Common monetary policy with transmission asymmetry and uncertainty: Is there a case for national data in EMU?

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This version:
June 2003

Abstract

In this paper we address the issue of how uncertainty could affect the choice between a federal monetary policy based on national data and one on area-wide aggregated data in a monetary union with asymmetry in the transmission of monetary policy. We find that the uncertainty about the transmission process increases the need to take into account information about national economies in the formulation of optimal monetary policies whereas the introduction of imperfect forecasts (and, thereby, of additive uncertainty) implies a trade-off between the relative accuracy of the (aggregated versus national) forecast and the asymmetry in the transmission of monetary policy. Under both cases however, a national based monetary policy is likely to be preferred compared to a strategy relying uniquely on Union-wide aggregates.

1 Introduction

The conduct of monetary policy in Euroland is made difficult because of the existence of asymmetries within the union. Asymmetries exist both at the level of the macroeconomic shocks to which members of the union are subjected and at the level of the transmission of monetary policies. Recent theoretical analysis has shown that the existence of asymmetries in the transmission of monetary policy actions of the ECB calls for a design of monetary policies that takes into account national data. Thus, in order for monetary policies to be set optimally it is not sufficient to use area-wide (euro) data on inflation and output gaps, but also to consider non-aggregated national data on these same variables if asymmetries in the transmission of monetary policies exist (see De Grauwe (2000) and Gros and Hefeker (2002)). Empirical evidence seems to support this view in the case of the Federal Reserve System of Central Banks in the United States (see Meade and Sheets (2002), and Heinemann and Hüfner (2002)).

The previous conclusion has been derived in the context of models in which there is no uncertainty surrounding the design of monetary policy. The issue that arises here is whether this conclusion continues to hold when uncertainty about the transmission process exists (multiplicative uncertainty) or when the policymaker has to decide upon the monetary policy on the basis of forecasts which reflect an imperfect knowledge of the shocks which may hit the economies (additive uncertainty).

Monetary policy transmission uncertainty is an important issue in the European context. According to several economists (see, among others, Dornbusch, Favero and Giavazzi (1998), Mihov (2001) and ECB (2001)), the creation of EMU is likely to have strengthened the degree of uncertainty surrounding the transmission of monetary policy measures within the Union.

The optimal design of monetary policy when transmission uncertainty exists has been analysed in detail in the theoretical literature. The main insight provided by this literature is that transmission uncertainty may call for more caution from the monetary authorities. Faced with this kind of uncertainty, the authorities will tend to stabilize less that when no such uncertainty exists (see Brainard (1967) for the original argument and Söderström (1999) and Peersman and Smets (1999) for an application to the European context)[1].

Regarding the use of forecasts in the implementation of monetary policy in EMU, several articles have emphasized the role played by the former in the Central Bank transparency issue (see Cuckierman (2001) or Svensson (1999)), especially for the ECB. But few address whether the national or Union-wide based content of such forecasts could impinge on the stabilisation properties of monetary policy in a monetary union, Some empirical studies (see, e.g., Massimiliano *et alii* (2003)) have shown that such a difference could matter however,

[1]Empirical evidence on the caution principle is more ambiguous (see European Central Bank (2001)).

by influencing the predictive power of the forecast at the Union level. Combining aggregation and uncertainty issues at a theoretical level may thus prove to be instructive in this respect and take advantage of this empirical evidence.

In this paper, we develop a model of a monetary union in which the transmission of policy induced changes in the interest rate is asymmetric. We do this when there is no uncertainty surrounding the design of monetary policy. Then we first extend the model allowing for uncertainty in the transmission process. In a second time, we address the issue of additive uncertainty by introducing forecast-based monetary policies. Under both cases, we are thus able to analyse how the uncertainty and asymmetry issues interact and how this interaction may impinge on the choice of a monetary strategy in EMU

2 National aggregation *versus* Union-wide aggregates : how to deal with transmission heterogeneity?

2.1 The modelling framework

We use a standard macroeconomic model and apply it to a monetary union framework. The asymmetry is introduced in the model by considering that the features of the national Phillips curves differ from one country to the other, so that:

$$U_i = U_i^* - a_i \cdot (\pi_i - \pi_i^e) + \varepsilon_i$$

i is the country $i = 1, 2, ..., N$

U_i is the unemployment rate in the country i and U_i^* is its natural counterpart. a_i denotes the transmission parameter of (unexpected) inflation impulses to the unemployment gap. As our objective is to analyse the implications of asymmetries in transmission, we assume that this coefficient differs across countries.

We will not assume asymmetry in the shocks. This has been done elsewhere (see De Grauwe (2000)). We focus on the asymmetries in the transmission process because this is where the uncertainty will arise (see *infra*). Thus, we suppose that $\varepsilon_i = \varepsilon$ for all i. Put differently, we intend to analyse a world of symmetric shocks that are transmitted asymmetrically.

π_i refers to the inflation rate of country i. It will be assumed that when the countries in the model form a monetary union the inflation rate is the same in all countries. We have two reasons to do this. First, as it is usually the case in the literature, we suppose that the monetary authorities directly control the inflation rate. Second, as monetary policy is determined in a centralised fashion when a monetary union exists, the member countries share common monetary conditions in the Union, which should lead to the same rates of inflation. There is of course evidence indicating that inflation rates in the eurozone differ across countries. However, it is likely that those inflation differentials are very much influenced by the Balassa-Samuelson effect. Since this is primarily a structural feature, it should not be very much influenced by monetary policy.

The single monetary policy in a monetary union can be designed in two ways.

1- First, the common (federal) Central Bank may choose to minimise a weighted average of national loss functions. We define this strategy as a **national aggregation** (**NA**) procedure. The national loss function depends on the squared deviations of inflation and output from target levels in the following way:

$$L_i \equiv (\pi_i)^2 + b \cdot (U_i - U_i^*)^2$$

where b denotes the relative weight of the unemployment gap with respect to inflation in the loss function. Note that for the sake of convenience we set the target rate of inflation equal to 0. In addition, we assume that the unemployment target of the authorities coincides with the natural unemployment. As a result, we disregard issues relating to credibility. Indeed, we want to emphasize that the monetary authorities are likely to face the heterogeneity in the transmission of monetary policy, regardless of their potential time-inconsistency.

In the (NA) scenario therefore, the central bank of the monetary union determines its optimal strategy by minimising the "average" of the loss functions of the member countries in the Union[2]:

$$\Lambda^{NA} \equiv \sum_{i=1}^{i=N} \mu_i \cdot L_i$$

μ_i is the weight associated to country i in the computation of the aggregate loss function. We have: $\sum_{i=1}^{i=N} \mu_i = 1$.

As the inflation rate is common to all the member countries, we may rewrite the former expression as:

$$\Lambda^{NA}(\pi) = (\pi)^2 + b \cdot \sum_{i=1}^{i=N} \mu_i \cdot (U_i - U_i^*)^2 \tag{1}$$

Since $(U_i - U_i^*)$ depends on the (rationally) unexpected component of the (common) inflation rate, Λ^{NA} will be a function of π (for a given value of the

[2]By convention, X_i^{NA} (resp. X_i^{EA}) will refer in the following to the value taken by the (endogenous) variable X_i when the so-called national-aggregation (resp. euro-aggregation) strategy is implemented by the Central Bank.

shock, ε). In the following, we define π^{NA} as the optimal inflation rate under the NA strategy (i.e. the one for which Λ^{NA} is minimal).

2- The second scenario refers to a strategy where the Central Bank minimizes a loss function defined in terms of Union-wide aggregate variables, i.e. an average inflation rate and an average unemployment rate. As we implicitly refer to the EMU case, we designate such a strategy as a **euro-aggregation** (**EA**) procedure, which we specify as follows:

$$\begin{aligned} U_E &\equiv \sum_{i=1}^{i=N} \mu_i \cdot U_i \\ U_E^* &\equiv \sum_{i=1}^{i=N} \mu_i \cdot U_i^* \\ \pi_E &\equiv \sum_{i=1}^{i=N} \mu_i \cdot \pi_i = \pi \end{aligned}$$

where the subscript E refers to a variable defined at the Union level.
The relevant loss function may then be defined as follows:

$$\begin{aligned} \Lambda^{EA} &\equiv (\pi)^2 + b \cdot (U_E - U_E^*)^2 \\ \Lambda^{EA}(\pi) &= (\pi)^2 + b \cdot \left[\sum_{i=1}^{i=N} \mu_i \cdot (U_i - U_i^*) \right]^2 \end{aligned} \tag{2}$$

Because of the linearity of the national Phillips relationships, the aggregation rule allows for the existence of a Union-wide Phillips "curve" between the aggregate inflation rate and the "mean" unemployment gap. Thus we have

$$U_E = U_E^* - a_E \cdot (\pi - \pi^e) + \varepsilon$$

with $a_E \equiv \sum_1^N \mu_i \cdot a_i$, which could be qualified as the *mean* transmission parameter.

