Assessing Money Supply Rules[1]

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Abstract

We provide evidence that Federal Reserve's money supply can be characterized by a simple rule, whereby the growth rate of nonborrowed reserves depends on expected inflation. The Volcker-Greenspan era is found to be associated with a negative inflation elasticity, whereas our estimates indicate that the Federal Reserve in the pre-1980 era supplied money in an accommodating way. While these results appear to be consistent with empirical evidence on interest rate rules, our theoretical analysis gives rise to novel insights. Applying a New Keynesian model, money supply rules are shown to ensure saddle path stability, indicating that they do not allow for self-fulfilling expectations. Further, optimal monetary policy can be implemented by a money supply rule with a negative inflation elasticity, implying that the pre-1980 regime was less efficient in dampening macroeconomic fluctuations. On the transmission of money supply shocks, we show that a negative inflation elasticity raises the likelihood of a liquidity effect and lowers the persistence of the output response.

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1 Introduction

Central bank policy is often summarized by supplying money according to a constant money growth rate. While this policy regime can be justified on theoretical grounds, e.g., by the so-called Friedman rule, in reality central banks have hardly taken this strategy literally. Though, some central banks have claimed to target a constant money growth rate, recent empirical contributions provide evidence that monetary policy can often be reasonably summarized by simple rules, whereby a short-run nominal interest rate is set contingent on macroeconomic variables (see, e.g. Clarida et al., 1999, 2000). Given that the short-run nominal interest rate generally serves as an operating target rather than an exogenous policy instrument, central bank policy might alternatively be described by a contingent rule, according to which high-powered money is supplied in open market operations. While empirical contributions to the monetary policy literature have repeatedly emphasized that the supply of narrow monetary aggregates, i.e., nonborrowed reserves, rather than the federal funds rate can actually be controlled by the central bank (see, e.g., Eichenbaum, 1992, or Strongin, 1995), the implied reaction function has not yet been taken into consideration in empirical and theoretical contributions to business cycle analysis. In this paper it is shown that contingency of money supply rules contributes to some issues therein, in particular, to the alleged instability of monetary business cycle models, the implementation of optimal monetary policy and monetary transmission.

This paper extends the line of research on monetary policy rules, which, since Taylor (1993), primarily has focussed on rules for a short-run nominal interest rate. We provide evidence that the Federal Reserve policy can be summarized by the growth rate of nonborrowed reserves being set contingent on realizations of the inflation rate. In particular, we find that the inflation elasticity of the growth rate of nonborrowed reserves is significantly negative during the Volcker-Greenspan era, indicating a monetary policy regime that aims at stabilizing the rate of inflation. On the contrary, the inflation elasticity in the pre-Volcker era is estimated to be slightly positive, implying an accommodating money supply regime. While these results are consistent with broad empirical evidence on the shift in Federal Reserve policy, our theoretical analysis of money supply rules further suggests that the high and volatile inflation rates in the pre-Volcker era were primarily due to the Federal Reserve's inefficacy to mitigate macroeconomic fluctuations triggered by fundamental shocks (as opposed to non-fundamental shocks). Conversely, Clarida et al. (2000) claim that the particular interest rate setting substantially contributed to the high inflation volatility in the pre-Volcker era by allowing for expectations to become self-fulfilling.[4] By considering an alternative monetary policy instrument, i.e., nonborrowed reserves, our results indicate that the latter conclusion mainly relies on the application of simple interest rate rules, which are known to easily allow for indeterminacy of prices and real macroeconomic aggregates (see Kerr and King, 1996; Benhabib et al., 2001, or Carlstrom and Fuerst, 2001).

For the theoretical analysis we apply a conventional New Keynesian model, which allows for an analytical treatment and facilitates comparisons with related studies on interest rate rules. It is shown that the money supply rules of interest are associated with saddle stable equilibrium paths and, therefore, exclude multiple or unstable equilibria. This implies that, although, Federal Reserve money supply was accommodating in the pre-Volcker era, it did not allow for self-fulfilling expectations. We then proceed by assessing money supply rules with regard to their ability to stabilize fundamental business cycle fluctuations. In particular, we derive a central bank loss function, which penalizes output and inflation volatility, based on a second order approximation of households' welfare (see Woodford, 2002). Given the characterization of the optimal monetary policy under commitment, it is shown that a simple money supply rule with an negative inflation elasticity is in fact able to implement the optimal allocation.[5] It can therefore be concluded that the money supply regime of the Volcker-Greenspan era has been successful in stabilizing macroeconomic fluctuations, whereas the Federal Reserve policy in the pre-Volcker era was apparently less efficient, giving rise to higher macroeconomic volatility due to an accommodating money supply regime.

Regarding the transmission of money supply shocks, we examine if the departure from a Friedman-style constant money growth rule matters for the short-run effects of monetary policy shocks. In particular, we focus on two issues which are extensively discussed in the literature on the transmission of monetary injections, namely, the liquidity effect and the persistence of output responses (see, e.g., Chari et al., 2000). The liquidity effect, although, repeatedly found in the data (see, e.g., Hamilton, 1997), can hardly be produced in sticky price models (see Christiano et al. 1997, or Andres et al., 2002), having led to the notion of the 'liquidity puzzle'. We show that smaller (negative) values for the inflation elasticity raise the likelihood for a liquidity effect to occur. We additionally find that the persistence of output response to a monetary policy shock relies on the central bank's reactiveness, in that a higher inflation elasticity slows down the recovery of output in response to a monetary injection.

[4] According to this view, the economy evolves according to a non-fundamental solution for the rational expectations equilibrium, where sunspot shocks are able to alter macroeconomic aggregates.

[5] To be more precise, the latter actually matches the so-called targeting rule for the optimal monetary policy. See Svensson (2003) for a comprehensive discussion and comparison of targeting and instrument rules.

The remainder of the paper is set out as follows. Section 2 provides empirical evidence on simple money supply rules. In Section 3 we develop a simple sticky price model, which facilitates an analytical assessment of money supply rules. Section 4 presents the analysis of the model's local dynamics and provides a welfare analysis of money supply rules. Section 5 examines the transmission of money supply shocks and relates the contingency of money supply to the likelihood of a liquidity effect and the degree of output persistence. Section 6 concludes.

2 Empirical Evidence

It is commonly agreed that the Federal Reserve adopted a more well managed and proactive stance after Volcker's appointment as Fed Chairman (see, e.g., Friedman and Kuttner, 1996; Taylor, 1998; or Clarida et al., 2000). For example, Clarida et al. (2000), estimating a forward-looking Taylor rule, provide evidence for a strong anti-inflationary stance during the Volcker-Greenspan period. Conversely, they find monetary policy in the 20 years prior Volcker to be more accommodative.[6] The objective of the present empirical section is to investigate whether similar findings apply to a contingent money supply rule that takes the following form:

$$\mu_t = \sum_{i=1}^{m} \rho_i \mu_{t-i} + \alpha E_t \{\pi_{t+n}\} + \varepsilon_t, \tag{1}$$

where μ_t denotes the growth rate of a monetary aggregate, $E_t \{\pi_{t+n}\}$ is the expected inflation rate in $t+n$, and the error term ε_t is assumed to be independently and identically distributed (*iid)* Gaussian. In accordance with the view that a central bank essentially controls the supply of high powered money (rather than the short-run nominal interest rate), μ_t denotes the growth rate of nonborrowed reserves. This choice is consistent with the approach in several contributions to the literature on monetary transmission, where monetary policy shocks are identified with innovations to nonborrowed reserves (see Eichenbaum, 1992; Strongin, 1995; Hamilton, 1997; or Christiano et al., 1999). By specifying the growth rate of nonborrowed reserves as in (1), we aim at disclosing the structural dependence between the supply of reserves and the growth rate of an aggregate price level.[7] In addition to lagged growth rates of nonborrowed reserves, the monetary policy rule (1) includes a forward looking component, allowing money supply to be related to expected future inflation. This formulation corresponds to variants of the Taylor (1993) rule, in which the federal funds rate responds

[6] Accommodating in the sense that on average the Federal Reserve let real short term interest rates decline as anticipated inflation rose.

[7] It should be noted that we do not presume the Federal Reserve to follow a specific target for the growth rate of nonborrowed reserves.

to expected movements in inflation, reflecting the aim of Federal Reserve policy to stabilize future inflation rates. The above equation is the basic building block of the empirical analysis and serves as the main novelty of the theoretical analysis in the subsequent sections.

