Modelling Long Memory and Risk Premia in Latin American Sovereign Bond Markets

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Abstract

A family of credit risk models is proposed to capture three salient features of Latin American (LA) Sovereign Bond Markets: individual Long Range Dependence in volatility—Long Memory (LM)—, high fractional comovement and time varying risk premia. Evidence in favor of LM is uncovered and the extent of Default Risk Contagion in these markets during the nineties is measured. Among others, the results suggest that the response of bond spread changes to volatility shocks is not statistically different, indicating that a common source may be driving the market. Also, the extent of fractional comovement is high and the magnitude of the risk premia for investing in these bond markets is substantial. Our suggested family of bivariate Fractional Integrated GARCH-in-Mean models is preferred to Brunetti (2000) and Teyssière (1998) processes as indicated by Schwartz Information Criteria and Likelihood Ratio tests.

Keywords: Financial Stability, Credit Risk, Default Risk Contagion, Long Memory, Bivariate FIGARCH(1,d,1)-in-Mean.

JEL: C14, C32, F34, F42, G12, G15.

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1 Introduction

Emerging Bond Markets (EBM) have become one of the largest and most liquid international markets. The amount of debt outstanding is more than $US 300 billion from which the issuance by four countries—Argentina, Brazil, Mexico and Venezuela—accounts for 90% or more of the total market debt in Latin America (LA). These four issues also drive around 50 or 60% of most international Emerging Market Bond Indexes (EMBI).

Interpreted as measures of default or credit risks EMBI have been associated to capital flows, market sentiment and fundamentals (GDP, stock markets, good prices, interest rates and various other variables.)[1]

This paper analyzes daily sovereign credit spreads in LA as proxied by the individual EMBI of JP Morgan. The main feature of this work with respect to previous studies is the suggestion that excess holding returns in these markets are mainly driven by conditional time varying risk premia. Research in EMBI has not explored yet the possibility of excess returns being predicted by their own conditional volatilities which we assume fully reflect information about fundamentals.

Some studies have found some degree of predictability in EMBI but have failed to take it into account at the modeling stage.[2] We find here that spreads are in fact fractionally integrated, hence indicating a high degree of predictability in these markets. Long Memory (LM) or long range dependency has not been investigated in EBM.

Some authors have documented already the high degree of cross correlation in this market as well as a remarkable comovement and potential common shocks in levels and volatility—see Mauro, Sussman & Yafeh (2000), Jostova (2002) and Fiess (2003). In our view, these studies have been nonetheless unsuccessful to capture the strong *fractional* long run cross dependence in these markets.

The contribution of this paper is twofold. First, the stationary LM properties of the volatility in these markets are investigated for the first time. In contrast with some studies suggesting that EMBI exhibit unit roots, this study provides strong evidence of stationarity and Long Memory. Second, we propose a new family of bivariate long memory models for credit risk that take into account risk premia, the high degree of correlation, individual persistency and co-persistency.

In econometric terms we provide the empirical literature with a new bivariate Fractional Integrated Generalized Autoregressive Conditional Heteroskedasticity model with in-Mean terms (FIGARCH-in-Mean). Quasi-Maximum Likelihood Estimation (QMLE) results as well as the Schwartz

[1] See Min (1998), Ferrucci (2003), Eichengreen & Mody (1998) and Fiess (2003).

[2] A notable exception is Jostova (2002) who finds that credit spreads are non stationary variables and from this carries out a cointegration analysis with fundamentals. Deviations from long-run equilibrium are then used to predict excess returns over US treasuries.

Information Criterion for these markets suggest that these new models give superior estimates compared to previous bivariate FIGARCH specifications. There is indeed a significant gain of information by including the risk premium in the modeling of sovereign default spreads.

The following section provides a descriptive analysis of EMBI spreads and its autocorrelation functions. The Long Memory and stationarity properties of volatility are also investigated. Interestingly, a battery of both heuristic and semiparametric methods confirm the existence of LM stationarity in volatility.

Univariate Long Memory models and the econometric models of Teyssière (1997) and Brunetti & Christopher (2000) are described in detail in section three. These models are extended to the new bivariate FIGARCH(1,d,1)-in-Mean specification. QMLE results are reported in section four. Policy implications and the significance of our findings to the understanding of Sovereign Emerging Bond Markets are discussed in the conclusions.

2 Descriptive analysis and Long Run Dependence

2.1 Descriptive analysis of Default Spreads in EBM

Figure 1 shows the EMBI spreads[3] in logs for Brazil, Mexico and Venezuela from December 31^{st} 1990 to July 10^{th} 2002. The Argentinean sample begins on April 30, 1993. EMBI spreads, or S_t, provide a single measure of pure sovereign default risk and may be readily interpreted as excess returns over US treasuries.[4] A relatively high spread may indicate a greater risk of default[5] and also a lower return on risk-free investments. EMBI spreads are also commonly regarded as the premium for holding defaultable sovereign instruments.

A salient feature of these markets is the high degree of comovement. Some authors have already documented a high degree of interdependence between EMBI spreads, common trends and common shocks—see Mauro et al. (2000) and Fiess (2003) for instance. In addition, the data suggests that risk premia are time varying and also that the relative spread difference between two given countries is not stable.[6] For all these reasons, the kinetics

[3] The data has been kindly provided by EcoWin.

[4] The EMBI for each country is calculated as the weighted average spread of all Brady bonds with similar properties. For further discussion on the actual calculation of credit spreads see Jostova (2002).

[5] Spreads also reflect various other risks: exchange rate risk, interest rate risk, liquidity risk and difult risk. Benczur (2001) notices that the first of these should be almost nil, while liquidity risk is usually the result of market conditions, volatility components or asymmetries.

[6] This conjecture is in direct contrast with the analysis of Forbes & Rigobon (2000) who by analyzing a shorter sample and the difference between any two given spreads suggested a relatively constant risk premium.

of EMBI should be analyzed in a multivariate setting rather than on an individual basis.

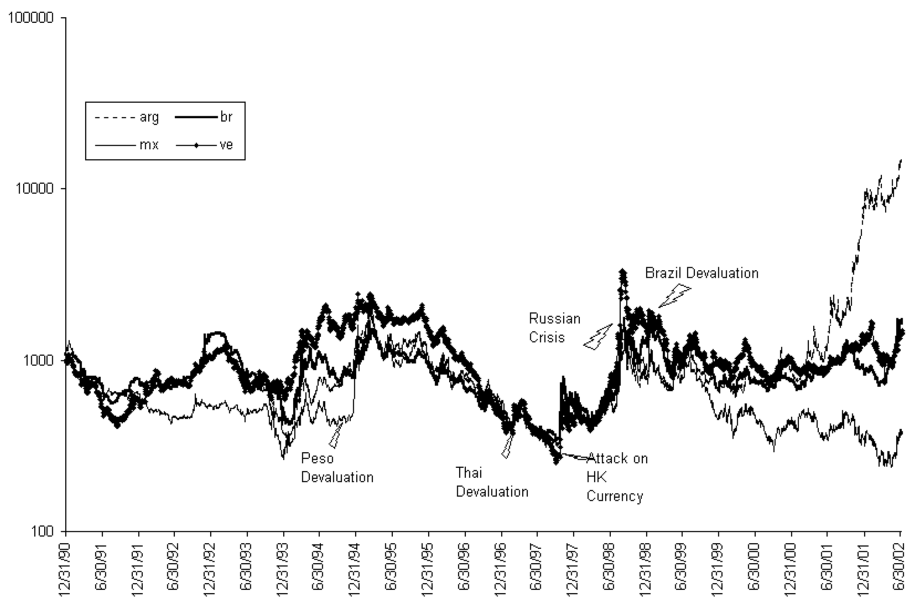

Figure 1: EMBI spreads, 31 Dec. 1990 - 30 Jun. 2002.

The first differences of the spreads in logs, i.e., named here as s_t, are presented in Figure 2. By definition, spread changes can be interpreted as changes in *excess returns* over US treasuries. These reflect general Emerging Market (EM) prospects and hence the credit risk[7] attached to EM assets—see Cunninham, Dixon & Hayes (2001).

It is interesting to observe that individual spread changes seem to exhibit a common response to shocks. For instance, periods of distress such as the Tequila Crisis in December 1994 not only affected the Mexican default risk and volatility, but also affected other Latin American bond indexes. What is more, these markets also seem to respond very rapidly to turmoils generated in other latitudes, e.g., Hong Kong or Russia. The common response to shocks in levels and volatility implies there may be a common factor driving these markets in the same direction.[8]

Figure 2 also shows that individual credit risks share some of the stylized facts of financial returns such as the presence of clusters and some degree of time dependence. There is no reason to rule out a priori the potential for high comovement and time varying volatility. Summary descriptive statistics

[7] Credit risk in this context may be interpreted as a measure of a sovereign government ability to meet its principal and/or interests.

[8] Cunninham et al. (2001) have also noted this in their study.