Again we observe that Λ^{EA} is a function of π. In the following we define π^{EA} as the optimal inflation rate under the EA strategy (i.e. the one for which Λ^{EA} is minimal).

Finally, both strategies have to be compared using a common welfare measure. As we consider an explicit heterogeneity in the transmission mechanisms, the benchmark used is the weighted average of the ex ante (expected) national losses obtained under each of the two alternatives. Thus welfare is defined as:

$$W \equiv E_\varepsilon \left[\sum_{i=1}^{i=N} \mu_i \cdot L_i \right] \tag{3}$$

where E_ε is the expectation operator taken with respect to the distribution of ε. We posit $E_\varepsilon[\varepsilon] = 0$ and $E_\varepsilon[\varepsilon^2] = \sigma_\varepsilon^2$. Note that $W = E_\varepsilon[\Lambda^{NA}]$.

2.2 Comparison of the strategies

We now examine the properties of the two strategies in more detail.

1- Under the first scenario, the Central bank determines π^{NA} such that it minimises Λ^{NA} subject to the constraint of national Phillips "curves" prevailing in the member countries while taking as given the value of the shock (ε) and the private sector's expectations of the inflation rate implemented under this strategy (π^e). Thus, we have

$$\begin{aligned}\pi^{NA} &= \arg\min_{\pi} \Lambda^{NA} \\ & \quad s.t. \left\{ \begin{array}{c} \pi^e,\ \varepsilon\ given \\ U_i = U_i^* - a_i \cdot (\pi - \pi^{e,}) + \varepsilon,\ \text{for } i = 1, 2, ..., N \end{array} \right.\end{aligned}$$

Solving this program (including the computation of the rational expected inflation rate at equilibrium) leads to:

$$\begin{aligned}\pi^{NA} &= \frac{ba_E}{1 + b\left(a_E^2 + \theta_{a_E}^2\right)}\varepsilon \\ &= \Omega_{NA} \cdot \varepsilon\end{aligned}$$

with: $\Omega_{NA} \equiv \dfrac{ba_E}{1 + b\left(a_E^2 + \theta_{a_E}^2\right)}$ and $\theta_{a_E}^2 \equiv \sum_1^N \mu_i \cdot (a_i - a_E)^2$ which is a measure of the dispersion in the national transmission parameters. Thus, $\theta_{a_E}^2$ measures the asymmetry in the transmission process. Besides, we have the following relation: $\overline{a_E^2} = a_E^2 + \theta_{a_E}^2$ with $\overline{a_E^2} \equiv \sum_1^N \mu_i \cdot a_i^2$.

We observe that when the asymmetry in the transmission process increases, the authorities' optimal inflation rate reacts less to shocks. The counterpart of this lessening in the inflation variability may be an increasing volatility of the national unemployment rate with the size in the transmission asymmetry[3]. This is seen from the following expression of the equilibrium unemployment rate (obtained by substituting the optimal inflation rate in the Phillips curve):

$$U_i^{NA} = U_i^* + (1 - \Omega_{NA} \cdot a_i) \cdot \varepsilon$$

2- Under the second scenario, the Central Bank minimises the loss function based on the Union-wide unemployment and inflation rates. The constraint is then given by the Union-wide Phillips relationship. We obtain:

[3]It is important to note that this trade-off does not necessarily prevail however. A sufficient condition to ensure its existence would be that the sensitivity of the unemployment gap to monetary policy in the country considered has to be smaller than the one which can be contemplated at the Union-wide level, that is $a_i < a_E$.

$$\begin{aligned} \pi^{EA} &= \arg\min_{\pi} \Lambda^{EA} \\ & \quad s.t. \left\{ \begin{array}{c} \pi^e,\ \varepsilon\ given \\ U_E = U_E^* - a_E \cdot (\pi - \pi^e) + \varepsilon \end{array} \right. \end{aligned}$$

which leads to:

$$\begin{aligned} \pi^{EA} &= \frac{ba_E}{1+ba_E^2}\varepsilon \\ &= \Omega_{EA} \cdot \varepsilon \end{aligned}$$

with $\Omega_{EA} \equiv \dfrac{ba_E}{1+ba_E^2}$.

Note that this is the same optimal inflation rate which would be obtained if the model had been applied to the case of a single country (whose role is played in our framework by the monetary union).

As in the first scenario the equilibrium level of the national unemployment rates can be obtained by substituting the optimal inflation rate into the national Phillips curves, i.e.:

$$U_i^{EA} = U_i^* + (1 - \Omega_{EA} \cdot a_i) \cdot \varepsilon$$

From these results we conclude that under a strategy which aims at minimising the variability of *Union-wide* variables, the asymmetry in the transmission of the common supply shocks does not act as a motive for changing the inflation rate and, thereby, for affecting the variability of the national unemployment rates.

A welfare comparison of both strategies goes through the computation of the weighted average of expected national losses after having substituted the relevant values of the inflation and unemployment rates in (3). We obtain

$$W^{NA} = \left[(\Omega_{NA})^2 + b \cdot \sum_{i=1}^{i=N} \mu_i \cdot (1 - \Omega_{NA} \cdot a_i)^2 \right] \cdot \sigma_\varepsilon^2 \qquad (4)$$

$$W^{EA} = \left[(\Omega_{EA})^2 + b \cdot \sum_{i=1}^{i=N} \mu_i \cdot (1 - \Omega_{EA} \cdot a_i)^2 \right] \cdot \sigma_\varepsilon^2 \qquad (5)$$

The relative benefits of a national aggregation strategy *versus* a Union-wide procedure are thus given by the differential loss, $\Delta W \equiv \frac{1}{\sigma_\varepsilon^2}\left(W^{EA} - W^{NA}\right)$, ie:

$$\begin{aligned}
\Delta W &= (\Omega_{EA})^2 - (\Omega_{NA})^2 + b \cdot \sum_{i=1}^{i=N} \mu_i \cdot \left[(1-\Omega_{EA}\cdot a_i)^2 - (1-\Omega_{NA}\cdot a_i)^2\right] \\
&= \left(1 + b\cdot \overline{a_E^2}\right)\left[(\Omega_{EA})^2 - (\Omega_{NA})^2\right] + 2\cdot b\cdot a_E \cdot [(\Omega_{NA}) - (\Omega_{EA})] \qquad (6)
\end{aligned}$$

Simplifying this expression[4] leads to:

$$\Delta W \equiv [(\Omega_{EA}) - (\Omega_{NA})]\cdot \Omega_{EA}\cdot b\cdot \theta_{a_E}^2 \qquad (7)$$

which is positive as $\Omega_{EA} > \Omega_{NA}$.

Thus adopting a national aggregation perspective is better than relying on a Union-wide strategy.

The comparison of the two loss functions may enlight the reasons why the NA strategy has to be favored[5].

Let define the national unemployment gap, U_i^g, as: $U_i^g \equiv U_i - U_i^*$. Manipulating the loss functions, given by equations (1) and (2), we obtain[6]:

$$\begin{aligned}
\Lambda^{NA}(\pi) &= \Lambda^{EA}(\pi) + b\cdot\left(\sum_{i=1}^{i=N} \mu_i \cdot (U_i^g - U_E^g)^2\right) \qquad (8) \\
&= \Lambda^{EA}(\pi) + b\cdot \theta_{U^g}^2
\end{aligned}$$

with $\theta_{U^g}^2 \equiv \sum_{i=1}^{i=N} \mu_i \cdot (U_i^g - U_E^g)^2$, which can be considered as a measure of the dispersion between the national unemployment rates. At equilibrium, this component depends on the inflation rate which is chosen, thus $\theta_{U^g}^2$ may be written as a function of π, $\theta_{U^g}^2 \equiv \theta_{U^g}^2(\pi)$.

Deriving this expression leads to two interesting and interrelated properties:

- First, we observe that the two strategies are equivalent if and only if there is no dispersion between the unemployment gaps $\left(\theta_{U^g}^2 = 0\right)$ and/or there is no output goal in the loss function of the monetary authorities ($b = 0$).
- Second, given the framework we have retained, there is *only one* strategy which would satisfy the welfare maximising criteria we have imposed (eq. 3), namely the choice of the national aggregation procedure. Put differently, $\Lambda^{NA}\left(\pi^{NA}\right) < \Lambda^{NA}\left(\pi^{EA}\right)$.

[4] On this point, it seems that Gros and Hefeker (2002, p.10) have (mistakenly?) obtained conditions which are superfluous with respect to the result they derive.

