A Note on Timeless Perspective Policy Design*

Andrew P. Blake†
Centre for Central Banking Studies
Bank of England

Tatiana Kirsanova‡
University of Exeter

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Abstract

Working with a typical macroeconomic model we investigate the superiority of timeless perspective policy design. We show that when fiscal policy matters the timeless perspective policy design can cause instability of the economy.

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†CCBS, Bank of England, Threadneedle Street, London EC2R 8AH; e-mail: Andrew.Blake@bankofengland.co.uk.

‡Corresponding author; School of Business and Economics, University of Exeter, Streatham Court, Rennes Drive, Exeter, EX4 4PU; e-mail T.Kirsanova@exeter.ac.uk.

1 Introduction

The concept of timeless perspective policy design has attracted a lot of attention in the recent monetary policy literature, see Clarida, Galí, and Gertler (1999), Woodford (1999b), Woodford (1999a), McCallum and Nelson (2000) amongst many others. The timeless perspective (TP) policy design is specifically designed to avoid dynamic inconsistency. It has been shown that TP solutions are superior to those that would result from discretionary policymaking with respect to the unconditional loss criterion for many cases. Although Blake (2001) and Jensen and McCallum (2002) show the non-optimality of a TP policy in general by providing counter-examples, they leave the reader with an impression that the concept is nevertheless innocuous in its implications for policy. Indeed, in all examples they present, the loss from the TP policy is higher then the one from the time consistent (TC) discretionary policy, however the ratio of the losses is still reasonably close to one. Moreover, in all these cases if one would compare the loss from the time inconsistent optimal plan (TI), discretionary optimisation and the timeless perspective, one would find that all three policies deliver similar values of the loss function (see, for example, Blake (2001) who presents such a table). An obvious question that arises is, is this a result of particular (and peculiar) dynamic properties of a given system?

In a very recent paper Giannoni and Woodford (2002) give a formal account of the concept where they try to rule out all possible complications with either existence or non-optimality of the timeless perspective policy. In this paper we assess how wide a range of possible dynamic models with which we typically work in monetary economics for which the timeless perspective policy can be a very inferior policy option. Here we work with a typical macroeconomic model with a standard dynamic structure that presents no problem for conventional policymaking. In this model the timeless perspective policy, if implemented, can bring disastrous results—the economy is not asymptotically stable. The previous discussion in available literature about the TP policy is predominantly based on an optimising model that includes a New Keynesian Phillips curve and an intertemporal IS curve. We add effects of fiscal policy in this model. This is sufficient to cause the model under the timeless perspective policy control to have very undesirable properties.

The paper is organised as follows. We discuss the concept of timeless perspective policy design in Section 2. As in Giannoni and Woodford (2002) we characterise it as a departure from the fully optimal time inconsistent plan, so we start with a brief description of the concept. In Section 3 we present an example of a model where Ricardian equivalence fails. We derive the timeless perspective policy and show that it can yield an unstable solution. We conclude in Section 4.

2 The 'timeless perspective' policy

All the authors cited in the introduction derive a timeless perspective policy in a model-specific way using very similar basic macroeconomic models. In this section, following Giannoni and Woodford (2002), we give a general interpretation of what is actually done in all these examples. The TP policy is easy to explain departing from the well-known concept of 'fully optimal time inconsistent plan'.

Consider a typical macroeconomic model which constitutes the mainstream modelling approach. To have an analytically tractable model, we usually have two agents in a policy game, characterised as the private sector and the monetary policy authorities. We solve out the maximisation problem of one player (the private sector) and explain its behaviour as a reduced form system of first order conditions, which are the relevant Euler equations. One might think of a system comprising a Phillips curve and an intertemporal IS curve. We might also have additional equations explaining the evolution of additional state variables of interest, for example the wealth accumulation equation. Each of these equations describe behaviour of a single player. Then, in this mainstream setup, we solve the remaining player's optimisation problem subject to this behavioural model.

