Measuring the Effect of Money: Test, Estimation and Identification

Mau-Ting Lin*

Department of Economics, National University of Singapore

June 17, 2003

Abstract

This paper provides a new approach of measuring the effects of money in both the long-run and the short-run horizons. The key identifying assumptions used to identify and measure the effect of money are long-run neutrality and long-run homogeneity.

The first chapter shows that both long-run propositions imply certain linear restrictions to be imposed on the cointegrating space. By testing the validness of such linear restrictions, both long-run propositions are tested. Compared with the previous long-run tests, the cointegration test in this paper does not depend heavily on the auxiliary assumptions, including identification restrictions and the correct selection of macroeconomic variables to be included in the empirical work.

The second chapter shows that the linear restrictions imposed by the long-run propositions can be used to identify the monetary shocks when the long-run proposition is overidentifying. In such a case, it is proved that the monetary shocks can be identified with the following three identifying assumptions: (1) a monetary shock is long-run neutral and homogeneous; (2) monetary shocks are not correlated with other structural shocks; and (3) the long-run effect of money is linearly independent from the long-run effects of other structural shocks.

* Email: mtlin@bu.edu. This paper is the first two chapters of my dissertation in Boston University. I am indebted to Robert G. King, Pierre Perron and Simon Gilchrist for their advice throughout the writing of this paper, to John Chao for his assistance in PIC programming, and to Christopher Otrok for his data provision. I also wish to thank Zhongjun Qu and Ferhan Salman for helpful comments and discussion. The usual disclaimer applies.

Contents

List of Tables

List of Figures

CHAPTER 1

New Tests of Long-Run Monetary Neutrality and Homogeneity

1. Introduction

The effect of money on real activity is one of the central research topics in macroeconomics. Standard macroeconomic models suggest short-run effects of money on both real output and the price level, with the longer-run effects on nominal variables but not on real variables. The research reported in this chapter devises new tests of long-run propositions about the effect of money on economic activity, in particular the long-run neutrality (LRN) and the long-run homogeneity (LRH). In this chapter, as in much prior work, long-run neutrality is defined as the implication that a once-for-all change in the level of money should not have a long-run impact on real variables. Similarly, long-run homogeneity is defined as the implication that such a permanent change in the level of money should affect all nominal variables proportionately in the long run.

There is a lengthy history of efforts to test LRN and LRH. One notable early strand of research on these two issues was at the Federal Reserve Bank of St. Louis during the 1970s. The St. Louis researchers ran regressions of the first difference of log output on the current and lagged values of money growth and then computed a long-run multiplier of money – the sum of the regression coefficients – as the basis for LRN and LRH tests. These tests were part of a larger effort by the St. Louis researchers to characterize the empirical effects of monetary and fiscal policy on macroeconomic activity, which was heavily criticized for its reduced form nature and its lack of concern about the direction of causality (see, e.g. Ando and Mogiliani (1990)). In the context of LRN and LRH, the force of this criticism is that a long-run multiplier which differs from its theoretical value – of zero for LRN and of one for LRH – may be a reflection of central bank policy response to economic conditions rather than a rejection of the long-run proposition. Later, in the early 1970s, Sargent and Lucas pointed out another important difficulty with LRN tests based on an estimated long-run multiplier: when the economy does not embody any long-run variation in money, the estimated long-run multiplier does not accurately capture the long-run effect of money. Their argument was a forerunner of the critique that Lucas subsequently made: if long-run variation in money is not a part of the environment that shapes the behavioral responses of economic agents, then a reduced form analysis – such as regressions or vector autoregressions – can never provide an answer about the effect of a long-run change in money.

Concern about causality and the Lucas critique cast a shadow over applied research on long-run (LR) tests for nearly two decades. However, Fisher and Seater (1993) pointed out that the pessimism was not necessarily justified if economists are concerned about whether a LR hypothesis held in a particular history and the historical data contained long-run variation in money. In the situation where money was nonstationary (integrated of order one), they showed how LRN can be tested via a long-run regression with proper identifying assumptions made to disentangle the causality between output and money. For convenience in the discussion below, we call tests based on these two ideas – integration and identification– *second generation* tests of LR propositions.[1] These second generation studies principally concerned bivariate relations between variables: they look at relations between money and output to test LRN and between money and the price level (or nominal income) to test LRH.

Two drawbacks of the long-run neutrality tests along second generation lines have been pointed out in the literature. One is that the results of the test are heavily dependent on identifying assumptions. More formally, identification involves making the correct mapping between the forecast errors and structural shocks, particularly the monetary shock. The

[1] Other related research is contained in Geweke (1986), who uses frequency domain methods, and King and Watson (1997), who use vector autoregression (VAR) methods.

importance of this set of assumptions (and the fragility of neutrality tests with respect to them) is most apparent in the VAR analysis of King and Watson (1997), where the sensitivity of the long-run neutrality tests to various identifying assumptions is graphically displayed. Yet, while such a VAR approach to neutrality testing has become popular[2], its application also requires that the researcher select the list of variables properly. This is because the vector of variables used in a second generation study must reveal the shocks in the economy correctly to the researcher. More formally, it must be possible to map between the forecast errors and the true structural shocks. If the researcher is studying a subvector of economic activity, then there are many reasons that this mapping may be infeasible. So accurate specification of the data vector is an essential part of second generation tests. Thus, any rejection of LRN or LRH in a VAR context can signal that the theory is wrong or that the identification and/or variable selection assumptions are incorrect.

In this chapter, I develop a LR test based on cointegration concepts which depends on a basic identifying assumption shared with the second generation tests: there must be an independent source of nonstationary variation in the monetary time series. But my test does not require either of the other maintained assumptions of the second generation tests: it can evaluate LRN and LRH without a parametric identifying assumption and without correct specification of the macroeconomic data vector.

Turning to the details, I follow the approach of second generation tests in working with a vector autoregression that is nonstationary in levels, so that there can be the stochastic trend in money that these studies and my approach both rely on. But, in contrast to these earlier studies, I suppose that there may be cointegration among the macroeconomic variables and study a system with three or more variables so as to test LRN and LRH. To be precise, I employ a vector error correction (VEC, henceforth) model of a form that is standard in cointegration analysis. With an application of the Granger representation theorem[3], I prove

[2]e.g. Bernanke and Mihov (1998), Serletis and Koustas (1998), and Bae and Ratti (2000).

[3]Please refer to Chapter 4 of Johansen(1995).

that any LR hypothesis can be interpreted as a set of linear constraints on the orthogonal cointegrating space, which in turn imposes restrictions on the cointegrating space. This orthogonality condition is independent of the conventional identifying assumptions employed in the second generation LR tests.

The LRN and LRH hypothesis constrain the cointegrating vector space, with a sacrifice in degrees of freedom which is dictated by the particular hypothesis. Exploiting this property, I show how to construct a likelihood ratio test for a particular LR hypothesis. When the degrees of freedom sacrificed is larger, the LR hypothesis is stronger, i.e., more constraining on the estimated model. Hence, the LR hypothesis that I derive in this chapter can be used to test LRN against LRH since, as I formulate these hypotheses, LRH is a stronger hypothesis involving a greater sacrifice in degrees of freedom.

I apply my LR tests to two different data sets for real output, nominal interest rate, the price level and nominal money stock: one is an annual data set based on the monetary history of Friedman and Schwartz (1982), which covers 1940-1975, and the other is a post WWII quarterly data set which covers 1959:1-2002:2. For the latter period, I split the data into two subsamples: one is a pre-Volcker sample and the other is a post-1983 sample. For all samples, I did not reject long-run neutrality (LRN). Long-run homogeneity was rejected in the pre-Volcker sample of quarterly data, but not in the other two samples.

2. An Overview of Long-Run Tests

There has long been interest in testing propositions about the long-run link between money and real or nominal variables, which are at the heart of classical macroeconomics. In this section, the history of such tests is reviewed and the alternative approach taken in this chapter is highlighted.

To begin, it is useful to review the two basic long-run propositions considered in this chapter. In the long run, a permanent change in the level of the money stock is assumed to affect nominal variables proportionately and not to affect real variables. In this chapter, these are called the long-run homogeneity (LRH) and long-run neutrality (LRN) propositions. For example, if the nominal variable is nominal income then the homogeneity proposition could be investigated via the regression

$$Y_t = bm_t + cx_t + e_t,$$

where Y_t is log nominal income, m_t is log money stock, x_t are other variables that affect nominal income and e_t is an error term. In this regression setting, the LRH hypothesis is that $b = 1$ since this implies that

$$\frac{\partial Y_t}{\partial m_t} = b = 1.$$

A comparable regression for the LRN hypothesis is

$$y_t = dm_t + gx_t + e_t,$$

where y_t is log real income and the other variables are as above, The neutrality hypothesis is that $d = 0$, since that implies

$$\frac{\partial y_t}{\partial m_t} = d = 0.$$

2.1. The beginning. In the 1960s, researchers at the Federal Reserve Bank of St. Louis began the empirical study of the relationship between nominal income and the money stock in a dynamic regression framework,

$$\Delta Y_t = B(L)\Delta m_t + C(L)x_t + \varepsilon_t \tag{2.1}$$

where $B(L) = \Sigma_{i=0}^{m} B_i L^i$ and $C(L) = \Sigma_{i=0}^{f} C_i L^i$ are polynomials in the lag operator L and $\Delta = 1 - L$ indicates a first difference here and below.

The St. Louis researchers were motivated to study such distributed lag models by Friedman's (1969) argument that there was a lag in the effect of monetary actions on the macro economy. In the well-known work of Anderson and Jordan (1968), the main focus of the regression analysis was two-fold. First, they sought to determine the nature of the lags in the effects of monetary policy in estimating the B coefficients. Anderson and Jordan found that there was a less than one-for-one short-run effect of money on nominal income, *i.e.*, $B_0 < 1$. To calculate the effect of a sustained change in the level of money on the path of nominal income, they calculated dynamic multipliers. Given that the regression is in first-difference form, they calculated the effect of a sustained change, beginning at t, on nominal income at date $t+s$ as $\partial Y_{t+s}/\partial m_t = \Sigma_{i=0}^{s} B_i$.[4] Second, they sought to test whether a specific set of x's, measures of fiscal policy, affected nominal variables as suggested by the prominent brand of Keynesian macroeconomics.

Later St. Louis analysis–Andersen and Karnosky (1972)—used this regression framework to test LRH as follows. They imagined a permanent change in the level of money beginning at date t. They calculated that the long-run multiplier attached to this change was

$$\lim_{s\to\infty} \frac{\partial Y_{t+s}}{\partial m_t} = \Sigma_{i=0}^{s} B_i = B(1),$$

so that they tested LRH by testing whether the sum of coefficients was equal to unity. They also implemented the comparable test for LRN, investigating whether $\lim_{s\to\infty} \partial y_{t+s}/\partial m_t = \Sigma_{i=0}^{s} B_i = B(1)$ was zero.

2.1.1. *Simultaneity.* The St. Louis approach was controversial. Notably, Ando and Modigliani (1990) criticized the St. Louis regression for not recognizing that nominal income and the money stock were simultaneously determined.

However, Sims (1972) provided some support in a bivariate context for the St. Louis regression, building on Granger's (1969) earlier work on testing for causality. Theoretically, Sims established that it was only legitimate to run the regression

$$A(L)Y_t = B(L)m_t + e_{Y,t},$$

[4]To understand this, we ignore $C(L)x_t$ and ε_t in (2.1). It follows that $\partial Y_{t+s} \equiv (Y_{t+s} - Y_{t-1}) = \Sigma_{j=0}^{s} \Delta Y_{t+j} = \Sigma_{j=0}^{s} \Sigma_{i=0}^{m} B_i \Delta m_{t+j-i}$. Conditional on time t money variation, *i.e.* setting $\Delta m_{t+i-j} = 0$ for all $i \neq j$, the long-run multiplier $\partial Y_{t+s}/\partial m_t \equiv \partial Y_{t+s}|\Delta m_t = \Sigma_{i=0}^{s} B_i$.

for the purposes of the St. Louis researchers if the reverse regression

$$C(L)m_t = D(L)Y_t + e_{m,t},$$

displayed D coefficients that were zero. Looking at nominal income and money empirically, he found evidence that the D coefficients were statistically insignificant.

However, subsequent studies produced a more ambiguous result in terms of Sims-Granger causality of money for real and nominal income, with causality being found in some data sets and for some variable lists. These ambiguous findings are the motivation for development of tests of LRH and LRN which can be employed in settings where there is a dynamic endogeneity.

2.1.2. *The rational expectation critique.* The interpretation of the St. Louis regressions was also called into question by the analyses of Sargent (1971) and Lucas (1972). Studying an economy in which only unanticipated monetary changes had real effects and which otherwise displayed the LRH and LRN properties, Lucas (1972) showed that restrictions on sums of coefficients did not provide a way of testing the classical propositions.[5] To illustrate Lucas's point, consider an economy in which the behavior of real and nominal income is given by

$$\begin{aligned} y_t &= \phi(m_t - E_{t-1}m_t) + e_{y,t} \\ Y_t &= m_t - \phi\theta(m_t - E_{t-1}m_t) + e_{Y,t} \end{aligned}$$

where ϕ is a positive parameter and $0 < \theta < 1$. That is: unanticipated monetary expansions raise real income and raise nominal income less than one-for-one. If the money supply is given by the first-order autoregression $m_t = \rho m_{t-1} + e_{m,t}$, it then follows that the rational expectations solutions for real and nominal output are

$$y_t = \phi m_t - \phi\rho m_{t-1} + e_{y,t} \tag{2.2}$$

$$Y_t = (1 - \phi\theta)m_t + \rho\phi\theta m_{t-1} + e_{Y,t} \tag{2.3}$$

[5]Sargent (1971) made a similar point in a Phillips curve framework.

Under Lucas's assumption that the money supply process is stationary($|\rho| < 1$), the sum of coefficients in the real output equation is inconsistent with neutrality($\phi(1-\rho) > 0$). These implications occur despite the fact that the model is one with a strong form of neutrality and homogeneity.

On the basis of this finding, Lucas and Sargent argued against evaluating the long-run effects of monetary policy on the basis of relatively unrestricted distributed lag models. Further, Lucas and Sargent argued that testing of the classical propositions required specification and estimation of a detailed structural model, so that the expectational and behavioral lags could be separated.

2.2. Second Generation LR Tests. A second generation of long-run tests was developed by Geweke (1986), Fisher and Seater (1992) and King and Watson (1997). Although these studies differ in the details, each was based on the core idea that long-run propositions are testable if there is suitable long-run variation in money. As an example, suppose that the money stock is assumed to be a random walk ($\rho = 1$) in the Lucas model just considered. This assumption means that all changes in money are unanticipated and permanent. Evaluating the expressions above at $\rho = 1$, it then follows that the sum of coefficients on the monetary variables in (2.2) is zero as suggested by prior neutrality tests and the sum in (2.3) is one as suggested by prior homogeneity tests.

Working from the assumption that there is exogenous long-run variation in money, the second generation LR tests all seek to determine how real or nominal variables respond to these variations in the long-run variation in money. If the study concerns real income, so that LRN is the hypothesis of interest, then the long-run effect should be zero. By contrast, if the study concerns nominal income or the price level, then this long-run effect should be unity. One way of determining these long-run effects is via considering the comovement of long-run variations in money and other variables.

2.2.1. *Long-run variations.* The long-run variation $\Delta_\infty x_{t-1}$ of a variable x is defined as $\lim_{k\to\infty}(x_{t+k} - x_{t-1})$, which is equal to $\lim_{k\to\infty} \Sigma_{i=0}^{k} \Delta x_{t+i}$. The second generation LR tests concern long-run variation conditional on a one-time shock, or, equivalently, the long-run response to a one-time shock. In our case, it would be the long-run response to a one-time monetary shock.

If x_t is I(1) stationary, then its long-run variation can be modeled by the following approximate autoregressive process of its first difference,

$$C(L)\Delta x_t = \varepsilon_t.$$

If we invert the process, $\Delta x_t = C(L)^{-1}\varepsilon_t$. Letting $C(L)^{-1} = \Sigma_{j=0}^{\infty}\psi_j L^j$, it then follows that $\lim_{k\to\infty} \Sigma_{i=0}^{k}\Delta x_{t+i} = \lim_{k\to\infty} \Sigma_{i=0}^{k}\Sigma_{j=0}^{\infty}\psi_j\varepsilon_{t+i-j}$. Accordingly, the long-run response of x to a one-time shock at time t can be calculated by setting all $\varepsilon_{t+i-j} = 0$ except $i = j$, which is

$$\begin{aligned} \Delta_\infty x_{t-1}|\varepsilon_t &= \lim_{k\to\infty} \Sigma_{i=0}^{k}\Sigma_{j=0}^{\infty}\psi_j\varepsilon_{t+i-j}|\varepsilon_{t+i-j} = 0 \text{ for all } i-j \neq 0 \\ &= (\Sigma_{j=0}^{\infty}\psi_j)\varepsilon_t \\ &= C(1)^{-1}\varepsilon_t. \end{aligned}$$

2.2.2. *Neutrality of long-run variations.* Within the Lucas model studied in the last section, the money supply process is $m_t = \rho m_{t-1} + e_{m,t}$ where $|\rho| \leq 1$.

$$\begin{aligned} \Delta m_t &= \rho\Delta m_{t-1} + (1-L)e_{m,t} \\ &= (1-\rho L)^{-1}(1-L)e_{m,t}. \end{aligned}$$

In this case, $C(L)^{-1} = (1-\rho L)^{-1}(1-L)$. Long-run variations in money will exist only if $C(1)^{-1} \neq 0$. This happens only when $\rho = 1$. In that case, the long-run response of m to one-time shock $e_{m,t}$ is $e_{m,t}$ itself.

From (2.2), it follows that the output response is

$$\begin{aligned}\Delta y_t &= \phi(1-L)\Delta m_t + (1-L)e_{y,t} \\ &= (1-L)(\phi e_{m,t} + e_{y,t}).\end{aligned}$$

The long-run response $\Delta_\infty y_{t-1}$ conditional on $e_{m,t}$ shock is zero, which is obtained by replacing 1 for L in the expression above. That is, in the Lucas model, there is neutrality of long-run variations in money: the shock $e_{m,t}$ has a long-run effect on m but not on y. As discussed above, when $\rho = 1$, $e_{m,t}$ produces the long-run variation in money, which the second generation tests require.

2.2.3. *VAR-based second generation methods.* Neutrality tests based on vector autoregressions are more complicated than the simple Lucas example on three dimensions. First, they allow real output to be potentially affected by real shocks in the long-run. Second, they allow for money growth to respond to its own lags and to lags of output growth. Third, they allow for short-run interactions of real and nominal variables. All of these considerations are reflected in the following structural vector autoregression,

$$\begin{aligned}\pi_{ry}\Delta y_t &= \pi_{rm}\Delta m_t + \Sigma_{i=1}^{p}\alpha_{yy,i}\Delta y_{t-i} + \Sigma_{i=1}^{p}\alpha_{ym,i}\Delta m_{t-i} + u_t^r \\ \pi_{mm}\Delta m_t &= \pi_{my}\Delta y_t + \Sigma_{i=1}^{p}\alpha_{my,i}\Delta y_{t-i} + \Sigma_{i=1}^{p}\alpha_{mm,i}\Delta m_{t-i} + u_t^m.\end{aligned} \tag{2.4}$$

In this structural VAR, there are two structural shocks u^r and u^m. The former refers to real productivity shocks. The latter refers to monetary shocks. In addition, both variables are treated endogenously, which mitigates the causality problems previously discussed in the context of the St. Louis regression. Short-run interactions of Δy_t and Δm_t are governed by the π coefficients, while the dynamic interactions are governed by the α coefficients

The long-run responses of y_{t-1} and m_{t-1} to u_t shocks, i.e. $\Delta_\infty y_{t-1}|u_t$ and $\Delta_\infty m_{t-1}|u_t$ respectively, are the solutions to the following system:

$$\begin{bmatrix} \pi_{ry} - \Sigma_{i=1}^{p}\alpha_{yy,i} & -(\pi_{rm} + \Sigma_{i=1}^{p}\alpha_{ym,i}) \\ -(\pi_{my} + \Sigma_{i=1}^{p}\alpha_{my,i}) & \pi_{mm} - \Sigma_{i=1}^{p}\alpha_{mm,i} \end{bmatrix} \begin{bmatrix} \Delta_\infty y_{t-1}|u_t \\ \Delta_\infty m_{t-1}|u_t \end{bmatrix} = \begin{bmatrix} u_t^r \\ u_t^m \end{bmatrix} \tag{2.5}$$

Inverting (2.5),

$$\begin{bmatrix} \Delta_\infty y_{t-1} \\ \Delta_\infty m_{t-1} \end{bmatrix} |u_t = \varphi \begin{bmatrix} \pi_{mm} - \Sigma_{i=1}^{p}\alpha_{mm,i} & \pi_{rm} + \Sigma_{i=1}^{p}\alpha_{ym,i} \\ \pi_{my} + \Sigma_{i=1}^{p}\alpha_{my,i} & \pi_{ry} - \Sigma_{i=1}^{p}\alpha_{yy,i} \end{bmatrix} \begin{bmatrix} u_t^r \\ u_t^m \end{bmatrix} \tag{2.6}$$

where $\varphi = 1/((\pi_{ry} - \Sigma_{i=1}^{p}\alpha_{yy,i})(\pi_{mm} - \Sigma_{i=1}^{p}\alpha_{mm,i}) - (\pi_{my} + \Sigma_{i=1}^{p}\alpha_{my,i})(\pi_{rm} + \Sigma_{i=1}^{p}\alpha_{ym,i}))$. The long-run effect of a monetary shock u^m is

$$\begin{bmatrix} \Delta_\infty y_{t-1} \\ \Delta_\infty m_{t-1} \end{bmatrix} |u_t^m = \varphi \begin{bmatrix} \pi_{rm} + \Sigma_{i=1}^{p}\alpha_{ym,i} \\ \pi_{ry} - \Sigma_{i=1}^{p}\alpha_{yy,i} \end{bmatrix} u_t^m.$$

This expression is at the heart of second-generation tests. Using it, the implication of LRN is that $(\pi_{rm} + \Sigma_{i=1}^{p}\alpha_{ym,i}) / (\pi_{ry} - \Sigma_{i=1}^{p}\alpha_{yy,i}) = 0$. If we replace real income (y) with nominal income (Y), the implication of LRH is that $(\pi_{rm} + \Sigma_{i=1}^{p}\alpha_{ym,i}) / (\pi_{ry} - \Sigma_{i=1}^{p}\alpha_{yy,i}) = 1$. That is: the second generation LR tests focused on the statistical behavior of the ratio $(\pi_{rm} + \Sigma_{i=1}^{p}\alpha_{ym,i}) / (\pi_{ry} - \Sigma_{i=1}^{p}\alpha_{yy,i})$, which Fisher and Seater (1993) called the long-run derivative (LRD) of y (or Y) with respect to m.

2.2.4. *Identification and Second-Generation LR tests.* By its nature, the LRD is a structural parameter, which requires identifying assumptions to estimate it. These identifying assumptions are most easily discussed if we follow the actual practice used in some of the prior literature. To begin, suppose that we estimate a reduced form VAR as

$$\begin{aligned} \Delta y_t &= \Sigma_{i=1}^{p} a_{yy,i}\Delta y_{t-i} + \Sigma_{i=1}^{p} a_{ym,i}\Delta m_{t-i} + \varepsilon_t^y \\ \Delta m_t &= \Sigma_{i=1}^{p} a_{my,i}\Delta y_{t-i} + \Sigma_{i=1}^{p} a_{mm,i}\Delta m_{t-i} + \varepsilon_t^m \end{aligned} \tag{2.7}$$

Comparing this expression with (2.4), we note a structural relationship between reduced form shocks (forecast errors, ε_t) and structural shocks u_t that takes the form

$$\begin{bmatrix} u_t^r \\ u_t^m \end{bmatrix} = \begin{bmatrix} \pi_{ry} & -\pi_{rm} \\ -\pi_{my} & \pi_{mm} \end{bmatrix} \begin{bmatrix} \varepsilon_t^y \\ \varepsilon_t^m \end{bmatrix}.$$

To compute the LRD, second generation LR tests imposed different identifying assumptions regarding π parameters.

Geweke assumed $\pi_{rm} = 0$, which implies that real output does not respond to money aggregate within the current period, according to the first equation in (2.4). Using this approach, he found an evidence supporting LRN for real output. However, when he studied real balances, the test result did not support LRN, since the estimated long-run effect of money on real balances was not zero.

Fisher and Seater assumed that money is long-run exogenous by imposing $-(\pi_{my} + \Sigma_{i=1}^{p}\alpha_{my,i}) = 0$. That is: the real shock u_t^r is assumed not to affect the variation in money in the long-run. (Formally, given this assumption, the second equation of (2.5) governing the long-run response of money implies that: the long-run response $\Delta_\infty m_{t-1}|u_t$ of money does not depend on the long-run response $\Delta_\infty y_{t-1}|u_t$ of output). Using this assumption, FS found an evidence against LRN for the U.S. real income, and the German real balance. But they did not reject LRH for the U.S. price level and nominal income.

King and Watson explored the relation between test results and identifying assumptions within the bivariate VAR setting. Varying π_{rm}, they found that the LRD tends to reject LRN when the assumed π_{rm} value is small, but does not when it is large. Varying π_{my}, they found that the LRD tends to reject LRN when the assumed value is large, but not when it is small.

Taken together, the results of these studies indicate that tests of LRN and LRH are quite sensitive to the identifying assumptions used by different researchers.

3. Why cointegration is a valuable basis for New LR Tests

The key insight of this chapter is that cointegration is a valuable basis for constructing new long-run tests. In this section, I provide a series of examples illustrating why this is so, working to generalize the examples of the last section. Then, in section 3.3 below, I provide formal proofs that show how to exploit the ideas that are exemplified in the current section.