[5] See Annex A for further results on this comparison.

[6] The properties of the mean operator imply that $\sum_{i=1}^{i=N} \mu_i \cdot (U_i^g - U_E^g) = 0$.

3 Introducing parameter transmission uncertainty in an heterogeneous monetary union

In the foregoing, we have shown that adopting a national aggregation perspective is unambiguously a better strategy to deal with asymmetries in the transmission mechanisms than to rely on Union-wide aggregates. The question that arises now is whether this conclusion is maintained when we introduce uncertainty about the transmission mechanisms.

3.1 Uncertainty at different levels of aggregation

The latter question is addressed in the model in the following way. Let suppose that the creation of the monetary Union modifies the Phillips relationship between national variables so that the coefficient a_i can no more be known with certainty by the authorities in charge of the common monetary policy but must be considered as a random variable.

In order to account for this change in regime and to distinguish from non-random variables in the model, we redefine the national Phillips curve slope parameter of country i as $\widetilde{a}_i$:

$$U_i = U_i^* - \widetilde{a}_i \cdot (\pi - \pi^e) + \varepsilon \tag{9}$$

We thus obtain N random variables which, to simplify the analysis, we suppose to be identically and independently distributed with:

$$\begin{aligned} E_a(\widetilde{a}_i) &= a_i, \quad \forall i = 1, ..., N \\ cov_a(\widetilde{a}_i, \widetilde{a}_j) &= \begin{cases} 0 & \text{if } i \neq j \\ \sigma_a^2 & \text{if } i = j \end{cases} \quad i, j = 1, ..., N \end{aligned}$$

where the subscript a refers to the common (marginal) distribution law of the system of the N random variables[7]. Furthermore we suppose that $\widetilde{a}_i$ and ε are not correlated (for all i)

By applying the aggregation rule on the transmission parameters, we are able to characterise the statistical properties of the Union-wide transmission coefficient (which thereby becomes a random variable), $\widetilde{a}_E \equiv \sum_1^N \mu_i \cdot \widetilde{a}_i$. Indeed, we obtain[8]:

$$E_a(\widetilde{a}_E) = a_E$$

[7]Assuming that the covariances between the $\widetilde{a}_i$ would not be equal to zero (and thus that some of the transmission mechanisms would be linked), would not change the results qualitatively. See why in Annex B.

[8]It is interesting to note thus that, in such a model, it is not possible to introduce parameter uncertainty at the Union level without taking it into account at the national level. Such an assumption would violate the aggregation principle. It would be possible to consider this distinction if the Phillips relationships were not linear. But in this case, solutions would be hardly tractable (see Bean (1997)).

$$
\begin{aligned}
var_a\left(\widetilde{a}_E\right) &= \sigma_a^2 \cdot \left(\sum_{i=1}^{i=N} \mu_i^2\right) \\
&\equiv \sigma_{a_E}^2
\end{aligned}
$$

Finally, the welfare criterium has to be adjusted to take the presence of uncertainty into account: it is thus based on the expectation of a weighted average of the national loss functions, with respect to both the distribution of the error term and the random coefficient (which we assume to be independent). Thus we use in the following, $\widetilde{W} \equiv E_{\varepsilon,a}\left[\sum_{i=1}^{i=N} \mu_i \cdot L_i\right]$.

1- Let look, first, at the Union-wide strategy (euro-aggregation). In an uncertain setting, the Central Bank considers the expected value of the loss function defined in terms of the Union-wide variables with respect to the distribution of $\widetilde{a}_E$. This reflects the assumption that the authorities manage optimally the uncertain effects of the policy they intend to design. Thus, the monetary authorities seek $\widetilde{\pi}^{EA}$ such as:

$$
\widetilde{\pi}^{EA} = \arg\min_{\pi} E_{a_E}\left[\Lambda^{EA}\right]
$$

subject to the constraint:

$$
U_E = U_E^* - \widetilde{a}_E \cdot (\pi - \pi^e) + \varepsilon
$$

and π^e and ε taken as given.

Solving this program leads to:

$$
\begin{aligned}
\widetilde{\pi}^{EA} &= \frac{ba_E}{1 + ba_E^2 + b\sigma_{a_E}^2}\varepsilon \\
&= \widetilde{\Omega}_{EA} \cdot \varepsilon
\end{aligned}
$$

with $\widetilde{\Omega}_{EA} \equiv \dfrac{ba_E}{1 + ba_E^2 + b\sigma_{a_E}^2}$.

To find out the value of the unemployment gap prevailing in country i, we have to substitute for the equilibrium values of π and π^e in the random, national Phillips curve equation (9). We obtain:

$$
\widetilde{U}_i^{EA} = U_i^* + \left(1 - \widetilde{\Omega}_{EA} \cdot \widetilde{a}_i\right) \cdot \varepsilon
$$

After the relevant substitutions, the expected value of the welfare loss function (with respect to both the distribution of ε and $\widetilde{a}_i$), obtains as follows:

$$\widetilde{W}^{EA} = \left[\left(\widetilde{\Omega}_{EA}\right)^2 \left(1 + b \cdot \sigma_a^2\right) + b \cdot \sum_{i=1}^{i=N} \mu_i \left(1 - \widetilde{\Omega}_{EA} \cdot a_i\right)^2\right] \cdot \sigma_\varepsilon^2$$

2- We now analyse the national aggregation strategy in an uncertain context. In this framework, the Central Bank takes uncertainty into account by considering the expected value of the weighted average of the national loss functions with respect to the common distribution law of the $\widetilde{a}_i$. Thus, the monetary authorities seek $\widetilde{\pi}^{NA}$ such as:

$$\widetilde{\pi}^{NA} = \arg\min_{\pi} E_a \left[\Lambda^{NA}\right]$$

subject to the constraint of the N national ("random") Phillips relationships:

$$U_i = U_i^* - \widetilde{a}_i \cdot (\pi - \pi^e) + \varepsilon_i = 1, 2, ..., N$$

and π^e and ε taken as given.

The optimal inflation rate is given by:

$$\begin{aligned} \widetilde{\pi}^{NA} &= \frac{ba_E}{1 + ba_E^2 + b\theta_{a_E}^2 + b\sigma_a^2}\varepsilon \\ &= \widetilde{\Omega}_{NA} \cdot \varepsilon \end{aligned}$$

with $\widetilde{\Omega}_{NA} \equiv \dfrac{ba_E}{1 + ba_E^2 + b\theta_{a_E}^2 + b\sigma_a^2}$ and $\theta_{a_E}^2$ still defined as $\theta_{a_E}^2 \equiv \sum_1^N \mu_i \cdot (a_i - a_E)^2$.

This leads to the following equilibrium unemployment and welfare loss :

$$\begin{aligned} \widetilde{U}_i^{NA} &= U_i^* + \left(1 - \widetilde{\Omega}_{NA} \cdot \widetilde{a}_i\right) \cdot \varepsilon \\ \widetilde{W}^{NA} &= \left[\left(\widetilde{\Omega}_{NA}\right)^2 \left(1 + b \cdot \sigma_a^2\right) + b \cdot \sum_{i=1}^{i=N} \mu_i \left(1 - \widetilde{\Omega}_{NA} \cdot a_i\right)^2\right] \cdot \sigma_\varepsilon^2 \end{aligned}$$

Whatever the strategy followed by the common central bank (euro *or* national aggregation), the introduction of uncertainty in the model has two effects.

- First, the uncertainty in the transmission process (measured by σ_a^2) has an ambiguous effect on welfare (either considered from the viewpoint of $\widetilde{W}^{NA}$ or $\widetilde{W}^{EA}$). On the one hand, it increases welfare through the presence of the term $b\sigma_a^2$ in the loss function. On the other hand it affects welfare negatively because $\widetilde{\Omega}_{EA}$ (or $\widetilde{\Omega}_{NA}$) depends negatively on σ_a^2. Thus, the net impact of transmission uncertainty on welfare depends on the relative strength of these two effects. This result is in accordance with the literature (see Letterie (1997)) and allows for looking at the optimal level of uncertainty with respect to welfare.

- We also find that $\widetilde{\Omega}_{EA} < \Omega_{EA}$ and $\widetilde{\Omega}_{NA} < \Omega_{NA}$. This means that in the case of transmission uncertainty the optimal inflation rate is less sensitive to shocks than in the absence of uncertainty. This reflects the so-called *brainardian principle* according to which the monetary authorities refrain from counteracting shocks too much if they know that such an intervention will add to the variability in the economy (instrumental variability) because of its random effectiveness. This smoothing effect prevails in the model, whatever the strategy followed by the monetary authorities.