The typical setup of such an optimisation problem for the monetary authorities can be written in the following form:

$$\min_{\{U_s\}_{s=t}^{\infty}} \frac{1}{2}\mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t} L_s \tag{1}$$

subject to:

$$\begin{bmatrix} Y_{t+1} \\ X_{t+1} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} Y_t \\ X_t \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} U_t + \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} \varepsilon_t. \tag{2}$$

Here L_s denotes a one-period loss function for the policymaker, Y_t is a vector of predetermined variables and X_t is a vector of non-predetermined or jump rational expectations variables. If we consider a representative model with 'persistent' Phillips curve and habit formation in the consumption equation, then $Y_t = (\pi_{t-1}, x_{t-1})$ and $X_t = (\pi_t, x_t)$. However most models used in the literature to illustrate the TP concept assume no persistence and no predetermined variables. If we introduce new variables $Z_t = (Y_t', X_t')'$ then the decision model can be rewritten as:

$$Z_{t+1} = AZ_t + BU_t. \tag{3}$$

Note that the (2) and (3) assume an identity matrix as a coefficient on Z_{t+1}. This representation can be always obtained and we term it 'canonical' because it differentiates equations with predetermined and non-predetermined variables. As will become apparent, this step may be crucial for the understanding of the formal concept of the TP policy.

The function L_s is typically a quadratic form of the government target variables G_t, $L_s = G_s' Q G_s$. The target variable G_t can be linked to the state variable Z_t as $G_t = CZ_t$, so that the criterion (1) can be re-written as:

$$\begin{aligned} \min_{\{U_s\}_{s=t}^{\infty}} \frac{1}{2} \mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t} G_s' Q G_s &= \min_{\{U_s\}_{s=t}^{\infty}} \frac{1}{2} \mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t} Z_s' C' Q C Z_s \\ &= \min_{\{U_s\}_{s=t}^{\infty}} \frac{1}{2} \mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t} Z_s' \mathcal{Q} Z_s \end{aligned}$$

where $\mathcal{Q} = C'QC$.[1]

The timeless perspective policy can be explained by comparing it with the fully optimal time inconsistent plan. We begin by discussing this concept to have a convenient frame of reference.

[1] We do not include instrument costs explicitly in the formulation of the cost function, but we could define a state equal to the instrument variable if we wished.

2.1 Optimal Commitment Plan

The fully optimal, time inconsistent policy requires the minimisation of the constrained loss function:

$$H = \min_{\{U_s\}_{s=t}^{\infty}} \mathcal{E}_t \sum_{s=t}^{\infty} H_s \tag{4}$$

where every term of H_s has the form:

$$\begin{aligned} H_s = & \frac{1}{2}\beta^{s-t}(X_s' \mathcal{Q}_{22} X_s + Y_s' \mathcal{Q}_{12} X_s + X_s' \mathcal{Q}_{21} Y_s + Y_s' \mathcal{Q}_{11} Y_s) \\ & + \lambda_{s+1}^{y\prime}(A_{11}Y_s + A_{12}X_s + B_1 U_s - Y_{s+1}) \\ & + \lambda_{s+1}^{x\prime}(A_{21}Y_s + A_{22}X_s + B_2 U_s - X_{s+1}) \end{aligned}$$

where λ^y is a vector of non-predetermined Lagrange multipliers (those associated with the predetermined variables Y) and λ^x is a vector of predetermined Lagrange multipliers (as those associated with the non-predetermined variables X). To derive the first order conditions we differentiate the constrained loss function with respect to X, Y, U, and λ to obtain the following system (we also used $\mu_s = \beta^{-s}\lambda_s$ to simplify notation):

$$\frac{\partial H}{\partial X_s} : \mathcal{Q}_{22} X_s + \mathcal{Q}_{21} Y_s + \beta A_{12}' \mu_{s+1}^y + \beta A_{22}' \mu_{s+1}^x - \mu_s^x = 0 \tag{5}$$

