variables in the RRA equation, $\gamma_t = \gamma_0 + \gamma_1 z_{1t} + \gamma_1 z_{2t}$, the factor vector is given by $\mathbf{f}_{t+1} \equiv (r^{CF}_{t+1}, z_{1t}r^{CF}_{t+1}, z_{2t}r^{CF}_{t+1}, r^h_{t+1})$, $\mathbf{b} \equiv (-\gamma_0, -\gamma_1, -\gamma_2, 1)$, and

$$\mathbf{\Sigma}_f \equiv \begin{bmatrix} \sigma^2_{CF} & \sigma_{CF,CFz1} & \sigma_{CF,CFz2} & \sigma_{CF,h} \\ & \sigma^2_{CFz1} & \sigma_{CFz1,CFz2} & \sigma_{CFz1,h} \\ & & \sigma^2_{CFz2} & \sigma_{CFz2,h} \\ & & & \sigma^2_h \end{bmatrix} \qquad (B.27)$$

leading to

$$(\lambda_{CF},\lambda_{CFz1},\lambda_{CFz2},\lambda_h)' = \begin{bmatrix} \gamma_0\sigma^2_{CF} + \gamma_1\sigma_{CF,CFz1} + \gamma_2\sigma_{CF,CFz2} - \sigma_{CF,h} \\ \gamma_0\sigma_{CF,CFz1} + \gamma_1\sigma^2_{CFz1} + \gamma_2\sigma_{CFz1,CFz2} - \sigma_{CFz1,h} \\ \gamma_0\sigma_{CF,CFz2} + \gamma_1\sigma_{CFz1,CFz2} + \gamma_2\sigma^2_{CFz2} - \sigma_{CFz2,h} \\ \gamma_0\sigma_{CF,h} + \gamma_1\sigma_{CFz1,h} + \gamma_2\sigma_{CFz2,h} - \sigma^2_h \end{bmatrix} \qquad (B.28)$$

Since we are dealing with multi-regression betas, the vector of betas risk prices $\boldsymbol{\lambda}$, depends not only on the SDF parameters, but also on the factors variances and covariances.

B.5. *Risk aversion and market returns*

Given the expression for risk aversion in equation (3), $\gamma_t \equiv -\frac{W_t V_{WW}(W_t,z_t)}{V_W(W_t,z_t)}$, and differentiating with respect to wealth, W_t, it follows,

$$\frac{\partial\gamma_t}{\partial W_t} = -\frac{W_t(V_{WWW}(.)V_W(.) - V_{WW}(.)^2) + V_{WW}(.)V_W(.)}{V_W(.)^2} \qquad (B.29)$$

In order to have $\frac{\partial\gamma_t}{\partial W_t} < 0$, the following condition must hold,

$$W_t > -\frac{V_{WW}(.)V_W(.)}{V_{WWW}(.)V_W(.) - V_{WW}(.)^2} \qquad (B.30)$$

With $V_W(.) > 0, V_{WW}(.) < 0, V_{WWW}(.) > 0$, if $V_{WWW}(.)V_W(.) - V_{WW}(.)^2 < 0$ holds, this is a sufficient condition for (B.30), since wealth is strictly positive.

B.6. *Pricing the market return*

Applying the pricing equation (23) to the market return, $r_{m,t+1}$, we have

$$E(r_{m,t+1} - r_{f,t+1}) + \frac{\sigma_m^2}{2} = \gamma_0 \sigma_{m,CF} + \gamma_1 \sigma_{m,CFz} - \sigma_{m,h} \qquad (B.31)$$

By expanding the variance and covariances, in the following way,

$$\sigma_{m,CF} \equiv cov(r_{m,t+1}, r_{t+1}^{CF}) = E[cov_t(r_{m,t+1}, r_{t+1}^{CF})] = E[cov_t(r_{m,t+1} - E_t(r_{m,t+1}), r_{t+1}^{CF})]$$

$$= E[cov_t(r_{t+1}^{CF} - r_{t+1}^{h}, r_{t+1}^{CF})] = E[Var_t(r_{t+1}^{CF}) - Cov_t(r_{t+1}^{h}, r_{t+1}^{CF})] = \sigma_{CF}^2 - \sigma_{CF,h} \qquad (B.32)$$

$$\sigma_{m,CFz} \equiv cov(r_{m,t+1}, r_{t+1}^{CF} z_t) = E[cov_t(r_{t+1}^{CF} - r_{t+1}^{h}, r_{t+1}^{CF} z_t)]$$

$$= E[Cov_t(r_{t+1}^{CF}, r_{t+1}^{CF} z_t) - Cov_t(r_{t+1}^{h}, r_{t+1}^{CF} z_t)] = \sigma_{CF,CFz} - \sigma_{h,CFz} \qquad (B.33)$$

$$\sigma_{m,h} \equiv cov(r_{m,t+1}, r_{t+1}^{h}) = E[cov_t(r_{t+1}^{CF} - r_{t+1}^{h}, r_{t+1}^{h})]$$

$$= E[Cov_t(r_{t+1}^{h}, r_{t+1}^{CF}) - Var_t(r_{t+1}^{h})] = \sigma_{CF,h} - \sigma_h^2 \qquad (B.34)$$

$$\sigma_m^2 = Var(r_{m,t+1}) = E[Var_t(r_{m,t+1})] = E[Var_t(r_{m,t+1} - E_t(r_{m,t+1})]$$

$$= E[Var_t(r_{t+1}^{CF} - r_{t+1}^{h})] = \sigma_{CF}^2 + \sigma_h^2 - 2\sigma_{CF,h} \qquad (B.35)$$

and substituting (B.32), (B.33), (B.34) and (B.35) in (B.31), we obtain the unconditional pricing equation for $r_{m,t+1}$,

$$E(r_{m,t+1} - r_{f,t+1}) = \gamma_0(\sigma_{CF}^2 - \sigma_{CF,h}) + \gamma_1(\sigma_{CF,CFz} - \sigma_{h,CFz}) + 0.5(\sigma_h^2 - \sigma_{CF}^2) \qquad (B.36)$$

C. *Econometric framework*

C.1. GMM *standard errors formulas for parameter estimates and moments*

The parameter estimates $\hat{\mathbf{b}}^*$ associated with GMM system (31), have variance formulas for first stage, second stage and HJ-distance given respectively by,

$$Var(\hat{\mathbf{b}}^*) = \frac{1}{T}(\mathbf{d}'\mathbf{I}_N\mathbf{d})^{-1}\mathbf{d}'\mathbf{I}_N\hat{\mathbf{S}}\mathbf{I}_N\mathbf{d}(\mathbf{d}'\mathbf{I}_N\mathbf{d})^{-1} \qquad (C.1)$$

$$Var(\hat{\mathbf{b}}^*) = \frac{1}{T}(\mathbf{d}'\hat{\mathbf{S}}^{-1}\mathbf{d})^{-1} \qquad (C.2)$$

$$Var(\hat{\mathbf{b}}^*) = \frac{1}{T}(\mathbf{d}'\mathbf{W}_{HJ}\mathbf{d})^{-1}\mathbf{d}'\mathbf{W}_{HJ}\hat{\mathbf{S}}\mathbf{W}_{HJ}\mathbf{d}(\mathbf{d}'\mathbf{W}_{HJ}\mathbf{d})^{-1} \qquad (C.3)$$

where $\mathbf{I}_N$ is a N order Identity matrix, $\mathbf{d} \equiv \frac{\partial g_T(\mathbf{b}^*)}{\partial \mathbf{b}^{*\prime}}$ represents the matrix of moments' sensitivities to the parameters, $\mathbf{W}_{HJ} = E(\mathbf{r}_t\mathbf{r}_t')^{-1}$ is the weighting matrix for the HJ-distance estimator, and $\hat{\mathbf{S}}$ is a estimator for the spectral density matrix $\mathbf{S}$, derived under the Newey-West procedure with 5 lags. The variance-covariance matrix for the moments is given by,

$$Var(\hat{\boldsymbol{\alpha}}) = \frac{1}{T}(\mathbf{I}_N - \mathbf{d}(\mathbf{d}'\mathbf{I}_N\mathbf{d})^{-1})\mathbf{d}'\mathbf{I}_N)\hat{\mathbf{S}}(\mathbf{I}_N - \mathbf{I}_N\mathbf{d}(\mathbf{d}'\mathbf{I}_N\mathbf{d})^{-1})\mathbf{d}') \qquad (C.4)$$

$$Var(\hat{\boldsymbol{\alpha}}) = \frac{1}{T}(\mathbf{I}_N - \mathbf{d}(\mathbf{d}'\hat{\mathbf{S}}^{-1}\mathbf{d})^{-1})\mathbf{d}'\hat{\mathbf{S}}^{-1})\hat{\mathbf{S}}(\mathbf{I}_N - \hat{\mathbf{S}}^{-1}\mathbf{d}(\mathbf{d}'\hat{\mathbf{S}}^{-1}\mathbf{d})^{-1})\mathbf{d}') \qquad (C.5)$$

$$Var(\hat{\boldsymbol{\alpha}}) = \frac{1}{T}(\mathbf{I}_N - \mathbf{d}(\mathbf{d}'\mathbf{W}_{HJ}\mathbf{d})^{-1})\mathbf{d}'\mathbf{W}_{HJ})\hat{\mathbf{S}}(\mathbf{I}_N - \mathbf{W}_{\mathbf{HJ}}\mathbf{d}(\mathbf{d}'\mathbf{W}_{HJ}\mathbf{d})^{-1})\mathbf{d}') \qquad (C.6)$$

for first-stage, second-stage and HJ-distance, respectively.

The distribution of the HJ-distance, δ is derived according to Appendix A in Hodrick and Zhang (2001).

C.2. *GMM robust standard errors in the cross-sectional regressions*

The GMM system equivalent to the time series-cross sectional regressions has a set of moments given by,

$$g_T(\boldsymbol{\Theta}) = \frac{1}{T}\begin{bmatrix} \sum_{t=1}^{T}(\mathbf{r}_t - r_{f,t}\mathbf{1}_N - \mathbf{a}^* - \boldsymbol{\beta}\mathbf{f}_t) \\ \sum_{t=1}^{T}(\mathbf{r}_t - r_{f,t}\mathbf{1}_N - \mathbf{a}^* - \boldsymbol{\beta}\mathbf{f}_t)\otimes\mathbf{f}_t \\ \sum_{t=1}^{T}(\mathbf{R}_t - R_{f,t}\mathbf{1}_N + \boldsymbol{\beta}\boldsymbol{\Sigma}_{\mathbf{f}}\mathbf{b}) \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \qquad (C.7)$$

where $\mathbf{R}_t(Nx1)$ is a vector of simple returns, $\mathbf{r}_t(Nx1)$ is a vector of log returns, $\mathbf{1}_N(Nx1)$ is a vector of ones, $\mathbf{a}^*(Nx1)$ is a vector of constants for the time series regressions, $\boldsymbol{\beta}(NxK)$ is a matrix of factor loadings for the N test assets, $\mathbf{f}_t(Kx1)$ is a vector of common factors used to price assets, $\mathbf{b}(Kx1)$ is a vector of parameters in the SDF equation, and $\otimes$ denotes the Kronecker product, and . The first two sets of moments identify the factor loadings (including the constant), and thus are equivalent to the time-series regressions, being exactly identified:

$N+NK$ orthogonality conditions and $N+NK$ parameters to estimate. The third set of moments corresponds to the cross-sectional regression and identifies the SDF parameters (prices of covariance risks), **b**. System (C.7) is different from the system presented in Cochrane (2001), chapter 12, in two aspects. First, it generalizes Cochrane (2001), for the case of K risk factors affecting the cross-section of returns. Secondly, the risk-prices associated with betas are not freely estimated in the cross-section, being constrained by $\boldsymbol{\lambda} \equiv -\Sigma_f \mathbf{b}$, since we are working with a theoretical derived asset pricing model that constraints $\boldsymbol{\lambda}$, and in addition only some of the parameters in the SDF equation, **b**, have to be estimated in the cross-section, which I will denote by $\mathbf{b}^*$. The number of covariance risk prices to be estimated (dimension of $\mathbf{b}^*$) is equal to K^*. Hence, the third set of moments have N moment conditions and K^* parameters to estimate, leading to $N-K^*$ overidentifying restrictions, which also correspond to the number of overidentifying conditions of the entire system.

The cross-sectional regression can be restated in the following way, with both $\boldsymbol{\Sigma}_\mathbf{f}$ and **b** being partitioned,

$$E(\mathbf{R}_t - R_{f,t}\mathbf{1}_N) = -\boldsymbol{\beta}\boldsymbol{\Sigma}_\mathbf{f}\mathbf{b} = -\boldsymbol{\beta}[\boldsymbol{\Sigma}_\mathbf{f}^{*}\boldsymbol{\Sigma}_\mathbf{f}^{**}]\begin{bmatrix} \mathbf{b}^* \\ \mathbf{b}^{**} \end{bmatrix} \qquad (C.8)$$

$\mathbf{b}^{**}$ represents the parameters in **b**, which are constrained by the asset pricing model, and hence are not estimated in the cross-section. In the case of the benchmark ICAPM model with $\mathbf{f}_t \equiv (r_t^{CF}, z_{t-1}r_t^{CF}, r_t^h)$ and $\mathbf{b} \equiv (-\gamma_0, -\gamma_1, 1)$, we have $\mathbf{b}^* \equiv (-\gamma_0, -\gamma_1)$ and $\mathbf{b}^{**} \equiv 1$. Similarly, $\boldsymbol{\Sigma}_\mathbf{f}^*$ represents the submatrix of $\boldsymbol{\Sigma}_\mathbf{f}$ that contains the variances of the factors which parameters are estimated in the cross-sectional regression. In our example, $\boldsymbol{\Sigma}_\mathbf{f}^* \equiv [\boldsymbol{\sigma}_{CF}, \boldsymbol{\sigma}_{CFz}], \boldsymbol{\Sigma}_\mathbf{f}^{**} \equiv [\boldsymbol{\sigma}_h]$, where $\boldsymbol{\sigma}_i$ denotes the column vector in $\boldsymbol{\Sigma}_\mathbf{f}$ that contains σ_i^2. By applying the product rule for partitioned matrices, we have,

$$E(\mathbf{R}_t - R_{f,t}\mathbf{1}_N) = -\boldsymbol{\beta}[\boldsymbol{\Sigma}_\mathbf{f}^*\mathbf{b}^* + \boldsymbol{\Sigma}_\mathbf{f}^{**}\mathbf{b}^{**}] = -\boldsymbol{\beta}\boldsymbol{\Sigma}_\mathbf{f}^*\mathbf{b}^* - \boldsymbol{\beta}\boldsymbol{\Sigma}_\mathbf{f}^{**}\mathbf{b}^{**} \qquad (C.9)$$

This formula will be useful in calculating the sensitivity of $g_T(\boldsymbol{\Theta})$ to $\mathbf{b}^*$, below.