3.2 Does parameter uncertainty reinforce the case for a national perspective?

We are now ready to assess how the presence of uncertainty may impinge on the choice between the two strategies we have envisaged so far.

1- We first compare how transmission uncertainty affects the optimal inflation rate under the two strategies.

Our main finding is that *transmission uncertainty has a stronger impact on the optimal inflation rate in the case of national aggregation than in the case of euro aggregation.*

Proof: $\left(\widetilde{\pi}^{EA}\right) - \left(\widetilde{\pi}^{NA}\right) > \left(\pi^{EA}\right) - \left(\pi^{NA}\right)$. This differential effect results from the fact that $\sigma_a^2 \geq \sigma_{a_E}^2$ (what is in turn implied by the aggregation rule as $\sum_{i=1}^{i=N} \mu_i^2 \leq 1$).

As a consequence, when uncertainty prevails, the impact of a shock on the optimal inflation rate is reduced more when the authorities follow a national aggregation procedure than when they use euro-aggregation (relative to the no uncertainty case). Thus transmission uncertainty makes the central bank more cautious under national than under euro aggregation. This result prevails even if the random national Phillips slope parameters are correlated (see Annex B).

This differential impact of uncertainty on the inflation rate can be explained as follows. When using the inflation rate as a stabilisation weapon, the monetary authorities know that they will add to the variability in the economy (besides the one implied by the supply shock) and thus try to counteract the consequences of this additional noise on the economy under both strategies. This leads them to react cautiously to the supply shock in terms of the inflation rate (that is less than when no uncertainty prevails). However the size of this instrumental variability will be higher when evaluated at the national level rather than at the Union level (because of the smoothing effect implied by the aggregation rule[9]). Therefore the lessening in the inflation rate will be larger under the NA strategy than under the EA procedure.

Those different results concerning the inflation rate may be summarized by the following inequality chain (for a positive value of the common shock):

$$\widetilde{\pi}^{NA} < \left(\widetilde{\pi}^{EA} <?> \pi^{NA}\right) < \pi^{EA}$$

2- Second we compare the welfare losses associated with the two strategies. Again, *this comparison favors the national aggregation procedure.*

Proof: let define the differential loss as $\Delta\widetilde{W} \equiv \frac{1}{\sigma_\varepsilon^2}\left(\widetilde{W}^{EA} - \widetilde{W}^{NA}\right)$. After substituting, we obtain,

$$\begin{aligned}\Delta\widetilde{W} \quad &= \quad \left(1 + b\sigma_a^2\right) \cdot \left[\left(\widetilde{\Omega}_{EA}\right)^2 - \left(\widetilde{\Omega}_{NA}\right)^2\right] \\ &\quad + b \cdot \sum_{i=1}^{i=N} \mu_i \cdot \left[\left(1 - \widetilde{\Omega}_{EA} \cdot a_i\right)^2 - \left(1 - \widetilde{\Omega}_{NA} \cdot a_i\right)^2\right]\end{aligned}$$

which simplifies to:

$$\Delta\widetilde{W} = \left[\left(\widetilde{\Omega}_{EA}\right) - \left(\widetilde{\Omega}_{NA}\right)\right] \cdot \widetilde{\Omega}_{EA} \cdot b \cdot \left[\theta_{a_E}^2 + \left(\sigma_a^2 - \sigma_{a_E}^2\right)\right]$$

This expression is unambiguously positive as $\left(\widetilde{\Omega}_{EA}\right) - \left(\widetilde{\Omega}_{NA}\right) > 0$ and $\left(\sigma_a^2 - \sigma_{a_E}^2\right) > 0$.

Compared to the certainty case (see equation (7)) the impact of uncertainty on the welfare loss operates at two levels:

- On the one hand, it affects the value of the reaction coefficient ($\widetilde{\Omega}_{NA}$ and $\widetilde{\Omega}_{EA}$) such that we have $\left(\widetilde{\Omega}_{EA}\right) - \left(\widetilde{\Omega}_{NA}\right) > (\Omega_{EA}) - (\Omega_{NA})$. This impact arises from two combined channels. First, there is the lessening effect on

[9] The uncertainty (and thus the instrumental variability) concerns the transmission of inflation impulses on the unemployment rate

the inflation rate under both strategies which reflects the cautious attitude of the monetary authorities faced with transmission uncertainty. Second, there is the smoothing effect of the aggregation rule when the monetary authorities take instrumental variability into account. The former implies that the presence of uncertainty acts more on the inflation rate under the NA strategy than under the EA procedure.

- On the other hand, it acts also in an additive way, through the difference between the variances of the Phillips curve slope parameters, $\left(\sigma_a^2 - \sigma_{a_E}^2\right)$ which is positive. This component is directly related to the additional variability which is introduced in the economy when transmission uncertainty surrounds the use of the inflation rate as a stabilisation device. The aggregation rule deadens the impact of this instrumental variability on welfare in the case of the NA strategy relative to the EA procedure.

These two effects increase the loss from using euro aggregation relative to the loss from using national aggregation. From the foregoing, we conclude that transmission uncertainty reinforces the result that in the presence of asymmetries in the transmission process the monetary authorities should use a national aggregation procedure rather than follow a euro aggregation strategy.

4 Additive uncertainty and information sharing in a heterogenous monetary union

In the foregoing, we have shown that transmission parameter uncertainty does not change the ranking of the optimal federal monetary policy in the presence of asymmetries. Rather it reinforces the need to base the monetary strategy on national variables.

In this section, we analyse the case of uncertainty which is not of the multiplicative but *of the additive type.* We introduce additive uncertainty in the model by assuming that the monetary authorities have an imperfect knowledge about the common (symmetric) supply shock which hits the different member economies[10]. We suppose that each of the strategies considered is associated with a specific forecasting process. With respect to the assumptions made previously, the monetary decisions taken at the Union level are set either on the basis of national variables (the NA strategy) or on the basis of of Union-wide aggregates (the EA strategy). Accordingly, under the NA-strategy, the forecasts are supposed to be based on the national information sets whereas under the (EA) procedure they are related to one Union-wide defined information set

[10]Another way to take additive uncertainty into account would have consisted in considering measurement errors on structural variables in the model (like the natural of unemployment or the unemployment gap).

which may or may not hinge on the national ones. Thus we have national based forecasts on the one hand and one direct, Union-wide forecast on the other hand.

Finally, we suppose that the forecast made by the Central Bank is private information under both strategies, ie. it is not revealed to the private sector before the formation of its expectations to be embedded into the nominal wage contracts. According to the classification used by Cukierman (2000), the monetary regime is one of limited transparency where the monetary authorities possess imperfect forecasts of the shock.

4.1 The role of national forecasts in the national aggregation strategy

Consider first the national aggregation strategy: we suppose that prior to the occurrence of the shock but before the decision setting stage, each of the national monetary authorities is able to obtain a forecast of the (symmetric) supply shock[11].

Thus, we have:

$$\varepsilon = f_i^{NA} + \nu_i^{NA} \tag{10}$$

where f_i^{NA} is the forecast implemented by the monetary authority of country i and ν_i^{NA} is the associated forecast (or *measurement*) error. The previous decomposition reflects the presence of additive uncertainty in the model: the Central Bank cannot observe the shock on the basis of which the monetary action should be decided, but can only use a noisy signal of it $\left(f_i^{NA}\right)$.

We further suppose that the forecast is rationally formed with respect to the relevant (national) information set (noted $I_{f_i^{NA}}$) and is unbiased. Thus, $f_i^{NA} \equiv E\left[\varepsilon \left| I_{f_i^{NA}} \right.\right]$ and $E\left[f_i^{NA}\right] = 0$. It follows that f_i^{NA} and ν_i^{NA} are not correlated (for a given i)[12] and thus that $cov\left(f_i^{NA}, \nu_i^{NA}\right) = 0$.

The statistical properties of the measurement error can be both expressed in terms of its unconditional and conditional (forecast-based) distribution. Given the no correlation assumption between f_i^{NA} and ν_i^{NA} and the fact that we assume that f_i^{NA} is unbiased, we obtain: $E\left[\nu_i^{NA}\right] = E\left[\nu_i^{NA} \left| I_{f_i^{NA}} \right.\right] = 0$.

Moreover, if we suppose that $var\left[\nu_i^{NA}\right] = var\left[\nu_i^{NA} \left| I_{f_i^{NA}} \right.\right] = \sigma_\nu^2$, it follows that $var\left[\varepsilon \left| I_{f_i^{NA}} \right.\right] = \sigma_\nu^2$ and, more importantly, that $var\left[f_i^{NA}\right] = \sigma_f^2$ with $\sigma_\varepsilon^2 = \sigma_f^2 + \sigma_v^2$.

[11]In what follows, we will assume that the underlying distribution of all the random variables is of the Normal type.

