The behaviour of the real exchange rate: Evidence from regression quantiles*

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April 2006

Abstract

We test for mean reversion in real exchange rates using a recently developed unit root test for non-normal processes based on quantile autoregression inference in semi-parametric and non-parametric settings. The quantile regression approach allows us to directly capture the impact of different magnitudes of shocks that hit the real exchange rate, conditional on its past history, and can detect asymmetric, dynamic adjustment of the real exchange rate towards its long run equilibrium. Our results suggest that large shocks tend to induce strong mean reverting tendencies in the exchange rate, with half lives less than one year in the extreme quantiles. Mean reversion is faster when large shocks originate at points of large real exchange rate deviations from the long run equilibrium. However, in the absence of shocks no mean reversion is observed. Finally, we report asymmetries in the dynamic adjustment of the RER.

JEL classification: F31.

Keywords: real exchange rate; purchasing power parity; quantile regression.

*__Acknowledgments:__ This paper was partly written while Kleopatra Nikolaou was a visiting researcher at the European Central Bank. The author is indebted for useful conversations and constructive comments to Lorenzo Cappiello, Domenico Gianone, Lutz Kilian, Roger Koenker, Simone Manganelli, Lucio Sarno and Bas Werker. The author alone is responsible for any errors that may remain and for the views expressed in the paper.

1 Introduction

1.1 Real exchange rate issues and related literature

Purchasing Power Parity (PPP) has long been considered as one of the fundamental arbitrage laws in international asset pricing. The building block of PPP is the Law of One Price (LOP), which contends that, in the absence of arbitrage, identical goods should be selling at the same price across countries. Aggregating across all tradable goods in an economy, we obtain PPP, which suggests that price levels between two countries should be equal, if expressed in the same currency. PPP, therefore, provides an equilibrium relationship for the real exchange rate (RER), which is the nominal exchange rate, adjusted for relative price levels. If PPP holds, the relative price levels and/or the bilateral nominal exchange rate would adjust in such a way so that the RER remain constant. In that sense, variations in the RER would suggest deviations from PPP.

Although intuitive theoretically, PPP lacks strong empirical support. In practice, the RER exhibits high variability over time and spends long periods away from its suggested PPP equilibrium. The ambiguity surrounding the persistency of the RER and the validity of PPP, is well summarized into two relevant puzzles. The first one directly investigates the persistency of the RER process. As long as the RER is reverting back to its PPP equilibrium, albeit slowly, this implies that PPP should, at least, be seen as a long term anchor for determining the RER equilibrium value, although it may not be holding at each point in time. However, if the deviations from the PPP are permanent, this suggests the absence of a unique, constant equilibrium. The second puzzle (Rogoff, 1996) is trying to rationalise the persistence of the RER and reconcile its extremely volatile nature in the short run with the extremely slow rate at which shocks appear to damp out. This puzzle raises the issue of the types and role of the shocks that hit the RER and how they impact on the RER mean reversion[1].

Given the importance of PPP in international finance and our limited understanding of the RER behaviour, an extensive amount of research is being dedicated to testing the unit root hypothesis in the RER. Evidence from early attempts was clearly rejecting PPP (for a summary exposition of early tests see Sarno and Taylor, 2003). Nevertheless, it soon became obvious that standard unit root tests have low power in rejecting the null of a unit root. This shortcoming is nurtured by the inability of these tests to take into account certain distributional stylised facts of the exchange rates in general, and the

[1] Namely, it would be difficult to rationalise the short-run variability of the RER with reference to real shocks only, because they are not so frequent and, in any case, would tend to induce permanent deviations. On the other hand, it would also be difficult to attribute RER behaviour to the effect of nominal shocks, because their effect would be apparent for a short period of time, only (Rogoff, 1996).

RER in particular[2]. More precisely, although the true RER distribution of the RER is not known, the notion that it is normally distributed is refuted, because the overall process appears to be better described by leptocurtic distributions (McLachlan and Peel, 2000). The non-normality of the RER distribution confounds standard unit root tests, by lowering their power (Perron, 1990; Kim, Nelson and Startz, 1998).

Parametric unit root tests of increased sophisticated and complex structures, which accounted for the non-normality of the RER, offered more robust alternatives (Pippenger and Goering, 1993, 2000; Michael, Nobay and Peel, 1997; Nelson, Piger and Zivot, 2001). These tests would typically result from regime switching models, where the RER is allowed to display different behaviour and, therefore, assume different speeds of adjustment at the different states. Several models were competing for the choice of the "correct" switching function, based on theoretical considerations about the nature of forces driving the RER behaviour (Leon and Najarian, 2005).

A big strand of non-linear unit root tests argues in favour of a discrete or smooth adjustment towards the PPP equilibrium, consistent with the limits to arbitrage theory. The latter relates to the existence of trade barriers and transaction costs (Dumas, 1992), which induce different dynamic adjustment of the RER towards its long run mean for different magnitudes of RER deviations from the PPP equilibrium. In case of discrete transition functions, fixed arbitrage costs create an implicit inaction band, within which the RER can float freely. The implication is that in this regime it is possible to observe a random walk in the RER. On crossing this threshold, however, arbitrage forces ensure that the RER process becomes mean reverting. Such behaviour is captured by a Threshold Autoregressive (TAR) model (Tong, 1990). Empirical application of a TAR model (Obstfeld and Taylor, 1997; Sarno, Taylor and Chowdhury, 2004, Leon and Najarian, 2005), provides support for the theory of discrete adjustments towards the PPP equilibrium and, thus, offers evidence in favour of the PPP.

However, advocates of smooth adjustment (Teräsvirta, 1994; Dumas, 1994; Bertola and Caballero, 1990) suggest a Smooth Transition Autoregressive (STAR) model (Teräsvirta, 1994) as an appropriate formulation[3]. This model assumes no explicit threshold, rather the speed of RER mean reversion to its

[2] In order to overcome the low power problem, other strands of the literature adopted long span studies or panel unit root studies in linear settings (Abuaf and Jorion, 1990; Lothian and Taylor, 1996; Taylor, 2002). Although both methods provided supportive evidence of the PPP condition, it is still contentious whether favourable outcomes using these methods are enough to validate PPP (Frankel and Rose, 1996; Lothian, 1997; Taylor and Sarno, 1998; Sarno and Taylor, 2003)

[3] Kilian and Taylor (2003) argue that a smooth adjustment of STAR type can also be due to the interaction of heterogenous agents in the foreign exchange market, namely economic fundamentalists, technical analysts and noise traders. As long as fundamentalists disagree about the level of the RER equilibrium, the traders will tend to act on information from the technical analysts. The latter follow trending techniques, which impart a unit root behaviour in the RER. However, as fundamentalists agree that the RER is far from its equilibrium, the tendency for the RER to revert back to its equilibrium

long run equilibrium increases as the degree of misalignment from the PPP equilibrium increases. Further (simulation) analysis reveals that the mean reversion rate also varies with both the size of the RER shock and the initial conditions, that is the degree of RER disequilibrium when a given magnitude of shock hits the RER (Taylor, Peel and Sarno, 2001)[4]. Empirical applications of STAR model variants provide strong evidence of non-linear mean reverting behaviour for large deviations from the PPP equilibrium (Michael, Nobay and Peel, 1997; Taylor, Peel and Sarno, 2001).

Finally, a growing strand of literature is using Markov-Switching (MS) functions to model the behaviour of the RER. Such models allow for the distribution of the RER to be approximated as a mixture of normal distributions, and can, thus, permit changes in the speed of reversion, the mean and the variance of the RER process. Such models have been typically used for long-span data analysis, but RER applications with encouraging results are also found for the recent float (Leon and Najarian, 2005). The various regimes can depend on the deviation of the RER from its PPP equilibrium (Sarno and Valente, 2005), or the volatility of the RER shock (Engel and Kim, 1999) [5].

By allowing for different RER behaviour at the different states, the afore mentioned literature implicitly raised a further relevant question. This concerns the potentially different speeds of adjustment for positive or negative deviations of the RER from its PPP equilibrium, i.e. the possibility of asymmetric mean reversion towards the RER equilibrium. There is a considerable division of feelings in the literature over this issue, as theoretical and empirical arguments can be found in support for both sides. On the one hand, if goods arbitrage is driving the impetus back towards the long run PPP equilibrium, it would be difficult to explain why the speed of adjustment should be different above or below the equilibrium (Taylor, Peel and Sarno, 2001). That is because, the limits to arbitrage theory, which motivates the specification of the TAR and STAR models, relies on the existence of symmetric transactions costs, and would, therefore, also require symmetric adjustment above or below the PPP equilibrium (Obstfeld and Taylor, 1997; Michael, Nobay and Peel, 1997; Taylor, Peel and Sarno, 2001, Sarno, Taylor and Chowdhury, 2004; Sarno and Valente, 2005).

On the other hand, a more recent strand of literature suggests that the limit to arbitrage theory

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is increasing.

[4] Notably, Taylor, Peel and Sarno (2001) provide further insight into the mean reverting process of the estimated non-linear (STAR) model, through a dynamic stochastic simulation. This allows the analysis of impulse response functions, where arbitrary magnitudes of shocks are imposed to drive the RER away from its equilibrium, in order to study the mean reverting path back to it and calculate half lives of shocks. Their findings suggest that if a shock of a given magnitude hits the RER and drives it further away from the equilibrium, the larger the shock, the faster the RER mean reversion. In that case half lives can fall just under one year (10 months).

[5] An extensive amount of literature has found that MS models are suitable for modelling the exchange rate behaviour (e.g. see Engel and Hamilton, 1990; LeBaron, 1992; Engel, 1994; Dueker and Neely, 2005 and Sarno and Valente, 2006).

cannot alone explain the dynamics of the RER. They bring forward the role of central bank intervention as an underlying force affecting the dynamic adjustment of the RER. In this context, asymmetries may arise as a result of intervention policies directed at the RER. Almekinders and Eijffinger (1996), provide evidence in favour of asymmetries in the intervention policies of the US and the German central banks in the post-Louvre period, by showing that the banks tried to counteract appreciations of their currency more strongly that depreciations. Taylor (2004) shows that net intervention from the same central banks could stabilise the RER, with the effect becoming bigger, the bigger the deviations of the RER from its equilibrium value. On the same note, Dutta and Leon (2002) argue that governments might want to defend an appreciation of the currency more or less rigorously than a depreciation, therefore inducing asymmetric dynamic adjustment behaviour. Finally, Leon and Najarian (2005) provide direct empirical support for the existence of asymmetries in the RER mean reverting behaviour across a wide range of countries[6].

In this paper, we address the issues confounding previous PPP tests and also assess the symmetric properties of the RER mean reverting behaviour with the aid of the recently developed methodology of quantile unit root inference. Our unit root test adopts an agnostic approach towards the potential RER distribution and allows the RER to assume different speeds of adjustment at different states, while naturally revealing asymmetries in the RER mean reversion process. As a result, the quantile unit root test provides an alternative approach for robust unit root inference. Our method effectively addresses the two PPP puzzles and further refines and enhances previous results in the PPP literature.

1.2 The quantile approach to the PPP puzzles

We investigate three major currencies (UK pound, Japanese yen and Euro versus the US dollar) using a recently developed, unit root test for non-normal processes based on the quantile autoregression (QAR) approach in both semi-parametric[7] (Koenker and Xiao, 2004a,b) and non-parametric (Koenker, Ng and Portnoy; 1994) settings. By using the more robust quantile unit root alternative we aim to refine previous results and shed further light into the PPP puzzle.