$$\frac{\partial H}{\partial Y_s} : \mathcal{Q}_{12} X_s + \mathcal{Q}_{11} Y_s + \beta A_{11}' \mu_{s+1}^y + \beta A_{21}' \mu_{s+1}^x - \mu_s^y = 0 \tag{6}$$

$$\frac{\partial H}{\partial U_s} : B_1' \mu_{s+1}^y + B_2' \mu_{s+1}^x = 0 \tag{7}$$

$$\frac{\partial H}{\partial \lambda_{s+1}^{y\prime}} : A_{11}Y_s + A_{12}X_s + B_1 U_s - Y_{s+1} = 0 \tag{8}$$

$$\frac{\partial H}{\partial \lambda_{s+1}^{x\prime}} : A_{21}Y_s + A_{22}X_s + B_2 U_s - X_{s+1} = 0 \tag{9}$$

This system must be solved using initial conditions for all predetermined variables (Y_0 and μ_0^x) and terminal conditions (transversality conditions) for all non-predetermined variables (X, μ^y and U). We observe Y_0, and the Pontryagin maximum principle requires that the initial conditions for the predetermined Lagrange multipliers should be set to zero,[2] $\mu_0^x = 0$. For a unique solution, the system (5)–(9) should have as many explosive

[2] For a very clear explanation see Currie and Levine (1993).

eigenvalues (i.e. absolute values outside the unit circle) as the number of non-predetermined variables.

Finally, the solution to the system can be written in a form:

$$\begin{bmatrix} Y_{s+1} \\ \mu^x_{s+1} \end{bmatrix} = Z_{11} S_{11}^{-1} T_{11} Z_{11}^{-1} \begin{bmatrix} Y_s \\ \mu^x_s \end{bmatrix} \tag{10}$$

$$\begin{bmatrix} U_s \\ X_s \\ \mu^y_s \end{bmatrix} = Z_{21} Z_{11}^{-1} \begin{bmatrix} Y_s \\ \mu^x_s \end{bmatrix} \tag{11}$$

where matrices Z, S and T are obtained by solving a particular generalised eigenvalue problem, see e.g. Söderlind (1999).

The requirement to set initial conditions $\mu^x_0 = 0$ highlights the problem of time-inconsistency associated with the fully optimal solution. As soon as optimisation is done at time t and μ^x_t is set to zero, this implies a certain time path for $\{\mu^x_s\}_{s=t}^{\infty}$ such that μ^x_s is not necessarily zero for any $s > t$. It immediately follows that given an option to re-optimise at any time $s > t$, the policymaker will choose to re-set μ^x_s to zero, reneging on the previously optimal plan. The optimal plan at time t is inconsistent from the perspective of any other time $s > t$.

2.2 Timeless perspective policy

This requirement of zero initial conditions and the implied time-inconsistency are typically explained in the literature using the observation that the system (5)–(9) is written for the time index $s = 1, 2, ...$ but equation (5) should be rewritten for $s = 0$. Thus:

$$\mathcal{Q}_{22} X_0 + \mathcal{Q}_{21} Y_0 + \beta A'_{12} \mu^y_1 + \beta A'_{22} \mu^x_1 = 0$$

while the rest of the system stays the same. We explicitly have one term less in equation (5) in the initial period, so the private sector can *observe* that in the initial period the government does something different from what it does in later on. Therefore, the solution is dynamically inconsistent.

A potential resolution to this problem, suggested by Woodford (1999b), is to design a policy such that the first order conditions (which govern the economy under policy control) would *look* the same for the private sector for every period of time, including $t = 0$. Intuitively, it means that we come to $t = 0$ having optimised our behaviour in the remote past and stick to this solution for all time. The effect of initial conditions should have 'worn away' by $t = 0$. Formally, it means that we should have no predetermined Lagrange multipliers in the system so that we have no associated problems with zero initial conditions.