The vector of parameters to estimate in this GMM system is given by

$$\Theta' = [\ \mathbf{a}^{*\prime} \quad \boldsymbol{\beta}^{*} \quad \mathbf{b}^{*\prime}\ ] \qquad (C.10)$$

where $\boldsymbol{\beta}^{*} \equiv vec(\boldsymbol{\beta}')'$, vec is the VEC operator that enables to stack the factor loadings for each of the N assets into a column vector. The matrix that chooses which moment conditions are set to zero in the first-order condition $\mathbf{a}g_T(\hat{\boldsymbol{\Theta}}) = 0$, is given by

$$\mathbf{a} = \begin{bmatrix} \mathbf{I}_N \bigotimes \mathbf{I}_{K+1} & \mathbf{0}_{(N(K+1)xN)} \\ \mathbf{0}_{(K^{*}xN(K+1))} & \boldsymbol{\gamma}^{*\prime} \end{bmatrix} \qquad (C.11)$$

where $\mathbf{I}_m$ denotes an identity matrix of order m. In the case of a OLS cross-sectional regression, we have $\boldsymbol{\gamma}^{*} = \boldsymbol{\beta}\boldsymbol{\Sigma}_{\mathbf{f}}^{*}$, whereas for GLS cross-sectional regression, $\boldsymbol{\gamma}^{*} = \boldsymbol{\Sigma}^{-1}\boldsymbol{\beta}\boldsymbol{\Sigma}_{\mathbf{f}}^{*}$, with $\boldsymbol{\Sigma} \equiv E(\boldsymbol{\epsilon}_t\boldsymbol{\epsilon}_t')$ representing the variance-covariance matrix associated with the pricing errors.

The sensitivity of the moment conditions to the parameters in this case is equal to

$$\mathbf{d} \equiv \frac{\partial g_T(\boldsymbol{\Theta})}{\partial \boldsymbol{\Theta}'} = -\begin{bmatrix} \mathbf{I}_N & \mathbf{I}_N \bigotimes (\frac{1}{T}\sum_{t=1}^{T} \mathbf{f}_t') & \mathbf{0}_{(NxK^{*})} \\ \mathbf{I}_N \bigotimes (\frac{1}{T}\sum_{t=1}^{T} \mathbf{f}_t) & \mathbf{I}_N \bigotimes (\frac{1}{T}\sum_{t=1}^{T} \mathbf{f}_t\mathbf{f}_t') & \mathbf{0}_{(NKxK^{*})} \\ \mathbf{0}_{(NxN)} & -\mathbf{I}_N \bigotimes (\mathbf{b}'\boldsymbol{\Sigma}_{\mathbf{f}}) & \boldsymbol{\beta}\boldsymbol{\Sigma}_{\mathbf{f}}^{*} \end{bmatrix} \qquad (C.12)$$

The variance-covariance matrix of the moments, S, has the following form

$$\mathbf{S} = \sum_{j=-\infty}^{\infty} E\left(\begin{bmatrix} \mathbf{r}_t - r_{f,t}\mathbf{1}_N - \mathbf{a}^{*} - \boldsymbol{\beta}\mathbf{f}_t \\ (\mathbf{r}_t - r_{f,t}\mathbf{1}_N - \mathbf{a}^{*} - \boldsymbol{\beta}\mathbf{f}_t) \bigotimes \mathbf{f}_t \\ \mathbf{R}_t - R_{f,t}\mathbf{1}_N + \boldsymbol{\beta}\boldsymbol{\Sigma}_{\mathbf{f}}\mathbf{b} \end{bmatrix} \begin{bmatrix} \mathbf{r}_{t-j} - r_{f,t-j}\mathbf{1}_N - \mathbf{a}^{*} - \boldsymbol{\beta}\mathbf{f}_{t-j} \\ (\mathbf{r}_{t-j} - r_{f,t-j}\mathbf{1}_N - \mathbf{a}^{*} - \boldsymbol{\beta}\mathbf{f}_{t-j}) \bigotimes \mathbf{f}_{t-j} \\ \mathbf{R}_{t-j} - R_{f,t-j}\mathbf{1}_N + \boldsymbol{\beta}\boldsymbol{\Sigma}_{\mathbf{f}}\mathbf{b} \end{bmatrix}' \right)$$

$$= \sum_{j=-\infty}^{\infty} E\left(\begin{bmatrix} \boldsymbol{\epsilon}_t \\ \boldsymbol{\epsilon}_t \bigotimes \mathbf{f}_t \\ \boldsymbol{\beta}(\mathbf{f}_t - \mathbf{E}(\mathbf{f}_t)) + \boldsymbol{\epsilon}_t \end{bmatrix} \begin{bmatrix} \boldsymbol{\epsilon}_{t-j} \\ \boldsymbol{\epsilon}_{t-j} \bigotimes \mathbf{f}_{t-j} \\ \boldsymbol{\beta}(\mathbf{f}_{t-j} - \mathbf{E}(\mathbf{f}_t)) + \boldsymbol{\epsilon}_{t-j} \end{bmatrix}' \right) \qquad (C.13)$$

where $\boldsymbol{\epsilon}_t \equiv \mathbf{r}_t - r_{f,t}\mathbf{1}_N - \mathbf{a}^{*} - \boldsymbol{\beta}\mathbf{f}_t$, and on the last equality one makes use of

$$\mathbf{R}_t - R_{f,t}\mathbf{1}_N + \boldsymbol{\beta}\boldsymbol{\Sigma}_{\mathbf{f}}\mathbf{b} = \mathbf{R}_\mathbf{t} - R_{f,t}\mathbf{1}_\mathbf{N} - \boldsymbol{\beta}\boldsymbol{\lambda} = \mathbf{R}_\mathbf{t} - R_{f,t}\mathbf{1}_\mathbf{N} - E(\mathbf{R}_\mathbf{t} - R_{f,t}\mathbf{1}_\mathbf{N})$$
$$= \mathbf{r}_\mathbf{t} - r_{f,t}\mathbf{1}_\mathbf{N} - E(\mathbf{r}_\mathbf{t} - r_{f,t}\mathbf{1}_\mathbf{N}) = \mathbf{r}_t - r_{f,t}\mathbf{1}_N - \mathbf{a}^{*} - \boldsymbol{\beta}E(\mathbf{f}_t) = \boldsymbol{\beta}(\mathbf{f}_t - \mathbf{E}(\mathbf{f}_t)) + \boldsymbol{\epsilon}_t \qquad (C.14)$$

S is estimated by the Newey-West procedure with 5 lags.

By using the general GMM formula for the variance-covariance matrix of the estimators,

$$Var(\hat{\mathbf{\Theta}}) = \frac{1}{T}(\mathbf{ad})^{-1}\mathbf{a}\hat{\mathbf{S}}\mathbf{a}'(\mathbf{ad})^{-1\prime} \qquad (C.15)$$

and the last K^* elements of the main diagonal give the variances of estimated covariance risk premia, used to calculate the respective t-statistics. In addition if we use the formula for the variance-covariance matrix of the GMM moments

$$Var(g_T(\hat{\mathbf{\Theta}})) = \frac{1}{T}(\mathbf{I}_{N(K+2)} - \mathbf{d}(\mathbf{ad})^{-1}\mathbf{a})\hat{\mathbf{S}}(\mathbf{I}_{N(K+2)} - \mathbf{d}(\mathbf{ad})^{-1}\mathbf{a})' \qquad (C.16)$$

we obtain the variance of the cross-sectional pricing errors ($\hat{\boldsymbol{\alpha}}$) from the last N elements of the diagonal, which will be used to conduct an asset-pricing test,

$$\hat{\boldsymbol{\alpha}}' var(\hat{\boldsymbol{\alpha}})^{-1}\hat{\boldsymbol{\alpha}} \sim \chi^2(N - K^*) \qquad (C.17)$$

C.3. *Shanken (1992) standard errors*

Shanken (1992) standard errors - which introduce a correction for the fact that the factor loadings are generated regressors in the cross sectional regression - can be derived as a special case of the above GMM "robust" standard errors derived above. If we assume that $\boldsymbol{\epsilon}_t$ is jointly i.i.d., $\boldsymbol{\epsilon}_t$ and $\mathbf{f}_t$ are independent, and finally that $\mathbf{f}_t$ has no serial correlation, then the spectral density matrix S specializes to,

$$S = E\left(\begin{bmatrix} \boldsymbol{\epsilon}_t \\ \boldsymbol{\epsilon}_t \otimes \mathbf{f}_t \\ \boldsymbol{\beta}(\mathbf{f}_t - E(\mathbf{f}_t)) + \boldsymbol{\epsilon}_t \end{bmatrix} \begin{bmatrix} \boldsymbol{\epsilon}_t \\ \boldsymbol{\epsilon}_t \otimes \mathbf{f}_t \\ \boldsymbol{\beta}(\mathbf{f}_t - E(\mathbf{f}_t)) + \boldsymbol{\epsilon}_t \end{bmatrix}' \right)$$
$$= \begin{bmatrix} \boldsymbol{\Sigma} & \boldsymbol{\Sigma} \otimes E(\mathbf{f}_t') & \boldsymbol{\Sigma} \\ \boldsymbol{\Sigma} \otimes E(\mathbf{f}_t) & \boldsymbol{\Sigma} \otimes E(\mathbf{f}_t\mathbf{f}_t') & \boldsymbol{\Sigma} \otimes E(\mathbf{f}_t) \\ \boldsymbol{\Sigma} & \boldsymbol{\Sigma} \otimes E(\mathbf{f}_t') & \boldsymbol{\beta}\boldsymbol{\Sigma}_f\boldsymbol{\beta}' + \boldsymbol{\Sigma} \end{bmatrix} \qquad (C.18)$$

By replacing (C.18) in formulas (C.16) and (C.17), we obtain Shanken corrected variances for estimated covariance risk premia and pricing errors.

Table I

Descriptive statistics for VAR state variables and determinants of time-varying risk aversion

This table reports descriptive statistics for the state variables used to predict market returns in the VAR (panel A) and the scaling variables used to explain time-varying risk aversion (panel B). The VAR state variables are the log earnings yield (EY), FED funds premium (FFPREM), term structure spread (TERM), and the log market return (r^m). The scaling variables are the market dividend yield (DY), default spread (DEF), smoothed log earnings yield (EY*), cyclical industrial production growth (IPG) and the value spread (VS). The original sample is 1954:07- 2003:09. ρ designates the first order autocorrelation. For details on the variables construction refer to section III.

Panel A (VAR state variables)							
			Correlations				
	Mean	**Stdev.**	**FFPREM**	**TERM**	**EY**	r_m	**ρ**
FFPREM	0.005	0.008	1.000	-0.440	0.433	-0.127	0.878
TERM	0.008	0.010		1.000	-0.357	0.131	0.967
EY	-2.780	0.376			1.000	0.002	0.997
r_m	0.004	0.044				1.000	0.073

Panel B (Risk-aversion determinants)								
			Correlations					
	Mean	**Stdev.**	**DY**	**DEF**	**EY***	**IPG**	**VS**	**ρ**
DY	0.032	0.010	1.000	0.429	0.916	-0.298	-0.605	0.989
DEF	0.010	0.004		1.000	0.639	-0.318	-0.082	0.972
EY*	-2.832	0.415			1.000	-0.271	-0.548	0.997
IPG	0.010	0.004				1.000	-0.010	0.888
VS	1.563	0.159					1.000	0.939

Table II

Estimating cash-flow and discount rate news: A VAR approach

Panel A presents the estimated coefficients (first column) and associated Newey-West t-statistics (second column) for the market excess return equation (r_m) in a first-order VAR. The VAR contains the FED funds premium (FFPREM), the term structure spread (TERM), log earnings yield (EY), and the value-weighted market log excess return (r_m). The original sample is 1954:08-2003:09. Underlined (bold) t-statistics denote significance at the 5% (1%) level. Adj. R^2 is the adjusted R^2.

Panel B shows the variance decomposition associated with cash-flow (r^{CF}) and discount rate news (r^h) implied by the VAR model, of panel A. The upper-right section shows the correlation between r^h and r^{CF}, with the standard error below the correlation coefficient. The upper-left section reports the variance decomposition of market excess returns, in terms of both news components. The lower-left section shows the correlations between shocks in each of the variables used in the VAR, with both r^h and r^{CF}. The lower-right section shows the functions ($e1'\rho A(I-\rho A)^{-1}$, $e1'+e1'\rho A(I-\rho A)^{-1}$) that map the VAR variables shocks into r^h and r^{CF}. The standard errors are computed from 5,000 bootstrapping simulations of the model, under the null of no predictability of excess returns.

Panel A

			Adj. R^2
FFPREM	-0.820	-2.490	0.031
TERM	0.349	1.974	
EY	0.018	**3.065**	
r_m	0.044	1.047	

Panel B

Variance decomposition		Correlations	r^h_{t+1}	r^{CF}_{t+1}
$Var(r^h_{t+1})$	0.721			
$Var(r^{CF}_{t+1})$	0.276	**r^h_{t+1}**	1.000	-0.003
$-2Cov(r^{CF}_{t+1},r^h_{t+1})$	0.002			0.556
Sum	1.000	**r^{CF}_{t+1}**		1.000

Shock correlations	r^h_{t+1}	S.E.	r^{CF}_{t+1}	S.E.	Funtions	r^h_{t+1}	S.E.	r^{CF}_{t+1}	S.E.
FFPREM	-0.142	0.369	-0.312	0.131	**FFPREM**	-1.748	1.986	-1.748	1.986
TERM	-0.187	0.651	-0.012	0.303	**TERM**	-0.017	5.750	-0.017	5.750
EY	0.954	0.656	0.191	0.194	**EY**	0.824	0.737	0.824	0.737
r_m	-0.851	0.597	0.528	0.112	**r_m**	-0.305	0.331	0.695	0.331

Table III

ICAPM with cyclical risk-aversion: Estimating covariance and beta factor risk-premia

This table reports the estimated covariance and beta factor risk prices for the ICAPM with time-varying risk aversion, as described in section III of the paper. Panel A presents the results for both the static ICAPM (BBGB) and the ICAPM scaled by dividend yield (DY). The ICAPM scaled by the default spread (DEF), smoothed earnings yield (EY*), cyclical industrial production growth (IPG) and value spread (VS), are presented in Panels B, C, D and E, respectively. For each model, there are 3 sets of test assets - the 25 size/book-to-market portfolios, 38 industry portfolios, and their combination. Each panel reports both the first stage and second stage GMM estimates. λ_{CF} and λ_{H} denote the beta risk prices estimates for the cash-flow and discount-rate news factors, respectively, while λ_{CFDY}, λ_{CFDEF}, λ_{CFEY^*}, λ_{CFIPG} and λ_{CFVS} refer to the scaled factor related with time-varying risk aversion. γ_0 and γ_1 denote the estimates of the covariance risk prices, which also correspond to the risk aversion coefficients. For each model the estimated risk prices are reported in line 1, while in line 2 are reported the associated t-statistics. $E(\gamma_t)$ is the average value of RRA. Test values (first row) and respective p-values (second row) for the asymptotic pricing error test are presented for each GMM estimation. The sample is 1954:08-2003:09. Italic, underlined and bold numbers denote statistical significance at the 10%, 5% and 1% levels respectively. The beta risk prices (λ) are multiplied by 100. For further details, refer to section III of the paper.