Quantile regression estimation (Koenker and Basset, 1978) allows one to estimate and conduct inference on a whole range of conditional quantile functions, that is models where quantiles of the conditional

[6] Leon and Najarian (2005) adopt both a time-varying TAR model and a smooth transition (STR) model. In the first case, the magnitudes, frequencies and durations of the deviations of the RER from its forecast are allowed to differ for depreciations and appreciations. In the case of the STR model asymmetric adjustment is allowed for middle and outer regimes.

[7] The method is semi-parametric, in that it only assumes a linear relationship between the dependent and explanatory variables, without making any distributional assumptions.

distribution of the response variable are expressed as functions of explanatory variables[8]. By making no prior distributional assumptions, the quantile regression examines quantiles of the conditional distribution, in order to uncover different stochastic dependencies in the different quantiles. It, therefore, provides a more complete and nuanced picture of how covariates influence the location and shape of the entire response variable distribution (Koenker and Xiao, 2004a).

More specifically, we consider QAR models, where the autoregressive (slope) parameters may vary with quantiles. In the case of the RER, different solutions in distinct quantiles may be interpreted as differences in the mean reverting behaviour of the RER at various quantiles of the conditional distribution of the RER, that is at different magnitudes of RER shocks. In that case, bigger (positive or negative) shocks correspond to more extreme (high or low) quantiles[9]. As a consequence, the quantile unit root test is modified to incorporate the effects of various sizes of RER shocks (Koenker and Xiao, 2004b), and is, therefore, more robust compared to standard unit root models. Furthermore, QAR unit root inference can reveal different patterns of mean reverting behaviour for positive or negative shocks to the RER and, thus, naturally expose asymmetries in both the distribution of RER shocks and the impact of these shocks in the dynamic adjustment process of the RER to its long run equilibrium[10].

Seen in a different way, the linear QAR model captures state dependencies in a way comparable to, but different from a non-linear MS, TAR or STAR model. The linear QAR model adopts a different characterization of states, by allowing for multiple discrete regimes, which are chosen on the basis of the conditional distribution of the RER (i.e. RER shocks). This procedure can effectively expose transient and/or permanent states (i.e. quantiles) in the RER adjustment process, thereby presenting a more compete and nuanced picture of the RER dynamic behaviour. In this sense, the linear QAR model bodes well with the spirit of the aforementioned non-linear models.

Overall, QAR inference has significant advantages in analysing dynamics and persistence in time series with non-Gaussian distributions and can, thus, provide a more robust alternative to the standard unit root tests, while sacrificing little efficiency under normality (Koenker and Xiao, 2004a,b). In the context

[8] This is in sharp contrast with the traditional conditional mean estimation procedure, which assumes normality in estimating a single measure of the conditional mean function. In cases of Gaussian distributions, the latter estimation method would adequately describe the whole conditional distribution and would, in fact, enjoy a certain optimality. Moreover, the coefficients of the dependent variables would be independent of the specified quantiles.

[9] A more refined analysis of what exactly we mean by "mean reversion at the different quantiles" is offered in section 2.2.3.

[10] Although limit to arbitrage models typically impose uniform or symmetric behaviour (Obstfeld and Taylor, 1997; Sarno, Taylor and Chowdhury, 2004; Taylor, Peel and Sarno, 2001), evidence on non-linear asymmetric dynamic adjustment, due to government policies (i.e. intervention) has been recently emerging in the literature (Dutta and Leon, 2002; Leon and Najarian, 2005).

of PPP, the QAR approach provides an alternative, robust way of looking at the validity of the PPP, while addressing the question of wether different magnitudes of shocks may generate different (symmetric or not) persistency patterns on the RER. Our application is, to the best of the author's knowledge, the first contribution of quantile regression in this context.

1.3 Contribution, main results and structure of the paper

The QAR analysis provides original insights in the RER behaviour because of its general, yet flexible formulation. In contrast to previous, parametric designs, the quantile framework adopts a more general approach. It remains agnostic about the underlying distribution of the RER, and, consequently, in the treatment of the causes and specification of the dynamic adjustment of the RER to its long run equilibrium. In other words, we may obtain evidence of dynamic adjustment, consistent with the previous parametric (non-linear) literature, but without specifying the nature of the parametric (non-linear) relationship. In this way, the quantile approach is nesting assumptions and results from previous parametric models, in an a-theoretical way, thus circumventing the need to discriminate across different parametric model formulations.

The generality of the quantile model is well exploited by a flexible estimating framework, where the researcher is allowed to choose the quantiles under investigation, and, therefore, determine the level of detailed analysis that needs be undertaken. In the context of the RER, the above qualities allow insights into the following: a) We are able to detect how different sizes of shocks affect the RER speed of adjustment, (irrespective or not of the RER disequilibrium point when the shock hits the RER). The shocks analysed are actual, observed shocks, whose sizes are determined endogenously by the model. This offers an original view into the role of shocks on the RER and enriches anecdotal evidence from previous literature (Taylor, Peel and Sarno, 2001; Engel and Kim, 1999) b) The quantile method is able to reveal asymmetries in both the distribution of RER shocks and their impact on the RER mean reverting behaviour in a simple, intuitive and yet effective way. In this way, we shed more light to the relevant debate, by providing evidence using an original and relatively more simple model. c) As a result from the above, the quantile unit root test is a more robust alternative in cases of non-gaussian innovations, compared to standard unit root tests. Overall, the quantile analysis sheds light into the two PPP puzzles by further refining and enhancing results previously obtained by, amongst others, Taylor, Peel and Sarno (2001), Leon and Najarian (2005).

More specifically, our results suggest that the RER is not a standard linear stationary or a constant unit root process. Namely, we find that: a) the dynamic behaviour of the RER is affected by the magnitude of RER shocks, with large RER shocks undermining the unit root behaviour of the RER and inducing potentially strong mean reverting tendencies. Half lives in that case can fall well below one year. b) When large shocks to the RER originate at large RER disequilibrium levels (i.e. far away from its PPP equilibrium), the effect can be even stronger. c) On the contrary, small shocks to the RER considerably weaken mean reversion tendencies, irrespective of the disequilibrium point of the RER at the time of the shock. d) There are marked asymmetries in the behaviour of the RER, i.e. extreme positive shocks can generate different reversion patterns than extreme negative shocks. Their extent also depends on the original condition of the RER with respect to its long run equilibrium.

The paper proceeds as follows. Section 2 introduces the quantile regression techniques employed in this paper. Section 3 describes the data and some preliminary data analysis. Section 4 presents the empirical results from the semi-parametric and non-parametric quantile approach, and Section 5 concludes.

2 Methodology

In this section we present the QAR framework in both its semi-parametric and non-parametric settings. We begin with the simple linear QAR(1) model and explain the estimation and inference procedure (i.e. quantile unit root tests within each quantile), as presented in Koenker and Xiao (2004b). We then proceed to a basic exposition of the non-parametric quantile estimation technique (Koenker, Ng and Portnoy, 1994).

The semi-parametric and non-parametric settings correspond to a general and a more refined analysis of shocks respectively. In the general (semi-parametric) analysis we consider different magnitudes of RER shocks in a linear QAR context in order to investigate their impact on the mean reversion of the RER. This methodology allows different speeds of adjustment for different magnitudes of RER shocks. However, this analysis does not consider the origin of the shock , i.e. the deviation of the RER from its RER long-run equilibrium when the shock occurs[11]. The limit to arbitrage theory offers plausible support for such considerations. We, therefore, further refine our results with a non-parametric quantile

[11] Note that there is an important difference between RER shocks and RER deviations from equilibrium. A shock hits the RER at a time t and has an observable impact on the RER at time $t+j$. A shock is equal to a RER deviation if they both occur at the same time interval studied and if the shock originates at equilibrium. However, shocks conditional on the past history of the RER can occur at any point of the RER distribution with respect to the equilibrium (i.e. can occur when the RER is below or above its long run equilibrium). Because of that, RER deviations can be the additive result of cumulative shocks to the RER and the two expressions are no longer tautologous. Overall, the effects of shocks on the RER can be variable, depending on the magnitude of the shock and the RER disequilibrium position at the impact.

model. In that context, we observe patterns of RER behaviour, which are identifiable primarily by the magnitude of shocks (size), but also by the level of RER disequilibrium when the shock occurred (origin). We can, therefore, gauge results about different speeds of adjustment when shocks of given magnitude hits the RER on, below or above its equilibrium.

2.1 Semi-parametric QAR model

Our semi-parametric analysis is founded on the recent extension of the theory of quantile regression to autoregressive models, which resulted in the linear QAR model (Koenker and Xiao, 2004b). We use a linear QAR estimation framework on the deviation of the real exchange rate from its equilibrium value and perform different quantile unit root tests in order to gain a more refined view of the RER dynamic behaviour.

2.1.1 Estimation of the QAR model

Let us consider a simple first order autoregressive, AR(1), model of the type

$$y_t = \alpha y_{t-1} + \varepsilon_t, \tag{1}$$

where $y_t = q_t - \mu$, with q_t denoting the logarithm of the RER and μ being the long run equilibrium level of q_t, i.e. the unconditional mean of q_t. Following the standard literature, the RER is defined as $q_t \equiv s_t - p_t + p_t^*$, where s_t is the logarithm of the nominal exchange rate (domestic price of foreign currency) and p_t and p_t^* denote the logarithms of the domestic and foreign price levels respectively. Hence, y_t represents the deviations of the real exchange rate from its equilibrium value. Finally, ε_t is an error term. In this traditional conditional mean function, standard unit root theory suggests the existence of a unit root in the RER, if the autoregressive coefficient, α, equals unity. In that case, deviations from the long run RER equilibrium are permanent. However, if the autoregressive coefficient is smaller than unity, the real exchange rate is a stationary process, suggesting that any deviations from equilibrium are transitory.

Following the methodology set out by Koenker and Xiao (2004b), the equivalent τ^{th} quantile representation takes the form:

$$Q_{y_t}(\tau \mid y_{t-1}) = Q_{\varepsilon_t}(\tau) + \alpha(\tau) y_{t-1}, \tag{2}$$

where $Q_{y_t}(\tau \mid y_{t-1})$ is the τ^{th} conditional quantile of y_t, conditional on y_{t-1}, and $Q_{\varepsilon_t}(\tau)$ is the τ^{th}

conditional quantile of ε_t. In other words, the τ^{th} conditional quantile function of the dependant variable y_t is expressed as a linear function of its own lagged value. $\alpha(\tau)$ is the autoregressive coefficient, which measures the persistence of the real exchange rate deviations within each quantile and is dependent on the τ^{th} quantile under investigation.

Estimation of the linear QAR model involves solving a minimisation problem of weighted residuals, where all the observations are considered, but are being weighted in such a way, so that the residuals fall into the selected quantile:

$$\min_{\alpha \in \mathbb{R}^2} \sum_{t: y_t \geq x_t^\intercal \alpha} \rho_t(y_t - x_t^\intercal \alpha(\tau)), \tag{3}$$

where $\rho_t(\varepsilon) = \varepsilon(\tau - I(\varepsilon < 0))$ is a check function with I denoting an indicator taking the value of 1 if the expression in parentheses is true and 0 otherwise, $x_t = (1, y_{t-1})$ and $\alpha(\tau) = (Q_\varepsilon(\tau), \alpha(\tau))$. Thus, equation (3) is equivalent to:

$$\min_{\alpha \in \mathbb{R}^2} \sum_{t: y_t \geq x_t^\intercal \alpha(\tau)} \tau(y_t - x_t^\intercal \alpha(\tau)) + \sum_{t: y_t < x_t^\intercal \alpha(\tau)} (\tau - 1)(y_t - x_t^\intercal \alpha(\tau)). \tag{4}$$

In our case the QAR model was estimated in the "quantreg" package included in R, using a modified simplex algorithm of Barrodale and Roberts (Koenker and d'Orey, 1987, 1994). This package offers the possibility to estimate a whole range of conditional quantile functions and computes bootstrapped standard errors for the parameters. In our case the number or replications employed were 2000.

