FIXED EXCHANGE RATES, MONETARY POLICY AND MACROECONOMIC INTERDEPENDENCE[1]

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Abstract

We revisit the issue of independent monetary policies and macroeconomic interdependence between two economies acting to fix their exchange rate. This paper builds upon the 'new open-economy macroeconomics' (NOEM) literature and can be seen as a fixed exchange rate variant of the seminal Obstfeld and Rogoff (1995) exchange rate dynamics paper. The paper constructs a fixed exchange rate model in the spirit of Mundell's two country model—see Mundell (1968, Chapter 18)—in which the money supply in each country is endogenously determined under perfect capital mobility following an attempt at a unilateral credit expansion. Unlike the traditional approach however, since the model is derived from intertemporal optimisation, we are able to analyse the long-run stock implications of the equilibrating reserve flows necessary to bring about the short-run equilibrium. As with its flexible exchange rate cousin, the welfare results in this fixed exchange rate model can be surprising, and stand in stark contrast to the implied spillovers in the early ad hoc models.

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1 Introduction

What are the linkages between economies fixing their exchange rate? Absent coordination between monetary policymakers, what are the spillovers across regions of independent policy choices within a fixed exchange rate regime? Until recently, issues of macroeconomic interdependence in general, and fixed exchange rates in particular, have relied upon variants of the workhorse Mundell-Fleming model. In the two-country context, this is spelled out most clearly in Mundell (1968), Mussa (1979) and Dornbusch (1980). However, advances in modeling techniques and the need to provide these earlier ad hoc models with fully articulated microfoundations has seen an outpouring of research in the sphere of open-economy macroeconomics following the lead of the closed-economy, new Keynesian approach exemplified by Blanchard and Kiyotaki (1987).

The seminal paper in this 'new open-economy macroeconomics' (NOEM) literature is the *Redux* paper of Obstfeld and Rogoff (1995). This contribution can be summarised as follows:

First, building on a monopolistically competitive market structure so as to provide a justification for explicit price setting behaviour and—under assumed one-period nominal rigidities and since prices will be set above marginal cost—demand determined output, monetary policy can have important welfare effects in the open economy as it pushes output away from the suboptimal decentralised equilibrium. This much is familiar from the closed-economy new Keynesian models.

Second, in the open-economy, with agents and the government obeying their intertemporal budget constraints in making choices through time, the accumulation of claims on the other region creates an important channel through which the stock implications of equilibrating flows in open economies can have long-lasting effects. Since Lucas' influential critique (Lucas, 1976) the importance of modeling individuals as acting in a forward looking and optimising manner is clear, and the NOEM achieves this in the same way as the intertemporal approach to the current account of the late 1970s and early 1980s, however without discarding the important earlier assumption of sticky prices.

Finally, with a model constructed from microeconomic foundations, the welfare effects of shocks can be explicitly considered. On the welfare front, within their special model, Obstfeld and Rogoff (1995) were able demonstrate how a monetary expansion and exchange rate depreciation could have a positive welfare spillovers. Tille (2001) generalises this result, focussing on the role of consuption substitutability, showing circumstances in which the positive welfare spillover need not obtain.

Since this early contribution, the NOEM has moved at a fast pace. One application has been to the issue of market segmentation and the possibility of pricing-to-market behaviour by firms. In particular, local currency pricing (LCP) by firms generates the empirically realistic scenario that there are significant deviations from the law of one price (LOOP) when the nominal exchange rate changes. And one strong implication of LCP behaviour is that the exchange rate need not play the expenditure switching role emphasised in the traditional

literature. A number of studies have therefore concluded that, absent expenditure switching due to market segmentation and LCP, a fixed exchange rate regime can be preferable to a float. However, recent NOEM models have been somewhat vague about the question of foreign exchange market intervention and the consequences for the central bank's reserves.

In this paper, a fixed exchange rate regime is developed which addresses the issue of central bank intervention. As in the traditional Mundell (1968) two-country fixed exchange rate model, the central banks simply commit to buying and selling foreign exchange at a given fixed rate. And as with Mundell, following an attempted monetary expansion by either of the countries there is a flow of reserves between the central banks making the actual money supplies endogenous. Unlike Mundell's model however, our intertemporal framework allows us to analyse the long run stock implications of these reserve flows.

Although, with the exception of the common currency in Europe, floating exchange rates have dominated the international monetary system since the collapse of Bretton-Woods, there are important reasons to examine the issue of macroeconomic interdependence in a fixed exchange rate world. For example, there has been constant unease with this floating experience. Despite the view of Friedman (1953) that speculation in foreign exchange markets under floating would be a stabilising force, the evidence from the last thirty years has been vastly different. The opinion that exchange rate behaviour cannot be explained by fundamentals alone has been popular. For example Frankel and Froot (1986, 1990) advanced this view for the final 20 percent or so of dollar appreciation up until 1985. Their perspective, based on the possible dominance of 'chartist' analysis in forex trading, is more closely associated with the view of the world during the interwar documented by Nurkse (1944).

The 1980s experience caused many commentators to review the efficacy of flexible exchange rates and call for a return to an international monetary system based on more stable exchange rates. To take Krugman's (1989, original emphasis) perspective:

> For most of my professional career, I believed that freely floating exchange rates represented the best system available... I have now changed my mind... I am now an advocate of an *eventual* return to a system of more or less fixed rates subject to discretionary adjustment.

A more recent perspective also demonstrates discontent with exchange rate behaviour, and has been associated with a move towards protectionism in the United States. This was summarised by former U.S. Treasury Secretary Paul O'Neill as he tried to explain to Congress early in 2002 what the dollar policy should be: "Since protectionist pressures in this country are mounting rapidly as a result of the overstrong dollar, we should be encouraging the markets to take the currency down."[2] Benign neglect has been causing real concern.

[2] As quoted in *The Observer*, Sunday 5 May 2002.

If the solution to these concerns is to move towards a system of more rigid rates of exchange, we should be prepared to dust off and refine our models of interdependence in a fixed exchange rate world which were so influential in the 1960s. Indeed, this paper's main contribution is that we utilise the current state-of-the-art in open economy macroeconomics to rigorously analyse the case of spillovers in a symmetric fixed exchange rate world. The paper can be thought of as complementing Obstfeld and Rogoff's (1995) *Redux* model and its generalisation provided by Tille (2001). We develop an apparatus for thinking about the symmetric fixed exchange rate regime within the modern literature on macroeconomic interdependence. The model takes explicit account of the reserve flows across countries committed to maintaining the fix in the manner of Mundell (1968). As per our discussion above, this not only fills a gap in the NOEM literature, but also carries potentially strong implications for thinking about future exchange rate arrangements.

The paper is organised as follows. The following section revisits Mundell's two-country model, providing some background for what follows. In Section 3 we set out the model, much of which will be familiar from the NOEM literature. Section 4 contains most of the original contribution of this model as it spells out the nature of monetary policy in the fixed exchange rate setting. Section 5 solves the model in relative per capita terms before in Section 6 a diagrammatic illustration of the model's properties is employed. Section 7 then solves for the per capita variables under the assumptions of nonsterilisation and sterilisation by the monetary authorities of the region not attempting to expand its monetary base. Finally, in Section 8 we consider the welfare spillovers in this model where it will be seen that the long run net asset implications of short run equilibrating reserve flows generate results which contrast with the flexible exchange rate literature on spillovers within the NOEM framework. Section 9 offers some concluding remarks and thoughts on future extensions and improvements to this work.

2 Mundell's Two Country Model

Robert Mundell presented his analysis of the effectiveness of monetary and fiscal policy under a fixed or a floating exchange rate regime in the early 1960s[3]. We consider here his results regarding international spillovers due to independent monetary policies in a two-country symmetric fixed exchange rate setting under

[3] Mundell (1968), Chapter 18—see especially the appendix where the two country case is set out. This model is also examined in Dornbusch (1980), Chapter 10. In this verbal sketch, we skim over the additional spillovers on country specific output through the trade balance in these models. Mundell (2001) provides an interesting discussion of the early development of the "Mundell-Fleming" model by himself and Marcus Fleming at the IMF in the early 1960s—although, as he points out, many of these ideas were already in place before he arrived at the Fund. In particular he notes how he extended his thinking to the two-country case following "a critical comment" published the year following his original 1963 small country model.

perfect capital mobility[4]. The fix is symmetric in the sense that each country can unilaterally pursue their own monetary policy by manipulating the domestic credit component of the central bank's balance sheet. However, ultimately the overall money supply is endogenously determined through changes in the central bank's reserves following financial market portfolio reallocation[5]. A key insight of this model in the two-country setting is that, unlike in the small country case where monetary policy is entirely ineffective as a stabilisation device, a monetary expansion by one country will result in a reduction in the world interest rate, an increase in demand, and an expansion in output worldwide.