[12]This reflects the assumption according to which the national central Bank does not make a systematic forecast error when predicting over the size of the shock.

Given that the supply shock is not observed but only forecasted we specify the loss function under the (NA) strategy as the weighted average of the *expected* national losses. The latter are computed conditionally on the national central bank's information sets, $I_{f_i^{NA}}$:

$$\Lambda_f^{NA} \equiv \sum_{i=1}^{i=N} \mu_i \cdot E\left[L_i \left| I_{f_i^{NA}} \right.\right]$$

The federal monetary authority then solves the following program:

$$\begin{aligned} \pi_f^{NA} &= \arg\min_{\pi} \Lambda_f^{NA} \\ & s.t. \left\{ \begin{array}{c} \pi^e,\ \varepsilon\ given \\ U_i = U_i^* - a_i \cdot (\pi - \pi^{e,}) + \varepsilon,\ \text{for } i = 1, 2, ..., N \end{array} \right. \end{aligned}$$

This leads to the optimal inflation rate:

$$\begin{aligned} \pi_f^{NA} &= \frac{ba_E}{1 + b\left(a_E^2 + \theta_{a_E}^2\right)} f_E^{NA} \\ &= \Omega_{NA} \cdot f_E^{NA} \end{aligned} \tag{11}$$

where f_E^{NA} is computed as the weighted average of national forecasts and may be defined *as the average (national) forecast*:

$$f_E^{NA} \equiv \sum_{i=1}^{i=N} \mu_i \cdot f_i^{NA}$$

The result just derived (equation (11)) requires two assumptions:

1- First, we suppose that the correlation between the national forecasts $\left(f_i^{NA}\right)$ and the transmission parameters of monetary policy (a_i) is zero. Given that the shocks, and thus the forecasts themselves have an expected value of zero, there is little reason to expect that there is any relation between economic disturbances (either fully or imperfectly forecasted) and monetary policy[13].

2- Second, when the private sector forms its expectation of the inflation rate, it does not know the value of f_i^{NA}, nor that of the measurement error, which are private information. (and, as we have implicitly assumed so far, it cannot

[13]In the first order condition of the program it is thus possible to replace the expression $\left(\sum_1^N \mu_i \cdot a_i \cdot f_i^{NA}\right)$ by $a_E \cdot f_E^{NA}$ under the assumption that the correlation term between the a_i and the f_i is zero, that is: $\sum_1^N (a_i - a_E) \cdot \left(f_i^{NA} - f_E^{NA}\right) = 0$

observe the realisation of the shock ε either). In such a context, the value of the inflation rate which is rationally expected by the private sector is zero.

Thus from (11) it follows that the optimal monetary policy (given by the coefficient Ω_{NA}) is identical to the case of no uncertainty about the common supply shock. The only difference is that in the uncertainty case the central bank reacts to the forecast of the shock, while in the certainty case it reacts to the realisation of the shock.

The solution is thus in line with the certainty equivalence principle with the proviso that the variable about which uncertainty exists, has been replaced by its certain equivalent (whose role is played by the forecast). This contrasts with the result we obtained when the uncertainty is multiplicative and involves the transmission process (see the previous section).

The welfare benchmark is given by the unconditional expectation of the weighted average of the national losses. Thus we retain the following expression for evaluating the strategies:

$$W_f \equiv E_{unc}\left[\sum_{i=1}^{i=N} \mu_i \cdot L_i\right] \tag{12}$$

where the expectation operator $E_{unc}[.]$ applies for the (unconditional) distribution of ε, f_i^{NA} and ν_i.

At this stage it is necessary to assess the statistical properties of the average forecast f_E^{NA}. We obtain:

$$\begin{aligned} E\left[f_E^{NA}\right] &= 0 \\ var\left[f_E^{NA}\right] &= \sigma_f^2 \cdot \left(\sum_{i=1}^{i=N} \mu_i^2\right) \\ &\equiv \sigma_{f_E^{NA}}^2 \end{aligned}$$

The last equality prevails if we assume that the forecasts are performed independently from one country to another[14], ie. that $cov\left(f_i^{NA},\ f_j^{NA}\right) = 0$ for $i \neq j$. The stringency of this assumption will be discussed later when we examine the relationship between the Union-wide and average forecasts.

After having substituted the relevant values of the inflation and unemployment rates in (12), we find:

[14] In a related way, we could also define a average measurement error, ν_E^{NA} such that $\varepsilon = f_E^{NA} + \nu_E^{NA}$. We have: $E\left[\nu_E^{NA}\right] = 0$ and $var\left[\nu_E^{NA}\right] = \sigma_\nu^2 + \left(\sum_{i=1}^{i=N} \mu_i^2 - 1\right)\sigma_f^2$.

It is important to note however that ν_E^{NA} and f_E^{NA} are **correlated** in this case: $cov\left(f_E^{NA}, \nu_E^{NA}\right) = \left(1 - \sum_{i=1}^{i=N} \mu_i^2\right) \cdot \sigma_f^2 \neq 0$.

$$W_f^{NA} = (\Omega_{NA})^2 \left(1 + b\overline{a_E^2}\right) \cdot \sigma^2_{f_E^{NA}} + b \cdot \sigma^2_\varepsilon - 2 \cdot b \cdot \Omega_{NA} \cdot a_E \cdot \underbrace{E\left[\varepsilon \cdot f_E^{NA}\right]}_{\sigma_f^2}$$

which can be rewritten as:

$$\begin{aligned} W_f^{NA} &= (\Omega_{NA})^2 \left(1 + b\overline{a_E^2}\right) \cdot \sigma^2_{f_E^{NA}} + b \cdot \sigma^2_\varepsilon - 2 \cdot b \cdot \Omega_{NA} \cdot a_E \cdot \sigma^2_{f_E^{NA}} \\ &\quad -2 \cdot b \cdot \Omega_{NA} \cdot a_E \cdot (1 - x) \cdot \sigma_f^2 \end{aligned} \tag{13}$$

with $x \equiv \sum_{i=1}^{i=N} \mu_i^2$, and $0 < x \leq 1$.

We observe that both the transmission asymmetry $\left(\theta^2_{a_E}\right)$ and the forecast quality $\left(\sigma^2_{f_E^{NA}}\right)$ affect the value of social welfare under the NA strategy.

Over and above those two elements, we also note the role played by the aggregation rule in the smoothing of the forecast variance (at the Union level) - $\sigma^2_{f_E^{NA}} \equiv x \cdot \sigma_f^2 \leq \sigma_f^2$–, whose impact positively impinges on the choice of the NA strategy by reducing the welfare loss (see the last term of equation (13)). This lessening effect is proper to the NA strategy insofar as the weighting average of the national losses implies, as a byproduct, that the monetary policy decisions are relying on the average forecast.

4.2 Euro-aggregation and the computation of a union-wide forecast

Under the euro-aggregation procedure, the federal Central Bank elaborates its own aggregate forecast of the common supply shock by adopting a (direct) Union-wide perspective. Later we examine the possible links with the national-based forecasting procedure related to the NA strategy.

Similarly to the foregoing, we define f_E^{EA} as the rational (unbiased) forecast implemented by the Central Bank on the basis of one Union-wide aggregate information set, $I_{f^{EA}}$, $f_E^{EA} \equiv E\left[\varepsilon \left| I_{f^{EA}}\right.\right]$ with $E\left[f_E^{EA}\right] = 0$.

The related measurement error is ν_E^{EA} and we have: $\varepsilon = f_E^{EA} + \nu_E^{EA}$ with $cov\left(f_E^{EA},\, \nu_E^{EA}\right) = 0$ (f_E^{EA} and ν_E^{EA} are supposed to be uncorrelated). It ensues that $E\left[\nu_E^{EA}\right] = E\left[\nu_E^{EA} \left| I_{f^{EA}}\right.\right] = 0$. We further suppose that $var\left[\nu_E^{EA}\right] = var\left[\nu_E^{EA} \left| I_{f^{EA}}\right.\right] = \sigma^2_{\nu_E^{EA}}$, which leads to $var\left[\varepsilon \left| I_{f^{EA}}\right.\right] = \sigma^2_{\nu_E^{EA}}$ and $var\left[f_E^{EA}\right] = \sigma^2_{f_E^{EA}}$ with $\sigma^2_\varepsilon = \sigma^2_{f_E^{EA}} + \sigma^2_{\nu_E^{EA}}$

In line with the assumptions made previously about the EA strategy, the federal central Bank minimises the expected value of the aggregate loss function which is computed conditionally on the relevant information set. Thus, the Central bank minimises $\Lambda_f^{EA} \equiv E\left[\Lambda^{EA}\left|I_{f^{EA}}\right.\right]$ with respect to π and taking as given the aggregate Union-wide Phillips "curve" and the level of private sector's inflation expectations.