2.1.2 Quantile unit root tests

A general analysis of the unit root behaviour based on the quantile approach involves examining the unit root property over a range of quantiles. The relevant statistic for testing the null of a constant unit root process over a range of quantiles is a Kolmogorov-Smirnov (KS) test based on the regression quantile process over a range of $\tau \in T$. Koenker and Xiao (2004a,b) suggest

$$QKS = \sup |t(\tau)|, \tag{5}$$

where $t(\tau)$ is the t-statistic of the autoregressive coefficient at the τ^{th} quantile. In practice, we may calculate $t(\tau)$ at $\tau \in T$ and construct the QKS statistic by taking the maximum statistic value over $\tau \in T$. The limiting distribution can be approximated by resampling methods, as explained below.

A more detailed examination of the unit root properties of the series is by examining the unit root property in each quantile separately. This allows for a closer look at the dynamics of the series and also permits the detection of possible asymmetries in the process. The relevant unit root test involves a simple t-statistic test, $t(\tau)$ for the null of a unit root. In other words, we are testing that the autoregressive coefficient in the specific quantile, $\alpha(\tau)$, will be equal to unity. Given that $\alpha(\tau)$ depends on τ, it is possible to have different mean reverting behaviour in the different quantiles. This implies that it is possible to observe sequences of innovations that reinforce the unit root behaviour of the series, followed by occasional realisations that induce mean reversion and thus undermine the persistency of the whole process.

For both types of tests we base our inference on a resampling (bootstrap) exercise, as described by Koenker and Xiao (2004b)[12], which was coded in R. The main idea of this exercise is to generate a distribution for the relevant statistic values and observe where our actual statistic values lies with respect to the bootstrapped distribution. For this purpose, we construct dependent variables (y_t) under the null of a unit root in the RER data generating process, by resampling from the original data. We then estimate the same quantile regression specification under the null and get the relevant t-statistic values. We repeat this procedure 2000 times. We, thus, create the distribution of the $t(\tau)$ test and generate the distribution of the QKS. We then compare the statistic value of the original (true) regression with the distribution under the null (of a unit root). The percentage amount of times that the statistic value will be above the bootstrapped statistic value gives us the probability of rejecting the null hypothesis of a unit root, within each quantile. In this study, we investigate a range of quantiles for $\tau = (0.01, 0.05, 0.1, 0.25, 0.5, 0.75, 0.9, 0.95, 0.99)$[13].

Koenker and Basset (2004b) by means of a Monte-Carlo analysis, compare the power of the OLS and QAR models for the case of Gaussian and Student-t innovations. Their results show that the quantile-based tests have superior power than the simple Augmented Dickey-Fuller (ADF) and Phillips-Perron tests in cases of Student-t innovations. In turn, the $t(\tau)$ test has more power than the QKS test, albeit marginally.

2.1.3 Interpretation of the quantiles and quantile mean reversion

In order to interpret our results from the QAR model, it is important to consider first the meaning of each quantile, i.e. what exactly the quantiles capture, and second the meaning of quantile mean reversion. As regards the first issue, looking at the QAR specification and the estimation procedure (equations 2-4), it becomes obvious that the quantile approach estimates quantiles of the conditional distribution of the

[12] Methods of asymptotic inference are also available for the $t(\tau)$ test. The asymptotic distribution is not the conventional Dickey–Fuller distribution, but rather a linear combination of the Dickey–Fuller distribution and the standard normal.

[13] For an explicit technical description of the procedure see Koenker and Xiao (2004b).

RER, conditional on its own past values, i.e. it estimates quantiles of the error term. Therefore, in the simple case of a QAR(1) model the quantiles capture the magnitude of shocks from period $t-1$ to period t[14]. That is, one-off shocks of similar magnitude, which are classified as falling into the same quantile are, in effect, the shocks that determine the fit of this quantile. The magnitude of these shocks is summarised by the constant term, $Q_{\varepsilon}(\tau)$. Therefore, the more extreme the quantile the more extreme the shocks that hit the RER in the same quantile.

The quantile methodology has the potential to reveal different localised mean reverting patterns, by explicitly testing for a unit root at the different quantiles (i.e. locally). More specifically, RER mean reversion at a specific quantile suggests that shocks of similar magnitude, that fall into this quantile, tend to undermine the persistency of the series and induce mean reversion tendencies on the RER. On the contrary, unit root behaviour within a quantile suggests the existence of innovations of a certain magnitude, which reinforce the persistency of the RER. It is, therefore, possible for a series to exhibit localised unit root behaviour (i.e. unit root in certain quantiles), followed by mean reverting occasions (i.e. mean reversion in other quantiles) capable of inducing stationarity in the overall process (i.e. globally).

2.2 Non-parametric QAR estimation

In the next step of our analysis we move to a non-parametric QAR framework. Namely, we investigate if the impact of different magnitudes of RER shocks is further affected by initial conditions (i.e. the level of RER disequilibrium when the shock hits the RER). According to the limits to arbitrage argument, should large deviations from the PPP equilibrium affect mean reversion, then the linear fit should not be a good approximation of the quantile process and instead we should observe kinks (i.e. different slopes) in each of the different quantile fits. We, therefore, employ a non-parametric model in an effort to allow for a more flexible functional form within each quantile compared to a semi-parametric one. Our aim is to expose distinct linear sub-segments, i.e. linear sub-segments with different gradients, within each quantile.

The preferred non-parametric estimation technique, is the method of quantile smoothing splines with total variation roughness penalty (Koenker, Ng and Portnoy, 1994). If y, x and τ are defined as above, the idea underlying this method is to derive the quantile smoothing spline estimator of $g_{\tau}(x)$, as the solution to a trade-off problem between "fidelity" and "roughness", i.e. between a fit the bears a reasonable degree of fidelity to the observed points and a fit with a plausible degree of smoothness:

[14] In a case of a higher order QAR model, of the type specified by Koenker and Xiao (2004a,b), they would capture the cumulative effect of the $t-n$ periods to period t, where n is the number of lags allowed for.

$$\min_{g} \text{"}fidelity\text{"} - \lambda \text{"}roughness\text{"} \tag{6}$$

where,

$$\text{"}fidelity\text{"} = \sum_{y_i - g(x_i) \geqslant 0}^{n} \tau(y_i - g(x_i)) + \sum_{y_i - g(x_i) < 0}^{n} (\tau - 1)(y_i - g(x_i)) \tag{7}$$

and

$$\text{"}roughness\text{"} = V(g'). \tag{8}$$

Therefore, the quantile smoothing spline estimator is the solution to

$$\min \sum_{i=1}^{n} \rho_\tau(y_i - g(x_i)) - \lambda V(g') \tag{9}$$

where g is a smooth function with a uniformly continuous first derivative g' and bounded second derivative g'' . In our approach, lambda (λ) penalises the total variation of function g', which we denote as $V(g')$, with $V(g') = \int_a^b |g''(x)|\, dx$. λ is a regularisation parameter, or the roughness penalty, that balances the trade off between fidelity and roughness and therefore determines the smoothness of the fitted function. As λ increases the penalty prevails until, for very high values of λ, the roughness penalty is maximised and we get a perfectly smooth line, matching the semi-parametric linear fit. The solutions are piecewise linear functions with knots at x_i (Koenker, 2005).

The estimation techniques for this type of non-parametric fit depend on the dimensionality of the vector of conditioning variables x. Our QAR(1) case corresponds to a univariate case of non-parametric smoothing and for this purpose the quantile model was estimated using the COBS (Constrained B-Splines Smoothing) algorithm of He and Ng (1999)[15].

The quantile smoothing spline methodology is appealing in our case, both technically and intuitively, since it provides a direct comparison with the semi-parametric linear fit, while also allowing a role for the limits to arbitrage theory. Namely, in each quantile we are testing the robustness of the linear fit, thereby investigating the validity of the limits to arbitrage argument. In the quantiles where deviations from the RER do not affect the mean reversion properties of the RER, the graphical results should deliver the same linear quantile fit found using the semi-parametric methodology. In the opposite case, however, where large RER deviations from its equilibrium value induce mean reversion, the linear quantile fit

[15] The COBS package in R permits the implementation of this algorithm and also enables the calculation of confidence intervals, based on the asymptotic results of He and Shi (1998).

should change to a piecewise linear quantile fit. If the limits to arbitrage theory is supported by our data, we would expect to find sub-segments with less than unity slope at the left and right hand side of a particular quantile, while the middle part could preserve a unity slope. Such a result would suggest that, when a given magnitude of shock is originated at a high disequilibrium level, the mean reversion of the RER is stronger.

3 Data and preliminary data analysis

The data sources used to construct our RER data set are the International Monetary Fund (IMF)'s, International Financial Statistics (IFS) and the Organisation for Economic Co-operation and Development (OECD)'s, Main Economic Indicators (MEI). The countries analysed include the euro area, the UK and Japan with the US as the reference country, for a period from January 1973 to December 2004. For each country, we obtained the relevant nominal bilateral (end-of-period) exchange rates vis-à-vis the US dollar. These were the euro (EU), the UK pound (GBP) and the Japanese yen (JY) denominated in US dollar (USD) terms. In order to prolong the EU nominal exchange rate series, euro-dollar values before the introduction of the euro were proxied with Deutsche mark-dollar data. CPI (total index) monthly data for the five countries were collected from MEI. The final times series - monthly RER (deviations from the long run mean) in logarithmic terms (y_t) - were constructed following the methodology in Section 3.1.1..

As a preliminary exercise we compared the sample moments of the RER deviations of the three exchange rates in question, and performed normality tests (Table 1). For the individual series in levels, the summary statistics confirm evidence of leptokurtosis and non-normality. The formal Jarque-Bera test rejects normality in every case, adding support for using quantile regression.

4 Empirical results

In this section we report estimation results from the semi-parametric linear QAR model and the non-parametric quantile smoothing method. Results are complemented with calculations of the relevant half lives. The semi-parametric method provides some evidence of mean reversion across a range of quantiles. A more detailed and instructive view is taken by focusing on the specific quantiles, where the mean reversion becomes much stronger in the extreme quantiles (i.e. for extreme RER shocks). The non-parametric test further reveals that the behaviour in each quantile is exacrabated when extreme shocks combine with extreme deviations from the RER long run equilibrium.

4.1 Estimation, unit root tests and half lives

As a first step, we had to choose the order of the AR process. Towards this, we followed previous practices from Granger and Teräsvirta (1993), Teräsvirta (1994) and Taylor, Peel and Sarno (2001) and focused on the partial autocorrelation function. In our case (results not reported, but available upon request) this analysis clearly reveals that only the first partial autocorrelation coefficient is significantly different from zero at the five percent level. Overall, in all cases a simple AR(1) model sufficiently captures the dynamics involved. We enhance this result with a test for residual correlation (F_{RS} test in Table 2, Panel A), where we find that we can reject the hypothesis of serial correlation at the five percent level for the AR(1) specification. We then estimate a conventional conditional mean specification, i.e. a simple AR(1) model using OLS, and a QAR(1) model for $\tau = (0.01, 0.05, 0.1, 0.25, 0.5, 0.75, 0.9, 0.95, 0.99)$. For both specifications we performed (quantile) unit root tests (Table 2, Panels A and B). Our analysis is completed with the estimation of half lives[16] for both the AR(1) and the QAR(1) models (Table 3, Panels A and B).