Consider a two country world consisting of Home and Foreign. A Home monetary expansion involves open market operations whereby the central bank purchases domestic securities by printing money. This demand for Home securities reduces their return, causing an incipient capital outflow. This portfolio reallocation then puts pressure on the Home exchange rate to depreciate vis-a-vis Foreign. Under its obligation to fix the exchange rate the Home central bank is committed to meeting this demand for Foreign currency and in doing so runs down some of its reserves—in the process offsetting some of the expansion in the Home monetary base[6].

The portfolio reallocation occurs instantaneously bringing the return on Home and Foreign assets in line. Since the Home country is not a small player in the world there is only a partial offset of the monetary expansion at Home due to a reserve loss. Instead the monetary base both at Home and in Foreign expands, the interest rate falls worldwide, and the ensuing increase in aggregate demand results in a positive spillover due to the monetary expansion in a fixed exchange rate world—output expands both at Home and in Foreign.

Whilst this is a familiar enough story, there are two points worth making about the model which will prove important in what follows: First, the analysis is very much short term, as acknowledged by Mundell[7]. The model makes use of reserves *flows* as agents respond to arbitrage opportunities in a world

[4] Perfect capital mobility requires: (i) assets to be perfect substitutes and (ii) instantaneous adjustment.

[5] Mundell's model, as with this paper, ignores the problem associated with reserves held by the central banks being finite. Essentially, we assume the central bank's reserves are never exhausted. Since our model will rely on linearisations applicable only for "small" shocks, the assumption that reserves will not be exhausted seems innocuous.

[6] Actually, there is much implicit in this analysis: (i) To begin with we restrict the central bank's credit expansion to require the bank only purchase Home securities. In reality they can hold Foreign securities as part of their balance sheet. (ii) We are also supposing that the Home central bank purchases the securities off Home agents who then reallocate their portfolios to Foreign securities. In practise of course either Home or Foreign agents could initially hold the domestic securities, and during the credit expansion it is the holders of these original securitites—regardless of nationality—who undertake a portfolio reallocation. As we shall see, however, the assumption implicit in this discussion that these asset holders are Home residents facilitates comparison with the model developed later where initially the net asset position across the countries is assumed to be zero. In this case, following the monetary expansion, it is the Home residents who purchase Foreign securities.

[7] As Mundell (1968, p.271) writes: "It should also be apparent that the analysis is short-run in character and this neglects long-run consideration of changes in the capital stock and the level of indebtedness."

of perfect capital mobility to bring about the expansion in Foreign monetary base and subsequently output. However, it fails to take into account the *stock* implications of the portfolio reallocation, and in this respect fails to provide a suitable narrative of the long run impact of such portfolio adjustments by private agents. Second, the model relies upon the central bank controlling the monetary base through open market operations. This means the central bank purchases government bonds held by the private sector by printing money. In much of the NOEM literature however, since Ricardian equivalence holds, it is assumed for simplicity that the government does not issue bonds.

In terms of what follows, the first of these issues represents an opportunity: In modelling a fixed exchange rate world in a dynamic context can we account, as in the flexible exchange rate NOEM models, for both the short run and the long run due to the portfolio reallocation by the private sector? The second of these issues is a modeling challenge: Are we able to fully map the government's policy choices into a world where, due to Ricardian equivalence, we retain the assumption that government bonds are not issued? With these preliminaries in mind, we turn to the model itself.

3 The Model

In the following section preferences of the individual, the market structure of the economy and the individual's budget constraint are used to generate the first order conditions for optimisation.

3.1 Individual Preferences, the Budget Constraint and Market Structure

3.1.1 Utility, Consumption Preferences and Demand

There is a continuum of yeoman-farmers on the unit interval in the world economy. These individuals, indexed by $j \in [0,1]$, are divided into two countries, Home and Foreign, where $h \in [0,n]$ whilst $f \in (n,1]$. Each individual j's utility, shown in (1), is additively separable through time, increasing in consumption, $C(j)$, and real money balances, $\frac{M(j)}{P}$—which enter in logarithmic form—and decreasing in output, $y(j)$—which enters in a quadratic manner:

$$U_t(j) = \sum_{s=t}^{\infty} \beta^{s-t} \left\{ \log C_s(j) + \chi \log \frac{M_s(j)}{P_s} - \frac{\kappa}{2} y_s(j)^2 \right\} \tag{1}$$

An analogous utility function exists for a Foreign resident, with variables distinguished using an asterisk. β is the discount factor, common to both countries, which can alternatively be replaced by $\frac{1}{1+\delta}$ where $\delta > 0$ is the rate of time

preference. κ is a parameter which can be interpreted as a shift factor on the disutility of work effort, or an inverse indicator of productivity. We focus upon this specific utility function in order to dwell upon other aspects of the model—such as a varying intratemporal elasticity of substitution in consumption. The results are easily generalisable however.

$C_t(j)$ is a consumption index which aggregates the consumption of goods produced at Home and those produced by Foreign. We will allow for a very general form for this consumption index aggregator as shown in (2)

$$C_t(j) = \left[n^{\frac{1}{\rho}} C_{H,t}(j)^{\frac{\rho-1}{\rho}} + (1-n)^{\frac{1}{\rho}} C_{F,t}(j)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \tag{2}$$

where an identical expression exists for the consumption index of a foreign consumer. This CES aggregator allows $\rho > 0$ to vary reflecting the within-period substitution between the goods in consumption. When $\rho < 1$ the goods are complements in consumption. The $\rho = 1$ case is the Cobb-Douglas formulation made familiar in the NOEM literature by Corsetti and Pesenti (2001), whilst the case where $\rho > 1$ reflects the goods produced in each country being substitutes where the limit $\rho \to \infty$ obtains when the goods are perfect substitutes[8]. This form of consumption aggregator of course implies residents of Home and Foreign have identical preferences over goods produced in each country. Thus, it does not allow for the empirical realism of home bias in consumption or the existence of nontraded goods[9]. The coefficients on C_H and C_F in (2) will be seen to provide a convenient way, together with the assumptions below, of ensuring that the demand for each good is independent of the size of the country in which the yeoman-farmer resides.

We assume that the law of one price (LOOP) holds in this model such that each good will sell at the same price, regardless of the country in which it is being consumed, after its price is converted to any given currency. Since LOOP obtains, given our assumption of indentical preferences over consumption indexes, purchasing power parity (PPP) will hold through time such that the consumer price indexes will be equated when converted into a common currency. Furthermore, since we will not be considering the possibility of currency devaluation in this model, without loss of generality we set the nominal exchange rate to unity and consequently we can associate Home and Foreign goods prices as being identical in each country, $P_{H,t} = P^*_{H,t}$, $P_{F,t} = P^*_{F,t}$ and therefore $P_t = P^*_t$ $\forall\ t$. As such we will drop the asterisk from the Foreign price index henceforth. As usual, this does not imply the terms of trade are constant.

[8] In allowing for varying degrees of substitution between goods produced in each country our results are comparable to those attained in Tille (2001) for the flexible exchange rate case. In a more general framework, whether the goods are complements or substitutes would depend also upon the intertemporal elasticity of substitution which is set to unity here.

[9] We will also see that this specification also carries implications for price level stability under sterilisation by either of the monetary authorities. That is, sterilisation of the monetary base does not imply price level stability.

Given our assumptions, the consumption based price index for both Home and Foreign, is defined as the minimum nominal expenditure required to purchase one unit of the consumption index:

$$P_t(j) = \left[nP_{H,t}(j)^{1-\rho} + (1-n)P_{F,t}(j)^{1-\rho}\right]^{\frac{1}{1-\rho}} \tag{3}$$

The parameter ρ not only reflects the intratemporal elasticity of substitution, but also reflects the price-elasticity of demand for each countrywide consumption aggregator—e.g. $C_{H,t}$—when the price is measured with respect to the price index in (3).