We apply the aforementioned money supply rule to different episodes of Federal Reserve policy. The data are obtained from a database provided by the Federal Reserve Bank of St. Louis and are of monthly frequency.[8] As has been highlighted in several studies, monetary policy was allegedly less well managed in the twenty years prior to Volcker. Hence, the first period we examine, covers the time horizon January 1960 to September 1979 and is referred to as the pre-Volcker era. We further explore monetary policy in the Volcker-Greenspan era (January 1983 to January 2003).[9] We expect substantial differences in the estimated policy rule across the two periods. In particular, the response to the forward-looking component, as measured by the coefficient α, is expected to differ across time. Monetary policy is considered reactive for $\alpha < 0$, implying that higher expected inflation leads to lower money supply. Conversely, a money supply rule characterized by $\alpha \geq 0$ is seen as accommodating.

We estimate our money supply rule with two inflation measures - consumer price inflation and producer price inflation (for all commodities). While the former serves as a more comprehensive price measure, we alternatively consider the latter price index as an early indicator of a nascent rise in aggregate prices. The inflation rate and the growth rate of nonborrowed reserves are constructed as year-on-year percentage changes. Figure 1 displays the growth rate of nonborrowed reserves together with consumer price inflation for the Volcker-Greenspan period. The inverse relationship between both variables is evident and appears to hold throughout key phases of the sample period.[10] This back-of-the-envelope evidence suggests that inflation is negatively related to nonborrowed reserves growth during the Volcker-Greenspan period. It is complemented in the following by estimating the money supply rule (1) using generalized methods of moments (GMM).[11]

The starting point of any GMM estimation is a theoretical relation that the parameters should satisfy, which is described by orthogonality conditions between some function of the

[8] This database is known as Federal Reserve Economic Data (FRED).

[9] Although Volcker was appointed Chairman of the Board of Governors of the Federal Reserve System in 1979, we refrain from including the first three years of his mandate in the sample period because this might lead to biased estimates for the Volcker-Greenspan period (see Clarida et al. 2000). Indeed, for a brief period at the start of the Volcker era, the Fed seemed to pursue a policy of nonborrowed reserves targeting (see Goodfriend, 1991). In addition, the period until the end of 1982 is usually regarded as the 'Volcker disinflation' episode, in which inflation was brought down to 4 percent from previously 10 percent in 1980. This was an exceptional period as inflation thereafter was steadily more stable.

[10] This finding also holds if one considers producer price inflation.

[11] GMM is now a widely used technique to estimate monetary policy rules. Clarida et al. (2000), for example, apply GMM to estimate a forward looking Taylor rule.

parameters $f(\theta)$ and a set of instrumental variables z_t:

$$E_t\left(f(\theta)\, z_t\right) = 0, \tag{2}$$

where θ are the parameters to be estimated. Let $f(\theta) = \mu_t - \sum_{i=1}^{m} \rho_i \mu_{t-i} - \alpha E_t\left\{\pi_{t+n}\right\}$ and assuming rational expectations we can write

$$E_t\left\{\left(\mu_t - \sum_{i=1}^{m} \rho_i \mu_{t-i} - \alpha \pi_{t+n}\right) z_t\right\} = 0. \tag{3}$$

The GMM estimator selects parameter estimates such that the sample correlation between function f and the instruments is close to zero.[12] For each estimation our vector of instruments includes 6 lags of inflation and 6 lags of nonborrowed reserves. Since not all current information may be available to the public at the time they form expectations, contemporary variables are not used as instruments.

Table 1 summarizes the results for the pre-Volcker period. Notably, the estimated coefficient α on the inflation rate is significantly positive suggesting that monetary policy during the pre-Volcker period was accommodating - higher expected inflation led to a mild increase in the money supply. Similar findings are obtained when the estimations are carried out using producer price inflation. In the baseline case the monetary policy rule is estimated using a forward-looking horizon for inflation of 1 month, that is $n = 1$. Alternatively, we allow for different target horizons, i.e., 3 and 6 months. Note, however, that the impulse of the forward looking component on the growth rate of the monetary aggregate remains unchanged even at different target horizons. The growth rate of nonborrowed reserves enters the money supply rule with a lag length of three. We only report the sum of the estimated autoregressive coefficients, which is found to be positive and less than unity in each case.

Next we estimate a similar money supply rule using data for the Volcker-Greenspan period. The growth rate of nonborrowed reserves enters the monetary policy rule with a lag of three. The most striking result discovered for the Volcker-Greenspan period concerns the inflation elasticity, which is now found to be significantly negative at all inflation target horizons (see Table 2), suggesting that monetary policy as measured by a simple contingent money supply rule appeared to be more reactive during the investigated period. This finding confirms earlier results reported by Clarida et al. (2000) who associate the Volcker-Greenspan era with a more aggressive monetary policy regime. It should be noted that the long-run inflation elasticity defined as $\mu_\pi \equiv \alpha/(1 - \Sigma\rho_i)$ is strictly smaller than unity for all specifications.

[12] The parameter estimates are obtained using a criterion function, that is of the following nature: $J(\theta) = (f(\theta)\, z)' \, W \, (f(\theta)\, z)$, where W is a weighting matrix.

Table 1. GMM Estimation Results: Pre-Volcker Period

Estimated Money Supply Rule: $\mu_t = \Sigma_{i=1}^{3}\rho_i\mu_{t-i} + \alpha E_t\{\pi_{t+n}\} + \varepsilon_t$

	CPI			PPI		
	$n=1$	$n=3$	$n=6$	$n=1$	$n=3$	$n=6$
$\Sigma\rho_i$	0.90	0.90	0.90	0.90	0.90	0.89
α	0.06	0.06	0.05	0.05	0.05	0.05
	(0.04)	(0.04)	(0.04)	(0.02)	(0.02)	(0.03)
R^2	0.85	0.85	0.85	0.85	0.85	0.84
J	0.32	0.32	0.35	0.17	0.18	0.19
$AR(3)$	0.23	0.21	0.19	0.10	0.08	0.05
$AR(6)$	0.48	0.45	0.43	0.25	0.21	0.16
$ARCH(3)$	0.35	0.37	0.39	0.25	0.26	0.32
$ARCH(6)$	0.60	0.64	0.67	0.50	0.53	0.63

Notes: Figures in parentheses below coefficient estimates denote p-values. R^2 denotes the coefficient of determination; J is a test statistic for the null hypothesis that the overidentifying restrictions are satisfied; $AR(j)$ is a Lagrange multiplier test statistic for up to jth-order serial correlation in the residuals, $ARCH(j)$ is a Lagrange multiplier test statistic for up to jth-order autoregressive conditional heteroskedasticity in the residuals. For J, $AR(j)$ and $ARCH(j)$ we only report p-values. Note that coefficient estimates of the autoregressive components are statistically significant. The associated p-values are not reported here but are available from the authors upon request.

In general, the goodness-of-fit statistics are satisfactory for both subperiods, with the coefficient of determination ranging from 0.62 for the Volcker-Greenspan period to 0.85 for the pre-Volcker era. We also carry out a number of diagnostic tests. We first compute Hansen's J-statistic to test the validity of overidentifying restrictions. In each case the null hypothesis that overidentifying restrictions are satisfied could not be rejected. Standard diagnostic tests generally indicate no evidence of serial correlation and autoregressive conditional heteroskedasticity in the residuals, suggesting that the residuals are well behaved.[13]

[13] We did carry out GMM estimations that consider different model specifications. In particular, we allowed alterantive lag specification and for contemporary variables in the vector of instruments. Our estimates, although qualitatively similar to the ones just reported, were not found to be statistically significant and the resulting diagnostic tests often indicated possible misspecification of the estimated models.

Table 2. GMM Estimation Results: Volcker-Greenspan Period

Estimated Money Supply Rule: $\mu_t = \rho\mu_{t-3} + \alpha E_t\{\pi_{t+n}\} + \varepsilon_t$

	CPI			PPI		
	$n=1$	$n=3$	$n=6$	$n=1$	$n=3$	$n=6$
ρ	0.88	0.88	0.89	0.89	0.90	0.86
α	-0.35	-0.35	-0.34	-0.34	-0.36	-0.41
	(0.01)	(0.02)	(0.01)	(0.001)	(0.001)	(0.001)
R^2	0.62	0.62	0.64	0.64	0.64	0.66
J	0.17	0.17	0.23	0.35	0.36	0.22
$AR(3)$	0.12	0.18	0.36	0.28	0.42	0.55
$AR(6)$	0.30	0.39	0.57	0.52	0.52	0.58
$ARCH(3)$	0.36	0.30	0.18	0.10	0.07	0.08
$ARCH(6)$	0.28	0.23	0.15	0.11	0.10	0.12

Notes: See notes to Table 1.