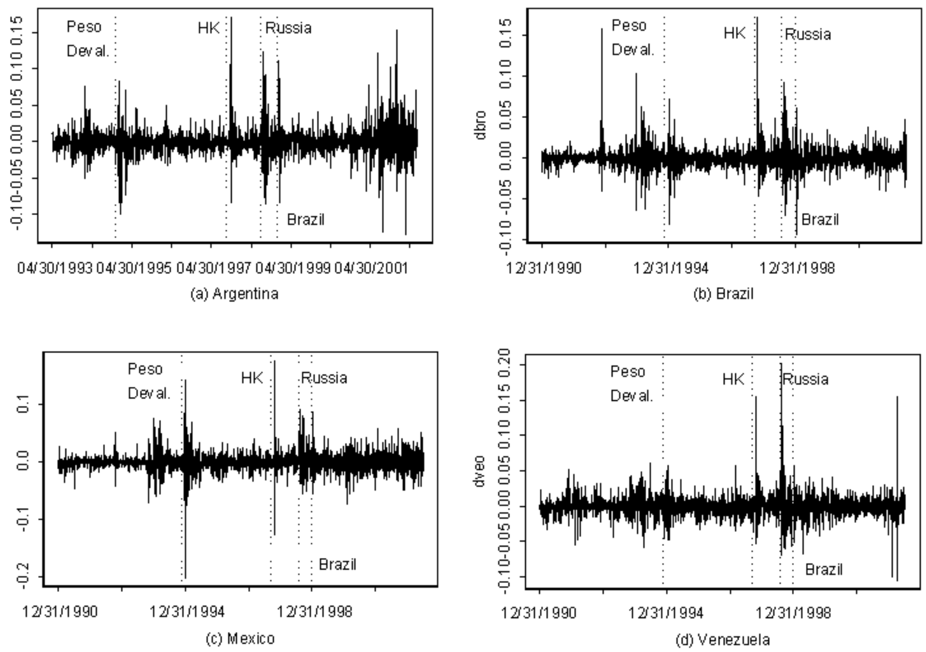

Figure 2: EMBI, spread log differences 31 Dec. 1990 - Jul. 10^{th} 2002.

are presented in Table 1. The series show a similar risk-return relationship, the returns exhibit high kurtosis and there is indeed evidence of time series dependence as indicated by the 20th order Ljung-Box statistic.

Table 1: EMBI, descriptive statistics, daily spread changes (s_t) in logs.

s_t^a	$\overline{x}$	σ	S^b	K^c	JB^d	$Min.$	$Max.$	n	LB(20)^e
Argentina	0.0005	0.0188	0.8775	11.61	13,780	-0.1274	0.1709	2,398	67.61*
Brazil	0.0001	0.0130	1.9415	24.60	77,712	-0.0934	0.1725	3,007	96.25*
Mexico	-0.0002	0.0164	0.3330	18.95	45,041	-0.2031	0.1764	3,007	82.05*
Venezuela	0.0001	0.0147	1.7089	24.60	77,299	-0.1045	0.2020	3,007	101.69*

*Significant at the 1% level. ^{a} s_t=log(S_t)-log(S_{t-1}) where S_t is the EMBI spread; ^{b}Skewness; ^{c}Kurtosis; ^{d}Jarque-Bera statistic; ^{e}Ljung-Box Statistic, order in brackets.

2.2 Long Memory (LM) in EBM

To examine the autocorrelation pattern further, Figure 3 graphically analyzes the dependence structure of individual EBM spreads. The autocorrelation functions (ACF) of spread changes (s_t) and of its absolute transformation ($|s_t|$) are bounded by a 95% confidence interval.[9] It is observed, with no exception, that the ACF associated with s_t presents an exponential decay rate—see lighter lines—, i.e., significant autocorrelations are reported only for the first lags, while the rate of convergence of the ACF for $|s_t|$ is much slower (see dark lines). Most of the countries present significant absolute value autocorrelations for more than one hundred lags, while Brazil—panel (b)—has the first negative autocorrelation not earlier than lag 350. This is fully consistent with Ding & Granger (1996) who have suggested that a time series shows LM if the rate of decay of the estimated conditional variances seems hyperbolic rather than exponential.

LM is also present in combinations of portfolios. To illustrate, Figure 4 shows the linear cross correlation of a portfolio consisting of two assets: Brazil and Mexico. The ACF of simple spread changes is denoted by $\rho_{mx,br}$ in grey lines, while absolute transforms, i.e., $\rho_{|brt,mxt-i|}$, are in dark lines. Panels (a) and (b) show the ACF up to lag 400, while panels (c) and (d) show a 60-day zoom of the cross correlations.

As with individual ACF, cross correlograms in Figure 4 suggest that simple cross correlations decay exponentially, while the ACFs of absolute transforms exhibit a hypergeometric decay rate. The graphical examination suggests strong and significant long run volatility cross dependencies in the absolute values of spread changes—see dark lines panels (a) and (b).

In Table 2 we present a summary of the cross long range dependence properties for the series under analysis. The first column shows the lead/lag relationships of the six portfolios as represented by the cross correlation coefficients $\rho_{it,jt-i}$ and $\rho_{jt,it-i}$. There is an equal contemporaneous response between countries only for the first lags and apparent asymmetric impacts afterwards—see first column of panels (a) and (b) in this Table. Except for the pair Argentina Mexico, all countries show significant cross correlations at least until lag fifty.

In the last column of Table 2 we show the point at which the last significant positive correlation takes place and label it as "spillover". This measure indicates the dependence of contemporaneous spreads with lagged values. As we observe in Panels (a) and (b) the volatility dependence structure is in general highly symmetric: volatility shocks disseminate at similar rates from one country to another.

Overall, this section has shown evidence of LM in the credit risk series of EBM. The high persistency may be explained, in agreement with Jostova (2002), as the result of financial market rigidities and informational deficiencies. Portfolio re-allocation after a sudden liquidity crunch for instance may not be immediate due to the way institutional investors operate in EBM[10]

[9] Confidence intervals are depicted by the dotted lines and are calculated as $\frac{2}{\sqrt{n}}$, where n is the sample size.

[10] The minimun transaction size in Brady markets is of $2 milion leaving room only to large investors such as mutual, endowment and pension funds—Jostova (2002).

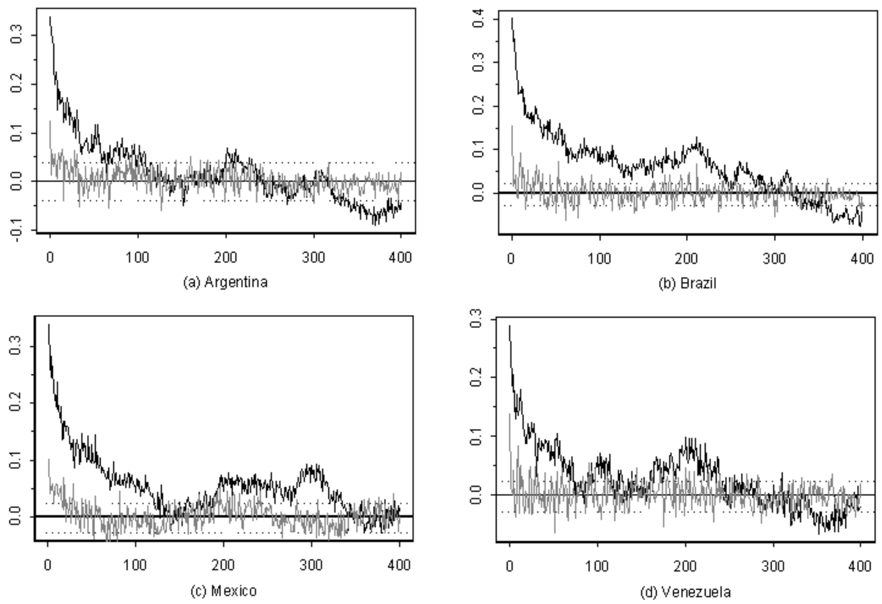

Figure 3: EMBI, Autocorrelation of $|s_t|$ and s_t from high to low, daily log differences 31 Dec. 1990-10 Jul. 2002.

and the lack of 'noise traders' among other factors. Also, dedicated investors may react more slowly to market signals since they pursue returns relative to a benchmark.

The rigorous reader may note that the LM diagnosis based on the graphical inspection of auto and cross correlation functions may be spurious. It can reasonably be argued that LM may be in fact not other thing but the result of structural changes in the data and monotonic trends—see Lobato & Robinson (1998) and Giraitis, Leipus & Philippe (2002). To rule out this possibility, we have formally tested for the presence of LM on (s_t), on spread volatility proxies ($|s_t|$ and $|s_t|^d$) and on nonlinear transforms ($|s_{it} \cdot s_{jt}|$) using semiparametric methods. The results, not presented here to save space, are qualitatively similar and reassuringly confirm the presence of LM in EBM.[11]

3 Parametric Long Memory in volatility

We have shown in the previous section that the volatility of credit risks in EBM exhibits LM. To model this property we need a process that is not

[11] The tests include Lobato & Robinson (1998) t test, the Modified Rescaled test of Lo (1991) and the rescaled variance V/S test of Giraitis et al. (2002). Estimation results and calculation details are readily available upon request.