The consumption indexes for each country's overall product themselves aggregate the consumption goods produced by the continuum of yeoman-farmers in each region. These aggregates, in the by now familiar manner in the macroeconomics literature, confer on each producer in the economy a degree of monopoly power in the production of their particular good. That is, the parameter $\theta > 1$ in below is an inverse indicator of monopoly power possessed by each consumer producer

$$C_{H,t}(j) = \left[n^{-\frac{1}{\theta}} \int_0^n c_t(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}} \qquad C_{F,t}(j) = \left[(1-n)^{-\frac{1}{\theta}} \int_n^1 c_t(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}} \tag{4}$$

As $\theta \to \infty$ the market structure becomes perfectly competitive. θ is also the price elasticity of demand for each good when its price is measured relative to the consumption based price index for the respective consumption index. The price indexes, defined in an analogous manner to that above, are given by

$$P_{H,t}(j) = \left[\frac{1}{n} \int_0^n p_t(i)^{1-\theta} di\right]^{\frac{1}{1-\theta}} \qquad P_{F,t}(j) = \left[\frac{1}{(1-n)} \int_n^1 p_t(i)^{1-\theta} di\right]^{\frac{1}{1-\theta}} \tag{5}$$

The coefficients in the consumption aggregators in (4) are convenient in that they have a counterpart in (2) which, as noted above, ensure that the demand for each individual good be independent of the size of the country in which the good is produced. Furthermore, from (5) we can associate the price index for the good produced in each country as being equal to the price set by any one yeoman-farmer in that region—within each region, prices will be identically set.

Box A

$$c_t(h) = \frac{1}{n}\left[\frac{p_t(h)}{P_{H,t}}\right]^{-\theta} C_{H,t} \qquad h \in [0,n]$$

$$c_t^*(h) = \frac{1}{n}\left[\frac{p_t(h)}{P_{H,t}}\right]^{-\theta} C_{H,t}^*$$

$$c_t(f) = \frac{1}{1-n}\left[\frac{p_t(f)}{P_{F,t}}\right]^{-\theta} C_{F,t} \qquad f \in (n,1]$$

$$c_t^*(f) = \frac{1}{1-n}\left[\frac{p_t(f)}{P_{F,t}}\right]^{-\theta} C_{F,t}^*$$

Using (4) we can calculate the demand for each individual Home good as a function of overall consumption of the Home aggregator, and likewise for Foreign, see Box A. By similar argument, using (2), the demand for the Home and Foreign goods as a function of overall consumption are calculated in Box B.

Box B

$$C_{H,t} = n\left[\frac{P_{H,t}}{P_t}\right]^{-\rho} C_t$$

$$C_{H,t}^* = n\left[\frac{P_{H,t}}{P_t}\right]^{-\rho} C_t^*$$

$$C_{F,t} = (1-n)\left[\frac{P_{F,t}}{P_t}\right]^{-\rho} C_t$$

$$C_{F,t}^* = (1-n)\left[\frac{P_{F,t}}{P_t}\right]^{-\rho} C_t^*$$

Combining the equations in Boxes A and B, and aggregating over all individuals we get the following demand functions faced by each yeoman-former in each region

$$y_t(h) = \left[\frac{p_t(h)}{P_{H,t}}\right]^{-\theta}\left[\frac{P_{H,t}}{P_t}\right]^{-\rho} C_t^w \qquad y_t^*(f) = \left[\frac{p_t(f)}{P_{F,t}}\right]^{-\theta}\left[\frac{P_{F,t}}{P_t}\right]^{-\rho} C_t^w \qquad (6)$$

where $C_t^w = nC_t + (1-n)C_t^*$. Each yeoman-farmer takes into account the above demand functions when price setting. However given that there are infinitely many producers in this economy, each takes the overall price index as given—individually, the yeoman-farmer is too small to affect the overall indexes.

3.1.2 The Individual's Budget Constraint

There is a nominal bond, denominated in either currency, available in this economy—denoted B_t. Holdings of the bond will reflect the net asset position between the two regions in that the bond will be in zero net supply. At each moment individuals carry forward bonds and money—$(1+i_t)B_t(j) + M_{t-1}(j)$—to which they add the nominal earnings on their product—$p_t(j)y_t(j)$. Out of this the individual must make a consumption decision, pay taxes (or accept a transfer) and carry forward a new quantity of money and bonds—$M_t(j) + B_{t+1}(j)$.

$$M_t(j) + B_{t+1}(j) \leq (1+i_t)B_t(j) + M_{t-1}(j) + p_t(j)y_t(j) - P_tC_t(j) - P_tT_t(j) \tag{7}$$

3.1.3 First Order Conditions

Solving for each individual's optimisation by maximising (1) subject to (6) and (7) gives us the usual first order conditions. That is, rearranging the demand equations (6) in $p(j)$ for $j \in [0,n]$ and substituting into the budget constraint (7) we can obtain an expression for $C_t(j)$ which, when used in the utility function (1), gives us an unconstrained optimisation problem. The solution to which yields for Home, and by eqivalent argument for Foreign, the following FOCs:

Box C: First order conditions

$$C_{t+1}(h) = \beta(1+r_{t+1})C_t(h) \qquad C^*_{t+1}(f) = \beta(1+r_{t+1})C^*_t(f)$$

$$\frac{M_t(h)}{P_t} = \left[\frac{\chi(1+i_{t+1})}{i_{t+1}}\right] C_t(h) \qquad \frac{M^*_t(f)}{P_t} = \left[\frac{\chi^*(1+i_{t+1})}{i_{t+1}}\right] C^*_t(f)$$

$$y_t(h) = \left(\frac{\theta-1}{\kappa\theta}\right)\frac{p_t(h)}{P_t}C_t^{-1}(h) \qquad y^*_t(f) = \left(\frac{\theta-1}{\kappa^*\theta}\right)\frac{p^*_t(f)}{P_t}C_t^{*-1}(f)$$

These equations have the usual interpretations. The first are the consumption Euler equations which reflects the optimal intertemporal allocation

of consumption for each country. The next line in Box C gives the money demand equations which illustrate how, with this money-in-the-utility-function approach, the demand for real balances is decreasing in the nominal interest rate—the opportunity cost of holding money—and increasing in real consumption. The final expressions are the labour-leisure trade-offs. Since individuals derive disutility from putting forth effort for production purposes, as consumption increases output in the economy decreases as people substitute from consumption into leisure—a normal good. Output increases with the price of the product since output can then provide for more consumption and therefore utility. Rearranging and using (6), they also demonstrate how the price of each good is set as a mark up over the ratio of the marginal disutility to labour effort divided by the marginal utility of consumption.

4 Monetary Policy and the Symmetric Fixed Exchange Rate

As discussed above, in the Mundell story holdings of government bonds by the monetary authority constituted the domestic credit component of the central bank balance sheet. It was through open market operations to alter the central banks holdings of government securities that domestic credit was affected, and therefore the monetary base in the economy expanded or contracted. However, in this class of deterministic NOEM models, Ricardian neutrality holds—and the literature conveniently assumes that governments do not issue bonds.

Whilst one approach to resolve this problem would be to analyse a model in which government debt matters—for example by developing an overlapping generations structure—in the first instance a useful exercise would be to see whether a symmetric fixed exchange rate model can still be developed within the simpler framework in which bonds are not issued. There are two reasons for this: (i) Such a model would be much more tractable, and hence would avoid many further technical details and (ii) this model would then allow a more direct comparison with the first generation of the 'new open economy macroeconomics' literature. It is to such a setting we now turn our attention[10].

4.1 Definition of the Monetary Base

We assume that the monetary authorities in both regions hold some reserves of the other regions' currency—which they surely will in order to meet the obligation to fix the exchange rate—such that if M_t' represents the total amount of domestic money issued by the Home central bank in per capita terms, and if the Foreign central bank holds the amount of domestic currency R_t^*, again in (Foreign) per capita terms, then total domestic currency in Home private agents' hands is by definition (and likewise for Foreign)

[10] Henceforth we drop the notation denoting the individual j.

$$M_t = M'_t - \left(\frac{1-n}{n}\right) R^*_t \qquad\qquad M^*_t = M'^*_t - \left(\frac{n}{1-n}\right) R_t \qquad (8)$$

Thus the per capita holdings of Home (Foreign) currency by domestic (Foreign) residents is the total issue of that currency minus the holdings of Home (Foreign) currency by the other central bank.