The optimal inflation rate obtained from this minimisation procedure has the same features as the one derived in the case where the supply shock is observed with certainty, except for the fact that, under the present scenario, the authorities react to their forecast :

$$\begin{aligned}\pi_f^{EA} &= \frac{ba_E}{1+ba_E^2} f_E^{EA} \\ &= \Omega_{EA} \cdot f_E^{EA}\end{aligned} \tag{14}$$

Finally, substituting for the (equilibrium) national unemployment rates and the inflation rate into (12) leads to:

$$W_f^{EA} = (\Omega_{EA})^2 \left(1 + b\overline{a_E^2}\right) \cdot \sigma^2_{f_E^{EA}} + b \cdot \sigma_\varepsilon^2 - 2 \cdot b \cdot \Omega_{EA} \cdot a_E \cdot \underbrace{E\left[\varepsilon \cdot f_E^{EA}\right]}_{\sigma^2_{f_E^{EA}}} \tag{15}$$

Again we observe that the value of the loss function depends on the transmission asymmetry and the union-wide forecast accuracy.

When we compare both strategies, we note that the relative variability of the inflation rates depends on the relative properties of the forecasts, no matter *the problem of heterogeneity* (see equations (11) and (14)).

This means, in turn, that the volatility of the national unemployment rate is also contingent upon the relative accuracy of the forecasts of the supply disturbance as the equilibirum level of the unemployment gap depends on the stabilisation performance provided by the monetary authority (through the manipulation of the inflation rate). This can be seen from the following expressions of the equilibrium national unemployment rates (obtained by substituting the optimal inflation rates in the Phillips curve):

$$U_i^{NA} = U_i^* - a_i \cdot \Omega_{NA} \cdot f_E^{NA} + \varepsilon \tag{16}$$

$$U_i^{EA} = U_i^* - a_i \cdot \Omega_{EA} \cdot f_E^{EA} + \varepsilon \tag{17}$$

4.3 Forecast accuracy *versus* transmission heterogeneity: characterising the trade-off

The overall comparison of the two strategies under additive uncertainty has to be made with respect to the benchmark welfare function. Making use of equations (13) and (15) we obtain the following expression:

$$\begin{aligned} W_f^{EA} - W_f^{NA} &= \left(1 + b\overline{a_E^2}\right) \cdot \left[(\Omega_{EA})^2 \cdot \sigma^2_{f_E^{EA}} - (\Omega_{NA})^2 \cdot \sigma^2_{f_E^{NA}} \right] \\ &\quad -2 \cdot b \cdot a_E \cdot \left((\Omega_{EA}) \cdot \sigma^2_{f_E^{EA}} - (\Omega_{NA}) \cdot \sigma_f^2 \right) \end{aligned}$$

which can be rewritten as follows:

$$\begin{aligned} W_f^{EA} - W_f^{NA} &= b \cdot a_E \cdot \Omega_{EA} \cdot \sigma^2_{f_E^{EA}} \cdot \left[\frac{b \cdot \theta^2_{a_E}}{(1 + b a_E^2)} - 1 \right] \\ &\quad + b \cdot a_E \cdot \Omega_{NA} \cdot \sigma^2_{f_E^{NA}} + 2 \cdot b \cdot a_E \cdot \Omega_{NA} \cdot (1 - x) \cdot \sigma_f^2 \end{aligned}$$

As expected, the sign of the differential loss function is ambiguous because several factors play a role in the setting of optimal monetary policies and may operate in different directions.

1. The first element (appearing through the comparison between Ω_{EA} and Ω_{NA}) relates to the presence of asymmetries in the transmission process. *Ceteris paribus*, we have shown that the NA strategy provided the best way (in terms of welfare) to tackle the prevailing heterogeneity $(\Omega_{EA} - \Omega_{NA} > 1)$.

2. The second factor refers to the quality of the forecasts associated to each of the strategies considered. This problem enters the differential loss *via* the comparison between $\sigma^2_{f_E^{EA}}$ and $\sigma^2_{f_E^{NA}}$ which can be considered as an indication of the relative accuracy of the union-wide forecast *versus* the average one.

3. The third element to take into account relates to the smoothed variability of the average forecast (with respect to the common variance of the national ones). As previously mentionned, this component is proper to the NA strategy and favors the choice of the former, no matter the two other elements.

All in all, the choice between the two strategies depends on the relative strength of the asymmetry and the forecast accuracy effects. To investigate how the trade-off performs between these two components, we denote by ϕ the coefficient reflecting *the relative accuracy of the union-wide forecast* with respect to the average forecast, ie. $\phi \equiv \left(\sigma^2_{f_E^{EA}} \Big/ \sigma^2_{f_E^{NA}}\right)$. When $\phi > 1$, the forecast based on the area-wide information set has a more predictive power than the averaging of the national forecasts.

We may thus rewrite the differential loss function as:

$$\begin{aligned}
W_f^{EA} - W_f^{NA} &= b \cdot a_E \cdot \Omega_{EA} \cdot \sigma^2_{f_E^{NA}} \cdot \left[\phi \cdot \left(\frac{b \cdot \theta^2_{a_E}}{(1+ba_E^2)} - 1\right) + \frac{\Omega_{NA}}{\Omega_{EA}} \cdot \left(\frac{2-x}{x}\right)\right] \\
&= b \cdot a_E \cdot \Omega_{EA} \cdot \sigma^2_{f_E^{NA}} \cdot \left[G\left(\phi \; ; \; \theta^2_{a_E}\right) - \phi\right] \qquad (18)
\end{aligned}$$

with $G\left(\phi \; ; \; \theta^2_{a_E}\right) \equiv \phi \cdot \dfrac{b \cdot \theta^2_{a_E}}{(1+ba_E^2)} + \dfrac{\Omega_{NA}}{\Omega_{EA}} \cdot \left(\dfrac{2-x}{x}\right)$

Whether the national or euro aggregation strategy has to be chosen depends on whether $G\left(\phi \; ; \; \theta^2_{a_E}\right) \lesseqgtr \phi$. The following results obtain (see Annex C):

1. If the dispersion in the transmission parameters is large, the NA procedure is always associated with a lower social loss than the EA strategy, *whatever* the value of ϕ.
2. If the asymetry is of a small extent, there is a range over the values taken by ϕ for which the choice of the EA procedure enhances welfare (with respect to the NA strategy). In those circumstances, ϕ is always more than one.

To illustrate further how the trade-off operates, it may be useful to analyse separetely the role played by the forecast accurracy and the transmission asymmetry on the sign of the differential loss. To this aim, two polar cases are considered, one where no asymmetry prevails between the countries (so that the choice will depend on the relative accuracy of the forecasts) and one where the forecasts have the same (statistical) properties so that the sign of the welfare differential loss will essentially depend on the stance of the dispersion between the national transmission processes.

1- Let suppose, first, that there is no asymmetry between the countries. It follows that $\theta^2_{a_E} = 0$ and $\Omega_{EA} = \Omega_{NA} = \dfrac{b \cdot a_E}{1+ba_E^2}$. As a consequence, the differential loss becomes:

$$W_f^{EA} - W_f^{NA} = b \cdot a_E \cdot \Omega_{EA} \cdot \left[\sigma^2_{f_E^{NA}} - \sigma^2_{f_E^{EA}} + 2 \cdot (1-x) \cdot \sigma^2_f\right] \qquad (19)$$

The relative gain in terms of welfare that the EA strategy provides over the NA procedure is clearly contingent upon the quality of the forecasting procedure to which it is related. The more accurate the union-wide forecast is[15], the less costly is the choice of the EA strategy.

Note however, that, even if the volatility of the forecasts were the same under both strategies (i.e. $\sigma^2_{f_E^{NA}} = \sigma^2_{f_E^{EA}}$), it would be beneficial to use the NA procedure. This could be explained by the fact that this strategy implies, as a byproduct of the aggregation rule, a lessening in the aggregate volatility of the national forecasts $\left(x \cdot \sigma^2_f \leq \sigma^2_f\right)$, what is positively valued in terms of welfare (compared to the volatility of one direct union-wide forecast under the EA strategy).