In the discussion of second-generation tests that we just completed, we saw that the LR tests based on the LRD rely heavily on two sets of maintained assumptions. First, it is clear that these tests depend on explicit identifying assumptions. Following Sims's(1980) discussion of the subtlety of identifying assumptions, we know that making right identifying assumptions is difficult. Second, the second-generation procedures rely importantly on the correct specification of the vector autoregression system: the analyst needs to be able to map between forecast errors and structural shocks. But, suppose that the macroeconomic data is really generated by a VAR in three variables, while the second generation researchers studied a system with only two variables. Then, it is quite likely that there is no way to map between structural shocks and forecast errors in the way that is required for the LRD tests.

A good econometric testing methodology should require the weakest possible auxiliary assumptions. My belief is that cointegration provides a valuable basis for weakening these maintained assumptions and I begin by displaying a series of examples that show why this is so.

3.1. The simplest example. To begin with the simplest example, we use the Lucas example of (y_t, m_t) to demonstrate our cointegration test. The model is

$$\begin{aligned} y_t &= \phi(m_t - E_{t-1}m_t) + u_t^y \\ m_t &= m_{t-1} + u_t^m. \end{aligned}$$

Recall that the solution to this model is:

$$\begin{aligned} y_t &= \phi \Delta m_t + u_t^y \\ \Delta m_t &= u_t^m. \end{aligned}$$

With recursive iteration, it follows that $m_t = m_0 + \Sigma_{i=1}^t u_i^m$, so that the solution can be expressed as

$$y_t = \phi u_t^m + u_t^y \tag{3.1}$$

$$m_t = m_0 + \Sigma_{i=1}^t u_i^m. \tag{3.2}$$

From this solution, it is not difficult to tell that y_t is stationary and m_t is I(1) stationary; and there is an I(1) component in m_t that is $\Sigma_{i=1}^t u_i^m$.

To implement a neutrality test empirically, we need to think of describing the behavior of output under the alternative of nonneutrality. In this case, we would need to append a term so that (3.1) becomes

$$y_t = \phi u_t^m + u_t^y + b(m_0 + \Sigma_{i=1}^t u_i^m) \tag{3.3}$$

In this case, both y_t and m_t are I(1) stationary, but they are cointegrated. It is clear that the cointegrating vector is $(1, -b)$, i.e., that $y_t - bm_t$ is stationary within this extended model.

To provide some further understanding of the content of the cointegrating vector, consider the long-run variation $(\Delta_\infty y_{t-1}, \Delta_\infty m_{t-1})$ of (y, m) to a one-time monetary shock at time t

$$\begin{bmatrix} \Delta_\infty y_{t-1} \\ \Delta_\infty m_{t-1} \end{bmatrix} | u_t^m = \begin{bmatrix} b \\ 1 \end{bmatrix} u_t^m$$

in line with our previous discussion in Section 2.2.1. The long-run response vector $(b, 1)'$ captures the contribution of the I(1) component coming from monetary shocks to each variable.[6] Hence, estimation of the cointegrating vector provides one way of isolating the

[6] The contributions of the I(1) component from monetary shocks to y_t is $b\Sigma_{i=1}^t u_i^m$ as in (3.3), and to m_t is $\Sigma_{i=1}^t u_i^m$ as in (3.2).

long-run effect of monetary shocks in this example. First, one determines that cointegrating vector. Second, one calculates the long run response vector, $(b,1)'$, which is orthogonal to the estimated cointegrating vector.

Importantly, though, this same strategy can be used to test long-run neutrality. LRN is a hypothesis about the long-run response of (y_t, m_t) to monetary shock, which says that

$$\begin{bmatrix} \Delta_\infty y_{t-1} \\ \Delta_\infty m_{t-1} \end{bmatrix} |u_t^m = \begin{bmatrix} 0 \\ g \end{bmatrix} u_t^m.$$

where we can always normalize g to one. Then, if LRN is true, the cointegrating vector of (y_t, m_t) should be orthogonal to $(0,1)$.

The geometry behind this testing strategy is illustrated in Figure 1. The long-run response vector that is implied by LRN is (0,1). Under long-run neutrality, the cointegrating vector is $\beta_1 = (1,0)$ which is orthogonal to the LRN long-run response vector (0,1). Other potential cointegrating vectors can be written in the form $\beta_2 = (b_1, b_2)$ and are not compatible with LRN because $\beta_2 * (0,1)' = b_2$ is non-zero.

This example highlights the general point that LRN imposes orthogonality restrictions on a cointegrating vector or, more generally, on the space of cointegrating vectors as we will see later in this chapter. If the orthogonality condition is violated, then LRN can be rejected. This example also illustrates that a LRN test based on cointegration does not require the identifying assumption imposed in second-generation LRN: it is not necessary to take a stand on the π coefficients in order to extract monetary shocks. We will see that this property extends to other LR tests, such as long-run homogeneity, and to larger systems.

3.2. Subtleties of LR tests based on cointegration. In the two variable example that we just studied, there is only one permanent shock, which is a monetary shock. For a bivariate model with one permanent shock, the cointegrating rank is one. However, when we investigate models with three or more variables, there can be other permanent shocks in addition to the monetary shock even if there is cointegration. This section accordingly studies a series of three variable examples, in which the variables are real output (y_t), the price level (p_t) and the money stock (m_t). We use this example to illustrate the subtleties encountered in LR tests when multiple permanent shocks are present.

Extending the earlier example, suppose that our three variables are governed by the following equations:

$$y_t = \theta(p_t - E_{t-1}p_t) + u_t^y \tag{3.4}$$

$$p_t = gm_t - y_t + u_t^d \tag{3.5}$$

$$m_t = \alpha m_{t-1} + u_t^m \tag{3.6}$$

where u_t^y is a supply or productivity shock; u_t^d is a money demand shock; and u_t^m is money supply shock. The economic interpretation of our three equations is that (3.4) represents a Lucas supply curve; that (3.5) is money demand equation; and that (3.6) is money supply equation. We assume that u_t^y, u_t^d, and u_t^m mutually independent. It is apparent that this simple model embeds LRN as a result of the Lucas supply curve and rational expectations. It also embeds LRH if $g = 1$.[7] These properties are reflected in the solution[8]:

$$\begin{aligned}
y_t &= \pi g u_t^m + \pi(u_t^d - E_{t-1}u_t^d) - \theta(u_t^y - E_{t-1}u_t^y) + (1+\theta)u_t^y \\
m_t &= \alpha^t m_0 + \Sigma_{i=1}^t \alpha^{t-i} u_i^m \\
p_t &= g\alpha^t m_0 + g\Sigma_{i=1}^t \alpha^{t-i} u_i^m \\
&\quad -\pi g u_t^m - \pi(u_t^d - E_{t-1}u_t^d) + \theta(u_t^y - E_{t-1}u_t^y) - (1+\theta)u_t^y + u_t^d
\end{aligned}$$

where $\pi = \theta/(1+\theta)$.

In particular, given this solution, we can explore three different cases, which differ in terms of the permanent structural shocks. In all three cases, we assume $\alpha = 1$. That is, in

[7] Though LRN and LRH certaintly do not necessarily require the Lucas supply curve and rational expectations, this simple model will help make more transparent the cointegration implications of LRN and LRH.

[8] Please refer to the appendix.

common with the second generation tests, we require that at least one of the permanent shocks is a monetary shock u_t^m. Hence the solution can be reduced to

$$y_t = \pi g u_t^m + \pi(u_t^d - E_{t-1}u_t^d) - \theta(u_t^y - E_{t-1}u_t^y) + (1+\theta)u_t^y \tag{3.7}$$

$$m_t = m_0 + \Sigma_{i=1}^t u_i^m \tag{3.8}$$

$$\begin{aligned} p_t &= g m_0 + g\Sigma_{i=1}^t u_i^m \\ &\quad -\pi g u_t^m - \pi(u_t^d - E_{t-1}u_t^d) + \theta(u_t^y - E_{t-1}u_t^y) - (1+\theta)u_t^y + u_t^d \end{aligned} \tag{3.9}$$

We now explore a series of cases.

3.2.1. *One permanent shock.* When there is only one permanent shock and it is a monetary shock, both productivity shock and money demand shock are stationary. For simplicity in the algebra, but without loss of generality, we assume both u^y and u^d are iid mean zero. Then $E_{t-1}u_t^d = E_{t-1}u_t^y = 0$. Consider the difference $y_{t+k} - y_{t-1}$, $m_{t+k} - m_{t-1}$ and $p_{t+k} - p_{t-1}$. From (3.7) to (3.9),

$$\begin{aligned} y_{t+k} - y_{t-1} &= \pi g(u_{t+k}^m - u_{t-1}^m) + \pi(u_{t+k}^d - u_{t-1}^d) + (u_{t+k}^y - u_{t-1}^y) \\ m_{t+k} - m_{t-1} &= \Sigma_{i=t}^{t+k} u_i^m \\ p_{t+k} - p_{t-1} &= g\Sigma_{i=t}^{t+k} u_i^m - \pi g(u_{t+k}^m - u_{t-1}^m) - (u_{t+k}^y - u_{t-1}^y) + (1-\pi)(u_{t+k}^d - u_{t-1}^d) \end{aligned}$$

Therefore, the long-run responses of y, m and p are:

$$\begin{bmatrix} \Delta_\infty y_{t-1} \\ \Delta_\infty m_{t-1} \\ \Delta_\infty p_{t-1} \end{bmatrix} | u_t = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & g \end{bmatrix} \begin{bmatrix} u_t^y \\ u_t^d \\ u_t^m \end{bmatrix}. \tag{3.10}$$

As discussed above, this confirms that u_t^m produces a long-run effect but not the other two structural shocks. But, more importantly, it shows that the long-run effect of u_t^m on (y, m, p) is $(0, 1, g)$.

Since there is only one permanent shock, the cointegrating rank should be two for this three variable system. That is: there are two cointegration vectors, which we now work to determine. From the solution, there is no I(1) component in y_t under LRN, just as in the example that we explored earlier. Accordingly, one of the cointegrating vectors is $(1, 0, 0)$. Yet, m_t has the I(1) component $\Sigma_{i=1}^t u_i^m$ and p_t has the I(1) component $g\Sigma_{i=1}^t u_i^m$. If $(\beta_y, \beta_m, \beta_p)$ is a cointegrating vector, then $\beta_y y_t + \beta_m m_t + \beta_p p_t$ should not contain any I(1) components. That is

$$\beta_y \times 0 + \beta_m \Sigma_{i=1}^t u_i^m + \beta_p (g\Sigma_{i=1}^t u_i^m) = 0.$$

It is easily verified that vector $(0, g, -1)$ satisfies this restriction, as well as $(1, 0, 0)$.

Hence, in this example, the origin of the I(1) components in y_t, m_t and p_t is the permanent monetary shock. The contribution of this I(1) component to each variable is $(0, 1, g)$ which is exactly the long-run response of (y, m, p) to a one-time monetary shock as in (3.10). A cointegrating vector gives a weight of linear combination of (y, m, p) such that the long-run response of this linear combination is zero to any one-time permanent shock.

In conclusion, both of the cointegrating vectors must be orthogonal to the (non-zero) long-run response vector in (3.10). It follows that the other way of finding cointegrating vectors is to find the vectors that are orthogonal to the long-run response matrix in (3.10). From this representation, we can easily see that $(1, 0, 0)$ and $(0, 1, -g)$ are one version of the cointegrating vectors.

Figure 2 shows the orthogonality conditions in this case. The cointegrating space consistent with long-run neutrality and long-run homogeneity is space B_1. It is comprised of all vectors that are orthogonal to the long-run response $(0, 1, 1)$. By contrast, the cointegrating space B_2 is orthogonal to the long-run response vector (1,1,2), which is inconsistent with both LRN and LRH.

While we have worked out the cointegration implications of LRN and LRH for a specific model above, let's now suppose we do not know the detailed structure of the economy. Under LRN, the I(1) component from money, *i.e.* $\Sigma_{i=1}^t u_i^m$, enters m_t and p_t, but not y_t. Let the weights it contribute to the nonstationarity of (y_t, m_t, p_t) be $(0, 1, g)$ which is the long-run response of (y_t, m_t, p_t) to a one-time permanent monetary shock. If $(\beta_y, \beta_m, \beta_p)$ is a cointegrating vector, by definition, $\beta_y y_t + \beta_m m_t + \beta_p p_t$ can not have I(1) component from money, *i.e.* $\beta_y \times 0 + \beta_m \times 1 + \beta_p \times g = 0$ that is the cointegrating vector should be orthogonal to $(0, 1, g)$. This orthogonality condition must be true under LRN without knowing the entire economic structure. The cointegration test of LRN is to test whether the cointegrating vector of (y, m, p) is orthogonal to $(0, 1, g)$ for some g. A test of LRH is a test of whether the cointegrating vector of (y, m, p) is orthogonal to $(0, 1, 1)$, *i.e.* the special case of $g = 1$. Once the orthogonality property is rejected, LR hypotheses are rejected.

3.2.2. *Two permanent shocks.* While the previous example is a helpful one, it is more realistic to assume that there are multiple permanent shocks. Hence, we consider two cases: (i) where there is a stochastic trend in real output; and (ii) where there is a stochastic trend in money demand.

u_t^y shock is I(1) stationary:

Let $u_t^y = u_{t-1}^y + u_t^r$ where u_t^r is an iid mean zero shock. Therefore, $u_t^y = u_0^y + \Sigma_{i=1}^t u_i^r$, $E_{t-1} u_t^y = u_{t-1}^y$ and $E_{t-1} u_t^d = 0$. The solution to (3.4) to (3.6) is then:

$$
\begin{aligned}
y_t &= \pi g u_t^m + \pi u_t^d - \theta u_t^r + (1+\theta)(u_0^y + \Sigma_{i=1}^t u_i^r) \\
m_t &= m_0 + \Sigma_{i=1}^t u_i^m \\
p_t &= g m_0 + g \Sigma_{i=1}^t u_i^m - \pi g u_t^m \\
&\quad -\pi g u_t^m - \pi u_t^d + \theta u_t^r - (1+\theta)(u_0^y + \Sigma_{i=1}^t u_i^r) + u_t^d.
\end{aligned}
$$

The long-run response of (y, m, p) is:

$$
\begin{bmatrix} \Delta_\infty y_{t-1} \\ \Delta_\infty m_{t-1} \\ \Delta_\infty p_{t-1} \end{bmatrix} | u_t = \begin{bmatrix} 1+\theta & 0 & 0 \\ 0 & 0 & 1 \\ -(1+\theta) & 0 & g \end{bmatrix} \begin{bmatrix} u_t^r \\ u_t^d \\ u_t^m \end{bmatrix}.
$$

From the solution, the I(1) components in y_t, m_t and p_t are $\Sigma_{i=1}^t u_i^r$, $\Sigma_{i=1}^t u_i^m$ and $g\Sigma_{i=1}^t u_i^m - \Sigma_{i=1}^t u_i^r$ respectively. If $(\beta_y, \beta_m, \beta_p)$ is a cointegrating vector, then $\beta_y y_t + \beta_m m_t +$

$\beta_p p_t$ should have no I(1) component left. That is

$$\beta_y(1+\theta)\Sigma_{i=1}^t u_i^r + \beta_m \Sigma_{i=1}^t u_i^m + \beta_p(g\Sigma_{i=1}^t u_i^m - (1+\theta)\Sigma_{i=1}^t u_i^r) = 0.$$

It is easily verified that vector $(-1, g, -1)$ satisfies this property. The solution $(\beta_y, \beta_m, \beta_p)$ must be able to remove both nonstationary components $\Sigma_{i=1}^t u_i^r$ and $\Sigma_{i=1}^t u_i^m$. The cointegrating vector can also be found through the space that is orthogonal to the long-run response matrix.

Now suppose we do not know the structure of the economy. Under LRN, the long-run response of (y_t, m_t, p_t) to a one-time monetary shock is $(0, 1, g)$ which shows the contribution of monetary shock to the I(1) component embedded in each variable. If $(\beta_y, \beta_m, \beta_p)$ is a cointegrating vector, by definition, $\beta_y y_t + \beta_m m_t + \beta_p p_t$ must remove all I(1) components, including the I(1) component from monetary shock. Therefore, $\beta_y \times 0 + \beta_m \times 1 + \beta_p \times g = 0$. The cointegrating vector should be orthogonal to $(0, 1, g)$. This orthogonality condition must be true under LRN. Of course, it is apparent that if the economy behaves according to (3.4) to (3.6), the estimated cointegrating vector will converge to $(-1, g, -1)$ which is orthogonal to $(0, 1, g)$. Data series generated from this process will satisfy the LRN orthogonality condition.

It is important to note that the additional shock changes the cointegrating rank from two to one, but it does not change the essence of the cointegration test for LRN or LRH. LRN requires the cointegrating vector to be orthogonal to vector $(0, 1, g)$, while LRH requires it to be orthogonal to $(0, 1, 1)$. This is the same as the case of one permanent shock.

u_t^d is I(1) stationary:

Similar to the previous case, let $u_t^d = u_{t-1}^d + u_t^v$. Then $E_{t-1}u_t^d = u_{t-1}^d$ and $E_{t-1}u_t^y = 0$. This can be interpreted as the income velocity of money being nonstationary. The model solution will be:

$$\begin{aligned} y_t &= \pi g u_t^m + \pi u_t^v + u_t^y \\ m_t &= m_0 + \Sigma_{i=1}^t u_i^m \\ p_t &= g m_0 + g \Sigma_{i=1}^t u_i^m \\ & \quad -\pi g u_t^m - \pi u_t^v - u_t^y + u_0^d + \Sigma_{i=1}^t u_i^v. \end{aligned}$$

The long-run response of (y, m, p) is:

$$\begin{bmatrix} \Delta_\infty y_{t-1} \\ \Delta_\infty m_{t-1} \\ \Delta_\infty p_{t-1} \end{bmatrix} |u_t = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & g \end{bmatrix} \begin{bmatrix} u_t^y \\ u_t^v \\ u_t^m \end{bmatrix}.$$

The cointegrating rank is one with cointegrating vector being $(1, 0, 0)$ up to a scale adjustment. Apparently this vector is orthogonal to $(0, 1, g)$. For LRH, it should be orthogonal to $(0, 1, 1)$. That is: the orthogonality properties are the same irrespective of which permanent shock is added.

The contribution of the I(1) component from monetary shock is still $(0, 1, g)$. For any cointegrating vector $(\beta_y, \beta_m, \beta_p)$, $\beta_y y_t + \beta_m m_t + \beta_p p_t$ must remove all I(1) components including the component from monetary shock. Therefore, $\beta_y \times 0 + \beta_m \times 1 + \beta_p \times g = 0$. This orthogonality condition is the same as before. Even though the additional permanent shock is now from money demand shock and it creates different I(1) component than the real shock example, the orthogonality condition imposed by LRN or LRH is still the same.

3.2.3. *Three permanent shocks.* This means that both u_t^y and u_t^d are nonstationary. We maintain the assumptions that their stochastic processes are $u_t^y = u_{t-1}^y + u_t^r$ and $u_t^d =$

$u^d_{t-1} + u^v_t$ as before. The model solution will be:

$$\begin{aligned} y_t &= \pi g u^m_t + \pi u^v_t - \theta u^r_t + (1+\theta)(u^y_0 + \Sigma^t_{i=1} u^r_i) \\ m_t &= m_0 + \Sigma^t_{i=1} u^m_i \\ p_t &= g m_0 + g\Sigma^t_{i=1} u^m_i \\ &\quad -\pi g u^m_t - \pi u^v_t + \theta u^r_t - (1+\theta)(u^y_0 + \Sigma^t_{i=1} u^r_i) \\ &\quad + u^d_0 + \Sigma^t_{i=1} u^v_i. \end{aligned}$$

The long-run response of (y, m, p) is:

$$\begin{bmatrix} \Delta_\infty y_{t-1} \\ \Delta_\infty m_{t-1} \\ \Delta_\infty p_{t-1} \end{bmatrix} | u_t = \begin{bmatrix} 1+\theta & 0 & 0 \\ 0 & 0 & 1 \\ -(1+\theta) & 1 & g \end{bmatrix} \begin{bmatrix} u^r_t \\ u^v_t \\ u^m_t \end{bmatrix}.$$

When there are three permanent shocks, the cointegrating rank is zero. The only possible cointegrating vector is zero vector which is orthogonal to $(0, 1, g)$ under LRN, and orthogonal to $(0, 1, 1)$ under LRH. The orthogonality properties are still the same as before even now we have two extra permanent shocks than monetary shock. However, in this case, the orthogonality properties are not informative in testing LRN or LRH since zero vector is orthogonal to any vector. Thus, there is a limitation of the cointegration test in that there must be a proper cointegrating rank. However, in practice, this has not turned out to be important: we can use a larger vector system to avoid this problem. We will return to this issue later in our analysis as well.

The conclusion from our examples so far is that: no matter how many different permanent shocks are present in the system, cointegrating vectors give linear combinations that remove all I(1) components from each different shock, including the monetary shock. The reason that they can remove all I(1) components is that they are orthogonal to the long-run response of the variables in the system. LRN and LRH impose certain restrictions on the long-run responses to the monetary shock. That is: cointegrating vectors must be orthogonal to the long-run response of a monetary shock that is constrained by LRN or LRH.

These constrained orthogonality conditions imposed by LRN and LRH can be used to test LRN and LRH. Because the entire test relies on cointegration, the cointegrating rank must be greater than zero.

3.3. General Approach. We now turn to discussing the general approach which is taken in this chapter, displaying two important points alluded to in the introduction. First, we show that the long-run implications of hypotheses like LRN and LRH are unaffected by identification restrictions like those used in the second generation tests. Second, we show that our LRN and LRH tests may be executed even if it is not possible to make a structural interpretation of errors because the researcher has chosen a data vector that contains only a portion of the relevant data.

3.3.1. *VECM background.* If a vector of series X_t is I(1) stationary and is cointegrated with cointegrating vectors β, so that $\beta' X_t$ is I(0), then the Granger representation theorem (e.g., Hamilton (1994), page 582) indicates that a vector error correction model is a suitable empirical specification for capturing the dynamics of nonstationary variables. The VEC model then takes the form

$$\Delta X_t = D + \alpha\beta' X_{t-1} + \Sigma_{i=1}^{p} \Gamma_i \Delta X_{t-i} + \varepsilon_t. \tag{3.11}$$

For our current purposes, it is important to recognize that the Granger representation theorem does not guarantee a structural interpretation of the ε_t: these are just the one-step-ahead forecast errors for ΔX_t given the variables $\beta' X_{t-1}$ and ΔX_{t-i}.

An additional consequence of Granger's theorem is

Lemma 1. *The vector moving average solution of (3.11) is*

$$X_t = C(Dt + \Sigma_{i=1}^{t} \varepsilon_i) + C(L)(D + \varepsilon_t) + P_{\beta_\perp} X_0$$

where

1. $C = \beta_\perp (\alpha'_\perp \Gamma \beta_\perp)^{-1} \alpha'_\perp$ *and* $\Gamma = I - \Sigma_{i=1}^{p} \Gamma_i$*;*

2. $P_{\beta_\perp}$ *is a projection matrix projecting vectors into* $sp(\beta_\perp)$*;*
3. $C(L) = \Sigma_{i=0}^{\infty} C_i L^i$ *is a matrix polynomial with* C_i *matrices absolutely summable.*[9]

This solution makes clear that the long-run response of X to ε_t shock is:

$$\Delta_\infty X_{t-1} | \varepsilon_t = C\varepsilon_t$$

which is orthogonal to $sp(\beta)$. This corresponds to our previous discussion about cointegrating vectors; they give linear combinations that eliminate all I(1) components from different shocks.

3.3.2. *Irrelevance of traditional identification assumptions.* Now, following the practice in second-generation tests, let us assume that there is a possible structural interpretation of the VECM and its associated vector moving average solution. For this purpose, we assume that the forecast errors ε_t are linearly related to the structural shocks u_t according to $\varepsilon_t = \Pi u_t$,

$$\Delta_\infty X_{t-1} | u_t = C\Pi u_t.$$

The long-run response to structural shocks is simply a projection from Π to the range space of C which by nature falls in $sp(\beta_\perp)$, and is orthogonal to $sp(\beta)$ regardless the structural relation Π. Therefore we have the following lemma:

Proposition 1. *The long-run response to structural shocks must be orthogonal to cointegrating vector space regardless of identifying structure* Π*.*

LRN and LRH are hypotheses about certain columns of $C\Pi$. In the (y, m, p) example, LRN implies that one column of $C\Pi$ looks like $(0, 1, g)$ with g unknown[10] and LRH implies that one column of $C\Pi$ looks like $(0, 1, 1)$. According to Proposition 2, LRN imposes the restriction that $(0, 1, g) \in sp(\beta_\perp)$, and LRH imposes the restriction that $(0, 1, 1) \in sp(\beta_\perp)$.

[9]The exact expression of C_i is complex, and out of the purpose of this paper. For details, please refer to Chapter 4 of Johansen(1995).

[10]This is equivalent to saying that one column of $C\Pi$ looks like $(0, g_m, g_p)$ with both g_m and g_p unknown but normalizing g_m to one.

Therefore, regardless of any identifying structure Π, β in (3.11) should be either orthogonal to some vector like $(0, 1, g)$ under LRN or orthogonal to $(0, 1, 1)$ under LRH.

The orthogonality between long-run hypothesis and cointegrating vectors is always true regardless of the identifying assumption Π. Due to this property, we are able to construct a likelihood ratio test to evaluate whether data support the orthogonality conditions implied by LRN or LRH without imposing conventional identifying assumptions.