We now make use of the government budget constraint to consider the policy options available within this framework.

4.2 The Government's Budget Constraint

We deal throughout with the budget constraint of the fiscal and monetary policymakers together. This consolidated constraint relates the change in money supply in any one period to nominal taxes and the change in reserve holdings by the Home central bank. It is written as

$$R_t - R_{t-1} - P_t T_t = M'_t - M'_{t-1} \qquad (9)$$

An analogous expression holds for the Foreign central bank. Ignoring reserves in this expression we observe how this just gives the usual budget constraint whereby a monetary expansion results in a transfer (negative tax) to Home individuals[11]. We use (9) to demonstrate the policy options available to the consolidated government.

Suppose there is an increase in Home reserves though no change in taxes in a given period. The reserve increase might be due to Home residents presenting Foreign exchange at the central bank which they wish to exchange for Home currency. The central bank must, under its obligation to fix the exchange rate, clearly exchange Foreign for Home currency. For the case where there is no tax change, this budget constraint demonstrates how this increase in R_t must by definition be met through increase in money issued by the Home central bank. Thus, the increase in reserves is *unsterilised* in the sense that the central bank allows this to occur through an increase in the monetary base. Whether this actually increases the holdings of Home currency by private individuals depends on the actions of the Foreign central bank also. However, through its accumulation of reserves, there will be an increase in the currency issued by the Home central bank.

There is another policy option however. Once again suppose the Foreign central bank remains inactive under the same inflow of reserves described. The Home central bank can *sterilise* the force acting towards the expansion in the

[11] See, for example, Obsfeld and Rogoff (1996, p.664).

Home monetary base. Consider, for example, the central bank after it meets the presentation of Foreign currency by private individuals—as it is obliged—by exchanging it for the Home currency. The consolidated government also has tax raising powers, and in order to sterilise the expansionary effect of this on the monetary base the central bank can increase taxes to offset this increase. This is equivalent to the consolidated government meeting this demand at the doors of the central bank for Home currency, though simultaneously allowing the taxman to extract currency holdings from these individuals in an equal and offsetting manner.

The consolidated government budget constraint therefore demonstrates the range of policy options—sterilisation or nonsterilisation—open to governments as being available in this setting, even without the existence of government bonds. Consider the following identity familiar from the central bank's balance sheet

$$M'_t \equiv D_t + R_t \qquad\qquad M'^{*}_t \equiv D^*_t + R^*_t \tag{10}$$

The issue of currency by the Home central bank is equal to the sum of 'domestic credit'—which in usual terms this represents the securities purchased by the central bank and sitting on the asset side of the bank's balance sheet—and the holdings of reserves. Consider now expression (9) above rearranged as

$$M'_t = [M'_{t-1} - R_{t-1} - P_t T_t] + R_t$$

This provides a useful comparison with the representation of the central bank's balance sheet in (10). The expression in square brackets plays, in some sense, the role of 'domestic credit'. An issue of money by the Home central bank can either be used to purchase reserves of the other currency or can be injected into the economy by means of a tax cut which is equivalent, in this model, to a 'domestic credit' expansion.

In the model we will be interested in the government acting to control the variable D_t, given above in (10), through time. This will be the exogenous policy instrument, and can be interpreted as a tax cut for the case of a domestic credit expansion, or a tax increase for a credit contraction. However, as in Mundell (1968), the reserves held by the central bank and the overall money supply will be endogenously determined[12]. That is, given the fixed exchange rate the

[12] Mundell (2001) discusses the development of thinking about monetary policy in the open economy. In his early, flexible exchange rate papers—for example Mundell (1961)—monetary policy was associated with varying interest rates. But as Mundell (2001, p.224) recalls: "Later, however, when I made the assumption of perfect capital mobility, monetary policy had to be redefined and was correctly treated as an open market operation, or change in domestic credit. The money supply is an endogenous variable under fixed exchange rates."

government stands prepared to buy or sell foreign currency at a preannounced price. Thus, even after acting upon the domestic credit term as given here, the actual overall money supply in private hands will be determined by the net effect after R_t and R_t^* have been endogenously determined.

Together, (8) and (10), along with its Foreign analogue, imply that the per capita holdings of currency by the respective agents of the Home and Foreign countries is

$$M_t = D_t + R_t - \left(\frac{1-n}{n}\right) R_t^* \qquad\qquad M_t^* = D_t^* + R_t^* - \left(\frac{n}{1-n}\right) R_t \qquad (11)$$

These identities imply that domestic currency in Home private agents' hands is equal to the domestic credit term, D_t, plus the difference between the reserves of Foreign currency held by the Home central bank, R_t, and those holdings by the Foreign central bank of domestic currency reserves weighted to reflect per capita terms, R_t^*. Likewise for Foreign. From these expressions we observe how the act of satisfying a private demand to exchange currecies has a symmetric effect on money supplies. Consider the Home central bank as conducting an expansionary monetary policy by injecting Home currency into the economy through D_t—a transfer to private hands through their budget constraint (7). If domestic residents are not content to hold this currency, equilibrium in the money market cannot be achieved in the usual way under flexible exchange rates by a depreciation of the domestic currency, and instead individuals will attempt to substitute away from holding this currency. In this model therefore, under the obligation of the central bank to fix the exchange rate, individuals can exchange their currency one-for-one for Foreign at (either) central bank. Thus, as part of the fixed exchange rate arrangement either the Home central bank supplies this Foreign currency by running down its reserves, R_t, or the Foreign central bank accumulates Home currency, R_t^*. Either way, the act of fixing the exchange rate by either central bank not only offsets somewhat the total holdings of Home currency in private hands, M_t, but also, absent sterilisation, simultaneously expands the holdings of Foreign currency in the hands of Foreign private agents[13].

[13] A natural question at this point, and one to which we return below, is: If Home private agents are exchanging this Home currency for Foreign, how is it we are cetain this currency finds itself in the hands of Foreign agents? The answer is that since Foreign currency does not enter the utility function of Home residents, these agents do not wish to hold this currency—which is dominated by any interest bearing assets. In this sense, a domestic credit expansion by the Home central bank will, say, result in an incipient outflow of reserves as domestic agents, in an analogous manner to that described by Mundell (1968), exchange Home for Foreign currency in order to alter their portfolio holdings in favour of Foreign interest bearing assets. This will have the simultaneous symmetric effect of expanding the Foreign monetary base. We return to this later when we consider the balance of payments equations in this model.

5 Solving the Model

In this section we solve the model by employing the technique introduced by Obstfeld and Rogoff (1995). That is, since in general this class of models does not yield an explicit closed form solution, we solve for an unanticipated shock to the policy variables—domestic credit as controlled by the central banks—about an initial symmetric steady-state. The most unique feature of this model is the symmetric fix. Thus, we spell out the solution to this lineariseation in close detail. The more familiar aspects of this section are relegated to an appendix.

5.1 Linearised Money Demand

Into the money demand equations given in Box C we use our expression for the individual's holdings of nominal money (11):

$$\frac{D_t + R_t - \left(\frac{1-n}{n}\right) R_t^*}{P_t} = \left[\frac{\chi(1+i_{t+1})}{i_{t+1}}\right] C_t$$

$$\frac{D_t^* + R_t^* - \left(\frac{n}{1-n}\right) R_t}{P_t} = \left[\frac{\chi(1+i_{t+1})}{i_{t+1}}\right] C_t^*$$

The symmetric steady-state about which we linearise, due to the additional complications in the fixed exchange rate model, will be slightly more restrictive than the flexible exchange rate version where the zero net asset assumption is utilised. The initial symmetric steady-state for this model will be defined as that which obtains when (i) $R_0 = \left(\frac{1-n}{n}\right) R_0^*$, (ii) $D_0 = \left(\frac{1-n}{n}\right) D_0^*$ and (iii) $B_0 = 0$. Together, (i) and (ii) imply that the initial per capita domestic credit to reserves ratio of the central banks are identical, or that $M_0 = D_0$ and $M_0^* = D_0^*$. Alternatively, (i) suggests that the total reserves of a small economy is identical to the total reserves of a large economy. Whilst this does not seem entirely realistic, we utilise this assumption without apology since it will allow us to solve the model.