4.1.1 Conditional mean (OLS) specification results

The naive conditional mean estimate of the autoregressive coefficient in the AR(1) model confirms the stylised facts of a unit root in the RER, with all estimated coefficients very close to unity and the relevant unit root tests in levels suggesting that the coefficient values are not statistically different from unity at the five percent level of significance. Overall, evidence from the two unit root tests employed, the Phillips-Perron (1988) and the Ng and Perron (2001) tests, on the levels and first differences indicate that, while changes in the real exchange rate are stationary, the level of the real exchange rate contains a unit root. These findings replicate well established results in the literature.

4.1.2 QAR specification results

We now reconsider these series using quantile unit root tests. In particular, we first apply the QKS test based on the QAR model for a range of quantiles $T = (0.01, 0.99)$. This test is gives us a general idea of the unit root behaviour of the series in question. Results are reported in Table 2, Panel B. Contrary to the conventional unit root tests presented above, QAR unit root tests provide some evidence in favor of mean reversion for the GBP and the EU, at the 10% significance level. For the JY, we cannot reject the null of a unit root. This is not overall unexpected for the JY, which has, in fact been notorious for such type of

[16] Half lives for a simple AR(1) model are computed based on the formula $\log(0.5)/\log(a)$, where α is the autoregressive coefficient under consideration.

behaviour. This could be the result of the Japanese catching up after the WWII, creating productivity differentials which determine the long run equilibrium of the RER (Harrold-Balassa-Samuelson effects).

By and large we find the comparison between the two sets of results encouraging. For a more refined investigation, we turn our attention towards the behaviour of specific quantiles (Table 2, Panel B). The first striking observation is a varied behaviour across the different quantiles, both for the intercept (τ_0) and the autoregressive coefficient ($\alpha(\tau)$). As noted above, the intercept captures the magnitude of the typical, observed RER shock in each quantile (negative signs suggesting negative shocks, loosely interpreted as appreciations and positive signs suggesting positive shocks, loosely interpreted as depreciations). The τ_0 coefficients present a monotonically ascending, symmetric behaviour, i.e. the absolute magnitudes of positive and negative RER shocks are quite similar for a given set of complementary (symmetric) quantiles (e.g. the 1% and the 99% or the 25% and 75% quantiles), therefore the magnitudes of shocks hitting the RER appear to be symmetric. We also observe that the magnitudes of shocks are similar across currencies, with the biggest shocks in absolute value deviating from the long run equilibrium by approximately 0.035 log units.

However, the most interesting results are the values of the autoregressive (slope) coefficients $\alpha(\tau)$ and the relevant unit root tests in the QAR(1) model, which determine the mean reverting behaviour of the RER in each quantile. A careful look reveals a distinct pattern and gives clear support for mean reversion in certain quantiles and unit root behaviour in others. In particular, in the middle quantiles we observe coefficient values very close to unity (above and below), and in fact not different from unity in statistical terms, suggesting a unit root in the RER[17]. However, in the extreme quantiles coefficients appear to be lower with p-values rejecting the null of a unit root in conventional significance levels, suggesting that the persistence in the RER drops. A graphical representation of the above results is produced in Figure 1, where the values of the autoregressive coefficient for the different quantiles are displayed. It is possible to see an inverse U-shaped pattern, suggesting that the coefficient is smaller in the extreme quantiles than in the mean quantiles. This heterogeneity in the slope coefficients suggests a dynamic adjustment towards the long-run PPP equilibrium. In fact, our main conclusion is that in the presence of small and medium shocks, the RER does not adjust towards its PPP equilibrium value, but extreme shocks seem to have the potential to induce mean reversion. These results are in line with evidence from Taylor, Peel and Sarno (2001) and also relate to relevant evidence from Engel and Kim (1999).

Our results from the $t(\tau)$ test identify that the series under consideration are not constant unit root

[17] It is also worth noting that the estimated autoregressive coefficient (α) in the conditional mean model assumes values very close to the conditional median quantile estimates.

processes. Nevertheless, the inconsistency with the QKS test, for the case of the JY raises questions about the global properties of this series. This inconsistency could be due to the lower power of the QKS test. However, our series do not follow a Student-t distribution, therefore evidence on the comparison of the two unit root tests in terms of power (Koenker and Xiao, 2004b) is weak. Overall, our results suggest that there are cases (i.e. quantiles) where the RER is mean reverting, and these tend to be cases where big shocks hit the RER (i.e. the most extreme quantiles). We can say with some certainty (90%) that this is enough for the whole process to revert back to its long run mean (apart from the case of the JY).

Turning our attention to the estimated half lives (Table 3, Panels A and B), in the simple AR(1) model, half lives are equal to infinity, because a unit root behaviour dominates the results. However, in the QAR(1) model, for the mean reverting quantiles we get different results. Namely, in the very extreme quantiles (99%) we get surprisingly low half lives, ranging from 5 to 8 months. Half lives increase, but still remain quite low in the 95% quantile, ranging from 10 to 14 months and only in the 90% quantile we can see half lives of more than one year. These findings are well below the four year average suggested by Rogoff (1996) and also below the findings of the non-linear literature.

We, therefore, see that the simple linear quantile model, in its ability to conduct analysis on the different magnitudes of RER shocks, can give signs of mean reversion at the different quantiles, consistent with the PPP.

4.1.3 Asymmetric dynamics

It is interesting, however, to note that only in the case of the GBP this effect appears symmetric, i.e. the RER is a less persistent process in both extreme positive and negative shocks (although more so for extreme positive shocks). In the case of the EU and the JY the RER appears to be mean reverting only in cases of extreme positive shocks. In statistical terms, this asymmetry suggests a shortage of extreme values in the low or high quantiles with the potential to induce mean reversion. This might be the case either because extreme shocks do not occur or because the shocks of different signs weight differently. Given the symmetric magnitudes of shocks, as reported from the constant term values (τ_0), it is more plausible to assume the latter explanation. A potential reason for such asymmetries might lie on monetary policy choices and official intervention that impact asymmetrically on exchange rates (Dutta and Leon,2002; Leon and Najarian, 2005). Overall, the semi-parametric quantile analysis suggests that, although the shocks that hit the RER are of symmetric magnitudes, they impact asymmetrically on the mean reversion of the RER.

4.2 A graphical representation of the RER behaviour

A closer look at Figure 2, should provide a clearer intuition on the focus and results of the QAR(1) model. In the graph we plot the realisations of the RER on the lagged value of the RER for the GBP. Given that our data are monthly, the graph plots the realisations (dots) of the UK pound RER this month against its value in the previous month. The straight diagonal line is the 45 degree, $x = y$ axis, suggesting that the RER has not changed since last month, i.e. implying an autoregressive coefficient of unity and therefore a unit root process. All the dots above the diagonal line suggest negative shocks (depreciations) to the RER, because a deviation at time t is followed by a bigger deviation at time $t + 1$. Alternatively, all realisations below the diagonal suggest positive shocks (appreciation), because a deviation at time t is followed by a smaller one at time $t + 1$.

A first look at the realisations can give a deceptive unit root impression, since most realisations lie across the diagonal line. A closer look, however, reveals different patterns, namely that the centre of the graph is more dense, i.e. most realisations lie close and around the long run equilibrium, whereas the tails of the unconditional distribution (top right hand and bottom left hand corner) are not only more sparsely populated but also have relatively bigger deviations from equilibrium, i.e. there appear to be either large positive or negative shocks.

The dotted line is the mean (OLS) and the dashed line is the median (50% quantile) fit. It is obvious that the slopes of both lines are very similar to each other and are, in fact, difficult to discern from the diagonal (long-slash) line, suggesting that the OLS and median quantile outcome will favor a unit root behaviour. However, the image changes when we look at the outer slashed lines, which are the fits on the 1% quantile (lowest line) and the 99% quantile (highest line), representing extreme negative and positive shocks to the RER. In our case, the slopes of the extreme quantile fits are definitely smaller than the slope of the diagonal, suggesting that extreme shocks tend to induce mean reversion in the RER.

Finally, it is important to note that, for a given quantile, the slope that represents the RER adjustment process, is determined by RER realisations that are close to (points in the middle part of the quantile fitted line) or far away from (points at the two ends of the quantile fitted line) the PPP equilibrium. That is, each quantile fit depends on realisations (shocks) that hit the RER at various RER points with respect to its PPP equilibrium. A linear fit suggests that it is only the magnitude of the shocks and not the original conditions of the RER at the time of the shock that affect the fit. However, the limit to arbitrage theory suggests that original conditions can impact on the mean reversion of the RER. In order not to ignore potentially richer dynamics, that might result when a shock occurs far away from the

PPP equilibrium, we accommodate such considerations in the non-parametric part of our analysis.

4.3 Non-parametric results

In this section we present the results of the piecewise linear fit, obtained by non-parametric quantile smoothing, using total variation regularisation, following the methodology set out by Koenker, Ng and Portnoy (1994). A graphical representation of the results is presented in Figure 3, Panels A to C. For each currency we impose the 1%, 50% and 99% quantiles of the piecewise linear fit on a line with unity slope and a constant equal to the respective quantile τ_0 coefficient, so that any discrepancy between the unit root case and the non-parametric fit is easier to detect. Figure 4, Panels A to C graphically presents the various slopes in the individual quantiles under consideration, corresponding to the piecewise linear plots in Figure 3. Finally, the analysis is complemented with Table 3, Panel C, where we show the slope coefficients and the relevant half lives for each subsegment of the piecewise linear fit.

Looking at the extreme quantiles we observe distinct departures from the linear QAR model. The multiplicity of linear sub-segments within the same quantile stresses the difference between the semi-parametric and the non-parametric method and, moreover, offers support to the limits to arbitrage theory. A careful look will reveal that the left and right end of the 99% and 1% quantiles are, in the majority of cases, associated with strong mean reverting RER behaviour for all currencies involved. Figure 4 and Panel C of Table 3 gives ample support to that observation, with half lives recording very fast mean reversion, as low as 1.3 months (GBP in 1% quantile). This outcome is much stronger compared to the previous results of the literature, and even stronger than our results in the semi-parametric model. For the middle part of the extreme quantile fits, however, we get evidence of an autoregressive coefficient close to unity in most cases. In line with the limits to arbitrage argument, evidence from the extreme quantiles suggests that large shocks, which originate at large disequilibrium levels, tend to induce strong RER mean reversion. Mean reversion tendencies in the presence of large shocks are much weaker around the RER long run equilibrium.

A quite novel insight comes from looking at the behaviour of the median quantiles, which differs significantly from the one mentioned above. In the median (50% quantile) the fit appears to be the same as in the linear case. Note that in the median quantiles the shock to the RER is minimal. This leads us to conclude that, in the absence of shocks, the dynamic behaviour of the RER is not affected, irrespective of the RER deviation from the equilibrium.

As regards asymmetric dynamic adjustment patterns, compared to the linear, semi-parametric QAR model, we find evidence of mean reversion in both extreme quantiles for all currencies, although by no means exactly symmetric. However, a more careful examination reveals a pattern. Asymmetries in the adjustment dynamics of the RER are more pronounced when large shocks hit the RER at points far away from its equilibrium. Asymmetries become less pronounced, or even disappear when large shocks hit the RER near its equilibrium value. Finally, in the absence of shocks, for any disequilibrium level, we cannot establish asymmetric dynamic adjustment patterns.