Overall, the empirical analysis provides evidence for the supply of nonborrowed reserves to react to expected inflation during the past four decades of Federal Reserve policy. The empirical results demonstrate that monetary policy during the Volcker-Greenspan era had indeed a proactive stance towards targeting inflation. Our findings thus seem to be consistent with the results in Clarida et al. (2000) who characterize the Volcker-Greenspan era as a highly sensitive monetary policy regime. Conversely, in the pre-Volcker period, supply of nominal balances appeared to be mildly accommodating, lending support to the view that the anti-inflationary stance of the Fed was weaker during that period. In any case, the Fed has controlled real balances not to rise with the expected inflation rate as the long-run inflation elasticity $\mu_\pi \equiv \alpha/(1-\Sigma\rho_i)$ was estimated never to exceed one. As will be shown in the theoretical analysis, a money supply regime is in fact destabilizing only if $\mu_\pi > 1$, implying that the central bank raises the growth rate of real balances when inflation rises.

3 A sticky price model

In this section we develop a New Keynesian model where the central bank sets the growth rate of money. We abstract from specifying different monetary aggregates for convenience, implicitly assuming that money multipliers are constant.[14]

[14] Interest rate policy, on the other hand, implies the real Federal Funds Rate.to govern consumption growth.

Households Throughout the paper nominal (real) variables are denoted by upper-case (lower-case) letters. There is a continuum of households indexed with $j \in (0,1)$. They are identical except for their idiosyncratic working time l_j, which is monopolistically supplied to firms. Hence, the indexation of households' variables with j can be omitted except for labor market variables. The objective of household j is given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - v(l_{jt})\right], \quad \text{with } u(c_t) \equiv \frac{c_t^{1-\sigma}}{1-\sigma}, \quad v(l_{jt}) \equiv \frac{l_{jt}^{1+\vartheta}}{1+\vartheta}, \quad \text{and } \sigma, \vartheta > 0, \tag{4}$$

where c denotes consumption, β the subjective discount factor ($0 < \beta < 1$), and E_0 the expectation operator conditional on the information in period 0. At the beginning of each period households are endowed with money M_{t-1} and risk-free government bonds B_{t-1}. Before households enter the goods market in period t, they are not able to adjust their asset holdings such that they rely on their predetermined asset holdings M_{t-1} and B_{t-1}. Households are assumed to hold checkable accounts at a financial intermediary. After goods are produced labor income is credited on this account, while it is charged for wage outlays of firms which are owned by the households. Entering the goods market, consumption expenditures are therefore restricted by the following liquidity constraint:

$$P_t c_t \leq M_{t-1} + \left(P_t w_{jt} l_{jt} - P_t w_t \int_0^1 l_{it} di\right) + P_t \tau_t, \tag{5}$$

where $w_j (w)$ denotes the idiosyncratic (aggregate) real wage rate and τ_t denotes lump sum transfers. The conventional cash-in-advance constraint is augmented by allowing for net wage earnings, i.e., the term in round brackets in (5), to be accepted as a means of payment. Hence, an individual labor income, which exceeds the average wage payments of final goods producing firms indexed with $i \in (0,1)$ employing l_i, leads to an relaxation of the cash constraint (5). This assumption, which is adopted from Jeanne (1998), is introduced to avoid the cash-credit good distortion between consumption and leisure. It further facilitates a comparison with related studies as the model's reduced form representation will be isomorphic to the standard New Keynesian model (see, e.g., Clarida et al., 1999, or Galí et al. 2001).[15] Obviously, we obtain a standard cash-in-advance specification in equilibrium.

We assume that households monopolistically supply differentiated labor services as in Clarida et al. (2002). The differentiated labor services l_j are transformed into one type of labor input l, which can be employed for the production of the final good. The transformation is conducted via the aggregator: $l_t^{1-1/\eta_t} = \int_0^1 l_{jt}^{1-1/\eta_t} dj$, with $\eta_t > 1$. The elasticity

[15] Otherwise, the nominal interest rate would enter the aggregate ressource contraint (the 'New Keynesian Phillips curve'). However, the local stability properties remain unchanged (see Schabert, 2003).

of substitution between differentiated labor services η_t is allowed to vary exogenously over time,[16] leading to changes in the labor market conditions which affects the costs of final goods producing firms. Cost minimization with respect to differentiated labor services then leads to following demand schedule for l_j:

$$l_{jt} = \left(\frac{w_{jt}}{w_t}\right)^{-\eta_t} l_t, \qquad \text{with} \qquad w_t^{1-\eta_t} = \int_0^1 w_{jt}^{1-\eta_t} dj, \tag{6}$$

where l denotes aggregate labor services. The households own final goods producing firms and, thus, receive their profits ω_t. Moreover, they receive wage payments and the government transfer. The budget constraint of household j is given by

$$P_t c_t + B_t + M_t \leq R_t B_{t-1} + M_{t-1} + P_t w_{jt} l_{jt} + P_t \tau_t + P_t \omega_t, \tag{7}$$

where R_t denotes the gross nominal interest rate on government bonds.[17] Maximizing the objective (4) subject to the cash-in-advance constraint (5), the budget constraint (7), labor demand (6) and a no-Ponzi-game condition, $\lim_{i\to\infty} E_t[(B_{t+i} + M_{t+i})\,\Pi_{v=1}^{i} R_{t+v}{}^{-1}] \geq 0$, for given initial values B_0 and M_0 leads to the following first order conditions:

$$c_t^{-\sigma} = \lambda_t + \psi_t, \quad l_{jt}^{\vartheta}\varphi_t = (\lambda_t + \psi_t)\, w_{jt}, \tag{8}$$

$$\frac{1}{\beta}\lambda_t = E_t\left[\frac{R_{t+1}}{\pi_{t+1}}\lambda_{t+1}\right], \quad \frac{1}{\beta}\lambda_t = E_t\frac{\lambda_{t+1}}{\pi_{t+1}} + E_t\frac{\psi_{t+1}}{\pi_{t+1}}, \tag{9}$$

$$\psi_t \geq 0, \quad \psi_t\left[m_{t-1}\pi_t^{-1} + w_{jt}l_{jt} - w_t\int_0^1 l_{it}di - c_t + \tau_t\right] = 0, \tag{10}$$

and (5), where $\varphi_t = \frac{\eta_t}{\eta_t - 1}$ denotes the markup over the perfectly competitive real wage, λ the shadow price of wealth, ψ the Lagrange multiplier on the cash-in-advance constraint, $m_t \equiv M_t/P_t$ real balances, and $\pi_t \equiv P_t/P_{t-1}$ the inflation rate. Furthermore, the budget constraint (7) holds with equality and the following transversality condition must be satisfied

$$\lim_{i\to\infty} E_t\left[\lambda_{t+i}\beta^{t+i}\left(b_{t+i} + m_{t+i}\right)\right] = 0, \tag{11}$$

where $b_t \equiv B_t/P_t$. Note that the cash constraint (5) avoids a cash-credit distortion between consumption and leisure such that the consumption/leisure decision satisfies $c_t^{\sigma} l_t^{\vartheta} = w_{jt}/\varphi_t$. We further assume that the 'cost push' shocks are generated by: $\varphi_t = \overline{\varphi}^{1-\rho_\varphi}\varphi_t^{\rho_\varphi}\exp(\varepsilon_{\varphi t})$, where $\overline{\varphi} > 1$ and $\varepsilon_{\varphi t}$ are i.i.d. with $E_{t-1}\varepsilon_{\varphi t} = 0$.

[16] For example, a decline in η_t leads to an exogenous increase in the competitiveness reducing the market power of the supply side.

[17] Thus, households have access to nominal state contingent debt.