Given condition (i), we can log-linearise[14] the above money demand equations about the initial steady state:

[14] Substituting out for the nominal interest rate after linearising the Fisher equation, $1 + i_{t+1} = \frac{P_{t+1}}{P_t}(1 + r_{t+1})$.

$$\frac{dD_t}{D_0} + \frac{dR_t}{D_0} - \left(\frac{1-n}{n}\right)\frac{dR_t^*}{D_0} = -\frac{1}{\delta}\left[\frac{dP_{t+1}}{P_0} - (1+\delta)\frac{dP_t}{P_0}\right] - \frac{1}{1+\delta}\frac{dr_{t+1}}{\delta} + \frac{dC_t}{C_0}$$

$$\frac{dD_t^*}{D_0^*} + \frac{dR_t^*}{D_0^*} - \left(\frac{n}{1-n}\right)\frac{dR_t}{D_0^*} = -\frac{1}{\delta}\left[\frac{dP_{t+1}}{P_0} - (1+\delta)\frac{dP_t}{P_0}\right] - \frac{1}{1+\delta}\frac{dr_{t+1}}{\delta} + \frac{dC_t^*}{C_0}$$

Next, given (ii) we can conveniently rewrite these equations as

$$d + \psi - \psi^* = -\frac{1}{\delta}\left[\overline{p} - (1+\delta)p\right] - \frac{r}{1+\delta} + c$$

$$d^* + \psi^* - \psi = -\frac{1}{\delta}\left[\overline{p} - (1+\delta)p\right] - \frac{r}{1+\delta} + c^*$$

where $d = \frac{dD_t}{D_0}$, $d^* = \frac{dD_t^*}{D_0^*}$, $\psi = \frac{dR_t}{D_0}$, $\psi^* = \frac{dR_t^*}{D_0^*}$, and $\overline{r} = \frac{dr_{t+1}}{\delta}$ etc. such that we follow the convention that lower case letters reflect deviations from their symmetric steady state and furthermore those lower case letters without an overbar are the initial period deviation, while those with an overbar the long run deviation. Note lower case variants of most upper case variables are used except in the case of reserves—R and R^*—to which we assign the Greek symbols ψ and ψ^* to avoid confusion with the real interest rate. As in Obstfeld and Rogoff (1995) ,the economy reaches its long run steady state in the period immediately after the shock, and consequently variables with an overbar count as the second period and long run deviations from the initial symmetric steady-state.

Subtracting the second from the first of these equations gives

$$\psi - \psi^* = \frac{1}{2}(c - c^*) - \frac{1}{2}(d - d^*) \tag{12}$$

Equation (12) demonstrates how an increase in relative domestic credit, $d - d^* > 0$, can be accommodated for through two channels: Either as part of the increase in relative consumption and therefore relative money demand which might occur, given one-period rigidities, because of the monetary expansion or

through the residents running down the relative reserve position of the central banks, $\psi - \psi^* < 0$.

In practise both these effects take place. Relative consumption will increase if, given nominal rigidities, output expands following the monetary expansion. At the same time, the relative reserve position will change as individuals bring about money market equilibrium by reducing their money holdings. Whilst a foray to the central bank to exchange unwanted money balances is an important channel which brings about money market equilibrium, this is not the end of the story. We now turn to the question of what happens to these reserves.

5.2 The Balance of Payments

As outlined above, when individuals are not prepared to hold any amount of a domestic credit expansion, they have an outlet through which they can reduce their holdings of domestic currency—by presenting it at the central bank and exchanging it for the currency of the other region. However, as Foreign money does not enter into the Home individual's utility function and vice versa, there is no incentive for the individual to hold this currency either—especially since this money is strictly dominated by interest bearing assets. Consequently, Home individuals have the option of using their excess liquidity to purchase bonds. Consolidating the budget constraints (7) and (9), and using (8) we get the balance of payments equation for the Home economy. An analogous expression exists for Foreign. This expression gives the short run accumulation of assets as the difference between the current account and changes in reserves in each region, or alternatively

$$B_{t+1} = (1+i_t)B_t + p_t(h)y_t(h) - P_tC_t - \left[(R_t - R_{t-1}) - \left(\frac{1-n}{n}\right)(R_t^* - R_{t-1}^*)\right] \tag{13}$$

Observe how, beginning at the initial symmetric steady state defined above, the current $(P_HY_H - PC)$ and capital $(-B)$ accounts of the private sector, when they do not sum to zero, must be offset by a change in the official reserve holdings of the central bank $(R - R_0) - \left(\frac{1-n}{n}\right)(R_t^* - R_{t-1}^*)$. Of course bonds are in zero net supply such that $\int_0^n B(j)dj + \int_n^1 B^*(j)dj = nB + (1-n)B^* = 0$ or $B^* = -\left(\frac{n}{1-n}\right)B$. Linearising (13) and its Foreign analogue about our initial symmetric steady state and subtracting we get:

$$\frac{b}{1-n} = (y_H - y_F) - (c - c^*) - 2(\widetilde{\psi} - \widetilde{\psi}^*)$$

where $\widetilde{\psi} = \frac{dR_t}{P_0 C_0}$ and $\widetilde{\psi}^* = \frac{dR_t^*}{P_0 C_0^*}$ to distinguish from above where $\psi = \frac{dR_t}{D_0}$ and $\psi^* = \frac{dR_t^*}{D_0^*}$. In order to make the balance of payments equations compatible with (12) it is necessary to spell out the relation between the steady state about which has been linearised. For this reason, consider the steady state money demand equations given our initial conditions, rearranged as:

$$D_0 = \left[\frac{\chi(1+\delta)}{\delta}\right] P_0 C_0 \qquad\qquad D_0^* = \left[\frac{\chi^*(1+\delta)}{\delta}\right] P_0 C_0^*$$

where if we further assume that $\chi = \chi^*$, and write $\left[\frac{\chi(1+\delta)}{\delta}\right] = v$—a measure of 'inverse velocity'—and make use of the fact that within a fixed exchange rate regime there is no expenditure switching effect such that $(y_H - y_F) = 0$, we can use these expressions to substitute in above as:

$$\frac{b}{1-n} = -(c - c^*) - 2v(\psi - \psi^*) \tag{14}$$

This is now in terms of the deviations of reserves from the inital steady state comparable with those in our linearised money demand equations. (14) demonstrates how the net asset position of the countries is given by two terms. The first is the relative consumptions of the two regions, and represents the current account channel in action. The second is the relative reserve position. Thus in this model, unlike in the flexible exchange rate models of, for example, Obstfeld and Rogoff (1995), the net asset position following a shock does not reflect the counterpart to the initial current account position alone, but also the portfolio reallocation due to reserve flows. This represents an important departure from the flexible exchange rate model which we will consider in more detail later—and is an important improvement on the ad hoc macro models of the 1960s which failed to spell out explicitly the constraints faced by the economies and therefore the stock implications of equilibrating reserve flows.

5.3 Connecting the Short- with the Long-Run

In order to solve the model we require an equation describing the long run equilibrium of the model and an equation connecting this long run equilibrium to the impact effect of the shock as derived above. The former is obtained by linearising both the labour leisure trade-off equations above and long run

equilibrium conditions[15], while the latter comes from the linearised consumption Euler equations. Together, after some manipulation, we are able to write

$$\overline{c} - \overline{c}^* = \left[\frac{\delta(1+\rho)}{2\rho(1-n)}\right]\overline{b} \tag{15}$$

This equation represents the fact that in the long run output is endogenous in this model, and consequently for $\overline{b} > 0$ relative consumption would increase by less when the goods are substitutes, $\rho > 1$, as individuals substitute towards leisure. As usual, the impact effect on all relative variables is equal to the long run value of their relative deviations from the symmetric steady state such that $\psi - \psi^* = \overline{\psi} - \overline{\psi}^*$ and $c - c^* = \overline{c} - \overline{c}^*$ and also $b = \overline{b}$. We are now in a position to think about the effect of a credit shock in this model.

