Model Persistence and the Role of the Exchange Rate and Instrument Inertia in Monetary Policy.

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Abstract

This paper addresses two of the unsettled issues in the design of monetary policy in small open economies: the importance of the exchange rate and the interpretation of the observed inertia in the policy interest rate. In doing so, this paper derives an optimizing macroeconomic model that introduces habit formation in the consumer's utility function and inertia in the inflation equation. As a consequence, aggregate demand and supply shocks will have a persistent effect on output and inflation. In this framework, we assess the performance of monetary policy rules under different degrees of habit formation and inflation inertia. We conclude that an inflation based rule, containing also a response to output and the exchange rate, is optimal. This rule performs particularly well in the presence of some domestic and foreign shocks. When less persistence is introduced in the economy, that rule is still the optimal one. Finally, the gains of adopting an inertial policy rule are directly related to the degree of inflation persistence in the model.

1 Introduction

Inflation targeting has been implemented successfully in small open economies. However, an important and still unsettled issue for monetary policy is how much of an interest-rate reaction there should be to the exchange rate (Taylor 2001). Research to date indicates that monetary policy rules that react directly to the exchange rate, as well as to inflation and output, do not work much better in stabilizing the economy than simple rules that do not react directly to the exchange rate[1].

Empirical evidence presented in Clarida et al (1997) suggests that monetary authorities in Japan, Germany, UK, and France do respond to exchange rate fluctuations. For Latin American economies, similar results are found in Caputo (2001) and Schmidt-Hebbel and Werner (2002). The advantages of responding to those fluctuations are, however, not very clear. In fact, Taylor (1999) using a seven-country model for the G7, concludes that if the European Central Bank reacts to the exchange rate France and Italy may increase their welfare, however, Germany may experience welfare losses. In a different study Batini et al (2003), calibrate a two-sector open economy model for the UK. They find that an inflation-forecast-based rule, containing a separate response to the level of the real exchange rate, improves stabilization only marginally. Finally, Parrado and Velasco (2002) conclude that responding to exchange rate fluctuations may improve the performance of the Chilean economy in the face of nominal domestic shocks. However, for foreign and real shocks the reverse is true.

In more theoretically oriented studies conclusions are similar; there are only marginal gains from responding to exchange rate fluctuations. Using a simple backward-looking model, Ball (1999) finds that policy rules that react to the exchange rate, as well as to output and inflation, generate a small reduction in the volatility of inflation, while keeping constant the volatility of output. Therefore, in this case, there are small gains from responding to exchange rate fluctuations. Svensson (2000) develops an alternative model that includes both backward and forward-looking behavior. In this setup, Svensson (2000) concludes that a policy rule that reacts to changes in the real exchange rate may reduce inflation volatility. This reduction comes, however, at the cost of inducing a higher output variance. In this way, if output volatility has an important weight in the social loss function, such a rule may induce welfare losses.

Another unsettled issue for monetary policy is the interpretation of the observed inertia in the policy instrument (interest rate). In fact, there is a general agreement that lagged interest rate is a significant variable in estimated reaction functions in open and closed economies[2]. However, there is some debate about why this is the case.

Rudebusch (2002) suggests that the observed inertia is just reflecting serially correlated shocks that the central bank faces. According to this view, the omission of those shocks, from the empirical estimations, gives the impression that the central bank is smoothing interest rates. Hence, the

[1] For an up-to date survey see Leitemo and Soderstrom (2003).

[2] Clarida et al (1997) find a degree of interest rate inertia of more than 90% in four European countries, Japan and the USA. For Chile, Caputo (2001), also finds a degree of interest rate inertia of around 90%.

observed inertia is, in Rudebusch's (2002) words, just an illusion. A different interpretation is given by English *et al* (2002). They argue that the lagged interest rate enters the policy reaction function in its own right and does play an important role in the dynamics of the interest rate. In their view, serially correlated shocks explain only a small proportion of the observed interest rate inertia.

In this context, the objective of this paper is twofold. First, we assess the role of the exchange rate in the design of monetary policy in a small open economy. Second, we investigate, as suggested by Rudebusch (2002), the link between the persistence in the economy and the inertia in the policy interest rate. To address those issues, we assess the performance of simple instrument rules, or policy rules, in a general equilibrium (GE) model of a small open economy. In particular, for reasons given below, we consider a GE model with endogenous persistence.

Many of the New Keynesian GE models for small open economies include only forward-looking components in the aggregate demand and supply equations. This is at odds with empirical evidence suggesting that both output and inflation present an important degree of persistence[3]. To overcome this problem we introduce endogenous persistence in the economy. In doing so, we extend the open economy framework of Gali and Monacelli (2002) and Parrado and Velasco (2002). In particular, we allow for habit formation in the consumer's utility function. As a result, the aggregate demand equation we derive contains both leads and lags of the output gap. In practice, this implies that demand shocks will be transmitted with some inertia to output, and consequently to prices. On the other hand, as in Svensson (2000), we introduce inertia into the supply equation[4] to allow for lagged output and inflation to affect current inflation. Finally, we explicitly derive structural equations for the rest of the world, allowing for habit formation and inflation persistence. The model developed here has the advantage of introducing persistence in a structural way. In fact, the dynamic response of output and inflation to domestic and foreign shocks will depend on the degree of habit formation and inflation inertia, both in the domestic economy and in the rest of the world.

Once the model has been derived, we investigate the degree of inertia and exchange rate reaction in the optimal policy rule. In order to obtain this rule, it is a standard practice to rely on an loss criterion reflecting the preferences of a central bank that targets inflation in a flexible way, as in Svensson (1997). This criterion, denominated *inflation targeting* loss criterion[5], penalizes inflation, output, and interest rate variability. In addition to this, we explicitly derive a utility-based criterion in the presence of habits. Therefore, and unlike most of the studies in this matter, we derive two set of optimal policy rules; rules that minimize an *inflation targeting* loss criterion and those that minimize a utility-based one.

We conclude that a simple monetary policy rule, including a separate response to output, expected inflation and the real exchange rate, is the optimal one. This result is independent of the

[3] See Fuhrer 2000.

[4] As in Svensson (2000) the introduction of endogenous inflation persistence may seem to be *ad-hoc*. However, as proved by Clarida and Gertler (1999), inflation peristence may emerge rationally if some proportion of the price-setting firms in the economy is backward-looking.

[5] This labeling follows Leitemo (2003).

loss criterion used. The gains from responding to the exchange rate are, for an all shock scenario, small. However, when the economy is facing some specific domestic and foreign shocks that rule delivers substantial welfare gains.

On the other hand, when less persistence is introduced in the model, by assuming a lower degree of habit formation and/or inflation inertia, a rule that responds to the exchange rate is still the optimal one. Furthermore, we find a direct link between the persistence in the economy and the degree of inertia in the optimal policy rule. In particular, when the degree of inflation persistence is low, the degree of interest rate smoothing decreases considerably. This last result is in line with Rudebusch's (2002) assertion that, in practice, the illusion of monetary policy inertia evident in the estimated policy rules likely reflects the persistent shocks that central banks face[6].

This paper is organized as follows; Section 2 derives the open economy aggregate demand and supply equations in the presence of habit and inflation inertia. It is shown that the standard open economy model is a limiting case of this more general specification. Section 3 describes the parametrization procedure and the algorithm used to solve the model. In Section 4 the model is stochastically simulated. Then, optimal policy reaction functions, according to both an inflation targeting and a utility-based loss criterion, are derived. Section 5 analyses the policy implications of introducing different degrees of inertia in the economy. Finally, Section 6 concludes.

2 Open Economy Model

In this section we extend the open economy framework developed by Gali and Monacelli (2002) and Parrado and Velasco (2002). First, we introduce habits in the consumers utility function. In doing so, we follow Kozicki and Tinsley (2002) and derive an Euler equation for consumption from a CRRA utility function with additive habits. Second, we derive a supply equation under the assumption that firms have some monopolistic power, and face a constant probability of resetting prices each period, as in Calvo (1983). We then characterize the equilibrium in this open economy. Finally we close the model by specifying a Central Bank reaction function, a long-term real interest rate, and a real interest rate parity condition.

2.1 Households

The small open economy is inhabited by a representative household who seeks to maximize expected utility from consumption and leisure over time. A household maximizes

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[u\left(C_t, C_{t-1}\right) - V(N_t) \right] \right\} \tag{1}$$

where $u(.)$ represents the utility from consumption. In this general specification, as we will see later on, past consumption can affect current utility. The $V(N_t)$ function is the disutility from

[6] Rudebusch (2002) confines his analysis to persistent monetary policy shocks. In this paper persitence is introduced in the economy through the output and inflation equations.

supplying N_t hours of labor, β is the discount factor and C_t is a composite consumption index defined by

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} (C_{H,t})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (C_{F,t})^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{2}$$

where $\alpha \in (0,1)$ is the share of consumption allocated to imported goods and $\eta > 0$ is the elasticity of substitution between domestic, $C_{H,t}$, and foreign, $C_{F,t}$, consumption goods. The two consumption subindexes, $C_{H,t}$ and $C_{F,t}$, are symmetric and are defined by the CES aggregators

$$C_{H,t} = \left(\int_0^1 C_{H,t}(j)^{\frac{\theta-1}{\theta}} dj\right)^{\frac{\theta}{\theta-1}} \quad ; \quad C_{F,t} = \left(\int_0^1 C_{F,t}(j)^{\frac{\theta-1}{\theta}} dj\right)^{\frac{\theta}{\theta-1}}$$

where θ is the elasticity of substitution within each category, and $j \in (0,1)$ indexes the type of good consumed.

The maximization of (1) is subject to a sequence of intertemporal budget constraints that can be represented as

$$\int_0^1 [P_{H,t}(j)C_{H,t}(j) + P_{F,t}(j)C_{F,t}(j)] + B_t \leq (1-\tau_1)W_t N_t + R_t^1 B_{t-1} + T_t \tag{3}$$

where B_t represent a portfolio investment (which includes shares in firms) with a payoff in period $t+1$. On the other hand, R_t^1 represents the one-period real gross return to a portfolio held at the beginning of t. It is assumed, as in Gali and Monacelli (2002), that households have a complete set of contingent claims, traded internationally. On the other hand, τ_1 is a proportional tax on nominal labor income, and T_t are nominal lump-sum transfers from the government.

Money does not appear in either the budget constraint or the utility function. In this case, money is playing a role only as a unit of account. As noted by Kozicki and Tinsley (2002), money can be omitted from the analysis for three reasons. First, money balances are small proportion of households wealth. Second, household utility specifications are often separable in real money balances. Finally, when the central bank pursues an interest rate policy in order to stabilize a nominal anchor, which is the case in this model, money balances are functionally irrelevant in the model.

The optimal allocation of any given expenditure, within each category of goods, yields the demand functions:

$$C_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\theta} C_{H,t} \quad ; \quad C_{F,t}(j) = \left(\frac{P_{F,t}(j)}{P_{F,t}}\right)^{-\theta} C_{F,t}$$

where $P_{H,t} = \left(\int_0^1 P_{H,t}(j)^{1-\theta} dj \right)^{\frac{1}{1-\theta}}$ and $P_{F,t} = \left(\int_0^1 P_{F,t}(j)^{1-\theta} dj \right)^{\frac{1}{1-\theta}}$ are the price indexes for domestic and foreign goods, both expressed in units of home currency. Using the definition of total consumption, equation (2), we derive the optimal allocation of expenditures between home and foreign goods as

$$C_{H,t} = (1-\alpha) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t \quad ; \quad C_{F,t} = \alpha \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} C_t \tag{4}$$

where the consumer price index (CPI) is given by

$$P_t \equiv \left[(1-\alpha) (P_{H,t})^{1-\eta} + \alpha (P_{F,t})^{1-\eta} \right]^{\frac{1}{1-\eta}} \tag{5}$$

Plugging equation (4) into the budget constraint, equation (3), gives a new expression for the latter in terms of the composite good:

$$P_t C_t + B_t = (1-\tau_1) W_t N_t + R_t^1 B_{t-1} + T_t \tag{6}$$

Then, the home's agent problem is to choose paths of consumption and output of good j to maximize expected utility in (1). In what follows, we specify a CRRA utility function that allows for habit formation, as in Kozicki and Tinsley (2002)

$$U(C_t, C_{t-1}) = \frac{(C_t C_{t-1}^{-\gamma})^{1-\sigma}}{1-\sigma} \tag{7}$$

where $\gamma \in (0,1)$ is the habit formation coefficient, and $\sigma > 0$ is the curvature of the utility function. In the limiting case in which $\gamma = 0$, σ represents the coefficient of relative risk aversion.