3.3.3. *Irrelevance of accurate specification of the data vector.* The other advantage of a cointegration approach for constructing LR tests is that the researcher does not need to accurately specify the data vector. To see this, suppose that the true data vector in the economy is Z_t, which contains X_t as its first m of n elements, with the remaining elements being W_t. Suppose further that all of the elements of Z_t are I(1) stationary and that the cointegration restrictions take the form

$$\begin{bmatrix} \beta'_{xx} & 0 \\ \beta'_{xw} & \beta'_{ww} \end{bmatrix} \begin{bmatrix} X_t \\ W_t \end{bmatrix}$$

Given Lemma 1 and the Granger representation theorem, Z_t can be modeled as

$$Z_t \equiv \begin{bmatrix} X_t \\ W_t \end{bmatrix} = \begin{bmatrix} \Psi_{xm} & \Psi_{x\tilde{}m} \\ \Psi_{wm} & \Psi_{w\tilde{}m} \end{bmatrix} \Sigma_{i=1}^{t} \begin{bmatrix} u_i^m \\ u_i^{\tilde{}m} \end{bmatrix} + \tilde{Z}_0$$

where $\tilde{Z}_0$ is a projection of initial values, u^m and $u^{\tilde{}m}$ are monetary and non-monetary shocks respectively. Without loss of generality, we have ignored the transitory components for convenience and simplicity in the discussion.

Consider a partial system consisting of X_t only. Orthogonality between cointegrating vectors and the long-run response matrix implies that

$$\beta'_{xx}\Psi_{xm} = 0. \tag{3.12}$$

Therefore, β_{xx} which serves as a cointegrating matrix for the partial system still maintains the orthogonality property imposed by the long-run response function. The restrictions imposed by LRN or LRH on Ψ_{xm} still impose restrictions on the cointegrating matrix β_{xx}

of X_t according to the orthogonality condition (3.12). In addition, the X_t nonstationary process with cointegrating matrix β_{xx} can be approximated by a VEC model according to the Granger representation theorem. Hence, a researcher following my methodology can still test LRN and LRH even though the system does not include the complete list of macro variables. Accurate specification of the data vector is not necessary.

By contrast, for the second generation LR tests, there is an even greater problem if the researcher studies only a partial system. The Wold decomposition theorem guarantees only that the forecast errors are uncorrelated with lagged ΔX's. But some forecast errors are typically weighted averages of the innovations to Z at date t and prior dates. Therefore, any shocks identified from these forecast errors are unlikely to include an accurate monetary shock. My method does not have this problem since I do not have to extract the monetary shock from the model forecast errors.

3.3.4. *Permanent Monetary Shocks and Identifying Assumptions.* The LR tests, both cointegration test and the second generation test, require the assumption of the existence of permanent monetary shocks. If monetary shocks are transitory as the discussion of Lucas and Sargent studies, then the only way to test LRN and LRH is to specify the deep structure of the economy.

In this section, I want to alert the reader that an important identifying assumptions regarding the long-run variation of money is being made in doing a LR cointegration test. To illustrate this point, take the Lucas supply curve from Section 2.1.2 as an example,

$$y_t = \phi(m_t - E_{t-1}m_t) + u_t^y.$$

If $m_t = m_{t-1} + u_t^m$, then the solution to this model is

$$\begin{aligned} y_t &= \phi\Delta m_t + u_t^y \\ m_t &= m_{t-1} + u_t^m. \end{aligned}$$

The solution is also expressible in terms of a structural VEC model as (3.11), which is

$$\begin{bmatrix} \Delta y_t \\ \Delta m_t \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ m_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^y \\ \varepsilon_t^m \end{bmatrix}$$

with

$$\begin{bmatrix} \varepsilon_t^y \\ \varepsilon_t^m \end{bmatrix} = \begin{bmatrix} 1 & \phi \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_t^y \\ u_t^m \end{bmatrix}.$$

In this model, the adjustment coefficient $\alpha = (1, 0)'$ and the cointegrating vector $\beta = (-1, 0)'$. The identifying assumption π_m regarding monetary shocks is $(\phi, 1)'$. According to Lemma 1, the long-run response of (y, m) to monetary shocks is equal to $\beta_\perp(\alpha'_\perp \beta_\perp)^{-1} \alpha'_\perp \pi_m$ which is $(0, 1)$. Thus, the long-run variation in money creates evidence of LRN. More generally, the LR approach presumes that there is exogenous long-run variation in m. But a virtue of this approach is that LR propositions can be tested without uncovering this trend in money.

To make this point more dramatically, suppose we consider another money supply rule. In particular, monetary authority targets real income with the rule $m_t = y_t + u_t^m$ and the productivity shock u_t^y is I(1) stationary with $u_t^y = u_{t-1}^y + u_t^r$.[11] The structural VEC solution to this model is[12]

$$\begin{bmatrix} \Delta y_t \\ \Delta m_t \end{bmatrix} = \begin{bmatrix} \eta \\ (1+\eta) \end{bmatrix} \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ m_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^y \\ \varepsilon_t^m \end{bmatrix}$$

with

$$\begin{bmatrix} \varepsilon_t^y \\ \varepsilon_t^m \end{bmatrix} = \begin{bmatrix} 1 & \eta \\ 1 & (1+\eta) \end{bmatrix} \begin{bmatrix} \tilde{u}_t^r \\ u_t^m \end{bmatrix}$$

where $\eta = \phi/(1-\phi)$ and $\tilde{u}_t^r = \eta \Delta u_t^r + u_t^r$.[13] In this model, $\alpha = (\eta, (1+\eta))'$ and $\beta = (1, -1)'$. The identifying structure π_m regarding the monetary shocks is $\pi_m = (\eta, (1+\eta))'$. Notice

[11] In this example, monetary shock u_t^m is a transitory shock. To highlight the identification issue, I maintain one permanent shock which is the productivity shock u_t^y here.

[12] Please refer to the appendix.

[13] Note that the structural error $\tilde{u}_t$ is serially correlated. In practice, we can put in lags of $(\Delta y_t, \Delta m_t)$ into the model to correct the serial correlation. The forecast errors, $(\varepsilon_t^y, \varepsilon_t^m)$'s, can be consistently estimated.

that $\pi_m \in sp(\alpha)$ in this case. Therefore, the long-run response of (y, m) to monetary shocks is $\beta_\perp(\alpha'_\perp\beta_\perp)^{-1}\alpha'_\perp\pi_m = 0$. Thus, the monetary shocks are transitory shocks and LR tests do not work under this policy rule.

In a more general setting, the cointegration LR test rules out the identifying structure that the contribution π_m of monetary shocks to the system errors belong to $sp(\alpha)$. According to Lemma 1, if $\pi_m \in sp(\alpha)$, then the monetary shock is transitory. Therefore, by requiring the existence of permanent monetary shocks, certain monetary policy rules have been ruled out.

In summary, the cointegration (LR) tests require the existence of permanent monetary shocks, just as the second generation tests did. Hence, they are based on the crucial assumption that observed trends in m partly reflect autonomous variation in the monetary authority's behavior rather than solely monetary response to trends originating elsewhere in the macroeconomy. However, given that maintained assumption, the LR tests can be conducted under weaker auxiliary assumptions concerning details of identification and accurate specification of the data vector.

4. Estimation with Long-Run Neutrality Restrictions

LR hypotheses impose restrictions on the cointegrating space through an orthogonality property between the cointegrating space and the long-run response functions. Based on this observation, I want to construct a likelihood ratio statistic to test both LRN and LRH. In order to do this, I need to compute the maximal likelihood values with these LR restrictions.

4.1. Hypothesis Setting. The nature of a likelihood ratio test requires us to compute the likelihood value of a VEC model with constraints implied by a specific LR hypothesis. LRN and LRH hypotheses concern the long-run response of variables to a monetary shock. As discussed above, LRN imposes zero restrictions on the long-run response of real variables;

LRH imposes further restrictions on the long-run response for all nominal variables. Consider the three variable(y, p, m) example in Section 2. Under LRN, the long-run response of (y, p, m) to a monetary shock should be $(0, g_m, g_p)$. Under LRH, $g_m = g_p$ in addition to the zero restriction. Let us call the long-run response vector to monetary shock γ. Since $\gamma \in sp(\beta_\perp)$, a restricted VEC model that accommodates LRN must have the estimated cointegrating vector β orthogonal to γ–that is $\gamma'\beta = 0$. This looks like a standard linear restriction on β, the estimation of which could be dealt with easily. However, the value of γ is unknown when testing LRN.

Since the γ is unknown, the maximum likelihood estimation of a VEC model accommodating LRN needs to find the MLE of both γ and β with (i) some elements of γ satisfying zero restrictions; and (ii) γ and β orthogonal to each other. Instead of estimating both γ and β with restrictions, we propose a method that transforms all restrictions on γ into restrictions on β. By doing so, the estimation problem will be converted into a problem of estimating a restricted β. We will show also that the restrictions on β look like standard linear restrictions and can be dealt with traditional estimation methods. In addition, given an estimated β, the MLE of γ can be obtained by solving a linear equation system.

First, notice that most long-run effects γ can be written in the following form:

$$\gamma = \mathbf{H}\psi$$

with H pre-specified and full rank and with ψ unknown. In the example above, for LRN, $\gamma = [0, g_p, g_m]'$ which can be expressed as

$$\gamma = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} g_p \\ g_m \end{bmatrix}.$$

For LRH, $\gamma = [0, g_m, g_m]'$ which can be expressed as

$$\gamma = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} [g_m] .$$

A VEC model satisfying LRN or LRH should be able to admit the long-run response as above in its orthogonal cointegrating space $sp(\beta_\perp)$. We can describe these LR hypotheses in terms of the following:

H_0 : The VEC model admits long-run effect γ with $\gamma = \mathbf{H}\psi$ and $\mathbf{H}(n \times s)$ being prespecified and full rank.

In this expression, s is the number of free parameters to be estimated in the long-run response vector that is implied by the specific LR hypothesis. Let $\Theta(H_0)$ be the set of parameters–consisting of $\{\alpha, \beta, D, \Gamma_i,$the variance covariance matrix Ω of $\varepsilon\}$ following the notation of (3.11)–that are admissible under the null hypothesis H_0. For any β admissible to $\Theta(H_0)$, there must exist at least one solution ψ such that $\beta' H\psi = 0$. It is clear that if we can estimate β properly, then the MLE of ψ is simply the solution to the orthogonality condition, *i.e.* $\beta' H\psi = 0$. Therefore, to find the MLE of the VEC model within the $\Theta(H_0)$ parameter space, we do not have to estimate β and γ jointly.

We are going to show that the parameter set $\Theta(H_0')$ generated by the following hypothesis,

H_0' : There are at least $r - s + 1$ cointegrating vectors lying in the space $sp(\mathbf{H}_\perp)$,

is equivalent to $\Theta(H_0)$. That is: $\{\alpha, \beta, D, \Gamma_i, \Omega\}$ in $\Theta(H_0')$ must insure the existence of at least one solution ψ such that $\beta' H\psi = 0$. Under H_0', the cointegrating matrix $\beta(n \times r)$ can be decomposed into two parts: $\beta_1(n \times s - 1)$ and $\beta_2(n \times r - s + 1)$ with no restriction on β_1 and $H'\beta_2 = 0$.

If we interpret the hypothesis in this way, then all restrictions are imposed on β and are linear. Given that β is estimated, the MLE of ψ can be obtained from the orthogonality condition, $\beta' H\psi = 0$. One potential problem is that: given any β satisfying H_0', it is possible to find the other base $\tilde{\beta}$ of $sp(\beta)$ such that $\tilde{\beta}$ is a linear transformation of β but it does not satisfy H_0'. However, what important is the vector space itself, not the base we choose to represent the cointegrating matrix. Therefore we have to define admissibility properly to ensure that $\tilde{\beta}$ is also admissible to $\Theta(H_0')$. Notice that two VEC models, say $\{\alpha, \beta, D, \Gamma_i, \Omega\}$ and $\{\tilde{\alpha}, \tilde{\beta}, D, \Gamma_i, \Omega\}$, are observationally equivalent if there is a nonsingular matrix R such that $\tilde{\alpha} = \alpha R'^{-1}$ and $\tilde{\beta} = \beta R$. If this is true, then the products of the adjustment coefficients and cointegrating vectors are the same for both models–that is $\tilde{\alpha}\tilde{\beta}' = \alpha\beta'$.

Definition 1. *The cointegrating matrix β is admissible with respect to the hypothesis H_0' if one of its observationally equivalent versions satisfies the hypothesis.*

4.1.1. *The Hypothesis Equivalence Theorem and Its Proof.* Now we are ready to prove the following Hypothesis Equivalence Theorem.

Theorem 1. *Given that the cointegrating rank of the VEC model is equal to r, the following two hypotheses are equivalent:*

H_0 : *The VEC model admits a long-run effect $\gamma(n \times 1)$ with $\gamma = \mathbf{H}\psi$ and $\mathbf{H}(n \times s)$ prespecified and full rank.*

H_0' : *There are at least $r - s + 1$ cointegrating vectors lying in the space $sp(\mathbf{H}_\perp)$.*

Proof. The proof is done in two parts. First, we prove that $\Theta(H_0) \supseteq \Theta(H_0')$. Then, that $\Theta(H_0) \subseteq \Theta(H_0')$.

If $(\alpha, \beta) \in \Theta(H_0')$,[14] then β can be divided into two groups $\beta_1(n \times s-1)$ and $\beta_2(n \times r-s+1)$ such that $\beta = [\beta_1, \beta_2]$ and $\beta_2 \in sp(H_\perp)$. To prove that $\beta \in \Theta(H_0)$, it is sufficient to show that there exists ψ such that $\beta \perp \gamma$ given $\gamma = H\psi$. By the definition of β_2, $\beta_2 \perp \gamma$ regardless

[14]The parameters in the model include not only the error correction coefficients (α, β), but also other short run parameters, Γ_i. Since these other parameters will not be constrained under either form of our null hypotheses, to keep the expressions terse, we treat (α, β) as the only parameters in our model for notation at simplicity.

of any choice of ψ so we only need to check the orthogonality condition for β_1. To ensure the existence of a nonzero ψ, $\beta_1' H$ must have rank less than s. Since β_1 has dimension $n \times s-1$, $rank(\beta' H) \leq s-1 < s$.

If $(\alpha, \beta) \in \Theta(H_0)$, then there exists a nonzero ψ such that given $\gamma = H\psi$, $\beta \perp \gamma$. This means that $\beta' H$ has rank less than s. Because $[H, H_\perp]$ form a base for $\Re^n$, β can be decomposed into $\beta = H\phi_1 + H_\perp \phi_2$. Since $rank(\phi_1' H' H) < s$ and $H'H$ is invertible, $rank(\phi_1) < s$; there exists a nonsingular square matrix R such that $\phi_1 R = [\phi_{1a}(n \times s-1), 0(n \times r-s+1)]$ and $\phi_2 R = [\phi_{2a}(n \times s-1), \phi_{2b}(n \times r-s+1)]$. Define $\tilde{\beta} = \beta R$, then $\tilde{\beta} = [H\phi_{1a} + H_\perp \phi_{2a}, H_\perp \phi_{2b}]$; at least $r-s+1$ cointegrating vectors in $\tilde{\beta}$ lie in $sp(H_\perp)$. Let $\tilde{\alpha} = \alpha R'^{-1}$. It follows that $\alpha\beta'$ and $\tilde{\alpha}\tilde{\beta}'$ are observationally equivalent. $(\tilde{\alpha}, \tilde{\beta}) \in \Theta(H_0')$ by definition 1 this implies that $(\alpha, \beta) \in \Theta(H_0')$. ∎

Hypothesis H_0' shows the constraints of the LR hypothesis imposed on cointegrating space in terms of the sacrifice of the degrees of freedom for the choice of cointegrating space. To under this point clearly, we use the bivariate (y, m) and trivariate (y, m, p) models as examples to demonstrate the loss of degrees of freedom in the choice of cointegrating space.

For the bivariate model with LRN, the long-run response of (y, m) to the monetary shock is

$$\begin{bmatrix} \Delta_\infty y_{t-1} \\ \Delta_\infty m_{t-1} \end{bmatrix} | u_t^m = \begin{bmatrix} 0 \\ 1 \end{bmatrix} [g_m] .$$

Therefore, the matrix H is equal to the vector $(0,1)'$. Suppose the cointegrating rank is one. Hypothesis H_0' says that at least one cointegrating vector should belong to $sp(H_\perp)$ where $H_\perp = (1,0)'$ in this example. A cointegrating vector that satisfies LRN is like β_1 in Figure 3, which should be orthogonal to the long-run response vector to the monetary shock that is H. Without imposing LRN, no orthogonality condition is imposed on the cointegrating vector space. Therefore, the choice of cointegrating vector is free in the (y, m) plane. In addition, any other cointegrating vector in this plane, such as β_2, is a rotation of the cointegrating vector β_1. Therefore, by imposing LRN, the researcher sacrifices one degree of freedom in choosing cointegrating space. In Figure 3, it means that Rotation 1 is prohibited.

For the trivariate model with LRH, the long-run response of (y, m, p) to the monetary shock is

$$\begin{bmatrix} \Delta_\infty y_{t-1} \\ \Delta_\infty m_{t-1} \\ \Delta_\infty p_{t-1} \end{bmatrix} |u_t^m = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} [g_m] . \tag{4.1}$$

Therefore, the matrix H is equal to the vector $(0, 1, 1)'$ and $s = 1$. Suppose the cointegrating rank r is two in this example. Then the possible cointegrating space allowed to choose is the cointegrating space B_1 in Figure 4, which is orthogonal to H. However, if there is no LR restriction imposed, then it is free to choose the cointegrating space from the (y, m, p) space. From the Figure, any cointegrating vector space can be represented as a combination of Rotation 1 and Rotation 2 of the cointegrating space B_1. By imposing LRH, these two rotations are prohibited. This is exactly equal to $r - s + 1$ in this example. Also, from the Figure, both cointegrating vectors are in $sp(H_\perp)$. This is exactly what hypothesis H_0' suggests, since $r - s + 1 = 2 - 1 + 1 = 2$.

Now, continue with the previous example, but impose LRN instead. Then the long-run response of (y, m, p) to the monetary shock is

$$\begin{bmatrix} \Delta_\infty y_{t-1} \\ \Delta_\infty m_{t-1} \\ \Delta_\infty p_{t-1} \end{bmatrix} |u_t^m = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} g_m \\ g_p \end{bmatrix} . \tag{4.2}$$

It follows that

$$\mathbf{H} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

and $s = 2$. In this case, any rotation of the B_1 cointegrating space in the direction of Rotation 1 around y-axis will produce a cointegrating space that satisfies LRN. Therefore, by imposing LRN, the researcher sacrifices only Rotation 2. There is only one degree of freedom in choosing cointegrating vector space is sacrificed. Again, this is consistent with hypothesis H_0', since $r - s + 1 = 2 - 2 + 1 = 1$. In addition, hypothesis H_0' requires that one cointegrating vector lie in $sp(H_\perp)$ where $H_\perp = (1, 0, 0)'$ in this example. The reader can verify from Figure 4 that the vector $(1, 0, 0)$ is always contained in the cointegrating space that satisfies LRN, which is also implied by hypothesis H_0'.

The reader might ask: what happens if $r \leq s-1$? Consider $rank(\beta' H)$. When $r \leq s-1$, $rank(\beta' H) < s$ regardless of any choice of the cointegrating matrix. There always exists a nonzero ψ such that $\gamma = H\psi$ and $\gamma \perp sp(\beta)$: the long-run effect hypothesis does not impose any restriction on the VEC model. Using the bivariate and trivariate examples as before, $r \leq s-1$ implies zero cointegrating rank ($r = 0$) for the bivariate example with LRN ($s = 1$) and for the trivariate example with LRH ($s = 1$). In both cases, the only possible cointegrating vector is zero vector which is orthogonal to any vector. Therefore it can not provide an informative cointegration test. For the trivariate example with LRN ($s = 2$), $r \leq s - 1$ implies $r \leq 1$. There are two possible situations: One is $r = 0$, the other is $r = 1$. $r = 0$ implies zero cointegrating vector. Therefore it does not provide an informative cointegration test. $r = 1$ is more subtle. Let $(\beta_y, \beta_m, \beta_p)$ be the cointegrating vector. Under LRN, the long-run response of (y, m, p) to the monetary shock can be normalized as $(0, 1, g_p)$. Orthogonality condition implies that $\beta_m + \beta_p g_p = 0$ which actually imposes no restriction on the cointegrating vector at all since g_p is also a free parameter. Therefore, $r = 1$ does not provide formative cointegration test either.

4.2. Estimation. The estimation procedure we use here follows section 7.2.3 in Johansen (1995) with slight modifications to fit our specific requirements. We are dealing with a maximum likelihood estimation of the following VEC model,

$$\text{(4.3)} \qquad \Delta X_t = D + \alpha\beta' X_{t-1} + \Sigma_{i=1}^{p}\Gamma_i \Delta X_{t-i} + \varepsilon_t$$
$$\varepsilon_t \overset{iid}{\sim} N(0, \Omega)$$

with restrictions that $\beta(n \times r) = [\beta_1(n \times s-1), H_{\perp}b_2]$ where $H_{\perp}(n \times n-s)$ is pre-specified and $b_2(n-s \times r-s+1)$ is to be estimated.

According to the Frisch-Waugh-Lovell theorem, we can concentrate (4.3) into

$$R_{0t} = \alpha_1\beta_1' R_{1t} + \alpha_2 b_2' \mathbf{H}_{\perp}' R_{1t} + \varepsilon_t$$

with

R_{0t} = the orthogonal projection error of ΔX_t on a constant and $\Delta X_{t-1}, \ldots, \Delta X_{t-p}$

R_{1t} = the orthogonal projection error of X_{t-1} on a constant and $\Delta X_{t-1}, \ldots, \Delta X_{t-p}$.

Let $\beta_2 = H_{\perp}b_2$ and define

$R_{i.\beta_j.t}$ = the orthogonal projection error of R_{it} on $\beta_j' R_{1t}$

$$S_{ij.\beta_k} = \Sigma_t R_{i.\beta_k.t} R_{j.\beta_k.t}/T$$
$$S_{ij} = \Sigma_t R_{it} R_{jt}/T.$$

The following lemma excerpted from Theorem 7.4 in section 7.2.3 of Johansen (1995) gives necessary conditions to solve for the MLE of β with restrictions.

LEMMA 2. *The maximized value $L_{\max}$ of the likelihood function is given by*

$$L_{\max}^{-2/T} = |S_{00}|\Pi_{i=1}^{m_1}(1-\rho_i)\Pi_{j=1}^{r-m_1}(1-\lambda_j)$$

where $1 > \lambda_1 > \cdots > \lambda_{r-m_1}$ are defined as the largest $r - m_1$ solutions to the eigenvalue problem:

$$\text{(4.4)} \qquad |\lambda \mathbf{H}_{\perp}' S_{11.\beta_1}\mathbf{H}_{\perp} - \mathbf{H}_{\perp}' S_{10.\beta_1} S_{00.\beta_1}^{-1} S_{01.\beta_1}\mathbf{H}_{\perp}| = 0,$$

and $1 > \rho_1 > \cdots > \rho_{m_1}$ are defined as the largest m_1 solutions to the eigenvalue problem:

$$\text{(4.5)} \qquad |\rho S_{11.\beta_2} - S_{10.\beta_2} S_{00.\beta_2}^{-1} S_{01.\beta_2}| = 0.$$

The eigenvectors corresponding to $\lambda_1, \ldots, \lambda_{r-m_1}$ *constitute the MLE* $\hat{b}_2$ *of* b_2*; the MLE* $\hat{\beta}_2$ *of* β_2 *is then* $\hat{\beta}_2 = H_\perp \hat{b}_2$*. The eigenvectors corresponding to* $\rho_1, \ldots, \rho_{m_1}$ *constitute the MLE of* β_1*.*

Equations (4.4) and (4.5) give necessary conditions to solve for the MLE of β_1 and β_2. However, both eigenvalue problems are mutually dependent. Johansen proposed an iterative method to compute the estimators, which is summarized in the appendix and is used in my applied work.

5. Test Statistic

5.1. Computation of the Likelihood Ratio Test Statistic. Let $L^u_{\max}$ and $L^r_{\max}$ be the maximal likelihood values with and without LR restrictions respectively. The likelihood ratio test statistic Q defined as

$$Q = 2(\log L^u_{\max} - \log L^r_{\max})$$

is the likelihood ratio test statistic for the LR hypothesis. The computation of $L^r_{\max}$ is reviewed in Section 3.2. As to the computation of $L^u_{\max}$, which is more standard, the reader is referred to the appendix.

5.2. The Asymptotic Distributions. For both restricted and unrestricted models, we maintain the same cointegrating rank and number of lags. Therefore the distribution of the likelihood ratio statistics Q will be χ^2 asymptotically.[15] The only work we have to do here is to compute the correct degrees of freedom for our test statistic.

The LR hypothesis imposes restrictions on the cointegrating vector space only, but not on other short-run parameters in the VEC model. The degrees of freedom of the statistic is thus determined by the difference of the dimensions of the tangent space of the $\alpha\beta'$ term in both VEC models–one without restrictions and one with restrictions.

[15]For the detail of the asymptotic distribution, please refer to Chapter 13 of Johansen(1995).

THEOREM 2. *Consider the long-run effect hypothesis*

$$H_0 \;:\; \text{The model admits a long-run effects with each effect } \gamma \text{ expressible as } \mathbf{H}\psi \text{ with } \mathbf{H}(n \times s) \text{ prespecified and full rank.}$$

Given the cointegrating rank of r, the likelihood ratio test statistic Q of the model with long-run effect restrictions H_0 against the model without restrictions follows a χ^2 asymptotic distribution with degrees of freedom equal to $r - s + 1$.

PROOF. According to Theorem 1, H_0 imposes on the cointegrating matrix $\beta(n \times r)$ the restrictions that $r-s+1$ cointegrating vectors should be in $sp(\mathbf{H}_\perp)$. According to Theorem 3 in Johansen and Juselius (1992), the degree of freedom is $r - s + 1$. ∎

According to the discussion following the Hypothesis Equivalence Theorem in Section 4.1.1, the degrees of freedom here is exactly the sacrifice of the degrees of freedom in choosing cointegrating space.

By the properties of likelihood ratio tests, our test can be easily extended to test two forms of the LR hypothesis if one is nested in the other. Consider the following hypotheses

$$H_0 \;:\; \text{The VEC model admits a long-run effects with each effect } \gamma \text{ expressible as } \mathbf{H}_A\psi_A \text{ with } \mathbf{H}_A(n \times s_A) \text{ prespecified and full rank.}$$

$$H_1 \;:\; \text{The VEC model admits a long-run effects with each effect } \gamma \text{ expressible as } \mathbf{H}_B\psi_B \text{ with } \mathbf{H}_B(n \times s_B) \text{ prespecified and full rank.}$$

with H_A being $n \times s_A$ and H_B being $n \times s_B$. The following proposition ensures that H_0 is nested in H_1.