6 Diagrammatic Analysis of a Relative Credit Shock

In order to provide some intuition for the results which will follow, we will proceed by providing a diagrammatic analysis of a relative credit shock hitting the economy. Equations (15), (16) and (17) represent three equations in three unknowns plus an exogenous forcing variable, the relative credit shock. Combining these three equations we can write two equations as a function of relative per capita reserve positions of the central banks, relative consumption and the relative credit components of the monetary base

$$\psi - \psi^* = -\left[\frac{2\rho + \delta(1+\rho)}{2v\delta(1+\rho)}\right](c - c^*) \tag{AA}$$

$$\psi - \psi^* = \frac{1}{2}(c - c^*) - \frac{1}{2}(d - d^*) \tag{MM}$$

These two equations are depicted in Figure 1. In the pre-shock equilibrium each of the lines in $(\psi - \psi^*)$, $(c - c^*)$ space cut through the origin. Relationship

[15] The steady state response to is reached in the period following the shock. Consequently, the long run steady state conditions are, with overbars representing the long run, $\overline{PC} = \overline{p}(h)\overline{y}(h) + \delta\overline{B}$ and $\overline{PC}^* = \overline{p}(f)\overline{y}(f) - \delta\left(\frac{n}{1-n}\right)\overline{B}$ such that nominal consumption must equal real income, and there are no reserve flows. Also, the labour-leisure trade-offs must be combined with the demand for each good given in (10) to generate (15).

AA gives the locus of points along which relative reserves and consumptions bring about a balance of payments equilibrium consistent with forward looking behaviour. The locus of points given by MM reflects the combinations of $(\psi - \psi^*)$ and $(c - c^*)$ where there obtains equilibrium in the money market, given $(d - d^*)$. The greater the relative consumption position, the larger the relative demand for money, and hence the relative reserve position between the central banks must increase in order to increase the relative money supplies and bring about money market equilibrium. MM is also a function of the relative credit shock. For a positive shock to $(d - d^*)$, Home individuals not willing to hold all the increase in relative credit run down reserves at the central bank—it is this immediate portfolio reallocation, and implicitly capital account deficit, has important long run effects in this model.

This figure allows us to think about the role of v in influencing the results in this model. v, which can be thought of as the inverse of the velocity of money circulation, affects the slope of AA. As the velocity of circulation decreases, and v becomes larger, the slope of the AA schedule in absolute terms decreases. And following a relative credit expansion in the model, there is a larger relative consumption and smaller relative reserve response. Since the velocity of money circulation is smaller, a larger amount of any monetary expansion is required to achieve money market equilibrium. Consequently, reserves are seen to flow less.

It is interesting also to consider how, in justifying focussing upon the real components of the utility function in performing welfare analyses, other studies have suggested that χ might be considered arbitrarily small. This model demonstrates that this is not necessarily a good assumption since χ—via v—plays an important role in affecting the real variables and utility through the money demand equation in this fixed exchange rate model. It is, of course, still useful to focus upon the real components of the utility function when thinking about welfare, and can be justfied by arguing that money is simply being a facilitator of consumption. The point is simply this: it might not be justified to make arbitrary assumptions about the size of the parameter χ in a fixed exchange rate context since, in this model at least, χ itself is an important determinant of the real variables. Finally, Figure 1 is directly comparable with the flexible exchange rate version of this model presented in Obstfeld and Rogoff (1996) where instead of the relative reserve position the nominal exchange rate features on the vertical axis[16].

Solving these equations we can calculate the response of relative per capita variables to the unanticipated relative credit shock

$$\overline{\psi} - \overline{\psi}^* = \psi - \psi^* = -\frac{2\rho + \delta(1+\rho)}{4\rho + 2\delta(1+\rho)(1+v)}(d - d^*) \tag{16}$$

[16] See Obstfeld and Rogoff (1996), Figure 10.2, page 680.

$$\overline{c} - \overline{c}^* = c - c^* = \frac{v\delta(1+\rho)}{2\rho + \delta(1+\rho)(1+v)}(d - d^*) \tag{17}$$

$$\overline{b} = b = \frac{2v\delta(1-n)}{2\rho + \delta(1+\rho)(1+v)}(d - d^*) \tag{18}$$

In the usual way, we can use these results to calculate the impact and long-run effects on country variables. In order to focus the remainder of the discussion however, henceforth we will restrict our attention to the case of an unexpected, permanent credit expansion in the Home region only. Therefore we focus upon the case where $d = \overline{d} > 0$ but $d^* = \overline{d}^* = 0$. Later we will consider the case of sterilisation by the Foreign monetary authority, whereby $d^* = \overline{d}^* < 0$ and is adjusted so as to sterlise completely the effect on the Foreign monetary base of the Home monetary expansion.

7 Spillovers of the Domestic Credit Shock

In this section we report the per capita effect on country specific variables of the domestic monetary expansion in both countries in both the short and long run. We consider in turn the cases of sterilisation and nonsterilisation. We begin with the former.

7.1 Nonsterilisation

Table 1 reports the impact and long run effect on country per capita variables. Following the credit expansion in the Home country, the real interest rate declines. That is, the expected jump in the future price level reduces the real interest rate and causes individuals to reduce their saving and increase consumption on impact[17]. This is common to both countries. However, the consumption response is not identical on impact. As Table 1 reports, although the per capita output response is identical in each region—given the fixed exchange rate which does not bring about a change in relative prices—the relative per capita consumption response, as we have seen, is positive. This is entirely due to the forward looking nature of consumption choices in this model. As the previous section highlighted, due to the fixed exchange rate regime in place monetary equilibrium is achieved by running down the relative reserve position

[17] In common with the flexible exchange rate class of models, there is no liquidity effect in this model—that is, no nominal interest rate response to monetary policy changes. This can be demonstarted by linearising the Fisher equation, see footnote 13, and substituting in from Table 1.

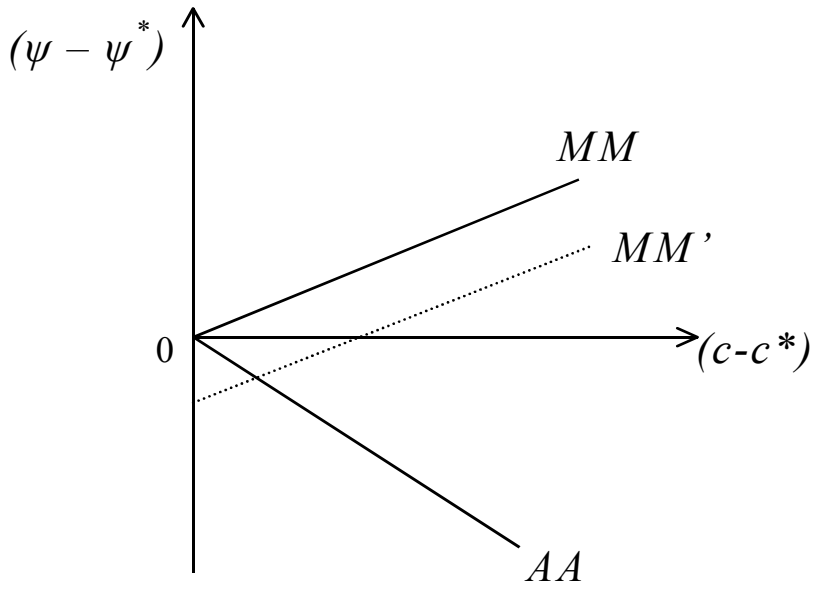

Figure 1: A relative Home credit expansion shifts down MM—the locus of points depicting equilibrium in the money market. The new intersection with AA demonstrates the relative reserve change required to maintain the fixed exchange rate and also illustrates the positive relative consumption response due to the policy shock.

of the Home central bank. These reserves are used to purchase assets—claims on future Foreign output—and goods—current Foreign output—and is the process through which the Foreign monetary base is itself expanded in the short run. As intertemporal optimisers, Home (and Foreign) agents are aware of the future flow of resources due to their portfolio reallocation. Consequently, consumption smoothing drives a wedge between the country specific per capita consumption response—resulting in a current account deficit at Home and a surplus abroad. As evidence of this consumption smoothing mechanism, consider how the Home deficit on current account, $y - c$, is identical to the long run deviation of Home consumption from the inital steady state, and likewise the current surplus in the Foreign region is identical to their long run consumption decrease.

In other words, due to forward looking agents, although the output response in each region is identical following the monetary expansion, microfoundations demonstrate the role of the resulting net asset position in bringing about a differential consumption response in this model—something which cannot be asserted using the ad hoc fixed exchange rate models.