When solving system (5)–(9), if we set $\mu_0^x \neq 0$ we may obtain a solution to the system within a class of possible trajectories consistent with our decision model. The natural question is whether it is always possible to find such initial conditions that the system would 'look the same'? In other words, are we always able to *substitute out* the predetermined Lagrange multipliers such that the dynamic properties of the solution would hold.[3] Even if it is so, such a solution will not deliver the minimum to the loss function (1). A further question is how much do we lose in terms of optimality?

3 An example economy with active fiscal policy

To examine the questions just raised, it is simplest to consider a model with rich enough dynamics.

3.1 The model and optimal commitment plan

In this section we discuss the optimising model in an economy, still without persistence, but with a predetermined state variable. We consider example of an economy where fiscal policy matters. For example, let us suppose that the Ricardian equivalence fails because consumers are not infinitely lived.

We consider the typical model with optimising forward-looking private sector, whose optimisation behaviour can be reduced to the Phillips curve

[3] We should comment here that as soon as non-predetermined Lagrange multipliers are concerned, nothing prevents us from keeping them in the system. They do not cause time-inconsistency so they should not be eliminated, following the main argument. Additionally, the final solution which will look like the system (10)–(11), except that without μ^x, has μ_s^y as a solution which is separated from the other variables. In essence we treat them as additional rational expectations variables.

and the intertemporal IS curve with a wealth effect:

$$\pi_t = \beta\pi_{t+1} + \kappa x_t + \varepsilon_{1t} \tag{12}$$

$$x_t = x_{t+1} - \sigma(i_t - \pi_{t+1}) + \nu b_t + \varepsilon_{2t} \tag{13}$$

where π is domestic inflation, x is real output (consumption), b is real domestic debt and i is the nominal interest rate, which is an instrument of the Central Bank. Equation (12) is a New Keynesian Phillips curve (Clarida, Galí, and Gertler 1999), and (13) is an intertemporal IS curve (McCallum and Nelson 1999) modified to include wealth effects.[4]

We have to close the model with a debt accumulation equation, that can be linearised as (Woodford 1996):

$$b_{t+1} = i_t + \frac{1}{\beta}(b_t - \pi_{t+1}) + \frac{1}{\beta}\Delta_t + \varepsilon_{3t}$$

where Δ_t is real primary deficit. For simplicity, we assume that the fiscal authorities control the real primary deficit with a feedback rule, $\Delta_t = -\tau b_t$, so the debt accumulation equation is:

$$b_{t+1} = i_t + \frac{1}{\beta}((1-\tau)b_t - \pi_{t+1}) + \varepsilon_{3t} \tag{14}$$

We can write the system (12), (13) and (14) in a matrix form and bring it to the canonical form of representation (2), so we would not have a mixture of predetermined and jump variables in the left hand side. We are therefore able to differentiate between predetermined and non-predetermined Lagrange multipliers. The predetermined state is $Y_t = b_t$, and the non-predetermined state is $X_t = (\pi_t, x_t)'$, with the matrices are given in Appendix A.

The Central Bank is minimising the following objective function

$$\min_{\{i_s\}_{s=t}^{\infty}} \frac{1}{2}\mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t}(\pi_s^2 + \omega x_s^2)$$

so the vector of target variables is $G_t = (\pi_t, x_t)$.

[4] We discuss the derivation of the parameter ν below.

The first order conditions can now be written as:

$$\frac{\partial H}{\partial i_s} : \sigma \mu^{xx}_{s+1} + \mu^{yb}_{s+1} = 0 \tag{15}$$

$$\frac{\partial H}{\partial b_s} : (1-\tau)\mu^{yb}_{s+1} - \frac{\nu}{\beta}\mu^{x}_{s+1} = \mu^{yb}_{s} \tag{16}$$

$$\frac{\partial H}{\partial \pi_s} : \mu^{x\pi}_{s+1} - \sigma\mu^{xx}_{s+1} - \frac{1}{\beta}\mu^{yb}_{s+1} = \mu^{x\pi}_{s} - \pi_s \tag{17}$$