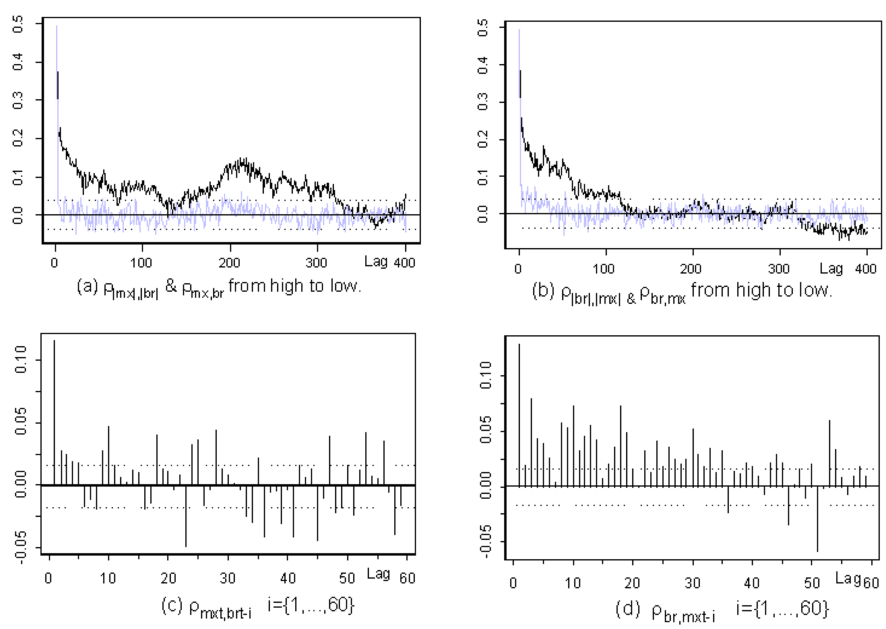

Figure 4: EMBI, Cross Correlogram Mexico vs. Brazil for $|s_t|$ and s_t.

Table 2: EMBI, cross correlations at different lag values.

$lag(i)$	1	10	50	100	150	200	300	Neg.[a]	Spill[b]
Panel (a): $\boldsymbol{\rho}_{1t,2t-i}$									
$\rho_{art,brt-i}$	0.5864	0.1087	0.0232	0.0074	-0.0145	0.0526	-0.0077	65	55
$\rho_{art,mxt-i}$	0.5041	0.1341	0.0544	0.0000	-0.0441	-0.0013	0.0198	67	59
$\rho_{art,vet-i}$	0.4340	0.0688	0.0165	-0.0064	-0.0648	-0.0119	-0.0065	47	40
$\rho_{brt,mxt-i}$	0.4917	0.1702	0.1411	0.0559	0.0214	0.0265	0.0015	124	116
$\rho_{brt,vet-i}$	0.4862	0.1416	0.1218	0.0652	0.0516	0.0442	0.0118	158	151
$\rho_{mxt,vet-i}$	0.4204	0.1233	0.0669	0.0596	0.0242	0.0565	0.0638	139	130
Panel (b): $\boldsymbol{\rho}_{2t,1t-i}$									
$\rho_{brt,art-i}$	0.5864	0.0958	0.0412	0.0068	-0.0062	0.0015	0.0030	63	57
$\rho_{mxt,art-i}$	0.5041	0.1254	0.0469	0.0243	-0.0077	0.0125	0.0259	66	58
$\rho_{vet,art-i}$	0.4340	0.0579	0.0158	0.0193	-0.0009	0.0613	-0.0180	36	30
$\rho_{mxt,brt-i}$	0.4917	0.1734	0.0580	0.0679	0.0590	0.1319	0.0696	128	120
$\rho_{vet,brt-i}$	0.4862	0.1395	0.0840	0.0823	0.0613	0.1378	-0.0009	290	286
$\rho_{vet,mxt-i}$	0.4204	0.1157	0.0632	0.0424	0.0027	0.0549	-0.0105	80	70

[a] Lag at which the first negative value is observed. [b]Lag at which the autocorrelation function first crosses the horizontal axis. Note: Standard errors (x2) for pairs with Argentina 0.0417; all others 0.0365.

only able to capture individual long run dependence, but also allows us to take into account fractional comovement, the response to common shocks and also provides estimates of risk premia. The aim of this section is to introduce and develop such family of credit risk models.

3.1 Univariate FIGARCH

Arguably, the most popular parametric process that replicates the strong dependence in volatility, i.e., hypergeometric decay of the autocorrelation function, is the Fractional Integrated Generalized Autoregressive Conditional Heteroskedastic (FIGARCH) model of Bollerslev & Mikkelsen (1996).[12]

To begin with, let us assume for simplicity that excess returns in EBM for each individual case follow a random walk process plus drift (c):

$$s_t = c + \varepsilon_t \tag{1}$$

To capture the time varying conditional volatility of the residuals ε_t it may be proposed that:

$$\varepsilon_t|\Omega_{t-1} = \eta_t\sqrt{h_t} \tag{2}$$

[12]Among other alternatives that generate slow decay of the autoccorelation function of returns are the Two Component model of Ding and Granger (1993) and Ding and Granger (1996).

where η_t is an independent identically distributed ($i.i.d.$) random process with mean equal to zero and variance equal to unity. Notice that $E(\varepsilon_t|\Omega_{t-1}) = 0$ and $Var(\varepsilon_t|\Omega_{t-1}) = h_t$.

Baillie, Bollerslev & Mikkelsen (1996) have proposed the following Fractionally Integrated Generalized Autoregressive Conditionally Heteroskedastic (FIGARCH) process for h_t:

$$\varepsilon_t^2(1-\phi)(1-L)^d = w + (1-\beta L)v_t \tag{3}$$

where L is the lag operator, $\phi = (\alpha + \beta)$ the mean reverting parameter, $v_t = \varepsilon_t^2 - h_t$, $(1-L)$ is the first differencing operator and d is the fractional integration parameter.

This model reduces to the parsimonious GARCH(1,1) process of Bollerslev (1986) when $d = 0$ and to the IGARCH(1,1) process of Engle & Bollerslev (1986) when $d = 1$. In the first case, shocks to the conditional variance decay at an exponential rate; while in the second, shocks remain important for forecasts of all horizons. In a FIGARCH process, shocks to the variance decay at a hyperbolic rate replicating well the behavior observed in the second section of this paper.

The fractional differencing operator has a binomial expansion that can be conveniently expressed in terms of the hypergeometric function:

$$\begin{aligned}(1-L)^d &= F(-d,1,1;L) \\ &= \sum_{k=0}^{\infty}\Gamma(k-d)\Gamma(k+1)^{-1}\Gamma(-d)^{-1}L^k \\ &= \sum_{k=0}^{\infty}\pi_k L^k\end{aligned} \tag{4}$$

The infinite ARCH representation of this $FIGARCH(1,d,1)$ process, is obtained by rearranging equation (3):

$$\begin{aligned}h_t &= \frac{w}{1-\beta(1)} + \left[1 - \frac{(1-\phi L)(1-L)^d}{1-\beta L}\right]\varepsilon_t^2 \\ h_t &= \frac{w}{1-\beta(1)} + \lambda(L)\varepsilon_t^2\end{aligned} \tag{5}$$

where $\lambda(L)$=$\lambda_1 L + \lambda_2 L + \ldots$. To ensure positiveness of the conditional variance all the coefficients in the infinite ARCH representation must be non-negative. In particular, for the case of the $FIGARCH(1,d,1)$ process in equation (5) the conditions for the process to be well-defined and positive have been given by Baillie et al. (1996) and Bollerslev & Mikkelsen (1996). These can be observed if we re-express the lag polynomial $\lambda(L)$ in (5) as:

$$
\begin{aligned}
\lambda_1 &= \phi - \beta + d \\
\lambda_k &= \beta\lambda_{k-1} + \left[(k-1-d)k^{-1} - \phi\right]\delta_{k-1} \text{ for } k \geq 2 \\
\delta_k &\equiv \delta_{k-1}(k-1-d)k^{-1},\ k = 2, 3, ...
\end{aligned} \tag{6}
$$

where δ_k are the coefficients in the Maclaurin's series expansion. From here, Bollerslev & Mikkelsen (1996) show sufficient conditions under which all corresponding ARCH parameters are nonnegative:

$$
\begin{aligned}
\beta - d &\leq \phi \leq (2-d)/3 \\
d\left[\phi - (1-d)/2\right] &\leq \beta(\phi - \beta + d)
\end{aligned} \tag{7}
$$

Baillie et al. (1996) explain that for $0 < d \leq 1$ the hypergeometric function evaluated at $L = 1$ equals 0 so that $\lambda(1) = 1$, and for this reason the second moment of the unconditional distribution of ε_t is infinite, and the FIGARCH process is clearly not weakly stationary. However, by extending the properties of IGARCH processes, Nelson (1991) and Bougerol & Picard (1992) show the FIGARCH process is still strictly stationary and ergodic.