Now, from the first order conditions of the maximization of (1) subject to the budget constraint (6) we obtain the following log-linear specification for consumption in the presence of habits[7]

$$c_t = a_1 \beta E_t c_{t+1} + a_1 c_{t-1} - a_2 n \rho_{t,n} + \epsilon_{c,t} \tag{8}$$

where $a_1 = \frac{\gamma(\sigma-1)}{\sigma+\gamma\beta[\gamma(\sigma-1)-1]} > 0$ and $a_2 = \frac{1-\gamma\beta}{\sigma+\gamma\beta[\gamma(\sigma-1)-1]} > 0$, and c_t represents log-deviations of consumption from the steady state. The $\rho_{t,n}$ variable is the n period real interest rate (measured as deviation from a constant mean) and $\epsilon_{c,t}$ is a random aggregate demand shock. In this setup an increase in habit formation, γ, increases the degree of consumption persistence, a_1, while reducing the sensitivity of consumption to the n period real interest rate, a_2. In the particular case in which habits are not present, $\gamma = 0$, equation (8) collapses to

$$c_t = -\frac{1}{\sigma} n \rho_{t,n} + \epsilon_{c,t} \tag{9}$$

which is the forward-looking expression for consumption derived in Gali and Monacelli (2002).

[7] See Appendix 2 for derivation.

2.2 Firms

In an imperfect competition environment, firms can set prices in order to maximize profits. We assume, following Calvo (1983), that each period a fraction 1-ϕ of firms is offered the opportunity to choose a new price, while the remaining suppliers have to maintain whichever price they set before. Moreover, suppliers are drawn randomly and independently of their own history.

We assume suppliers face the same demand function for domestic output, $\widetilde{y}_{t+k}(j)$:

$$\widetilde{y}_{t+k}(j) = \left(\frac{p_{H,t+k}(j)}{P_{H,t+k}}\right)^{-\theta} Y_{t+k} \tag{10}$$

where Y_{t+k} represents the aggregate level of output.

Following Svensson (2000), we assume that suppliers who can set a price today choose the same price, denoted by $\widetilde{p}_{H,t}$, in order to maximize expected profits

$$E_t \left\{ \sum_{k=0}^{\infty} (\phi\beta)^k \lambda_{H,t+k} \left[\frac{\widetilde{y}_{t+k}(j)\widetilde{p}_{H,t}}{P_{H,t+k}} - \frac{W_{t+k}}{P_{H,t+k}} \frac{F\left(\widetilde{y}_{t+k}(j)\right)}{A_{t+k}} \right] \right\} \tag{11}$$

where ϕ is the probability that consumers-producers will keep the same price as the previous period, $\lambda_{H,t+k}$ is the marginal utility of domestic goods, $\frac{\widetilde{y}_{t+k}(j)\widetilde{p}_{H,t}}{P_{H,t+k}}$ is the real revenue from selling a unit of good j when the price is $\widetilde{p}_{H,t}$, $\frac{W_{t+k}F\left(\widetilde{y}_{t+k}(j)\right)}{A_{t+k}}$ is the cost of producing $\widetilde{y}_{t+k}(j)$. Finally, the input requirement function, $\frac{F\left(\widetilde{y}_{t+k}(j)\right)}{A_{t+k}}$, is the same for all goods j and A_{t+k} is a wide economy productivity shock.

From the first order conditions for the maximization of (11) and assuming, as in Svensson (2000), that the cost of the input, W_{t+k}, evolves according to the following log-linear representation (lower case letters denote the variables in log deviations)

$$w_t = (1-\alpha)p_{H,t} + \alpha p_{F,t}$$

it is possible to obtain the New Keynesian Phillips curve[8]

$$\pi_{H,t} = \beta E_t\left(\pi_{H,t+1}\right) + k(\varpi y_t + \alpha q_t) \tag{12}$$

where $\pi_{H,t}$ is the domestic inflation, $k = \frac{(1-\phi)(1-\beta\phi)}{\phi(1+\varpi\theta)} > 0$, $\varpi > 0$ is the elasticity of F' with respect to $y_{t+k}(j)$, and q_t is the real exchange expressed as a percentage deviation from steady state.

[8] For a complete derivation see Svensson (1998) pp.33-35.

2.3 Inflation, Terms of Trade, Real Exchange Rate: Some Identities

Following Gali and Monacelli (2002), we present some of the basic relationships that will be used in deriving the equilibrium. The aim of this procedure is to obtain an expression for the CPI inflation as a function of both the terms of trade and the real exchange rate. In what follows, all variables with an * superscript will represent foreign variables, and as before lower case letters denote the variables in log deviations from steady-state.

Domestic inflation, $\pi_{H,t}$, can be expressed as $\pi_{H,t} = \log(\frac{P_{H,t+1}}{P_{H,t}})$. The CPI inflation, π_t, is defined as $\pi_t = \log(\frac{P_{t+1}}{P_t})$.

We define terms of trade, the price of foreign goods in terms of home goods, as $S_t = \frac{P_{F,t}}{P_{H,t}}$. On the other hand, the ratio of domestic to CPI prices is defined as $H_t = \frac{P_{H,t}}{P_t}$. After log-linearizing the CPI formula, equation (5), we obtain the following relationship in log-deviations from the steady state; $h_t = -\alpha s_t$. Then, we take logs to the H_t expression and first differentiate it to obtain the following expression for the CPI inflation:

$$\pi_t = \pi_{Ht} + \alpha \Delta s_t \tag{13}$$

Hence, changes in terms of trade will impact CPI inflation proportionally to the degree of openness, α.

Now, we derive a relationship between the real exchange rate and terms of trade. Given that the share of imports in the rest of the world is negligible, it follows that $P_t^* = P_{F,t}^*$, and consequently $\pi_t^* = \pi_{F,t}^*$ for all t. Then, the real exchange rate can be expressed as $Q_t = \frac{e_t P_t^*}{P_t} = S_t H_t$, where e_t is the nominal exchange rate defined as the domestic price of foreign currency. This implies the following relationship in log-deviations

$$\begin{aligned} q_t &= s_t + h_t \\ &= (1-\alpha) s_t \end{aligned} \tag{14}$$

Then, there is a direct link between the terms of trade, s_t, and the real exchange rate, q_t. The CPI inflation can also be expressed as a function of the real exchange rate by substituting (14) into (13) to obtain

$$\pi_t = \pi_{Ht} + \frac{\alpha}{(1-\alpha)} \Delta q_t \tag{15}$$

2.4 Aggregate Demand

Market clearing in the small economy requires that $Y_t(j) = C_{H,t}(j) + C_{H,t}^*(j)$, for all $j \in (0,1)$. The government collect taxes and makes transfers to the private sector, and it balances its budget each period. Hence, the government does not consume final goods. Now, log-linearizing the above expression around the steady state with balanced trade, and aggregating over j, implies that at an aggregate level

$$y_t = (1-\alpha)c_{H,t} + \alpha c^*_{H,t} \tag{16}$$

On the other hand, an expression for total consumption is obtained by log-linearizing equation (2) around the steady state:

$$c_t = (1-\alpha)c_{H,t} + \alpha c_{F,t} \tag{17}$$

Combining equations (16) and (17) gives the following expression for domestic output:

$$y_t = (1-\alpha)c_t + \alpha c^*_t + \alpha\left[\left(c^*_{H,t} - c^*_t\right) - (c_{F,t} - c_t)\right] \tag{18}$$

Now, we log-linearize equation (4) and use the identity in (14) to obtain $c_{F,t} - c_t = -\eta q_t$, and by analogy, $c^*_{H,t} - c^*_t = \frac{\eta^* q_t}{(1-\alpha)}$. Also, because the world economy is assumed to have a negligible weight on the goods imported from the small economy, the market clearing conditions in the rest of the world imply that $y^*_t = c^*_t$. Hence, from equation (18) we get the aggregate demand equation

$$y_t = (1-\alpha)c_t + \alpha y^*_t + \phi_1 q_t \tag{19}$$

where $\phi_1 = \frac{\alpha(\eta^*+\eta-\alpha\eta)}{(1-\alpha)} > 0$, and we allow for the possibility that the elasticity of substitution between domestic and foreign goods, η, is not equal across countries. The impact of consumption, c_t, on output depends on the degree of openness, α. The closer the economy, the larger the impact on output of any deviation in domestic consumption. Furthermore, any degree of persistence in consumption will be transmitted to output. On the other hand, the impact of foreign demand, y^*_t, on domestic output depends directly on the degree of openness. In this case, foreign demand shocks will have a persistent effect on domestic output if we allow for habit formation in the rest of the world.

In this setup, a real exchange depreciation has a positive impact on domestic output. A real depreciation makes foreign goods relatively more expensive, shifting foreign and domestic consumption towards domestically produced goods. The coefficient that links exchange rate fluctuations to output, ϕ_1, depends on both the degree of openness in the economy, α, and the foreign and domestic elasticities of substitution between goods; η^* and η. In particular, if an economy has a non diversified export sector (i.e. faces a higher η^*) or is relatively more open (i.e. faces a higher α), the impact of exchange rate fluctuations on output will be exacerbated.

In the particular case in which no habit formation is present, $\gamma = 0$, and countries have the same elasticity of substitution between foreign and domestic good, $\eta^* = \eta$, the above model collapses to the standard forward looking equation derived in previous studies[9]:

$$y_t = -\frac{(1-\alpha)n}{\sigma}\rho_{t,n} + \alpha y^*_t + \frac{\alpha\eta(2-\alpha)}{(1-\alpha)}q_t \tag{20}$$

[9] See Parrado and Velasco (2002).

2.5 Aggregate Supply

Following Svensson (2000), we assume there is a proportion $(1-\delta)$ of firms that are forward-looking. Their behavior can be described by the New Keynesian Phillips equation in (12). On the other hand, a proportion δ are backward-looking, and their behavior is described just by lagged inflation. Accordingly, domestic inflation, $\pi_{H,t}$, can be expressed as;

$$\pi_{H,t} = (1-\delta)\left\{\beta E_t\left(\pi_{H,t+1}\right) + k(\varpi y_t + \alpha q_t)\right\} + \delta\pi_{H,t-1} + \epsilon_{\pi,t} \tag{21}$$

where $\epsilon_{\pi,t}$ is a cost-push shock.

Now, combining equations (21) and (15) gives the following expression for CPI inflation;

$$\pi_t = (1-\delta)\left\{\beta E_t\left(\pi_{H,t+1}\right) + k(\varpi y_t + \alpha q_t)\right\} + \delta\pi_{H,t-1} + \frac{\alpha}{(1-\alpha)}\Delta q_t \tag{22}$$

The real exchange rate impacts CPI inflation in three different ways. First, a real depreciation shifts foreign and domestic demand towards domestically produced goods. This increases aggregate demand, y_t, and consequently domestic inflation. Second, a depreciation increases the input cost to the firms, αq_t, which in turn impacts domestic inflation. Finally, a depreciation increases the domestic price of imports, increasing CPI inflation directly.

2.6 Monetary Policy and Interest Rate Parity Condition

The central bank reaction function is expressed as an inflation-forecast-based rule (IFB), that is, an instrument rule that reacts to expected future inflation deviations from target. In addition, we allow for the possibility that the central bank may react to output, y_t, and the real exchange rate, q_t. Finally, we capture the tendency of central banks to smooth changes in interest rates by assuming that the actual rate partially adjusts to the target. Under those assumptions, the monetary policy reaction function can be expressed as[10]

$$r_t = \rho r_{t-1} + (1-\rho)\left(\rho_\pi E_t\left(\pi_{t+\tau}\right) + \rho_y y_t + \rho_q q_t\right) + \epsilon_{r,t} \tag{23}$$

where r_t is the short-term nominal interest rate set by the central bank, $\rho \in (0,1)$ is the interest rate smoothing coefficient, $\epsilon_{r,t}$ is a monetary policy shock, and τ is the horizon over which the central bank targets inflation. Instead of assuming an arbitrary value for τ we derive it optimally (see Section 4).

In this model, the monetary policy instrument, the short-term nominal interest rate, r_t, impacts aggregate demand through the term structure channel. In particular, domestic consumption reacts to the n period real interest rate, $\rho_{t,n}$, rather than to r_t. To find a relationship between short-term and long-term real interest rates, we proceed as in Fuhrer and Moore (1995a pp.223); we make use

[10] The simple specification in equation (23) has been used in Clarida et al (1998) to describe, empirically, the conduct of monetary in the major developed countries.

of the intertemporal arbitrage condition that equalizes the expected real holding-period yields on a long-term bond and the real return on a short-term central bank instrument:

$$\rho_{t,n} = n\left\{E_t\left(\rho_{t+1,n}\right) - \rho_{t,n}\right\} + \left\{r_t - E_t\left(\pi_{t+1}\right)\right\} \tag{24}$$

where n represents the duration of the bond, which we assume to be ten years, i.e. $n = 40$ in a quarterly basis. In this way, any change in the short-term real interest rate is transmitted to the long-term real interest rate through equation (24).