By and large, our results in this section offer support to the limits to arbitrage theory, put forth by non-linear TAR and STAR methodologies. In the mean time, by taking into account both the effect of different magnitudes of RER shocks and the original disequilibrium condition of the RER we manage to find half lives significantly smaller compared to the previous literature. Overall, our results suggest the following about the driving forces behind the RER mean reverting behaviour: a) When a big shock hits the RER at a point already far from its equilibrium level, this shock tends to induce mean reversion. b) Big shocks that originate at points near the PPP equilibrium have much reduced mean reversion abilities. c) Small shocks either around or away from the RER equilibrium do not appear to induce mean reversion. d) We find asymmetries in the adjustment dynamics of the RER when large positive or negative shocks of the same absolute magnitude hit the RER at large disequilibrium points. Asymmetries become less pronounced when a big shock hits the RER near its equilibrium value or in the absence of shocks, despite the disequilibrium level of the RER.

5 Conclusions

This paper elaborates on the long standing PPP puzzles. Earlier literature seeked answers by employing unit root tests with different levels of sophistication. Amongst those, the ones which accommodated the non-Gaussian behaviour of the RER, seemed to have better power in detecting reversion towards the PPP equilibrium. In this paper, we present QAR semi-parametric and non-parametric methods as an alternative approach for robust inference in non-gaussian series. The quantile approach adopts an agnostic and yet flexible framework for the analysis of the RER behaviour, thus sidestepping the need to specify theory-consistent driving forces of the RER dynamic adjustment process. More precisely, the quantile framework makes no assumptions about the underlying distribution of the RER, while allowing for different (symmetric or asymmetric) persistence patterns at the different quantiles. It this sense, it is possible to observe sequences of unit-root behaviour, while occasional mean reverting tendencies can undermine the persistence of the whole process. By taking into account the different adjustment processes at the different quantiles, the quantile approach offers a more robust unit root test than standard alternatives.

More importantly, the QAR analysis and inference sheds light into both PPP puzzles. As concerns the first one, our methodology offers some support for the PPP, by providing evidence in favour of a mean reversion in the RER from two different quantile unit root tests. Our approach also addresses the second PPP puzzle by undertaking a detailed analysis of the impact of different magnitudes of actual shocks on the RER. We rationalise the high persistence of the RER behaviour, by suggesting that different magnitudes of shocks can induce different speeds of adjustment to the RER, while maintaining consistency to the limit to arbitrage theory.

More specifically, our evidence from two different quantile unit root tests in semi- and non-parametric settings suggests that the RER is not a constant unit root process across quantiles. We find that the bigger the shock to the RER (i.e. the bigger the quantile), the faster the mean reversion back towards its long run equilibrium, with half lives comfortably less than a year, in the case of extreme shocks. Our results are further enhanced when large shocks hit the RER at points already far from its equilibrium. In such cases half lives can fall significantly less than a year. However, the mean reversion ability of large shocks is diminished in cases when the RER is around its equilibrium value. Finally, in the absence of shocks, mean reversion cannot be established irrespective of the RER disequilibrium level. In addition, our method captures asymmetric dynamic adjustment of the RER, i.e. positive shocks have different impact than negative shocks. Our results offer novel insights on the RER mean reverting behaviour and further refine and enhance previous evidence in the PPP literature.

References

[1] Abuaf, N. and P. Jorion (1990), "Purchasing Power Parity in the Long Run," *Journal of Finance*, 45, 157-74.

[2] Almekinders, G.J. and S.C.W. Eijffinger (1996), " A Friction Model of Daily Bundesbank and Federal Reserve Intervention," *Journal of Banking and Finance*, 20, 1365–80.

[3] Barrodale, I. and F. Roberts (1974), "Solution of an Overdetermined System of Equations in the ℓ_1 Norm," *Communication of the ACM*, 17, 319-320.

[4] Bertola, G. and A.C. Caballero (1990), "Kinked Adjustment Costs and Aggregate Dynamics," in O.J. Blanchard and S. Fischer, eds., NBER Macroeconomics Annual (Cambridge Massachusetts: MIT Press).

[5] Dueker, M. and C.J. Neely (2005), "Can Markov Switching Models Predict Excess Foreign Exchange Returns?" Federal Reserve Bank of St. Louis, Working Paper No. 21B.

[6] Dumas, B. (1992), "Dynamic Equilibrium and the Real Exchange Rate in Spatially Separated World," *Review of Financial Studies*, 5, 153-80

[7] Dumas, B. (1994), "Partial Equilibrium versus General Equilibrium Models of the International Capital Market," in F. Van Der Ploeg, ed., The Handbook of International Macroeconomics (Oxford: Blackwell).

[8] Dutta, J. and H. Leon (2002), "Dread of depreciation: measuring real exchange rate interventions," IMF Working Paper, WP/02/63, Washington, DC.

[9] Engel, C. (1994), "Can the Markov Switching Model Forecast Exchange Rates?" *Journal of International Economics*, 36, 151-165.

[10] Engel, C. and C.J. Kim (1999), "The Long-Run U.S./U.K. Real Exchange Rate," *Journal of Money, Credit, and Banking*, 31, 335-356.

[11] Engel, C. and J.D. Hamilton (1990), "Long Swings in the Dollar: Are They in the Data and Do Markets Know It?" *American Economic Review*, 80, 689-713.

[12] Frankel, J.A and A.K. Rose (1996), "A Panel Project on Purchasing Power Parity: Mean Reversion Within and Between Countries," *Journal of International Economics*, 40, 209-24.

[13] Granger, C. W. J. and T. Teräsvirta (1993), "Modelling Nonlinear Economic Relationships," Oxford: Oxford University Press, 1993.

[14] He, X. and P. Ng (1999), "COBS: Qualitatively Constrained Smoothing via Linear Program," *Computational Statistics*, 14, 315–337.

[15] He, X. and P. Shi (1998), "Monotone B-spline Smoothing," *Journal of the American Statistical Association*, 93, 643-650.

[16] Kilian, L. and M.P. Taylor (2003), "Why is it so Difficult to Beat the Random Walk Forecast of Exchange Rates?" *Journal of International Economics*, 60(1), 85-107.

[17] Kim, C.J., C. R. Nelson and R. Startz (1998), "Testing for Mean Reversion in Heteroskedastic Data Based on Biggs-Sampling Augmented Randomization," *Journal of Empirical Finance*, 5, 131-54.

[18] Koenker, R. (2005), "Quantile Regression," Econometric Society Monographs, 38, Cambridge University Press.

[19] Koenker, R. and G. Basset (1978), "Regression Quantiles," *Econometrica*, 46, 33-50.

[20] Koenker, R., P. Ng. and S. Portnoy (1994), "Quantile Smoothing Splines," *Biometrika*, 81, 673-680.

[21] Koenker, R. and d'Orey (1987), "Computing Regression Quantiles," *Applied Statistics*, 36, 383–393.

[22] Koenker, R. and d'Orey (1994), "Computing Regression Quantiles," *Applied Statistics*, 43, 410–414.

[23] Koenker, R. and Z. Xiao (2004a), "Quantile Autoregression," unpublished manuscript.

[24] Koenker, R. and Z. Xiao (2004b), "Unit Root Quantile Autoregression Inference," *Journal of the American Statistical Association*, 99, 775-787.

[25] LeBaron, B. (1992), "Forecast Improvements Using a Volatility Index," *Journal of Applied Econometrics*, 7, 137-49.

[26] Leon, H. and S. Najarian (2005), "Asymmetric Adjustment and Nonlinear Dynamics in real Exchange Rates," *International Journal of Finance and Economics*, 10, 15-39.

[27] Lothian, J.R. and M.P Taylor. (1996), "Real Exchange Rate Behavior: the Recent Float from the Perspective of the Past Two Centuries," *Journal of Political Economy*, 104, 488-510.

[28] Lothian, J.R. (1997), "Real Exchange Rate Behavior: The Problem of Power and Sample Size," *Journal of International Money and Finance*, 16, 945-54.

[29] McLachlan, G.J., and D. Peel. (2000), "Finite Mixture Models", New York: John Wiley & Sons.

[30] Michael, P., A.R. Nobay and D.A. Peel (1997), "Transaction Costs and Non-Linear Adjustment in Real Exchange Rates: An Empirical Investigation," *Journal of Political Economy*, 105, 862-79.

[31] Nelson, C.R., J. Piger and E. Zivot (2001), "Markov regime-switching and unit root tests," *Journal of Business and Economic Statistics*, 19, 404–415.

[32] Ng, S. and P. Perron (2001), "Lag length selection and the construction of unit root tests with good size and power," *Econometrica*, 69, 1519-1554.

[33] Obstfeld, M. and A.M. Taylor (1997), "Nonlinear Aspects of Goods-Market Arbitrage and Adjustment: Heckscher's Commodity Points Revisited," *Journal of the Japanese and International Economies*, 11, 441-79.

[34] Perron, P. (1990), "Testing for a Unit Root in a Time Series With a Changing Mean," *Journal of Business and Economic Statistics*, 8, 153-162.

[35] Phillips, P.C.B. and P. Perron (1988), "Testing for a unit root in time series regression," *Biometrika*, 75, 335–346.

[36] Pippenger, M.K. and G.E. Goering (1993), "A note on the empirical power of unit root tests under threshold processes," *Oxford Bulletin of Economics and Statistics*, 55, 473–481.

[37] Pippenger, M.K. and G.E. Goering (2000), "Additional results on the power of unit root and cointegration tests under threshold processes," *Applied Economics Letters*, 7, 641–644.

[38] Rogoff, R. (1996), "The Purchasing Power Parity Puzzle," *Journal of Economic Literature*, 34, 647-68.

[39] Sarno, L. and M. P. Taylor (2003), "The Economics of Exchange Rates," Cambridge and New York: Cambridge University Press.

[40] Sarno, L., M.P. Taylor and I. Chowdhury (2004), "Nonlinear Dynamics in Deviations From the Law of One Price: A Broad-Based Empirical Study," *Journal of International Money and Finance*, 23, 1-25.

[41] Sarno, L. and G. Valente (2006), "Deviations from Purchasing Power Parity Under Different Exchange Rate Regimes: Do They Revert and, If So, How?" *Journal of Banking and Finance*, (forthcoming).

[42] Taylor, A.M. (2002), "A Century of Purchasing-Power Parity," *Review of Economics and Statistics*, 84, 139-150.

[43] Taylor, M.P. (2004), "Is Official Exchange Rate Intervention Effective?" *Economica*, 71, 1-12.

[44] Taylor, M.P., D.A Peel and L. Sarno (2001), "Non-Linear Mean Reversion in Real Exchange Rates: Towards a Solution to Purchasing Power Parity Puzzles," *International Economic Review*, 42, 1015-42.

[45] Taylor, M.P. and L. Sarno. (1998), "The Behaviour of Real Exchange Rates During the Post-Bretton Woods Period," *Journal of International Economics*, 46, 281-312.

[46] Teräsvirta, T. (1994), "Specification, Estimation and Evaluation of Smooth Transition Autoregressive Models," *Journal of the American Statistical Association*, 89, 208-18.

[47] Tong, H. (1990), "Non-linear Time Series: A Dynamical System Approach," (Oxford: Clarendon Press).