3.2 Bivariate Fractional GARCH

The cross-correlation coefficient has become the widely preferred statistic to examine the comovement of sovereign spreads since this measure provides information on the speed, degree and direction of contagion.[13] Even though this seems a very convenient measure for short term cross market dependencies, this indicator is subject to at least three major criticisms: it does not take into account the fractional comovement of these markets, it does not consider the time varying volatility[14] and it disregards the existence of risk premia.

By relying on the flexibility of Bollerslev (1990) Constant Correlation model (CCC), it is possible to use the correlation coefficient as a measure of interdependence while simultaneously taking into account the individual behavior of volatilities. Such parametrization assumes that the correlation matrix of the vector of residuals is constant—a very strong assumption in EBM—but at the same time allows the individual volatilities contained in

[13] See Cunninham et al. (2001) and the Quarterly Global Financial Stability Report of the IMF for more on this. Previous litterature using correlation coefficients as measures of contagion include Forbes & Rigobon (1999) and Forbes & Rigobon (2000).

[14] Baig & Goldfajn (2000) suggest indeed that excluding volatilities from the analysis may be misleading. Contagion for instance is the result of panic, margin calls, thin markets, etc., factors which are at the same time responsible for changes in volatility.

the conditional *variance-covariance* matrix (H_t) to be time varying. In addition, by employing Brunetti & Christopher (2000) bivariate specification,[15] the individual conditional variances $h_{1,t}$, $h_{2,t}$ are assumed to take FIGARCH processes as follows:

$$\epsilon_t|_{t-1} \sim N(0, H_t), \qquad \{H_t\}_{ij} = h_{ij,t} \tag{8}$$

$$\begin{aligned} h_{ii,t} &= \frac{\omega_i}{1-\beta_{ii}(1)} + \lambda_{ii}(L)\varepsilon_{i,t}^2; \text{ for } i = 1,2 \\ h_{12,t} &= \rho_{12}\sqrt{h_{11,t}}\sqrt{h_{22,t}}, \end{aligned} \tag{9}$$

where $\lambda_{ii}(L) = \left[1 - \frac{(1-\phi_{ii}L)(1-L)^{d_i}}{1-\beta_{ii}L}\right]$ for $i = 1,2$ and $\epsilon_t = [\varepsilon_1, \varepsilon_2]$.

In order for H_t to be positive definite it is required that $h_{11,t}$ and $h_{22,t}$ are positive and the conditional correlation matrix is positive definite. Bollerslev (1990) noticed that under the assumption of time invariant correlations, the Maximum Likelihood Estimate (MLE) of the correlation matrix is equal to the sample correlation matrix of the standardized residuals. Hence, this is a parsimonious specification where positive definiteness of the variance covariance matrix is ensured provided that $|\rho|<1$, $\beta_{ii} - d_i \leq (1/3)(2 - d_i)$ and $d_i[\phi_{ii} - 1/2 \cdot (1 - d_i)] \leq \beta_{ii}(\phi_{ii} - \beta_i + d_i)$. Also the conditional variance of the system is stationarity for all $0 \leq d_i \leq 1$—see Brunetti & Christopher (2000).

Analogous to the individual representation in (3), Brunetti & Christopher (2000) present the CCC-FARIMA representation in terms of the squared residuals as

$$\Phi(L)\begin{pmatrix} (1-L)^{d_1} & 0 \\ 0 & (1-L)^{d_2} \end{pmatrix}\begin{pmatrix} \varepsilon_{1t}^2 \\ \varepsilon_{2t}^2 \end{pmatrix} = w + B(L)v_t \tag{10}$$

where $\Phi_0 = B_0 = I$. From this form, it is clear that a direct testing for common orders of LM, i.e., $d_1 = d_2$, is possible.

A Constant Correlation assumption may be far too restrictive given the episodes of distress in Emerging Bond Markets during the nineties. More importantly perhaps, none of the parameters in (10) captures the time varying interdependence evident in the descriptive section before.

In order to explore the possibility of time varying correlation, Teyssière (1997) relaxed the constancy assumption by allowing the conditional *covariance* to be represented by a FIGARCH process too. Assuming a random walk plus drift for the mean we get:

$$\begin{aligned} s_{it} &= c_i + \varepsilon_{it} \\ s_{jt} &= c_j + \varepsilon_{jt} \end{aligned} \tag{11}$$

[15] Brunetti & Christopher (2000) used a bivariate CCC-FIGARCH framework to model fractional cointegration in oil markets.

$$h_{ij} \quad = \quad w_{ij} + \left(1 - \frac{(1-\phi_{ij}L)(1-L)^{d_{ij}}}{1-\beta_{ij}L}\right)\varepsilon_{it-k}\varepsilon_{jt-k} \qquad i,j = 1,2$$

This innovative extension allows to capture and test the presence of long range cross dependencies. In particular the parameter d_{ij} becomes a natural measure of volatility contagion and fractional comovement. It also captures the effect of volatilities and is fully consistent with the perception of Forbes & Rigobon (1999) who argue that contagion is not the result of changing autocorrelations but derives from changing volatilities.

As pointed out by Teyssière (1997) there is no analytical set of conditions for ensuring positive definiteness of the conditional variance-covariance so this has to be implemented numerically in the estimation algorithm.

3.3 Bivariate Fractional GARCH-in-Mean process

We propose in this and the following sections our new models. As before, it is assumed that credit risks follow a random walk plus drift. The key contribution to the existing bivariate FIGARCH models is the inclusion of risk premia as regressors of spread changes:

$$\begin{array}{lllll} s_{1t} & = & c_1 & + & \gamma_{11}h_{1t} \quad +\gamma_{12}h_{2t} + \varepsilon_{1t} \\ s_{2t} & = & c_2 & + & \gamma_{21}h_{1t} \quad +\gamma_{22}h_{2t} + \varepsilon_{2t} \\ h_{1t} & = & \frac{w_1}{1-\beta_{11}L} & & \left(1 - \frac{(1-\phi_{11}L)(1-L)^{d_1}}{1-\beta_{11}L}\right)\varepsilon^2_{1,t} \\ & & & + & \\ h_{2t} & = & \frac{w_2}{1-\beta_{22}L} & & \left(1 - \frac{(1-\phi_{22}L)(1-L)^{d_2}}{1-\beta_{22}L}\right)\varepsilon^2_{2,t} \\ \multicolumn{5}{l}{h_{12,t} = \rho_{12}\sqrt{h_{11,t}}\sqrt{h_{22,t}}} \end{array} \qquad (12)$$

As an additional feature, in the empirical section we will not only employ the cross conditional variances (h_{1t} or h_{2t}) in (12) as regressors but also the conditional covariances, i.e., h_{12t}, in order to capture the association between spreads changes and fundamentals as in Baillie & Bollerslev (1990).

Notice also that similar to Teyssière (1997) this model can easily be extended by relaxing the assumption of time invariant correlation, in which case we would have an extra equation in the variance process describing the dynamics of the conditional covariances and hence an estimate of the fractional comovement parameter (d_{12}) as follows:

$$\begin{aligned} s_{1t} &= c_1 + \gamma_{11}h_{1t} + \gamma_{12}h_{2t} + \varepsilon_{1t} \\ s_{2t} &= c_2 + \gamma_{21}h_{1t} + \gamma_{22}h_{2t} + \varepsilon_{2t} \\ h_{1t} &= \frac{w_1}{1-\beta_{11}L} + \left(1 - \frac{(1-\phi_{11}L)(1-L)^{d_1}}{1-\beta_{11}L}\right)\varepsilon^2_{1,t} \qquad (13) \end{aligned}$$

$$
\begin{aligned}
h_{2t} &= \frac{w_2}{1-\beta_{22}L} + \left(1 - \frac{(1-\phi_{22}L)(1-L)^{d_2}}{1-\beta_{22}L}\right)\varepsilon_{2,t}^2 \\
h_{12t} &= \frac{w_{12}}{1-\beta_{12}L} + \left(1 - \frac{(1-\phi_{12}L)(1-L)^{d_{12}}}{1-\beta_{12}L}\right)\varepsilon_{1,t}\varepsilon_{2,t}
\end{aligned}
$$

Model (13) can also include cross conditional covariances in-Mean replacing individual variances. In this case, as well as in Teyssière (1997)'s model, the positivity conditions have not been derived analytically and hence they have to be imposed during estimation.