We close the model by specifying a relationship for the real exchange rate, q_t. In doing so, we assume the nominal exchange rate fulfills the uncovered interest parity condition (UIP)

$$r_t - r_t^* = E_t\left(e_{t+1}\right) - e_t + \varphi_t \tag{25}$$

where r_t^* is the foreign nominal interest rate, and φ_t is the foreign risk premium reflecting portfolio preferences, credibility effects, etc. Substracting the current real exchange rate from the expected real exchange rate and using the UIP condition, equation (25), we get an expression for the real exchange rate

$$q_t = E_t\left(q_{t+1}\right) - \left\{r_t - E_t\left(\pi_{t+1}\right)\right\} + \left\{r_t^* - E_t\left(\pi_{t+1}^*\right)\right\} + \varphi_t \tag{26}$$

As noted by Svensson (2000), the real UIP condition, equation (26), may give the impression that the real exchange rate is a nonstationary variable. However, in equilibrium all real variables, inflation rates and interest rates are stationary, implying that the real exchange rate is itself stationary.

2.7 World Economy

It is a standard practice to model foreign inflation and output with stationary univariate autoregressive processes. In that case, it is assumed that the foreign central bank follows a Taylor-type rule, that is, a rule that is a linear function of foreign inflation and output.

In this paper, we adopt an alternative approach; we model the rest of the world aggregate demand and supply equations allowing for habit formation and inflation inertia. Furthermore, we assume the foreign central bank pursues inflation targeting, and to that end it follows an IFB policy rule. In this framework, the structure of overseas shocks is not imposed in an arbitrary way; all shocks have dynamic properties that are entirely determined by the structure of the world economy. The advantage of this approach is that, by changing the structural coefficients of the foreign economy, we can alter the path of all foreign shocks faced by the small economy.

To derive a model for the world economy, we consider the small open economy model and set the openness coefficient, α^*, to zero. As a result, the world economy is characterized by the following four equations;

$$y_t^* = a_1^*\beta^* E_t\left(y_{t+1}^*\right) + a_1^* y_{t-1}^* - a_2^* n^* \rho_{t,n}^* + \epsilon_{y^*,t} \tag{27}$$

$$\pi_t^* = (1-\delta^*)\left(\beta E_t \pi_{t+1}^* + k^* \varpi^* y_t^*\right) + \delta^* \pi_{t-1}^* + \epsilon_{\pi^*,t} \tag{28}$$

$$r_t^* = \rho^* r_t^* + (1-\rho^*)\left(\rho_{\pi^*}^* E_t\left(\pi_{t+\tau^*}^*\right) + \rho_{y^*}^* y_t^*\right) + \epsilon_{r^*,t} \tag{29}$$

$$\rho_{t,n}^* = n^*\left\{E_t\left(\rho_{t+1,n}^*\right) - \rho_{t,n}^*\right\} + \left\{r_t^* - E_t\left(\pi_{t+k}^*\right)\right\} \tag{30}$$

Equation (27) is the aggregate demand in the world economy where, analogously to the small open economy, $a_1^* = \frac{\gamma^*(\sigma^*-1)}{\sigma^*+\gamma^*\beta^*[\gamma^*(\sigma^*-1)-1]} > 0$ and $a_2^* = \frac{1-\gamma^*\beta^*}{\sigma^*+\gamma^*\beta^*[\gamma^*(\sigma^*-1)-1]} > 0$. On the other hand, $\epsilon_{y^*,t}$ represent an overseas demand shock and, as before, the duration of the foreign bond, n^*, is assumed to be 40 quarters.

Equation (28), represents the CPI inflation in the rest of the world, where $k^* = \frac{(1-\phi^*)(1-\beta^*\phi^*)}{\phi^*(1+\omega^*\theta^*)} > 0$, $\varpi^* > 0$, and $\epsilon_{\pi^*,t}$ is an overseas cost-push shock. The δ^* coefficient represents the proportion of foreign backward-looking firms.

Equation (29), is the central bank's policy reaction function. We determine the optimal coefficients in such a rule by minimizing an inflation targeting loss criteria (see Section 4). The foreign monetary policy shock is represented by $\epsilon_{r^*,t}$.

Finally, equation (30) links the foreign short-term real interest rate and the long-term real return on a foreign bond, $\rho_{t,n}^*$.

3 Parametrization and Model Solution

All relevant coefficients governing the dynamics of the domestic and world economy depend on a few structural parameters. In the baseline parametrization we assume, for simplicity, that those parameters are the same for both the domestic and foreign economies. The model presented here is, however, very general and therefore alternative parametrizations, in which foreign a domestic coefficients are not the same, are also possible.

Fuhrer (2000, pp.377), reports the results of jointly estimating, by FIML, the degree of habit formation, γ, and the curvature of the utility function, σ. The results are $\gamma = 0.8$ and $\sigma = 6.11$. Accordingly, we use those values for both the domestic and world economy. Although Fuhrer's estimates are based on US data, the degree of habit formation observed in other countries is not very different. In fact, Batini et al (2003), set the degree of habit formation to 0.7, arguing that this value tends to fit the UK path of consumption. On the other hand, a value for σ of 6.11 may appear to be high, however, it is in line with estimates of the elasticity of substitution $(1/\sigma)$ for developed and developing countries reported in Agenor and Montiel (1999, pp.468).

As it is common in the literature, we set the quarterly discount factor, β to 0.99. This implies a riskless annual return of about 4% in steady-state. The degree of openness, α, is set to 0.3 as in

Parrado and Velasco (2002). This value is equivalent to the average share of imports to GDP for a small open economy like the Chilean one. This is consistent with Gali and Monacelli (2002) who set that coefficient to 0.4. The elasticity of substitution between home and foreign goods is set to $\eta = 1.5$, as assumed by Chari, Kehoe and McGrattan (2000) for the US economy and by Parrado and Velasco (2002) for Chile.

Following Rotemberg and Woodford (1997), we set the probability of keeping prices unchanged, ϕ, to 0.66 and the elasticity of substitution within each good category, θ, to 7.88. Those values imply that the frequency of price adjustment, 1/(1-ϕ), is three quarters, and the average mark-up in goods market is 15% $\left(\frac{\theta}{\theta-1} = 1.15\right)$. The degree of inflation persistence, δ, is set to 0.5, as in Furher and Moore (1995b). This is higher than the value reported in Gali an Gertler (1999), however, we also use less persistent specification for inflation (see Section 4). Finally, we set the elasticity of the disutility function v with respect to work, ϖ , to 0.6 as in Svensson (2000). For convenience, a summary with the baseline parametrization is presented in Table 1.

Table 1 Baseline Parametrization

Coefficient	Value	Source
Degree of habit formation, γ=γ^*	0.80	Fuhrer (2000)
Curvature of the utility function, σ=σ^*	6.11	Fuhrer (2000)
Discount factor, β=β^*	0.99	Rotemberg and Woodford (1997).
Degree of Openness, α	0.30	Parrado and Velasco (2002)
Elasticity between foreign and domestic goods, η=η^*	1.50	Chari et al (2000)
Probability of keeping prices unchanged, ϕ=ϕ^*	0.66	Rotemberg and Woodford (1997)
Elasticity of substitution within each good category, θ=θ	7.88	Rotemberg and Woodford (1997)
Proportion of backward looking firms, δ=δ^*	0.50	Fuhrer and Moore (1995b).
Elasticity of F' with respect to $y_{t+k}(j)$	0.60	Svensson (2000)

3.1 Solving the Model

The model, consisting of the domestic and world economy, is a linear perfect foresight one. The vector of domestic variables, $x_{H,t}$, contains seven elements; $x_{H,t} = (c_t, y_{t,} \pi_{H,t}, \pi_t, r_t, \rho_{n,t}, q_t)$, and the vector of foreign variables, $x_{F,t}$, contains four ; $x_{F,t} = (y^*_{t,} \pi^*_t, r^*_t, \rho^*_{n,t})$. The state representation of the whole system can be cast in the format

$$\sum_{i=1}^{\vartheta} H_i E_t x_{t+i} + \sum_{i=-k}^{0} H_i x_{t+i} = \epsilon_t \qquad (31)$$

where $x_t = (x_{H,t}, x_{F,t})'$ is the vector of the variables in the system, the H_i are square coefficient matrices, and $\epsilon_t = (\epsilon_{c,t}, 0, \epsilon_{\pi_H,t}, 0, \epsilon_{r,t}, 0, \varphi_t, \epsilon_{y^*,t}, \epsilon_{\pi^*,t}, \epsilon_{r^*,t}, 0)'$ is the vector of structural shocks. In equation (31), the coefficients ϑ and k represent the maximum number of leads and lags in the whole system, respectively.

As in Fuhrer and Moore (1995b), we assume $E_t(\epsilon_{t+i}) = 0$ for $i > 0$ and use the generalized saddlepath procedure of Anderson and Moore (1985) to solve the system in (31). For a given set of initial conditions, if the system has a unique solution that grows no faster than a given upper bound, this procedure generates a representation of the model that is called the observable structure[11]

$$S_0 x_t = S_{-1} x_{t-1} + \epsilon_t \tag{32}$$

Equation (32) is a structural representation of the model because it is driven by the structural disturbance vector, ϵ_t. The coefficient matrix S_0 contains the contemporaneous relationships among the elements of x_t. This is an observable representation of the model because it does not contain unobservable expectations.

Now it is possible to generate the reduced form of the structural model. In fact, premultiplying equation(32) by S_0^{-1} gives the autoregression

$$x_t = B_{-1} x_{t-i} + S_0^{-1} \epsilon_t \tag{33}$$

In order to generate impulse-responses functions of the estimated model, we use the VAR representation in (33), and the fact that S_0^{-1} and $B_{-1} \equiv S_0^{-1} S_{-1}$ are known, to compute the response of a variable i to a structural disturbance j, $\frac{\partial x_{i,t}}{\partial \epsilon_{jt}}$. In this way, all shocks are identified. This implies that the impulse-responses functions we compute have a structural interpretation.

4 Instrument Rules and the Exchange Rate

After solving the model, we generate 1000 stochastic shocks for each of the elements in ϵ_t[12]. Then, we perform a grid search[13] to determine the coefficients in the policy reaction function that minimize a given loss criteria. We optimize over the whole set of coefficients in equation (23). The only restriction we impose is that $\rho_\pi > 1$. This condition, known as Taylor Principle, ensures the system is not explosive.

4.1 Inflation Targeting Loss Criterion

Following Woodford (1999), we assume the central bank minimizes a loss criterion that depends on inflation, output and interest rate variability:

$$L = 2\sigma_\pi^2 + \sigma_y^2 + 0.5\sigma_r^2 \tag{34}$$

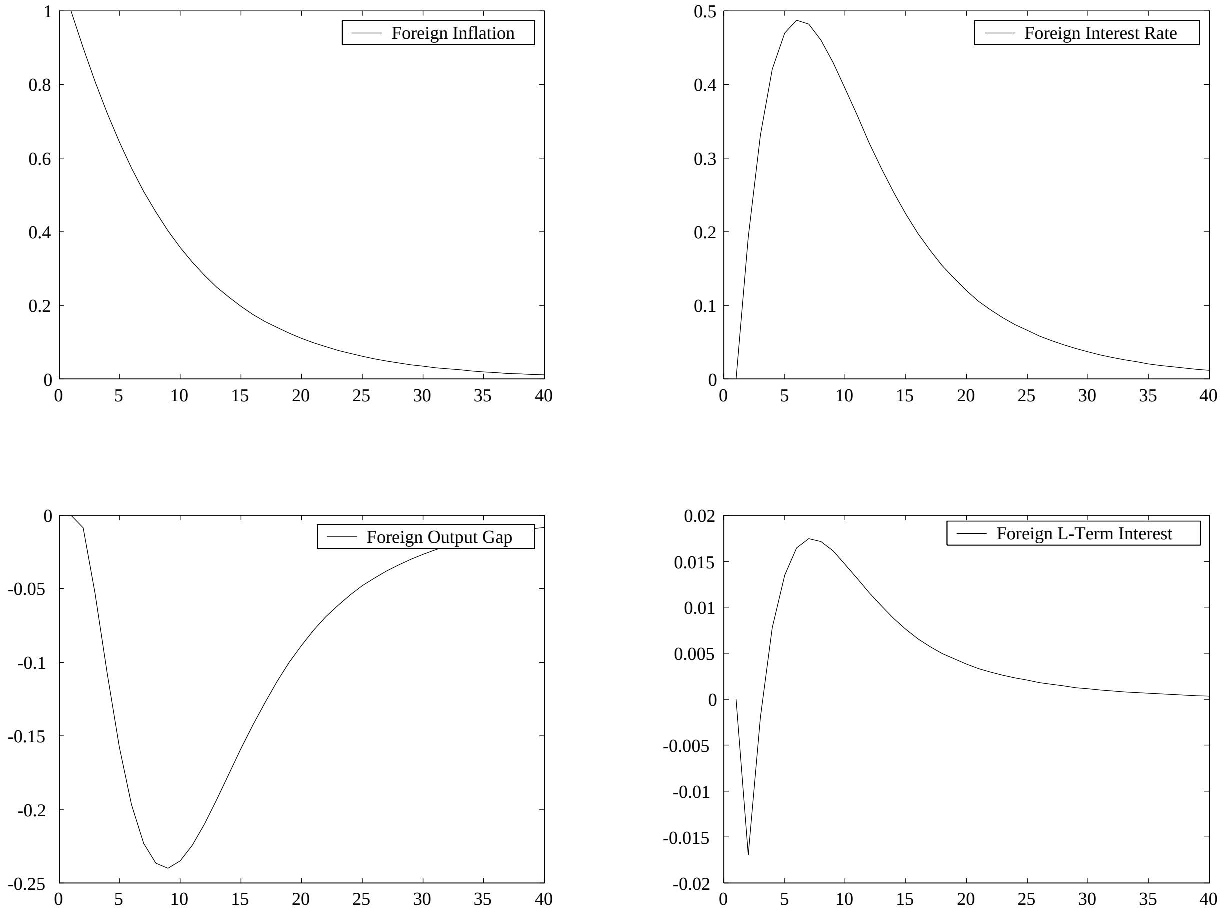
The above criterion is similar to Batini's et al (2003) one, and reflects the fact that inflation variability is the main concern of the central bank. In particular, the central bank penalizes CPI