Table 1. Descriptive statistics

	GBP	JPY	CHF	EUR
Mean	1.04E-10	-1.30E-11	-1.30E-11	1.08E-02
St. Dev.	0.057	0.093	0.071	0.143
Skewness	0.325	0.061	0.305	-0.538
Kyrtosis	3.251	2.108	2.750	2.055
Jarque Bera	7.829 [0.019]**	13.178 [0.001]*	7.371 [0.025]**	33.247 [0.000]*

Notes. The table presents the results from the descriptive statistics (the first four moments) and the Jarque-Bera normality test. Values in brackets are asymptotic p-values. One and two asterisks denote significance in the 1% and 5% level respectively.

Table 2. Autoregression estimation and unit root tests

Panel A) Conditional mean (OLS) specification (lags=1)

OLS	GBP	JY	EUR
α	0.974 (0.012)	0.981 (0.018)	0.988 (0.005)
F_{RS}	[0.089]*	[0.087]*	[0.088]*
	Unit root tests in levels		
PP	−2.276 [0.180]	−2.394 [0.144]	−2.322 [0.165]
MZ_a	-2.954	-0.474	-0.614
	Unit root tests in first differences		
PP	−18.145 [0.000]***	−18.091 [0.000]***	−18.245 [0.000]***
MZ_a	-78.483	-20.594	-23.139

(Table 2 continues...)

(...Table 2 continued)

Panel B) Quantile autoregressive linear specification (lags=1)

Quantile	GBP		JY		EUR	
	QKS unit root test over a range of quantiles (1% to 99%)					
1%-99%	[0.073)]		[0.492]		0.093	
	Quantile estimation and unit root tests within each quantile					
	τ_0	$a(\tau)$	τ_0	$a(\tau)$	τ_0	$a(\tau)$
1%	-0.034 [0.000]***	0.875 [0.023]**	-0.039 [0.000]***	0.960 [0.138]	-0.035 [0.000]***	0.992 [0.541]
5%	-0.022 [0.000]***	0.958 [0.455]	-0.028 [0.000]***	0.985 [0.600]	-0.021 [0.000]***	0.988 [0.261]
10%	-0.017 [0.000]***	0.964 [0.292]	-0.020 [0.000]***	0.986 [0.671]	-0.014 [0.000]***	0.988 [0.359]
25%	-0.009 [0.000]***	0.987 [0.715]	-0.008 [0.000]***	0.986 [0.455]	-0.007 [0.000]***	0.994 [0.680]
50%	-0.000 [0.472]	0.988 [0.690]	0.000 [0.349]	0.992 [0.292]	0.001 [0.070]*	0.993 [0.427]
75%	0.008 [0.000]***	1.009 [0.957]	0.008 [0.000]***	0.984 [0.179]	0.010 [0.000]***	0.992 [0.461]
90%	0.016 [0.000]***	0.984 [0.613]	0.016 [0.000]***	0.977 [0.079]*	0.020 [0.000]***	0.973 [0.283]
95%	0.021 [0.000]***	0.953 [0.075]*	0.021 [0.000]***	0.953 [0.065]*	0.027 [0.000]***	0.935 [0.060]*
99%	0.032 [0.000]***	0.898 [0.010]***	0.032 [0.000]***	0.912 [0.055]*	0.034 [0.000]***	0.918 [0.015]**

Notes: *Panel A)* The table shows the estimated values of the autoregressive (α) coefficient of a simple AR(1) model with the correspondent standard errors in parenthesis, the p-values of the residual correlation F-test (F_{RS}), and two unit root tests, the Philips-Perron (PP) (test statistic and p-values in brackets) and the Ng and Perron (MZ_a) test statistic. Unit root tests are reported for the level and the first difference of each series in question. The critical values for the MZ_a test are -13.800, -8.100 and -5.700 for the 1%, 5% and 10% level respectively. *Panel B.* The table shows the bootstrapped p-values (2000 replications), calculated using the pair-wise bootstrap method for the Kolmogorov-Smirnov (QKS) test, for the null of a unit root over a range of quantiles $\tau \in T$, where $T = (0.1, 0.99)$. It also shows the estimated values of the constant term (τ_0) and autoregressive ($a(\tau)$) coefficient of a QAR(1) model, for $\tau = \{0.01, 0.05, 0.1, 0.25, 0.5, 0.75, 0.9, 0.95, 0.99\}$. Numbers in brackets are bootstrapped p-values (2000 replications), calculated using the pair-wise bootstrap method. For the constant term we are testing the null of zero statistical significance, whereas for the slope coefficients we are testing the null of a unit root. One, two and three asterisks denote statistical significance at the 10, 5 and 1 percent level respectively.

Table 3. Autoregressive coefficients and estimated half lives

Panel A) Autoregressive coefficients and half lives (lag=1)

OLS	GBP	JY	EUR
α	0.974	0.981	0.988
	(∞)	(∞)	(∞)

Panel B) Quantile autoregressive coefficients and half lives (lags=1)

Quantile	GBP	JY	EUR
	$a(\tau)$	$a(\tau)$	$a(\tau)$
1%	0.875	0.960	0.992
	(5.191)	(∞)	(∞)
5%	0.958	0.985	0.988
	(∞)	(∞)	(∞)
10%	0.964	0.986	0.988
	(∞)	(∞)	(∞)
25%	0.987	0.986	0.994
	(∞)	(∞)	(∞)
50%	0.988	0.992	0.993
	(∞)	(∞)	(∞)
75%	1.009	0.984	0.992
	(∞)	(∞)	(∞)
90%	0.984	0.977	0.973
	(∞)	(29.78)	(∞)
95%	0.953	0.953	0.935
	(14.398)	(14.398)	(10.313)
99%	0.898	0.912	0.918
	(6.443)	(7.525)	(8.101)

(Table 3 continues...)

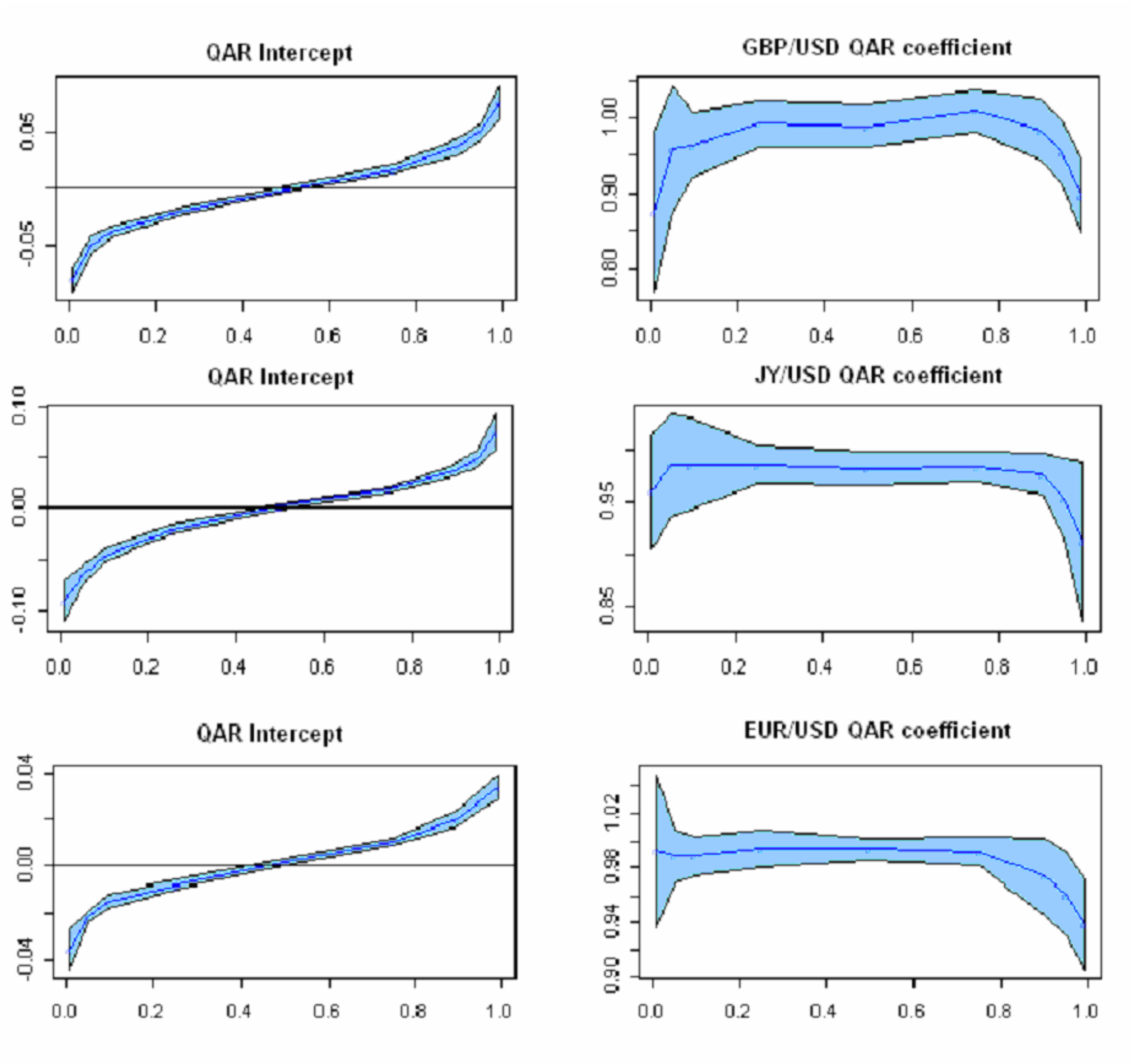
(...Table 3 continued)

Panel C) Non-parametric quantile autoregressive coefficients and half lives (lags=1)

	GBP/USD			JY/USD			EU/USD		
Quantile	1%	50%	99%	1%	50%	99%	1%	50%	99%
	0.597 (1.344)	0.988 (∞)	0.789 (2.925)	1.050 (∞)	0.992 (∞)	0.855 (4.425)	0.925 (∞)	0.994 (∞)	0.719 (2.101)
	1.070 (∞)		1.004 (∞)	1.051 (∞)		0.950 (13.513)	0.954 (∞)		1.084 (∞)
$a(\tau)$	1.124 (∞)			0.898 (6.443)		1.006 (∞)	1.188 (∞)		0.761 (2.538)
	0.667 (1.712)			1.094 (∞)		0.937 (10.652)	1.053 (∞)		0.934 (10.152)
				1.004 (∞)		0.771 (2.665)	0.926 (9.016)		0.992 (∞)
				0.774 (2.706)			0.665 (1.699)		0.894 (6.186)