Production Sector The final consumption good is an aggregate of differentiated goods produced by monopolistically competitive firms indexed with $i \in (0,1)$. The CES aggregator of differentiated goods is defined as

$$y_t^{1-1/\epsilon} = \int_0^1 y_{it}^{1-1/\epsilon} di, \quad \text{with} \quad \epsilon > 1, \tag{12}$$

where y is the number of units of the final good, y_i the amount produced by firm i, and ϵ the constant elasticity of substitution between these differentiated goods. Let P_i and P denote the price of good i set by firm i and the price index for the final good. The demand for each differentiated good is derived by minimizing the total costs of obtaining y : $y_{it} = (P_{it}/P_t)^{-\epsilon} y_t$, with $P_t^{1-\epsilon} = \int_0^1 P_{it}^{1-\epsilon} di$. A firm i produces good y_i employing a technology which is linear in the labor: $y_{it} = l_{it}$. We introduce a nominal stickiness in form of staggered price setting as developed by Calvo (1983) and Yun (1996). Each period firms may reset their prices with the probability $1-\phi$ independent of the time elapsed since the last price setting. The fraction ϕ of firms are assumed to adjust their previous period's prices according to $P_{it} = \overline{\pi} P_{it-1}$, where $\overline{\pi}$ denotes the average inflation rate. The linear approximation to the corresponding aggregate supply constraint at the steady state, given by

$$\widehat{\pi}_t = \chi \widehat{mc}_t + \beta E_t \widehat{\pi}_{t+1}, \qquad \text{with } \chi = (1-\phi)(1-\beta\phi)\phi^{-1} > 0, \tag{13}$$

can be found in Yun (1996). Note that $\widehat{x}$ denotes the percent deviation from the steady state value $\overline{x}$ of a generic variable x, $\widehat{x} = \log(x_t) - \log(\overline{x})$, and mc the real marginal costs. The demand for aggregate labor input in a symmetric equilibrium relates the real marginal costs to the real wage:

$$mc_t = w_t. \tag{14}$$

Public Sector The public sector consists of a monetary and a fiscal authority. The latter is assumed to issue one-period bonds, earning the net interest $(R_t - 1)B_{t-1}$, while the former issues money. The consolidated flow budget constraint of the public sector is given by $B_t + M_t = R_t B_{t-1} + M_{t-1} + P_t \tau_t$. The public sector is assumed to satisfy: $\lim_{i\to\infty}(B_{t+i} + M_{t+i}) E_{t+i} \Pi_{v=1}^{i} (1 + i_{t+v})^{-1} = 0$. This specification of the solvency constraint, which includes all government labilities, is taken from Benhabib et al. (2001) and characterizes a Ricardian policy regime. As government bonds are irrelevant for the focus of this paper, we assume that they are issued at a net supply equal to zero.[18]

[18] In fact, this prevents bonds to enter the consolidated cash constraint, which could alternatively be avoided by assuming that only seignorage, $P_t \tau_t^s = M_t - M_{t-1}$, is transferred in form of cash, such that $P_t \tau_t^s$ instead of $P_t \tau_t$ enters the right hand side of (5).

The central bank controls the money growth rate according to the following simple rule

$$\mu_t \equiv M_t/M_{t-1} = \mu(\pi_t, \varepsilon_t), \quad \mu_t \geq 1, \tag{15}$$

where the innovation ε has an expected value of zero and is serially uncorrelated. It should be noted that the money growth rule in (15) is specified in a simpler way than the rule estimated in section 2. For example, the inflation rate π_t does not enter the policy rule in form of its expected value, as the central bank is assumed to be able to adjust money supply after aggregate shocks are realized. In fact, the main results derived in the remainder of the paper will be demonstrated to be robust when the current period inflation rate is replaced by the expected future inflation rate. Generalizing the case of a constant money growth policy ($\partial\mu_t/\partial\pi_t = 0$), which can be found in the majority of studies applying money growth rules, we allow the growth rate μ_t to depend on the realization of the inflation rate. A similar money growth rule featuring (lagged) inflation rates can, for example, be found in McCallum (1999). Though, we allow the money growth rate to vary in the short-run, we certainly ensure that it is constant in the long-run equilibrium. We assume that the condition $\mu(\overline{\pi}, 0) = \overline{\pi}$ has a solution for a steady state inflation rate such that the steady state nominal interest rate, satisfying $\overline{R} = \overline{\pi}/\beta > 1$, is strictly larger than zero. We further restrict the realizations of ε (and of ε_φ) to be sufficiently small, such that the gross interest rate always exceeds one in the neighborhood of this steady state.

The *rational expectations equilibrium* of the model with $R_t > 1$ is a set of sequences $\{\pi_t$, w_t, m_{t-1}, λ_t, ψ_t, R_t, c_t, l_t, mc_t, $b_t\}_{t=0}^{\infty}$ satisfying (i) the household's first order conditions (8)-(9) together with the consolidated cash constraint, $c_t = m_t$, (ii) the optimal pricing condition approximated by (13) and the aggregate labor demand (14); (iii) the money growth rule (15) and a net bond supply of zero ($b_t = 0$); (iv), the aggregate resource constraint, $l_t = c_t$, and the transversality condition of the households (11) for given initial values of the nominal money stock M_0 and the aggregate price level P_0.

4 Stability and welfare properties

In general, money growth rules differ from Taylor-type interest rate rules with regard to the determinacy properties (see Carlstrom and Fuerst, 2003) and the implied fundamental solution of rational expectations (or perfect foresight) models (see Schabert, 2003). In this section we assess the stability and welfare implications of contingent money supply rules, applying methods, which are commonly used for the analysis of interest rate rules (see, e.g., Clarida et al., 1999, or Giannoni and Woodford, 2002). Given that a log-linear approximation of our model delivers – except for the policy rule – the standard New Keynesian model, we can immediately compare our findings with existing results on interest rate rules. Before we examine the welfare implications of contingent money supply rules, we first derive the conditions for saddle path stability. Hereby, it is shown that a central bank can implement the optimal allocation under commitment by applying a simple money supply rule.

The linearized model In the remainder of this paper we restrict our attention to cases where the nominal interest rate is always strictly larger than one, $R_t > 1$, such that the cash-in-advance constraint always binds. We focus on the properties of the model at a target steady state with an inflation rate $\overline{\pi} : \overline{\pi} \geq \beta$.[19] The model is then log-linearized at the steady state and reduced, such that output, the inflation rate, real balances and the nominal interest rate remain to be determined. The log-linear version of the money growth rule is given by: $\widehat{m}_t - \widehat{m}_{t-1} + \widehat{\pi}_t = \mu_\pi \widehat{\pi}_t + \varepsilon_t$. Its non-linear representation might, for example, take the form: $\mu(\pi_t, \varepsilon_t) = \kappa_\mu \pi_t^{\mu_\pi} \exp(\varepsilon_t)$, where the parameter κ_μ is calibrated contingent on the realizations of the inflation elasticity μ_π to match the steady state condition for a particular steady state inflation rate $\overline{\pi} : \kappa_\mu = \overline{\pi}^{1-\mu_\pi}$. A rational expectations equilibrium of the linear model is defined as follows.

Definition 1 *A rational expectations equilibrium of the linear approximation to the model at the steady state with $R_t > 1$ is a set of sequences $\{\widehat{\pi}_t,\ \widehat{m}_t = \widehat{y}_t, \widehat{R}_t\}_{t=0}^{\infty}$ satisfying*

$$\widehat{\pi}_t = \omega \widehat{m}_t + \beta E_t \widehat{\pi}_{t+1} + \chi \widehat{\varphi}_t, \quad \textit{with} \quad \widehat{\varphi}_t = \rho_\varphi \widehat{\varphi}_{t-1} + \varepsilon_{\varphi t} \quad \textit{and} \quad \omega \equiv \chi(\vartheta + \sigma) > 0, \tag{16}$$

$$\widehat{m}_t = \widehat{m}_{t-1} + (\mu_\pi - 1)\widehat{\pi}_t + \varepsilon_t, \tag{17}$$

$$\sigma \widehat{m}_t = \sigma E_t \widehat{m}_{t+1} - \widehat{R}_t + E_t \widehat{\pi}_{t+1}. \tag{18}$$

and (11) given sequences of shocks $\{\varepsilon_t,\ \varepsilon_{\varphi t}\}_{t=0}^{\infty}$ and an initial value $m_0 = M_0/P_0$.

The first equilibrium condition (16) in definition 1 is derived from the aggregate supply curve, while the second condition (17) is the monetary policy rule. Equation (18) is the consumption Euler equation, where consumption is replaced by real balances using the cash-constraint. This equation residually determines the equilibrium sequence of the nominal interest rate for given sequences of inflation and real balances. Replacing the money supply rule (17) by an interest rate rule and using $\widehat{y}_t = \widehat{m}_t$ delivers Clarida et al.'s (2000) model.