PROPOSITION 2. *If $sp(H_A) \subset sp(H_B)$, then H_0 is nested in H_1.*

PROOF. For $\beta \in \Theta(H_0)$, there exists a nonzero $\psi(s_A \times 1)$ such that $\beta' H_A \psi = 0$. Since $sp(H_A) \subset sp(H_B)$, there exists a nonzero Γ such that $H_A = H_B\Gamma$. Therefore, $\beta' H_B(\Gamma\psi) = 0$. It is evident that $\beta \in \Theta(H_1)$. Hence H_0 is nested in H_1. ∎

Suppose Q_A is the Q statistic of testing H_A with degrees of freedom df_A; and Q_B is the Q statistic of testing H_B with degrees of freedom df_B. Then the likelihood ratio test of H_A against H_B has Q statistics equal to $Q_A - Q_B$ with degrees of freedom $df_A - df_B$. Hence we have the following proposition.

PROPOSITION 3. *For the nested hypothesis testing*

$$\begin{aligned} H_0 \quad &: \quad \textit{The VEC model admits a long-run effects with each effect } \gamma \\ &\quad\ \textit{expressable as } \mathbf{H}_A\psi_A \textit{ with } \mathbf{H}_A(n \times s_A) \textit{ prespecified and full rank.} \\ H_1 \quad &: \quad \textit{The VEC model admits a long-run effects with each effect } \gamma \\ &\quad\ \textit{expressable as } \mathbf{H}_B\psi_B \textit{ with } \mathbf{H}_B(n \times s_B) \textit{ prespecified and full rank} \end{aligned}$$

with H_A being $n \times s_A$, H_B being $n \times s_B$ and $sp(H_A) \subset sp(H_B)$. Then, the Q-statistic $Q_{A\subset B}$ is computed as $Q_{A\subset B} = Q_B - Q_A$ and has a χ^2 distribution asymptotically with degrees of freedom, $s_B - s_A$.

PROOF. We only need to prove its degrees of freedom. According to Theorem 2, the degrees of freedom for Q_A is $r - s_A + 1$. The degrees of freedom for Q_B is $r - s_B + 1$. Their difference is $s_B - s_A$ with $s_B > s_A$ since $sp(H_A) \subset sp(H_B)$. ∎

This proposition is important for testing LRH since LRH is a special case of LRN. Hence it is nested in LRN. With this proposition, the LRH test can be easily conducted. Taking (y, m, p) trivariate model as an example, the LRH in (4.1) has an H matrix, say H_A, equal to

$$\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}.$$

and LRN in (4.2) has an H matrix, say H_B, equal to

$$\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

According to Proposition 2, the LRH hypothesis is nested in LRN hypothesis.

6. Empirical Application

In this section, we apply our cointegration test for LRN and LRH[16] to two different U.S. macro data sets: annual data from Friedman and Schwartz (1982) and post-WWII quarterly data. In addition to the test results for our method, we also provide results of the FS test and the Geweke test.

The Friedman and Schwartz data set spans 1869 to 1975. However, we focus only on the post-1939 period, *i.e.* the years after the Great Depression. The reason for this sample selection is the previous evidence of structural breaks provided by Boschen and Otrok (1994) and Haug and Lucas (1997). For the post-WWII data set, we split it into three subsamples: one is from 1959:1 to 1978:4 (the pre-Volcker period), the other is post-1983 period (which will be referred to post-Volcker experiment period), and another is the sample between these two periods. We drop the last subsample due to its small sample size (only twenty observations.)

Before we test LRN and LRH, we need to test whether money stock has a unit root, which is required for any LR test. Our unit root tests we used are based on three different test statistics proposed by Ng and Perron (2001). Test results are in Table 1. For the Friedman and Schwartz data, we rejected a unit root for money at the 10 percent level, but not at the 5 percent level. Given that FS and Geweke both regarded money to be I(1) stationary in their LR tests and that the unit root is not strongly rejected, we maintain the

[16]LRN refers to long-run neutrality but allows non-homogeneity. LRH refers to both long-run neutrality and homogeneity. Therefore LRH is a special case of LRN

unit root assumption of money. For both post-WWII quarterly data sets, we did not reject a unit root for money at both 5 percent and 10 percent testing sizes.

6.1. Friedman and Schwartz Data. To begin, we further describe the econometric methods of FS and Geweke. This discussion concentrates on the essence of each test and highlights the identification assumptions in these two second generation LR tests.

6.1.1. *The LR test of Fisher and Seater.* Consider the reduced form VAR

$$B(L)\begin{bmatrix}\Delta X_t \\ \Delta m_t\end{bmatrix} = \begin{bmatrix}\varepsilon_{x,t} \\ \varepsilon_{m,t}\end{bmatrix}. \tag{6.1}$$

Then, the long-run responses of X and m with respect to time t forecast errors will be

$$\begin{bmatrix}\Delta_\infty X_{t-1} \\ \Delta_\infty m_{t-1}\end{bmatrix} = B(1)^{-1}\begin{bmatrix}\varepsilon_{y,t} \\ \varepsilon_{m,t}\end{bmatrix}.$$

where, as above, $\Delta_\infty X_{t-1} = \lim_{k\to\infty}(X_{t+k} - X_{t-1})$ and $\Delta_\infty m_{t-1} = \lim_{k\to\infty}(m_{t+k} - m_{t-1})$. The identifying assumptions considered are

$$\begin{bmatrix}\varepsilon_{x,t} \\ \varepsilon_{m,t}\end{bmatrix} = \Pi\begin{bmatrix}u_{r,t} \\ u_{m,t}\end{bmatrix}, \tag{6.2}$$

where u_m is the monetary shock and u_r is the real shock. The long-run responses of X and m with respect to structural shocks are then

$$\begin{bmatrix}\Delta_\infty X_{t-1} \\ \Delta_\infty m_{t-1}\end{bmatrix} = B(1)^{-1}\Pi\begin{bmatrix}u_{r,t} \\ u_{m,t}\end{bmatrix}. \tag{6.3}$$

To consider alternative identifying assumptions, it is convenient to partition the matrix $B(1)^{-1}\Pi$ into four blocks,

$$B(1)^{-1}\Pi = \begin{bmatrix}\gamma_{x,r} & \gamma_{x,m} \\ \gamma_{m,r} & \gamma_{m,m}\end{bmatrix}. \tag{6.4}$$

As an identifying assumption, FS imposed the long-run exogeneity of money, which is that long-run response of m does not depend on the long-run response of X. This means that $\gamma_{m,r} = 0$.[17] They also assumed that $u_{r,t}$ and $u_{m,t}$ are not correlated.

The consequences of these identifying assumptions was that FS could produce regression tests of LRN and LRH. The regression coefficient β of $\Delta_\infty X_t$ on $\Delta_\infty m_t$ is equal to $cov(\Delta_\infty X_t, \Delta_\infty m_t)/var(\Delta_\infty m_t) = \gamma^2_{x,m}/\gamma^2_{m,m}$ for a bivariate VAR. Following FS, some authors also call β the long-run derivative $LRD_{x,m}$ of X with respect to m. If X is a nominal variable, then the monetary shock should move X and m proportionately in the long-run under LRH, i.e. $\gamma_{x,m} = \gamma_{m,m}$ so that $\beta = 1$. If X is a real variable, then the monetary shock has no impact on X in the long-run under LRN, i.e. $\gamma_{x,m} = 0$ so that $\beta = 0$.

In principle, the estimation of β can be done through the following regression

$$\Delta_\infty X_t = a + b\Delta_\infty m_t + e_t.$$

In practice, it is impossible to take an infinite long-horizon difference. FS therefore took a finite long-horizon difference instead, using the regression model

$$\Delta_k X_t = a + b\Delta_k m_t + e_t \tag{6.5}$$

where $\Delta_k X_t = X_{t+k} - X_t$ and $\Delta_k m_t = m_{t+k} - m_t$. In theory, if k increases with sample size at a proper speed, then the estimator b converges to the true value.

Replicating the FS results, we run the regression (6.5) for k from one to thirty. Figure 5 displays the estimates of b for different k and their 95 percent confident intervals[18]. When

[17] In a bivariate VAR, we can invert $B(1)^{-1}\Pi$ in (6.3) into the following representation

$$\frac{1}{(\gamma_{x,r}\gamma_{m,m} - \gamma_{m,r}\gamma_{x,m})} \begin{bmatrix} \gamma_{m,m} & -\gamma_{x,m} \\ -\gamma_{m,r} & \gamma_{x,r} \end{bmatrix} \begin{bmatrix} \Delta_\infty X_t \\ \Delta_\infty m_t \end{bmatrix} = \begin{bmatrix} u_{r,t} \\ u_{m,t} \end{bmatrix}.$$

If the long-run response $\Delta_\infty X_t$ does not enter the the long-run response function of $\Delta_\infty m_t$, it means $\gamma_{m,r} = 0$.

[18] The confidence intervals are constructed with consistent estimates of the variance-covariance matrix using the approach of Newey and West(1987). The Bartlett window is used with truncation lags determined by the criterion of Newey and West(1994). The t-statistic of b estimator follows a t distribution with degrees of freedom $[T/k]$ where T is the sample size and [ ] rounds T/k to the nearest interger.

k approaches thirty, the 95 percent confident interval includes 1 when X is nominal income and when X is the price level. Hence, LRH is not rejected. When X is replaced with real income, the confidence interval for large k includes zero. Hence, LRN is not rejected, either.

6.1.2. *The LR test of Geweke.* Consider the reduced form VAR (6.1) and the identifying framework (6.2). Given a block diagonal variance-covariance matrix of structural shocks u,

$$\begin{bmatrix} \Sigma_r & 0 \\ 0 & \Sigma_m \end{bmatrix},$$

the matrix spectral density $s(\omega)$ of $[X_t, m_t]'$ at frequency $\omega = 0$ is equal to

$$s(0) = \frac{1}{2\pi} B(1)^{-1} \Pi \begin{bmatrix} \Sigma_r & 0 \\ 0 & \Sigma_m \end{bmatrix} \Pi' B(1)^{-1\prime}. \tag{6.6}$$

Following (6.4), this spectral density can be expressed as

$$s(0) = \frac{1}{2\pi} \begin{bmatrix} \gamma_{r,r}\Sigma_r\gamma_{r,r}' + \gamma_{r,m}\Sigma_m\gamma_{r,m}' & \gamma_{r,r}\Sigma_r\gamma_{m,r}' + \gamma_{r,m}\Sigma_m\gamma_{m,m}' \\ \gamma_{m,r}\Sigma_r\gamma_{r,r}' + \gamma_{m,m}\Sigma_m\gamma_{r,m}' & \gamma_{m,r}\Sigma_r\gamma_{m,r}' + \gamma_{m,m}\Sigma_m\gamma_{m,m}' \end{bmatrix}.$$

Note that the upper left block of $s(0)$ is the long-run variance of X_t or the variance of $\Delta_\infty X_t$ equivalently.

Studying LRN, Geweke tested if the following parameter, called $f_{m\to X}(0)$, is equal to zero.

$$f_{m\to X}(0) = \log(\left|\gamma_{r,r}\Sigma_r\gamma_{r,r}' + \gamma_{r,m}\Sigma_m\gamma_{r,m}'\right| / \left|\gamma_{r,r}\Sigma_r\gamma_{r,r}'\right|).$$

Three aspects of this approach need to be discussed. First, to estimate $f_{m\to X}(0)$, the Π matrix must be identified. Geweke adopted a recursive identifying scheme, assuming that the ordering of the vector $[X_t, m_t]'$ was from the most endogenous to the most exogenous. This assumption makes Π an upper triangular matrix. In addition, Geweke assumed that all diagonal elements of Π were normalized to one. With these assumptions, $f_{m\to X}(0)$ measures the long-run feedback from m to X. If money is neutral in the long-run, its long-run feedback to X should be zero. Second, the statistic $f_{m\to X}$ by nature is always greater than zero. Therefore, whether LRN is confirmed depends on whether the confidence interval of the estimated $f_{m \to X}(0)$ close to zero or not.

The estimate of $f_{m \to X}(0)$ can be constructed with the estimates of $B(1)$, Π, Σ_r and Σ_m which can be obtained through traditional ordinary least squares regression of the VAR (6.1). However, Geweke alternatively used the extended Yule-Walker algorithm following Whittle (1963) and our replication adopts his practice.[19]

The estimation results are shown in Table 2. Eighty percent confidence intervals[20] are constructed by the R-fold replication method, the details of which are contained in an appendix to this chapter. The estimated long-run feedback from money to the real income and to the income velocity of money is close to zero. Their confidence intervals are close to zero, too. Real income and the income velocity of money gave support for LRN.[21] If we combine these two results together, it also shows the support for LRH. However, when we replace the X variables with real money balance, or both real income and real money balance together, the estimated long-run feedbacks are not close to zero. Their confidence intervals are not close to zero either. LRN is not strongly supported in this case.

6.1.3. *Cointegration test.* In this section, we report results of our cointegration test for two different VEC models. The first uses real income(y), money stock(m), and the price level(p) so as to make for ready comparability with the results of Fisher-Seater and Geweke. All variables are in logarithms. The second augments this basic system with the nominal interest rate, so as to produce a model similar to that studied in many VAR analyses.

[19]Geweke opted for this approach in order to ensure the invertibility of the estimated matrix polynomial $B(L)$.

[20]The small confidence intervals are used because of the replication method typically gives wide confidence intervals. We follow Geweke in using the eighty percent confidence interval.

[21]By construction, the parameter $f_{m \to X}(0)$ is always greater than zero. The lower bound of the confidence interval will be greater than zero, too. Therefore the conclusion regarding LRN and LRH is based on whether the confidence interval is close to zero enough.

6.1.4. *Results for three variable system.* Let $X_t = [y_t, m_t, p_t]'$ and assume that it is described by a VEC model,

$$\Delta X_t = C + \alpha\beta' X_{t-1} + \Sigma_{i=1}^{k} \Delta X_{t-i} + \varepsilon_t$$

To apply our LR test, we have to determine the number of lags k and the proper cointegrating rank. We use the Posterior Information Criterion(PIC, hereafter) proposed by Phillips and Ploberger (1996) to select model lags and cointegrating rank simultaneously.[22] The VEC model selected by PIC involves a cointegrating rank of one and indicates that it is necessary to include only one lag of ΔX, i.e., that $k = 1$.

First, consider the LRH hypothesis under which the long-run response of (y, m, p) to monetary shocks should be

$$\begin{bmatrix} \Delta_\infty y_{t-1} \\ \Delta_\infty m_{t-1} \\ \Delta_\infty p_{t-1} \end{bmatrix} |u_t^m = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} [g_m] \tag{6.7}$$

in the $\gamma = H\psi$ form. The H matrix is equal to the vector $(0, 1, 1)'$ here. Without imposing any restriction, we find that the estimated cointegrating vector is $(16, 4466, -11, 6244, 5, 9478)'$ which is not orthogonal to the H vector. Now we imposed LRH restrictions on the MLE of the cointegrating vector. The restricted estimate of the cointegrating vector is $(8.7955, -14.4571, 14.4571)$ which is orthogonal to H through the orthogonality restriction imposed by LRH. This might be interpreted as a long-run money demand function with an income elasticity of $8.8/14.5 \simeq 0.6$, since it would imply $m - p - 0.6y$ is stationary. The geometry relations between restricted and unrestricted estimates of the cointegrating vectors are shown in Figure 6. In Figure 6, β_1 is the unrestricted estimate, and β_2 is the restricted one. β_2 is orthogonal to H vector, but not β_1. The likelihood ratio test result

[22]The conventional approach is to use a model selection criterion–such as BIC or AIC–to select lags first, and then to choose cointegrating rank, for example by the likelihood ratio test of Johansen (1995) or the multivariate unit root test of Stock and Watson (1988). However, as pointed out by Johansen (1992), such a sequential model selection may be inconsistent. Chao and Phillips (1999) compared AIC, BIC and PIC; and found some finite sample evidence in favor of PIC.

of LRH is reported in Table 4(a). The Q statistic is 1.5688 which is smaller than both 5 percent and 10 percent critical values. Therefore, we do not reject LRH.

However, in this trivariate VECM, the LRN hypothesis is not testable. Recall that the definition of LRN is that the long-run response of (y, m, p) to monetary shocks can be expressed as

$$\begin{bmatrix} \Delta_\infty y_{t-1} \\ \Delta_\infty m_{t-1} \\ \Delta_\infty p_{t-1} \end{bmatrix} |u_t^m = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} g_m \\ g_p \end{bmatrix}$$

for some nonzero g_m. Let $\beta_1 = (\beta_{1y}, \beta_{1m}, \beta_{1p})$. To satisfy LRN, $\beta_{1m} g_m + \beta_{1p} g_p = 0$ for some (g_m, g_p). When there is only one cointegrating vector, given any (β_{1m}, β_{1p}) value we can always find a (g_m, g_p) value to fulfill the $\beta_{1m} g_m + \beta_{1p} g_p = 0$ condition.[23] Therefore, LRN does not impose any restriction on the cointegrating space. Refer back to the discussion on page 33. It is not testable because $r \leq s - 1$ with $r = 1$ and $s = 2$ in line with discussion above.

6.1.5. *Results for a four-variable system.* In order to test LRH against LRN, we need cointegrating rank larger than the rank of H matrix minus one as discussed in Section 3.1. Therefore, we expand the system. Many studies of nominal and real interactions using VAR methods concern systems that augment the basic three variable model with the nominal interest rate, as we do in this system. The cointegrating rank and the number of lags in this four variable VEC model selected by PIC are two and one respectively, so that we have a enriched set of testing possibilities.

There are two types of LR test we are interested. One is that monetary shock is long-run neutral, but may not be long-run homogeneous, which implies a long-run effect of money

[23] In our trivariate model, the unrestricted estimated β_1 is orthogonal to the long-run response function under LRN with $g_m = 5.9478$ and $g_p = 11.6244$.

like:

$$
\begin{bmatrix} \Delta_\infty y_{t-1} \\ \Delta_\infty R_{t-1} \\ \Delta_\infty m_{t-1} \\ \Delta_\infty p_{t-1} \end{bmatrix} |u_{m,t} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} g_m \\ g_p \end{bmatrix}. \tag{6.8}
$$

The other is that monetary shock is both long-run neutral and homogenous, which implies a long-run effect like:

$$
\begin{bmatrix} \Delta_\infty y_{t-1} \\ \Delta_\infty R_{t-1} \\ \Delta_\infty m_{t-1} \\ \Delta_\infty p_{t-1} \end{bmatrix} |u_{m,t} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} [g_m]. \tag{6.9}
$$

The test of (6.8) is LRN test; the test of (6.9) is LRH test. Both of them are testable since the cointegrating rank in this system is larger than the rank of H matrix minus one.

One thing worth mentioning is the long-run effect on the nominal interest rate. In a system where money is I(1), it should be zero under LRN. That is: the change in money level affects only the long-run price level but not the inflation rate and the real interest rate should not be affected under LRN, the long-run response of nominal rate should be zero.

Before we go to the test results, it is useful to investigate the geometric relations between cointegrating space and long-run hypotheses. Let $\beta_1 = (\beta_{1y}, \beta_{1R}, \beta_{1m}, \beta_{1p})'$ and $\beta_2 = (\beta_{2y}, \beta_{2R}, \beta_{2m}, \beta_{2p})'$ be two linear independent cointegrating vectors in this four-variable VECM. If the model admits LRH, then both of them should be orthogonal to the vector $(0, 0, 1, 1)'$. This orthogonality condition is equivalent to the requirement that the space spanned by $(\beta_{1y}, \beta_{1m}, \beta_{1p})'$ and $(\beta_{2y}, \beta_{2m}, \beta_{2p})'$ is orthogonal to the vector $(0, 1, 1)'$, and the space spanned by $(\beta_{1R}, \beta_{1m}, \beta_{1p})'$ and $(\beta_{2R}, \beta_{2m}, \beta_{2p})'$ is orthogonal to the vector

$(0,1,1)'$. In Figure 7(a) and 7(b)[24], the space B_0^Y is spanned by the unrestricted estimates of $(\beta_{1y}, \beta_{1m}, \beta_{1p})'$ and $(\beta_{2y}, \beta_{2m}, \beta_{2p})'$; the space B_0^R is spanned by the unrestricted estimates of $(\beta_{1R}, \beta_{1m}, \beta_{1p})'$ and $(\beta_{2R}, \beta_{2m}, \beta_{2p})'$.[25] Both of them do not satisfy the orthogonality condition for LRH. Therefore, the unrestricted estimated VECM can not admit LRH. In Figure 7(c) and 7(d), the space B_1^Y is spanned by the LRN-restricted estimates of $(\beta_{1y}, \beta_{1m}, \beta_{1p})'$ and $(\beta_{2y}, \beta_{2m}, \beta_{2p})'$; the space B_1^R is spanned by the LRN-restricted estimates of $(\beta_{1R}, \beta_{1m}, \beta_{1p})'$ and $(\beta_{2R}, \beta_{2m}, \beta_{2p})'$.[26] As the reader can tell, both of them are very close to the spaces that admit LRH. This aspect will show in the test result below that: when we test LRH against LRN, LRH is not rejected.

The test results are in Table 4(b). Both LRN and LRH are not rejected under 5 percent and 10 percent testing size. Then we further test LRH against LRN. The testing Q-statistic value is simply the difference of the test values of LRN and LRH, and its degrees of freedom are also the difference of the degrees of freedom of LRN and LRH tests. The Q-statistics value is 1.1946 with one degree of freedom, which is not significant under both 5 percent and 10 percent size. That is: the long-term Friedman-Schwartz data support LRH.

6.2. post-WWII Quarterly Data. In this section, we use post-WWII quarterly data from 1959:1 to 2002:2. The sample is split into three subsamples: one is the pre-Volcker period (from 1959:1 to 1978:4), the other is post-1983 period(refer to post-Volcker experiment period), another is the sample between these two periods. We drop the last subsample due to its small sample size (only twenty observations.)

6.2.1. *LR test of Fisher and Seater.* The long regression results are in Figure 6. For the pre-Volcker sample, the long horizon estimate(refers to large k) of the LRD is close to

[24] In Figure 7, the space with normal vector $(0,1,1)$ is graphed. This space is the space that can admit LRH. It provides contrast to visualize the difference between unrestriced estimates and restricted estimates.

[25] The unrestricted estimate of $\beta_1 = (-20.1803, 0.3418, 13.1949, -6.2688)$ and $\beta_2 = (19.6202, -1.8818, -4.7390, 5.9697)$.

[26] The LRN-restricted estimate of $\beta_1 = (-0.0074, -0.0740, 0.8110, -0.5803)$ and $\beta_2 = (-0.9951, 0.0991, 0, 0)$

one for nominal income and price, so that LRH is not rejected. It is close to zero for real income, so that LRN is not rejected. Note that this finding is consistent with my earlier results using the Friedman and Schwartz annual data.[27]

When we move the sample period from pre-Volcker to post-1983, the results using the FS method change dramatically. Though the long horizon estimate of LRD is close to zero for real income, it is far from one for nominal income and price. Thus: LRN is not rejected, but LRH is rejected.

The change of the FS testing results can be rationalized from two possible angles: one is that money is not LRH after 1983, the other is that the monetary policy changed after 1983, which changed the identifying structure.

6.2.2. *LR test of Geweke.* The estimate of the long-run feedback $f_{m \to X}(0)$ for the post-WWII quarterly data is reported in Table 5, which displays more mixed results than the FS test. For the pre-Volcker data, the estimate is close to zero for real balances. Thus, LRH is not rejected. However, the estimate is far from zero for real income or the joint test of real income and real balances. Hence, LRN is rejected. These results are not consistent with each other, since if LRN is rejected, LRH can not be true. For the post-1983 data, LRN is rejected for real income, real balance, or the joint test of both. But it is not rejected for the income velocity of money.

6.2.3. *Cointegration test.* The VEC model employs the same four variables as previously used for the Friedman and Schwartz data. After PIC model selection, we select a zero lag and a cointegrating rank equal to two for pre-Volcker sample; we select one lag and a cointegrating rank equal to two for post-1983 sample.

The test results for the pre-Volcker data are reported in Table 6(a). When testing LRN, we do not reject LRN at 5 percent and 10 percent testing size. However, when testing LRH,

[27]This is no surprise, since the pre-Volcker period is covered in the annual data we used. The only difference is its data frequency.

it was significantly rejected at both testing sizes. Given the test results, if we believe in LRN, we can test LRH against LRN. We found that LRH is rejected. Therefore money is long-run neutral but not homogenous in this period. This is different from the test results of the annual data from Friedman and Schwartz. In the annual data, LRH is not rejected.

For the pre-Volcker data, since LRH is not significantly supported, the long-run response of price may not be proportionate to the long-run response of money to LRN monetary shock. It would be interesting to know the long-run response of price to monetary shock– that is the estimate of g_p in (6.8). We normalized g_m to one, and found the estimated g_p to be 2.8489. Therefore, in this period, for a one-time increase of LRN permanent monetary shock that increases money stock by one percent in the long-run, price level will increase by 2.85 percent in the long-run. This implies that permanent monetary shock has long-run impact on the income velocity of money, which will increase by a 1.85 percent response in the long run.

The testing results for the post-1983 data are reported in Table 6(b). Both LRN and LRH are not rejected at 10 percent size. When testing LRH against LRN, we do not reject LRH under 10 percent size. In this period, long-run proportional movement of nominal variables in response to LRN monetary shock is supported.

In summary: LRN is not rejected for both pre-Volcker and post-1983 data. As to LRH, it is not strongly supported in pre-Volcker period, but is strongly supported in post-1983 data. We found that for the pre-Volcker data, a one-time increase of permanent LRN monetary shock that increases money stock by one percent in the long-run, price level increases by 2.85 percent in the long-run. The long-run increase of the income velocity of money is then 1.85 percent.