[11] The number of lags in equation (32) is the same as the maximum number of lags in the whole system, k. In the model $k = 1$.

[12] Each shock is normally and independently distributed with mean of zero and a standard error of 1%.

[13] See Appendix 1 for a description of the algorithm.

Figure 1: Response of Foreign Variables to a 1% Cost-Push Shock



inflation volatility twice as heavily as output volatility. This criteria reflects, also, a preference for interest rate smoothing.

Before deriving the optimal reaction function for the small open economy, we find the optimal coefficients in the overseas' policy rule, equation (29). The coefficients, $\rho^* = 0.90$, $\rho^*_{\pi^*} = 5.9$, $\rho^*_{y^*} = 2.3$ and $\tau^* = 4$ minimize a criterion analogous to (34) for the world economy. Under this policy the dynamics of the foreign variables, when faced with a cost-push shock, are described in Figure 1. The foreign interest rate (nominal and real) increases in response to this shock. As a consequence, output contracts and foreign inflation adjust towards its equilibrium. The timing is roughly consistent with Rotemberg and Woodford (1997) results for the US, although in our specification both foreign output and the interest rate respond with more delay.

4.1.1 Optimal Rules

Given the overseas' reaction function, and the vector of shocks, it is now possible to derive the coefficients in the domestic reaction function[14], equation (23), that minimizes the criterion in (34). We perform the grid search for three possible values of τ; four quarters, one quarter and contemporaneous targeting ($\tau = 0$). We use two measures of inflation in the monetary policy rule; CPI and domestic inflation. The results are presented in Table 2

Table 2. Optimal Coefficients under Inflation Targeting Loss Criteria

Horizon and Inflation Measure	ρ	ρ_π	ρ_y	ρ_q	Loss Function
τ=4 and CPI Inflation	0.90	16.8	8.8	0.0*	8.349
	0.90	16.4	6.9	2.5	8.261
τ=1 and CPI Inflation	0.90	11.4	10.2	0.0*	8.397
	0.90	11.6	8.4	2.9	8.285
τ=0 and CPI Inflation	0.92	7.8	9.1	0.0*	8.713
	0.92	7.7	7.6	2.2	8.626
τ=4 and Domestic Inflation	0.90	17.6	8.6	0.0*	8.306
	0.90	17.1	7.1	1.9	8.263
τ=1 and Domestic Inflation	0.90	10.2	8.7	0.0*	8.312
	0.90	10.0	7.4	1.8	8.259
τ=0 and Domestic Inflation	0.92	7.3	9.3	0.0*	8.942
	0.92	7.1	7.6	2.2	8.849

(*) Restricted to be zero.

All policy rules perform better if a separate response to the exchange rate is included. Also IFB rules outperform Taylor-type ones. This result is robust to the value of τ and the measure of inflation used.

In general, policy rules present an important degree of persistence and respond more strongly to inflation than to output misalignments. Furthermore, the response to the real exchange rate is positive, but it is less important than the response to inflation and output.

Our qualitative results are in line with empirical evidence for six European countries and the USA presented in Clarida et al (1998 and 2000). In fact, they find a high degree of interest rate persistence, a stronger response to expected inflation, when compared to output, and a positive response to the exchange rate whose quantitative effects are, however, small. On the contrary, our quantitative results do differ from the estimates in Clarida et al (1998 and 2000); in our case, all

[14]We assess the performance of a policy rule that reacts to changes in the exchange rate, Δq_t, rather than to the level. This rule, however, implies a more aggressive policy response towards inflation and exchange rate. On the other hand, this policy has a better performance only in two of the seven potential shocks that may hit the economy: supply and exchange rate innovations.

the coefficients in the rule are bigger. In a calibrated model for the UK Batini et all (2003) also find bigger response to expected inflation, one period ahead, of $\rho_\pi = 10.15$. On the other hand, McCallum (2001), argues that a stronger response to output deviations, ρ_y, is desiderable when there is no uncertainty about the current level of output, which is the case in this model.

One explanation for the size of ρ_π and ρ_y is that they imply too much interest rate variability. In other words, the magnitude of ρ_π and ρ_y may reflect a low weight on the interest rate variance in the inflation targeting loss criterion. To see whether this is the case or not, we compare the simulated interest rate variance with the historical interest rate variance for the USA. The simulated variance is 3.93 whereas the variance of the FED's funds rate, during the Greenspan period (1987.09 to 2003.05), is 4.23. Hence, the simulated interest rate is not more volatile than the observed interest rate in the USA.

Finally, the size of ρ_π increases with the horizon to which the central bank targets inflation, τ. This fact is a consequence of the way in which cost-push shocks[15] are transmitted to inflation. In fact, the biggest impact of such a shock, in this model, occurs in the first quarters. As a consequence the required increase in the interest rate is met, for short horizons, with a relatively lower response to expected inflation, ρ_π.

4.1.2 Performance to Individual Shocks

The rule that minimizes the inflation targeting loss criterion is an IFB rule that targets domestic inflation with $\tau = 1$ (see Table 2). In this case, the overall gain from responding to the exchange rate is modest. However, for some specific shocks the gains are substantial (see Table 3). In particular, when the economy faces either an aggregate demand or a foreign cost-push shock, responding to exchange rate reduces welfare losses by 28% and 10% respectively.

Table 3. Performance of the Optimal Rule under Inflation Targeting Loss Criteria[16]
(loss for the rule with τ=1 and Domestic Inflation)

Shocks	Loss for ρ_q=0.0	Loss for ρ_q>0.0	Difference
Cost-Push, $\epsilon_{\pi,h}$	6.285	6.275	-0.2%
Interest Rate, ϵ_r	0.156	0.147	-5.7%
Demand, ϵ_c	0.067	0.048	-28.3%
Overall Domestic Shocks	6.496	6.468	-0.4%
Foreign Supply, ϵ_{π^*}	0.208	0.187	-10.0%
Foreign Interest Rate, ϵ_{r^*}	0.686	0.699	1.9%
Foreign Demand, ϵ_{y^*}	0.281	0.271	-3.8%
Real Exchange Rate, ϵ_q	0.293	0.289	-1.3%
Overall Domestic Shocks	1.553	1.533	-1.3%
Total Loss	8.312	8.259	-0.6%

[15] As we will see later on, cost-push shocks account for a significant proportion of the total loss.
[16] The results in Table 3 hold for the rest of rules.

Aggregate Demand Shocks What can explain the good performance[17] of such a rule in the face of aggregate demand shocks? The answer is that, by reacting to exchange rate deviations, the central bank is able to induce less volatility in output, CPI inflation and nominal interest rates. The precise mechanism can be understood by analyzing the impulse-response functions generated after a demand shock hits the economy (Figure 2).

In this case, an aggregate demand expansion induces an increase in both the short term interest rate ($\rho_y > 0$) and the long-term real interest rate, $\rho_{n,t}$. Because the real UIP condition holds, an increase in the real interest rate induces a real exchange rate appreciation. Both, the real exchange rate appreciation and the increase in real interest rate have a negative impact on output.

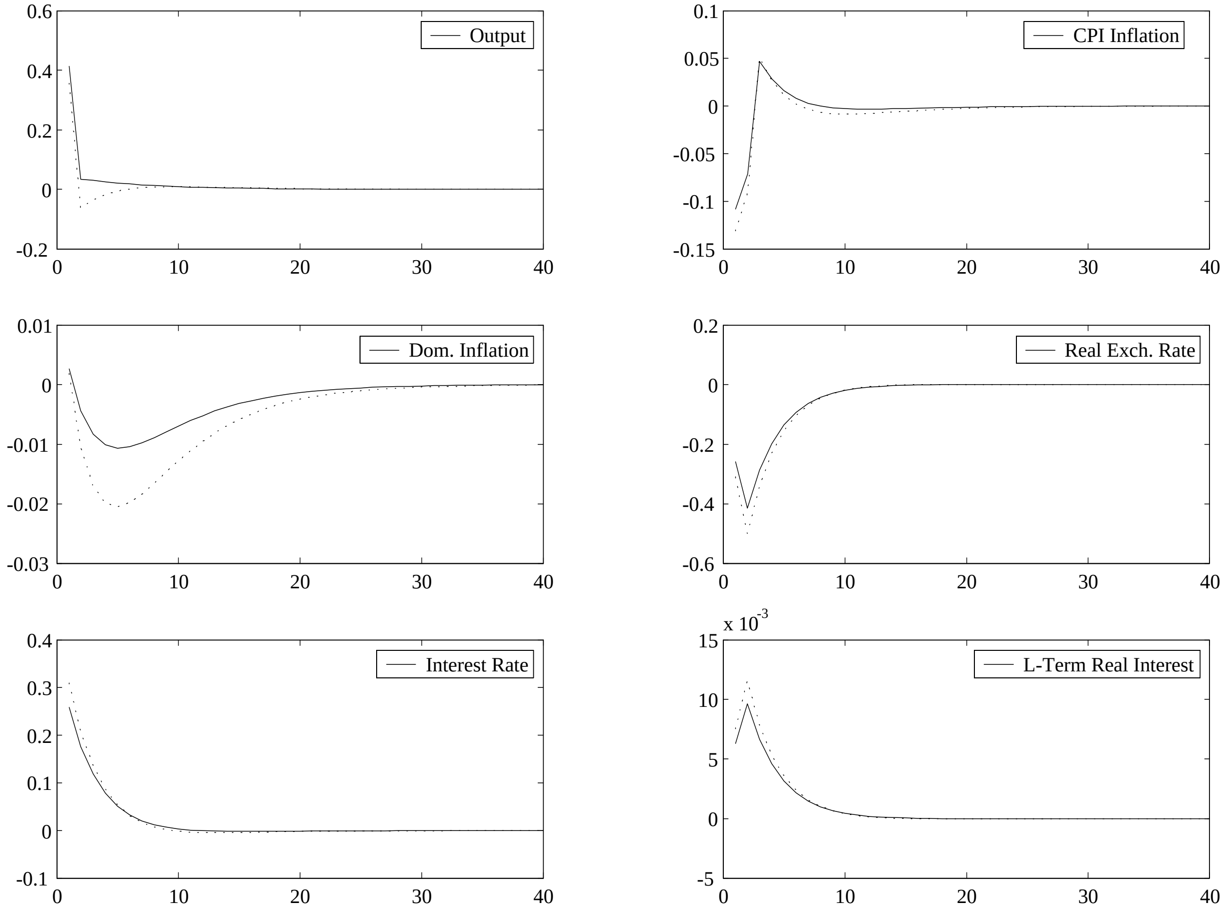
Now, if the central bank reacts to the exchange rate (solid line in Figure 2), the increase in interest rate and the real exchange rate appreciation will be lower. This is because the central bank offsets the real appreciation by partially reducing the interest rate. As a consequence, output will not contract as much and therefore its variability will be lower (first panel in Figure 2).