Panel A (BBGB + Dividend yield)

First stage GMM								**Second stage GMM**						
Row	λ_{CF}	λ_{CFDY}	λ_{H}	γ_0	γ_1	$E(\gamma_t)$	$\alpha'\Sigma^{-1}\alpha$	λ_{CF}	λ_{CFDY}	λ_{H}	γ_0	γ_1	$E(\gamma_t)$	$\alpha'\Sigma^{-1}\alpha$
25 Size/book-to-market portfolios														
1	0.552		-0.136	10.771		10.771	23.266	0.299		-0.135	5.823		5.823	23.668
	2.517		**-139.519**	2.516			0.504	*1.874*		**-190.463**	*1.872*			0.481
2	0.583	0.045	-0.093	-96.650	3562.644	15.746	28.114	0.312	0.024	-0.113	-50.025	1850.827	8.366	18.532
	2.652	**4.517**	**-8.355**	**-3.444**	**3.862**		0.211	*1.954*	**3.553**	**-16.838**	**-2.966**	**3.340**		0.728
38 Industry portfolios														
3	0.445		-0.136	8.683		8.683	25.940	0.319		-0.135	6.224		6.224	25.298
	2.140		**-146.638**	2.139			0.766	2.206		**-209.895**	2.204			0.794
4	0.426	0.017	-0.129	-8.143	542.519	8.973	25.578	0.337	0.013	-0.131	-4.920	378.655	7.026	23.557
	2.097	2.132	**-21.772**	-0.550	1.081		0.741	2.252	2.285	**-33.940**	-0.512	1.155		0.828
25 Size/book-to-market portfolios + 38 Industry portfolios														
5	0.493		-0.136	9.604		9.604	60.973	0.324		-0.135	6.312		6.312	66.218
	2.325		**-144.219**	2.323			0.335	2.460		**-230.930**	2.459			0.189
6	0.478	0.022	-0.125	-18.410	914.441	10.439	63.418	0.313	0.014	-0.128	-12.852	625.110	6.870	65.631
	2.273	**2.670**	**-20.944**	-1.231	*1.824*		0.231	2.325	**2.858**	**-40.612**	-1.632	2.339		0.178

Panel B (Default spread)

First stage GMM								**Second stage GMM**						
Row	λ_{CF}	λ_{CFDEF}	λ_{H}	γ_0	γ_1	$E(\gamma_t)$	$\alpha'\Sigma^{-1}\alpha$	λ_{CF}	λ_{CFDEF}	λ_{H}	γ_0	γ_1	$E(\gamma_t)$	$\alpha'\Sigma^{-1}\alpha$
25 Size/book-to-market portfolios														
1	0.173	-0.006	-0.385	73.316	-6543.156	10.502	23.782	0.005	-0.008	-0.390	71.510	-6681.787	7.365	19.528
	0.826	-0.988	-2.549	*1.815*	*-1.654*		0.416	0.029	-2.512	**-5.616**	**3.832**	**-3.675**		0.670
38 Industry portfolios														
2	0.490	0.006	-0.097	-1.345	1018.752	8.435	25.664	0.362	0.005	-0.096	-3.964	1030.799	5.932	25.241
	2.323	2.004	*-1.648*	-0.083	0.661		0.737	2.363	2.001	-2.075	-0.322	0.850		0.757
25 Size/book-to-market portfolios + 38 Industry portfolios														
3	0.429	0.003	-0.185	22.088	-1284.595	9.756	63.564	0.270	0.001	-0.180	17.717	-1164.471	6.538	66.054
	2.154	0.974	**-2.867**	1.212	-0.762		0.227	2.012	0.714	**-5.206**	*1.872*	-1.288		0.168

Panel C (Smoothed earnings yield)

First stage GMM								Second stage GMM						
Row	λ_{CF}	λ_{CFEY*}	λ_H	γ_0	γ_1	$E(\gamma_t)$	$\alpha'\Sigma^{-1}\alpha$	λ_{CF}	λ_{CFEY*}	λ_H	γ_0	γ_1	$E(\gamma_t)$	$\alpha'\Sigma^{-1}\alpha$
25 Size/book-to-market portfolios														
1	0.657	-0.754	-0.004	248.936	82.746	14.628	33.946	0.347	-0.521	-0.080	105.625	34.646	7.521	23.140
	2.978	-1.119	-0.098	**3.681**	**3.523**		0.066	2.154	-1.107	**-3.778**	**2.773**	**2.632**		0.453
38 Industry portfolios														
2	0.436	-1.039	-0.111	51.935	15.221	8.834	25.087	0.356	-0.843	-0.115	43.867	12.939	7.228	24.084
	2.120	*-1.851*	**-5.502**	1.378	1.196		0.764	2.369	-2.046	**-8.713**	*1.781*	1.562		0.807
25 Size/book-to-market portfolios + 38 Industry portfolios														
3	0.497	-1.117	-0.100	73.491	22.356	10.187	60.963	0.325	-0.726	-0.111	49.118	14.992	6.666	65.886
	2.341	*-1.894*	**-4.986**	1.977	*1.773*		0.302	2.409	*-1.948*	**-10.820**	2.544	2.316		0.172

Panel D (Industrial production growth)

First stage GMM								Second stage GMM						
Row	λ_{CF}	λ_{CFIPG}	λ_H	γ_0	γ_1	$E(\gamma_t)$	$\alpha'\Sigma^{-1}\alpha$	λ_{CF}	λ_{CFIPG}	λ_H	γ_0	γ_1	$E(\gamma_t)$	$\alpha'\Sigma^{-1}\alpha$
25 Size/book-to-market portfolios														
1	0.835	-0.008	0.204	123.658	-11838.436	2.550	32.421	0.354	0.000	-0.069	27.915	-2315.824	4.224	24.778
	3.473	-1.586	*1.687*	**3.070**	**-2.816**		0.092	2.078	0.069	-1.141	1.366	-1.106		0.362
38 Industry portfolios														
2	0.462	0.003	-0.102	19.796	-1189.156	7.631	26.201	0.324	0.002	-0.125	9.497	-351.785	5.899	25.347
	2.145	0.910	*-1.646*	0.934	-0.553		0.712	2.136	1.414	**-3.621**	0.782	-0.292		0.752
25 Size/book-to-market portfolios + 38 Industry portfolios														
3	0.530	0.002	-0.077	28.842	-2040.375	7.969	60.218	0.305	0.003	-0.130	7.602	-182.327	5.737	65.199
	2.413	0.741	-1.300	1.411	-0.985		0.326	2.222	1.635	**-4.336**	0.718	-0.174		0.187

Panel E (Value spread)

First stage GMM								Second stage GMM						
Row	λ_{CF}	λ_{CFVS}	λ_H	γ_0	γ_1	$E(\gamma_t)$	$\alpha'\Sigma^{-1}\alpha$	λ_{CF}	λ_{CFVS}	λ_H	γ_0	γ_1	$E(\gamma_t)$	$\alpha'\Sigma^{-1}\alpha$
25 Size/book-to-market portfolios														
1	0.064	-0.384	-0.404	407.498	-252.024	13.592	26.199	0.040	-0.224	-0.294	241.169	-149.128	8.086	7.480
	0.307	-1.057	**-7.426**	**4.994**	**-4.952**		0.292	0.256	-0.831	**-8.565**	**4.691**	**-4.648**		0.999
38 Industry portfolios														
2	0.360	0.511	-0.174	64.799	-35.841	8.781	25.114	0.309	0.429	-0.172	62.086	-34.785	7.718	24.120
	1.932	*1.677*	**-4.654**	1.149	-1.030		0.763	2.124	*1.841*	**-6.840**	1.624	-1.484		0.806
25 Size/book-to-market portfolios + 38 Industry portfolios														
3	0.332	0.392	-0.214	125.196	-73.652	10.079	62.726	0.217	0.234	-0.198	99.725	-59.248	7.121	64.757
	1.722	1.248	**-5.695**	2.203	-2.102		0.250	*1.662*	1.112	**-10.085**	**3.346**	**-3.240**		0.198

Table IV

ICAPM with time-varying risk-aversion: Adding the market return

This table reports the estimated covariance and beta risk prices for the ICAPM scaled by the market return (Panel A), and the ICAPM scaled by the dividend yield plus the market return (Panel B), as described in section III of the paper. For each model, there are 3 sets of test assets - the 25 size/book-to-market portfolios, 38 industry portfolios, and their combination. Each panel reports both the first stage and second stage GMM estimates. λ_{CF} and λ_{H} denote the beta risk prices estimates for the cash-flow and discount-rate news factors, respectively, while λ_{CFDY} and λ_{CFRM} refer to the scaled factor related with time-varying risk aversion. γ_0, γ_1 and γ_2 denote the estimates of the covariance risk prices, which also correspond to the risk aversion coefficients. For each model the estimated risk prices are reported in line 1, while in line 2 are reported the associated t-statistics. $E(\gamma_t)$ is the average value of RRA. Test values (first row) and respective p-values (second row) for the asymptotic pricing error test are presented for each GMM estimation. The sample is 1954:08-2003:09. Italic, underlined and bold numbers denote statistical significance at the 10%, 5% and 1% levels respectively. The beta risk prices (λ) are multiplied by 100. For further details, refer to section III of the paper.

Panel A (Market excess return)														
First stage GMM								**Second stage GMM**						
Row	**λ_{CF}**	**λ_{CFRM}**	**λ_{H}**	**γ_0**	**γ_1**	**$E(\gamma_t)$**	**$\alpha'\Sigma^{-1}\alpha$**	**λ_{CF}**	**λ_{CFRM}**	**λ_{H}**	**γ_0**	**γ_1**	**$E(\gamma_t)$**	**$\alpha'\Sigma^{-1}\alpha$**
25 Size/book-to-market portfolios														
1	1.113	0.190	0.374	30.653	1559.854	37.422	24.867	0.505	0.069	0.052	13.132	571.726	15.613	43.985
	4.017	**3.530**	**2.632**	**4.254**	**3.591**		0.357	**2.802**	**2.880**	0.819	**3.155**	**2.956**		**0.005**
38 Industry portfolios														
2	0.257	-0.071	-0.316	1.859	-550.725	-0.531	23.661	0.186	-0.057	-0.278	1.120	-437.549	-0.778	23.026
	1.293	**-2.589**	**-4.402**	0.447	<u>-2.519</u>		0.824	1.264	**-3.845**	**-7.216**	0.364	**-3.714**		0.848
25 Size/book-to-market portfolios + 38 Industry portfolios														
3	0.429	-0.026	-0.196	7.304	-183.295	6.508	61.296	0.264	-0.028	-0.203	3.957	-205.719	3.064	66.144
	<u>2.099</u>	-1.181	**-3.433**	*1.771*	-1.055		0.292	*1.998*	**-2.782**	**-7.767**	1.489	**-2.586**		0.166

Panel B (Dividend yield + market excess return)									
First stage GMM									
Row	λ_{CF}	λ_{CFDY}	λ_{CFRM}	λ_{H}	γ_0	γ_1	γ_2	$E(\gamma_t)$	$\alpha'\Sigma^{-1}\alpha$
25 Size/book-to-market portfolios									
1	0.534	0.045	-0.012	-0.137	-102.679	3738.070	-137.680	13.592	26.501
	2.163	**4.469**	-0.388	*-1.809*	**-3.341**	**3.784**	-0.574		0.231
38 Industry portfolios									
2	0.273	0.010	-0.060	-0.291	-4.332	344.883	-482.151	0.737	23.772
	1.330	1.257	-2.545	**-4.650**	-0.292	0.685	**-2.587**		0.782
25 Size/book-to-market portfolios + 38 Industry portfolios									
3	0.462	0.021	-0.004	-0.140	-18.545	911.098	-46.968	9.633	63.210
	2.222	2.573	-0.193	-2.417	-1.245	*1.813*	-0.273		0.209
Second stage GMM									
Row	λ_{CF}	λ_{CFDY}	λ_{CFRM}	λ_{H}	γ_0	γ_1	γ_2	$E(\gamma_t)$	$\alpha'\Sigma^{-1}\alpha$
25 Size/book-to-market portfolios									
4	0.221	0.023	-0.029	-0.193	-59.784	2128.068	-254.696	4.283	19.134
	1.310	**3.397**	-1.499	**-3.905**	**-3.336**	**3.663**	-1.642		0.637
38 Industry portfolios									
5	0.196	0.007	-0.052	-0.270	-1.648	203.237	-415.358	-0.243	22.897
	1.268	1.155	**-3.397**	**-6.662**	-0.171	0.614	**-3.447**		0.819
25 Size/book-to-market portfolios + 38 Industry portfolios									
6	0.283	0.013	-0.013	-0.165	-13.014	617.750	-113.277	5.109	65.805
	2.089	**2.615**	-1.306	**-6.170**	*-1.652*	2.311	-1.411		0.151

Table V

Average pricing errors: 25 size/book-to-market portfolios

This table reports the average pricing errors (stated in percentage points) across the book-to-market (Panel A) and size (Panel B) quintiles associated with the 25 size/book-to-market portfolios. The models are the static ICAPM (BBGB), and the ICAPM scaled by the market dividend yield (DY), default spread (DEF), smoothed log earnings yield (EY*), cyclical industrial production growth (IPG), value spread (VS), and market return plus dividend yield (DY+RM). Panel C reports t-statistics for the individual pricing errors. SBV_{ij} denotes the portfolio with ith size and jth book-to-market quintiles. S1 and BV1 denote the lowest size and book-to-market quintiles, respectively. The sample is 1954:08-2003:09. Underlined and bold numbers denote statistical significance at the 5% and 1% levels respectively. For further details, refer to section III of the paper.

Panel A (Book-to-market quantiles)							
	BBGB	**DY**	**DEF**	**EY***	**IPG**	**VS**	**DY+RM**
BV1	-0.430	-0.136	-0.410	-0.175	-0.269	-0.042	-0.137
BV2	-0.064	-0.046	-0.050	-0.056	-0.046	-0.060	-0.043
BV3	0.142	0.053	0.131	0.062	0.113	-0.001	0.046
BV4	0.292	0.140	0.264	0.181	0.204	0.095	0.141
BV5	0.303	0.059	0.315	0.085	0.156	0.060	0.063

Panel B (Size quantiles)							
S1	0.127	0.262	0.012	0.271	0.177	0.137	0.263
S2	0.072	0.052	0.031	0.056	0.082	-0.052	0.049
S3	0.071	-0.003	0.052	0.010	0.083	-0.103	0.001
S4	0.025	-0.069	0.079	-0.073	-0.025	0.028	-0.066
S5	-0.052	-0.171	0.076	-0.167	-0.159	0.042	-0.176

Panel C (t-statistics)							
	BBGB	**DY**	**DEF**	**EY***	**IPG**	**VS**	**DY+RM**
SBV11	**-3.618**	-1.208	**-4.651**	<u>-2.002</u>	**-3.579**	-0.692	-1.204
SBV12	0.126	**3.006**	<u>-2.040</u>	**3.209**	1.412	1.404	**3.644**
SBV13	<u>1.985</u>	<u>2.428</u>	1.715	<u>2.491</u>	**2.632**	0.917	<u>2.372</u>
SBV14	**4.646**	**4.285**	**5.349**	**4.757**	**4.781**	**2.860**	**4.066**
SBV15	**4.334**	**3.601**	**4.904**	**3.856**	**4.145**	**2.876**	**3.443**
SBV21	**-4.551**	**-4.113**	**-4.815**	**-4.539**	**-4.915**	**-3.420**	**-4.141**
SBV22	-0.843	-0.690	-1.134	-0.844	-0.269	-1.534	-0.572
SBV23	**3.444**	1.943	**3.425**	1.910	<u>2.448</u>	-0.473	1.666
SBV24	**4.181**	**3.758**	**3.423**	**3.996**	**4.044**	1.728	**3.422**
SBV25	**3.946**	<u>2.281</u>	**3.546**	**2.849**	**3.942**	0.930	<u>2.283</u>
SBV31	**-3.503**	<u>-2.285</u>	**-3.373**	**-2.584**	**-2.989**	<u>-2.119</u>	<u>-2.343</u>
SBV32	0.172	1.046	-0.504	0.758	1.826	-1.204	1.032
SBV33	1.576	-0.897	1.317	-0.368	-0.062	**-2.637**	-0.889
SBV34	**3.792**	**2.810**	**2.842**	**3.350**	**3.733**	0.428	**2.904**
SBV35	**3.116**	-0.248	**3.168**	0.228	**2.802**	<u>-2.417</u>	-0.052
SBV41	<u>-2.558</u>	0.680	<u>-2.428</u>	0.356	-0.122	**3.781**	0.733
SBV42	<u>-1.972</u>	**-3.970**	-0.776	**-4.094**	**-3.168**	**-3.251**	**-4.087**
SBV43	1.889	0.352	1.811	0.351	<u>2.005</u>	-0.193	-0.009
SBV44	**2.932**	1.161	**3.638**	1.463	1.430	<u>2.302</u>	1.395
SBV45	1.880	<u>-2.485</u>	**2.751**	<u>-2.337</u>	-1.544	-0.162	**-2.614**
SBV51	-1.633	-1.352	-0.541	-1.335	-0.897	0.609	-1.586
SBV52	-1.305	<u>-2.173</u>	0.594	<u>-2.426</u>	<u>-2.197</u>	-0.250	<u>-2.353</u>
SBV53	0.240	-0.834	1.466	-0.728	-0.211	0.752	-0.859
SBV54	0.451	**-2.934**	1.755	<u>-2.548</u>	-1.952	-0.533	**-2.834**
SBV55	0.197	-0.963	1.212	-1.068	**-2.731**	1.009	-1.016

Table VI

Average pricing errors: 38 industry portfolios

This table reports the individual average pricing errors associated with the 38 industry portfolios. The models are the static ICAPM (BBGB), and the ICAPM scaled by the market dividend yield (DY), default spread (DEF), smoothed log earnings yield (EY*), cyclical industrial production growth (IPG), value spread (VS), and market return plus dividend yield (DY+RM). For each model the first column presents the average pricing errors (stated in percentage points), and the second column presents the respective t-statistics. The sample is 1954:08-2003:09. Underlined and bold numbers denote statistical significance at the 5% and 1% levels respectively. For further details, refer to section III of the paper.