7. Conclusion

Previous tests of long-run monetary propositions concerning neutrality and homogeneity based on vector autoregressions require estimating the effects of permanent monetary shocks on real and nominal variables. If the estimated monetary effect on a real variable is not significantly different from zero, then long-run neutrality is not rejected. If the estimated effect on a nominal variable is not different from one, then long-run homogeneity is not rejected.

These tests have a serious drawback: their measures of the long-run effects of money are heavily dependent on identifying assumptions and other maintained assumptions including the choice of variables included in the VAR. Thus, for example, any rejection of long-run neutrality or homogeneity can signal that the theory is incorrect or that the identifying assumptions are wrong.

This chapter uses different approach to test long-run neutrality and homogeneity. I show that these propositions can be cast in terms of linear restrictions on cointegrating space, which is independent of any traditional identifying assumption. Based on this argument, the likelihood ratio test of linear restrictions on cointegrating vectors can be applied to testing long-run neutrality and homogeneity. The test is then applied to three different data sets: the post Great Depression annual data set from Friedman and Schwartz; the post-WWII pre-Volcker quarterly data set; and the post-1983 quarterly data set of the U.S. The test results do not reject long-run neutrality for all three data sets.

Besides long-run neutrality, I also tested long-run homogeneity which is not rejected in the annual data set and the post-1983 quarterly data set, but is rejected in the pre-Volcker quarterly data set. Given that long-run homogeneity is rejected in the pre-Volcker period, I estimated the long-run response of price to a one-time permanent monetary shock. The estimated results is: for a one-time increase of permanent long-run neutral monetary shock that increases money stock by one percent in the long-run, price level increases by 2.85

percent in the long-run. Long-run increase of the income velocity of money is then 1.85 percent.

CHAPTER 2

Long-Run Identification When the Long-Run Proposition is Over-Identifying

1. Introduction

Since the work of Sims in 1980, vector autoregressions have been widely used in economic profession to study the dynamic effects of structural shocks in the economy. Researchers first estimate a reduced form vector autoregression (VAR), then try to recover its structural form through some identifying assumptions. There are two types of identification methods generally employed: one is long-run identification; and the other is short-run identification.

Short-run identification orders variables in the system according to the assumptions regarding their degree of contemporaneous exogeneity. The identifying assumptions allow the researcher to transform a structural VAR estimation into a sequence of recursive regressions. Take money and output as an example. If the researcher believes that money reacts to output contemporaneously but not vice versa, then money is more endogenous than output. Some examples of structural VAR study using short-run identification are Christiano, Eichenbaum and Evans (1994), Eichenbaum and Evans (1995), and Bernanke and Mihov (1998). Short-run identification, though easy to use, has a problem of choosing proper ordering. It usually relies on the researcher's subjective belief. Cochrane (1994) criticizes short-run identification for the analysis of monetary shocks, arguing that, "empirical researchers typically fish for VAR specifications to produce impulse-responses that capture qualitative monetary dynamics, ..." Beside the subjective ordering criticism, if all variables in the system are allowed to react to each other contemporaneously, then short-run identification can not be used.

The other type of identification identifies structural shocks using assumptions regarding the shocks' long-run effects. For an example, if the researcher believes that money is long-run neutral, then the idea of long-run identification is to use this long-run proposition to identify monetary shocks. The long-run neutrality of money is a widely accepted proposition among economists and can be tested without strong assumptions.[1] Therefore, using long-run identification to identify monetary shocks is attractive for many researchers. Some papers falling in this category are Blanchard and Quah (1989); King, Plosser, Stock and Watson (1991); and Jang and Ogaki (2001). However, such works on long-run identification has a problem: without choosing variables carefully, the unrestricted estimate of a VAR may not be able to accommodate the long-run proposition. That is, there may not exist any identifying structure that produces an identified structural shock consistent with the long-run proposition. In other words, the long-run proposition may be over-identifying.

When the long-run proposition is over-identifying, long-run identification requires proper restrictions on model estimation. In my previous chapter, I developed a restricted estimation approach that ensures the estimated system will admit long-run proposition. With the tool developed in chapter one, I show that the long-run proposition combined with two more identifying assumptions provides necessary and sufficient conditions to identify the permanent structural shock up to a scale adjustment. These two extra identifying assumptions are: (i) the structural shock to be identified must be uncorrelated with other structural shocks; and (ii) the long-run effect of the structural shock to be identified must be linearly independent from the long-run effects of other structural shocks.

This chapter uses an open economy VAR model to demonstrate the application of the method and then identifies U.S. monetary shocks. Because the model includes two countries' interest rates and exchange rates, I can discuss one issue in the International Finance literature: the uncovered interest parity(UIP) puzzle. UIP says that a one percent increase in the interest rate differential between home and foreign countries should predict

[1]See chapter one.

a one-percent home currency depreciation. However, many empirical studies – such as Fama(1984) and Froot and Frankel(1989) – widely reject this parity. These authors find currency depreciation is not one-percent. But, more strikingly, for most foreign currencies, exchange rates even respond with the wrong sign. In addition, they show that the excess returns of the foreign investment tend to be serially correlated, which also violates the implication of UIP. In this chapter, I use a structural VAR based on long-run identification to examine whether U.S. monetary shocks are important in accounting for UIP deviations.

2. An Overview of Long-run Identification

There has long been interest in identifying structural shocks based on the assumption regarding their long-run effect. For instance, to identify monetary shock based on the assumption that its long-run effect is neutral to all real variables. In this section, the history of such an identifying approach is reviewed and the alternative approach taken in this chapter is highlighted.

2.1. The Blanchard and Quah Approach. The frontier work in long-run identification was done by Blanchard and Quah in 1989. They used a bivariate VAR model to identify demand shocks based on the assumption that such shocks do not produce a long-run effect on real output.[2] To be more clear, they model the stochastic process of real output growth rate (Δy) and unemployment rate (u) with a VAR as follows:

$$C(L)X_t = \varepsilon_t$$

where $X_t = [\Delta y_t, u_t]'$ and $C(0) = I$. This is a reduced form unless structurally both Δy and u do not respond to each other contemporaneously. Since these two variables are stationary, $C(L)$ is invertible. The effect of the forecast errors ε_t on X_t is simply

$$X_t = C(L)^{-1}\varepsilon_t.$$

[2]There are two more identifying assumptions needed to be imposed: the structural shocks are not correlated; and the variance of demand shocks is equal to one.

Now, assume that there is a linear structural relation between the forecast errors ε_t and the structural shocks u_t which can be expressed as $\varepsilon_t = \Pi u_t$. Then the effect of structural shocks on X_t are

$$X_t = C(L)^{-1}\Pi u_t.$$

Let $u_t = [u_t^d, u_t^s]'$ where u^d represents the demand shock and u^s represents the supply shock. Given that Δy_t involves the first difference of real output, the upper left corner of $C(1)^{-1}\Pi$ presents the long-run effect of a demand shock on real output level. BQ impose a long-run restriction on the upper left value, requiring it to be zero. This implies that demand shocks have no long-run effect on the real output level.[3] The long-run restriction that they impose can help the researcher to identify the first column of the identifying structure Π, which identifies demand shocks.

2.2. The King, Plosser, Stock and Watson Approach. The long-run identification proposed by Blanchard and Quah, though simple to apply, has some major problems or limits. First it is limited to a specific type of bivariate model: one variable must be first difference stationary, the other must be level stationary. This makes the interpretation of the identified shock difficult. In most macro models, there are more than two structural shocks. Second, the model does not allow the cointegration of nonstationary variables to present. In a more general stochastic model, cointegration appears frequently.

King, Plosser, Stock and Watson (KPSW, hereafter) in 1991 developed another long-run identification method. They use some structural cointegration relations – that are agreed by many economists – within a vector error correction model (VECM) to identify those shocks that have long-run effect on the system. These shocks are called permanent shocks. Mathematically, the core of their idea lies in the observation that the long-run effect of permanent shocks must be in the orthogonal cointegrating space. To understand this,

[3]In this model, both structural shocks have no long-run effect on the unemployment rate since by nature it is stationary.

consider the following VECM.

$$\Delta X_t = D + \alpha\beta' X_{t-1} + \Sigma_{i=1}^{p}\Gamma_i \Delta X_{t-i} + \varepsilon_t. \tag{2.1}$$

According to an auxiliary lemma to Granger Representation Theorem stated below, the stochastic process of X_t as in (2.1) can be expressed in terms of a vector moving average process with initial values.

LEMMA 3. *The vector moving average solution of (2.1) is*

$$X_t = C(Dt + \Sigma_{i=1}^{t}\varepsilon_i) + C(L)(D + \varepsilon_t) + P_{\beta_\perp} X_0$$

where

1. $C = \beta_\perp(\alpha'_\perp \Gamma \beta_\perp)^{-1}\alpha'_\perp$ *and* $\Gamma = I - \Sigma_{i=1}^{p}\Gamma_i$*;*
2. $P_{\beta_\perp}$ *is a projection matrix projecting vectors into* $sp(\beta_\perp)$*;*
3. $C(L) = \Sigma_{i=0}^{\infty} C_i L^i$ *is a matrix polynomial with* C_i *matrices absolutely summable.*[4]

Given this moving average representation, the forecast error in time t has a long-run effect on X with the magnitude C, the space spanned by which lies in the orthogonal cointegrating space $sp(\beta_\perp)$. Suppose the structural relationship between the forecast errors and the structural shocks is $\varepsilon_t = \Pi u_t$. Then the long-run effect of structural shocks is $C\Pi u_t$. Each column in $C\Pi$ presents the long-run effect of a unit increase of a specific structural shock. Apparently, it must lie in $sp(\beta_\perp)$.

KPSW exploit the fact that since the long-run effect vector of a permanent structural shock must lie in $sp(\beta_\perp)$. They can estimate $sp(\beta_\perp)$ and then choose a specific base, each vector of which has some economic interpretation. If they can identify the shocks that produce long-run effect like the vectors they choose for the base, then they can give the identified shocks structural meanings according to the economic interpretation to the base vector. For example, if one column vector of the base has long-run neutrality property

[4] The exact expression of C_i is complex, and not relevant to the purpose of this paper. For details, please refer to Chapter 4 of Johansen(1995).

which would be the property of the long-run effect of money under long-run neutrality proposition, then an identified shock that produces long-run effect as that column vector is called monetary shock. For example, consider the six-variable VECM they used: the variables are real output (y), real consumption (c), real investment (i), real money balance ($m-p$), nominal interest rate (R) and inflation rate (Δp). Based on economic reasoning, three cointegration relations are imposed: $(c-y) = \phi_1 (R - \Delta p)$, $(i-y) = \phi_2 (R - \Delta p)$, and $m - p = \beta_y y - \beta_R R$. The first two cointegration relations describe the effects on the consumption share and investment share of a change in real interest rate. The third relation is a money demand equation. There are three stochastic trends. Let $X_t = (y, c, i, m-p, R, \Delta p)'$. KPSW then select a base, say $\tilde{A}$, that is orthogonal to these three cointegrating vectors so as to represent the long-run effects of three different permanent structural shocks.

$$\tilde{A} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & \phi_1 \\ 1 & 0 & \phi_2 \\ \beta_y & -\beta_R & -\beta_R \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}. \tag{2.2}$$

The permanent shocks that produce the long-run effect corresponding to each column of $\tilde{A}$ respectively are called the balanced-growth shock, the neutral inflation shock and the real-interest-rate shock. The naming of these three shocks is based on the property of each column of $\tilde{A}$. The shock that produces long-run effect as the first column of $\tilde{A}$ leads to a unit of long-run increase in y, c and i. That is why it is called the real-balance-growth shock. The shock that produces the second column of $\tilde{A}$ is called the neutral inflation shock since it produces no long-run effect on y, c and i but moves nominal interest rate R and inflation rate Δp proportionately in the long-run. The third column corresponds to real-interest-rate shock since the shock produces a long-run effect on real interest rate.

If the contemporaneous impact of these three permanent shocks on X_t is Π_P, then the long-run effects of these three permanent shocks should be equal to $C\Pi_P$. Ideally, $C\Pi_P$ should be equal to $\tilde{A}$ in order to preserve the economic interpretations of these three shocks. However, in general $C\Pi_P = \tilde{A}$ is over-identifying as in the KPSW paper. Instead of imposing restriction on the VECM estimation, KPSW supplement more structural parameters to be identified in order to make the system just-identifying. These extra structural parameters are contained in a square matrix denoted as π which is assumed to be lower triangular with the diagonal elements being normalized to one. They then impose the identifying restriction that $C\Pi_P = \tilde{A}\pi$ instead. The long-run effects of three permanent shocks will be

$$\tilde{A}\pi = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & \phi_1 \\ 1 & 0 & \phi_2 \\ \beta_y & -\beta_R & -\beta_R \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ \pi_{12} & 1 & 0 \\ \pi_{31} & \pi_{32} & 1 \end{bmatrix}. \tag{2.3}$$

2.3. The Jang and Ogaki Approach. By throwing in more structural parameters in π, the KPSW approach creates a major problem: since the shocks they identified produce long-run effects as $\tilde{A}\pi$ instead of $\tilde{A}$, those identified structural shocks no longer carry the properties that determine the interpretation of each shock. For example, the second shock – if producing the long-run effect like the second column of $\tilde{A}$ – is called the neutral inflation shock. However, its long-run effect under the $C\Pi_P = \tilde{A}\pi$ identifying assumption is not equal to the second column of $\tilde{A}$ but to $\tilde{A}$ times the second column of π, which is a linear combination of the second and third columns of $\tilde{A}$. Consequently, the second shock no longer maintains the long-run neutral property unless the identified π_{32} fortuitously takes on a value of zero.

Jang and Ogaki (JO, hereafter) investigate (2.3) further and find an improvement on the KPSW approach. To understand the JO variation, notice that

$$\tilde{A}\pi = \left[\tilde{A}\pi_{(1.2)}, \tilde{A}_{(3)}\right]$$

where $\pi_{(1,2)}$ denotes the first and second columns of π and $\tilde{A}_{(3)}$ is the third column of $\tilde{A}$. The identified shock corresponding to the third column of Π_P will always produce the long-run effect equal to $\tilde{A}_{(3)}$. It maintains the economic interpretation from $\tilde{A}_{(3)}$. They show that the researcher can just-identify the third shock if the following three identifying assumptions are used: (i) the long-run effect of the shock, which is $\tilde{A}_{(3)}$ here, should comply with some long-run proposition; (ii) the long-run proposition imposes zero restrictions but no other types of restrictions on $\tilde{A}_{(3)}$; and (iii) given the estimated $sp(\beta_\perp)$, the vector in $sp\,(\beta_\perp)$ that can be chosen to represent $\tilde{A}_{(3)}$ is unique up to a scale adjustment.

JO use the federal funds rate (R_{ff}), the nonborrowed reserve ratio ($NBRX$), U.S. real output (y_{us}), U.S. price (p_{us}), foreign real output (y_{for}), foreign interest rate (R_{for}), and the real exchange rate (e, dollar/foreign) in a VECM to identify monetary shocks. In this seven variable system, they first conclude that the cointegrating rank is equal to three, which implies four permanent shocks – and monetary shocks are one of them. Therefore, $sp(\beta_\perp)$ has the dimension equal to four. Let $X_t = (y_{us}, y_{for}, e, R_{ff}, R_{for}, NBRX, p_{us},)'$. JO try to identify U.S. monetary shocks, assuming that the long-run effect of the shocks should be long-run neutral to y_{us}, y_{for} and e. Since the dimension of $sp(\beta_\perp)$ is four, the unrestricted estimate of it will be able to admit a base $\tilde{A}$ generically as below

$$\tilde{A} = \begin{bmatrix} \times & \times & \times & 0 \\ \times & \times & \times & 0 \\ \times & \times & \times & 0 \\ \times & \times & \times & 1 \\ \times & \times & \times & \times \\ \times & \times & \times & \times \\ \times & \times & \times & \times \end{bmatrix}.$$

Notice that the last column of it is uniquely determined and satisfies the long-run neutral assumption regardless of the researcher's choice for the first three linear independent columns.

2.4. The Admissibility Problem. Both the KPSW and JO approaches have a problem in choosing a vector to represent the long-run effect of a structural shock: though the long-run proposition suggests a reasonable vector to be chosen, without imposing proper restrictions on β estimation, the researcher may not be able to choose that vector since it may not be in $sp(\beta_\perp)$. When this happens, $sp(\beta_\perp)$ does not admit the long-run proposition. KPSW realize this problem: that is why they impose restrictions on β estimation, even though with these restrictions, $C\Pi_P = \tilde{A}$ is still over-identifying. For the JO approach, it looks like there is no restriction on β estimation needed since the long-run proposition they use imposes only zero restriction on the long-run effect vector but not other types of restrictions. However, even zero restriction can cause an *admissibility problem.*

The admissibility problem in the JO paper can be highlighted once we notice that the long-run neutrality proposition they impose is not truly long-run neutral. First, the monetary shocks they identify will affect the money level in the long-run but not its growth rate. Therefore, its long-run impact on inflation rate should be zero. Since nominal rate is the sum of the expected real rate and the expected inflation rate, and the long-run effect of a monetary shock on inflation rate is zero, the long-run effect of a monetary shock on interest rates (R_{us} and R_{for}) should be zero under the long-run neutrality assumption. Therefore, if the last column $\tilde{A}_{(4)}$ of $\tilde{A}$ represents the long-run effect of monetary shocks which is long-run neutral, then it should be $(0, 0, 0, 0, 0, 1, \times)'$. Generically the base of an unrestricted estimated $sp(\beta_\perp)$ can be normalized as the following echelon form

$$\tilde{A} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \times & 1 & 0 & 0 \\ \times & \times & 1 & 0 \\ \times & \times & \times & 1 \\ \times & \times & \times & \times \\ \times & \times & \times & \times \\ \times & \times & \times & \times \end{bmatrix},$$

which unfortunately can not admit a vector like $(0, 0, 0, 0, 0, 1, \times)'$. Thus, the JO method will fail whenever the number of zero restriction imposed by the long-run neutrality is larger than or equal to the number of stochastic trends.

Due to the limited number of stochastic trends found in most macro empirical studies, it is very likely that long-run neutrality will impose more zero restrictions than the number of stochastic trends. In this case, the long-run proposition requires some restrictions to be imposed on $sp(\beta_\perp)$ estimation. When this happens, long-run identification is over-identifying. Besides, if the researcher believes in long-run homogeneity, then all nominal variables should have a proportional long-run response to a monetary shock. Therefore, there are more restrictions on the choice of vector to represent the long-run effect of monetary shocks, which in turn implies more restrictions to be imposed on $sp(\beta_\perp)$ estimation. This chapter is designed to resolve this problem, and to bring long-run identification and estimation into a more compact framework.

3. Long-Run Identification When the Long-Run Proposition is Over-Identifying

This chapter provides a new long-run identification method to partially identify one permanent structural shock based on three identifying assumptions. The first is that the permanent structural shock to be identified satisfies some long-run proposition that is over-identifying; the second is that the shock to be identified is uncorrelated with other structural shocks; the third is that there is no linear combination of other structural shocks which can produce the same long-run effect as the shock to be identified.

3.1. Over-identifying Long-run Proposition. One novelty of this chapter is to extend the long-run identification method to the case that the long-run proposition is over-identifying. I will first show in this section the conditions under which a long-run proposition is over-identifying. I will then determine what kind of restrictions on estimation should be imposed in this situation.

3.1.1. *When is the long-run proposition over-identifying?* In section 2.4, I briefly discussed the situation in which a long-run proposition that imposes zero restrictions is over-identifying. Here, I want to extend the discussion to a more general type of long-run proposition: the type that imposes linear restrictions on the long-run effect. Let us call this type of long-run proposition a linear long-run proposition(a linear LR proposition, hereafter).

Consider a VECM as in (2.1) with n variables and γ the long-run effect vector of some permanent shock that we want to identify. A linear LR proposition imposes linear restrictions on the long-run effect vector γ, which can be expressed as that this vector must lie in some subspace of R^n. To understand the form of a linear LR proposition, take the example of a four-variable VECM in real output (y), the nominal interest rate (R), the price level (p), and a money aggregate (m). Suppose we are interested in identifying permanent monetary shocks u^m which are the autonomous unexpected movement from the central bank that has long-run impact on at least m. Let $X_t = (y, R, m, p)'$; and $\gamma = (g_y, g_R, g_m, g_p)'$ be the vector representing the long-run effect of u_t^m on these four variables. Without imposing any long-run proposition on γ, γ can be any vector in R^4. Suppose the researcher believes that monetary shocks must be long-run neutral, then u_t^m has no long-run effect on y and

R, i.e. $g_y = g_R = 0$. The long-run effect γ that is long-run neutral can be expressed as

$$\gamma = H\psi$$

with

$$H = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } \psi = \begin{bmatrix} g_m \\ g_p \end{bmatrix}. \tag{3.1}$$

Apparently $\gamma \in sp(H) \subset R^4$. It is true that any linear restriction on γ can be expressed as limiting the choice of γ to some subspace of R^4 since these restrictions can be expressed in $\gamma = H\psi$ form with H being some known matrix and ψ being some vector denoting the free parameters in γ that are not constrained by the linear LR proposition.[5] For later discussion convenience, I define a linear long-run proposition as follows:

Definition 2. *Given X_t being $n \times 1$, a linear long-run proposition on a long-run effect vector γ $(n \times 1)$ can be expressed as*

$$\gamma = H\psi \tag{3.2}$$

where H is $n \times s$ and full rank and ψ is $s \times 1$ for some $s \leq n$.

In line with the discussion in section 2.2, any long-run effect must be orthogonal to the cointegrating vector space, that is $\beta'\gamma = 0$. Suppose the cointegrating rank is r. Therefore β is $(n \times r)$. If the long-run identified permanent shock can produce a long-run effect expressible as (3.2), then $\beta' H\psi = 0$, then this implies that $\beta' H$ with dimension $(r \times s)$ must have a rank less than s. Otherwise, $\beta' H\psi = 0$ has no solution[6]: there is no way to identify a permanent shock that can produce a long-run effect expressible as $H\psi$. Apparently, the rank of $\beta' H$ is related to whether the estimated $sp(\beta)$ can admit a linear LR proposition.

[5]Linear LR proposition imposes restrictions on γ like $B'\gamma = 0$ which is equivalent to $\gamma = H\psi$ where $H = B_\perp$.

[6]We do not consider zero vector as a solution since the shock to be identified is a permanent shock which should produce long-run effect. The shocks that have long-run effect expressible as $H\psi$ with $\psi = 0$ are not permanent shocks by definition.

Proposition 4. *Given the cointegrating matrix $\beta(n \times r)$ and the $H(n \times s)$ implied by a linear LR proposition, if both of them are full rank, then $sp(\beta)$ (or $sp(\beta_\perp)$) can admit this linear LR proposition if and only if $rank(\beta' H) < s$.*

The proof is rather straightforward. If $rank(\beta' H) < s$, then the dimension of the null space of $\beta' H$ is at least one. There exists non-zero ψ such that $\beta' H \psi = 0$. This means that it is possible to choose some long-run effect vector γ from $sp(\beta_\perp)$ that is consistent with the linear LR proposition characterized by the H matrix.

3.1.2. *A restricted estimation when the long-run proposition is over-identifying.* Macroeconomic empirical studies frequently detect a fairly small number of stochastic trends. Therefore, when the VECM system is large or the long-run proposition imposes strong restrictions expressible as a small dimension of $sp(H)$, it is very likely that $r \geq s$. Generically an unrestricted estimate of β will produce the case that $\beta' H$ has full rank. Under the $r \geq s$ situation, $rank(\beta' H) = s$ which by Proposition 4 implies that $sp(\beta)$ can not admit a linear LR proposition. Therefore, a linear LR proposition will be over-identifying.

In terms of the literature discussed above – the KPSW and the JO paper, each provides over-identifying examples. In the KPSW paper, $X_t = (y, c, i, m-p, R, \Delta p)'$ as mentioned in section 2.2. The long-run effect of a neutral inflation shock is $\gamma = (0, 0, 0, g_{m-p}, g_R, g_{\Delta p})'$ with $g_R = g_{\Delta p}$ [7] can be expressed as

$$\gamma = H\psi \tag{3.3}$$

[7] $g_R = g_{\Delta p} = 1$ in KPSW paper. This can be interpretted as some normalization. Here g_{m-p} represents $-\beta_R$.

with

$$H = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad \psi = \begin{bmatrix} g_{m-p} \\ g_R \end{bmatrix}.$$

Since there are three stochastic trends, the cointegrating matrix β is (6×3). The cointegrating rank r is 3 and the dimension s of $sp(H)$ is 2. Generically, an unrestricted estimate of $\beta' H$ will have a rank equal to 2 which is not less than s. According to Proposition 4, an unrestricted estimate of $sp(\beta)$ can not admit a neutral inflation shock unless some restrictions are imposed on β estimation.

In the JO paper, $X_t = (y_{us}, y_{for}, e, R_{ff}, R_{for}, NBRX, p_{us},)'$. As argued in section 2.4, a long-run neutral monetary shock should produce long-run effect $\gamma = (0, 0, 0, 0, 0, g_{NBRX}, g_{pus})'$ which can be expressed as (3.3) with

$$H = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad \psi = \begin{bmatrix} g_{NBRX} \\ g_{pus} \end{bmatrix}.$$

The dimension s of $sp(H)$ is 2. In their paper, the cointegrating rank $r = 3$. Without imposing restrictions on β estimation, generically $\beta' H$ would have rank equal to 2. Since this is not less than s, long-run neutrality of money is an over-identifying proposition.

If the unrestricted estimate of β can not admit the long-run proposition, then some restrictions must be imposed on the VECM estimation. As proved in chapter one, the following Hypothesis Equivalence Theorem holds.