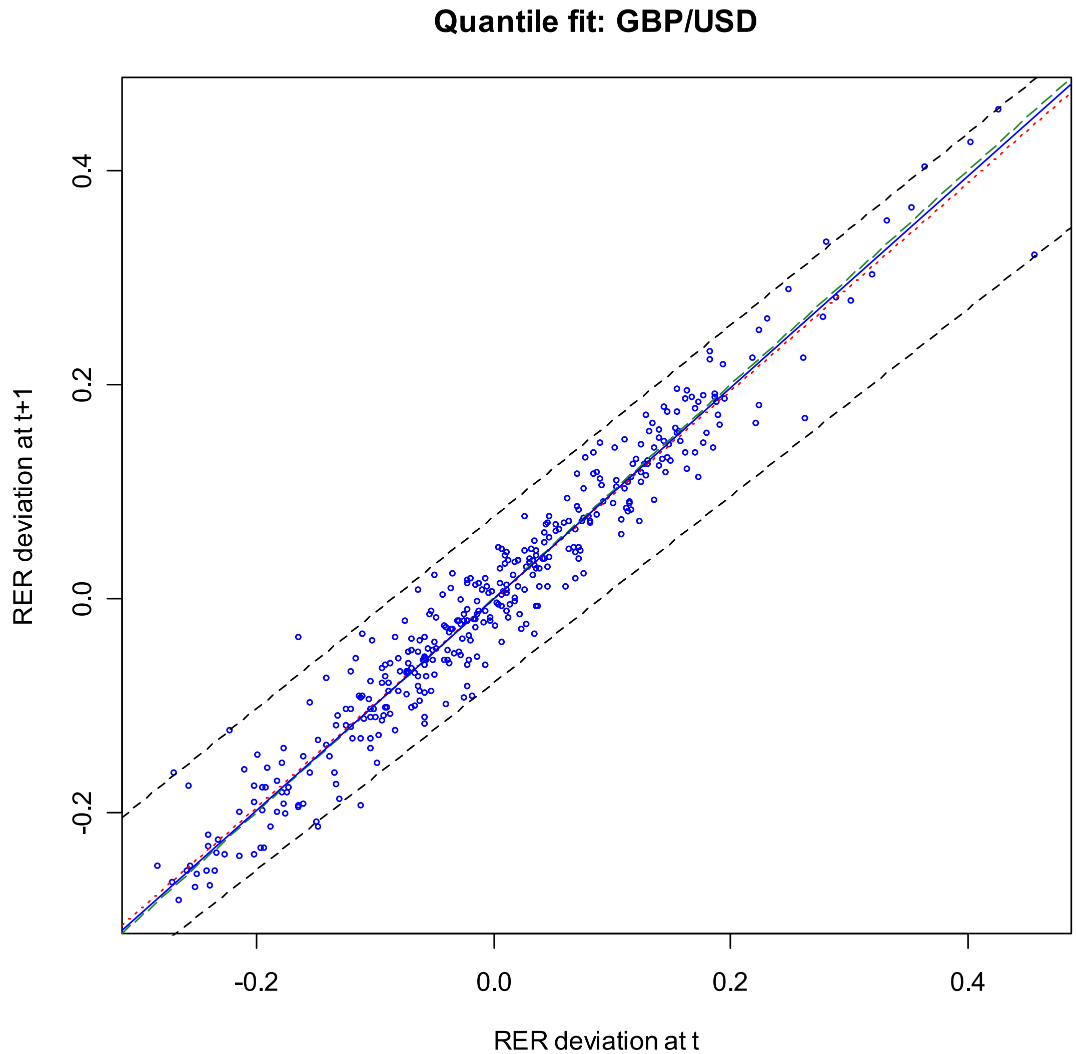
Notes: *Panel A)* The table presents the estimated values of the autoregressive (α) coefficient of a simple AR(1) model with the correspondent half lives in parenthesis, for each of the currencies under consideration. *Panel B)* The table shows the estimated values of the autoregressive ($a(\tau)$) coefficient of a QAR(1) model and the correspondent half lives in parenthesis, for $\tau = \{0.01, 0.05, 0.1, 0.25, 0.5, 0.75, 0.9, 0.95, 0.99\}$. *Panel C)* The table shows the estimated values of the autoregressive coefficient ($a(\tau)$), as they result from the non-parametric total variation penalty, quantile smoothing method, and their correspondent half lives in parenthesis for $\tau = \{0.01, 0.5, 0.99\}$. *For all panels* only the mean reverting coefficients (i.e. smaller than unity) were assigned a half live, whereas for the case of coefficients either bigger than unity or statistically not different from unity, half lives are set to infinity ∞. The significance of the coefficients with respect to unity, for the case of the non-parametric fit, was determined using asymptotic inference methods (He and Ng, 1999).

Figure 1. Quantile intercept and autoregressive (QAR) coefficients



Notes: The figures plot the quantile process of the intercept (right plots) and QAR coefficients (left plots) for each one of the major currencies. The vertical axis measures the values of the coefficients and the horizontal axis represents the values of the quantiles, ranging from *0.0* to *1.0*. The nine points on the plots are the coefficient (intercept and slope) estimates at $\tau=\{0.01,0.05,0.10,0.25,0.5,0.75,0.90,0.95,0.99\}$. The grey areas indicate the 95% confidence band.

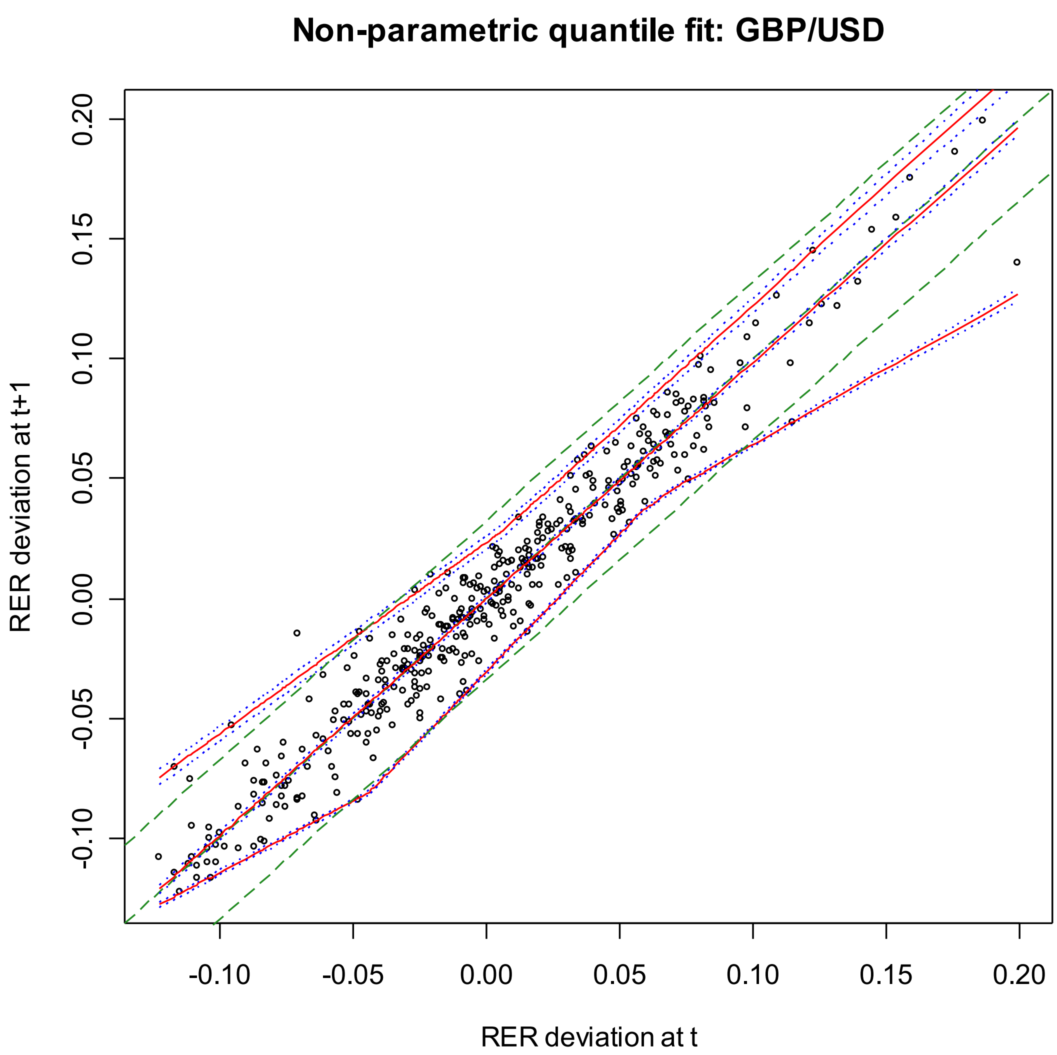
Figure 2. OLS and quantile fits



Notes: The figures present the realisations of the logged RER deviations (period *t-1* against period *t*) from Jan 1973 to Dec 2004 for the four currencies under consideration. The horizontal and vertical axes represent degrees of dis-equilibrium of the RER. The long-slash line represents the diagonal axis (x=y) and superimposed on that are the (OLS) mean and median fits, dotted and slashed lines respectively. The outer slashed lines represent the fit in the 1% (lower) and 99% (higher) quantile respectively.

Figure 3. Non-parametric quantile fit

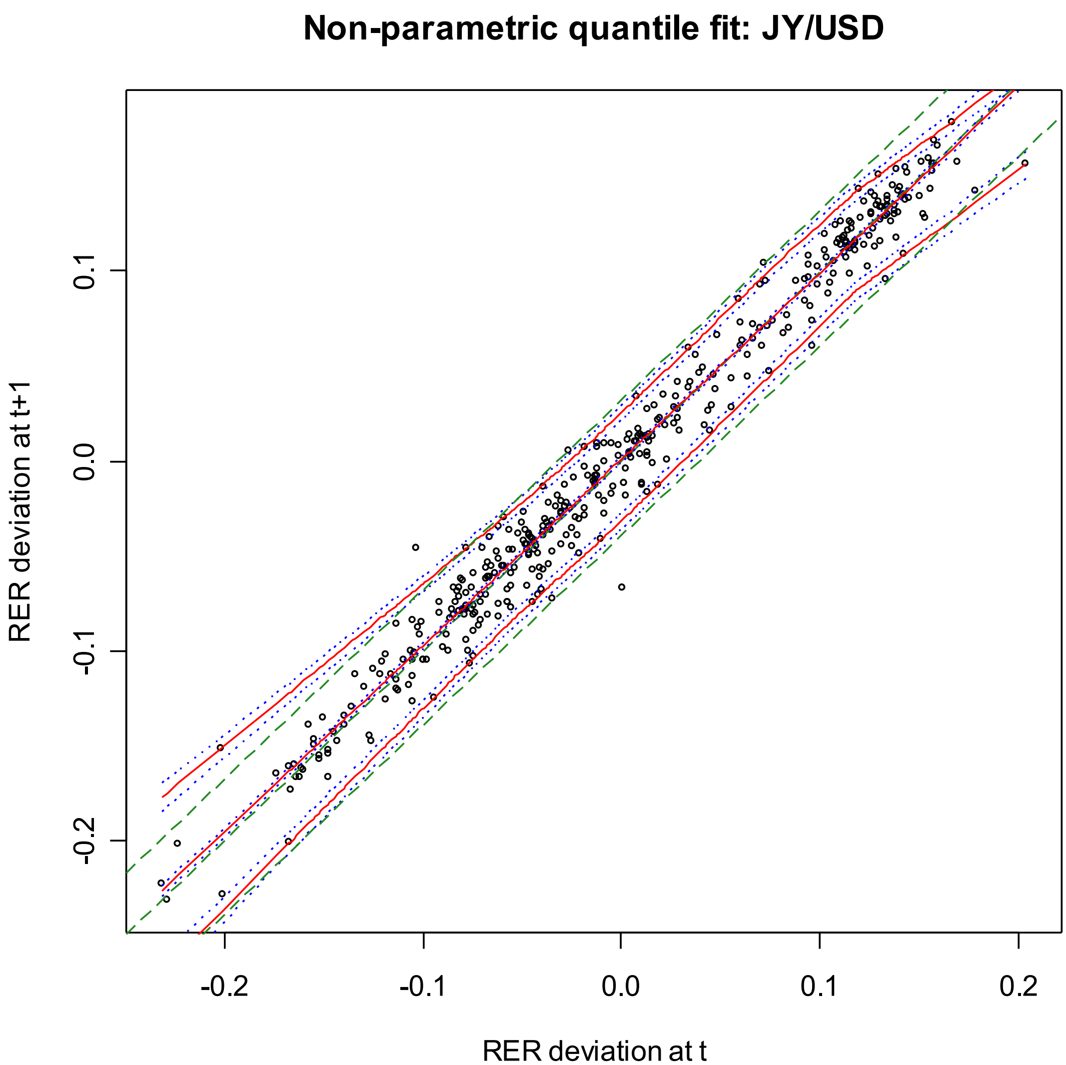
Panel A) British Pound



(Figure 3 continues…)