Table 1 also demonstrates the long run effect of the Home monetary expansion. The equilibrating short run reserve flows and concomitant positive net asset position (capital account deficit) immediately following the shock mean that Home receives a flow of resources through time with which they increase their long run consumption and substitute away from production—leisure is a normal good. The reverse happens in the Foreign region. Being obliged to transfer resources to Home, they reduce their long run consumption and increase production relative to the steady state. Since prices can change, Table 1 reports the increase in the price level and also the terms of trade reponse—the Home good becomes relatively more expensive in the long run.

Table 1: Home Monetary Expansion Under Fixed Exchange Rates (No Sterilisation)

Impact Effect

$$r = -\left(\frac{1+\delta}{\delta}\right)\left[\frac{2\rho+\delta(1+\rho)(1+2nv)}{\Theta}\right]d < 0$$

$$c = \left[\frac{2\rho+\delta(1+\rho)(1+2v)}{\Theta}\right]d = m = \overline{m} > 0$$

$$c^* = \left[\frac{2\rho+\delta(1+\rho)}{\Theta}\right]d = m^* = \overline{m}^* > 0$$

$$y = y^* = \left[\frac{2\rho+\delta(1+\rho)(1+2nv)}{\Theta}\right]d > 0$$

$$b = \left[\frac{(1-n)4v\rho}{\Theta}\right]d > 0 \qquad \psi - \psi^* = -\left[\frac{2\rho+\delta(1+\rho)}{\Theta}\right]d < 0$$

Long Run Effect

$$\overline{c} = \left[\frac{(1-n)2v\delta(1+\rho)}{\Theta}\right]d > 0 \qquad \overline{c}^* = -\left[\frac{2nv\delta(1+\rho)}{\Theta}\right]d < 0$$

$$\overline{y} = -\left[\frac{(1-n)2v\delta\rho}{\Theta}\right]d < 0 \qquad \overline{y}^* = \left[\frac{2nv\delta\rho}{\Theta}\right]d > 0$$

$$\overline{p}_H - \overline{p}_F = \left[\frac{2v\delta}{\Theta}\right]d > 0 \qquad \overline{p} = \left[\frac{2\rho+(1+\rho)(1+2nv)}{\Theta}\right]d > 0$$

where $\Theta = 4\rho + 2\delta(1+\rho)(1+v)$.

One feature of the model which it would be interesting to account for is the country specific reserve response. Table 1 reports the relative per capita reserve position in each country, which decreases as we would expect. Unfortunately, since this model is silent on the issue of precisely who intervenes, we are unable to make this transparent. This is clearly an unsatisfactory feature of the model and deserves further attention.

7.2 Sterilisation

The results under the case of Foreign sterilisation of the effect on its monetary base are reported in Table 2. Sterilisation here is defined as the act of the

Foreign monetary authority altering credit to leave the overall Foreign money supply unchanged. Note, the denominator in these expressions differs from that above. Here, $\Omega < \Theta$. As a result of the sterilisation, there is no increase in Foreign per capita consumption on impact—sterilising the monetary base means money market equilibrium, given no liquidity effect, is achieved with no change in Foreign consumption. However, Home continues to enjoy an increase in consumption on impact. Infact, the increase in Home consumption, Home and Foreign output, along with the decrease in the real interest rate, are all greater than in our previous case of no sterilisation. Here there is no increase in the Foreign monetary base and therefore no increase in Foreign output through the Foreign consumption channel. Instead, the increase in Foreign output comes about entirely through the demand from the Home region.

The intuition for these results is that the Foreign credit contraction serves to augment the credit expansion by the Home authority in terms of the net asset position of the Home residents—which is even larger due to the sterilisation. Thus, Home consumption in a forward looking manner, increases by more on impact which itself requires a larger fall in the real interest rate and a larger increase in Home and Foreign output on impact.

Table 2: Home Monetary Expansion Under Fixed Exchange Rates (Sterilisation)

Impact Effect

$$r = -\left(\frac{1+\delta}{\delta}\right)\left[\frac{2n\delta v(1+\rho)}{\Omega}\right] d < 0$$

$$c = \left[\frac{2\delta v(1+\rho)}{\Omega}\right] d = m = \overline{m} > 0$$

$$c^* = 0 = m^* = \overline{m}^*$$

$$y = y^* = \left[\frac{2n\delta v(1+\rho)}{\Omega}\right] d > 0$$

$$b = \left[\frac{(1-n)4v\rho}{\Omega}\right] d > 0 \qquad d^* = \psi - \psi^* = -\left[\frac{2\rho + \delta(1+\rho)}{\Omega}\right] d < 0$$

Long Run Effect

$$\overline{c} = \left[\frac{(1-n)2v\delta(1+\rho)}{\Omega}\right] d > 0 \qquad \overline{c}^* = -\left[\frac{2nv\delta(1+\rho)}{\Omega}\right] d < 0$$

$$\overline{y} = -\left[\frac{(1-n)2v\delta\rho}{\Omega}\right] d < 0 \qquad \overline{y}^* = \left[\frac{2nv\delta\rho}{\Omega}\right] d > 0$$

$$\overline{p}_H - \overline{p}_F = \left[\frac{2v\delta}{\Omega}\right] d > 0 \qquad \overline{p} = \left[\frac{2nv\delta(1+\rho)}{\Omega}\right] d > 0$$

where $\Omega = 2\rho + \delta(1+\rho)(1+2v)$.

Thus, the Home net asset position and the change in the relative reserve position in the case of sterilisation are unambiguously larger than in the no sterilisation case. As a result, all of the long run variables, except the increase in the long run price level, are unambiguously larger due to the short run capitial account deficit. Home enjoys greater long run consumption and leisure while Foreign residents reduce consumption more and increase their work effort and output.

7.3 Comparison with Flexible Exchange Rate Models

In the first generation NOEM models, exchange rate depreciation in the face of a monetary shock plays the Keynesian expenditure switching role whereby

the relative price change increases the demand for the product of the country undergoing the monetary expansion. The result of this is that agents in the region in which the expansion occurs, for values of $\rho > 1$, run a current account surplus, or a deficit for $\rho < 1$. In the former case, the surplus occurs because the increase in output is temporary. Forward looking agents wish to smooth their consumption through time, and this they achieve by consuming less upon impact than the increase in output, saving the rest via the current account surplus. Latterly, a deficit occurs on impact since the goods are compliments.

Thus, in the flexible exchange rate case the overall current account and capital account position sums to zero. Hence, under the assumption that the internationally traded goods are substitutes, the current account surplus implies an identical positive net foreign asset position which agents use to increase consumption in the future. This current account surplus under flexible exchange rates contrasts with our result for the fixed exchange rate case that a current account *deficit* obtains upon impact for all values of ρ. This result is driven by the fact that in this model the change in the foreign exchange reserves at the central bank drives a wedge between the current account and capital account of the private sector. Consider for example the balance of payments for this fixed exchange rate model.

Balance of Payments Accounts:

$P_H Y_H - PC$	current account surplus
$+ \quad - \quad B$	capital account surplus
$(R - R_0) - \left(\frac{1-n}{n}\right)(R^* - R_0^*)$	overall BoP surplus

Given the fixed exchange rate, the portfolio reallocation following the monetary shock results in an accumulation of Foreign assets, given by a positive B. Forward looking agents aware of this future flow of income wish to smooth this consumption flow through time, and run a current account deficit, $P_H Y_H - PC < 0$. The balance of payments position has been financed by running down reserves at the central bank. This brings out an important contrast between the flexible and fixed exchange rate models, and demonstrates an important channel often ignored in discussions of fixed exchange rate regimes in the second generation NOEM models.

8 Welfare Analysis

Of course, one of the advantages of the NOEM literature is that the models themselves, based on explicit utility functions, suggest a natural welfare metric with which policy interdependence can be evaluated. In this section we proceed by focussing upon the real component of the utility function—the consumption

and output components. Using the results presented in the previous section we can caculate the present discounted flow of utility through time for Home and Foreign agents given different parameter values. In what follows we consider a unit credit expansion by the Home country and allow the within period elasticity of substitution, ρ, to vary[18]. We assume $\delta = 0.06$ which is equivalent to setting $\beta \approx 0.94$. We set $\theta = 6$ which is equivalent to a mark up of around 20%. We also set the inverse of velocity, $v = 1$ and consider various country sizes. Given the functional form we assume in the utility function and these additional parameter restrictions, these results are comparable to those reported for the flexible exchange rate case by Tille (2001).