A similar argument explains why CPI inflation is also less volatile; a lower contraction in both output and the real exchange rate attenuate the reduction in domestic inflation (first column, second row in Figure2). This and the fact that the initial real appreciation is lower, contributes to a less volatile CPI inflation, in particular, in the first three quarters after the shock (second column, first row in Figure 2).

Finally, because the central bank offsets the real appreciation by partially reducing the interest rate, nominal and real interest rates become less volatile.

[17] The variance of output, inflation and interest rate is reduced when the central bank reacts to the real exchange rate.

Figure 2: Response of Domestic Variables to a 1% Aggregate Demand Shock (dotted line: Rule with $\rho_q = 0$; solid line: Rule with $\rho_q > 0$)



Foreign Inflation Shocks In the face of an overseas cost-push shock, a policy rule that reacts to the real exchange rate reduces the nominal interest rate and CPI inflation volatility. This comes, however, at the cost of inducing a higher output variance.

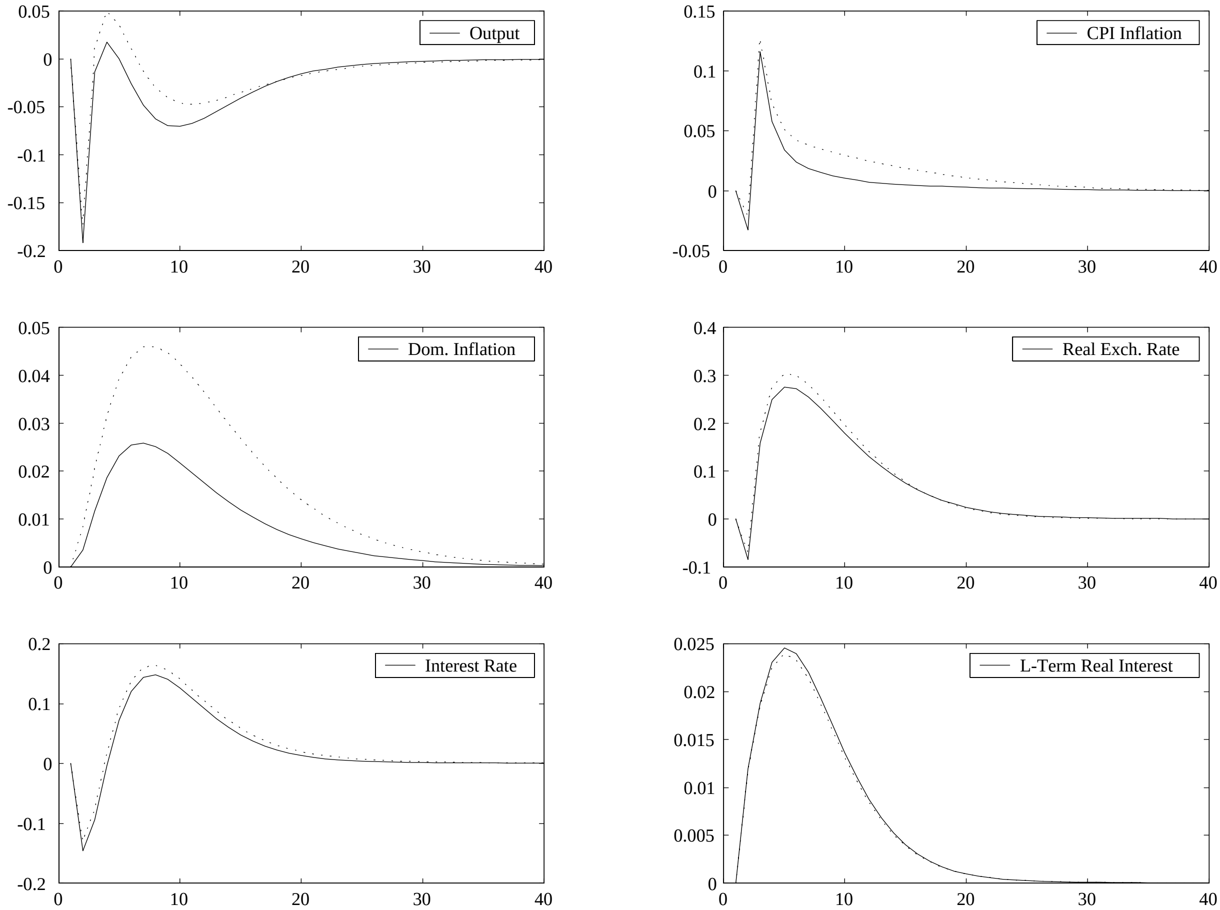
An overseas cost-push shock generates an endogenous increase in the nominal and real foreign interest rates (see Figure 1). As a consequence, the real exchange rate depreciates, increasing both domestic and CPI inflation (Figure 3).

Now, if the central bank reacts to this depreciation, it can attenuate the inflationary impacts of the cost-push shock. In fact, the central bank's reaction generates a lower real depreciation and a higher contraction in output. Both effects contribute to a lower domestic and CPI inflation.

This reaction has, however, a negative impact on output volatility. In fact, because the real interest rate is higher and the exchange rate is lower, output is always kept below the level it would reach in the absence of a central bank reaction. As a result, output becomes more volatile.

Finally, and somehow unexpected, the level of nominal interest rate is lower in the case in which the central bank does react to exchange rate. The reason is that, expected inflation is always lower in this case. This means that any increase in the real interest rate is met with a lower nominal rate. A confirmation of this argument is the fact that the real interest rate is consistently higher when the central bank does respond to real exchange rate (second column, third row in Figure 3)

Figure 3: Response of Domestic Variables to a 1% Foreign Inflation Shock (dotted line: Rule with $\rho_q = 0$; solid line: Rule with $\rho_q > 0$)



4.2 Utility-Based Loss Criterion

The inflation targeting loss criterion in (34) is often used as a metric to assess the performance of alternative policy rules. However, if not derived from first principles, this criterion may be an arbitrary one. In fact, it may not reflect the utility loss of households, and hence, its validity as a loss criterion may be undermine. To overcome this problem, we derive a utility-based criterion by taking a second order approximation of the consumer's utility function, equation (1). As a result, we obtain the following loss criterion[18];

$$W = L_1\sigma^2_{c_t} + L_2\sigma^2_{\pi_{H,t}} + L_3\sigma^2_{q_t} - L_4\sigma_{c_tc_{t-1}} + L_5\sigma_{c_tq_t} + L_6\sigma_{c_ty^*_t} + L_7\sigma_{q_ty^*_t} \tag{35}$$

where $L_i > 0$ for $i = 1....7$, and the L_i coefficients depend on the structural parameters of the economy.

The utility-based loss criterion penalizes the variability in both consumption and domestic inflation. A similar result is also found in the utility-based loss criterion derived for both closed and open economies; Woodford (2002) and Batini et al (2003). The relative importance of each component, in the baseline parametrization, is $L_1 = 8.5$ and $L_2 = 8.8$.

The real exchange variance, $\sigma^2_{q_t}$, enters the welfare criterion. The reason is that real exchange rate variability increases the volatility of output and total work in the economy, and this increases the welfare losses of the representative agent. The importance of this component is, however, modest when compared to consumption and domestic inflation; $L_3 = 0.4$. On the other hand, and as expected, the importance of this element depends on the degree of openness, α. In fact, $L_3 = (1-\gamma)(1+\omega)\left[\frac{\alpha(\eta^*+\eta-\alpha\eta)}{(1-\alpha)}\right]^2$ is an increasing function of α and goes to zero in a closed economy.

The autocovariance in consumption, $\sigma_{c_tc_{t-1}}$, enters the loss function with a negative sign. The importance of this element depends positively on the degree of habit formation;$L_4 = 2\gamma(\sigma-1)$. The intuition behind this result is that as γ increases, and habits become more important, consumers are less willing to substitute consumption over time. Therefore they penalize more strongly negative correlations between current and past consumption. Another way to look at this argument is to remember that the elasticity of consumption with respect to the long-term real interest rate, a_2 in equation (8), is a decreasing function of γ. This means that as γ increases a higher real interest rate is required in order to induce less consumption today and more in the future. In the baseline parametrization, $L_4 = 8.2$.

Finally, the elements $\sigma_{c_tq_t}$, $\sigma_{c_ty^*_t}$, and $\sigma_{q_ty^*_t}$ enter the loss function with relatively small weights; $L_5 = 0.5$, $L_6 = 0.1$, and $L_7 = 0.2$, respectively. Those elements affect welfare for the same reason; a positive correlation tends to increase the volatility of output and total work.

[18] See appendix 2 for derivation.

4.2.1 Optimal Rules under Utility-Based Loss Criterion

The optimal policy rules, according to the utility-based loss criterion in (35), are presented in Table 4[19]. The qualitative results do not change from the previous exercise; there is a high degree of persistence, even though interest rate variability does not enter the utility-based loss criterion. There are small gains from including a response to the real exchange rate, and as the targeting horizon increases, the response to expected inflation goes up.

Table 4. Optimal Coefficients. under Welfare Based Criteria

Horizon and Inflation Measure	ρ	ρ_π	ρ_y	ρ_q	Loss Function
τ=4 and CPI Inflation	0.92	62.4	5.7	0.0*	17.648
	0.92	63.8	3.5	2.9	17.580
τ=1 and CPI Inflation	0.89	36.2	8.1	0.0*	17.583
	0.89	35.8	5.4	3.3	17.467
τ=0 and CPI Inflation	0.88	22.4	5.7	0.0*	16.904
	0.88	22.4	4.7	1.4	16.872
τ=4 and Domestic Inflation	0.91	86.6	4.5	0.0*	17.614
	0.91	87.2	4.6	0.1	17.614
τ=1 and Domestic Inflation	0.90	24.0	3.7	0.0*	17.585
	0.91	26.5	3.3	1.1	17.561
τ=0 and Domestic Inflation	0.92	9.9	2.7	0.0*	17.752
	0.92	10.6	2.1	1.1	17.652

(*) Restricted to be zero.

There are, however, some striking difference with the **inflation targeting** rules presented in Table 3. First, the rules that minimize the utility-based criterion are much more aggressive towards inflation; the coefficient ρ_π is comparatively bigger. The reason is that, a more aggressive response contributes to a substantial decline in domestic inflation variability, $\sigma^2_{\pi_{H,t}}$, and to an increase in the autocovariance in consumption, $\sigma_{c_t c_{t-1}}$. This comes at the cost of inducing a higher volatility in consumption[20], $\sigma^2_{c_t}$, and the real exchange rate, $\sigma^2_{q_t}$. Of course, the relative improvements offset the cost of adopting a more aggressive policy rule.

The second difference has to do with the targeting horizon, τ. Now, the optimal reaction function is one that targets domestic inflation with $\tau = 0$ (see Table 4). The relative advantage of this rule, when compared to the rest, is that it increases the consumption autocovariance, $\sigma_{c_t c_{t-1}}$, and hence reduces the welfare losses. If the importance of the consumption autocovariance is set

[19] Given that the utility-based criterion depends on consumption variability, it would seem natural to include a response to consumption in the policy rule, rather than to output. Doing so does not generate, however, any improvement.

[20] Adopting a more aggressive policy rule has also the cost of inducing a higher volatility in both output and the interest rate. However, those elements do not enter the utility-based loss criterion.

to zero, $L_4 = 0$, then an IFB rule with $\tau = 4$ is the optimal reaction function. The intuition behind this result is that, increasing the smoothness in consumption, and hence the consumption autocovariance, $\sigma_{c_t c_{t-1}}$, comes at the cost of allowing higher volatility in interest rates, and hence in real exchange rate. Therefore, when $\sigma_{c_t c_{t-1}}$ is not considered in the analysis, rules with $\tau > 0$ do perform better.

4.2.2 Performance to Individual Shocks

As before, responding to real exchange rate misalignments does not generate substantial improvements from a utility-based perspective. For domestic interest rate and foreign inflation shocks the performance is better if a response to the exchange rate is included, however, the overall performance improves only marginally (Table 5).