THEOREM 3. *Given that the cointegrating rank of the VECM is equal to r, the following two hypotheses are equivalent:*

H_0 : *The VECM admits a long-run effect $\gamma(n \times 1)$ with $\gamma = \mathbf{H}\psi$ and $\mathbf{H}(n \times s)$ prespecified and full rank.*

H_0' : *There are at least $r - s + 1$ cointegrating vectors lying in the space $sp(\mathbf{H}_\perp)$.*

This means that in KPSW empirical model, the restriction which needs to be imposed to ensure the admissibility of a neutral inflation shock is that two cointegrating vectors must be in $sp(H_\perp)$. In the JO empirical model, the restriction to ensure the admissibility of a long-run neutral monetary shock is that two cointegrating vectors must lie in $sp(H_\perp)$. The restricted estimation algorithm to ensure the admissibility is stated in chapter one, section 4.2.

4. Identifying Permanent Structural Shocks

A linear LR proposition requires that the long-run effect γ of the shock to be identified must be expressible as $\gamma = H\psi$, where H is known. In other words, it requires $\gamma \in sp(H)$. When the unrestricted estimate of β does not admit the linear LR proposition, applying the restricted estimation described in section 3.1.2 will ensure the existence of a long-run effect vector γ which complies with the proposition. To locate the γ that is consistent with the long-run proposition, we can solve the linear problem $\beta' H\psi = 0$ for ψ. Any such $H\psi$ – belonging to $sp(H)$ and orthogonal to $sp(\beta)$ – can serve as a candidate to represent the long-run effect of the shock to be identified.

In this section, I will show that given the estimated $sp(\beta)$ admitting the linear LR proposition, the researcher can identify a permanent structural shock by choosing one vector in $sp(H)$ to represent its long-run effect with the help of the following two additional identifying assumptions: (i) the shock to be identified is not correlated with other structural shocks; and (ii) there is no linear combination of other structural shocks which produces the same long-run effect as the shock to be identified.

4.1. Choosing a long-run effect vector. First, we have to choose one vector to represent the long-run effect of the shock to be identified. A reasonable candidate to represent the long-run effect of the shock should belong to $sp(H)$ and be orthogonal to $sp(\beta)$. As discussed before, it must be expressible as $H\psi$ where ψ is some solution to $\beta' H\psi = 0$. If the dimension of the null space of $\beta' H$ is one, then the choice of ψ is unique up to a scale adjustment. Given $\hat{\beta}$, we can solve for the estimate $\hat{\psi}$ of ψ by solving $\hat{\beta}' H\psi = 0$ with one of the parameters in ψ being normalized to one. Then the estimated long-run effect of the permanent structural shock to be identified is $\hat{\gamma} = H\hat{\psi}$. I will use $\hat{\gamma}$ to represent the long-run effect of the shock to be identified.

Given the estimated long-run effect of the shock to be identified, the identifying assumptions (i) and (ii) can identify the shock. For illustrative purpose, I call the shock monetary shocks, u^m, which I want to identify from a VECM as of (2.1). Suppose there is some linear identifying structure Π between the structural shocks u_t and the forecast errors ε_t, taking the form as $\varepsilon_t = \Pi u_t$. Then the long-run response of X_t to the structural shocks u_t is $\Delta_\infty X_{t-1}|u_t = C\Pi u_t = \beta_\perp(\alpha'_\perp \Gamma \beta_\perp)^{-1}\alpha'_\perp \Pi u_t$. Partition u_t into two different shocks: one is a permanent monetary shock u_t^m and the rest are $u_t^{\tilde{m}}$. That is

$$u_t = \begin{bmatrix} u_t^m \\ u_t^{\tilde{m}} \end{bmatrix}.$$

Accordingly, I can partition the identifying structure Π into two blocks corresponding to u_t^m and $u_t^{\tilde{}m}$, respectively, such that

$$\Pi = [\pi_m, \pi_{\tilde{}m}].$$

The goal of the entire identification is to identify π_m and the variance σ_m^2 of u_t^m. Let n be the number of variables in the VECM. Then I have n parameters from $\pi_m(n \times 1)$ and one from σ_m^2 to be identified. The next job is to count the number of identifying equations.

4.2. Identifying equations from the long-run proposition: $n-r$ equations. Let γ be the long-run effect vector we choose to represent the long-run response $\Delta_\infty X_{t-1}|u_t^m$. According to Lemma 1, we know $\Delta_\infty X_{t-1}|u_t^m = \beta_\perp(\alpha'_\perp \Gamma \beta_\perp)^{-1}\alpha'_\perp \pi_m$. Therefore,

$$\beta_\perp(\alpha'_\perp \Gamma \beta_\perp)^{-1}\alpha'_\perp \pi_m = \gamma. \tag{4.1}$$

Though there are n equations in (4.1), r of them are not linear independent from the rest of the equations. This is because by nature, $\gamma \in sp(\beta_\perp)$ which can be expressed as $\gamma = \beta_\perp h$ with h being $n - r \times 1$. Therefore, there are only $n - r$ linear independent equations that can be used to solve for π_m which is

$$(\alpha'_\perp \Gamma \beta_\perp)^{-1}\alpha'_\perp \pi_m = \mathbf{h}. \tag{4.2}$$

4.3. Identifying equations from the linear independent long-run effect assumption: r equations. Let us partition the long-run response of X_t to structural shocks u_t into two blocks: one is γ, the long-run response of X_t to the monetary shock u_t^m; the other is $\Lambda_{\tilde{}m}$, the long-run response matrix of X_t to other shocks. Therefore

$$\begin{aligned} \Delta_\infty X_{t-1}|u_t &= \gamma u_t^m + \Lambda_{\tilde{}m} u_t^{\tilde{}m} \\ &= \begin{bmatrix} \gamma & \Lambda_{\tilde{}m} \end{bmatrix} u_t. \end{aligned}$$

Given the identifying structure $\varepsilon_t = \Pi u_t$, we have $u_t = \Pi^{-1}\varepsilon_t$. Therefore, $\Delta_\infty X_{t-1}|u_t = \begin{bmatrix} \gamma & \Lambda_{\tilde{}m} \end{bmatrix} u_t = \begin{bmatrix} \gamma & \Lambda_{\tilde{}m} \end{bmatrix} \Pi^{-1}\varepsilon_t$. According to Lemma 1, the long-run response of X_t to the time t forecast errors ε_t is $\Delta_\infty X_{t-1}|\varepsilon_t = \beta_\perp(\alpha'_\perp \Gamma \beta_\perp)^{-1}\alpha'_\perp \varepsilon_t$. Hence, $\begin{bmatrix} \gamma & \Lambda_{\tilde{}m} \end{bmatrix} \Pi^{-1} =$

$\beta_\perp(\alpha'_\perp \Gamma \beta_\perp)^{-1}\alpha'_\perp$ and consequently $\begin{bmatrix} \gamma & \Lambda_{\check{}m} \end{bmatrix} \Pi^{-1}\alpha = 0$. Let us partition Π^{-1} into $[g'_m, G'_{\check{}m}]'$. Then, $\begin{bmatrix} \gamma & \Lambda_{\check{}m} \end{bmatrix} \Pi^{-1}\alpha = \gamma g_m \alpha + \Lambda_{\check{}m} G_{\check{}m}\alpha = 0$. If the long-run effect γ is linearly independent from the long-run effects $\Lambda_{\check{}m}$ of other structural shocks, then $\gamma g_m \alpha + \Lambda_{\check{}m} G_{\check{}m}\alpha = 0$ if and only if $g_m \alpha = 0$.

There are two things worth mentioning. First, given the partition of Π^{-1}, the contemporaneous relation between a monetary shock u_t^m and the forecast error ε_t is $u_t^m = g_m \varepsilon_t$, which can be interpreted as the within-period reaction function of the monetary authority to the unexpected change to the VAR variables. The orthogonality condition between g_m and α thus implies that, if the long-run effect of money is linearly independent from the long-run effects of other structural shocks, then the within-period reaction function g_m must be orthogonal to the adjustment coefficient α that is associated to the deviations from the long-run equilibrium cointegrating relations. Second, the structural shocks do not have to be divided into transitory shocks and permanent shocks as in the KPSW and JO papers. It is possible for all shocks to have permanent effects as long as that there are no linear combination of other shocks which produce the same long-run effect as u_t^m.

Now I want to use the orthogonality condition between g_m and α to produce r extra identifying equations. Since $\varepsilon_t = \Pi u_t$, the relation between the variance covariance matrix Σ_ε of ε_t and the variance covariance matrix Σ_u of u_t is $\Sigma_\varepsilon = \Pi \Sigma_u \Pi'$, i.e. $\Pi^{-1}\Sigma_\varepsilon = \Sigma_u \Pi'$. Therefore, $g_m \Sigma_\varepsilon = \sigma_m^2 \pi'_m$ since u_t^m is assumed to be uncorrelated with other structural shocks. Hence $g_m \alpha = 0$ implies $\sigma_m^2 \pi'_m \Sigma_\varepsilon^{-1}\alpha = 0$, i.e. $\pi'_m \Sigma_\varepsilon^{-1}\alpha = 0$ since $\sigma_m^2 > 0$. We have r extra identifying equations as below:

$$\alpha' \Sigma_\varepsilon^{-1} \pi_m = 0. \tag{4.3}$$

4.4. Identifying equation from the $\Pi^{-1}\Pi = I$: one equation. Notice that (4.2) and (4.3) give exactly n equations to identify π_m for monetary shocks, which is independent of identifying the volatility σ_m^2 of monetary shocks. To identify σ_m^2, we can use the nature that $\Pi^{-1}\Pi = I$ which implies that $g_m \pi_m = 1$. According to the previous discussion, we know $g_m = \sigma_m^2 \pi_m' \Sigma_\varepsilon^{-1}$. Therefore

$$\sigma_m^2 = 1/(\pi_m' \Sigma_\varepsilon^{-1} \pi_m). \tag{4.4}$$

Through the discussions of section 4.2 to 4.4, we have the following proposition regarding identifying permanent monetary shocks.

PROPOSITION 5. *Given that the estimated $sp(\beta)$ can admit a unique normalized γ which represents the long-run effect of the shock to be identified, we can partially identify this shock through restrictions (4.2) to (4.4) if the following two identifying assumptions are made:*

1. *The shock to be identified is not correlated with other structural shocks.*
2. *The long-run effect of the shock is linearly independent from the long-run effects of other structural shocks.*

5. VAR Model of International Monetary Transmission: An Application

In this section, I apply my method to identify U.S. monetary shock and study its international transmission to four different foreign countries: Germany (GM), France (FR), Italy (IT) and the United Kingdom (UK). There are four separate VAR models: always one foreign country v.s. U.S. For notational simplicity, I always use the European country name to denote each panel without referring to the U.S.

The selection of variables in a VAR is a subtle issue. The variables selected should permit us to extract the structural shock that we are interested in. In this chapter, I am interested in the U.S. monetary shock. In order to extract it, the reduced form VAR should be compact enough to include variables about which the monetary authority is concerned and therefore builds into its monetary policy rule. Here, I choose the U.S. real income (y), the price level (p), the nominal interest rate (R) and a money aggregate (m), since these four variables are the essential variables in most small-scale macro models. An objective of this chapter is to analyze the importance of U.S. monetary shocks in explaining uncovered interest rate parity deviations. Therefore, two more foreign variables, foreign nominal interest rates (R^*) and nominal exchange rates (s)[8], are included.

The VAR model in this chapter is

$$\Delta X_t = \alpha\beta' X_{t-1} + \Sigma_{i=1}^{p} \Gamma_i \Delta X_{t-i} + e_t \tag{5.1}$$

where $X_t = (y_t, p_t, R_t, m_t, R_t^*, s_t)'$. This is a VECM. The forecast error e_t is assume to be iid normally distributed with mean zero and $Ee_te_t' = \Omega$. To estimate this model, I need to determine the number of lags p and the proper cointegrating rank r. I use the Posterior Information Criterion(PIC, hereafter) proposed by Phillips and Ploberger (1996) to select model lags and cointegrating rank simultaneously.[9] The selection results are in Table 7.[10] All country panels have their cointegrating rank equal to two, which means that there are four stochastic trends in each system.

5.1. Identification of U.S. monetary shocks. I use the long-run identification method in this chapter to identify U.S. monetary shocks, based on three assumptions: first, monetary shocks should be long-run neutral and long-run homogeneous; second, monetary shocks are not correlated with other structural shocks; and third, the long-run effect of money is linearly independent from the long-run effects of other structural shocks.

5.1.1. *Long-run neutrality and homogeneity of money.* Let u_t^m be the U.S. monetary shocks, and define $\Delta_\infty X_{t-1}|u_t^m \equiv \lim_{k\to\infty}(X_{t+k} - X_{t-1})|u_t^m$ to be the long-run response

[8]Nominal exchange rates are defined as the U.S. dollar against the foreign currency. For the detail of the source of data, please refer to the appendix.

[9]The conventional approach is to use a model selection criterion, such as BIC or AIC, to select lags first, and then to choose cointegrating rank, for example, by the likelihood ratio test of Johansen (1995) or the multivariate unit root test of Stock and Watson (1988). However, as pointed out by Johansen (1992), such a sequential model selection may be inconsistent. Chao and Phillips (1999) compared AIC, BIC and PIC; and found some finite sample evidence in favor of PIC.

[10]The minimal cointegrating rank and lag are set to be zero. The maximal lag is set to be four.

of X to the time t monetary shock u_t^m. If money is long-run neutral(LRN) and homogeneous(LRH), then

$$(5.2) \qquad \gamma \equiv \Delta_\infty X_{t-1}|u_t^m = \begin{bmatrix} \Delta_\infty y_{t-1} \\ \Delta_\infty p_{t-1} \\ \Delta_\infty R_{t-1} \\ \Delta_\infty m_{t-1} \\ \Delta_\infty R^*_{t-1} \\ \Delta_\infty s_{t-1} \end{bmatrix} |u_t^m = \begin{bmatrix} 0 \\ g_m \\ 0 \\ g_m \\ 0 \\ g_s \end{bmatrix}.$$

There are several aspects of the long-run proposition formalized in (5.2) that are worth mentioning. First, the long-run response of the U.S. price is equal to the response of money because of LRH. Second, the long-run response of nominal interest rates is zero. This is because that nominal interest rate is the sum of the expected real interest rate and the expected inflation rate. Long-run neutrality requires the long-run response of real interest rates to be zero. In addition, the shock identified in this model produces a long-run effect only on the price level but not on its first difference (the inflation rate). Therefore, the long-run response of inflation rates should be zero too. Consequently, the long-run response of the nominal interest rate to a monetary shock is zero. Third, there is no restriction on the long-run response of the nominal exchange rate since it also depends on the long-run response of foreign money and the model does not include a foreign money variable. Fourth, the long-run proposition is over-identifying. If we write γ in the $H\psi$ format, then

$$H = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad \psi = \begin{bmatrix} g_m \\ g_s \end{bmatrix}.$$

The rank of $\beta' H$ is equal to two which is not less that the dimension of $sp(H)$. According to Proposition 4, the long-run proposition is over-identifying. Fifth, the long-run proposition is testable. From chapter one, we know that a long-run proposition is testable if the cointegrating rank(r) is larger than the number(k) of free parameter in ψ minus one. This condition holds here. Table 8 shows that the test results do not reject the long-run proposition (5.2) for all countries at both five and ten percent levels.

5.2. The Effects of U.S. Monetary Shocks. Estimating the VECM via the approach of section 3.1.2 and following the identifying approach in section 4, I can identify both π_m (the short-run impact of U.S. monetary shock) and σ_m (the variance of U.S. monetary shock). With the identified shocks, I can study the impulse response functions of all variables, including foreign interest rates and exchange rates. Then I examine whether U.S. monetary shock itself can generate the exchange rate dynamics which are consistent with the UIP deviations observed in the data.

5.2.1. *The impulse response functions.* In this section, I study the short-run and the intermediate-run effects of U.S. monetary shocks.

The impulse response functions to a U.S. monetary shock are shown in Figure 9. The graph is based on the normalized monetary shock which moves up the U.S. nominal interest rate by one percent in the short-run. The qualitative responses of U.S. variables are the same across all country panels. When there is an increase in the nominal interest rate, the price level and output will be contractionary. This fits the story of a contractionary monetary shock. The response of money is expansionary in the short-run, except the GM panel, which seems to be abnormal. However, the abnormality disappears (the response of money becomes contractionary) immediately after one month for all country panels. Therefore, I refer to the impulse responses given here as the impulse responses to a U.S. contractionary monetary shock.

The effects of a contractionary U.S. monetary shock are: (i) the U.S. nominal interest rate increases in the short-run, then falls back to its long-run level gradually; (ii) the price level decreases monotonically through time; and (iii) the money aggregate – though increases immediately when the shock happens – decreases through time and reaches a long-run contractionary level gradually. As to the output impulse response, I found that the contractionary effect reaches its maximum after ten months, with output going back to its long-run level gradually because of long-run neutrality of money.

The exchange rate response is similar across all country panels. Foreign currencies are depreciating over time, which is consistent with the steady decreasing pattern of U.S. money aggregate. Foreign nominal interest rates responses are different across countries in the short-run. For UK, the nominal interest rate decreases in the short-run, which can occur if the UK monetary authority responds to a U.S. contractionary monetary shock by expanding money supply. For GM, FR and IT, the nominal interest rate increases in the short-run, which can occur if their authorities respond with a contractionary money supply. However, in the intermediate-run, all foreign countries's nominal interest rates are above their long-run levels. If the adjustment of interest rates reflects the money supply pattern, this means that in the intermediate-run all foreign countries's money supply is contractionary. Put in the language of modern macroeconomics, they are strategically complementary to the U.S. money supply.

Given long-run neutrality and homogeneity of money, monetary shocks have a long-run impact only on money level and foreign exchange rates, with the long-run response of price being proportional to the long-run response of money. Since the two long-run propositions I use do not require foreign money to move proportionately to the U.S. money in the long-run, exchange rate response does not have to be proportional to the US money in the long-run either. The estimated long-run responses of foreign exchange rates are shown in Table 9. A U.S. monetary shock that increases U.S. money by one percent in the long-run will appreciate all foreign currencies in the long-run. This means that the long-run responses of these four foreign money supplies are less than that of the U.S. money supply.[11]

5.2.2. *Uncovered Interest Parity.* In this chapter, I also study the dynamics of exchange rates and interest rates in order to re-examine a classical proposition called uncovered interest parity (UIP). UIP implies

$$E_t(\Delta s_{t+1}) = \left(R_t - R_t^*\right)/1200. \tag{5.3}$$

The interest rate differentials are dividend by 1200 is because the interest rate data I use is in annual percentage. The division will convert the data series into monthly rates. If UIP holds – and taking into account of risk premium, then

$$\Delta s_{t+1} = \alpha + (R_t - R_t^*)/1200 + \xi_{t+1}.$$

Here the constant term α is the risk premium, and ξ_{t+1} can be interpreted as the excess returns of foreign asset investment.

UIP implies two different time-series properties of exchange rates. First, interest rate differentials should not predict the excess returns. Thus, if we consider the following regression.

$$\Delta s_{t+1} = \alpha + \beta(R_t - R_t^*)/1200 + \xi_{t+1}, \tag{5.4}$$

Then the β regression estimate should be equal to one. Otherwise, interest rate differentials are correlated with the excess returns. Using the data set in this chapter, the estimated β values are not consistent with this implication. Table 10 shows the estimated results. Except IT, all β estimates are significantly different from one and of the wrong sign, which is a standard finding discussed earlier in this chapter.

[11]The impulse response functions in Figure 9 are based on the normalization that the U.S. nominal interest rate increases by one percent in the short-run. The effects on the U.S. money level is however long-run contractionary. Table 9, however, is based on the normalization that the U.S. money level is long-run expansionary.

Second, if UIP holds, then the excess returns should not be serially correlated. This implies that the ex post UIP deviation η_{t+1}, where

$$\eta_{t+1} = \Delta s_{t+1} - (R_t - R_t^*)/1200,$$

should not be serially correlated. The sample estimated autocorrelation of the η_{t+1} are in Table 11. Except UK, all other foreign countries violate the non-serial-correlation implication of UIP.

There is also another implication of UIP if UIP holds. Dornbusch (1976) points out that a once-for-all unexpected change in money level should create exchange rate overshooting in the short-run. That is the domestic currency over-depreciates compared to its long-run equilibrium level. Though there are works such as Eichenbaum and Evans (1995) using VAR to study the exchange rate overshooting, to legitimately examine whether the exchange rate dynamics are consistent with Dornbusch's story, the monetary shocks we identify must create a once-for-all effect on the money level. In most VAR studies, unless the money supply equation is a first difference equation with no lag and does not depend on other variables, the identified monetary shocks generate feedback within the system. The movement of money level will be a gradual adjustment process instead of a once-for-all level change. Therefore, it is impossible to use the VAR system in this chapter to talk about exchange rate overshooting directly. However, the failure of the data compliance with the previous two implications of UIP is an evidence against the effectiveness of overshooting claim.

5.2.3. *How Important Are U.S. Monetary Shocks in Accounting for the UIP Regression Puzzle.* To understand the importance of U.S. monetary shocks in accounting for the UIP regression puzzle, we need to understand the regression implication and its relationship with structural shocks. Given the regression model (5.4), the UIP regression is a conditional expectation of Δs_{t+1} on interest rate differentials, that is[12]

$$E\Delta s_{t+1}|(R_t - R_t^*) = \alpha + (R_t - R_t^*) + E\xi_{t+1}|(R_t - R_t^*).$$

If β deviates from one, it implies that the current interest rate differential has predictive power for the future excess return. Consider only linear prediction, we will have $E\xi_{t+1}|(R_t - R_t^*) = \gamma_0 + \gamma_1(R_t - R_t^*) + \nu_{t+1}$, where $\gamma_1 = cov(\xi_{t+1}, R_t - R_t^*)/var(R_t - R_t^*)$. Therefore, the UIP regression coefficient β can be expressed as $\beta = 1 + \gamma_1$, where γ_1 measures the bias of regression coefficient due to the predictability of interest rate differentials to the future excess returns. Since both ξ_{t+1} and $R_t - R_t^*$ can be expressed as a moving average process of past structural shocks in a structural VAR system, γ_1 can be decomposed into different structural shocks. Suppose there are n structural shocks, i.e. $u_t^1, \ldots, u_t^n$, then

$$\begin{aligned}
\gamma_1 &= cov(\xi_{t+1}, R_t - R_t^*)/var(R_t - R_t^*) \\
&= \Sigma_{i=1}^n cov(\xi_{t+1}, R_t - R_t^*|u^i)/var(R_t - R_t^*) \\
&= \Sigma_{i=1}^n \frac{var(R_t - R_t^*|u^i)}{var(R_t - R_t^*)} \frac{cov(\xi_{t+1}, R_t - R_t^*|u^i)}{var(R_t - R_t^*|u^i)}.
\end{aligned}$$

Therefore the contribution to the UIP regression deviation from U.S. monetary shocks can be decomposed into two parts: one is its contribution to the volatility of interest rate differentials, i.e. $var(R_t - R_t^*|u^m)/var(R_t - R_t^*)$;[13] the other is the UIP regression coefficient conditional on a world with only the presence of the U.S. monetary shocks, i.e. $cov(\xi_{t+1}, R_t - R_t^*|u^m)/var(R_t - R_t^*|u^m)$. [14]

The estimated results of the conditional bias and the U.S. monetary shocks contributions are in Table 12. The results show that U.S. monetary shocks cause a downward bias in the UIP regression coefficient. However, their contribution is negligible. None of them is larger than one percent.

[12]For notation simplicity, interest rates here are referred to monthly rate already. Therefore, I omit the division of 1200.

[13]This part is the variance-covariance decompostion for interest rate differentials. The symbol "$|u^m$" does not mean that it is conditional on the information of u^m, but conditional on the information that other shocks are zero all the time.

[14]The computation of this conditional regression coefficients is in the appendix to this paper.

5.2.4. *How Important Are U.S. Monetary Shocks in Accounting for UIP Deviation Persistence.* Figure 10 shows the impulse responses of the UIP deviations. For all four countries, a U.S. monetary shock that increases U.S. interest rate by one percent in annual rate will decrease the future excess return of foreign asset investment. However, this excess return – in favor of U.S. investment – will disappear gradually through time.

The estimated conditional serial correlation of the excess returns for four countries are in Table 13.[15] Though they are consistent with the positive autocorrelation we saw in Table 11, the one-period lagged autocorrelations are too small. U.S. monetary shocks themselves do not generate the extent of serial correlation that we see in the data.

6. Conclusion

In this chapter, I construct a long-run identification method that can partially identify the permanent structural shock, when – as is commonly the case – we believe in a long-run proposition regarding its effect which is over-identifying. Isolating this structural shock requires two other assumptions: (i) the shock to be identified is uncorrelated with other structural shocks; and (ii) there is no linear combination of other structural shocks that produces the same long-run effect as the shock to be identified.

I applied my method to identify U.S. monetary shocks based on long-run neutrality and homogeneity. The application is used to study not only the internal effects of U.S. monetary shocks but also the international transmission of U.S. monetary shocks to four different foreign countries: Germany, France, Italy and the United Kingdom.

The structural VAR study in this chapter is also used to study the uncovered interest rate parity(UIP) puzzle. I analyze whether U.S. monetary shocks can account for two different aspects of deviations from UIP. One is the predictability of the interest rate differentials for the future excess returns of foreign investment; the other is the serial correlation of the

[15]The computation of the conditional serial correlation of excess return can be obtained through its impulse response function. For detail, please refer to the appendix.

excess returns. I found that though U.S. monetary shocks generate these UIP deviations with the right sign in each case, the contributions of monetary shocks to accounting for the general deviations are small.

Table 1. Unit root test for money stock

Ng-Perron test statistics		MZ_a	MZ_t	MSB
Friedman and Schwartz data: post Great Depression		-15.0378^*	-2.7243^*	0.1812^*
post WWII data: pre-Volcker period		-0.9696	-0.4369	0.4506
post WWII data: post-1983 period		-1.7246	-0.7655	0.4439
Asymptotic critical values:	5%	-17.3000	-2.9100	0.1680
	10%	-14.2000	-2.6200	0.1850

* significant at 10 percent level

† Null hypothesis is that the series has a unit root.