	BBGB		DY		DEF		EY*		IPG		VS		DY+RM	
AGRIC	-0.141	-0.630	-0.144	-0.645	-0.042	-0.306	-0.128	-0.568	-0.165	-0.748	-0.175	-0.790	-0.222	-1.034
APPRL	-0.178	-0.971	-0.197	-1.104	-0.153	-0.852	-0.186	-1.026	-0.183	-1.002	-0.224	-1.364	-0.261	-1.494
CARS	-0.013	-0.098	-0.012	-0.090	-0.032	-0.247	-0.020	-0.153	-0.014	-0.108	-0.003	-0.024	0.072	0.580
CHAIR	0.023	0.168	0.027	0.198	0.038	0.276	0.028	0.206	0.009	0.062	-0.027	-0.225	-0.060	-0.438
CHEMS	0.102	0.927	0.058	0.471	0.090	0.840	0.063	0.529	0.117	1.115	0.068	0.568	0.000	0.000
CNSTR	-0.216	-1.262	-0.273	-1.716	-0.249	-1.403	-0.289	-1.823	-0.228	-1.374	-0.249	-1.520	-0.237	-1.480
ELCTR	-0.190	-1.077	-0.057	-0.493	-0.183	-1.032	-0.042	-0.366	-0.145	-0.951	-0.040	-0.402	-0.049	-0.425
FOOD	0.341	**2.661**	0.306	2.413	0.351	**2.693**	0.307	2.446	0.358	**2.829**	0.267	2.092	0.257	2.158
GLASS	-0.207	-1.316	-0.117	-0.897	-0.197	-1.253	-0.097	-0.753	-0.197	-1.301	-0.119	-0.902	0.006	0.046
INSTR	0.073	0.511	0.070	0.489	0.050	0.355	0.068	0.472	0.109	0.847	0.096	0.703	0.070	0.484
LETHR	0.215	1.092	0.258	1.253	0.237	1.204	0.244	1.205	0.216	1.095	0.175	0.941	0.253	1.232
MACHN	-0.105	-0.634	-0.021	-0.168	-0.107	-0.652	-0.008	-0.061	-0.087	-0.566	0.000	0.001	-0.057	-0.441
MANUF	0.065	0.358	0.022	0.124	0.065	0.360	0.002	0.013	0.085	0.469	-0.002	-0.009	0.092	0.510
METAL	-0.375	-2.299	-0.362	-2.201	-0.380	-2.348	-0.357	-2.169	-0.351	-1.989	-0.348	-2.145	-0.190	-1.342
MINES	-0.040	-0.191	-0.139	-0.782	-0.044	-0.213	-0.143	-0.791	-0.064	-0.337	-0.124	-0.632	-0.050	-0.298
MONEY	0.055	0.518	0.057	0.537	0.071	0.651	0.060	0.565	0.049	0.460	0.060	0.561	-0.014	-0.143
MTLPR	0.014	0.148	-0.024	-0.268	0.007	0.071	-0.030	-0.335	0.020	0.216	-0.048	-0.526	-0.025	-0.279
OIL	-0.108	-0.476	-0.129	-0.584	-0.090	-0.399	-0.127	-0.570	-0.139	-0.674	-0.107	-0.475	-0.116	-0.526
PAPER	-0.002	-0.014	-0.053	-0.458	-0.021	-0.206	-0.052	-0.458	-0.013	-0.107	-0.040	-0.343	-0.138	-1.101
PHONE	-0.081	-0.391	0.063	0.409	-0.072	-0.345	0.069	0.446	-0.086	-0.410	0.064	0.436	-0.006	-0.043
PRINT	0.184	1.447	0.179	1.411	0.183	1.438	0.179	1.405	0.190	1.509	0.161	1.265	0.219	1.675
PTRLM	0.278	1.722	0.202	1.298	0.275	1.706	0.196	1.240	0.258	1.730	0.250	1.544	0.090	0.556
RTAIL	0.131	1.022	0.132	1.029	0.109	0.861	0.118	0.905	0.141	1.154	0.116	0.902	0.171	1.304
RUBBR	-0.081	-0.572	-0.110	-0.773	-0.102	-0.736	-0.112	-0.792	-0.062	-0.448	-0.128	-0.913	0.049	0.381
SMOKE	0.661	**2.754**	0.616	**2.692**	0.640	**2.675**	0.599	**2.654**	0.681	**2.797**	0.629	**2.729**	0.551	2.515
SRVC	-0.050	-0.368	0.058	0.523	-0.032	-0.246	0.063	0.575	-0.014	-0.113	0.051	0.440	-0.058	-0.584
STONE	0.311	1.112	0.196	0.827	0.321	1.161	0.198	0.811	0.187	1.360	0.218	0.850	0.133	0.574
TRANS	-0.129	-1.098	-0.192	-1.614	-0.146	-1.249	-0.197	-1.661	-0.167	-1.378	-0.163	-1.409	-0.177	-1.506
TV	0.250	1.400	0.311	1.920	0.252	1.413	0.318	1.965	0.253	1.431	0.335	2.109	0.343	2.116
TXTLS	0.040	0.220	0.027	0.153	0.059	0.331	0.033	0.185	0.052	0.283	-0.046	-0.315	0.000	-0.001
UTILS	0.149	1.138	0.115	0.867	0.154	1.175	0.112	0.854	0.122	0.888	0.144	1.105	0.160	1.168
WHLSL	0.078	0.879	0.049	0.541	0.064	0.716	0.043	0.484	0.071	0.785	0.062	0.708	-0.028	-0.320
WOOD	-0.003	-0.018	-0.103	-0.618	-0.046	-0.280	-0.125	-0.753	-0.027	-0.151	-0.090	-0.539	-0.103	-0.618

Table VII

ICAPM with cyclical risk-aversion: Incorporating estimation error in covariances

This table reports the estimated covariance and beta factor risk prices for the ICAPM with time-varying risk aversion, by using the augmented GMM system which takes into account the estimation error in covariances, as described in section III of the paper. Panel A presents the results for both the static ICAPM (BBGB) and the ICAPM scaled by dividend yield. The ICAPM scaled by the default spread, smoothed earnings yield, cyclical industrial production growth, value spread, and market return plus dividend yield, are presented in Panels B, C, D, E and F, respectively. For each model, there are 3 sets of test assets - the 25 size/book-to-market portfolios (SBV25), 38 industry portfolios (IND38), and their combination (SBV25+IND38). The estimates correspond to a first-stage GMM estimation. λ_{CF} and λ_{H} denote the beta risk prices estimates for the cash-flow and discount-rate news factors, respectively, while λ_{CFDY}, λ_{CFDEF}, λ_{CFEY*}, λ_{CFIPG} ,λ_{CFVS} and λ_{CFRM} refer to the scaled factor related with time-varying risk aversion. γ_0, γ_1 and γ_2 denote the estimates of the covariance risk prices, which also correspond to the risk aversion coefficients. For each model the estimated risk prices are reported in line 1, while in line 2 are reported the associated t-statistics. The sample is 1954:08-2003:09. Italic, underlined and bold numbers denote statistical significance at the 10%, 5% and 1% levels respectively. The beta risk prices (λ) are multiplied by 100. For further details, refer to section III of the paper.

Panel A (BBGB + Dividend yield)

Row	λ_{CF}	λ_{CFDY}	λ_H	γ_0	γ_1
SBV25					
1	0.552		-0.136	10.771	
	2.525		**-139.944**	2.524	
2	0.583	0.045	-0.093	-96.650	3562.644
	1.370	**2.897**	**-8.015**	**-3.276**	**3.687**
IND38					
3	0.445		-0.136	8.683	
	2.105		**-144.242**	2.104	
4	0.426	0.017	-0.129	-8.143	542.519
	1.748	2.210	**-78.763**	-1.559	**4.130**
SBV25 + IND38					
5	0.493		-0.136	9.604	
	2.344		**-145.421**	2.343	
6	0.478	0.022	-0.125	-18.410	914.441
	1.844	2.553	**-68.531**	**-3.413**	**5.621**

Panel B (Default spread)

Row	λ_{CF}	λ_{CFDEF}	λ_H	γ_0	γ_1
SBV25					
1	0.173	-0.006	-0.385	73.316	-6543.156
	0.521	-1.527	**-6.423**	**4.110**	**-4.178**
IND38					
2	0.490	0.006	-0.097	-1.345	1018.752
	1.993	2.360	**-10.771**	-0.286	**4.205**
SBV25 + IND38					
3	0.429	0.003	-0.185	22.088	-1284.595
	1.648	1.097	**-18.925**	**3.458**	**-5.262**

Panel C (10 year earnings yield)

Row	λ_{CF}	λ_{CFEY^*}	λ_H	γ_0	γ_1
SBV25					
1	0.657	-0.754	-0.004	248.936	82.746
	1.542	-0.613	-0.096	**3.602**	**3.479**
IND38					
2	0.436	-1.039	-0.111	51.935	15.221
	1.770	-1.476	**-16.524**	**4.036**	**3.670**
SBV25 + IND38					
3	0.497	-1.117	-0.100	73.491	22.356
	1.897	-1.503	**-14.722**	**5.348**	**5.254**

Panel D (Industrial production growth)

Row	λ_{CF}	λ_{CFIPG}	λ_H	γ_0	γ_1
SBV25					
1	0.835	-0.008	0.204	123.658	-11838.436
	1.473	-1.504	2.466	**3.925**	**-4.093**
IND38					
2	0.462	0.003	-0.102	19.796	-1189.156
	1.836	1.202	**-12.796**	**3.208**	**-4.186**
SBV25 + IND38					
3	0.530	0.002	-0.077	28.842	-2040.375
	1.961	0.927	**-7.527**	**3.970**	**-5.508**

Panel E (Value spread)

Row	λ_{CF}	λ_{CFVS}	λ_H	γ_0	γ_1
SBV25					
1	0.064	-0.384	-0.404	407.498	-252.024
	0.131	-0.461	**-4.794**	**3.237**	**-3.189**
IND38					
2	0.360	0.511	-0.174	64.799	-35.841
	1.421	1.249	**-17.074**	**4.036**	**-3.805**
SBV25 + IND38					
3	0.332	0.392	-0.214	125.196	-73.652
	1.197	0.887	**-14.098**	**5.186**	**-5.307**

Panel F (Dividend yield + Market return)

Row	λ_{CF}	λ_{CFDY}	λ_{CFRM}	λ_H	γ_0	γ_1	γ_2
SBV25							
1	0.534	0.045	-0.012	-0.137	-102.679	3738.070	-137.680
	1.282	**2.779**	-0.148	-0.665	**-3.487**	**3.746**	-0.221
IND38							
2	0.273	0.010	-0.060	-0.291	-4.332	344.883	-482.151
	0.828	0.881	*-1.863*	**-3.360**	-0.411	0.926	*-1.908*
SBV25 + IND38							
3	0.462	0.021	-0.004	-0.140	-18.545	911.098	-46.968
	1.843	2.565	-0.165	-2.079	**-3.486**	**5.611**	-0.231

Table VIII

ICAPM with cyclical risk-aversion: Hansen-Jagannathan distance

This table reports the estimated covariance and beta factor risk prices for the ICAPM with time-varying risk aversion, by using the HJ-distance approach, as described in section III of the paper. Panel A presents the results for both the static ICAPM (BBGB) and the ICAPM scaled by dividend yield. The ICAPM scaled by the default spread, smoothed earnings yield, cyclical industrial production growth, value spread, and market return plus dividend yield, are presented in Panels B, C, D, E and F, respectively. In this table, the test assets are the 38 industry portfolios. λ_{CF} and λ_{H} denote the beta risk prices estimates for the cash-flow and discount-rate news factors, respectively, while λ_{CFDY}, λ_{CFDEF}, λ_{CFEY^*}, λ_{CFIPG} ,λ_{CFVS} and λ_{CFRM} refer to the scaled factor related with time-varying risk aversion. γ_0, γ_1 and γ_2 denote the estimates of the covariance risk prices, which also correspond to the risk aversion coefficients. For each model the estimated risk prices are reported in line 1, while in line 2 are reported the associated t-statistics. Test values (first row) and respective p-values (second row) for the asymptotic pricing error test are presented for each GMM estimation. HJ denotes the value of the test that the HJ-distance is zero, with the respective p-value given below. The sample is 1954:08-2003:09. Italic, underlined and bold numbers denote statistical significance at the 10%, 5% and 1% levels respectively. The beta risk prices (λ) are multiplied by 100. For further details, refer to section III of the paper.