Requirements for local determinacy As can be seen from the conditions in definition 1, past realizations of real balances always enter the set of equilibrium conditions such that the model exhibits one predetermined, $\widehat{m}_{t-1}$, and one jump variable, $\widehat{\pi}_t$. It turns out that existence and uniqueness of a stable rational expectations equilibrium, which requires – according to Blanchard and Kahn (1980) – one stable and one unstable eigenvalue, depends on

[19] The steady state of the model in consumption, real balances, and inflation is characterized by the following conditions: $\overline{c}^{\sigma+\vartheta} = \frac{\varepsilon-1}{\varepsilon\overline{\varphi}}$, $\overline{m} = \overline{c}$, and $\overline{\pi} = \mu(\overline{\pi}, 0) = \beta\overline{R}$.

the value for the inflation elasticity μ_π. The following proposition presents the conditions for saddle path stability.

Proposition 1 (Equilibrium determinacy) *If the central bank sets the money growth rate such that $\mu_\pi < 1$, then the model is saddle path stable.*

Proof. To examine the local dynamics, we rewrite the model given in definition 1 in matrix form, where M_e is irrelevant given that the exogenous variables $(\widehat{\varphi}_t, \varepsilon_t)$ are stationary:

$$M_0 \begin{pmatrix} \widehat{m}_t \\ E_t\widehat{\pi}_{t+1} \end{pmatrix} = M_1 \begin{pmatrix} \widehat{m}_{t-1} \\ \widehat{\pi}_t \end{pmatrix} + M_e \begin{pmatrix} \widehat{\varphi}_t \\ \varepsilon_t \end{pmatrix}, \quad \text{with } M_0 \equiv \begin{pmatrix} \omega & \beta \\ 1 & 0 \end{pmatrix} \text{ and } M_1 \equiv \begin{pmatrix} 0 & 1 \\ 1 & \mu_\pi - 1 \end{pmatrix}.$$

As the model exhibits one endogenous state variable $(\widehat{m}_{t-1})$, we want to derive the conditions for the case where exactly one eigenvalue lies inside the unit circle, such that the model is saddlepoint stable and the state variable exhibits a positive autocorrelation. The characteristic polynomial of $(M_0)^{-1} M_1$ then reads

$$f(X) = X^2 - (\beta - \omega\mu_\pi + \omega + 1)\,\beta^{-1}X + 1/\beta. \tag{19}$$

Given that $f(0)$ is equal to $1/\beta$ and, therefore, strictly positive and $f(1) = \omega(\mu_\pi - 1)/\beta$ is strictly negative for $\mu_\pi < 1$, the model exhibits one stable and one unstable eigenvalue, i.e., $0 < X_1 < 1$ and $1 < X_2$, if $\mu_\pi < 1$, which establishes the claim made in the proposition. ■

According to proposition 1, a constant money growth policy ($\mu_\pi = 0$) ensures a uniquely determined stable equilibrium path. When money supply is, however, allowed to depend on the realizations of the inflation rate, saddle path stability is ensured as long as the elasticity μ_π is smaller than one. Otherwise, the central bank gives rise to multiple or unstable equilibrium paths. Whether the inflation elasticity μ_π is smaller or larger than one is decisive for higher inflation rates leading either to a decline or to a rise in real balances. Suppose, for example, that a cost push shock leads to a rise in inflation, which reduces the real value of money for $\mu_\pi < 1$. In this case, aggregate demand declines bringing inflation back to its steady state value. Otherwise, $\mu_\pi > 1$ causes real balances and, thus, aggregate demand to grow, which further leads to an upward pressure on prices. Hence, a money supply rule must be accompanied by an inflation elasticity smaller than one to avoid explosiveness and to ensure the economy to evolve on a saddle stable equilibrium path. In contrast, a simple interest rate rule, $\widehat{R}_t = \rho_\pi\widehat{\pi}_t$, has to feature a value for an inflation elasticity which is larger than one ($\rho_\pi > 1$) in order to rule out multiple equilibria in our model (see, e.g., Woodford, 2001). Moreover, the determinacy condition for an interest rate rule easily changes when the central bank responds to expected inflation, $\widehat{R}_t = \rho_\pi E_t\widehat{\pi}_{t+1}$, rather than to the current inflation rate (see Carlstrom and Fuerst, 2001, or Svensson and Woodford, 2003). On the contrary, the condition for saddle path stability remains unchanged ($\mu_\pi < 1$) when the money supply rule features the expected future inflation rate, $\widehat{\mu}_t = \mu_\pi E_t \widehat{\pi}_{t+1}$, as for example assumed in Section 2. The stability analysis for the latter rule is provided in appendix 8.1.

Now recall that the (long-run) inflation elasticities, i.e., $\alpha/(1-\sum_i \rho_i) = \mu_\pi$, estimated in Section 2, were found to be smaller than one for both eras and all model specifications.[20] According to these results, the money growth rules in the pre-Volcker and in the post-Volcker era reveal that money supply ensured the economy to evolve on a saddle stable equilibrium path. Hence, there is a unique solution of the rational expectations equilibrium of the model, i.e., the so-called fundamental solution or minimum-state-variable solution (see McCallum, 1999). The absence of non-fundamental solutions, i.e., solutions with extraneous state variables, implies that fluctuations in macroeconomic aggregates could not have been induced by non-fundamental (sunspot) shocks. This stands in clear contrast to the hypothesis stated by Clarida et al. (2000), which is based on their finding that interest rate policy has been passive ($\rho_\pi < 1$) in the pre-Volcker era, that the high and volatile inflation rates in this era were due to a monetary policy regime allowing for multiple solutions, including non-fundamental solutions, and, thus, for self-fulfilling expectations.

Optimal monetary policy Though the money supply rules for pre-Volcker and the Volcker-Greenspan era do not differ with regard to their determinacy implications, they clearly differ by their ability to reduce the volatility of endogenous variables (triggered by fundamental shocks), which will be demonstrated in the remainder of this section. In order to disclose the ability of different money supply rules to stabilize the economy, we consider a central bank loss function L_t which penalizes the volatility of relevant variables. Following Woodford (2002), we assume that monetary policy aims at maximizing household's welfare. Given that we restrict our attention to the model's local dynamics at the long-run equilibrium, we apply a second order approximation of the households' objective (4) at the undistorted steady state. For this it is implicitly assumed that the distortion arising from monopolistic competition in the goods market is eliminated in the steady state by an appropriate transfer scheme of the fiscal authority. Further, it is assumed that the central bank sets the long-run money growth rate equal to one (for example by setting $\kappa_\mu = 1 \Rightarrow \overline{\mu} = \overline{\pi} = 1$). Based on these assumption the objective of the central bank can be written as

$$-\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t L_t, \quad \text{with } L_t \equiv \widehat{\pi}_t^2 + \frac{\omega}{\epsilon} \widehat{y}_t^2. \tag{20}$$

[20] Specifically, the long-run inflation elasticity never exceeds 0.6.

The derivation of (20), which is closely related to the procedure in Woodford (2002, chap. 6) or Walsh (2003), can be found in appendix 8.2. We now proceed by deriving the optimal monetary policy under commitment, implying that the central bank does not re-optimize each period. To be more precise, we want to derive a simple money supply rule, which is able to implement the optimal allocation for a timeless perspective; the latter concept being for example applied in McCallum and Nelson (2000) and Giannoni and Woodford (2002).[21] The following proposition summarizes the outcome of the policy problem.

Proposition 2 (Optimal policy) *A central bank can implement the optimal monetary policy under commitment on a saddle stable path by setting the money growth rate according to*

$$\widehat{\mu}_t = \mu_\pi^* \widehat{\pi}_t, \quad \textit{with } \mu_\pi^* \equiv -(\epsilon - 1) < 0. \tag{21}$$

Proof. Using that the equilibrium conditions (17) and (18) are not binding for optimal monetary policy, the policy problem can be written as

$$\max_{\widehat{\pi}_t, \widehat{y}_t} -E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{2} \left(\widehat{\pi}_t^2 + \frac{\omega}{\epsilon} \widehat{y}_t^2 \right) + \phi_t \left[\widehat{\pi}_t - \omega \widehat{y}_t - \beta \widehat{\pi}_{t+1} - \chi \widehat{\psi}_t \right] \right\}.$$

The first order conditions for $t > 0$, given by $-\widehat{\pi}_t - \phi_t + \phi_{t-1} = 0$ and $-\frac{\omega}{\epsilon} \widehat{y}_t + \omega \phi_t = 0$, can be summarized to the so-called targeting rule

$$-\frac{1}{\epsilon}(\widehat{y}_t - \widehat{y}_{t-1}) = \widehat{\pi}_t \quad \forall t > 0 \tag{22}$$