$$\frac{\partial H}{\partial x_s} : -\kappa\mu^{x\pi}_{s+1} + (\sigma\kappa + \beta)\mu^{xx}_{s+1} + \frac{\kappa_1}{\beta}\mu^{yb}_{s+1} = \mu^{xx}_{s} - \omega x_s \tag{18}$$

where μ^{yb} denotes non-predetermined Lagrange multipliers (associated with the debt equation (b)) and μ^{xj} denotes two predetermined Lagrange multipliers (for the IS curve ($j = x$) and the inflation equation ($j = \pi$)). Additionally, we have our original system (12)–(14) trivially obtained by differentiating with respect to Lagrange multipliers.

To solve for the time-inconsistent optimal plan we should solve the system (15)–(18) and (12)–(14) with initial conditions for b and μ^{xj}. To minimise the cost-to-go we would need to impose $\mu^{xj}_0 = 0$.

3.2 Timeless perspective policy

As discussed above, in order to derive a timeless perspective policy we need to eliminate the predetermined μ^{xj} from system (15)–(18). Equation (15) can be used to solve for $\mu^{xx}_{s+1} = -\frac{1}{\sigma}\mu^{yb}_{s+1}$. Substitute μ^{xx} into the rest of the system we get:

$$\frac{\partial H}{\partial b_s} : \left(1 - \tau + \frac{\nu}{\sigma\beta}\right)\mu^{yb}_{s+1} = \mu^{yb}_{s} \tag{19}$$

$$\frac{\partial H}{\partial \pi_s} : \mu^{x\pi}_{s+1} + (1 - \frac{1}{\beta})\mu^{yb}_{s+1} = \mu^{x\pi}_{s} - \pi_s \tag{20}$$

$$\frac{\partial H}{\partial x_s} : -\kappa\mu^{x\pi}_{s+1} - \left((1-\frac{1}{\beta})\kappa + \frac{\beta}{\sigma}\right)\mu^{yb}_{s+1} = -\frac{1}{\sigma}\mu^{yb}_{s} - \omega x_s \tag{21}$$

Note that when we substituted $\mu^{xx}_s = -\frac{1}{\sigma}\mu^{yb}_s$ in the last equation we implicitly assumed that relationship which is valid for predetermined μ^{xx} for time $s+1$, is also valid for time s, so that it is *here* that we introduce the timeless perspective policy rule. We can now get rid of $\mu^{x\pi}$, by solving the last equation to get:

$$\mu^{x\pi}_{s+1} = -\left(1 + \frac{\beta}{\sigma\kappa}\right)\mu^{yb}_{s+1} + \frac{1}{\sigma\kappa}\mu^{yb}_{s} + \frac{\omega}{\kappa}x_s$$

and substitute $\mu^{x\pi}$ into (20):

$$-\left((1-\frac{1}{\beta})+\frac{\beta}{\kappa\sigma}\right)\mu^{yb}_{s+1}+\frac{1}{\sigma\kappa}\mu^{yb}_{s}+(1-\frac{1}{\beta})\mu^{yb}_{s+1}+\frac{\omega}{\kappa}x_s$$
$$=-\left((1-\frac{1}{\beta})+\frac{\beta}{\kappa\sigma}\right)\mu^{yb}_{s}+\frac{1}{\sigma\kappa}\mu^{yb}_{s-1}+\frac{\omega}{\kappa}x_{s-1}-\pi_s. \quad (22)$$

To simplify the last equation we can use equation (19) for μ^{yb}_{s-1} and get:

$$-\left[\frac{\beta}{\kappa\sigma}-\frac{1}{\sigma\kappa}(1-\tau+\frac{\nu}{\sigma\beta})\right]\mu^{yb}_{s+1}+\frac{\omega}{\kappa}x_s$$
$$=-\left[(1-\frac{1}{\beta})+\frac{\beta}{\kappa\sigma}-\frac{1}{\sigma\kappa}(1-\tau+\frac{\nu}{\sigma\beta})\right]\mu^{yb}_{s}+\frac{\omega}{\kappa}x_{s-1}-\pi_s. \quad (23)$$