2- Alternatively, we have to consider the case of a monetary union with dispersion between the transmission parameters $\left(\theta^2_{a_E} > 0\right)$ but where optimal monetary policy under both strategies is based on forecasts with similar statistical properties $\left(\sigma^2_{f_E^{NA}} = \sigma^2_{f_E^{EA}}\right)$. Then, the differential loss becomes:

$$\begin{aligned} W_f^{EA} - W_f^{NA} &= (\Omega_{EA} - \Omega_{NA}) \cdot \Omega_{EA} \cdot b \cdot \theta^2_{a_E} \cdot \sigma^2_{f_E^{NA}} \\ &\quad + 2 \cdot b \cdot a_E \cdot \Omega_{NA} \cdot (1 - x) \cdot \sigma^2_f \end{aligned} \tag{20}$$

The first term is equivalent to the one prevailing under the case with no uncertainty (cf. equation (7), $\sigma^2_{f_E^{NA}}$ replacing σ^2_ε in the expression). We note that there is a positive relationship between the the size of the dispersion coefficient and the need for a national agregation-based strategy[16].

The second term reflects the additional advantage related to the choice of the NA procedure (*cum* forecasting) that we have already mentionned: there is a smoothing in the variability associated to the use of the forecast when we agregate the national losses under the NA strategy, no matter the presence or the absence of dispersion.

At this stage, we may conclude that the national aggregation procedure has to be chosen because it is a better way to deal with the asymmetry in the transmission. But, on the other hand, such a strategy could be associated with a relatively poor forecast which would affect the stabilisation properties of the monetary policy so as to render the EA strategy more attractive, especially if the dispersion is of small amplitude. In other words to the extent that the forecast errors related to the euro-aggregation procedure are smaller than the

[15]Here, we have in mind that the accuracy of the forecast (which is a random variable) can be appreciated through the share that the forecast variance takes in the variance of the underlying disturbance, ε.

[16]given that $\Omega_{EA} > \Omega_{NA}$

aggregation of the forecast errors which is implicit in the national aggregation strategy, and that the asymmetry in the transmission is not too important, the benefit obtained from using national data in the monetary policy decisions may disappear.

This issue has been adressed in some empirical studies (see, *e.g.*, Massimiliano *et alii* (2003) and Fabiani and Morgan (2003)) which have compared the predictive power of two forecasting methods for EMU variables. Two ways for obtaining Euro-aggregate forecasts are generally considered either through the *"pooling country-specific forecasts"* or *"by directly forecasting the aggregate variables using other aggregate variables"*. In the case where the forecasts are pooled, the pooling has to be understood as the weighted average of the individual country-specific forecasts (GDP weights). In general this "pooling" method obtains better results in terms of forecast accuracy than one direct Union-wide forecasting.

According to our theoretical framework, this result would imply that the choice of the national aggregation strategy would be *unambiguously* better in terms of welfare than adopting the euro-aggregation procedure. The argument runs as follows.

First, a correspondence may be established beween the NA strategy and the pooling method on the one hand and the EA procedure and the "*direct aggregate forecasting method*" on the second hand (what we have called, in the model, the computation of a union-wide forecast). This link is explicit when we observe how the optimal inflation rate expressions are defined under the two strategies: π^{NA} depends on f_E^{NA} which is the weighted average of the national forecasts (one specific pooling method) whereas π^{EA} depends on f_E^{EA} which is the union-wide forecast.

Given this correspondence, the empirical evidence suggests to examine in the model the case where the relative accuracy of the union-wide forecast is less than one, $\phi < 1$. According to the results derived from the model (see *supra* and Annex C), the federal central bank would gain in this case to implement a national aggregation procedure, whatever the extent of the degree of asymmetry prevailing between the national transmission mechanisms[17].

5 Conclusion

The design of monetary policies in a monetary union is particularly challenging. One such challenge arises from the fact that the member countries have

[17]The case for the national aggregation procedure is all the more favored as a better relative accuracy of the federal forecast is not a sufficient condition to adopt the euro-aggregation procedure.

maintained many of their idiosyncrasies. These have the effect of creating asymmetries in the transmission of common shocks. In this paper we confirmed that when asymmetries in the transmission exist, the common central bank can improve the quality of monetary policy making by using national information about inflation and the output gap, instead of focusing only on the union-wide aggregates.

The main contribution of this paper consists in analysing whether this conclusion holds when uncertainty (of the additive or of the multiplicative type) surrounds the design of monetary policy. We found that usually the presence of uncertainty *reinforces* the need to use national data on inflation and output gaps. The insistence of the ECB to use only union-wide aggregated information about these variables is therefore likely to be suboptimal.

6 References

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7 Annexes

7.1 Annex A: loss comparison in the absence of uncertainty

As seen p.8, the comparison of the losses favors the choice of a national aggregation perspective with respect to one relying on a Union-wide strategy (see equation (8)). This result comes from two effects appearing in equation (6) and which may play in opposite directions:

- First, the euro-aggregation strategy implies a higher volatility in the inflation rate ($\Omega_{EA} \geq \Omega_{NA}$). Indeed, under this strategy, the Central Bank does not take the heterogenous structures of national transmission mechanisms in the Union into account which would otherwise play as an incentive to lessen the sensitivity of the optimal inflation rate to the supply shock. This result obtains whether or not the weight on output stabilisation in the loss function is more than one.
- The term $\sum_{i=1}^{i=N} \mu_i \cdot \left[(1 - \Omega_{EA} \cdot a_i)^2 - (1 - \Omega_{NA} \cdot a_i)^2 \right]$ in equation (6) can be positive or negative. It can be shown (see *infra*) that, in the case where the asymmetry in the transmission $\left(\theta_{a_E}^2\right)$ is large enough and the output weight (b) in the loss function not too small, this term is positive. In that case the national aggregation procedure contributes to reduce the unemployment variability relative to the euro-aggregation strategy[18].

To go further on the last result, let define the differential welfare loss from the viewpoint of unemployment variability associated with a Union-wide aggregation strategy as ΔW^{UN}.

$$\Delta W^{UN} \equiv \left(\sum_{i=1}^{i=N} \mu_i \cdot \left[(1 - \Omega_{EA} \cdot a_i)^2 - (1 - \Omega_{NA} \cdot a_i)^2 \right] \right)$$

[18] In that case, the two effects play in the same direction which favors the national aggregation procedure

If ΔW^{UN} is negative, such a procedure has to be favored. We may rewrite ΔW^{UN} as:

$$\begin{aligned} & (\Omega_{NA} - \Omega_{EA}) \cdot \left[2a_E - \overline{a_E^2} \cdot (\Omega_{EA} + \Omega_{NA})\right] \\ = \quad & (\Omega_{NA} - \Omega_{EA}) \cdot \frac{2a_E}{\left(1 + b\overline{a_E^2}\right)(1 + ba_E^2)} \cdot \left[1 + ba_E^2 - \frac{b^2}{2}\overline{a_E^2}\theta_{a_E}^2\right] \end{aligned}$$

As $(\Omega_{NA} - \Omega_{EA})$ is negative, the differential loss will be negative if and only if $\left[1 + ba_E^2 - \frac{b^2}{2}\overline{a_E^2}\theta_{a_E}^2\right]$ is positive.

$\left[1 + ba_E^2 - \frac{b^2}{2}\overline{a_E^2}\theta_{a_E}^2\right]$ may be defined in turn as a function of b. Indeed we have:

$$P(b) = -\frac{b^2}{2} \cdot \overline{a_E^2} \cdot \theta_{a_E}^2 + ba_E^2 + 1$$

As the discriminant $\left(\Delta \equiv \left(a_E^2\right)^2 + 2 \cdot \overline{a_E^2} \cdot \theta_{a_E}^2\right)$is positive, this second order polynomial has two roots

$$\begin{aligned} \Delta_1 &= \frac{a_E^2 + \sqrt{\Delta}}{\overline{a_E^2} \cdot \theta_{a_E}^2} > 0 \\ \Delta_2 &= \frac{a_E^2 - \sqrt{\Delta}}{\overline{a_E^2} \cdot \theta_{a_E}^2} < 0 \end{aligned}$$

Thus, $P(b)$ will be positive if and only if $b \in [0\ ;\ \Delta_1]$ as b can only take positive values.

At this stage, the question is to know how Δ_1 behaves when $\theta_{a_E}^2$ varies.