The cash constraint $\widehat{m}_t = \widehat{y}_t$ and $\widehat{\mu}_t = \widehat{m}_t + \widehat{\pi}_t - \widehat{m}_{t-1}$ leads to the instrument rule $\widehat{\mu}_t = (1 - \epsilon)\widehat{\pi}_t$. Given that $\epsilon > 1$ (see 12), we can conclude that optimal monetary policy under commitment can be implemented with a money growth rule featuring a strictly negative inflation elasticity ($\mu_\pi^* < 0$), which, by proposition 1, ensures saddle path stability. ■

As shown in proposition 2, a central bank can install the optimal allocation under commitment, if it sets the money growth rate contingent on current inflation with a negative value for the inflation elasticity, $\mu_\pi^* = 1 - \epsilon < 0$. Applying this particular money supply rule the central bank exactly implements the so-called targeting rule (22). Alternatively, the optimal allocation can also be implemented by a forward looking money supply rule.[22] The condition presented in proposition 2 further reveals that a constant money growth rule ($\mu_\pi = 0$) can be optimal in the limiting case where the deterministic distortion, which stems from the average price mark-up of monopolistically competitive firms, vanishes ($\epsilon \to 1$). It should, however,

[21] According to this approach, the central bank behaves as if it has implemented its policy plan infinite periods ago implying that the initial period can be neglected.

[22] For example, when $\rho_\varphi = 0$ a rule satisfying $\widehat{\mu}_t = (1 - \epsilon)\delta_m^{-1} E_t \widehat{\pi}_{t+1}$ is optimal, where δ_m is equal to the stable root X_1 of the characteristic polynomial in (19).

be noted that the so-called Friedman rule, $\mu_t = 1/\beta$, violates the condition $\overline{\pi} = 1$, which characterizes the undistorted steady state.[23] Nevertheless, the existence of price mark-ups ($\epsilon > 1$) implies that a money supply rule with a negative inflation elasticity is more efficient than constant or accommodating ($\mu_\pi > 0$) money growth regimes.

In contrast to the case where the nominal interest rate serves as the policy instrument (see Giannoni and Woodford, 2002, proposition 4), the particular type and the statistical properties of the exogenous disturbance has no impact on the conditions describing the optimal money supply. For example, the optimal money supply rule does not depend on ρ_φ and is unchanged when we allow for productivity shocks instead of cost push shocks. However, when money demand shock are considered, the particular money supply rule, which implements the optimal allocation, is not entirely deterministic. As in the case of the optimal interest rate rules derived in Clarida et al. (1999), optimal money supply would then also depend on stochastic disturbances, i.e., on the realizations of money demand shocks. In any case, the central bank does not need to solve for the equilibrium sequences under the targeting rule (22) to identify the optimal rule for its instrument. This property is not self-evident and does in general not apply for interest rate rules (see Giannoni and Woodford, 2002). Moreover, an optimal money supply rule under commitment guarantees the economy to evolve on a saddle stable path (see proposition 1). Thus, it rules out explosiveness and self-fulfilling expectations, which can easily arise for interest rate rules, which are aimed to implement the optimal allocation (see Svensson and Woodford, 2003).

The sign restriction for optimal money growth rules in (21) reveals the main difference between the policy regimes presented in the empirical analysis. While the pre-Volcker era was associated with a strictly non-negative inflation elasticity, Federal Reserve policy in Volcker-Greenspan era has lead to a significantly negative inflation elasticity. Given this clear evidence, we can immediately conclude that the latter regime was more successful in maximizing welfare and, thus, in stabilizing macroeconomic fluctuations induced by fundamental shocks.

5 Transmission of money supply shocks

In this section we aim at revealing the implications of money supply contingency for the transmission of monetary injections, which are usually analyzed for constant money growth regimes ($\mu_\pi = 0$). In particular, we derive the implications regarding two often discussed issues in the literature (see, e.g., Christiano et al., 1997, or Chari et al., 2000), namely, the existence of a liquidity effect and the persistence of output responses. Disregarding cost push

[23] The Friedman rule is in fact optimal if a standard cash-constraint applies, $P_t c_t \leq M_{t-1} + P_t \tau_t$, prices are flexible and households and firms are perfectly competitive such that $u_c(t) = v_l(t) R_t$.

shocks ($\psi_t = 1$), the generic form of the fundamental solution only features real balances $\widehat{m}_{t-1}$ and the policy shocks ε_t as the relevant state variables of the model:

$$\widehat{m}_t = \widehat{y}_t = \delta_m \widehat{m}_{t-1} + \delta_{m\varepsilon}\varepsilon_t \qquad \text{and} \qquad \widehat{\pi}_t = \delta_{\pi m}\widehat{m}_{t-1} + \delta_{\pi\varepsilon}\varepsilon_t. \tag{23}$$

The coefficients δ_i with $i \in \{m,\ m\varepsilon,\ \pi m,\ \pi\varepsilon\}$ are determined applying the method of undetermined coefficients (see, e.g., McCallum, 1999). Using the stability condition given in proposition 1, the following qualitative properties of the coefficients in (23) can easily be derived (see appendix 8.3).

Lemma 1 *Suppose that the central banks sets the growth rate of money according to (17) with $\mu_\pi < 1$. Then the coefficients of the fundamental solution (23) are characterized by $0 < \delta_m, \delta_{m\varepsilon} < 1$ and $\delta_{\pi m}, \delta_{\pi\varepsilon} > 0$.*

Hence, an unexpected rise in the money growth rate temporarily leads to higher real balances, output, and inflation rates ($\partial\widehat{m}_t/\partial\varepsilon_t = \partial\widehat{y}_t/\partial\varepsilon_t = \delta_{m\varepsilon} > 0$, $\partial\widehat{\pi}_t/\partial\varepsilon_t = \delta_{\pi\varepsilon} > 0$). The central relation between money growth rates and the nominal interest rate, which stems from the consumption Euler equation (18), reads $\widehat{R}_t = \sigma E_t(\widehat{\mu}_{t+1} - \widehat{\pi}_{t+1}) + E_t\widehat{\pi}_{t+1}$. The behavior of the nominal interest rate thus depends on the way money growth policy depends on the inflation rate. The fundamental solution for the interest rate reveals that a sufficiently small value for the inflation elasticity is able to ensure the existence of a liquidity effect.

Proposition 3 (Liquidity effect) *A shock to the money growth rule, $\varepsilon_t > 0$, leads to an immediate decline in the nominal interest rate ($\partial\widehat{R}_t/\partial\varepsilon_t < 0$) if $\mu_\pi < \overline{\mu_\pi}$, with $\overline{\mu_\pi} \equiv \frac{\sigma-1}{\sigma} < 1$.*

Proof. Combining (17) and (18) to $\widehat{R}_t = E_t\left[1 + \sigma\left(\mu_\pi - 1\right)\right]\widehat{\pi}_{t+1}$, and applying (23), gives the solution $\widehat{R}_t = \left[1 + \sigma\left(\mu_\pi - 1\right)\right]\delta_{\pi m}(\delta_{m\varepsilon}\varepsilon_t + \delta_m\widehat{m}_{t-1})$. Further using that $\delta_{m\varepsilon}$ and $\delta_{\pi m}$ are strictly positive (see lemma 1), we can conclude that the inflation elasticity must be sufficiently small, $\mu_\pi < (\sigma - 1)/\sigma$, to ensure $\partial\widehat{R}_t/\partial\varepsilon_t < 0$. ■

The result presented in proposition 3 shows that the likelihood for a money growth rule to lead to a liquidity effect rises with smaller elasticities μ_π and with higher values for the inverse of the elasticity of intertemporal substitution σ. For example, a constant money growth is sufficient for a liquidity effect as long as $\sigma > 1$. This corresponds to the finding of Christiano et al. (1997) in a model with one period preset prices and is qualitatively consistent with the results in Galí (2001) and Andrés et al. (2002) derived from simulations. Given that the reactiveness of money growth policy governs the commovement of real balances and the inflation rate, it further has a bearing on the persistence of the output responses to a monetary injection. In particular, the model predicts that the persistence is raised by higher values for the inflation elasticity. In order to derive this result we use that the persistence of output measured by $\partial\widehat{y}_{t+1}/\partial\widehat{y}_t$ equals the stable eigenvalue δ_m , as the cash-in-advance constraint demands output to be equal to real balances.

Proposition 4 (Persistence) *The persistence of the output response to a money supply shock rises for larger values for the inflation elasticity:* $\partial(\partial\widehat{y}_{t+1}/\partial\widehat{y}_t)/\partial\mu_\pi = \partial\delta_m/\partial\mu_\pi > 0$.