‡ All test statistics are constructed under spectral GLS-detrended(including constant, and linear trend) autoregression with regression lags determined by the Modified Schwartz Information Criterion. The maximal lag allowed is 6.

Table 2. Friedman and Schwartz data: Geweke test

X variables	$f_{m\to X}(0)$	80% C.I.	Lags in VARs*
y	0.00013	$(0.00000, 0.00030)$	4
$m-p$	0.18871	$(0.00204, 0.43581)$	4
$y, m-p$	0.15790	$(0.02022, 0.35669)$	1
$v^\dagger$	0.00215	$(0.00046, 0.00464)$	4

† $v = p + y - m$ is the income velocity of money.

* lags are selected by BIC with maximal lags allowed to be 5.

Table 3. Friedmand and Schwartz data: Unrestricted Estimation of the VECM

(a) Cointegrating vectors β

	y	m	p
$\hat{\beta}$	16.4466	−11.6244	5.9478

(b) Adjustment coefficients α

	y	m	p
$\hat{\alpha}$	−0.0024	−0.0156	−0.0034

Table 4. Friedmand and Schwartz data: Cointegration test

(a) Three Variable Case

H_0	H_1	Q statistic	d.f
LRN as (6.7)	not H_0	1.5688	1

(b) Four Variable Case

H_0	H_1	Q statistic	d.f
LRN as (6.8)	not H_0	2.216	1
LRH as (6.9)	not H_0	3.4106	2
LRH	LRN	1.1946	1

* significant at 10 percent level
** significant at 5 percent level

Table 5. Post-WWII Quarterly data: Geweke test

(a) Pre-Volcker Data

X variables	$f_{m\to X}(0)$	80% C.I.	Lags in VARs*
y	0.50099	$(0.09926, 0.90873)$	1
$m-p$	0.00322	$(0.00003, 0.00775)$	1
$y, m-p$	2.64590	$(0.83660, 4.76890)$	1
v	0	N.A.**	0

(b) Post-1983 Data

X variables	$f_{m\to X}(0)$	80% C.I.	Lags in VARs*
y	0.39254	$(0.04648, 0.84426)$	2
$m-p$	0.57986	$(0.02868, 1.31810)$	1
$y, m-p$	0.53938	$(0.17882, 0.94721)$	1
v	0.12082	$(0.00406, 0.28649)$	2

* lags are selected by BIC with maximal lags allowed to be 6.
** when there is no lag selected, $f_{m\to X}(0)$ is always zero under the recursive identifying assumptions.

Table 6. Post-WWII Quarterly data: Cointegration test

(a) Pre-Volcker Data

H_0	H_1	Q statistic	d.f
LRN as (6.8)	not H_0	0.8657	1
LRH as (6.9)	not H_0	8.9192**	2
LRH	LRN	8.0535**	1

(b) Post-1983 Data

H_0	H_1	Q statistics	d.f
LRN as (6.8)	not H_0	0.7159	1
LRH as (6.9)	not H_0	0.8176	2
LRH	LRN	0.1017	1

* significant at 10 percent level
** significant at 5 percent level

Table 7. VECM Selections

	lags p	cointegrating rank r
GM	1	2
UK	1	2
FR	1	2
IT	1	2

Table 8. Long-run neutrality test

Countries	Q statistics†	d.f.	Countries	Q statistics	d.f.
GM	0.6161	1	FR	0.0249	1
UK	0.5404	1	IT	0.0254	1

† The Q statistic follows a χ^2 distribution asympotically with $r-(k-1)$ degrees of freedom.
* significantly reject (5.2) at ten percent size.
** significantly reject (5.2) at five percent size.

Table 9. The estimated long-run responses of foreign exchange rates

Country	(%)	Country	(%)
GM	4.1174†	FR	5.1599
UK	4.4238	IT	2.1253

† The estimations are based on the normalization of the long-run response of the U.S. money to one percent increase.

Table 10. The UIP Regression

Country		Country	
GM	-1.8274 (0.71046)†	FR	-1.3781 (0.64268)
UK	-2.6152 (1.1619)	IT	0.17877 (0.45116)

† The numbers in the parentheses are the Newey-West consistent estimates of standard errors.

Table 11. Sample Estimated Serial Correlation of UIP Deviations

lags	GM	FR	UK	IT
1	0.152**	0.323**	0.096	0.374**
2	0.062	0.072	0.015	0.059
3	0.030	0.121*	0.009	0.079
4	0.013	0.026	0.038	0.052
5	0.022	0.051	0.024	0.062

* significant at ten percent
** significant at five percent

Table 12. UIP regression

Countries	conditional bias γ_1	weights of contribution (%)
GM	-9.20171	0.000
UK	-4.97988	0.063
FR	-12.8279	0.000
IT	-2.97444	0.131

Table 13. Serial Correlation of UIP Deviations Conditional on the U.S. Monetary Shocks

lags	GM	FR	UK	IT
1	0.0116	0.0112	0.0108	0.0047
2	0.0109	0.0107	0.0108	0.0052
3	0.0101	0.0101	0.0099	0.0050
4	0.0090	0.0091	0.0087	0.0045
5	0.0079	0.0079	0.0074	0.0039

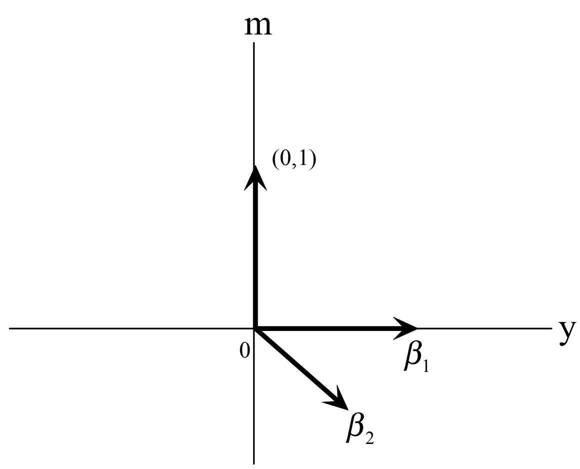

FIGURE 1. Cointegrating Vector Space and the Long-Run Effect of Money: A Bivariate Example

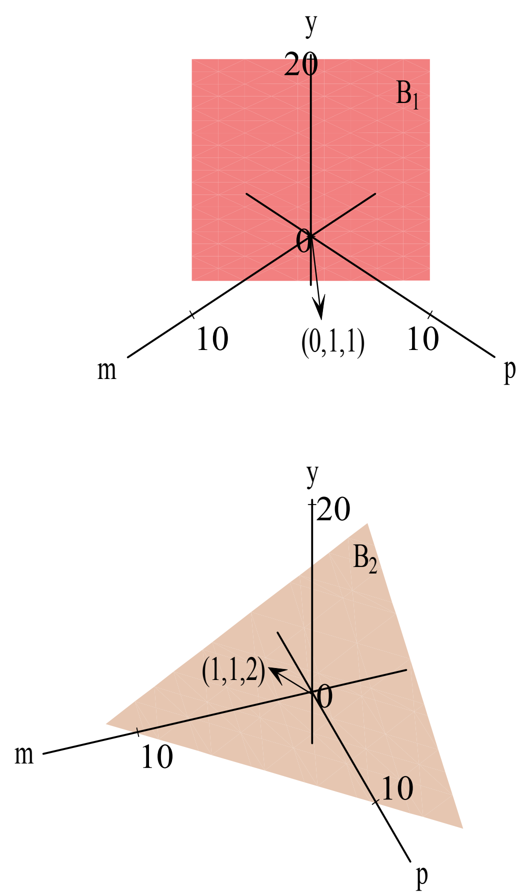

FIGURE 2. Cointegrating Vector Space and the Long-Run Effect of Money: A Trivariate Example

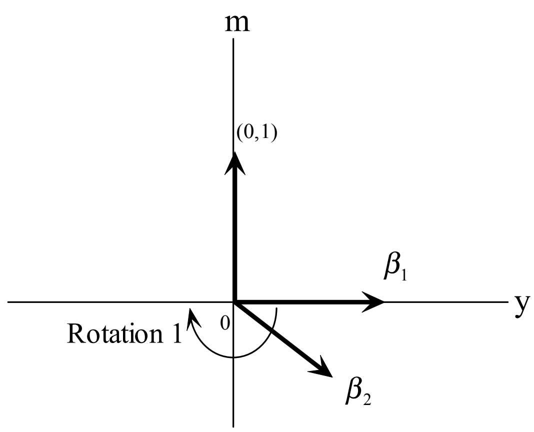

FIGURE 3. Long-Run Proposition Restriction on Cointegrating Vector Space: A Bivariate Example

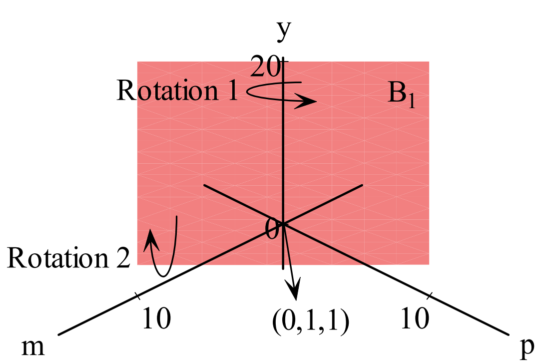

FIGURE 4. Long-Run Proposition Restriction on Cointegrating Vector Space: A Trivariate Example

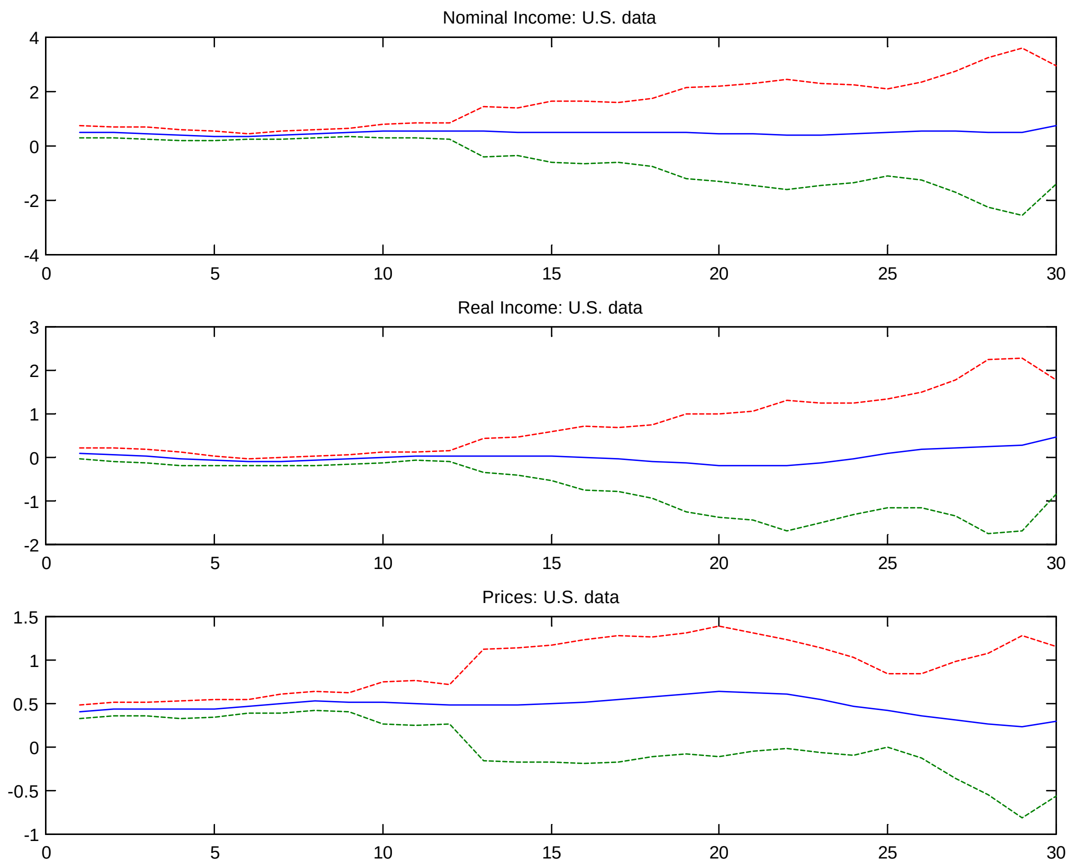

Figure 5. Long-Run Proposition Test on the Annual Data: Friedman and Schwartz Method

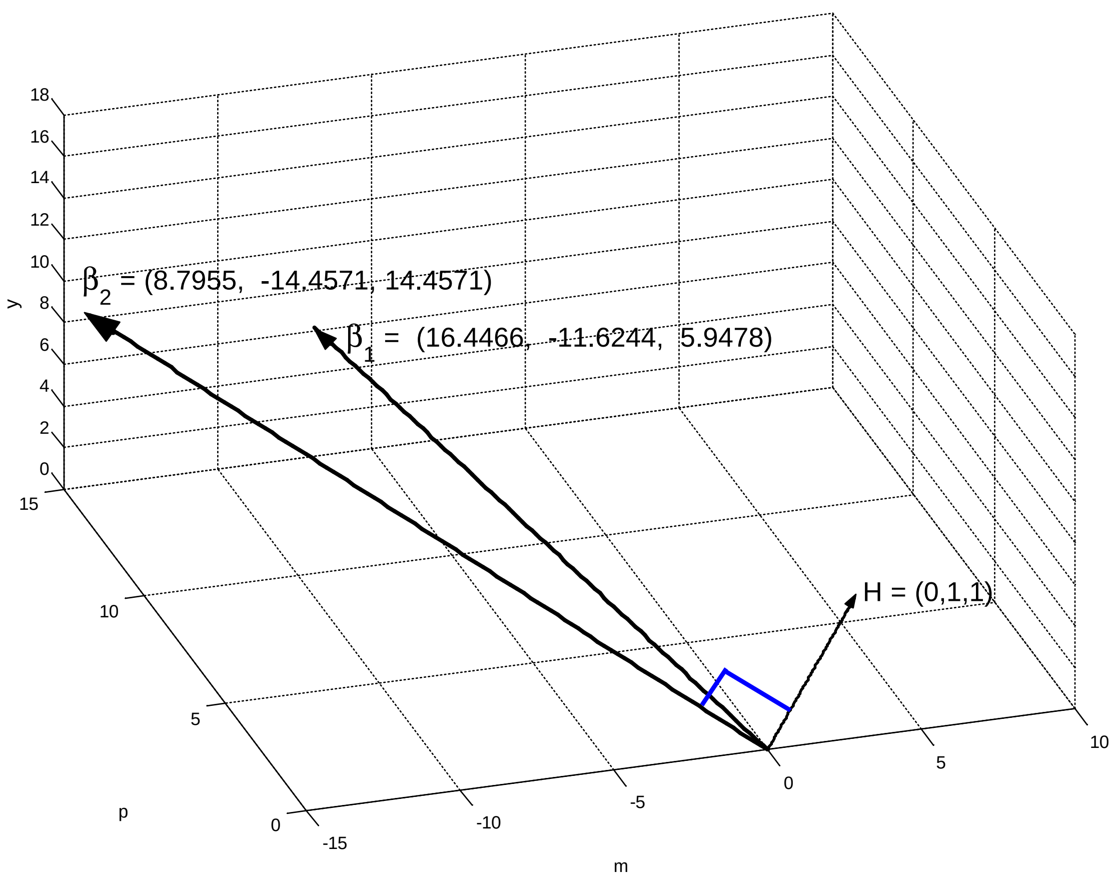

Figure 6. The Geometry Relations Between Restricted and Unrestricted Estimates of the Cointegrating Vectors: Trivariate Case

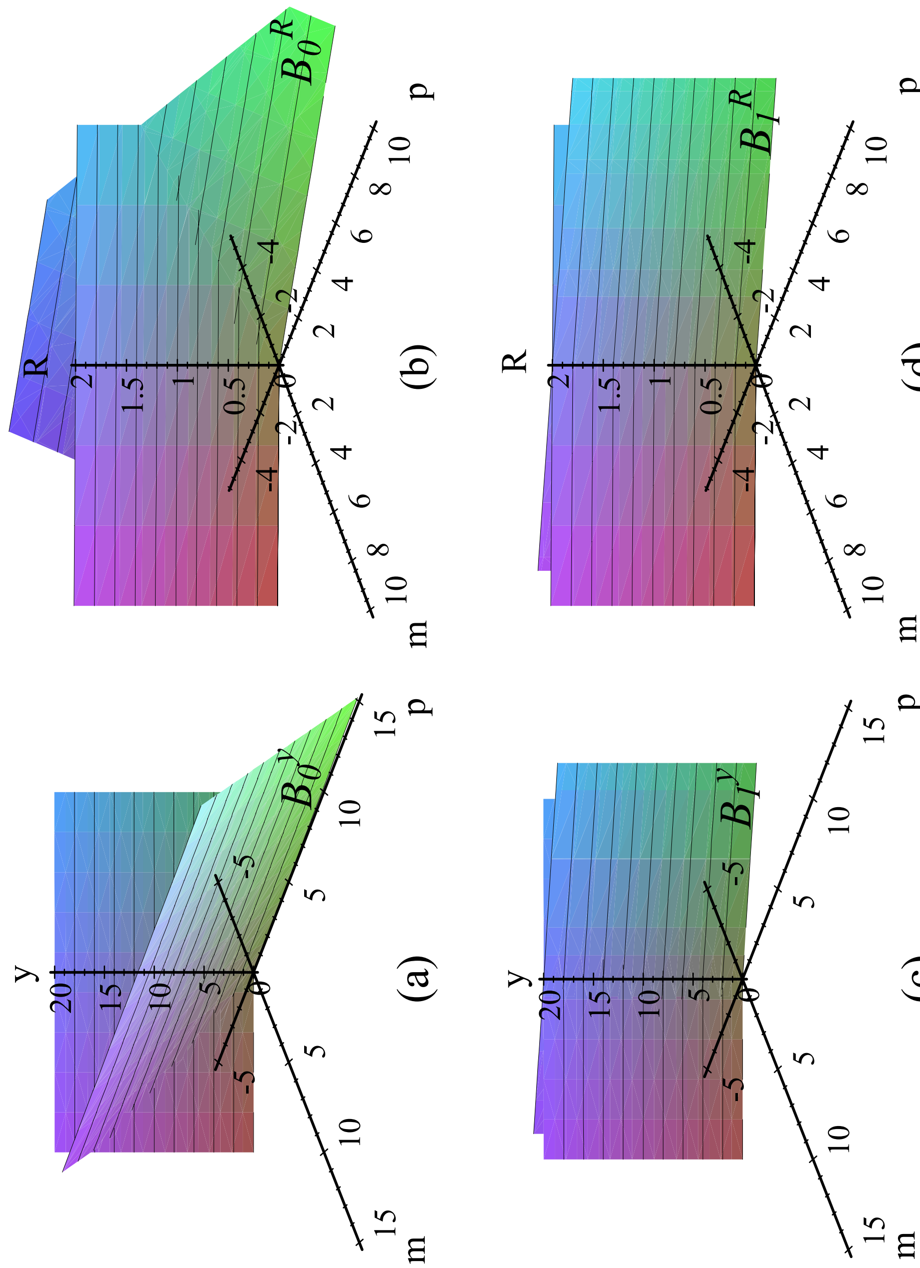

Figure 7. The Geometry Relations Between Restricted and Unrestricted Estimates of the Cointegrating Vectors: Four-variable Case

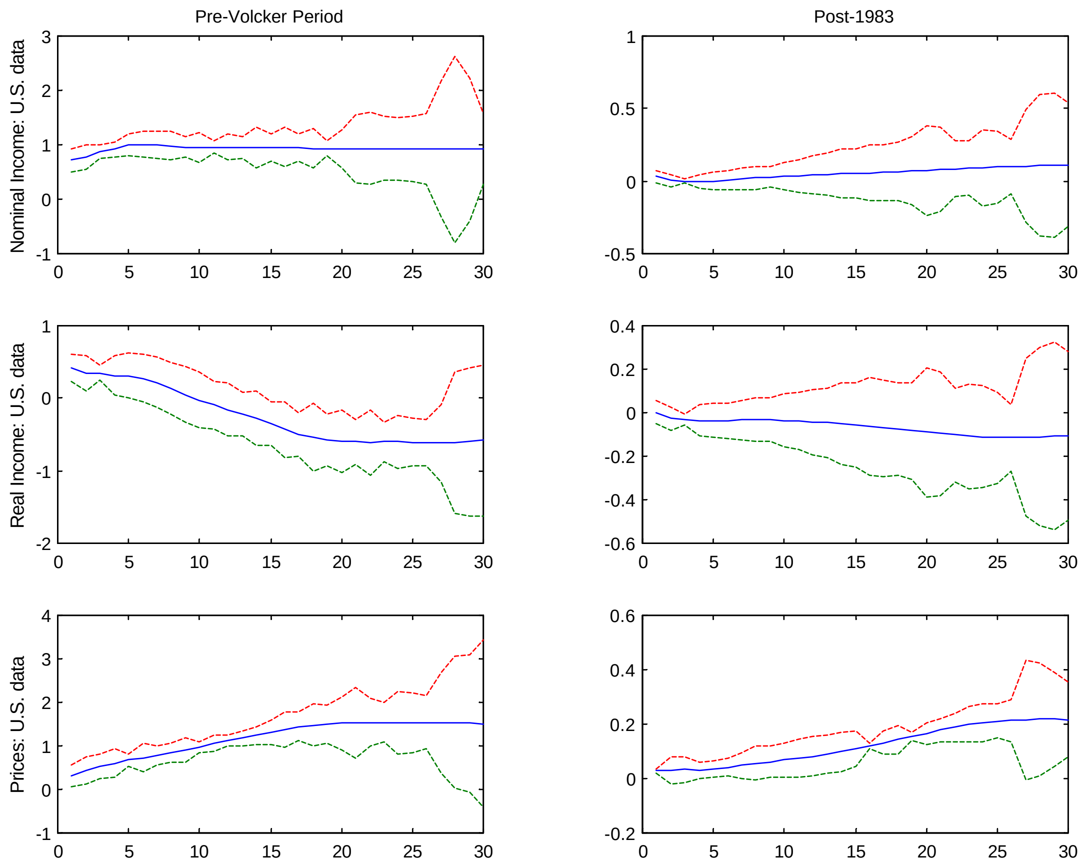

FIGURE 8. Long-Run Proposition Test on the Quarterly Data: Friedman and Schwartz Method