Empirical findings for high frequency financial returns suggest in general that $FIGARCH(1,d,1)$ innovations are usually non-normal and exhibit serial autocorrelation. To rule out such possibility in the estimation of these models we use the Quasi-Maximum Likelihood Estimation (QMLE) approach of Bollerslev & Wooldridge (1992).[16]

4 Estimation results

We now present the bivariate QMLE estimations for the credit risk models introduced in the previous section. We assume that investors hold pair-based portfolios and have four Latin American bonds to choose facing a total of six combinations.

4.1 Common Long Memory

Decision criteria prefer random walk models for the mean and *FIGARCH(1,d,1)* processes for the variance equation.[17] To capture long-run comovement between sovereign markets, Table 3 reports the QMLE estimations of equation (9),[18] the bivariate Constant Correlation long memory model.

[16] For further discussion about robustness, consistency, ergodicity and asymptotic normality properties under this estimation method see Baillie et al. (1996).

[17] We tried different $FARIMA(p,d,q) - FIGARCH(1,d,1)$ specifications and used the Log-likelihood value, Schwartz Information Criterion (SIC) and Akaike Information Criterion (AIC) to discriminate between models. As Teyssière (1997) points out, the statistical properties of the AIC and SIC have not been stablished for the class of Long Memory ARCH process; however we consider that these statistics provide good reasonable guidance. They are calculated herein as:

$$
\begin{aligned}
AIC &= -2ln(L(\widehat{\theta})) + 2 * n_\theta \\
SIC &= -2ln(L(\widehat{\theta})) + n_\theta * \ln(n)
\end{aligned}
$$

where $L(\widehat{\theta})$ is the optimized likelihood value, n_θ is the number of estimated parameters and n is the sample size.

[18] The initial estimation procedure was kindly provided by Celso Brunetti, University of Pennsylvania. The estimation strategy consisted in using starting values from univariate $FIGARCH(1,d,1)$ estimations. Convergence was achieved by reducing the effect of outliers to no more than three standard deviations. The original samples of Argentina-Brazil

All the estimates are highly significant. The behavior of the individual conditional variances (h_{11} and h_{22}) is fully described by the mean reversion (ϕ and β) and persistence (d) parameters.

Table 3: EMBI, CCC-FIGARCH(1,d,1) QMLE estimations.

	(s_{art}, s^a_{brt})	(s_{art}, s_{mxt})	(s_{art}, s_{vet})	(s_{brt}, s_{mxt})	(s_{brt}, s_{vet})	(s_{mxt}, s_{vet})
Conditional Mean						
μ_1	-0.0427	-0.0479	-0.0418	-0.0174	-0.0267	-0.0402
	(0.0227)[b]	(0.0240)	(0.0247)	(0.0128)	(0.0132)	(0.0176)
μ_2	-0.0657	-0.0577	-0.0644	-0.0270	-0.0317	-0.0332
	(0.0158)	(0.0224)	(0.0224)	(0.0169)	(0.0201)	(0.0202)
Conditional Variances						
ω_1	0.1428	0.1433	0.1318	0.0327	0.0374	0.0424
	(0.0133)	(0.0143)	(0.0245)	(0.0058)	(0.0059)	(0.0069)
β_1	0.4446	0.4878	0.2817	0.5004	0.5029	0.5451
	(0.0229)	(0.0288)	(0.1019)	(0.0222)	(0.0212)	(0.0204)
ϕ_1	0.3162	0.2686	0.0897	0.2820	0.2886	0.3041
	(0.0178)	(0.0201)	(0.0914)	(0.0149)	(0.0147)	(0.0160)
d_1	0.3675	0.4628	0.3371	0.4361	0.4229	0.3918
	(0.0355)	(0.0402)	(0.0351)	(0.0298)	(0.0295)	(0.0319)
ω_2	0.0941	0.1363	0.1067	0.0506	0.1569	0.1433
	(0.0173)	(0.0173)	(0.0231)	(0.0076)	(0.0204)	(0.0198)
β_2	0.2234	0.6218	0.4449	0.5469	0.3995	0.3915
	(0.1196)	(0.0592)	(0.0318)	(0.0226)	(0.0243)	(0.0264)
ϕ_2	0.0615	0.1437	0.3509	0.2964	0.3826	0.3789
	(0.1031)	(0.0381)	(0.0200)	(0.0172)	(0.0127)	(0.0133)
d_2	0.4296	0.6693	0.2981	0.4073	0.2348	0.2422
	(0.0353)	(0.0716)	(0.0400)	(0.0343)	(0.0255)	(0.0267)
ρ_{12}	0.7155	0.6078	0.5998	0.5189	0.5624	0.4944
	(0.0052)	(0.0074)	(0.0109)	(0.0099)	(0.0109)	(0.0108)

[a] (S_{it},S_{jt}) indicate the bond components in a given portfolio. [b] Robust standard errors in parenthesis.

As expected from the graphical analysis before, all the correlation coefficients are large and positive, indicating a greater systematic risk effect—i.e., the risk of the portfolio that cannot be diversified away—on these portfolios. The highest correlation is observed in Argentina-Brazil with 71.6 percent, while the correlation of Argentina with Mexico and Venezuela is 60.8 and 59.9% respectively. The correlation for the rest of the portfolios is no less than 49 percent.

and Argentina-Mexico converged satisfactorily without adjusting outliers. In all the estimations we used BFGS optimization algorithm although estimations via BHHH were quiet similar and usually less computing intensive.

As shown by the measures of long range volatility dependence in Table 3, and in line with the graphical inspection and semiparametric estimations before, the memory parameters—see d_1 and d_2 in each column—do not seem to significantly depart much from each other.

To assess this conjecture, we now formally test whether any two fractional differencing parameters are statistically similar. The bivariate *FIGARCH(1,d,1)* specifications are constrained by imposing $d_1 = d_2$. The penultimate row of Panel (a) in Table 5 shows the optimized mean log-likelihood of the constrained models. Except for the cases of Brazil-Venezuela and Mexico-Venezuela, a likelihood ratio cannot reject the hypothesis of common orders of fractional integration—see p-values in brackets.

In other words, despite the fact that each individual bond spread presumably follows its own individual volatility process, they both seem to be driven by a common information arrival process. This is consistent with the claim of Cunninham et al. (2001) who suggests that a single factor may drive all EBM spreads in the same direction. This finding also gives support to the view of Forbes & Rigobon (2000) for Latin American Brady markets in the sense that volatility is not driven by any individual country or subset of countries, but it is instead shared by all countries in the region. And finally, these conclusions add to the propositions of Kaminsky & Reinhart (2002) indicating that developed markets act as conduits between regions of developing countries.

4.2 Default Risk Contagion

The long term time invariant comovement assumption may seem far too restrictive given the number of distress episodes observed in Latin America during the nineties. In fact, time varying cross correlations have already been found in stock and sovereign bond markets—see Hausler (2003) and Cunninham et al. (2001).

Thus, to take this fact into account, in Table 4 we relax the constancy assumption and present the estimation results of the unrestricted bivariate *FIGARCH(1,d,1)* model introduced in equation (11) of section four. While the previous constant correlation model provided a good insight into the degree of association between any two bond spreads, the time variant specification will allows us to capture the extent of interdependence or fractional comovement in these markets.

The conditional covariance (h_{12}) is a measure of volatility in a given *portfolio*. It represents the risk perceived by the investor of holding two bonds. The joint long memory parameter, i.e., d_{12}, indicates the extent of long term default risk volatility contagion and, as shown, it is highly significant in all cases. We interpret *default risk contagion* as the situation in which the risk perception of default about one sovereign bond affects the risk perception of default of another bond in the same market.[19]

It is worth noticing in Table 4 that the orders of individual fractional integration, as well as their statistical significance, do not seem to be affected by the relaxation of the time invariant correlation assumption. What is more, the hypothesis of common long range dependencies cannot be rejected. The optimized likelihood functions resulting from imposing the constraint $d_1 = d_2$ are presented in the fourth rows of Panels (a) and (c) in Table 5 respectively and are labeled as $L(\theta)_{d_1=d_2}$. As indicated by the Likelihood Ratio (LR) tests—p-values in squared brackets—with the exception of the last two portfolios, sovereign exhibit decay rates that are not statistically different.

It was suggested in section two that the decay rate of individual volatilities was different to that of the joint volatility measures. To test this conjecture we re-estimate the model in equation (11) by imposing the restriction of common orders of fractional integration but now conditional covariances are included, i.e., $d_1 = d_2 = d_{12}$. The row labeled $L(\theta)_{d_1=d_2=d_{12}}$ in Panel (c) of Table 5 shows the optimized mean log-likelihood of these estimations. The results show a strong rejection of this hypothesis—see p-values of LR tests—indicating that even though countries may individually share the same degree of LM, interdependent shocks are propagated differently. That is, individual volatilities and volatility propagation in EBM—i.e., d_i and d_{ij}—may arise from different market sources.