Panel A (BBGB + Dividend yield)

λ_{CF}	λ_{CFDY}	λ_H	γ_0	γ_1	$E(\gamma_t)$	$\alpha'\Sigma^{-1}\alpha$	HJ
0.198		-0.135	3.860		3.860	25.852	0.256
1.245		**-190.130**	1.243			0.770	0.263
0.247	0.011	-0.130	-8.291	432.435	5.352	24.985	0.251
1.531	*1.764*	**-29.924**	-0.763	1.184		0.768	0.270

Panel B (Default spread)

λ_{CF}	λ_{CFDEF}	λ_H	γ_0	γ_1	$E(\gamma_t)$	$\alpha'\Sigma^{-1}\alpha$	HJ
0.227	0.003	-0.108	-2.921	687.429	3.679	25.677	0.255
1.361	1.185	-2.194	-0.220	0.530		0.737	0.248

Panel C (10 year earnings yield)

λ_{CF}	λ_{CFEY*}	λ_H	γ_0	γ_1	$E(\gamma_t)$	$\alpha'\Sigma^{-1}\alpha$	HJ
0.261	-0.577	-0.115	40.550	12.429	5.354	24.651	0.248
1.605	-1.279	**-7.908**	1.503	1.360		0.783	0.286

Panel D (Industrial production growth)

λ_{CF}	λ_{CFIPG}	λ_H	γ_0	γ_1	$E(\gamma_t)$	$\alpha'\Sigma^{-1}\alpha$	HJ
0.212	0.001	-0.119	8.954	-531.296	3.519	25.993	0.256
1.275	0.650	**-3.139**	0.668	-0.400		0.722	0.244

Panel E (Value spread)

λ_{CF}	λ_{CFVS}	λ_H	γ_0	γ_1	$E(\gamma_t)$	$\alpha'\Sigma^{-1}\alpha$	HJ
0.230	0.296	-0.176	65.895	-38.096	6.352	25.243	0.247
1.458	1.156	**-6.043**	1.500	-1.408		0.757	0.288

Panel F (Dividend yield + Market return)

λ_{CF}	λ_{CFDY}	λ_{CFRM}	λ_H	γ_0	γ_1	γ_2	$E(\gamma_t)$	$\alpha'\Sigma^{-1}\alpha$	HJ
0.165	0.007	-0.035	-0.223	-7.090	355.568	-279.016	0.765	23.214	0.238
0.990	1.152	-2.030	**-4.932**	-0.651	0.967	-2.073		0.806	0.328

Table IX

ICAPM with cyclical risk-aversion: Estimating covariance and beta factor risk-premia by time-series/cross-sectional regressions

This table reports the estimated covariance and beta factor risk prices for the ICAPM with time-varying risk aversion, using the time-series/cross-sectional regressions approach, as described in section IV of the paper. Panel A presents the results for the static ICAPM (BBGB), whereas the ICAPM scaled by dividend yield and value spread, are presented in Panels B and C, respectively. For each model, there are 3 sets of test assets - the 25 size/book-to-market portfolios, 38 industry portfolios, and their combination. Each panel reports estimates associated with both OLS and GLS cross-sectional regressions. λ_{CF} and λ_{H} denote the beta risk prices estimates for the cash-flow and discount-rate news factors, respectively, while λ_{CFDY} and λ_{CFVS} refer to the scaled factor related with time-varying risk aversion. γ_0 and γ_1 denote the estimates of the covariance risk prices, which also correspond to the risk aversion coefficients. For each model the estimated risk prices are reported in line 1, while in lines 2 and 3 are reported the associated t-statistics, calculated with type I and type II standard errors, respectively. $E(\gamma_t)$ is the average value of RRA. Test values (first row) and respective p-values (second row) for the asymptotic pricing error test, under type I and type II standard errors, are presented for each cross-sectional regression. The sample is 1954:08-2003:09. Italic, underlined and bold numbers denote statistical significance at the 10%, 5% and 1% levels respectively. The beta risk prices (λ) are multiplied by 100. For further details, refer to section IV of the paper.

Panel A (BBGB)

	OLS						**GLS**					
Row	λ_{CF}	λ_{H}	γ_0	$E(\gamma_t)$	$\alpha'\Sigma^{-1}\alpha$ I	$\alpha'\Sigma^{-1}\alpha$ II	λ_{CF}	λ_{H}	γ_0	$E(\gamma_t)$	$\alpha'\Sigma^{-1}\alpha$ I	$\alpha'\Sigma^{-1}\alpha$ II
25 Size/book-to-market portfolios												
1	0.555	-0.136	10.826	10.826	35.781	39.726	0.401	-0.136	7.816	7.816	36.917	58.206
	2.458	**-135.581**	2.457		0.058	0.023	2.180	**-165.684**	2.179		0.045	**0.000**
	2.723	**-150.172**	**2.722**				2.226	**-169.142**	2.224			
38 Industry portfolios												
2	0.447	-0.136	8.716	8.716	24.837	31.389	0.428	-0.136	8.341	8.341	25.056	32.442
	2.125	**-145.006**	2.123		0.813	0.497	2.369	**-168.858**	2.368		0.804	0.445
	2.280	**-155.644**	2.279				2.425	**-172.847**	2.424			
25 Size/book-to-market portfolios + 38 Industry portfolios												
3	0.000	0.000	9.646	9.646	61.742	109.674	0.406	-0.136	7.919	7.919	61.383	113.131
	2.291	**-141.527**	2.290		0.310	**0.000**	2.304	**-172.814**	2.302		0.322	**0.000**
	2.499	**-154.355**	2.498				2.319	**-173.941**	2.317			

Panel B (Dividend yield)

	OLS								GLS							
Row	λ_{CF}	λ_{CFDY}	λ_H	γ_0	γ_1	$E(\gamma_t)$	$\alpha'\Sigma^{-1}\alpha$ I	$\alpha'\Sigma^{-1}\alpha$ II	λ_{CF}	λ_{CFDY}	λ_H	γ_0	γ_1	$E(\gamma_t)$	$\alpha'\Sigma^{-1}\alpha$ I	$\alpha'\Sigma^{-1}\alpha$ II
25 Size/book-to-market portfolios																
1	0.586	0.046	-0.093	-97.869	3604.919	15.860	30.989	37.199	0.416	0.020	-0.123	-22.643	1014.025	9.348	40.622	41.205
	2.250	**2.911**	**-4.800**	-2.029	2.243		0.123	0.031	2.256	**2.720**	**-16.892**	-1.229	*1.680*		0.013	0.011
	2.659	**3.958**	**-6.513**	**-2.738**	**3.063**				2.289	**2.797**	**-19.891**	-1.456	*1.956*			
38 Industry portfolios																
2	0.427	0.017	-0.129	-8.303	548.747	9.009	24.319	31.188	0.442	0.018	-0.129	-8.964	579.659	9.323	24.321	30.167
	2.086	2.103	**-22.254**	-0.577	1.109		0.797	0.457	2.430	**2.607**	**-30.574**	-0.854	1.606		0.797	0.509
	2.234	2.305	**-25.698**	-0.665	1.279				2.474	**2.707**	**-32.628**	-0.907	*1.720*			
25 Size/book-to-market portfolios + 38 Industry portfolios																
3	0.480	0.022	-0.125	-18.700	925.291	10.492	60.051	107.133	0.416	0.015	-0.131	-2.398	346.302	8.527	60.167	107.592
	2.231	**2.618**	**-21.087**	-1.264	*1.851*		0.331	**0.000**	2.342	2.409	**-34.608**	-0.249	1.084		0.327	**0.000**
	2.436	**2.864**	**-23.583**	-1.415	2.065				2.347	2.404	**-37.437**	-0.270	1.158			

Panel C (Value spread)

	OLS								GLS							
Row	λ_{CF}	λ_{CFVS}	λ_H	γ_0	γ_1	$E(\gamma_t)$	$\alpha'\Sigma^{-1}\alpha$ I	$\alpha'\Sigma^{-1}\alpha$ II	λ_{CF}	λ_{CFVS}	λ_H	γ_0	γ_1	$E(\gamma_t)$	$\alpha'\Sigma^{-1}\alpha$ I	$\alpha'\Sigma^{-1}\alpha$ II
25 Size/book-to-market portfolios																
1	0.062	-0.393	-0.407	411.826	-254.739	13.677	22.025	27.970	0.303	0.303	-0.237	158.889	-94.911	10.546	26.001	55.818
	0.229	-0.744	**-4.140**	**2.808**	**-2.764**		0.519	0.217	*1.667*	0.979	**-5.448**	2.435	-2.336		0.301	**0.000**
	0.286	-1.006	**-6.279**	**4.246**	**-4.198**				*1.702*	1.041	**-7.466**	**3.320**	**-3.208**			
38 Industry portfolios																
2	0.361	0.512	-0.174	65.274	-36.123	8.815	24.367	30.500	0.377	0.513	-0.188	85.910	-48.732	9.743	25.018	29.468
	1.941	*1.684*	**-4.430**	1.099	-0.987		0.795	0.492	2.187	*1.850*	**-5.784**	*1.747*	-1.614		0.767	0.545
	2.062	*1.824*	**-5.682**	1.404	-1.268				2.208	*1.887*	**-7.275**	2.187	-2.035			
25 Size/book-to-market portfolios + 38 Industry portfolios																
3	0.333	0.392	-0.215	126.359	-74.366	10.126	55.871	101.108	0.365	0.493	-0.188	86.383	-49.171	9.531	57.809	99.519
	1.695	1.218	**-5.104**	1.987	*-1.895*		0.480	**0.000**	2.119	*1.767*	**-6.787**	2.059	*-1.906*		0.408	**0.000**
	1.844	1.353	**-6.760**	**2.620**	-2.515				2.133	*1.815*	**-8.507**	2.555	-2.400			

Table X

Average pricing errors: Market return

This table reports the average pricing errors associated with the market return, as described in section V of the paper. The models are the static ICAPM (BBGB), and the ICAPM scaled by the market dividend yield (DY), default spread (DEF), smoothed log earnings yield (EY*), cyclical industrial production growth (IPG), value spread (VS), and market return plus dividend yield (DY+RM). For each model the first row presents the average pricing error (stated in percentage points), and the second row presents the annualized error. The sample is 1954:08-2003:09. For further details, refer to section V of the paper.

	BBGB	DY	EY*	IPG	VS	DY + RM
25 Size/book-to-market portfolios						
α_m	-0.164	-0.152	-0.137	-0.106	0.056	-0.147
α_m*12	-1.972	-1.821	-1.642	-1.277	0.670	-1.761
38 Industry portfolios						
α_m	-0.057	-0.031	-0.023	-0.039	-0.010	-0.039
α_m*12	-0.683	-0.370	-0.277	-0.473	-0.119	-0.468
SBV25 + IND38						
α_m	-0.104	-0.078	-0.073	-0.083	-0.022	-0.078
α_m*12	-1.251	-0.939	-0.878	-0.997	-0.263	-0.932

Table XI

Average pricing errors for 25 size/book-to-market portfolios: Comparison with alternative factor models

This table reports the average individual pricing errors (stated in percentage points) associated with the 25 size/book-to-market portfolios (Panel A). The ICAPM models are the static ICAPM (BBGB), and the ICAPM scaled by the market dividend yield (DY), value spread (VS), and market return plus dividend yield (DY+RM). The alternative factor models are the CAPM, Fama-French 3 factor model (FF3) and Fama-French 4 factor model (FF4). Panels B and C, present the average pricing errors across size and book-to-market quintiles, respectively. ij denotes the portfolio with ith size and jth book-to-market quintiles. S1 and BV1 denote the lowest size and book-to-market quintiles, respectively. The sample is 1954:08-2003:09. For further details, refer to section V of the paper.

Panel A (25 Size/book-to-market portfolios)							
	BBGB	**DY**	**VS**	**DY+RM**	**CAPM**	**FF3**	**FF4**
11	-0.626	-0.111	-0.091	-0.108	-0.610	-0.356	-0.190
12	0.014	0.292	0.154	0.311	-0.008	0.038	-0.109
13	0.204	0.246	0.088	0.246	0.134	0.039	-0.080
14	0.496	0.448	0.253	0.445	0.431	0.251	0.122
15	0.553	0.435	0.282	0.433	0.476	0.175	0.187
21	-0.580	-0.274	-0.278	-0.282	-0.552	-0.178	-0.088
22	-0.054	-0.043	-0.097	-0.036	-0.070	-0.032	0.058
23	0.236	0.128	-0.028	0.117	0.194	0.093	0.051
24	0.351	0.272	0.084	0.265	0.292	0.081	-0.001
25	0.409	0.179	0.057	0.180	0.354	0.041	-0.060
31	-0.428	-0.177	-0.161	-0.181	-0.410	0.020	0.096
32	0.009	0.063	-0.071	0.062	-0.012	0.028	0.033
33	0.104	-0.044	-0.134	-0.045	0.084	-0.041	0.011
34	0.330	0.163	0.021	0.169	0.279	0.048	0.033
35	0.341	-0.018	-0.169	-0.003	0.301	-0.029	0.094
41	-0.315	0.063	0.253	0.075	-0.286	0.156	0.028
42	-0.153	-0.311	-0.261	-0.317	-0.155	-0.094	0.017
43	0.144	0.020	-0.013	0.000	0.129	0.039	0.093
44	0.250	0.067	0.173	0.082	0.214	0.028	0.069
45	0.204	-0.185	-0.014	-0.172	0.183	-0.121	-0.038
51	-0.223	-0.181	0.068	-0.196	-0.212	0.225	0.150
52	-0.141	-0.229	-0.026	-0.237	-0.131	0.015	-0.039
53	0.027	-0.085	0.082	-0.087	0.036	0.071	-0.088
54	0.053	-0.248	-0.058	-0.247	0.043	-0.130	-0.098
55	0.028	-0.114	0.144	-0.120	0.044	-0.239	-0.213
Panel B (Size quantiles)							
	BBGB	**DY**	**VS**	**DY+RM**	**CAPM**	**FF3**	**FF4**
S1	0.128	0.262	0.137	0.265	0.085	0.029	-0.014
S2	0.072	0.052	-0.052	0.049	0.044	0.001	-0.008
S3	0.071	-0.003	-0.103	0.000	0.048	0.005	0.053
S4	0.026	-0.069	0.028	-0.066	0.017	0.002	0.034
S5	-0.052	-0.171	0.042	-0.177	-0.044	-0.012	-0.058
Panel C (Book-to-market quantiles)							
	BBGB	**DY**	**VS**	**DY+RM**	**CAPM**	**FF3**	**FF4**
BV1	-0.434	-0.136	-0.042	-0.138	-0.414	-0.027	-0.001
BV2	-0.065	-0.046	-0.060	-0.044	-0.075	-0.009	-0.008
BV3	0.143	0.053	-0.001	0.046	0.115	0.040	-0.003
BV4	0.296	0.140	0.095	0.143	0.252	0.056	0.025
BV5	0.307	0.059	0.060	0.064	0.272	-0.035	-0.006

Table XII

SBV25 factor loading estimates: BBGB

This table reports in panel A the factor loadings estimates associated with the BBGB model, for the 25 size/book-to-market portfolios (SBV25). r^{CF} and r^{h} denote the cash-flow and discount-rate news factors, respectively. For each portfolio, the first row shows the estimated coefficients, and the second row presents the associated Newey-West t-statistics, calculated with 5 lags. SBV_{ij} denotes the portfolio with ith size and jth book-to-market quintiles. Average betas across the book-to-market and size quintiles are reported in Panels B and C, respectively. S1 and BV1 denote the lowest size and book-to-market quintiles, respectively. The sample is 1954:08-2003:09. Underlined and bold numbers denote statistical significance at the 5% and 1% levels, respectively. For further details, refer to section V of the paper.