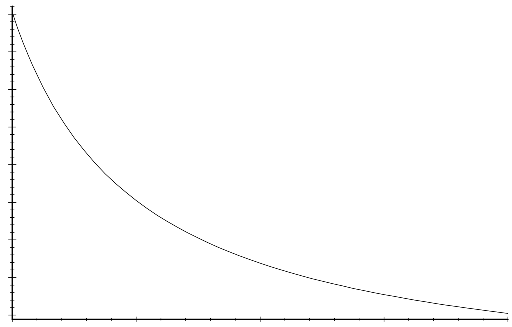
We have:

$$\begin{aligned} \lim_{\theta_{a_E}^2 \to 0^+} \Delta_1\left(\theta_{a_E}^2\right) &= +\infty \\ \lim_{\theta_{a_E}^2 \to +\infty} \Delta_1\left(\theta_{a_E}^2\right) &= 0 \end{aligned}$$

By the way, for $\theta_{a_E}^2$ and a_E^2 strictly positive, $\Delta_1\left(\theta_{a_E}^2\right)$ may be re-written as:

$$\Delta_1\left(\theta_{a_E}^2\right) = \frac{1 + \sqrt{1 + \frac{2 \cdot \theta_{a_E}^2}{a_E^2} \cdot \left(1 + \frac{\theta_{a_E}^2}{a_E^2}\right)}}{\theta_{a_E}^2\left(1 + \frac{\theta_{a_E}^2}{a_E^2}\right)}$$

From this expression we conclude that $\frac{\partial \Delta_1\left(\theta^2_{a_E}\right)}{\partial \theta^2_{a_E}} < 0$. Thus, $\Delta_1\left(\theta^2_{a_E}\right)$ is an hyperbolic, monotonically decreasing function of $\theta^2_{a_E}$.



Thus, for small values of $\theta^2_{a_E}$, $P(b)$ will be positive whatever the value of b. In this case, the (EA) strategy has to be favored. In the opposite case, when $\theta^2_{a_E}$ takes relatively large values, the interval on which $P(b)$ will be positive is of small magnitude. It is then possible that for relatively large values of b, the (NA) strategy delivers a smaller (aggregate) volatility of unemployment than the (EA) procedure.

7.2 Annex B: correlated Phillips curve slopes

Suppose that the distribution of the $\widetilde{a}_i$ has the following properties:

$$\begin{aligned} E_a\left(\widetilde{a}_i\right) &= a_i, \quad \forall i = 1, ..., N \\ cov_a\left(\widetilde{a}_i, \widetilde{a}_j\right) &= \begin{cases} \rho_{ij} & \text{if } i \neq j \\ \sigma^2_a & \text{if } i = j \end{cases} \quad i, j = 1, ..., N \end{aligned}$$

It ensues that the variance of $\widetilde{a}_E$ is now given by:

$$\begin{aligned} var_a\left(\widetilde{a}_E\right) &= \sigma^2_a \cdot \left(\sum_{i=1}^{i=N} \mu^2_i\right) + 2 \cdot \sum_{i \neq j} \mu_i \mu_j \rho_{ij} \\ &\equiv \sigma^{*2}_{a_E} \end{aligned}$$

The results obtained in the paper would be modified (qualitatively) by these new assumptions if $\sigma^{*2}_{a_E} > \sigma^2_a$. But, as we will see, this is not the case.

Proof: let rewrite ρ_{ij} in terms of the correlation coefficient r_{ij}: $\rho_{ij} = r_{ij} \cdot \sqrt{var(\widetilde{a}_i).var(\widetilde{a}_j)}$, that is, $\rho_{ij} = r_{ij} \cdot \sigma_a^2$. Moreover, we know that: $-1 \leq r_{ij} \leq 1$. Thus we may write:

$$\sigma_{a_E}^{*2} \leq \sigma_a^2 \cdot \left[\sum_{i=1}^{i=N} \mu_i^2 + 2 \cdot \sum_{i \neq j} \mu_i \mu_j \right]$$

which is equivalent to:

$$\sigma_{a_E}^{*2} \leq \sigma_a^2 \cdot \left(\sum_{i=1}^{i=N} \mu_i \right)^2$$

But $\sum_{i=1}^{i=N} \mu_i = 1$. Therefore,

$$\sigma_{a_E}^{*2} \leq \sigma_a^2$$

7.3 Annex C: forecast accuracy and transmission asymmetry

We consider how forecast accuracy (measured by the coefficient ϕ) and dispersion (measured by $\theta_{a_E}^2$) affect the differential welfare loss between the national and euro-aggregation strategies. To this aim, we define $\left(W_f^{EA} - W_f^{NA}\right)$ as a function of ϕ and $\theta_{a_E}^2$, such that $\left(W_f^{EA} - W_f^{NA}\right) \equiv H\left(\phi\ ;\ \theta_{a_E}^2\right)$. The expression of the former is given by:

$$\begin{aligned} H\left(\phi\ ;\ \theta_{a_E}^2\right) &= b \cdot a_E \cdot \Omega_{EA} \cdot \sigma_{f_E^{NA}}^2 \cdot \left[\phi \cdot \left(\frac{b \cdot \theta_{a_E}^2}{(1 + b a_E^2)} - 1 \right) + \frac{\Omega_{NA}}{\Omega_{EA}} \cdot \left(\frac{2-x}{x} \right) \right] \\ &= A \cdot \left[G\left(\phi\ ;\ \theta_{a_E}^2\right) - \phi \right] \end{aligned}$$

with $A \equiv b \cdot a_E \cdot \Omega_{EA} \cdot \sigma_{f_E^{NA}}^2$ and $G\left(\phi\ ;\ \theta_{a_E}^2\right) \equiv \phi \cdot \frac{b \cdot \theta_{a_E}^2}{(1 + b a_E^2)} + \frac{\Omega_{NA}}{\Omega_{EA}} \cdot \left(\frac{2-x}{x} \right)$.

Thus the sign of $\left(W_f^{EA} - W_f^{NA}\right)$ depends on whether $\left(G\left(\phi\ ;\ \theta_{a_E}^2\right) - \phi\right)$ is positive or negative. Note that only strictly positive values for ϕ have to be considered. By the way, in analysing this relationship, we take $\theta_{a_E}^2$ as the conditionning parameter. Under these conditions, we remark that:

- $\dfrac{\partial H\left(\phi\ ;\ \theta_{a_E}^2\right)}{\partial \phi} = A \cdot \left(\dfrac{b \cdot \theta_{a_E}^2}{(1 + b a_E^2)} - 1 \right) \lesseqgtr 0$
- $\lim\limits_{\phi \longrightarrow 0^+} H\left(\phi\ ;\ \theta_{a_E}^2\right) = A \cdot \dfrac{\Omega_{NA}}{\Omega_{EA}} \cdot \left(\dfrac{2-x}{x} \right) > 0$

- $H(\phi\ ;\ 0) = A \cdot \left[\left(\frac{2-x}{x}\right) - \phi\right] \lesseqgtr 0$

As a consequence, two cases can be disentangled.

1. $\theta^2_{a_E} \geq \frac{1+ba^2_E}{b} \equiv \widetilde{\theta^2_{a_E}}$. Then, the differential loss is a monotonically increasing function of ϕ with $\left(W^{EA}_f - W^{NA}_f\right) > 0, \quad \forall \phi \in R^*_+$.

2. $\theta^2_{a_E} < \widetilde{\theta^2_{a_E}}$. Then, the differential loss is a monotonically decreasing function of ϕ with $\left(W^{EA}_f - W^{NA}_f\right) \geq 0, \quad for\ \phi \in]0\ ;\ \phi^*]$ and $\left(W^{EA}_f - W^{NA}_f\right) < 0, \quad for\ \phi \in]\phi^*\ ;\ +\infty[$ with ϕ^* such that $G\left(\phi^*\ ;\ \theta^2_{a_E}\right) = \phi^*$.

Finally, we may examine the relationship between ϕ^* and $\theta^2_{a_E}$, as $\phi^*\left(\theta^{*2}_{a_E}\right)$. We have:

$$\begin{aligned}
\phi^* &= \frac{1+ba^2_E}{1+b\overline{a^2_E}} \cdot \left(\frac{2-x}{x}\right) \cdot \left[1 - \frac{b \cdot \theta^2_{a_E}}{(1+ba^2_E)}\right]^{-1} \\
&\equiv \phi^*\left(\theta^2_{a_E}\right)
\end{aligned}$$

This in particular implies :

- $\frac{\partial \phi^*(.)}{\partial \theta^2_{a_E}} > 0$
- $\phi^*(0) = \left(\frac{2-x}{x}\right) \geq 1$
- $\lim\limits_{\theta^2_{a_E} \longrightarrow \widetilde{\theta^2_{a_E}}^-} \phi^*(.) = +\infty$

As a consequence, $\phi^*\left(\theta^2_{a_E}\right)$ is an hyperbolic, monotonically increasing function of $\theta^2_{a_E}$ for $\theta^2_{a_E} \in \left[0\ ;\ \widetilde{\theta^2_{a_E}}\right[$. As $\phi^*(0) > 1$ (for $x \neq 1$), we conclude that $\phi^* > 1$ when $\theta^2_{a_E} \in \left[0\ ;\ \widetilde{\theta^2_{a_E}}\right[$. This means that for the euro-agregation strategy to be welfare-improving (with respect to the national agregation procedure), the union-wide forecast has to be more accurate than the average one (where accuracy is defined in terms of variance).