Table 5 Performance of the Optimal Rule under Welfare-Based Loss Criteria[21]
(loss for the rule with τ=0 and CPI Inflation)

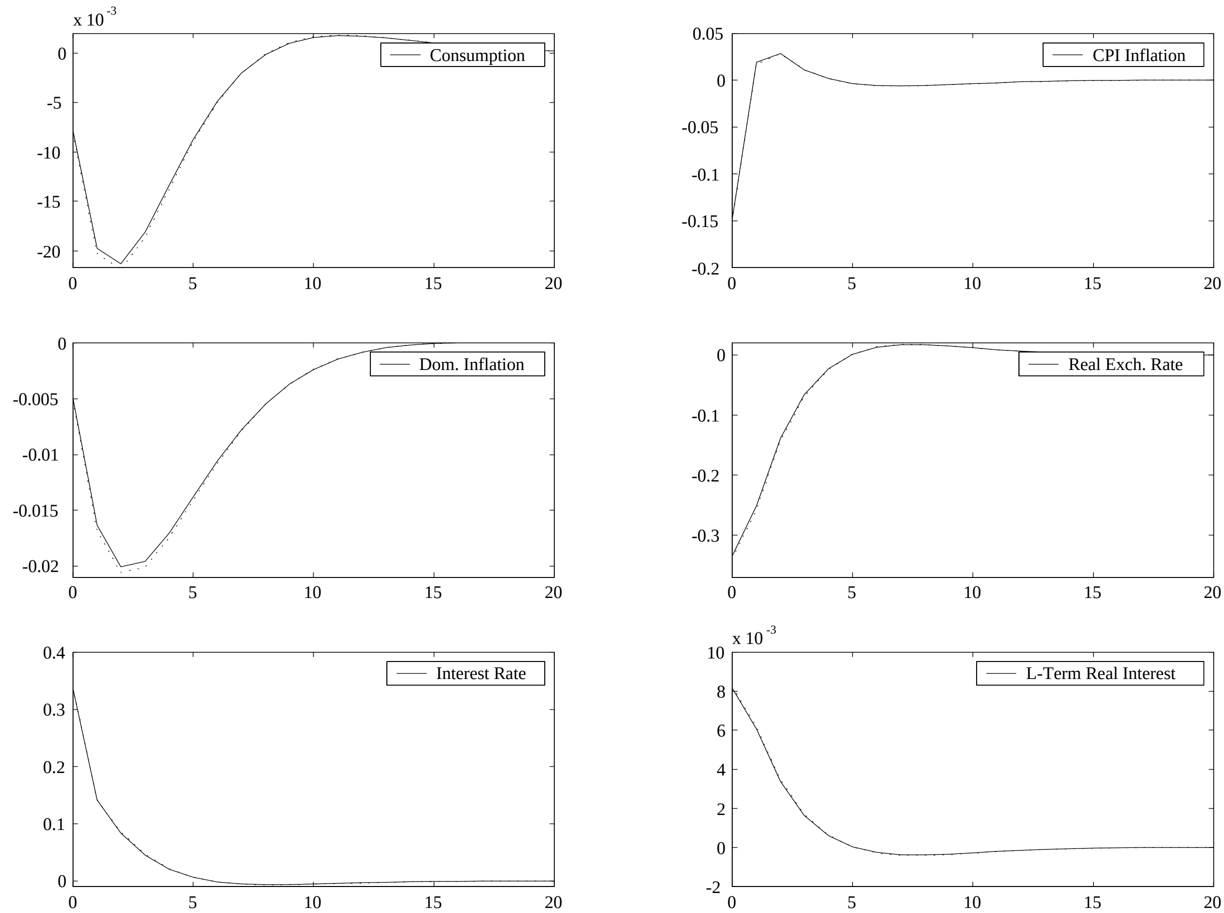
Shocks	Loss for ρ_q=0	Loss for ρ_q>0	Difference
Supply, $\epsilon_{\pi,h}$	13.031	13.011	-0.2%
Interest Rate, ϵ_r	0.067	0.064	-5.2%
Demand, ϵ_c	2.529	2.556	1.1%
Overall Domestic Shocks	15.689	15.704	0.1%
Foreign Supply, ϵ_{π^*}	0.293	0.277	-5.6%
Foreign Interest Rate, ϵ_{r^*}	0.268	0.273	1.8%
Foreign Demand, ϵ_{y^*}	0.220	0.222	1.0%
Real Exchange Rate, ϵ_q	0.0048	0.0052	7.0%
Overall Foreign Shocks	0.786	0.777	-1.2%
Total Loss	16.904	16.973	-0.2%

Overseas Inflation Shocks As discussed before, a foreign inflation shock induces a real depreciation. This generates an expansion in domestic inflation, the output level and the long-term real interest rate. Now, if the central bank reacts to this depreciation, domestic inflation and the real exchange will increase less. As a result, their volatility will be lower. The precise mechanism, in terms of the impulse-response analysis, is very similar to that described in Figure 3.

Interest Rate Shock In the face of monetary policy shocks, responding to the real exchange rate reduces the welfare losses from a utility-based perspective. To illustrate the mechanism behind this result, Figure 4, present the impulse-response functions after this shock hits the economy. Initially, an increase in the interest rate generates an real appreciation. However, if the central banks reacts to the real exchange rate, the appreciation and the increase in interest rate will be marginally lower. As a consequence, both domestic inflation and consumption contract less, and their variability is reduced. In addition, responding to the exchange rate increases the autocovariance in consumption.

[21] The results in Table 5 hold for the rest of the rules.

Figure 4: Response of Domestic Variables to a 1% Domestic Interest Rate Shock (dotted line: Rule with $\rho_q = 0$; solid line: Rule with $\rho_q > 0$)



5 Persistence Analysis

This section explores the consequences, in terms of the optimal policy rule, of different degrees of endogenous inertia in the economy. In particular, we induce less persistence in output and inflation and then search for the optimal policy reaction function.

We consider three scenarios. First, we induce less persistence in the consumption equation by reducing the habit formation coefficient from $\gamma = \gamma^* = 0.8$ to $\gamma = \gamma^* = 0.1$. The degree of inflation inertia is kept at its initial level, $\delta = \delta^* = 0.5$. The second scenario keeps the habit coefficient constant, $\gamma = \gamma^* = 0.8$, and reduces the inflation persistence to $\delta = \delta^* = 0.1$. Finally, we consider a scenario in which consumption and inflation inertia are reduced simultaneously; $\gamma = \gamma^* = \delta = \delta^* = 0.1$.

In the first exercise the **inflation targeting** loss criterion in (34) is minimized. In each scenario an IFB rule, targeting domestic inflation one quarter ahead, is used. The results are presented in Table 6. As a general result, adopting a rule that reacts to the real exchange rate reduces the loss function. When compared to the initial policy rule in Table 3, inducing less persistence attenuates the policy response to inflation, output and exchange rate. On the other hand, the degree of interest rate smoothing, ρ, is reduced considerably when inflation is less persistent.

Table 6. Optimal Coefficients under Inflation Targeting Loss Criteria
(rule with $\tau = 1$ and Domestic Inflation)

Alternative Scenario	ρ	ρ_π	ρ_y	ρ_q	Loss Function
δ^*=δ=0.5;γ^*=γ=0.1	0.86	7.6	6.1	0.0*	7.739
	0.85	6.7	4.5	1.1	7.709
δ^*=δ=0.1;γ^*=γ=0.8	0.53	24.9	4.5	0.0*	1.420
	0.36	1.1	3.5	1.6	1.356
δ^*=δ=γ^*=γ=0.1	0.25	25.3	4.0	0.0*	0.743
	0.00	1.1	1.0	1.5	0.678

(*) Restricted to be zero.

The second exercise minimizes the utility-based loss criterion in (35). A policy rule, targeting CPI inflation to an horizon of $\tau = 0$, is used. The results, presented in Table 7, are similar to those in the previous exercise. In most cases, adopting a rule that reacts to the real exchange rate reduces the loss function. The policy responses to inflation, output and exchange rate is, in most cases, lower than the responses in the baseline case, Table 5. Finally, the degree of interest rate inertia is also lower.

Table 7. Optimal Coefficients under Utility-Based Loss Criteria
(rule with $\tau = 0$ and CPI Inflation)

Alternative Scenario	ρ	ρ_π	ρ_y	ρ_q	Loss Function
δ^*=δ=0.5;γ^*=γ=0.1	0.79	11.7	2.7	0.0*	65.900
	0.83	14.7	2.3	1.6	65.386
δ^*=δ=0.1;γ^*=γ=0.8	0.60	1.1	12.3	0.0*	3.223
	0.60	1.1	12.3	0.0	3.223
δ^*=δ=γ^*=γ=0.1	0.32	1.1	7.2	0.0*	2.819
	0.28	1.1	1.9	0.9	2.809

(*) Restricted to be zero.

5.1 Interest Rate Inertia and Inflation Persistence

In the previous exercises, a lower degree of interest rate inertia is the consequence of a lower degree of inflation persistence. To understand this result we analyze the performance of two different rules. The first one, R1, is the rule that minimizes the inflation targeting criterion in the baseline scenario; δ^{*}=δ=0.5 and γ^{*}=γ=0.8. This rule has a degree of interest rate inertia of $\rho = 0.90$ (see Table 3). The second rule, R2, minimizes the same criterion when domestic inflation is less persistent environment; δ^{*}=δ=0.1 and γ^{*}=γ=0.8. This rule has a degree of interest rate inertia of $\rho = 0.36$ (see Table 6).

For the baseline scenario, we assess the relative performance of R1 and R2 to three different shocks[22]; cost-push, monetary policy and real exchange rate innovations. The results, in terms of the **inflation-targeting** loss criterion, are presented in Table 8;

Table 8. Loss under Alternative Rules in the Baseline Scenario
(Baseline Case:δ^{*}=δ=0.5;γ^{*}=γ=0.8)

Shocks	R1 (ρ=0.90)	R2 (ρ=0.36)
Supply, $\epsilon_{\pi,h}$	6.275	9.882
Interest Rate, ϵ_r	0.147	0.017
Real Exchange Rate, φ	0.289	0.034
Total Loss	8.259	11.820

R1: ρ=0.90, ρ_π=10.0, ρ_y=7.4 and ρ_q=1.8
R2: ρ=0.36, ρ_π=1.1, ρ_y=3.5 and ρ_q=1.6

The results in Table 8 indicate that a less inertial policy rule, R2, increases the loss function in the faces of cost-push shocks, $\epsilon_{\pi,h}$. On the contrary, that rule leads to a lower welfare loss in the face of interest rate, ϵ_r, and real exchange rate shocks, φ. The total loss is dominated by the cost-push shock. Therefore the benefits of adopting a more inertial rule, R1, offset the costs.

Now, we assess the relative performance of those rules, in a scenario with less inflation inertia. The results are presented in Table 9.

Table 9. Loss under Alternative Rules in the Less Persistent Inflation Scenario
(Less Persistent:δ^{*}=δ=0.1;γ^{*}=γ=0.8)

Shocks	R1 (ρ=0.90)	R2 (ρ=0.36)
Supply, $\epsilon_{\pi,h}$	0.028	0.029
Interest Rate, ϵ_r	0.157	0.017
Real Exchange Rate, φ	0.299	0.035
Total Loss	1.630	1.356

R1: ρ=0.90, ρ_π=10.0, ρ_y=7.4 and ρ_q=1.8
R2: ρ=0.36, ρ_π=1.1, ρ_y=3.5 and ρ_q=1.6

[22]For the remaining shocks, the relative performance changes only marginally. Hence, for simplicity, we do not present those results.

In this scenario, a more inertial rule, R1, is still the optimal one if the economy is hit by a cost-push shock. As before, this rule leads to higher welfare losses when an interest rate or exchange rate shock occurs. In this case, however, the total loss is dominated by the last two shocks, ϵ_r, and φ. Hence, the advantages of adopting a more inertial rule, R1, are offset by the costs.

As a general result, the advantages of adopting a more inertial policy rule depend on the degree of inflation inertia. In fact, if inflation is more persistent, supply shocks will take longer to die out. As a consequence, the variability in inflation and output increases[23]. This explains why, in absolute terms, the loss associated with a supply shock is higher when inflation is more persistent. On the contrary, the costs of a more persistent rule, in terms of the welfare losses associated with interest rate and real exchange rate shocks, change only marginally with inflation persistence. As a result, the advantages of adopting a persistent policy rule are directly related to the degree of persistence in the inflation equation. This result is also present in Batini and Haldane (1999), who pointed out that higher degrees of smoothing deliver more persistent interest rate responses who tend to reduce inflation volatility.

5.2 Model Persistence or Persistent Shocks?

In the model considered in this paper all structural shocks in the economy are white noise, and hence non persistent. Persistence is introduced through the inertial components in the consumption and inflation equations. In this subsection we explore the consequences of introducing persistence in a different way. In particular, we analyze whether persistent shocks generate similar results, in terms of the inertia in the instrument rule, than model persistence. In doing so, we assume that model persistence is low. In particular, we impose a low degree of habit formation and inflation inertia, δ^{*}=δ=γ^{*}=γ=0.1. On the other hand, we drop the assumption that aggregate demand and supply shocks are white noise. In particular, we model those innovations as autoregressive processes of order one. In this case those shocks evolve as

$$\xi_{\pi_H,t} = \varphi_\pi \xi_{\pi_H,t-1} + \epsilon_{\pi_H,t}$$

$$\xi_{c,t} = \varphi_c \xi_{c,t-1} + \epsilon_{c,t}$$

where $\epsilon_{\pi_H,t}$ and $\epsilon_{c,t}$ are the supply and aggregate demand shocks considered in the previous sections, and $\xi_{\pi_H,t}$ and $\xi_{c,t}$ are aggregate demand and supply shocks that are potentially persistent. Notice that in the previous sections we have implicitly assumed that $\varphi_c = \varphi_\pi = 0$. Now, we consider three alternative scenarios that generate persistent shocks. First, we introduce persistence only in the supply shocks. In particular, $\varphi_\pi = 0.9$ and $\varphi_c = 0$. In the second scenario, persistence is present only in the aggregate demand innovations, $\varphi_\pi = 0$ and $\varphi_c = 0.9$. Finally, we assume persistence in both shocks, $\varphi_c = \varphi_\pi = 0.9$. In each scenario we find the optimal instrument policy rule using both the inflation targeting and the utility-based loss criteria. The results, are presented in Table 10 and 11.