Panel A (SBV25)				
	Const.	**r^{CF}_{t+1}**	**r^{h}_{t+1}**	**Adj. R^2**
SBV11	0.000	1.397	-1.408	0.569
	0.136	**12.399**	**-21.300**	
SBV12	0.006	1.139	-1.223	0.580
	2.782	**11.789**	**-22.630**	
SBV13	0.007	0.885	-1.089	0.596
	3.598	**10.705**	**-21.950**	
SBV14	0.009	0.828	-1.015	0.598
	5.386	**9.969**	**-20.323**	
SBV15	0.010	0.835	-1.045	0.565
	5.020	**9.838**	**-19.444**	
SBV21	0.002	1.413	-1.405	0.703
	0.785	**17.250**	**-28.878**	
SBV22	0.005	1.086	-1.167	0.704
	3.325	**14.650**	**-25.277**	
SBV23	0.007	0.904	-1.042	0.699
	4.916	**12.522**	**-23.655**	
SBV24	0.008	0.830	-1.000	0.676
	5.243	**11.380**	**-23.822**	
SBV25	0.009	0.915	-1.084	0.644
	5.203	**11.082**	**-20.571**	
SBV31	0.003	1.331	-1.335	0.752
	1.787	**19.631**	**-30.993**	
SBV32	0.006	1.008	-1.101	0.772
	4.174	**16.229**	**-29.736**	
SBV33	0.006	0.905	-0.985	0.741
	4.588	**13.307**	**-25.823**	
SBV34	0.008	0.805	-0.947	0.698
	5.498	**11.609**	**-24.001**	
SBV35	0.008	0.880	-1.021	0.655
	5.063	**10.792**	**-19.445**	
SBV41	0.004	1.259	-1.232	0.831
	3.112	**23.498**	**-36.405**	
SBV42	0.004	1.021	-1.064	0.824
	3.624	**15.786**	**-31.262**	
SBV43	0.007	0.916	-0.989	0.783
	5.737	**13.999**	**-29.535**	
SBV44	0.007	0.833	-0.942	0.739
	5.914	**14.784**	**-26.305**	
SBV45	0.007	0.938	-1.010	0.653
	4.656	**12.450**	**-20.318**	
SBV51	0.004	1.022	-1.029	0.863
	4.041	**27.800**	**-35.798**	
SBV52	0.004	0.949	-0.950	0.839
	4.459	**22.822**	**-38.306**	
SBV53	0.005	0.850	-0.848	0.752
	5.096	**16.659**	**-28.943**	
SBV54	0.005	0.779	-0.814	0.666
	4.208	**12.560**	**-23.923**	
SBV55	0.005	0.882	-0.845	0.579
	3.535	**10.192**	**-19.057**	

Panel B (Book-to-market quantiles)			
	Const.	**r^{CF}_{t+1}**	**r^{h}_{t+1}**
BV1	0.002	1.284	-1.282
BV2	0.005	1.041	-1.101
BV3	0.006	0.892	-0.991
BV4	0.007	0.815	-0.943
BV5	0.008	0.890	-1.001

Panel C (Size quantiles)			
	Const.	**r^{CF}_{t+1}**	**r^{h}_{t+1}**
S1	0.006	1.017	-1.156
S2	0.006	1.030	-1.140
S3	0.006	0.986	-1.078
S4	0.006	0.994	-1.047
S5	0.005	0.896	-0.897

Table XIII

SBV25 factor loading estimates: ICAPM scaled by dividend yield

This table reports in Panel A the factor loadings estimates associated with the DY ICAPM model, for the 25 size/book-to-market portfolios (SBV25). r^{CF} and r^{h} denote the cash-flow and discount-rate news factors, respectively, whereas DYr^{CF} represents the factor related with time-varying risk aversion. For each portfolio, the first row shows the estimated coefficients, and the second row presents the associated Newey-West t-statistics, calculated with 5 lags. SBV_{ij} denotes the portfolio with ith size and jth book-to-market quintiles. Average betas across the book-to-market and size quintiles are reported in Panels B and C, respectively. S1 and BV1 denote the lowest size and book-to-market quintiles, respectively. The sample is 1954:08-2003:09. Underlined and bold numbers denote statistical significance at the 5% and 1% levels, respectively. For further details, refer to section V of the paper.

Panel A (25 Size/book-to-market portfolios)					
	Const.	r^{CF}_{t+1}	$r^{CF}_{t+1}DY_t$	r^h_{t+1}	**Adj. R^2**
SBV11	0.000	1.931	-17.601	-1.406	0.572
	0.193	**6.124**	-1.918	**-21.177**	
SBV12	0.006	1.423	-9.367	-1.222	0.581
	2.828	**4.903**	-1.080	**-22.408**	
SBV13	0.007	0.911	-0.876	-1.089	0.596
	3.605	**3.613**	-0.111	**-21.913**	
SBV14	0.009	0.760	2.218	-1.015	0.597
	5.395	**3.012**	0.300	**-20.320**	
SBV15	0.010	0.691	4.736	-1.045	0.565
	5.004	**2.860**	0.652	**-19.537**	
SBV21	0.002	1.721	-10.160	-1.404	0.704
	0.830	**8.116**	-1.516	**-28.515**	
SBV22	0.005	1.078	0.260	-1.167	0.703
	3.333	**4.728**	0.038	**-25.263**	
SBV23	0.007	0.769	4.442	-1.043	0.699
	4.922	**3.501**	0.654	**-23.666**	
SBV24	0.008	0.729	3.325	-1.000	0.675
	5.249	**2.912**	0.461	**-23.977**	
SBV25	0.009	0.647	8.816	-1.085	0.646
	5.181	<u>2.520</u>	1.098	**-20.815**	
SBV31	0.003	1.580	-8.238	-1.335	0.753
	1.832	**7.408**	-1.389	**-30.757**	
SBV32	0.006	1.048	-1.306	-1.100	0.771
	4.189	**5.260**	-0.223	**-29.685**	
SBV33	0.006	0.728	5.856	-0.986	0.741
	4.594	**2.983**	0.824	**-25.699**	
SBV34	0.008	0.609	6.481	-0.948	0.699
	5.509	<u>2.463</u>	0.916	**-24.219**	
SBV35	0.008	0.472	13.429	-1.022	0.659
	5.026	1.906	1.786	**-20.051**	
SBV41	0.004	1.653	-12.992	-1.231	0.835
	3.217	**9.876**	**-2.856**	**-35.329**	
SBV42	0.004	0.831	6.267	-1.065	0.825
	3.638	**3.803**	1.054	**-31.129**	
SBV43	0.007	0.767	4.921	-0.990	0.784
	5.762	**3.221**	0.711	**-29.502**	
SBV44	0.007	0.621	7.006	-0.942	0.741
	5.900	**3.189**	1.248	**-26.650**	
SBV45	0.007	0.500	14.430	-1.011	0.658
	4.627	<u>2.014</u>	<u>1.996</u>	**-20.702**	
SBV51	0.004	1.052	-0.998	-1.028	0.862
	4.039	**10.621**	-0.308	**-35.634**	
SBV52	0.004	0.839	3.636	-0.950	0.839
	4.430	**5.779**	0.903	**-37.895**	
SBV53	0.005	0.718	4.365	-0.848	0.752
	5.087	**4.009**	0.897	**-28.618**	
SBV54	0.005	0.439	11.203	-0.815	0.671
	4.193	1.948	1.776	**-23.933**	
SBV55	0.005	0.718	5.389	-0.845	0.579
	3.534	<u>2.194</u>	0.576	**-18.911**	

Panel B (Book-to-market quantiles)				
	Const.	r^{CF}_{t+1}	$r^{CF}_{t+1}DY_t$	r^h_{t+1}
BV1	0.003	1.588	-9.998	-1.281
BV2	0.005	1.044	-0.102	-1.101
BV3	0.006	0.779	3.741	-0.991
BV4	0.007	0.632	6.047	-0.944
BV5	0.008	0.606	9.360	-1.002

Panel C (Size quantiles)				
	Const.	r^{CF}_{t+1}	$r^{CF}_{t+1}DY_t$	r^h_{t+1}
S1	0.006	1.143	-4.178	-1.156
S2	0.006	0.989	1.337	-1.140
S3	0.006	0.887	3.244	-1.078
S4	0.006	0.875	3.927	-1.048
S5	0.005	0.753	4.719	-0.897

Table XIV

SBV25 factor loading estimates: ICAPM scaled by value spread

This table reports in Panel A the factor loadings estimates associated with the VS ICAPM model, for the 25 size/book-to-market portfolios (SBV25). r^{CF} and r^{h} denote the cash-flow and discount-rate news factors, respectively, whereas VSr^{CF} represents the factor related with time-varying risk aversion. For each portfolio, the first row shows the estimated coefficients, and the second row presents the associated Newey-West t-statistics, calculated with 5 lags. SBV_{ij} denotes the portfolio with ith size and jth book-to-market quintiles. Average betas across the book-to-market and size quintiles are reported in Panels B and C, respectively. S1 and BV1 denote the lowest size and book-to-market quintiles, respectively. The sample is 1954:08-2003:09. Underlined and bold numbers denote statistical significance at the 5% and 1% levels, respectively. For further details, refer to section V of the paper.

Panel A (25 Size/book-to-market portfolios)					
	Const.	r^{CF}_{t+1}	$r^{CF}_{t+1}VS_t$	r^{h}_{t+1}	**Adj. R^2**
SBV11	0.000	0.657	0.459	-1.411	0.569
	0.176	0.560	0.625	**-21.366**	
SBV12	0.006	1.431	-0.182	-1.222	0.580
	2.723	1.429	-0.289	**-23.059**	
SBV13	0.006	1.729	-0.523	-1.085	0.597
	3.508	<u>2.336</u>	-1.153	**-22.194**	
SBV14	0.009	2.056	-0.762	-1.009	0.601
	5.274	**2.986**	-1.776	**-20.525**	
SBV15	0.009	2.138	-0.809	-1.039	0.568
	4.952	**2.978**	-1.807	**-19.708**	
SBV21	0.001	1.474	-0.038	-1.405	0.702
	0.773	1.821	-0.077	**-29.285**	
SBV22	0.005	1.958	-0.540	-1.163	0.705
	3.249	**3.426**	-1.556	**-25.700**	
SBV23	0.007	2.312	-0.873	-1.035	0.704
	4.831	**4.329**	**-2.678**	**-23.981**	
SBV24	0.008	2.160	-0.825	-0.993	0.680
	5.174	**3.809**	<u>-2.292</u>	**-24.085**	
SBV25	0.009	2.590	-1.039	-1.076	0.650
	5.144	**3.961**	<u>-2.530</u>	**-21.042**	
SBV31	0.003	1.428	-0.061	-1.335	0.752
	1.767	<u>2.384</u>	-0.162	**-31.031**	
SBV32	0.006	1.932	-0.573	-1.096	0.774
	4.079	**4.614**	<u>-2.277</u>	**-30.232**	
SBV33	0.006	2.288	-0.858	-0.978	0.746
	4.536	**4.450**	**-2.610**	**-25.644**	
SBV34	0.008	2.286	-0.918	-0.940	0.705
	5.448	**4.622**	**-2.886**	**-24.024**	
SBV35	0.008	3.080	-1.365	-1.010	0.667
	4.969	**5.366**	**-3.775**	**-20.303**	
SBV41	0.004	0.326	0.579	-1.237	0.833
	3.182	0.592	1.630	**-35.640**	
SBV42	0.004	2.095	-0.666	-1.059	0.827
	3.562	**4.558**	<u>-2.293</u>	**-31.556**	
SBV43	0.007	2.037	-0.696	-0.984	0.787
	5.696	**4.296**	<u>-2.285</u>	**-29.240**	
SBV44	0.007	1.589	-0.469	-0.938	0.741
	5.858	**4.079**	-1.851	**-25.886**	
SBV45	0.007	2.274	-0.829	-1.003	0.657
	4.611	**3.307**	-1.859	**-19.973**	
SBV51	0.004	0.800	0.138	-1.030	0.863
	4.051	**2.807**	0.782	**-36.146**	
SBV52	0.004	1.263	-0.195	-0.948	0.839
	4.375	**2.726**	-0.647	**-38.027**	
SBV53	0.005	1.289	-0.272	-0.846	0.752
	4.996	<u>2.487</u>	-0.812	**-28.736**	
SBV54	0.005	1.682	-0.561	-0.809	0.669
	4.115	<u>2.456</u>	-1.259	**-23.338**	
SBV55	0.005	1.168	-0.178	-0.843	0.578
	3.500	1.165	-0.271	**-18.775**	

Panel B (Book-to-market quantiles)				
	Const.	r^{CF}_{t+1}	$r^{CF}_{t+1}VS_t$	r^{h}_{t+1}
BV1	0.003	0.937	0.216	-1.283
BV2	0.005	1.736	-0.431	-1.097
BV3	0.006	1.931	-0.644	-0.986
BV4	0.007	1.955	-0.707	-0.938
BV5	0.008	2.250	-0.844	-0.994

Panel C (Size quantiles)				
	Const.	r^{CF}_{t+1}	$r^{CF}_{t+1}VS_t$	r^{h}_{t+1}
S1	0.006	1.602	-0.363	-1.153
S2	0.006	2.099	-0.663	-1.134
S3	0.006	2.203	-0.755	-1.072
S4	0.006	1.664	-0.416	-1.044
S5	0.005	1.241	-0.214	-0.895

Table XV

Factor risk premia for book-to-market quintiles

This table reports the risk premium (beta times risk price) for each factor, across the book-to-market quintiles. The models are the BBGB (Panel A), dividend yield ICAPM (Panel B), and the ICAPM scaled by the value spread (Panel C). CAPM denotes the average pricing errors associated with the CAPM model, $E(r_m)$ denotes the average excess return for the book-to-market quintiles, and α represents the average pricing models associated with the ICAPM model. λ_{CF} and λ_H denote the risk premium estimates for the cash-flow and discount-rate news factors, respectively, while λ_{CFDY} and λ_{CFVS} refer to the risk premium associated with the scaled factors. All the values are presented in percentage points. BV1 denote the lowest book-to-market quintile. The sample is 1954:08-2003:09. For further details, refer to section V of the paper.

Panel A (BBGB)

	CAPM	$E(r_{mt})$	λ_{CF}	λ_H	α
BV1	-0.414	0.453	0.713	0.175	-0.434
BV2	-0.075	0.663	0.578	0.150	-0.065
BV3	0.115	0.773	0.495	0.135	0.143
BV4	0.252	0.877	0.452	0.129	0.296
BV5	0.272	0.937	0.494	0.136	0.307

Panel B (Dividend yield)

	CAPM	$E(r_{mt})$	λ_{CF}	λ_{CFDY}	λ_H	α
BV1	-0.414	0.453	0.930	-0.458	0.119	-0.137
BV2	-0.075	0.663	0.612	-0.005	0.102	-0.046
BV3	0.115	0.773	0.456	0.172	0.092	0.053
BV4	0.252	0.877	0.370	0.277	0.088	0.142
BV5	0.272	0.937	0.355	0.429	0.093	0.060

Panel C (Value spread)

	CAPM	$E(r_{mt})$	λ_{CF}	λ_{CFVS}	λ_H	α
BV1	-0.414	0.453	0.058	-0.085	0.522	-0.042
BV2	-0.075	0.663	0.107	0.170	0.447	-0.061
BV3	0.115	0.773	0.120	0.254	0.401	-0.001
BV4	0.252	0.877	0.121	0.278	0.382	0.096
BV5	0.272	0.937	0.139	0.332	0.405	0.061

Table XVI

Factor risk premia for book-to-market quintiles: Campbell and Vuolteenaho (2004) BBGB

This table reports the average betas (Panel A) and risk premium (beta times risk price) for each factor, across the book-to-market quintiles, for the BBGB model from Campbell and Vuolteenaho (2004). r^{CF} and r^{h} denote the cash-flow and discount-rate news factors, respectively. CAPM denotes the average pricing errors associated with the CAPM model, $E(r_m)$ denotes the average excess return for the book-to-market quintiles, and α represents the average pricing models associated with the ICAPM model. λ_{CF} and λ_H denote the risk premium estimates for the cash-flow and discount-rate news factors, respectively. All the values are presented in percentage points. BV1 denote the lowest book-to-market quintile. The sample is 1954:08-2003:09. For further details, refer to section V of the paper.