Proof. The stable eigenvalue, i.e., the smaller root of the characteristic polynomial in (19), is $\delta_m = \left(\Phi(\mu_\pi) - \sqrt{\Phi(\mu_\pi)^2 - 4\beta}\right)/2\beta > 0$, with $\Phi(\mu_\pi) \equiv \omega\,(1-\mu_\pi) + 1 + \beta$, and takes real values given that $\Phi(\mu_\pi)^2 - 4\beta = (1-\beta)^2 > 0$. Thus, the partial derivative $\frac{\partial\delta_m}{\partial\mu_\pi} = \frac{\partial\Phi(\mu_\pi)}{\partial\mu_\pi}\frac{\partial\delta_m}{\partial\Phi(\mu_\pi)}$ is strictly positive, as $\partial\Phi(\mu_\pi)/\partial\mu_\pi < 0$ and $\partial\delta_m/\partial\Phi(\mu_\pi) < 0$ given that $\beta > 0$. ■

The result presented in proposition 4 can be rationalized as follows. Suppose that an expansionary money growth shock hits the economy. Then real balances and the rate of inflation rises on impact as revealed by the signs of the impact multiplier in lemma 1. Hence, a positive feedback from inflation to the nominal money growth prolongs the time interval required for real balances to return to the steady state. Consequently, the convergence back to the steady state is speeded up by a negative inflation elasticity.

6 Conclusion

In this paper we argue that Federal Reserve policy can actually be summarized by simple money supply rules. We provide empirical evidence that the growth rate of non-borrowed reserves has been set contingent on expected inflation rates. Estimates for the pre-Volcker and the Volcker-Greenspan era reveal that the latter regime has been highly reactive indicated by a significantly negative inflation elasticity, whereas the former regime was in fact associated with an accommodating money supply (a positive inflation elasticity). While this finding supports related evidence on simple interest rate rules as well as common wisdom about the relative performances of the policy regimes, the results cast doubt on the hypothesis that high and volatile inflation rates in the pre-Volcker period were mainly brought about Federal Reserve interest rate policy, by allowing for self-fulfilling expectations. Considering the underlying money supply behavior, we show that saddle path stability was never abolished, and that monetary policy actually differed with regard to its effectiveness to stabilize the economy hit by fundamental (rather than non-fundamental) shocks. This conclusion is exactly supported by an analysis of the optimal monetary policy under commitment, which reveals that the optimal inflation elasticity must indeed be negative. On the transmission of money supply shocks, we further show that with smaller inflation elasticities the likelihood of a liquidity effect rises, while the persistence of output responses declines.

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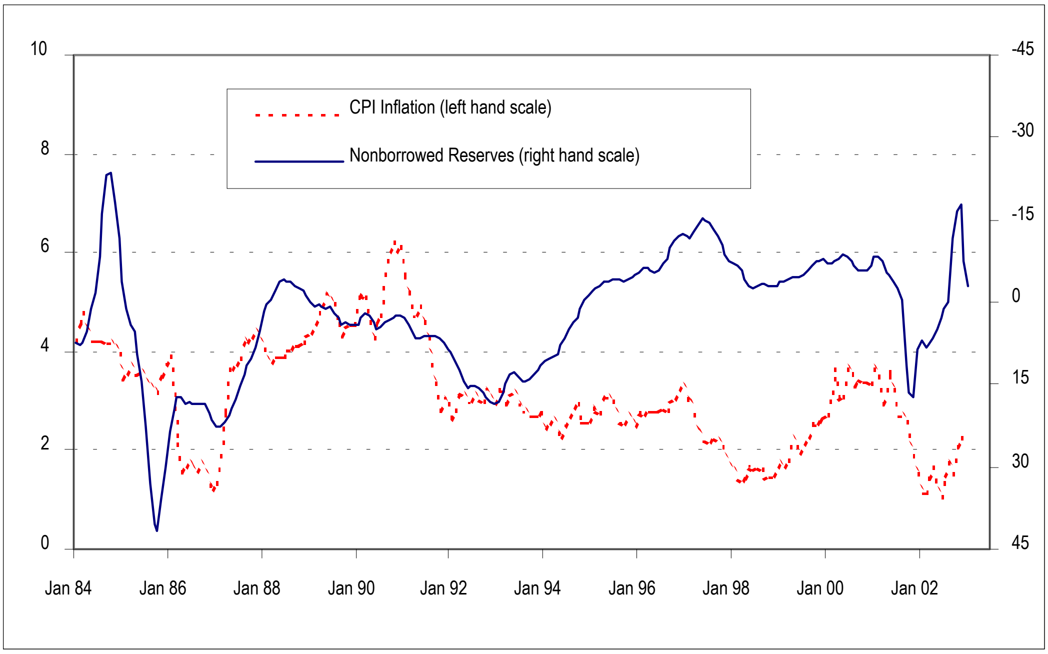
Figure 1: Nonborrowed Reserves Growth versus Consumer Price Inflation



Notes: The left hand axis refers to the inflation rate and the right hand axis measures the growth rate of nonborrowed reserves. The scale of the right hand axis has been inverted. Growth rates are calculated as year-on-year percentage changes.

8 Appendix

8.1 Forward looking money supply rules

In order to demonstrate the robustness of the determinacy condition presented in proposition 1, we consider the following forward looking money growth rule

$$\widehat{m}_t - \mu_\pi E_t \widehat{\pi}_{t+1} = \widehat{m}_{t-1} - \widehat{\pi}_t + \varepsilon_t. \tag{24}$$

Hence, the model, given by (16) and (24), now reads

$$M_0 \begin{pmatrix} \widehat{m}_t \\ E_t \widehat{\pi}_{t+1} \end{pmatrix} = M_1 \begin{pmatrix} \widehat{m}_{t-1} \\ \widehat{\pi}_t \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \varepsilon_t, \quad \text{with } M_0^{-1} M_1 = \begin{pmatrix} \omega & \beta \\ 1 & -\mu_\pi \end{pmatrix}^{-1} \begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix}.$$

The characteristic polynomial is given by $H(X) = X^2 - \frac{\beta+1+\omega}{\omega\mu_\pi+\beta} X + \frac{1}{\omega\mu_\pi+\beta}$, with $H(0) = (\omega\mu_\pi + \beta)^{-1}$ and $H(1) = \omega(\mu_\pi - 1)(\omega\mu_\pi + \beta)^{-1}$. Thus, $\mu_\pi > -\beta/\omega$ implies $H(0) > 0$ and $H(1) < 0$ if $\mu_\pi < 1$, such that there is exactly one stable root between zero and one. For $\mu_\pi < -\beta/\omega$, we know that $H(0) < 0$ and $H(1) > 0$ if $\mu_\pi < 1$, implying that saddle path stability is ensured by $\mu_\pi < 1$.

8.2 Derivation of the central bank's loss function

In this appendix we derive a linear-quadratic approximation of the average of households' objective $u(c_t) + \int_0^1 v(l_{jt})dj$ at the undistorted steady state. The derivation closely follows Woodford (2002, chap. 6) and Walsh (2003). Let $\widetilde{X}_t = X_t - \overline{X}$ and $\widehat{X}_t = \log X_t - \log \overline{X}$, where X denotes a generic variable. Thus, a linear-quadratic approximation implies $\frac{X_t}{\overline{X}} \approx 1 + \log \frac{X_t}{\overline{X}} + \frac{1}{2}\left(\log \frac{X_t}{\overline{X}}\right)^2 = 1 + \widehat{X}_t + \frac{1}{2}\widehat{X}_t^2$. The second order Taylor expansion for $u(c_t)$ is

$$u(c_t) = u(\overline{c}) + u_c(\overline{c})\widetilde{c}_t + \frac{1}{2} u_{cc}(\overline{c})\widetilde{c}_t^2 + O\left(\|a\|^3\right),$$

where $O(X^n)$ summarizes terms of order equal or higher than n and $\|a\|$ denotes a bound on the amplitude of exogenous disturbances. Using $\widetilde{c}_t \approx \overline{c}\left(\widehat{c}_t + \frac{1}{2}\widehat{c}_t^2\right)$ and $\sigma \equiv -\frac{\overline{u}_{cc}\overline{c}}{\overline{u}_c}$, we can therefore write

$$u(c_t) \approx u(\overline{c}) + u_c(\overline{c})\overline{c}\left[\widehat{c}_t + \frac{1}{2}(1-\sigma)\widehat{c}_t^2\right].$$