[23] Furher (1997) shows that the variability of the output gap and inflation increases with the degree of persistence in the economy. This is also a feature of the model presented in this paper.

Table 10. Optimal Coefficients under Inflation Targeting Loss Criteria
(rule with $\tau = 1$ and Domestic Inflation)

Alternative Scenario	ρ	ρ_π	ρ_y	ρ_q	Loss Function
φ_π=0.9 ; φ_c=0	0.44	9.3	7.4	0.0*	0.882
	0.00	1.1	1.1	1.6	0.806
φ_π=0 ; φ_c=0.9	0.24	23.5	1.1	0.0*	0.981
	0.00	1.1	0.2	1.5	0.714
φ_π=φ_c=0.9	0.29	14.2	1.6	0.0*	1.173
	0.00	1.1	0.3	1.6	0.843

(*) Restricted to be zero.

Table 11. Optimal Coefficients under Utility-Based Loss Criteria
(rule with $\tau = 0$ and CPI Inflation)

Alternative Scenario	ρ	ρ_π	ρ_y	ρ_q	Loss Function
φ_π=0.9 ; φ_c=0	0.62	2.4	5.9	0.0*	5.167
	0.38	1.1	1.2	1.1	5.143
φ_π=0 ; φ_c=0.9	0.19	1.1	8.2	0.0*	2.945
	0.25	1.1	1.7	1.0	2.942
φ_π=φ_c=0.9	0.11	3.4	8.8	0.0*	5.327
	0.38	1.1	1.2	1.1	5.279

(*) Restricted to be zero.

When the policy response to exchange rate, ρ_q, is set to zero persistent inflation shocks generate the higher level of policy inertia. This result is independent from the welfare loss criterion used. However, the degree of policy inertia induced by persistent inflation shocks is well below the level of inertia generated in the baseline scenario (see results in Table 2 and Table 4).

From the previous results we can concluded that model persistence is more important in explaining policy inertia than persistent shocks. One of the reasons behind this result, is that model persistence generates spillover effects from the other equations in the system that feedback to inflation. As a result, inflation volatility is higher under model persistence than under innovation persistence. To explain the intuition behind this argument, consider the impact of a supply shock. When model persistence is present a supply shock will impact inflation directly, on impact and also in the future (because the lagged value of inflation appears in the supply equation). This shock will generate, also, an indirect impact on inflation. In fact, if inflation deviates from target, as a consequence of this shock, interest rates, output and exchange rate will also move from their equilibrium levels. Those movements will feedback to the inflation equation and will be transmitted to future levels of inflation via the lagged value of inflation in the supply equation. On the contrary, when model persistence is zero, the indirect effects are not transmitted to inflation in a persistence way because the lagged inflation coefficient is not present in the supply equation. As a consequence inflation variability will be lower[24].

We believe that the fact that inflation is more volatile under model persistence, explains why there are advantages of adopting a more inertial policy rule. In fact, as it was showed in the previous subsection, a higher degree of policy inertia tends to reduce inflation volatility. Hence, the advantages of adopting a more inertial policy rule are more important when model persistence induces a higher level of inflation volatility.

6 Conclusions

This paper addresses two important questions about the design of monetary policy today: the role of the exchange rate, and the interpretation of the smoothness coefficient in the instrument policy rule. In doing so, we derive an optimizing open economy model that allows for endogenous persistence. In this framework, we derive optimal policy rules according to an inflation targeting and a utility-based welfare criterion.

In general, a monetary policy rule that reacts to inflation, output and exchange rate is the optimal one. The overall gains from responding to the real exchange rate are small. However, for some specific domestic and foreign shocks, those gains are substantial. In particular, when the economy faces aggregate demand and foreign supply shocks, responding to the exchange rate reduces the volatility in output, domestic inflation and interest rates.

When the deep parameters of the domestic economy are modified, in order to induce less persistence in the economy, the rules that react to the real exchange rate still perform well. Hence, a general result of this paper is that, independently of the loss criterion considered, the degree of model persistence or the functional form of the policy rule, there are gains from responding to real exchange rate deviations.

A second set of results, relates the inertial behavior in the policy rule to the persistence in the economy. In fact, in a less persistent environment, the monetary policy inertia is reduced. This is in line with the arguments presented in Rudebusch (2002), suggesting that interest rate inertia and persistent shocks are related. In particular, the advantages of adopting an inertial policy rule increases with the degree of inflation persistence. In this context, model persistence is more important than shock persistence in explaining the observed inertia in the policy interest rate.

A third contribution of this paper is that it develops an optimizing macroeconomic model in which persistence depends on two structural coefficients: the habit formation and the inflation persistence coefficients. In this way, different degrees of model persistence can be induced in a structural way, without resorting to ad-hoc assumptions. In this context, we explicitly derive

[24] Welfare losses are also lower in the absence of model persistence (compare the welfare losses in Table 10 and 11 to those in Table 2 and 4).

a utility-based welfare criterion consistent with the structure of the model. In particular, the coefficients in this criterion do depend on the structure of the economy, and have, therefore, a structural interpretation.

This model is calibrated according to standard values given in the related literature. The shocks that hit the economy are all assumed to be normally and independently distributed with the same variance. This is, however, not necessarily true in practice. In this sense, a direction for future research is to estimate empirically this type of structural models to analyze the performance of alternative policy rules in small open economies. An empirical estimation of this model may be useful to explore questions about the design of optimal policies rules and the relative importance of each of the shocks that hit the economy.

References

[1] Agenor and Montiel (1999) Development Macroeconomics. Princeton University Press.

[2] Anderson, G. and Moore, G (1985) "A Linear Algebraic Procedure for Solving Linear Perfect Foresight Models" Economic Letters, XVII, 247-52.

[3] Amato, J. and Laubach, T. (2001). "Implications of Habit Formation for Optimal Monetary Policy". Federal Reserve, Working Paper.

[4] Ball, L. (1999)"Policy Rules for Open Economies" in .Taylor, J. Monetary Policy Rules. The University of Chicago Press.

[5] Batini, N and Haldane, A.(1999) "Forward-Looking Rules for Monetary Policy" in .Taylor, J. Monetary Policy Rules. The University of Chicago Press.

[6] Batini, N., Harrison, R., and Millard, S. (2003). "Monetary Policy Rules for Open Economies".Journal of Economics Dynamics and Control. Forthcoming.

[7] Bernanke, B., Laubach, T., Mishkin, F. and Posen, A. (1999) Inflation Targeting. Lessons from the International Experience.Princeton University Press.

[8] Calvo, G.(1983). "Staggered Prices in a Utility Maximizing Framework". Journal of Monetary Economics, 12, pp.383-98.

[9] Caputo, R. (2002). "Exchange Rate, Inflation and Monetary Policy Objectives in Open Economies: The Chilean Experience"mimeo. Cambridge University.

[10] Chari, Kehoe and McGrattan (2000)"Can Sticky Prices Explains Exchange Rate Persistence?" FRB Working Papers.

[11] Clarida, R.; Gali, J.; Gertler, M.(1998) "Monetary Policy Rules in Practice: Some International Evidence". European Economic Review, June. pp.1033-67.

[12] Clarida, R.; Gali, J.; Gertler, M.(2000) "Monetary Policy Rules and Macroeconomic Stability: Theory and Evidence". Quarterly Journal of Economics, January. pp.147-180.

[13] Clarida, R., Gali, J., Gertler,M. (2001)."Open versus Closed Economies an Integrated Approach"American Economic Review. May.pp 263-67.

[14] English, W., Nelson, W. and Sack, B. (2002) "Interpreting the Significance of the Lagged Interest Rate in Estimated Monetary Policy Rules". Federal Reserve Working Paper N.224

[15] Fuhrer, J. (1997) "The (Un)Importance of Forward-Looking Behavior in Price Specifications" Journal of Money, Credit and Banking, Vol.29, N.3, pp.339-50.

[16] Fuhrer, J. (2000). "Habit Formation in Consumption and Its Implications for Monetary-Policy Models". American Economic Review, June, vol. 90. n3.

[17] Fuhrer, J. and G. R. Moore. (1995a) "Monetary Policy Trade-offs and the Correlation between Nominal Interest Rates and Real Output", American Economic Review, 1995, 85 No 1.

[18] Furher, J. and Moore, G. (1995b)"Inflation Persistence"Quarterly Journal of Economics.February.

[19] Gali and Gertler (1999). "Inflation Dynamics: A Structural Econometrics Analysis". Journal of Monetary Economics., vol.44, pp.195-222.

[20] Gali, J. and Monacelli, T. (2002), "Optimal Monetary Policy and Exchange Rate Volatility in a Small Open Economy". NBER Working Paper.

[21] Kozicki, S. and Tinsley, P. (2002) "Dynamic Specifications in Optimizing Trend-Deviation Macro Model". Journal of Economic Dynamics & Control.

[22] Leitemo, K. (2003) "Optimal Perception of Inflation Persistence at an Inflation-Targeting Central Bank". mimeo. Norwegian School of Management.

[23] Leitemo, K. and Soderstrom, U. "Simple Monetary Policy Rules and Exchange Rate Uncertainty" mimeo. Norwegian School of Management.

[24] McCallum, B. (2001). "Should Monetary Policy Respond Strongly to Output Gaps?"American Economic Review. May.pp 258-62.

[25] Rotemberg, J. and Woodford, M. (1997) "An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy: Expanded Version". NBER Technical Working Paper Series.

[26] Parrado, E. (2000) "Inflation Targeting and Exchange Rate Rules in an Open Economy". mimeo. New York University.

[27] Parrado, E. and Velasco, A. (2002)."Alternative Monetary Rules in the Open Economy: A welfare Based Approach". Fourth Annual Conference of the Central Bank of Chile.

[28] Rudebusch, G. (2002) "Term Structure Evidence on Interest Rate Smoothing and Monetary Policy Inertia". Journal of Monetary Economics, 49. 1161-1187.

[29] Schmidt-Hebbel, K. and Werner, A. (2002). "Inflation Targeting in Brazil, Chile and Mexico: Performance, Credibility, and the Exchange Rate". Journal of the Latin American and Caribbean Economic Association. Vol. 2 N.2 pp. 31-79.

[30] Svensson, L. (2000)."Open-Economy Inflation Targeting". Journal of International Economics, February.

[31] Svensson, L. (1998)."Open-Economy Inflation Targeting". Working Paper IISE.

[32] Svensson, L. (2001). "Discussion of Clarida, Gali and Gertler, Optimal Monetary Policy in Open versus Closed Economies: An Integrated Approach," American Economic Association Annual Meeting, January 5, 2001.[*http://www.princeton.edu/~svensson/papers/cggoh.pdf*].

[33] Taylor, J. (1999) "The Robustness and Efficiency of Monetary Policy Rules as a Guidelines for Interest Rate Setting by the European Central Bank." Journal of Monetary Economics, June. pp.655-79.

[34] Taylor, J. (2001). "The Role of the Exchange Rate in Monetary-Policy Rules" American Economic Review. May.pp 263-67.

[35] Woodford, M. (1999)."Optimal Monetary Policy Inertia". NBER Working Paper 7261.

[36] Woodford, M. (2002) "Inflation Stabilization and Welfare" mimeo Princeton University.

Appendix 1
Grid Search

The grid search procedure aims to obtain the coefficients in the policy reaction function that minimize a given loss criteria, W. In particular, given the instrument policy rule

$$r_t = \rho r_{t-1} + (1-\rho)\left(\rho_\pi E_t\left(\pi_{t+\tau}\right) + \rho_y y_t + \rho_q q_t\right)$$

the procedure finds the values of $\left(\rho, \rho_\pi, \rho_y, \rho_q\right)$ that minimize either the inflation targeting loss criteria in (34) or the utility-based loss criteria in (35).