Panel A (Average betas)

	Const.	r^{CF}_{t+1}	r^{h}_{t+1}
BV1	0.002	1.138	-1.340
BV2	0.005	1.022	-1.107
BV3	0.007	0.941	-0.969
BV4	0.008	0.918	-0.899
BV5	0.008	1.022	-0.944

Panel B (Average risk premium)

	CAPM	$E(r_{mt})$	λ_{CF}	λ_H	α
BV1	-0.414	0.453	0.703	0.128	-0.377
BV2	-0.075	0.663	0.631	0.105	-0.074
BV3	0.115	0.773	0.581	0.092	0.099
BV4	0.252	0.877	0.567	0.086	0.224
BV5	0.272	0.937	0.631	0.090	0.216

Table XVII

ICAPM with cyclical risk-aversion: Incorporating the HML factor

This table reports the estimated covariance and beta factor risk prices for the scaled ICAPM, augmented by the HML factor. The models are the ICAPM scaled by the dividend yield (Panel A) and the ICAPM scaled by the value spread (Panel B). For each model, there are 3 sets of test assets - the 25 size/book-to-market portfolios, 38 industry portfolios, and their combination. The estimates are obtained from first-stage GMM. λ_{CF} and λ_{H} denote the beta risk prices estimates for the cash-flow and discount-rate news factors, respectively, while λ_{CFDY} and λ_{CFVS} refer to the scaled factor related with time-varying risk aversion. γ_0 and γ_1 denote the estimates of the risk aversion coefficients. λ_{HML} and γ_{HML} represent the beta and covariance risk prices associated with the HML factor, respectively. For each model the estimated risk prices are reported in line 1, while in line 2 are reported the associated t-statistics. The sample is 1954:08-2003:09. Italic, underlined and bold numbers denote statistical significance at the 10%, 5% and 1% levels respectively. The beta risk prices (λ) are multiplied by 100. For further details, refer to section V of the paper.

Panel A (Dividend yield)

Row	λ_{CF}	λ_{CFDY}	λ_{HML}	λ_{H}	γ_0	γ_1	γ_{HML}
25 Size/book-to-market portfolios							
1	0.588	0.045	0.236	-0.090	-95.015	3512.164	0.643
	2.902	**4.191**	0.396	<u>-2.199</u>	**-3.096**	**3.420**	0.082
38 Industry portfolios							
2	0.458	0.018	0.394	-0.103	-4.556	447.177	4.827
	<u>2.380</u>	<u>2.214</u>	0.624	<u>-2.236</u>	-0.323	0.910	0.602
25 Size/book-to-market portfolios + 38 Industry portfolios							
3	0.571	0.022	1.020	-0.057	-4.314	516.509	12.725
	2.911	**2.739**	*1.890*	-1.452	-0.315	1.073	*1.855*

Panel B (Value Spread)

Row	λ_{CF}	λ_{CFVS}	λ_{HML}	λ_{H}	γ_0	γ_1	γ_{HML}
25 Size/book-to-market portfolios							
1	0.064	-0.385	0.331	-0.404	407.614	-252.100	-0.020
	0.342	-1.081	0.381	**-3.231**	**3.855**	**-3.799**	-0.002
38 Industry portfolios							
2	0.404	0.585	0.444	-0.138	56.769	-30.274	5.261
	<u>2.336</u>	<u>2.073</u>	0.650	<u>-1.975</u>	0.982	-0.848	0.601
25 Size/book-to-market portfolios + 38 Industry portfolios							
3	0.442	0.589	0.884	-0.136	99.137	-56.056	10.400
	2.604	<u>2.141</u>	1.455	<u>-1.969</u>	*1.653*	-1.517	1.321

Table XVIII

ICAPM with cyclical risk-aversion: Incorporating the UMD factor

This table reports the estimated covariance and beta factor risk prices for the scaled ICAPM, augmented by the UMD factor. The models are the ICAPM scaled by the dividend yield (Panel A) and the ICAPM scaled by the value spread (Panel B). For each model, there are 3 sets of test assets - the 25 size/book-to-market portfolios, 38 industry portfolios, and their combination. The estimates are obtained from first-stage GMM. λ_{CF} and λ_{H} denote the beta risk prices estimates for the cash-flow and discount-rate news factors, respectively, while λ_{CFDY} and λ_{CFVS} refer to the scaled factor related with time-varying risk aversion. γ_0 and γ_1 denote the estimates of the risk aversion coefficients. λ_{UMD} and γ_{UMD} represent the beta and covariance risk prices associated with the UMD factor, respectively. For each model the estimated risk prices are reported in line 1, while in line 2 are reported the associated t-statistics. The sample is 1954:08-2003:09. Italic, underlined and bold numbers denote statistical significance at the 10%, 5% and 1% levels respectively. The beta risk prices (λ) are multiplied by 100. For further details, refer to section V of the paper.

Panel A (Dividend yield)

Row	λ_{CF}	λ_{CFDY}	λ_{UMD}	λ_{H}	γ_0	γ_1	γ_{UMD}
25 Size/book-to-market portfolios							
1	0.427	0.035	-3.420	-0.033	-77.562	2848.660	-23.122
	<u>2.215</u>	**3.821**	**-2.881**	-1.431	**-2.639**	**2.985**	**-2.859**
38 Industry portfolios							
2	0.410	0.015	-0.777	-0.117	0.198	260.823	-5.337
	<u>2.054</u>	*1.934*	-1.141	**-8.590**	0.013	0.538	-1.151
25 Size/book-to-market portfolios + 38 Industry portfolios							
3	0.432	0.017	-1.382	-0.102	-6.307	492.260	-9.436
	<u>2.152</u>	<u>2.365</u>	<u>-2.035</u>	**-7.343**	-0.420	1.028	<u>-2.040</u>

Panel B (Value Spread)

Row	λ_{CF}	λ_{CFVS}	λ_{UMD}	λ_{H}	γ_0	γ_1	γ_{UMD}
25 Size/book-to-market portfolios							
1	0.066	-0.365	-0.494	-0.387	392.482	-242.657	-2.570
	0.311	-0.921	-0.404	**-4.920**	**4.180**	**-4.098**	-0.305
38 Industry portfolios							
2	0.389	0.598	-0.780	-0.134	28.612	-12.981	-5.330
	<u>2.091</u>	<u>1.966</u>	-1.280	**-2.945**	0.479	-0.352	-1.275
25 Size/book-to-market portfolios + 38 Industry portfolios							
3	0.351	0.466	-0.895	-0.171	87.006	-49.655	-5.984
	1.797	1.439	-1.615	**-4.077**	1.525	-1.403	-1.576

Figure 1

Time variation in the relative risk aversion coefficient

This figure plots estimates of the relative risk aversion coefficient (RRA) associated with the BBGB model, estimated for a 60-months rolling sample. The test assets are the 25 size/book-to-market portfolios (SBV25) and the 38 industry portfolios (IND38). The estimates are obtained from first-stage GMM. The whole sample is 1959:07-2003:09. For further details, refer to section III of the paper.

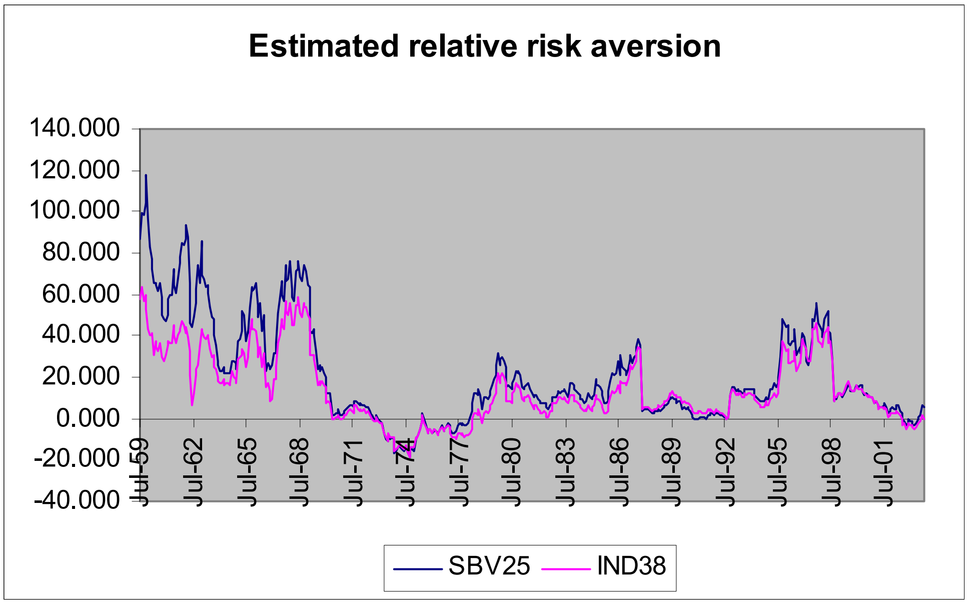
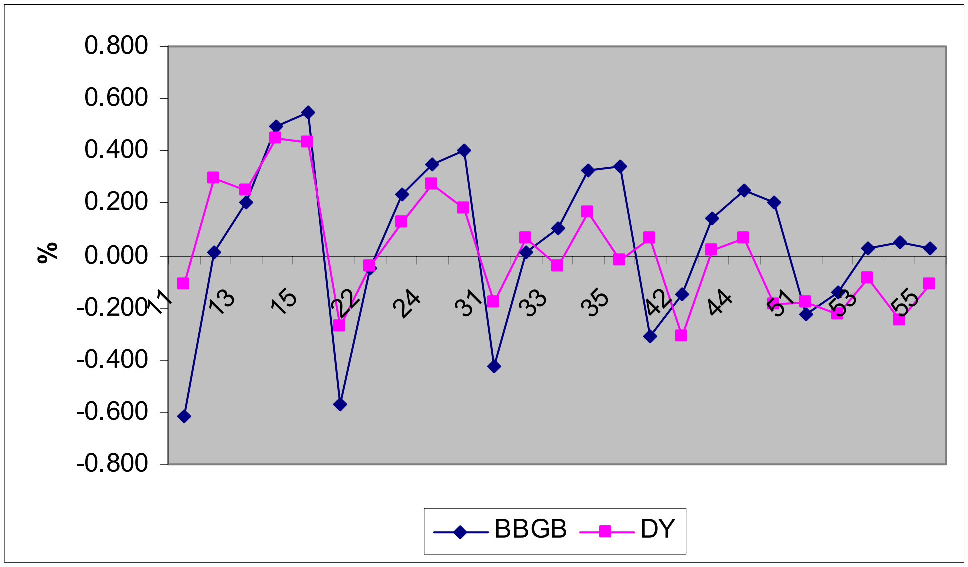
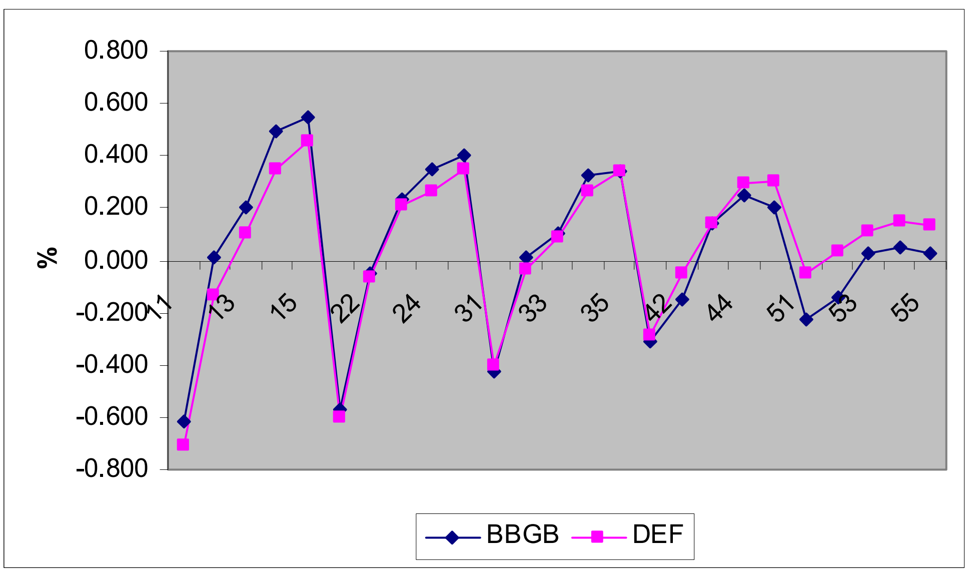


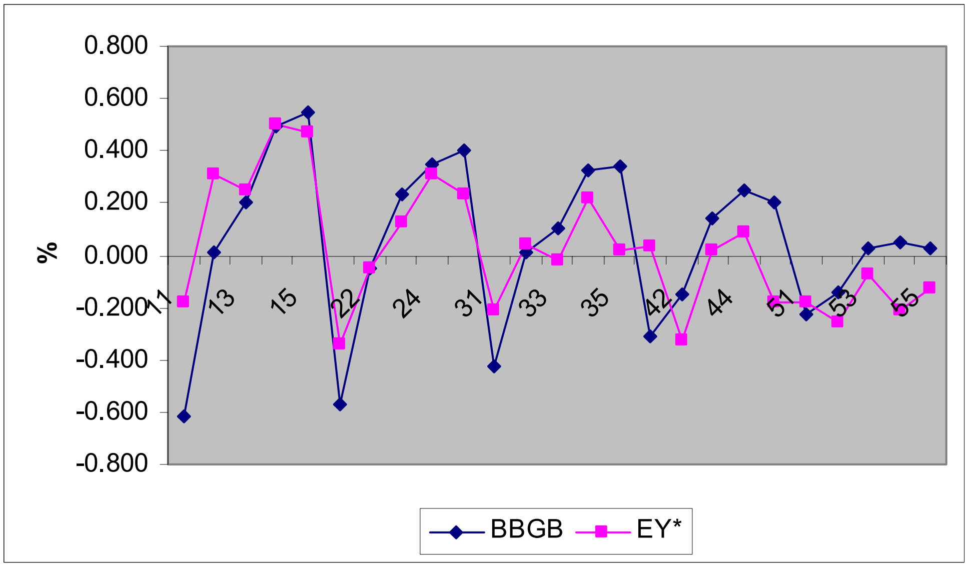
Figure 2

Average pricing errors: 25 size/book-to-market portfolios

This figure presents the average pricing errors (stated in percentage points) across the book-to-market quintiles associated with the 25 size/book-to-market portfolios. The models are the static ICAPM (BBGB), and the ICAPM scaled by the market dividend yield (DY), default spread (DEF), smoothed log earnings yield (EY*), cyclical industrial production growth (IPG), value spread (VS), and market return plus dividend yield (DY+RM). Panel C reports t-statistics for the individual pricing errors. ij denotes the portfolio with ith size and jth book-to-market quintiles. The sample is 1954:08-2003:09. For further details, refer to section III of the paper.