4.3 The risk premium

Spreads in EBM are usually taken as measures of risk premium. However, spreads per se leave out the compensation required by investors for holding volatile sovereign instruments. Hence, to take this volatility component into account we now include the conditional variances as regressors as proxies for time varying risk premia.[20]

Tables 6 and 7 show mixed results with respect to the significance of individual risk premium estimates—see γ_{ij}. This seems consistent with Eichengreen & Mody (1998) who have suggested that individual risk premiums often fail to reflect changing economic conditions and to respond to changes in the spreads of other countries. Nonetheless, LR tests against the no in-mean specifications—see Panels (a) to (d) in Table 5—and SIC strongly confirm that time varying risk premia do jointly drive the behavior of excess returns in Emerging Bond Markets.

[19] Definitions for credit contagion can be found in Avellaneda & Wu (2001) or Giesecke & Weber (2003).

[20] In addition to these specifications, we also considered the inclusion of two additional in mean functions: $g = \sqrt{H_t}$ and $g = \log(H_t)$. Preliminary results suggest that the qualitative conclusions remain unchanged. Also, the squared root transformations seem to perform better than simple in-mean effects.

Overall and as it would be expected, during the period under analysis a greater volatility has been reflected in higher risk premia. The extent of individual risk premium in one market differs greatly from portfolio to portfolio. For instance, if we consider the portfolio Argentina-Brazil in the first column of Table 6,[21] we observe that the magnitude of the risk premium required to hold Argentinean bonds (γ_{11}) is equivalent to the size of the mean return standard deviation. Similarly, the risk premium for holding Brazilian bonds (γ_{22}) in this portfolio is equivalent to more than two standard deviations of its mean return. These magnitudes are quite substantial and indicate a stronger risk aversion by investors during the sample period.

Hence, holders of Argentinean bonds claimed a considerable premium for investing in its own instrument. However they might also have adjusted for the effect of holding Brazilian bonds in their portfolio. The cross effects—(γ_{12})—measure the extra premium arising from holding the second bond in the portfolio.[22] The magnitude of γ_{12} in the first column of Table 6 indicates that the additional compensation required by Argentinian investors for holding Brazilian bonds is not only large—two thirds of standard deviation of mean return—but also statistically significant. In contrast, Brazilian investors however do not significantly adjust their premiums to account for their Argentinian holdings, suggesting that cross premia are not necessarily symmetric.

Additionally, the risk premia here obtained could be interpreted as a broad estimate of the probability of default attached to EBM. The substantial levels of compensation for investing in EBM reflect the investors' perception of increased credit risk and could also show the high probability of default expected by the market during the sample period.[23]

4.4 In-Mean shared default risk perceptions

The high frequency nature of our model has prevented us from including consumption, prices or other critical fundamentals as regressors in the mean equation. However, our models are fully consistent with Eichengreen & Mody (1998) who have also argued that risk premia in emerging debt markets are incapable of adjusting to reflect changing economic conditions, information and news from other markets. An alternative to take fundamentals into account has been proposed by Baillie & Bollerslev (1990) who have suggested the inclusion of the asset return covariances as proxies of the comovement between bond returns and consumption. In this section we follow this approach by including the conditional covariances as regressors.

[21] Similar conclusions may be drawn for the analysis of all the portfolios in Tables 6, 7 and 8. We consider only one case to save space.

[22] These could also be interpreted as short-term contagion measures.

[23] This is a natural interpretation given the many events of financial distress, devaluations, defaults and crisis in LA markets during the end of the nineties.

The estimations of equation (13) using covariances as well as the variance in-Mean terms as regressors are shown in Table 8 where γ_{12} and γ_{21} now measure the impact of the covariances on excess returns. The results using these cross premia proxies are at best weakly significant. In fact, from the information in Panels (d) and (e) of Table 5, we observe that the only case in which there is a clear preference for covariance in-mean terms—as indicated by the optimized Likelihood value and Schwartz Information Criteria (SIC)—is in the portfolio Argentina-Brazil.[24]

The results seem to go in line with Eichengreen & Mody (1998) who have observed that changes in fundamentals, here proxied by conditional covariances, may only explain a fraction of spread compression in periods of crisis.

4.5 Econometric Performance

Teyssière (1997) and Teyssière (1998) have shown that the optimized log-likelihood value of unrestricted bivariate FIGARCH(1,d,1) models is in general greater than the CCC-FIGARCH(1,d,1) of Brunetti & Christopher (2000). Selection criteria (AIC and SIC) overwhelmingly favor the unrestricted bivariate $FIGARCH(1, d, 1)$ model lending support for time varying correlation models. As can be observed in Table 5 this finding is verified for Latin American EBM—compare Panels (a) and (c).

Interestingly, by comparing Teyssière (1998) and Brunetti & Christopher (2000) models against our *in-Mean* versions we observe in general that the inclusion of in-mean terms improves the optimized Likelihood value. Also, Schwartz Information Criterion (that penalizes the inclusion of additional parameters) strongly prefers our new in-Mean models. We test the restriction $Ho : \gamma_{11} = \gamma_{12} = \gamma_{21} = \gamma_{22} = 0$ using standard LR tests and as it can be observed, there is a clear rejection of the null in favor of in-mean parameters—see p-values in brackets in Panels (b) and (d) respectively.[25]

Among the different proxies of risk premia AIC and SIC seem to prefer the Variance-in-Mean terms. The optimized Likelihood function however is greater for the unrestricted specifications.

In terms of stability, apart from risk premia, parameter estimates remain highly significant and the magnitudes and precision do not seem to be affected after the inclusion of in-mean terms. A simulation study is being undertaken in continuing work to assess the relative improvement/performance of the QMLE estimation of the fractional differencing parameters in small samples. The models are also extended to consider the effect of asymmetries in a bivariate FIEGARCH context.

[24] The weak significance of these parameters is also in agreement with the results of Baillie & Bollerslev (1990).

[25] Such testing framework however renders mixed results for the case of covariance in mean parameters—see Panel (e).

5 Conclusions and discussion

The contribution of this paper has been twofold: first we have uncovered the high degree of volatility persistency, strong fractional comovement and risk premia of Latin American EBM's and, second, we have proposed a new family of bivariate long memory models to capture these salient features. These new models not only allow to derive time varying estimates of volatilities, correlations and investors' risk aversion series, but also account for the high predictability observed in these markets.

Long Memory in EBM is explained as the result of financial market rigidities and informational deficiencies. A potential implication of this finding is that domestic policies—e.g., capital controls—oriented at constraining the effect of sudden destabilization shocks may only be of temporary use. In line with the arguments of Forbes & Rigobon (2000) short-run isolation strategies may be costly and only delay a country's adjustment to equilibrium.

The results also suggest a high degree of fractional *comovement* in these markets. The default risk contagion parameters are in all cases highly significant. This outcome is consistent with the claims of Mauro et al. (2000), Fiess (2003) and Cunninham et al. (2001) who report strong comovement of spreads in EBM.

We find that the individual degrees of LM are not statistically different. This lends support to the conjecture that a common market factor drives EBM spreads. Episodes of contagion for instance may have a common base being either Brady markets (Baig & Goldfajn (2000)) or any other developed financial market acting as the conduit between regions of developing markets—Kaminsky & Reinhart (2002). In order to deal with pervasive shock, LA countries should then *coordinate* their efforts rather than cope with turmoils in an individual basis.

Derivative markets on these instruments are not still fully developed. For this reason, focusing on volatilities implicit in market prices of option-like credit derivatives would not be sensible. Bielcki and Rutowski (2001) have already observed that the valuation of credit derivatives requires to take into account credit spread volatilities and, if several distinct assets are modeled simultaneously, credit spread correlations. The models proposed here provide estimates of these parameters.

In addition, policy makers and investors in LA sovereign bond markets might be interested in assessing the effect of contagious shocks on the volatility of its own market and on the market perception of default risk. The models presented here are capable of capturing such phenomena.

Another potential implication of the LM finding relates to the pricing of derivatives. Baillie et al. (1996) have already showed that valuation of derivatives under the presence of Long Memory may be highly unreliable and heavy loses are possible. A future line of research should investigate the effect of LM in the valuation of sovereign basket derivatives.

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Table 4: EMBI, unrestricted FIGARCH(1,d,1) QMLE estimations.