Finally, we have two equations (19) and (23), and the three original equations (12)–(14). This system can be used to find non-predetermined Lagrange multipliers μ^{yb}, and all economic variables (π, x, i and b) under the timeless perspective policy rule. This might at first sight seem odd, as we appear to have two equations for μ^{yb}. This is actually a result of two factors. Firstly, the second order difference equation for the Lagrange multiplier is re-written as (23) and so we need two first order equations. Secondly, the equation for the interest rate is only implicitly given.

It is clearly seen from the system that as soon as:

$$1-\tau+\frac{\nu}{\sigma\beta}=1 \quad (24)$$

we have a unit-root Lagrange multiplier μ^{yb} from (19). The system is not dynamically stable under the timeless perspective policy control for this particular regime and it is close to the unit-root process when $1-\tau+\frac{\nu}{\sigma\beta}$ is only slightly greater than one. From (23) we can see that the dynamics now include a term $\frac{\omega}{\kappa}\Delta x_s$. It is this that induces additional persistence in the model under TP control.

As an illustration, we simulate a typical dynamic path of inflation in the economy in area $1-\tau+\frac{\nu}{\sigma\beta}\gtrapprox 1$ under three regimes: commitment, discretion[5] and timeless perspective policy, see Figure 1.[6] The result is self-evident:

[5] We do not discuss here how to solve for a time consistent equilibrium. See Oudiz and Sachs (1985) or Söderlind (1999) for a recent version.

[6] Parameter details are given in Appendix B.

for the timeless perspective policy the inflation rate ranges over a far wider region, demonstrated by the radically different scales on the graphs. Figure 2 presents impulse responses to a debt shock for the three regimes. It can be demonstrated that the TP response does eventually return to zero, but after several hundred periods. We do not need to compute the unconditional loss function to demonstrate that the loss can be made very large. Note that both fully optimal plan and discretionary policy bring a unique stationary equilibrium and Lagrange multipliers in for the commitment regime are not unit root processes, they slowly converge to zero.

4 Caveats and conclusions

Our result assumes that the fiscal policy operates with a 'weak' feedback rule (although still ruling out the possibility of that debt is uncontrolled) and that consumers' mortality rate is sufficiently high to ensure inequality $1-\tau+\frac{\nu}{\sigma\beta} > 1$. This situation is feasible and both the fully optimal plan and optimal discretionary policy can easily stabilise this economy. The problem is that the area of reasonably calibrated parameters is not wide. The consumption-out-of-wealth coefficient, ν, can be derived as $\nu = \theta(\theta + \frac{1-\beta}{\beta})$ where θ is mortality rate and might reasonably be calibrated as 0.01 for yearly data. This leaves a very narrow feasible region for the feedback parameter τ.

Note that condition (24) can be one outcome of the pure fiscal theory of the price level, which assumes that consumers are infinitely lived ($\nu = 0$) and the real government deficit is uncontrolled ($\tau = 0$) so the price level can be affected by the level of government debt, and the fiscal policy matters for the economy. Leeper (1991), for example, has identified several non-conflicting regimes (our example is one of such regimes with 'active' fiscal policy and 'passive' monetary policy in his classification) and Evans and Honkapohja (2002) who have shown that these regimes are learnable so could logically exist. However, the timeless perspective policy is a unit-root policy under this version of the fiscal theory of the price level.