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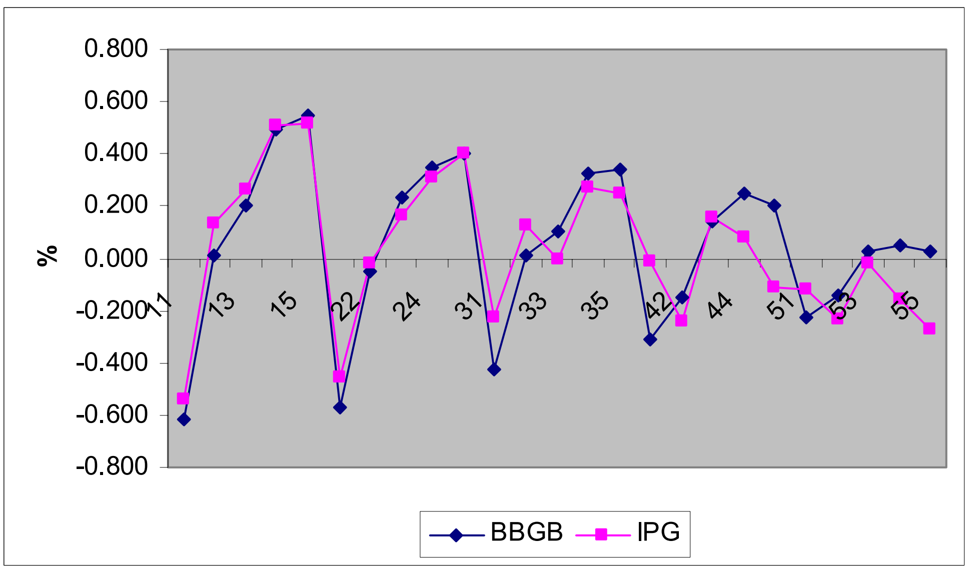
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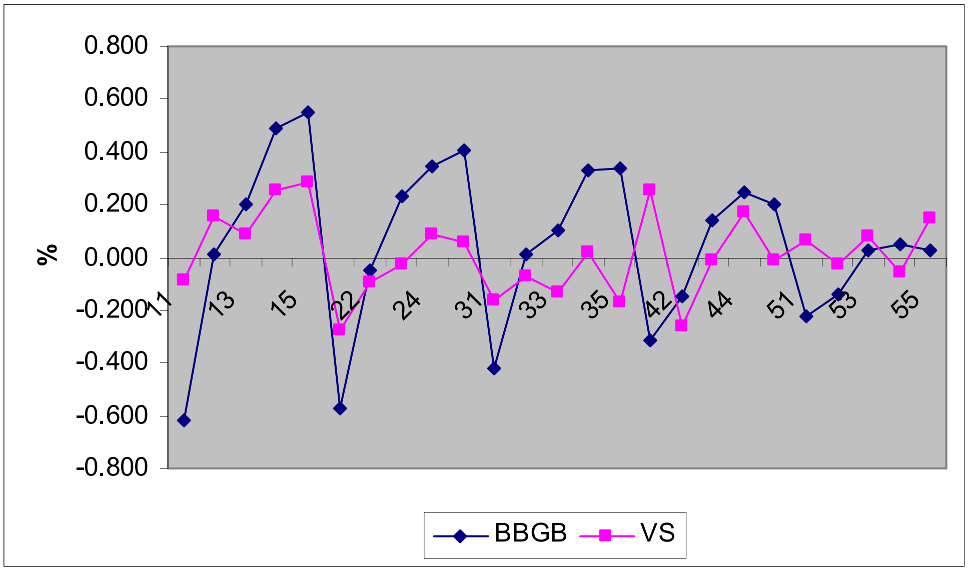
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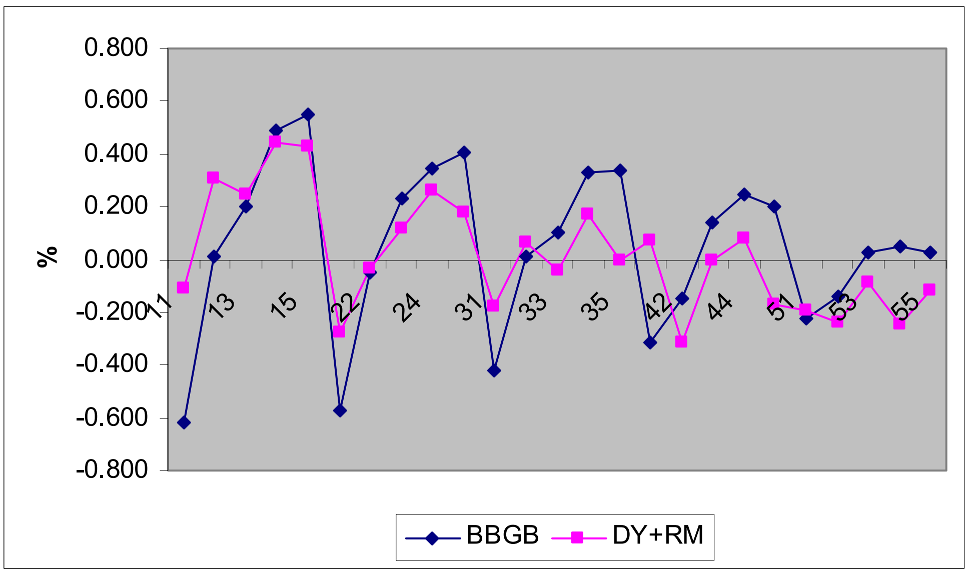
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BBGB
DY+RM

Figure 3

Estimating time-varying risk aversion

This figure plots time-series of the risk-aversion estimates from the ICAPM with time varying risk aversion. The models are the ICAPM scaled by the market dividend yield (DY), smoothed log earnings yield (EY*), value spread (VS), and market return plus dividend yield (DY+RM).
The test assets are the combination of the 25 size/book-to-market portfolios and the 38 industry portfolios. The estimates are obtained from first-stage GMM. The sample is 1954:08-2003:09. For further details, refer to section V of the paper.

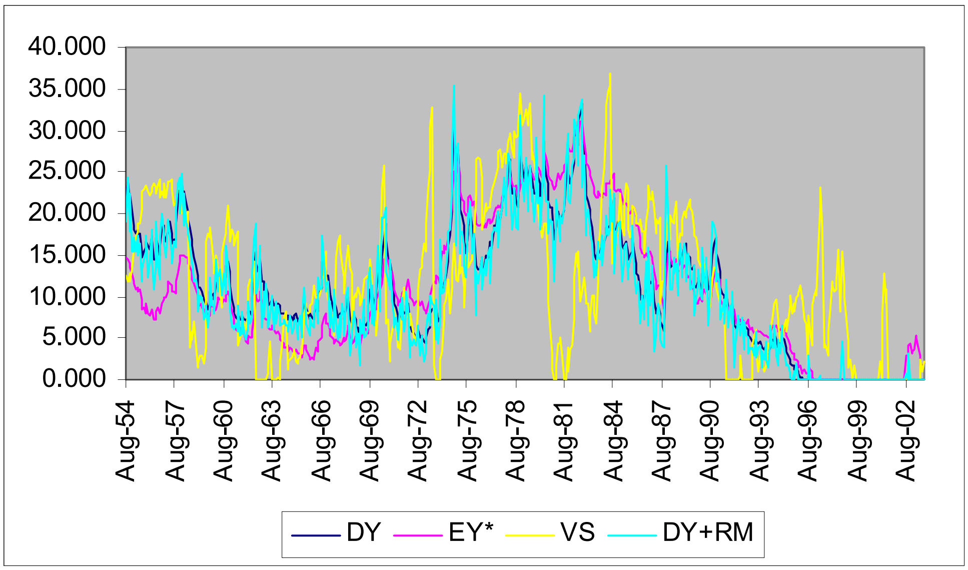
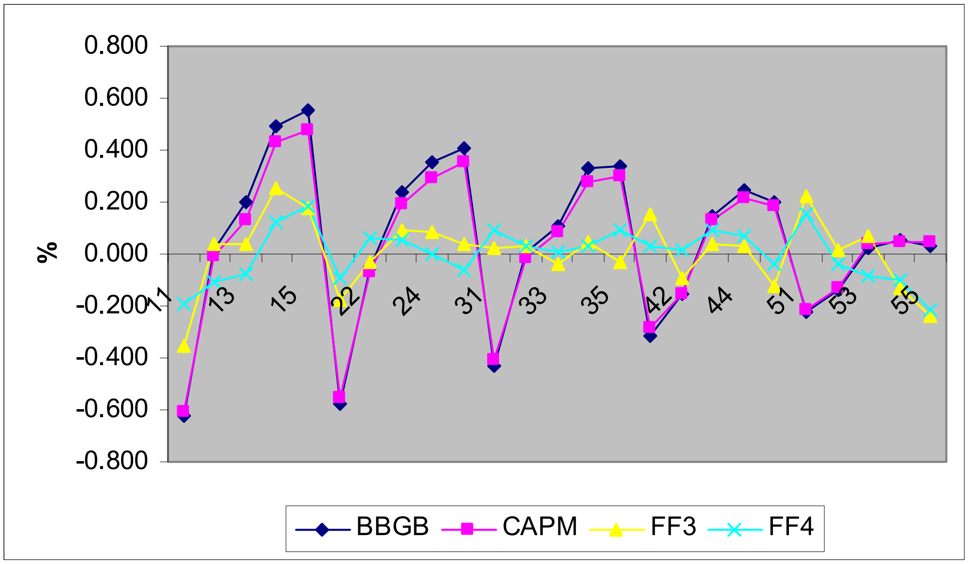
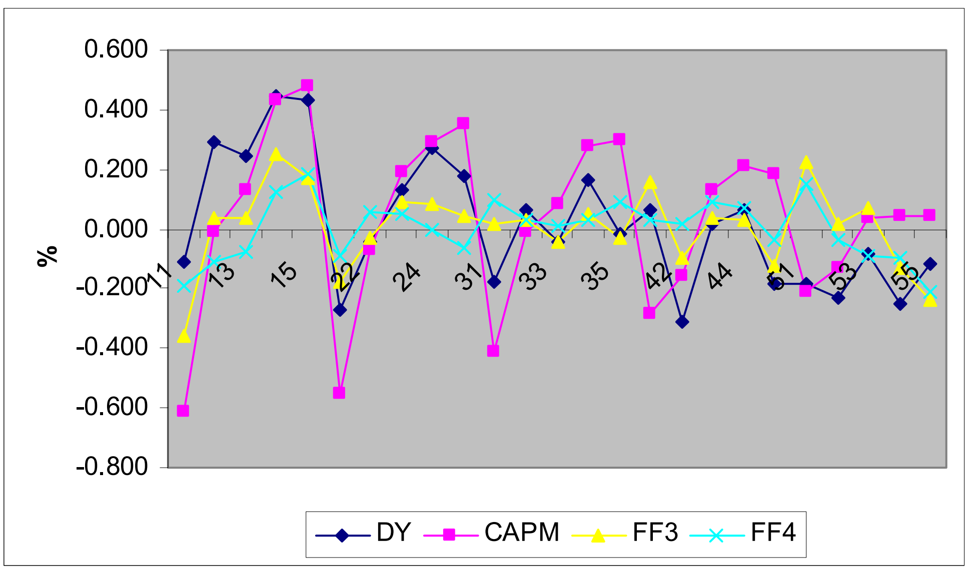


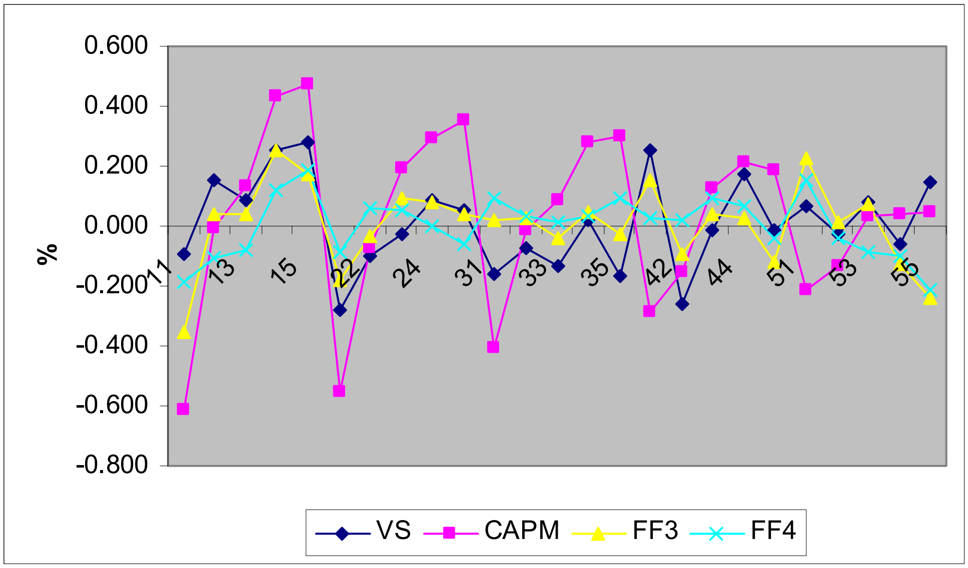
Figure 4

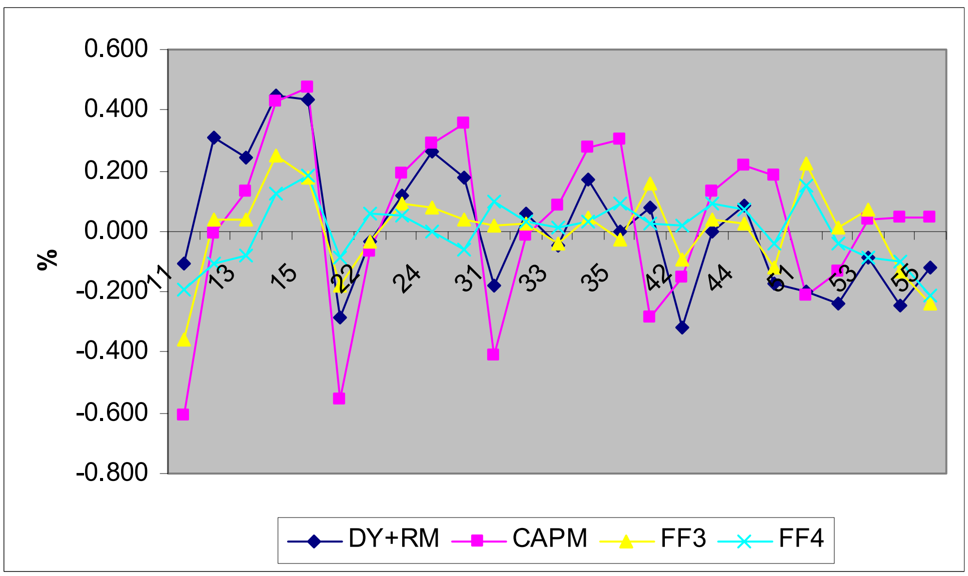
Average pricing errors: Comparison with alternative factor models

This figure presents the average pricing errors (stated in percentage points) across the book-to-market quintiles associated with the 25 size/book-to-market portfolios. The ICAPM models are the static ICAPM (BBGB), and the ICAPM scaled by the market dividend yield (DY), value spread (VS), and market return plus dividend yield (DY+RM). The alternative factor models are the CAPM, Fama-French 3 factor model (FF3) and Fama-French 4 factor model (FF4). The sample is 1954:08-2003:09. For further details, refer to section V of the paper.






0.600
0.400
0.200
0.000
-0.200
-0.400
-0.600
-0.800
%
11
13
15
22
24
31
33
35
42
44
51
53
55
VS
CAPM
FF3
FF4


0.600
0.400
0.200
0.000
-0.200
-0.400
-0.600
-0.800
%
11
13
15
22
24
31
33
35
42
44
51
53
55
DY+RM
CAPM
FF3
FF4