Proceeding with $\int_0^1 v(l_{jt})dj = v(l_t) = \frac{1}{1+\vartheta} l_t^{1+\vartheta}$, a second order expansion leads to

$$v(l_t) = v(\overline{l}) + v_l(\overline{l})\widetilde{l}_t + \frac{1}{2} v_{ll}(\overline{l})\widetilde{l}_t^2 + O\left(\|a\|^3\right).$$

Combining $\widetilde{l}_t = \int_0^1 \widetilde{l}_{it} di$, $\widetilde{l}_{it} \approx \bar{l}\left(\widehat{l}_{it} + \frac{1}{2}\widehat{l}_{it}^2\right)$ and $y_{it} = l_{it}$ to $\widetilde{l}_t = \overline{y} \int_0^1 \left(\widehat{y}_{it} + \frac{1}{2}\widehat{y}_{it}^2\right) di$, and using $\vartheta \equiv \frac{\overline{v}_{ll}\bar{l}}{\overline{v}_l}$, we can write

$$v(l_t) \approx v(\bar{l}) + v_l(\bar{l})\overline{y}\left(\int_0^1 \widehat{y}_{it} di + \frac{1}{2}\int_0^1 \widehat{y}_{it}^2 di + \frac{1}{2}\vartheta\left[\int_0^1 \widehat{y}_{it} di\right]^2\right),$$

Transforming the aggregator (12) to

$$\left(\frac{y_t}{\overline{y}}\right)^{1-\epsilon^{-1}} = \int_0^1 \left(\frac{y_{it}}{\overline{y}}\right)^{1-\epsilon^{-1}} di \quad \Leftrightarrow \quad \exp\left[\left(1-\epsilon^{-1}\right)\widehat{y}_t\right] = \int_0^1 \exp\left[\left(1-\epsilon^{-1}\right)\widehat{y}_{it}\right] di,$$

and using that $\exp\left[\left(1-\epsilon^{-1}\right)\widehat{X}_t\right] \approx 1 + \left(1-\epsilon^{-1}\right)\widehat{X}_t + \frac{1}{2}\left(1-\epsilon^{-1}\right)^2 \widehat{X}_t^2$, gives

$$\widehat{y}_t + \frac{1}{2}\left(1-\epsilon^{-1}\right)\widehat{y}_t^2 \approx \int_0^1 \widehat{y}_{it} di + \frac{1}{2}\left(1-\epsilon^{-1}\right)\int_0^1 \widehat{y}_{it}^2 di.$$

By neglecting terms of higher than second order, this implies

$$\widehat{y}_t \approx \int_0^1 \widehat{y}_{it} di + \frac{1}{2}\left(1-\epsilon^{-1}\right)\left(\int_0^1 \widehat{y}_{it}^2 di - \left[\int_0^1 \widehat{y}_{it} di\right]^2\right) \text{ and } \left[\int_0^1 \widehat{y}_{it} di\right]^2 \approx \widehat{y}_t^2.$$

Further applying $var_i\widehat{y}_{it} \equiv \int_0^1 \widehat{y}_{it}^2 di - \left[\int_0^1 \widehat{y}_{it} di\right]^2$, we obtain

$$\widehat{y}_t \approx \int_0^1 \widehat{y}_{it} di + \frac{1}{2}\left(1-\epsilon^{-1}\right) var_i\widehat{y}_{it} \text{ and } \int_0^1 \widehat{y}_{it}^2 di \approx \widehat{y}_t^2 + var_i\widehat{y}_{it}.$$

With these expressions $v(l_t)$ can be approximated by

$$v(l_t) \approx v(\bar{l}) + v_l(\bar{l})\overline{y}\left(\widehat{y}_t + \frac{1}{2}(1+\vartheta)\widehat{y}_t^2 + \frac{1}{2}\epsilon^{-1} var_i\widehat{y}_{it}\right).$$

Combining both parts of the utility function to

$$u(c_t) - v(l_t) \approx u(\overline{c}) - v(\bar{l}) + u_c(\overline{c})\overline{c}\left[\widehat{c}_t + \frac{1}{2}(1-\sigma)\widehat{c}_t^2\right] - v_l(\bar{l})\overline{y}\left[\widehat{y}_t + \frac{1}{2}(1+\vartheta)\widehat{y}_t^2 + \frac{1}{2}\epsilon^{-1} var_i\widehat{y}_{it}\right],$$

and using $v_l(\bar{l})/u_c(\overline{c}) = \overline{w}/\overline{\psi} = \overline{mc}/\overline{\psi} = (1-\Omega)$, where Ω measures the distortion due to monopolistic competition, we can collect terms to

$$u(c_t) - v(l_t) \approx u(\overline{c}) - v(\bar{l}) + u_c(\overline{c})\overline{y}\left[\Omega \cdot \widehat{y}_t - \frac{1}{2}\left[\sigma + \vartheta - \Omega(1+\vartheta)\right]\widehat{y}_t^2 - (1-\Omega)\frac{1}{2}\epsilon^{-1} var_i\widehat{y}_{it}\right],$$

where we used $c_t = y_t$. Assuming that $\Omega \to 0$, which can be interpreted as a fiscal authority being able to avoid the monopolistic distortion by an appropriate transfer system, gives

$$u(c_t) - v(l_t) \approx u(\overline{c}) - v(\bar{l}) - u_c(\overline{c})\overline{y}\frac{1}{2}(\sigma + \vartheta)\widehat{y}_t^2 - u_c(\overline{c})\overline{y}\frac{1}{2}\epsilon^{-1} var_i\left(\log y_{it} - \log\overline{y}\right).$$

Further using that the demand function $y_{it} = (P_{it}/P_t)^{-\epsilon}\, y_t$ implies $var_i \log y_{it} = \epsilon^2 var_i \log P_{it}$ and that $\sum_{t=0}^{\infty} \beta^t var_i \log p_{it} \approx \chi^{-1} \sum_{t=0}^{\infty} \beta^t \pi_t^2$ (see Woodford, 2002, chap. 6), we obtain

$$\frac{1}{\Xi} \sum_{t=0}^{\infty} \beta^t \left(U_t - \overline{U}\right) \approx -\frac{1}{2} \sum_{t=0}^{\infty} \beta^t L_t, \quad \text{with } L_t \equiv \widehat{\pi}_t^2 + \frac{\omega}{\epsilon} \widehat{y}_t^2 \quad \text{and } \Xi \equiv u_c(\overline{c})\overline{y}\frac{\epsilon}{\chi},$$

where we used that $\omega \equiv \chi\,(\vartheta + \sigma)$ and $\overline{\pi} = 1$.

8.3 Proof of lemma 1

In order to establish the claims made in the lemma, we apply the equilibrium conditions given in definition 1 and use the method of undetermined coefficients (see, e.g., McCallum, 1999). The aggregate supply curve (16) together with the general solution form (23) immediately leads to the following four conditions for the coefficients δ_i with $i \in \{m,\, m\varepsilon,\, \pi m,\, \pi\varepsilon\}$

$$\delta_{\pi m} - \beta \delta_{\pi m} \delta_m - \omega \delta_m = 0, \qquad \delta_m + (1 - \mu_\pi)\, \delta_{\pi m} - 1 = 0, \tag{25}$$

$$\delta_{\pi\varepsilon} - \beta \delta_{\pi m} \delta_{m\varepsilon} - \omega \delta_{m\varepsilon} = 0, \qquad \delta_{m\varepsilon} + (1 - \mu_\pi)\, \delta_{\pi\varepsilon} - 1 = 0. \tag{26}$$

Eliminating $\delta_{\pi m}$ in the equations in (25) yields a quadratic equation in δ_m, which equals the characteristic polynomial in (19), featuring a exactly one root between zero and one if $\mu_\pi < 1$ (see proposition 1). Given that this inequality is satisfied, such that $0 < \delta_m = X_1 < 1$ holds, the following expression, which is derived from the equations in (26), reveals that $\delta_{m\varepsilon}$ also lies between zero and one $0 < \delta_{m\varepsilon} = [1 + \beta(1 - \delta_m) + \omega\,(1 - \mu_\pi)]^{-1} < 1$. Therefore, we can conclude that $\delta_{\pi\varepsilon} = \frac{1-\delta_{m\varepsilon}}{1-\mu_\pi} > 0$ and $\delta_{\pi m} = \frac{1-\delta_m}{1-\mu_\pi} > 0$, which completes the proof of the lemma. ■