Formally, this property follows from the dynamic form of the debt accumulation equation. If debt is not sufficiently controlled and the central bank discounts the future at the social discount rate (which we assume is benevo-

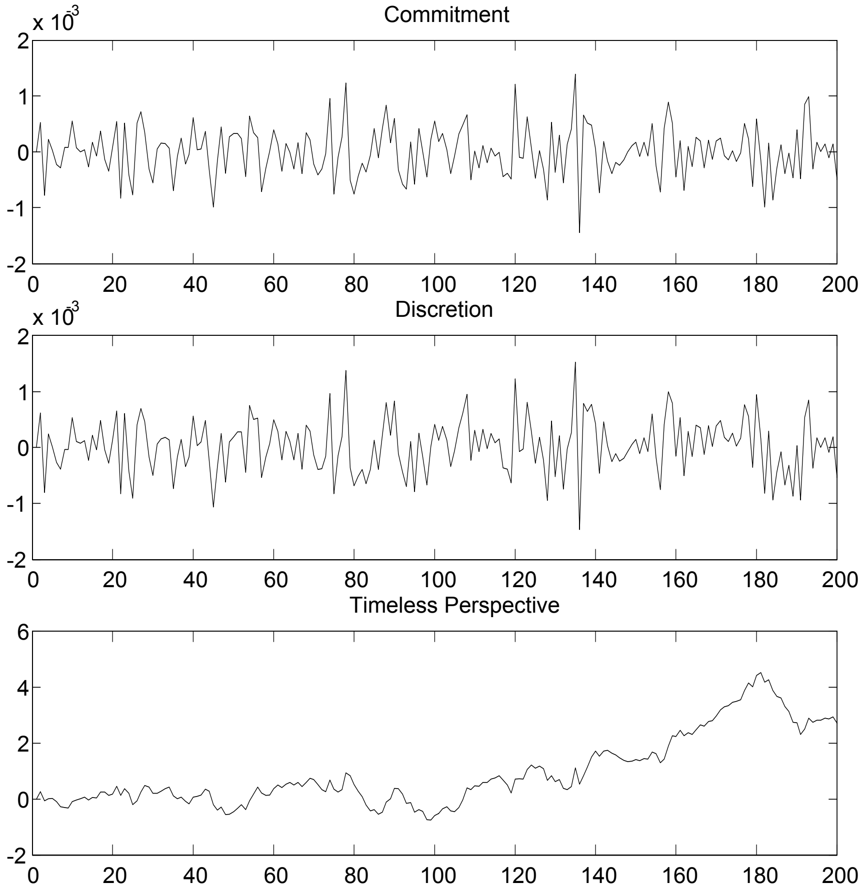

Figure 1: Time path of inflation under different policy regimes. Parameters: $\tau = 0.005$, $\nu = 0.003$.

lent) then the non-predetermined Lagrange multiplier on the predetermined debt is 'almost' a unit root variable as well as the non-predetermined Lagrange multiplier set on the debt constraint. There is nothing wrong with unit-root *non-predetermined* Lagrange multipliers—they are immaterial and can be separated from the system when it is solved for the TI policy. However, when deriving the TP policy, these unit root relationships are used, thus potentially 'infecting' the whole system with unit roots as we demonstrated in our example.