The grid search is an iterative process. For the initial values of the coefficients, $\rho = 0.0, \rho_\pi = 1.1, \rho_y = 0.0, \rho_q = 0.0$, it first finds the value of ρ_y that minimizes W. Then it updates the value of ρ_y and finds the value of ρ_q (when this coefficient is not restricted to be zero) that minimizes the given criterion. Once ρ_q has been updated, it finds the value of ρ. Finally, the algorithm search for the value of ρ_π given the all the previous coefficients. This process is then repeated until there is no change in the loss function. On average each iteration takes one minute and it may take up-to 200 iterations to find the coefficients in the policy rule.

There is no upper bound limit for the value that the coefficients can take. The only restriction that we impose is that ρ_π = 1.1. This avoid indeterminacy in the system.

Appendix 2
First Order Conditions of Utility Maximization

Individuals maximize

$$E_0\left\{\sum_{t=0}^{\infty}\beta^t\left[u\left(C_t, C_{t-1}\right) - V(N_t)\right]\right\}$$

where $u\left(C_t, C_{t-1}\right) = \frac{(C_t C_{t-1}^{-\gamma})^{1-\sigma}}{1-\sigma}$, subject to the budget constraint, expressed in real terms:

$$C_t + \frac{B_t}{P_t} = \frac{(1-\tau)W_t N_t}{P_t} + R_t^1\frac{B_{t-1}}{P_t} + \frac{T_t}{P_t}$$

The first order conditions (FOC) for the utility maximization, subject to the above budget constraint are:

$$E_t\left\{C_t^{-\sigma}C_{t-1}^{-\gamma(1-\sigma)} - \beta\gamma C_{t+1}^{1-\sigma}C_t^{-\gamma(1-\sigma)-1} - \Lambda_t\right\} = 0 \tag{36}$$

$$E_t\left\{-\Lambda_t + \beta R_t^1 \Lambda_{t+1}\right\} = 0 \tag{37}$$

$$-V_N + \Lambda_t\frac{W_t}{P_t} = 0 \tag{38}$$

Now, we log-linearize (37) around the steady state to obtain the following expression

$$\begin{aligned}
\lambda_t &= E_t\left\{\rho_t - \lambda_{t+1}\right\} \\
&= E_t \sum_{i=0}^{\infty} \rho_{t+i} \\
&\simeq E_t \sum_{i=0}^{n} \rho_{t+i} \\
&= n\rho_{t,n}
\end{aligned}$$

where $\rho_t = r_t - E_t\left(\pi_{t+1}\right)$ is the real, ex-ante, interest rate and $\rho_{t,n}$ is the real rate on a n-period bond, both expressed as a percentage deviation from the steady state. It is assumed that after n periods the interest rate does not deviate. Now, the above expression combined with the log-linearized equation (36) gives the following equation for consumption, equation (8) in the main text:

$$c_t = a_1 \beta E_t(c_{t+1}) + a_1 c_{t-1} - a_2 n \rho_{t,n}$$

where $a_1 = \frac{\gamma(\sigma-1)}{\sigma+\gamma\beta[\gamma(\sigma-1)-1]} > 0$ and $a_2 = \frac{1-\gamma\beta}{\sigma+\gamma\beta[\gamma(\sigma-1)-1]} > 0$.

Steady State From the FOC equations (36) to (38), we derive the following expressions in steady state

$$C^{\sigma\gamma-\sigma-\gamma}(1-\beta\gamma) = \Lambda \tag{39}$$

$$R^1 = \frac{1}{\beta} \approx 1 \text{ when } \beta = 0.99 \tag{40}$$

$$V_N = \Lambda \frac{W}{P} \tag{41}$$

$$C + \frac{B}{P} = \frac{(1-\tau)WN}{P} + R^1 \frac{B}{P} + \frac{T}{P} \tag{42}$$

Now, assuming a fiscal balance in steady state, $\tau WN = T$, the FOC in (42) can be expressed

$$\frac{C}{N} = \frac{W}{P}$$

which combined with (41) gives

$$
\begin{aligned}
V_N &= \Lambda \frac{W}{P} \\
&= \Lambda \frac{C}{N} \\
V_N N &= \Lambda C
\end{aligned}
$$

Finally, given that in steady state $u_c(1-\gamma) = \Lambda$, the above expression becomes

$$V_N N = u_c(1-\gamma)C$$

Utility-Based Loss Function

We derive a second order approximation of the representative consumer's utility function around the steady state

$$U_t = u\left(C_t, C_{t-1}\right) - V(N_t)$$

the first term on the right hand side, $u\left(C_t, C_{t-1}\right)$, can be approximated as

$$
\begin{aligned}
u\left(C_t, C_{t-1}\right) &= \bar{u} + u_{c_t}\widetilde{C}_t + u_{c_{t-1}}\widetilde{C}_{t-1} + \frac{u_{c_t c_t}\widetilde{C}_t^2}{2} + \frac{u_{c_{t-1}c_{t-1}}\widetilde{C}_{t-1}^2}{2} + \\
&+ \frac{u_{c_t c_{t-1}}\widetilde{C}_t \widetilde{C}_{t-1}}{2} + \frac{u_{c_{t-1}c_t}\widetilde{C}_{t-1}\widetilde{C}_t}{2} \qquad (43)
\end{aligned}
$$

where u_{c_t}, $u_{c_{t-1}}$, $u_{c_t c_t}$, $u_{c_{t-1}c_{t-1}}$, $u_{c_t c_{t-1}}$ and $u_{c_{t-1}c_t}$ are all evaluated at the steady state level of consumption, C. On the other hand, $\widetilde{C}_t$ is the deviation of consumption (in levels) from the steady state. Now, defining the log-linear deviation $c_t = \log(C_t/C)$ allows us to approximate $\widetilde{C}_t$ as

$$\widetilde{C}_t = C\left(c_t + \frac{1}{2}c_t^2 + 0\left(\|\xi\|^3\right)\right)$$

Therefore, equation (43) can be expressed as:

$$
\begin{aligned}
u\left(C_t, C_{t-1}\right) &= \bar{u} + u_{c_t}C\left(c_t + \frac{1}{2}c_t^2\right) + u_{c_{t-1}}C\left(c_{t-1} + \frac{1}{2}c_{t-1}^2\right) + \frac{u_{c_t c_t}C^2 c_t^2}{2} + \frac{u_{c_{t-1}c_{t-1}}C^2 c_{t-1}^2}{2} + \\
&+ u_{c_t c_{t-1}}C^2 c_t c_{t-1} \qquad (44)
\end{aligned}
$$

where the last line in (44) has made use of the fact that $u_{c_t c_{t-1}} = u_{c_{t-1}c_t}$.

Now, it can be proved that in steady state:

i) $u_{c_t c_t} C^2 = -\sigma u_{c_t} C^2$
ii) $u_{c_{t-1} c_{t-1}} C^2 = (\sigma\gamma - \gamma - 1)(-\gamma) u_{c_t} C$
iii) $u_{c_t c_{t-1}} C^2 = (\sigma\gamma - \gamma) u_{c_t} C$
iv) $u_{c_{t-1}} = -\gamma u_{c_t}$.

Using conditions (i) to (iv), and ignoring the constant $\overline{u}$ gives a more compact representation of (44);

$$u\left(C_t, C_{t-1}\right) = u_{c_t} C \left(c_t - \gamma c_{t-1} - \frac{1}{2}(\sigma - 1) c_t^2 - \frac{\gamma^2}{2}(\sigma - 1) c_{t-1}^2 + \gamma(\sigma - 1) c_t c_{t-1} \right) \tag{45}$$

Following Gali and Monacelli (2002), the term, $V(N_t)$, can be approximated as;

$$V(N_t) = \bar{V} + V_N \widetilde{N}_t + \frac{V_{NN} \widetilde{N}_t}{2} \tag{46}$$

where, as before,

$$\widetilde{N}_t = N \left(n_t + \frac{1}{2} n_t^2 + 0 \left(\|\xi\|^3 \right) \right)$$

implying that equation (46), can be rewritten as;

$$V(N_t) = \bar{V} + V_N N \left(n_t + \frac{1}{2} n_t^2 \right) + \frac{V_{NN} N^2}{2} n_t^2 \tag{47}$$

On the other hand, assuming as in Gali and Monacelli (2002) a technology with constant returns to scale, it is possible to show that the aggregate level of labor, N_t, evolves according to the following expression:

$$N_t = \left(\frac{Y_t}{A_t} \right) \int_o^1 \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\theta} di$$

now, log-linearizing the above expression around the steady state gives

$$n_t = y_t + u_t$$

where $u_t = \log \int_o^1 \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\theta} di$ is of second order[25]. Furthermore, assuming that $V(N_t) = \frac{N_t^{1+\omega}}{1+\omega}$[26], implies that $V_{NN}^2 N^2 = \omega V_N N$, and then equation (46) becomes;

[25] The proof is given in Gali and Monacelli (op.cit, pp.31).

[26] In the case of linear technology, it can be proved that the elasticitity of F' with respect to output, ϖ, is equal to the elasticity of V' with respect to labor, ω. See Rotemberg and Woodford (1997) pp.22-23.

$$V(N_t) = \bar{V} + V_N N\left(n_t + \frac{1}{2}(1+\omega)n_t^2\right)$$
$$= \bar{V} + V_N N\left(y_t + u_t + \frac{1}{2}(1+\omega)y_t^2\right) \quad (48)$$

where the last line in (41) makes use of the fact that u_t^2 and $u_t y_t$ are of order four and three respectively, and hence equal to zero. Finally, remembering that the aggregate demand can be expressed as $y_t = (1-\alpha)c_t + \alpha y_t^* + \phi_1 q_t$, and assuming that y_t^* is independent of the domestic monetary policy, we arrive to the following expression (ignoring the constant term)

$$V(N_t) = V_N N\left\{(1-\alpha)c_t + \phi_1 q_t + \frac{1}{2}(1+\omega)\left((1-\alpha)^2 c_t^2 + \phi_1^2 q_t^2 + 2(1-\alpha)\phi_1 c_t q_t\right)\right\}$$
$$... + V_N N\left\{\frac{1}{2}(1+\omega)\left(2(1-\alpha)\alpha c_t y_t^* + 2\alpha\phi_1 q_t y_t^*\right) + u_t\right\} \quad (49)$$

Now, substracting (49) from (45), and using the fact that $V_N N = u_c(1-\gamma)C$, and $u_t = \frac{1}{2}\frac{(1-\gamma)\theta\phi}{(1-\phi)(1-\beta\phi)}\sigma^2_{\pi_{H,t}}$ [27] we obtain a utility-based welfare criterion;

$$W = -\frac{1}{2}u_{c_t}\bar{C}\left\{\left[(\sigma-1)(\gamma^2+1) + (1-\gamma)(1+\omega)(1-\alpha)^2\right]\sigma^2_{c_t} + \frac{(1-\gamma)\theta\phi}{(1-\phi)(1-\beta\phi)}\sigma^2_{\pi_{H,t}}\right.$$
$$+(1-\gamma)(1+\omega)\phi_1^2\sigma^2_{q_t} - 2\gamma(\sigma-1)\sigma_{c_t c_{t-1}} + 2(1-\gamma)(1+\omega)(1-\alpha)\phi_1\sigma_{c_t q_t}..$$
$$\left.+2(1-\gamma)(1+\omega)(1-\alpha)\alpha\sigma_{c_t y_t^*} + 2(1-\gamma)(1+\omega)\alpha\phi_1\sigma_{q_t y_t^*} + \right\} + Z_t$$

Or, in a more compact way;

$$W = -\frac{1}{2}u_{c_t}\bar{C}\left\{L_1\sigma^2_{c_t} + L_2\sigma^2_{\pi_{H,t}} + L_3\sigma^2_{q_t} - L_4\sigma_{c_t c_{t-1}} + L_5\sigma_{c_t q_t} + L_6\sigma_{c_t y_t^*} + L_7\sigma_{q_t y_t^*}\right\}$$

where $Z_t = E_t(c_T) + E_t\sum(c_t - y_t) = 0$, because as in Batini et al (2003), we assume that the unconditional expectation of the first order terms is zero[28].

In particular, when $\gamma = \alpha = 0$ (closed economy without habits) and $\sigma = 1$, the above welfare function collapses to;

$$W = -\frac{1}{2}u_{c_t}\bar{C}\left\{(\sigma+\omega)\sigma^2_{y_t} + \frac{(1-\gamma)\theta\phi}{(1-\phi)(1-\beta\phi)}\sigma^2_{\pi_{H,t}}\right\}$$

which is the utility-based criterion derived in Gali and Monacelli (2002).

[27] See Gali and Monacelli (2002 pp.32) and Woodford (2002 pp.20-21) for a formal proof.
[28] All the results in the paper hold even if we drop this assumption an compute the realizations of Z_t.