(…Figure 3 continued)

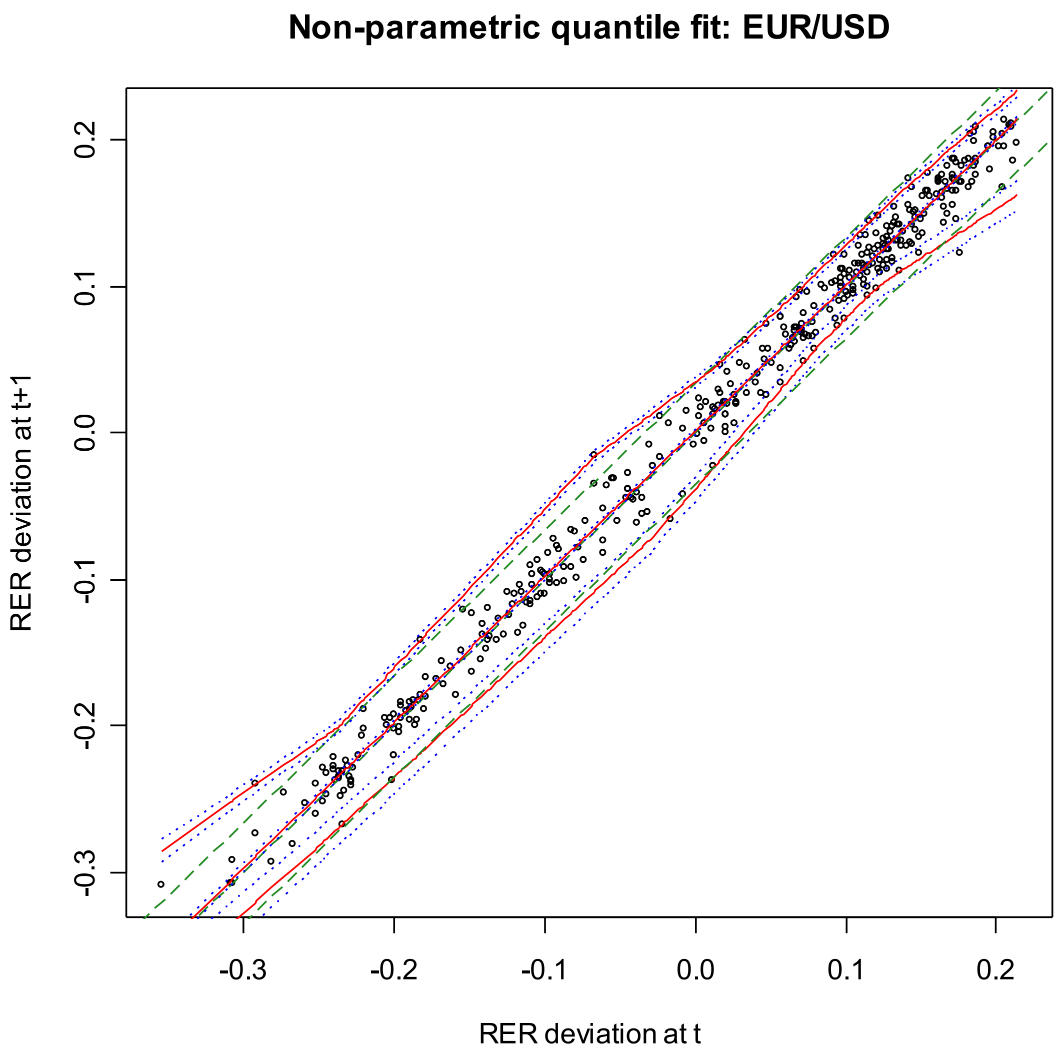
Panel B) Japanese Yen



(Figure 3 continues…)

(…Figure 3 continued)

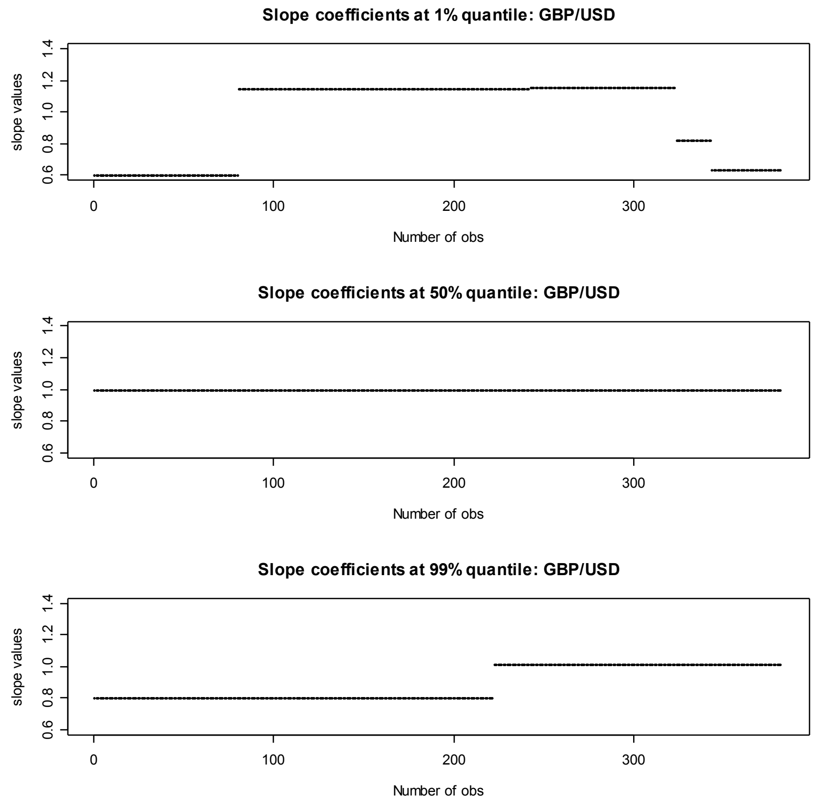
Panel C) Euro



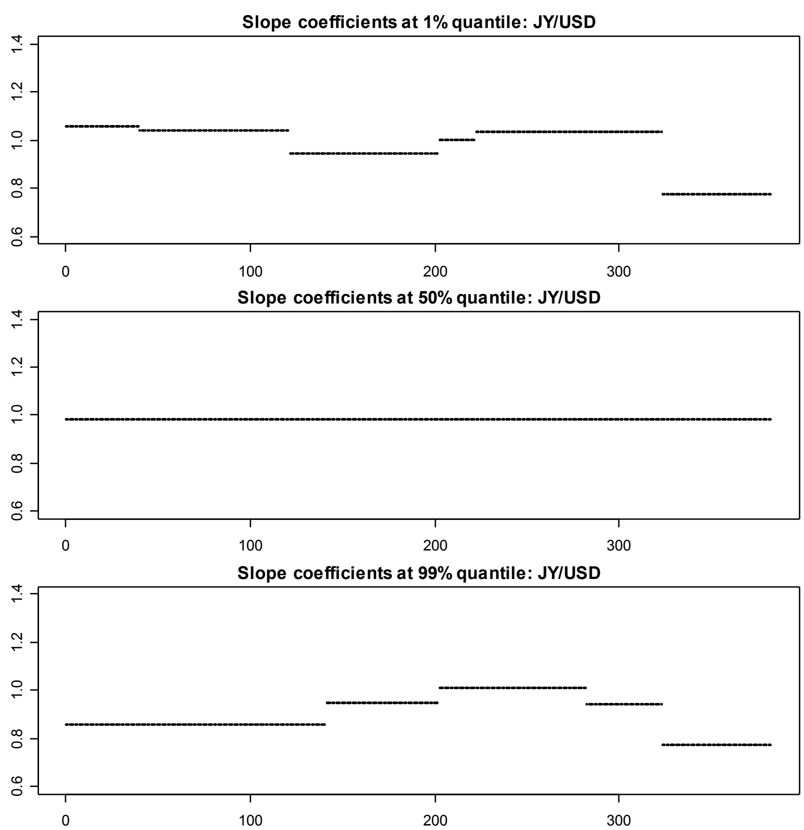
Notes: The figures present the realisations of the logged RER deviations (period *t-1* against period *t*) from Jan 1973 to Dec 2004 for the three currencies under investigation. The horizontal and vertical axes represent degrees of disequilibrium of the RER. Superimposed on the realisations are the fits of the regression quantiles smoothing splines for τ = *(0.01, 0.5, 0.99))* (solid lines), with standard error bands (dotted lines). The long-slash lines represent the diagonal axis (x=y) for the intercept values of the respective QAR fits (Table 2).

Figure 4. Non-parametric quantile fit– slope coefficients

Panel A: British pound



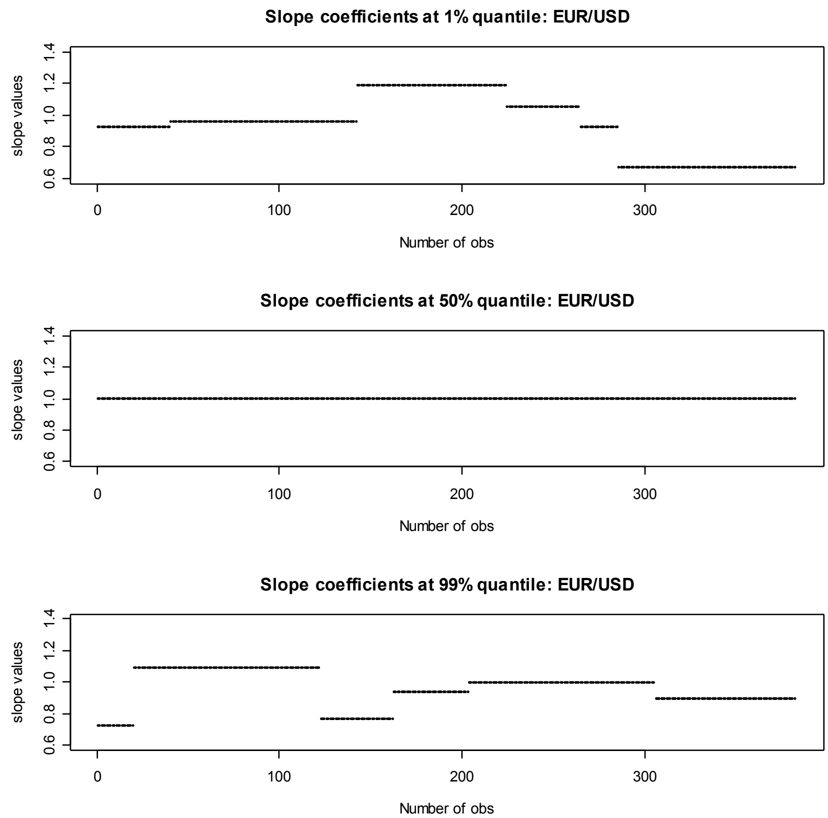
Panel B: Japanese Yen



(Figure 4 continues…)

(…Figure 4 continued)

Panel C: Euro



Notes: The figures present the correspondent slope coefficients at the different quantiles of the non-parametric quantile fits in Figure 3, Panel B, for τ = *(0.01, 0.5, 0.99)*. The vertical axis presents the range of slope coefficient values and the horizontal axis the number of observations (relating to the ordered values of the RER realisations in the horizontal axis of Figure 3).