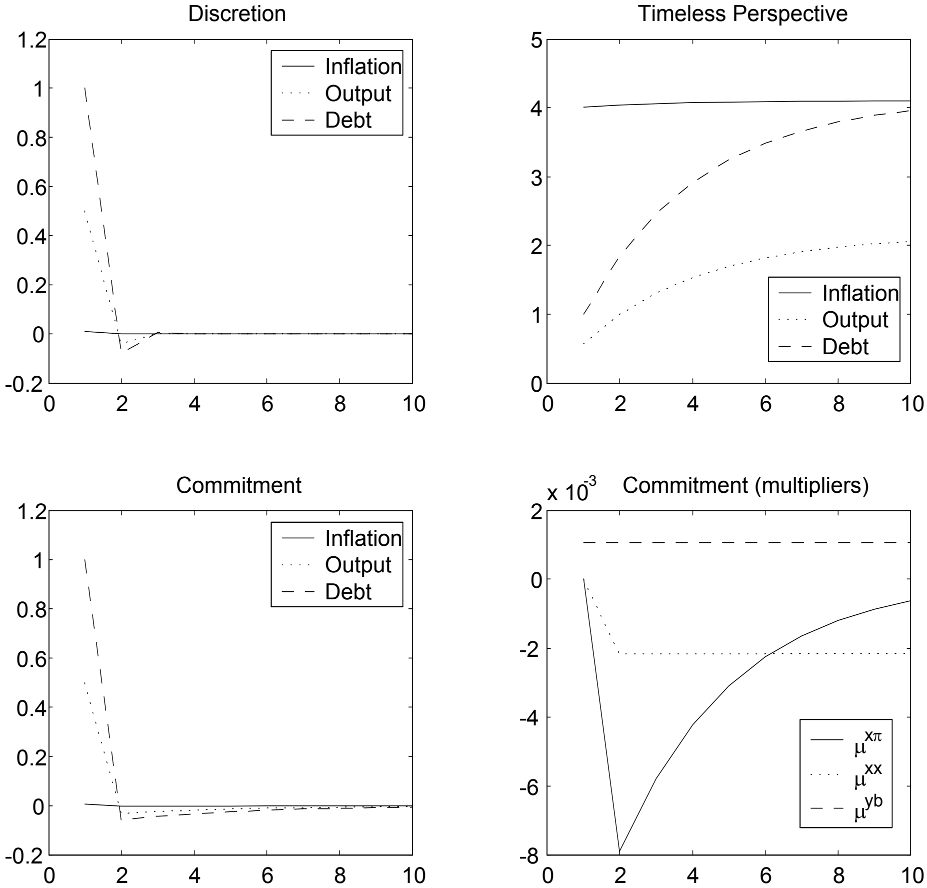

Figure 2: Impulse responses to a debt shock.

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A System matrices

The complete system (12)–(14) can be written in matrix form as:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \beta & 0 \\ 0 & \sigma & 1 \end{bmatrix} \begin{bmatrix} b_{t+1} \\ \pi_{t+1} \\ x_{t+1} \end{bmatrix} = \begin{bmatrix} \frac{1}{\beta}(1-\tau) & -\frac{1}{\beta} & 0 \\ 0 & 1 & -\kappa \\ -\nu & 0 & 1 \end{bmatrix} \begin{bmatrix} b_t \\ \pi_t \\ x_t \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ \sigma \end{bmatrix} i_t$$
$$+ \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \\ \epsilon_{3t} \end{bmatrix}.$$

Pre-multiplying both sides with the inverse of the left hand side matrix we obtain the canonical form of representation with the following matrix partitioning:

$$A_{11} = \begin{bmatrix} \frac{1}{\beta}(1-\tau) \end{bmatrix},\ A_{12} = \begin{bmatrix} -\frac{1}{\beta} & 0 \end{bmatrix},$$
$$A_{21} = \begin{bmatrix} 0 \\ -\frac{\nu}{\beta} \end{bmatrix},\ A_{22} = \begin{bmatrix} \frac{1}{\beta} & -\frac{\kappa}{\beta} \\ -\frac{\sigma}{\beta} & \frac{\sigma\kappa}{\beta}+1 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},\ B_2 = \begin{bmatrix} 0 \\ \sigma \end{bmatrix},$$
$$E_1 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix},\ E_2 = \begin{bmatrix} -\frac{1}{\beta} & 0 & 0 \\ \frac{\sigma}{\beta} & -1 & 0 \end{bmatrix}$$
$$Q = \begin{bmatrix} 1 & 0 \\ 0 & \omega \end{bmatrix},\ C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},\ \mathcal{Q} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \omega \end{bmatrix}.$$

B Calibration

We consider a microfounded model similar to that discussed by Rotemberg and Woodford (1997). Therefore the parameters κ and ω are endogenous. For the numerical experiment we assumed the probability that wage contracts are not reconsidered is $\gamma = 0.85$ and an intertemporal substitution factor of $\sigma = 0.5$. The discount factor is set to $\beta = 0.99$.