	(s_{art}, s_{brt}^{a})	(s_{art}, s_{mxt})	(s_{art}, s_{vet})	(s_{brt}, s_{mxt})	(s_{brt}, s_{vet})	(s_{mxt}, s_{vet})
Conditional Mean						
μ_1	-0.0475	-0.0552	-0.0348	-0.0223	-0.0183	-0.0415
	(0.0250)	(0.0250)	(0.0251)	(0.0134)	(0.0133)	(0.0176)
μ_2	-0.0572	-0.0481	-0.0569	-0.0309	-0.0316	-0.0372
	(0.0189)	(0.0250)	(0.0231)	(0.0170)	(0.0191)	(0.0202)
Conditional Variances						
ω_1	0.1133	0.1769	0.1549	0.0228	0.0223	0.0404
	(0.0112)	(0.0272)	(0.0269)	(0.0134)	(0.0050)	(0.0068)
β_1	0.4784	0.3588	0.2684	0.5063	0.5204	0.5483
	(0.0138)	(0.0807)	(0.0894)	(0.0603)	(0.0189)	(0.0176)
ϕ_1	0.3230	0.1809	0.0928	0.2946	0.3071	0.3121
	(0.0122)	(0.0703)	(0.0835)	(0.0581)	(0.0125)	(0.0136)
d_1	0.3539	0.3900	0.3019	0.3788	0.3858	0.3759
	(0.0245)	(0.0310)	(0.0268)	(0.0262)	(0.0251)	(0.0272)
ω_2	0.0748	0.1947	0.0972	0.0431	0.1007	0.1234
	(0.0131)	(0.0286)	(0.0188)	(0.0073)	(0.0149)	(0.0182)
β_2	0.1886	0.3424	0.4818	0.5456	0.4624	0.4395
	(0.0539)	(0.0733)	(0.0253)	(0.0202)	(0.0215)	(0.0224)
ϕ_2	0.0011	0.1454	0.3483	0.3018	0.3635	0.3768
	(0.0446)	(0.0601)	(0.0169)	(0.0152)	(0.0129)	(0.0123)
d_2	0.3441	0.4020	0.3034	0.3965	0.2729	0.2463
	(0.0232)	(0.0330)	(0.0338)	(0.0304)	(0.0258)	(0.0245)
Conditional Covariance						
ω_{12}	0.0573	0.1282	0.0837	0.0149	0.0201	0.0369
	(0.0093)	(0.0181)	(0.0191)	(0.0052)	(0.0061)	(0.0096)
β_{12}	0.3599	0.3842	0.4348	0.2976	0.5134	0.5215
	(0.0299)	(0.0677)	(0.0650)	(0.0199)	(0.0168)	(0.0169)
ϕ_{12}	0.1971	0.2311	0.2819	0.3055	0.3576	0.3780
	(0.0310)	(0.0666)	(0.0626)	(0.0408)	(0.0110)	(0.0116)
d_{12}	0.3599	0.3391	0.2582	0.4989	0.2848	0.2439
	(0.0299)	(0.0260)	(0.0249)	(0.0448)	(0.0220)	(0.0233)

[a] See notes in Table 3.

Table 5: EMBI, decision criteria FIGARCH(1,d,1) estimations.

	(s_{art}, s^a_{brt})	(s_{art}, s_{mxt})	(s_{art}, s_{vet})	(s_{brt}, s_{mxt})	(s_{brt}, s_{vet})	(s_{mxt}, s_{vet})
Panel (a): *Brunetti* $CCC - FIGARCH(1,d,1)$						
$L(\theta)^b$	-4,911.3	-5,989.1	-5,361.4	-5,624.1	-5,391.9	-6,389.3
AIC[c]	9,844.6	12,000.3	10,744.7	11,270.3	10,805.9	12,800.7
SIC[d]	9,908.2	12,063.8	10,808.4	11,336.3	10,871.9	12,866.7
$L(\theta_{d_1=d_2})^e$	-4,911.9	-5,991.3	-5,361.6	-5,624.3	-5,397.9	-6393.9
LR^f	[0.5488][g]	[0.1108]	[0.8187]	[0.8190]	[0.0025]	[0.0101]
Panel (b): *CCC-FIGARCH(1,d,1) Variance in-Mean*						
$L(\theta)$	-4,906.0	-5,982.5	-5,654.7	-5,619.9	-5,386.4	-6,648.6
LR	[0.0314][h]	[0.0103]	[—]	[0.0780]	[0.0266]	[—]
AIC	9,842.0	11,995.0	11,159.4	11,269.9	10,802.7	13,327.2
SIC	9,928.7	12,081.7	11,246.1	11,359.9	10,892.9	13,417.3
Panel (c): *Unrestricted FIGARCH(1,d,1)*						
$L(\theta)$	-4,737.7	-5,794.5	-5,268.4	-5,524.4	-5,300.8	-6,342.7
AIC	9,503.5	11,617.0	10,564.8	11,076.8	10,629.6	12,713.4
SIC	9,584.4	11,697.9	10,645.8	11,160.9	10,713.7	12,797.5
$L(\theta_{d_1=d_2})$	-4,737.9	-5,794.5	-5,268.4	-5,524.5	-5,304.5	-6,347.8
LR	[0.4966]	[0.9999]	[0.9999]	[0.9048]	[0.0247]	[0.0074]
$L(\theta_{d_1=d_2=d_{12}})$	-4,810.3	-5,840.3	-5,275.3	-5,540.1	-5,309.4	-6,357.1
LR	[—]	[0.0001]	[0.0032]	[0.0000]	[0.0006]	[0.0001]
Panel (d): *Unrestricted-FIGARCH(1,d,1) Variance in-Mean*						
$L(\theta)$	-4,731.0	-5,786.4	-5,260.2	-5,519.1	-5,297.6	-6,339.5
LR	[0.0095][i]	[0.0028]	[0.0025]	[0.0314]	[0.1712]	[0.1700]
AIC	9,498.1	11,608.8	10,556.5	11,074.2	10,631.2	12,715.0
SIC	9,602.1	11,712.9	10,660.5	11,182.4	10,739.4	12,823.2
Panel (e): *Unrestricted-FIGARCH(1,d,1) Covariance in-Mean*						
$L(\theta)$	-4,729.3	-5,787.4	-5,261.9	-5,522.4	-5,297.5	-6,339.9
LR	[0.0021][i]	[0.0067]	[0.0113]	[0.4060]	[0.1586]	[0.2311]
AIC	9,495.2	11,610.8	10,559.9	11,080.9	10,630.5	12,715.7
SIC	9,598.7	11,714.9	10,663.9	11,188.9	10,739.2	12,823.9

[a]Subindex i,j indicate the bond components in a portfolio. [b]Maximized Log-likelihood. [c]Akaike Information Criteria. [d]Schwartz Information Criterion. [e]Maximized likelihood function with restriction in sub-index. [f]LR stands for Likelihood Ratio and is computed as LR=-2×[L(θo)-L(θ)] where LR~χ^2_m with m being to the number of linear restrictions.[g]P-values corresponding to the Likelihood Ratio between the restricted model and the extended models respectively. [h] P-value of the LR between the in-mean version and the CCC-FIGARCH(1,d,1) model—Panels (b) and (a) respectively. [i]P-value of the LR between the in-mean version and Teyssière FIGARCH(1,d,1) model—Panels (d) and (e) against (c) respectively.

Table 6: EMBI, CCC-FIGARCH(1,d,1) Variance-in-Mean.

	(s_{art}, s_{brt})	(s_{art}, s_{mxt})	(s_{art}, s_{vet})	(s_{brt}, s_{mxt})	(s_{brt}, s_{vet})	(s_{mxt}, s_{vet})
Conditional Mean[a]						
μ_1	-0.0963	-0.1320	-0.1336	-0.0429	-0.0849	-0.0655
	(0.0277)	(0.0305)	(0.0283)	(0.0173)	(0.0179)	(0.0211)
μ_2	-0.1057	-0.0917	-0.1162	-0.0473	-0.1131	-0.0698
	(0.0184)	(0.0299)	(0.0273)	(0.0233)	(0.0328)	(0.0252)
γ_{11}	0.0283	0.0289	0.0294	-0.0234	-0.0137	0.0082
	(0.0143)	(0.0165)	(0.0139)	(0.0288)	(0.0252)	(0.00147)
γ_{12}	0.0176	0.0205	0.0321	0.0293	0.0547	0.0139
	(0.0115)	(0.0127)	(0.0071)	(0.0142)	(0.0140)	(0.0071)
γ_{21}	0.0010	0.0025	-0.0004	0.0349	0.0009	0.0037
	(0.0049)	(0.0068)	(0.0051)	(0.0278)	(0.0258)	(0.0097)
γ_{22}	0.0519	0.0203	0.0487	-0.0106	0.0644	0.0289
	(0.0189)	(0.0164)	(0.0167)	(0.0227)	(0.0315)	(0.0186)
Conditional Variances						
ω_1	0.1426	-0.0917	0.1236	0.0327	0.0363	0.0591
	(0.0134)	(0.0299)	(0.0124)	(0.0058)	(0.0058)	(0.0081)
β_1	0.4420	0.4741	0.4415	0.5002	0.4972	0.5041
	(0.0233)	(0.0755)	(0.0245)	(0.0220)	(0.0213)	(0.0528)
ϕ_1	0.3183	0.2586	0.3132	0.2858	0.2981	0.2749
	(0.0183)	(0.0589)	(0.0187)	(0.0147)	(0.0143)	(0.0464)
d_1	0.3634	0.4599	0.3736	0.4284	0.4037	0.4429
	(0.0366)	(0.0458)	(0.0373)	(0.0294)	(0.0286)	(0.0346)
ω_2	0.0929	0.1379	0.1653	0.0513	0.1567	0.1813
	(0.0179)	(0.0177)	(0.0218)	(0.0076)	(0.0207)	(0.0177)
β_2	0.2093	0.6167	0.3278	0.5442	0.3976	0.2958
	(0.1279)	(0.0597)	(0.0287)	(0.0229)	(0.0239)	(0.0242)
ϕ_2	0.0593	0.1399	0.3338	0.2968	0.3854	0.3561
	(0.1115)	(0.0377)	(0.0168)	(0.0176)	(0.0125)	(0.0131)
d_2	0.4169	0.6631	0.3324	0.4064	0.2293	0.2879
	(0.0349)	(0.0715)	(0.0337)	(0.0351)	(0.0249)	(0.0262)
ρ_{12}	0.7154	0.6088	0.5943	0.5198	0.5627	0.4982
	(0.0053)	(0.0080)	(0.0089)	(0.0099)	(0.0108)	(0.0101)

[a] See notes in Table 3.