Table 3 reports the welfare implications of the unit Home credit expansion under the cases of no sterilisation by Foreign and then under sterilisation. The Table also reports, for comparison, the flexible exchange rate spillovers from Tille (2001). Consider the upper part of the Table which demonstrates the role of consumption substitutability and country size in the propagation of shocks. Note how, regardless of ρ the Home country always benefits from the monetary expansion. That is, with the exchange rate fixed—thus imparting no expenditure switching effect of monetary policy through the exchange rate—the Home country always benefits from the expansion due to the porfilio reallocation which is required to bring about short run equilibrium. In fact, as the goods become worse substitutes, the Home country benefits more from the expansion, which contrasts with the flexible exchange rate case, where Home welfare is increasing with ρ—opposite to the results under the fixed exchange rate regime.

[18] A unit, $d = 1$, expansion in credit does not accord well with our assertion that the shocks should be "small" for our solution to hold. However, this figure is chosen for simplicity, and the values of spillovers for shocks can be calculated for, say, a 1% domestic credit expansion by multiplying the figures presented by 10^{-2}. The unit expansion assumption in the flexible exchange rate case was also utilised in Tille (2001) with which we compare our model.

TABLE 3: WELFARE RESULTS UNDER NO-STERILISATION

		ρ	1	2	3	4	5	6	7	8	9
FIXED											
No Steril.	$n = 0.25$	u	1.07	0.92	0.86	0.83	0.82	0.81	0.80	0.79	0.79
	$n = 0.5$	u	0.74	0.64	0.60	0.59	0.57	0.57	0.56	0.56	0.55
	$n = 0.75$	u	0.42	0.36	0.34	0.34	0.33	0.33	0.32	0.32	0.32
	$n = 0.25$	u^*	-0.25	-0.20	-0.18	-0.17	-0.16	-0.16	-0.16	-0.16	-0.15
	$n = 0.5$	u^*	-0.58	-0.47	-0.44	-0.42	-0.41	-0.40	-0.40	-0.39	-0.39
	$n = 0.75$	u^*	-0.90	-0.75	-0.70	-0.67	-0.65	-0.64	-0.63	-0.63	-0.62
FIXED											
Steril.	$n = 0.25$	u	1.88	1.60	1.51	1.46	1.42	1.41	1.39	1.38	1.37
	$n = 0.5$	u	1.26	1.07	1.01	0.98	0.96	0.94	0.93	0.92	0.92
	$n = 0.75$	u	0.64	0.54	0.51	0.49	0.48	0.48	0.47	0.47	0.46
	$n = 0.25$	u^*	-0.62	-0.53	-0.50	-0.48	-0.47	-0.47	-0.46	-0.46	-0.45
	$n = 0.5$	u^*	-1.24	-1.06	-1.00	-0.96	-0.94	-0.93	-0.92	-0.91	-0.91
	$n = 0.75$	u^*	-1.86	-1.59	-1.50	-1.45	-1.42	-1.40	-1.38	-1.37	-1.36
FLOATING											
	$n = 0.5$	u	-0.33	-0.16	-0.07	-0.01	0.04	0.08	0.12	0.16	0.20
	$n = 0.5$	u^*	0.50	0.33	0.24	0.18	0.13	0.08	0.04	0.01	-0.03

Moreover, Table 3 makes clear that there are negative spillovers to the Foreign region for all parameter values reported. However, the negative spillover is increasing in the size of the Home country. Since the per capita negative net asset position taken on by the Foreign country is proportional to the size of the Home region, as Foreign becomes larger, the long run burden of making payments to the Home region becomes, in per capita terms, smaller. As a result, the Foreign region, when sufficiently large, will benefit from the monetary expansion which has a first order effect on welfare as the economy is pushed towards the efficient production level in the short run. The general expression for the value of n for which there is exactly no welfare spillover on the Foreign country is

$$n = \frac{2v\left[\delta(1+\rho)+\rho\right] + 2\rho + \delta(1+\rho)}{2v\theta\left[\delta(1+\rho)+1+2\rho\right]}$$

which, for the parameter values utilised in the above calibration, suggests $n \approx 0.12$ for $\rho = 1$ and $n \approx 0.16$ for $\rho = 9$. Thus, the boundary at which Foreign

overall utility does not change for a Home monetary expansion occurs when the Home country is less than one fifth of the overall size of the two economies, and Foreign therefore benefits in welfare terms so long as the Home country is very small.

As reported in Table 3, sterilisation by Foreign is a beggar-thy-self policy. That is, although the Foreign region can ensure no impact on its monetary base through sterilisation, in doing so Foreign gains no utility from a short run consumption bonus, and in fact takes on a larger liability due to the capital flows. Consequently, Foreign increases long run ouput and reduces consumption more which means, for any given monetary expansion by Home, the negative spillovers are greater. And the spillover is *always* negative under Foreign sterilisation, regardless of the Home region size. Thus, a central bank not playing by the rules-of-the-game, if you like, in this symmetric fix is instigating a beggar-thyself policy at all times.

Of course, there is one important feature of welfare that this analysis is unable to account for. The fact that the Foreign monetary authority has accumulated large reserves of the Home currency means at some point in the future the Foreign monetary authority could run down these reserves to achieve a consumption boom. Thus, this analysis is unable to account for the long-run welfare implications of changes in reserves at the central banks. And although this cannot be explicitly considered in this model due to the restrictions required in solving the model, it is likely to be an important feature in real world considerations. For one thing, following the accumulation of reserves, the Foreign central bank is likely to be able to expand monetary policy in the future more within the fix before pushing up against its reserve constraint.

9 Concluding Remarks

In this paper we develop a model analogous to the fixed exchange rate ad hoc two country model of Mundell which complements the flexible exchange rate NOEM models. The main results of this work can be summarised as being fourfold:

First, stock implications of portfolio adjustments by private agents following a credit shock play an important role in the spillover of credit shocks across countries participating in a symmetric fixed exchange rate regime. These welfare spillovers are likely to be, following a positive credit shock in one region, negative—which stands in stark contrast to the implied spillover in the ad hoc version of the model.

Second, one consequence of a model built upon explicit microeconomic foundations is that the region experiencing a positive credit shock runs a current account deficit in smoothing consumption through time. This contrasts with the flexible exchange rate NOEM models in which the region where a monetary expansion occurs runs a current account surplus or deficit depending upon the intratemporal elasticity of substitution. The result for this fixed exchange rate model obtains due to the role of foreign exchange reserves allowing a capital

account deficit on the balance of payments to be achieved at the expense of official reserve holdings at the central bank, and consequently forward looking agents run a current account deficit by way of consumption smoothing.

Third, the symmetric Foreign money supply expansion comes about thorough two channels: (i) The portfolio reallocation mechanism emphasised in the ad hoc models. (ii) The relative consumption and therefore current account channel which cannot be identified within the ad hoc framework.

Finally, sterilisation by the monetary authority not instigating the expansionary policy turns out to be a beggar thy-self policy.

We note here two particular drawbacks to this analysis however:

First, since our model is unable to distinguish between precisely which central bank is intervening, a potentially important feature of the real world is left unexplored. Indeed, if the intervention is shared between the central banks, it would be useful to be precise about the nature of this sharing arrangement. It is possible that this could carry strong implications for the robustness of the results in this paper.

Second, the reserve differential which occurs following the Home credit shock implies that in per capita terms the Foreign residents hold a greater quantity of reserves than Home—in reserve terms, a positive net asset position for Foreign agents. This differential is a result of the nonstationarity of the net asset and reserve positions of the central banks following a shock in this class of models. Clearly, a more satisfactory dynamic model in which the central banks return their relative reserve position to their initial position would account for the longer run reserve requirements of participating in a fixed exchange rate regime. This dynamic response, through the balance of payments equation (and, for simplicity, holding the net asset position of the private sector constant) implies a relative consumption response in favour of Foreign since reserves are a claim on future Home output. One reason for our startling welfare results must therefore be that we are unable to account for the long run relative reserve position of the central banks which are themselves an asset of the country involved.

There are clearly a number of lines of research which require further thought, and suggest themselves to future research. However, it is hoped that this framework offers an alternative view of fixed exchange rate modeling within the NOEM literature. Finally, an important and novel approach of this paper has been to spell out precisely the nature of a symmetric fixed exchange rate regime in a microfounded model. This framework could be modified in future work to allow researchers to explore other important issues relating to the fixing of exchange rates in microfounded models.

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