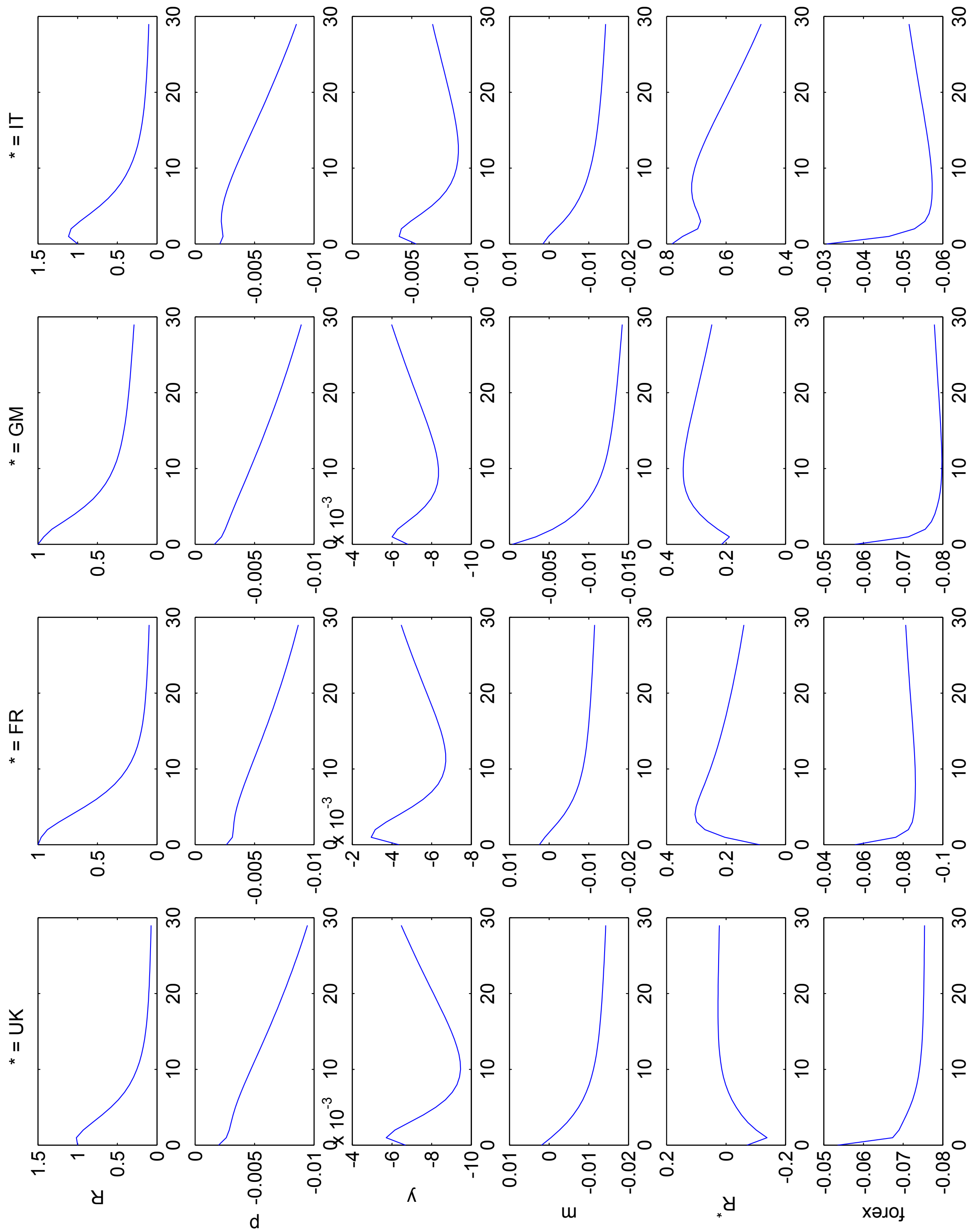

Figure 9. The Impulse Response Functions

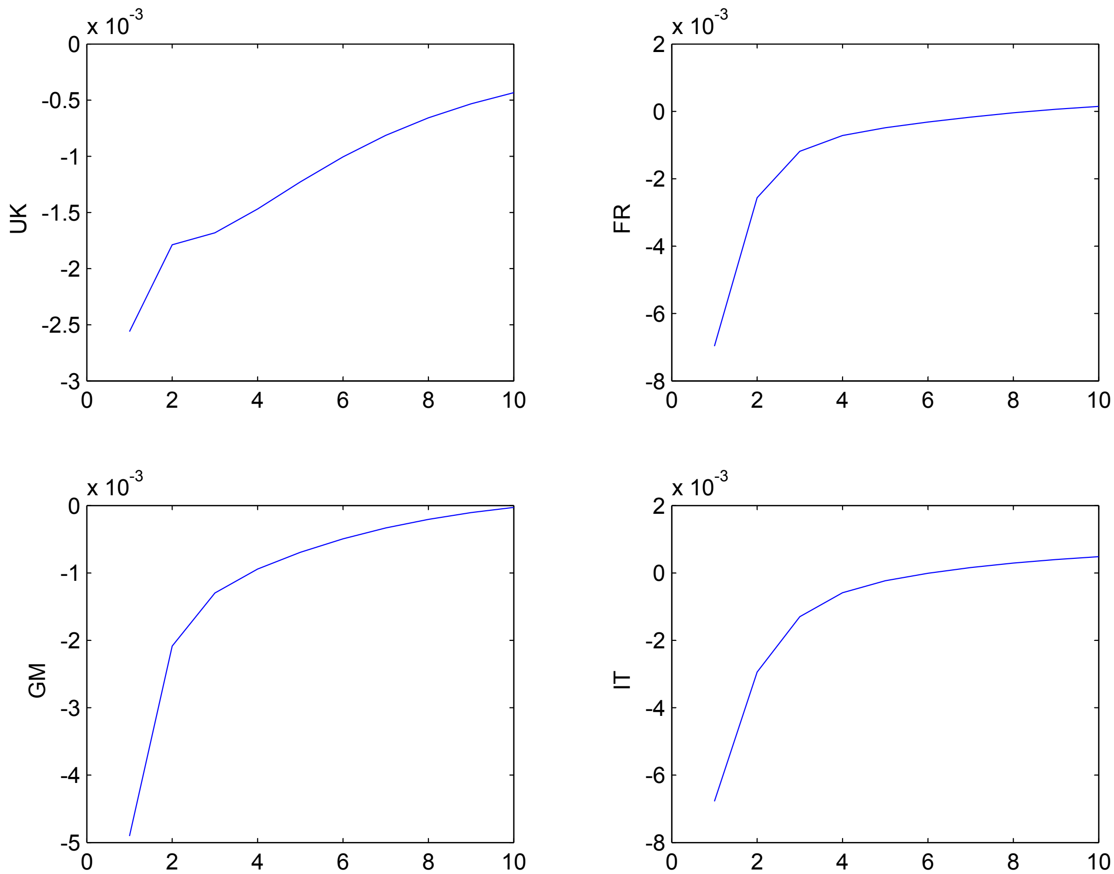

FIGURE 10. The Impulse Response Functions of UIP Deviations

Appendices to Chapter 1

1. Solution to a Simple Macro Model

$$y_t = \theta(p_t - E_{t-1}p_t) + u_t^y \tag{1.1}$$

$$p_t = gm_t - y_t + u_t^d \tag{1.2}$$

$$m_t = \alpha m_{t-1} + u_t^m \tag{1.3}$$

First, replace p in (1.1) with (1.2); replace all m with (1.3). Output is then determined by

$$\begin{aligned} y_t &= \theta\left[\left(g\alpha m_{t-1} + gu_t^m - y_t + u_t^d\right) - (g\alpha m_{t-1} - E_{t-1}y_t + E_{t-1}u_t^d)\right] + u_t^y \\ &= \pi(E_{t-1}y_t + gu_t^m + u_t^d - E_{t-1}u_t^d) + u_t^y \end{aligned}$$

where $\pi = \theta/(1+\theta)$. Taking condition expectation E_{t-1} on both sides, we find $E_{t-1}y_t = (1+\theta)E_{t-1}u_t^y$. Therefore,

$$y_t = \pi g u_t^m + \pi(u_t^d - E_{t-1}u_t^d) - \theta(u_t^y - E_{t-1}u_t^y) + (1+\theta)u_t^y. \tag{1.4}$$

With recursive iteration, the motion of m_t is:

$$m_t = \alpha^t m_0 + \Sigma_{i=1}^t \alpha^{t-i} u_i^m. \tag{1.5}$$

Combined with (1.4) and (1.5), the motion of p_t is:

$$\begin{aligned} p_t &= g\alpha^t m_0 + g\Sigma_{i=1}^t \alpha^{t-i} u_i^m \\ &\quad -\pi g u_t^m - \pi(u_t^d - E_{t-1}u_t^d) + \theta(u_t^y - E_{t-1}u_t^y) - (1+\theta)u_t^y + u_t^d. \end{aligned}$$

2. Solution to the Simple Macro Model in Section 3.3.4

$$y_t = \phi(m_t - E_{t-1}m_t) + u_t^y \tag{2.1}$$

$$m_t = y_t + u_t^m \tag{2.2}$$

$$u_t^y = u_{t-1}^y + u_t^r \tag{2.3}$$

Replace m_t in (2.1) with (2.2), then $y_t = \phi(y_t + u_t^m - E_{t-1}y_t) + u_t^y$. Taking the expectation E_{t-1} on both sides, we get $E_{t-1}y_t = E_{t-1}u_t^y = u_{t-1}^y$. Hence $y_t = \eta u_t^m + \eta u_t^r + u_t^y$ where $\eta = \phi/(1-\phi)$. If we take the first difference for both sides, then $\Delta y_t = \eta \Delta u_t^m + \eta \Delta u_t^r + u_t^r$. Δu_t^m can be replaced with $\Delta m_t - \Delta y_t$ according to (2.2). Also (2.2) can be expressed as $\Delta m_t = \Delta y_t + y_{t-1} - m_{t-1} + u_t^m$. Hence

$$\begin{bmatrix} 1+\eta & -\eta \\ -1 & 1 \end{bmatrix} \begin{bmatrix} \Delta y_t \\ \Delta m_t \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ m_{t-1} \end{bmatrix} + \begin{bmatrix} \tilde{u}_t^r \\ u_t^m \end{bmatrix}$$

where $\tilde{u}_t^r = \eta \Delta u_t^r + u_t^r$. Inverting the matrix associated with $(\Delta y_t, \Delta m_t)$, we obtain

$$\begin{bmatrix} \Delta y_t \\ \Delta m_t \end{bmatrix} = \begin{bmatrix} \eta \\ 1+\eta \end{bmatrix} \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ m_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^y \\ \varepsilon_t^m \end{bmatrix}$$

where

$$\begin{bmatrix} \varepsilon_t^y \\ \varepsilon_t^m \end{bmatrix} = \begin{bmatrix} 1 & \eta \\ 1 & 1+\eta \end{bmatrix} \begin{bmatrix} \tilde{u}_t^r \\ u_t^m \end{bmatrix}.$$

3. Algorithm for the Maximum Likelihood Value of Lemma 2

1. Estimate β unrestricted. And solve the following eigenvalue problem, given the unrestricted estimates $\hat{\beta}$,

$$\left| \lambda \hat{\beta}' \hat{\beta} - \hat{\beta}' \mathbf{H}_\perp (\mathbf{H}'_\perp \mathbf{H}_\perp)^{-1} \mathbf{H}'_\perp \hat{\beta} \right| = 0 \tag{3.1}$$

for eigenvectors $(v_1, \cdots, v_{r-s+1})$ corresponding to the largest $r-s+1$ eigenvalues. Let $\hat{\beta}_2 = \hat{\beta}\,[v_1, \cdots, v_{r-s+1}]$

2. Fix $\beta_2 = \hat{\beta}_2$; estimate β_1 with $\hat{\beta}_1 = [b_1, \cdots, b_{s-1}]$ where $b_1, \cdots, b_{s-1}$ are the eigenvectors associated with the $s-1$ largest eigenvalues to the following eigenvalue problem.

$$\left|\rho S_{11.\beta_2} - S_{10.\beta_2} S_{00.\beta_2}^{-1} S_{01.\beta_2}\right| = 0.$$

3. Fix $\beta_1 = \hat{\beta}_1$; estimate φ with $\hat{\varphi} = \left[\varphi_1, \cdots, \varphi_{r-s+1}\right]$ where $\varphi_1, \cdots, \varphi_{r-s+1}$ are the eigenvectors associated with the $r-s+1$ largest eigenvalues to the following eigenvalue problem

$$\left|\lambda \mathbf{H}'_{\perp} S_{11.\beta_1} \mathbf{H}_{\perp} - \mathbf{H}'_{\perp} S_{10.\beta_1} S_{00.\beta_1}^{-1} S_{01.\beta_1} \mathbf{H}_{\perp}\right| = 0.$$

   This gives an updated estimate $\hat{\beta}_2 = H_{\perp}\hat{\varphi}_2$.

4. Continue with 2 and 3 until convergence. Then compute the log-likelihood function value under the converged β_1 and β_2 estimates. This gives the $L_{\max}$ value of Lemma 2.

Two steps–steps 1 and 4–are worth explained here. The first step is to find a reasonable initial β_2 value for further iterations. Given unrestricted estimates $\hat{\beta}$, step one finds an initial value $\beta_2^{(0)}$ whose vectors are linear combinations of $\hat{\beta}$ and are as close to $sp(H_{\perp})$ as possible. The solutions to the eigenvalue problem(3.1) solve for the problem. The fourth step requires researchers to set up some convergence criteria for the algorithm to stop in a finite step. The criteria we used in this chapter is to stop the program when the likelihood function value is still climbing but its incremental magnitude is small. Therefore the Q values we get in practice are approximates. A user-friendly Matlab program can be received from the author upon request. In this program, users can ignore all econometric problems stated in this subsection, and only need to specify the H matrix for the program to compute Q value.

4. Unrestricted Maximum Likelihood Estimation

This is excerpted from Theorem 6.1 of Johansen(1995). The maximized value $L_{\max}^{u}$ of the likelihood function of the VEC model–whose cointegrating rank is equal to r–without any restriction is given by

$$(L^u_{\max})^{-2/T} = |S_{00}|\Pi^r_{i=1}(1-\lambda_i)$$

where $1 > \lambda_1 > \cdots > \lambda_r$ are defined as the largest r solutions to the eigenvalue problem: $|\lambda S_{11} - S_{10}S_{00}^{-1}S_{01}| = 0$.

5. R-Fold Replications

Given the estimates $\hat{B}(L)$ of $B(L)$ in (6.1) and the estimated variance covariance matrix $\hat{\Omega}$ of the error terms, we use the following data generating process to generate each replication sample.

$$\hat{B}(L)\begin{bmatrix}\Delta X_t \\ \Delta m_t\end{bmatrix} = \varepsilon_t$$

with $\varepsilon_t \sim N(0,\hat{\Omega})$. Let p be the number of lags in the system.

1. Using first p observations of $(\Delta X_t, \Delta m_t)$ as initial values
2. Sampling T observations of error terms from $N(0,\hat{\Omega})$
3. Generate T observations of $(\Delta X_t, \Delta m_t)$
4. Compute estimated $f_{m\to X}(0)$, call it $\hat{f}^i_{m\to X}(0)$.

Repeat 2 to 4 for R times (R=200 here). Let $\bar{f}_{m\to X}(0) = \Sigma^R_{i=1}\hat{f}^i_{m\to X}(0)/R$. We compute the percentage bias b^R of the $f_{m\to X}(0)$ estimate. $b^R = \hat{f}_{m\to X}(0)/\ \bar{f}_{m\to X}(0)$ where $\hat{f}_{m\to X}(0)$ is the estimate from the true sample. The percentage-bias adjusted estimate of $f_{m\to X}(0)$ is then $b^R\hat{f}_{m\to X}(0)$ which is the number we reported in the table. Let $l = [\alpha R/2]$ and $u = R - [\alpha R/2]$, and $\hat{f}^{(i)}_{m\to X}(0)$ denote a typical order statistic. The $100(1-\alpha)$ percent confidence interval is $(b^R\ \hat{f}^{(l)}_{m\to X}(0), b^R\ \hat{f}^{(u)}_{m\to X}(0))$.

6. Data Source: Friedman and Schwartz Data

All series are from Friedman and Schwartz (1982) from 1940 to 1975 , Table 4.8.

MNY=Money Stock (Billion \$)

NI=Nominal Income (Billion \$)

RI=Real Income (Billion 1929 \$)

PRICE=Implicit Price Deflator 1929=100

POP=Populations (Millions)

INT=Short-Term Commercial Paper Rate (Annual Percentage)

1. Time series in the FS test:

   $m = \log(\text{MNY})$

   $y = \log(\text{RI})$

   $Y = \log(\text{NI})$

2. Time series in the Geweke test[16]:

   $m = \log(\text{MNY/POP})$

   $y = \log(\text{RI/POP})$

   $Y = \log(\text{NI/POP})$

3. Time series in the cointegration test

   m, y, and Y are the same as in the Geweke test.

   $R = \text{INT}$.

7. Data Source: Post-WWII Quarterly Data

Sample period: 1959:1 to 2002:2

MNY=M1 (Billion \$), seasonally adjusted, Federal Reserve Board of Governors: H.6 Release

[16]FS series are not divided by the population to be consistent with Fisher and Seater (1993).

NI=Gross Domestic Product (Billion \$), seasonally adjusted, U.S. Department of Commerce, Bureau of Economic Analysis

RI=Real Gross Domestic Product (Billion of chained 1996 \$), seasonally adjusted, U.S. Department of Commerce, Bureau of Economic Analysis

PRICE=Implicit Price Deflator 1996=100, seasonally adjusted, U.S. Department of Commerce, Bureau of Economic Analysis

POP=Civilian Noninstitutional Population, end of month (Thousands), U.S. Department of Labor, Bureau of Labor Statistics

INT=3-Month Treasury Bill Secondary Market Rate, (Annual Percentage), Federal Reserve Board of Governors: H.15 Release

1. Time series in the FS test:

   $m = \log(\text{MNY})$

   $y = \log(\text{RI})$

   $Y = \log(\text{NI})$

2. Time series in the Geweke test:

   $m = \log(\text{MNY}/\text{POP})$

   $y = \log(\text{RI}/\text{POP})$

   $Y = \log(\text{NI}/\text{POP})$

3. Time series in the cointegration test

   m, y, and Y are the same as in the Geweke test.

   $R = \text{INT}/400.$

Appendices to Chapter 2

1. Computation of the β Conditional on Monetary Shocks

Suppose $E(\xi_{t+1}|(R_t - R_t^*)/1200) = \gamma_0 + \gamma_1 (R_t - R_t^*)/1200$, then $E(\Delta s_{+1}|(R_t - R_t^*)/1200) = (\alpha + \gamma_0) + (1 + \gamma_1)(R_t - R_t^*)/1200$. By definition,

$$\gamma_1 = cov(\xi_{t+1}, (R_t - R_t^*)/1200)/var((R_t - R_t^*)/1200).$$

If we compute the UIP regression conditional on the monetary shocks only, then the conditional UIP regression is

$$E(\Delta s_{+1}|(R_t - R_t^*)/1200, u^m) = (\alpha + \gamma_0) + (1 + \tilde{\gamma}_1)(R_t - R_t^*)/1200$$

where $\tilde{\gamma}_1 = cov(\xi_{t+1}, (R_t - R_t^*)/1200|u^m)/var((R_t - R_t^*)/1200|u^m)$. An easy way to compute $\tilde{\gamma}_1$ is to use the impulse response functions of UIP deviations and interest rate differentials. Suppose

$$\begin{aligned} \xi_t|u^m &= \psi_0 u_t^m + \psi_1 u_{t-1}^m + \psi_2 u_{t-2}^m + \cdots \\ (R_t - R_t^*)/100|u^m &= \phi_0 u_t^m + \phi_1 u_{t-1}^m + \phi_2 u_{t-2}^m + \cdots \end{aligned}$$

then

$$\tilde{\gamma}_1 = \left(\Sigma_{j=0}^{\infty} \phi_j \psi_{j+1}\right) / \Sigma_{j=0}^{\infty} \phi_j^2.$$

2. Computation of the Persistence of UIP Deviations

The impulse response function of the ξ_t to the forecast errors in (2.1) can be expressed as

$$\xi_t = \varepsilon_t + \mathbf{B}_1 \varepsilon_{t-1} + \mathbf{B}_2 \varepsilon_{t-1} + \cdots .$$

Given the identified π_m, the impulse response functions of ξ_t conditional on monetary shocks is

$$\xi_t | u_j^m \text{ for } j \leq t = \psi_0 u_t^m + \psi_1 u_{t-1}^m + \psi_2 u_{t-2}^m + \cdots$$

with $\psi_i = B_i \pi_m$ where $B_0 = I$. Therefore,

$$\begin{aligned} cov(\xi_{t+k}, \xi_t | u_j^m, j &\leq t+k) = \left(\Sigma_{j=0}^{\infty} \psi_{k+j} \psi_j\right) \sigma_m^2 \\ var(\xi_{t+k} | u_j^m, j &\leq t+k) = \left(\Sigma_{j=0}^{\infty} \psi_j^2\right) \sigma_m^2 \\ var(\xi_t | u_j^m, j &\leq t+k) = \left(\Sigma_{j=0}^{\infty} \psi_j^2\right) \sigma_m^2. \end{aligned}$$

It follows that

$$\rho_k = \left(\Sigma_{j=0}^{\infty} \psi_{k+j} \psi_j\right) / \left(\Sigma_{j=0}^{\infty} \psi_j^2\right).$$

3. Data Source

All data are from IFS, September 2002, disk unless otherwise specified. Data are monthly data. The sample period is from 1979:01 to 2001:05.

$INDP$ = U.S. industrial production, data series number is 11166..IZF.

MM = U.S. money aggregate (billion \$). Data series number is 11159MACZF.

CPI =U.S. CPI. Data series number is 11164...ZF.

R =U.S. nominal interest rates (annual percentage). Data series number is 11160LD-CZF.

R^* =Foreign nominal interest rates (annual percentage). Data series numbers are 11260EA.ZF for UK; 13260C..ZF for FR; 13460B..ZF for GM; and 13660B..ZF for IT.

$FOREX$ =Foreign exchange rate (US \$ / Foreign Currency). Data are from the Federal Reserve Bank of St. Louis.

$y = \log(INDP)$

$m = \log(MM)$

$p = \log(CPI)$

$s = \log(FOREX)$

To take into account the structural change due to the reunification of West and East Germany on July 1, 1990, R^* and s series for GM have been transformed to be orthogonal to the dummies $\{d_t, d_{t-1}, \ldots, d_{t-6}\}$ where $d_t = 1$ if t =1990:07; and $d_t = 0$ otherwise.

Bibliography

[1] Andersen Leonall C. and Jordan, Jerry L.,"Monetary and Fiscal Actions: A Test of Their Relative Importance in Economic Stabilization," Federal Reserve Bank of St. Louis Review, November 1968, 68, 11-24.

[2] Andersen Leonall C. and Karnosky Denis S., "The Appropriate Time Frame for Controlling Monetary Aggregates: The St. Louis Evidence," in Controlling Monetary Aggregate II: The Implementation, Federal Reserve Bank of Boston, 1972.

[3] Ando, Albert and Modigliani, Franco, "The Relative Stability of Monetary Velocity and the Investment Multiplier," in Critical Assessments of Contemporary Economists series Milton Friedman: Critical Assessments Volume 1. London and New York: Routledge 1990; 228-66. (Previously published: 1965.)

[4] Bae, Sang-Kun and Ratti, Ronald A., "Long-Run Neutrality, High Inflation, and Bank Insolvencies in Argentina and Brazil," Journal of Monetary Economics, December 2000, 46(3), 581-604.

[5] Bernanke, Ben S. and Mihov, Ilian, "Measuring Monetary Policy," Quarterly Journal of Economics, August 1998, 113(3), 869-902.

[6] Bernanke, Ben S. and Mihov, Ilian, "The Liquidity Effect and Long-Run Neutrality," Carnegie Rochester Conference Series on Public Policy, December 1998, 49(0), 149-194.

[7] Blanchard, Olivier Jean and Quah, Danny, "The Dynamic Effects of Aggregate Demand and Supply Disturbances," American Economic Review. September 1989. 79(4), 655-73.

[8] Boschen, John F. and Otrok, Christopher M., "Long-Run Neutrality and Superneutrality in an ARIMA Framework: Comment," American Economic Review, December 1994, 84(5), 1470-73.

[9] Chao, John C. and Phillips, Peter C.B., "Model Selection in Partially Nonstationary Vector Autoregressive Processes with Reduced Rank Structure," Journal of Econometrics, August 1999, 91, 227-71.

[10] Christiano, Lawrence J., Eichenbaum, Martin and Evans, Charles, "The Effects of Monetary Policy Shocks: Some Evidence From the Flow of Funds," National Bureau of Economic Research Working Paper: 4699 April 1994.

[11] Cochrane, John. "Shocks," National Bureau of Economic Research Working Paper: 4698 April 1994.

[12] Dornbusch, Rudiger, "Expectations and Exchange Rate Dynamics," Journal of Political Economy, December 1976, 84(6), 1161-76.

[13] Eichenbaum, Martin and Evans, Charles L., "Some Empirical Evidence on the Effects of Shocks to Monetary Policy on Exchange Rates" Quarterly Journal of Economics, November 1995, 110(4), 975-1009.

[14] Fama, Eugene F., "Forward and Spot Exchange Rates" Journal of Monetary Economics, November 1984, 14(3), 319-38.

[15] Fisher, Mark E. and Seater, John J., "Long-Run Neutrality and Superneutrality in an ARIMA Framework," American Economic Review, June 1993, 83, 402-415.

[16] Friedman, Milton. The Optimum Quantity of Money and Other Essays. Chicago: Aldine. 1969.

[17] ——— and Schwartz, Anna J., Monetary Trends in the United States and the United Kingdom. NBER, 1982.

[18] Froot, Kenneth A. and Frankel, Jeffrey A., "Forward Discount Bias: Is It an Exchange Risk Premium?" Quarterly Journal of Economics, February 1989, 104(1), 139-61.

[19] Geweke, John, "The Superneutrality of Money in the United States: an Interpretation of the Evidence," Econometrica, January 1986, 54, 1-21.

[20] Granger, Clive W.J., "Investigating Causal Relations by Econometric Models and Cross-Spectral Methods," Econometrica, July 1969, 37, 424-438.

[21] Hamilton, James D., Time Series Analysis, 1994, Princeton University Press.

[22] Haug, Alfred A. and Lucas, Robert F., "Long-Run Neutrality and Superneutrality in an ARIMA Framework: Comment," American Economic Review, September 1997, 87(4), 756-59.

[23] Jang, Kyungho and Ogaki, Masao, "The Effects of Monetary Policy Shocks on Exchange Rates: A Structural Vector Error Correction Model Approach" Working Paper No.01-02, Ohio State University, March 2001.

[24] Johansen, Soren, "Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models," Econometrica, November 1991, 59, 1151-1580.

[25] ———, "Determination of Cointegrating Rank in the Presence of a Linear Tread." Oxford Bulletin of Economics and Statistics, 1992, 54, 383-397.

[26] ———, Likelihood-Based Inference in Cointegrated Vector Autoregressive Models, Oxford University Press, 1995.

[27] Johansen, Soren, and Juselius, Katarina, "Testing Structural Hypotheses in a Multivariate Cointegration Analysis of the PPP and the UIP for the UK," Journal of Econometrics. July-Sept. 1992, 53(1-3): 211-44.

[28] King, Robert G., Plosser, Charles I., Stock, James H., and Watson, Mark W. "Stochastic Trends and Economic Fluctuations," American Economic Review, September 1991, 81, 819-840.

[29] King, Robert G. and Watson, Mark W., "Testing Long-Run Neutrality," Economic Quarterly, Summer 1997, 83, 69-101.

[30] Lucas, Robert E., Jr. "Econometric Testing of the Natural Rate Hypothesis." in The Econometrics of Price Determination Conference, ed. by Otto Eckstein. Washington: Federal Reserve Board, 1972.

[31] Muth, John F. "Rational Expectations and the Theory of Price Movements," Econometrica, July 1961, 29, 315-335.

[32] Newey, Whitney K. and West, Kenneth D., "A Simple, Positive Semi-definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix," Econometrica, May 1987, 55, 703-8.

[33] ———, "Automatic Lag Selection in Covariance Matrix Estimation," the Review of Economics Studies, October 1994, 61(4), 631-653.

[34] Ng, Serena and Perron, Pierre, "Lag Length Selection and the Construction of Unit Root Tests with Good Size and Power," Econometrica, November 2001, 69, 1519-1554.

[35] Phillips, Peter C.B. and Ploberger, Werner, "An Asymptotic Theory of Bayesian Inference for Time Series," Econometrica, March 1996, 64, 381-412.

[36] Sargent, Thomas J. "A Note on the 'Accelerationist' Controversy," Journal of Money, Credit, and Banking, August 1971, 3(3), 721-725.

[37] Serletis, Apostolos and Koustas, Zisimos, "International Evidence on the Neutrality of Money," Journal of Money, Credit, and Banking, February 1998, 30(1), 1-25.

[38] Sims, Christopher A. "Money, Income, and Causality," American Economic Review, September 1972, 62(4), 540-52.

[39] ———, "Macroeconomic and Reality," Econometrica, January 1980, 48, 1-48.

[40] Stock, James H. and Watson, Mark W, "Testing for Common Trends," Journal of the American Statistical Association, December 1988, 83, 1097-1107.

[41] Whittle, P, "On the Fitting of Multivariate Autoregressions, and the Approximate Canonical Factorization of a Spectral Density Matrix," Biometrika, 1963 (1/2), 129-134.