Table 7: EMBI, unrestricted FIGARCH(1,d,1) Variance-in-Mean.

	(s_{art}, s_{brt})	(s_{art}, s_{mxt})	(s_{art}, s_{vet})	(s_{brt}, s_{mxt})	(s_{brt}, s_{vet})	(s_{mxt}, s_{vet})
Conditional Mean[a]						
μ_1	-0.1063	-0.1556	-0.1712	-0.0598	-0.0652	-0.0759
	(0.0307)	(0.0317)	(0.0400)	(0.0213)	(0.0196)	(0.0267)
μ_2	-0.1169	-0.1115	-0.1832	-0.0605	-0.0920	-0.1165
	(0.0234)	(0.0329)	(0.0387)	(0.0268)	(0.0303)	(0.0339)
γ_{11}	0.0249	0.0294	0.0479	-0.0139	-0.0084	0.0163
	(0.0153)	(0.0154)	(0.0218)	(0.0298)	(0.0254)	(0.0211)
γ_{12}	0.0262	0.0273	0.0472	0.0338	0.0429	0.0136
	(0.0197)	(0.0121)	(0.0286)	(0.0149)	(0.0170)	(0.0231)
γ_{21}	0.0013	-0.0001	0.0149	0.0439	0.0102	0.0139
	(0.0062)	(0.0083)	(0.0127)	(0.0308)	(0.0274)	(0.0168)
γ_{22}	0.0656	0.0479	0.0779	-0.0087	0.0405	0.0502
	(0.0184)	(0.0366)	(0.0271)	(0.0230)	(0.0304)	(0.0292)
Conditional variances						
ω_1	0.1116	0.1709	0.1490	0.0243	0.0227	0.0408
	(0.0111)	(0.0262)	(0.0269)	(0.0089)	(0.0050)	(0.0068)
β_1	0.4788	0.3639	0.2550	0.4872	0.5186	0.5479
	(0.0140)	(0.0839)	(0.0947)	(0.0809)	(0.0189)	(0.0178)
ϕ_1	0.3237	0.1883	0.0847	0.2812	0.3114	0.3127
	(0.0125)	(0.0728)	(0.0878)	(0.0740)	(0.0125)	(0.0140)
d_1	0.3526	0.3884	0.2939	0.3685	0.3772	0.3746
	(0.0250)	(0.0323)	(0.0269)	(0.0354)	(0.0249)	(0.0281)
ω_2	0.0703	0.1832	0.0908	0.0436	0.0991	0.1191
	(0.0127)	(0.0276)	(0.0192)	(0.0114)	(0.0150)	(0.0183)
β_2	0.1984	0.3638	0.4811	0.5433	0.4632	0.4436
	(0.0538)	(0.0693)	(0.0243)	(0.0242)	(0.0211)	(0.0222)
ϕ_2	0.0146	0.1649	0.3535	0.3016	0.3644	0.3774
	(0.0453)	(0.0571)	(0.0163)	(0.0173)	(0.0128)	(0.0122)
d_2	0.3393	0.3992	0.2930	0.3965	0.2713	0.2451
	(0.0237)	(0.0339)	(0.0326)	(0.0346)	(0.0256)	(0.0244)
Conditional Covariance						
ω_{12}	0.0555	0.1152	0.0785	0.0162	0.0205	0.0370
	(0.0086)	(0.0165)	(0.0191)	(0.0081)	(0.0061)	(0.0096)
β_{12}	0.3618	0.4258	0.4308	0.4850	0.5123	0.5215
	(0.0301)	(0.0614)	(0.0692)	(0.0596)	(0.0168)	(0.0168)
ϕ_{12}	0.1998	0.2687	0.2855	0.2921	0.3597	0.3789
	(0.0319)	(0.0628)	(0.0668)	(0.0536)	(0.0110)	(0.0117)
d_{12}	0.3072	0.3419	0.2502	0.2952	0.2807	0.2422
	(0.0194)	(0.0268)	(0.0245)	(0.0260)	(0.0219)	(0.0234)

[a] See notes in Table 3.

Table 8: EMBI, unrestricted FIGARCH(1,d,1) Covariance-in-Mean.

	(s_{art}, s_{brt})	(s_{art}, s_{mxt})	(s_{art}, s_{vet})	(s_{brt}, s_{mxt})	(s_{brt}, s_{vet})	(s_{mxt}, s_{vet})
Conditional Mean[a]						
μ_1	-0.1188	-0.1424	-0.1381	-0.0390	-0.0368	-0.0638
	(0.0306)	(0.0312)	(0.0378)	(0.0197)	(0.0173)	(0.0244)
μ_2	-0.1222	-0.0939	-0.1549	-0.0382	-0.0863	-0.1095
	(0.0231)	(0.0322)	(0.0368)	(0.0253)	(0.0305)	(0.0329)
γ_{11}	0.0450	0.0457	0.0409	-0.0303	-0.0777	0.0010
	(0.0219)	(0.0187)	(0.0255)	(0.0398)	(0.0429)	(0.0288)
γ_{12}	0.0080	0.0137	0.0543	0.0775	0.1499	0.0425
	(0.0355)	(0.0258)	(0.0501)	(0.0512)	(0.0592)	(0.0611)
γ_{21}	0.0700	0.0517	0.0396	0.0663	-0.0038	-0.0046
	(0.0317)	(0.0344)	(0.0485)	(0.0566)	(0.0556)	(0.0546)
γ_{22}	0.0031	-0.0058	0.0544	-0.0210	0.0508	0.0662
	(0.0346)	(0.0261)	(0.0395)	(0.0280)	(0.0382)	(0.0374)
Conditional Variances						
ω_1	0.1127	0.1716	0.1553	0.0243	0.0242	0.0407
	(0.0112)	(0.0260)	(0.0275)	(0.0089)	(0.0057)	(0.0068)
β_1	0.4765	0.3657	0.2538	0.4904	0.5044	0.5482
	(0.0143)	(0.0821)	(0.0948)	(0.2812)	(0.0474)	(0.0178)
ϕ_1	0.3236	0.1899	0.0841	0.2812	0.2969	0.3121
	(0.0127)	(0.0715)	(0.0880)	(0.0739)	(0.0431)	(0.0139)
d_1	0.3528	0.3858	0.2946	0.3734	0.3776	0.3758
	(0.0253)	(0.0319)	(0.0271)	(0.0359)	(0.0266)	(0.0278)
ω_2	0.0712	0.1830	0.0947	0.0436	0.1004	0.1213
	(0.0128)	(0.0276)	(0.0193)	(0.0113)	(0.0150)	(0.0185)
β_2	0.1928	0.3727	0.4790	0.5451	0.4627	0.4414
	(0.0538)	(0.0697)	(0.0247)	(0.0243)	(0.0211)	(0.0222)
ϕ_2	0.0074	0.1731	0.3529	0.3011	0.3648	0.3784
	(0.0449)	(0.0580)	(0.0164)	(0.0174)	(0.0127)	(0.0122)
d_2	0.3412	0.4036	0.2942	0.3987	0.2704	0.2431
	(0.0237)	(0.0343)	(0.0329)	(0.0348)	(0.0254)	(0.0243)
Conditional covariance						
ω_{12}	0.0562	0.1168	0.0825	0.0162	0.0213	0.0372
	(0.0087)	(0.0165)	(0.0194)	(0.0082)	(0.0062)	(0.0096)
β_{12}	0.3581	0.4276	0.4312	0.4867	0.5121	0.5219
	(0.0302)	(0.0609)	(0.0692)	(0.0599)	(0.0166)	(0.0168)
ϕ_{12}	0.1974	0.2703	0.2867	0.2928	0.3587	0.3789
	(0.0319)	(0.0625)	(0.0668)	(0.0536)	(0.0110)	(0.0116)
d_{12}	0.3082	0.3428	0.2508	0.2971	0.2825	0.2420
	(0.0195)	(0.0269)	(0.0246)	(0.0263)	(0.0219)	(0.0233)

[a] See notes in Table 3.