The consumption-real exchange rate anomaly*

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Abstract

This paper addresses the Backus-Smith puzzle regarding the absence of a close cross-correlation between relative consumption and real exchange rates, in a simple dynamic general equilibrium open economy model. Following Backus and Smith (1993), we show that a very simple form of market incompleteness combined with wealth effects is sufficient in generating the observed cross-correlation. A key role is played by the steady-state net foreign asset position.

JEL classification F3, F41

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1 Introduction

One of the well known puzzles in international finance is the so called Backus-Smith puzzle (see Backus and Smith, 1993). Under market completeness and supply shocks, there is a perfect correlation between real exchange rates and relative consumption across countries. This model's feature is in sharp

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contrast with empirical evidence that suggests no clear path in the aforementioned cross-correlations.

In his comments on Obstfeld and Rogoff (2000), Engel (2000) wonders to what extent market incompleteness is needed in order for a model with deviations from purchasing power parity (PPP) to explain the empirical evidence.

In this work we give a simple answer to this question: we propose a very simple international real business cycle model with market incompleteness in which only one nominal risk-less bond is traded across the border. This simplest form of market incompleteness is sufficient for addressing the so called Backus-Smith puzzle so that our model is able to reproduce the observed behaviour of the data for a wide range of plausible parameters values.

We highlight the role of two factors in generating the observed cross-correlation. Under supply shocks, a country's net foreign asset position is an important determinant in understanding the observed evidence.[1] With demand shocks, our calibration predicts a negative cross-correlation between the real exchange rate and the consumption differential.

From a modelling perspective, we illustrate which features a simple model requires to replicate some important stylised facts associated with the behaviour of both real exchange rates and consumption. From a policy perspective, our results highlight the role of wealth effects in the transmission mechanism of real shocks, as well as the importance of the source of shocks hitting the economy for the joint behaviour of the real exchange rate and relative consumption.

Our model is a simple two-country stochastic dynamic open economy model: markets are incomplete by only allowing households to trade internationally in a risk-less foreign nominal bond, prices are flexible and households consume domestic as well as foreign-produced consumption goods.[2] Deviations from PPP are obtained by imposing a home-bias toward home-produced goods and we introduce a cost for the home household in holding foreign bonds as in Benigno, P. (2001) in order to pin down a well defined

[1] Lane and Milesi-Ferretti (2001) have shown that net foreign assets over GDP varies across countries and is different from zero.

[2] Market incompleteness in our context refers to the lack of state-contingent claims.

steady state for consumption and assets. Our calibration is standard (see Chari, Kehoe and McGrattan, 2002) and we do not impose any structure on the shocks hitting the economy by focusing only on white noise processes to highlight the underlying mechanisms.

The remainder of the paper is structured as follows: In section two, we discuss the nature of the Backus-Smith puzzle and what has been proposed to solve it. Section three presents the basic structure of the model and sets out the log-linearized equilibrium conditions. Section four outlines the basic mechanism behind our results. The model is calibrated and the results are discussed in sections five and six, respectively. Section seven concludes by summarising our main findings and pointing out the policy relevance of these results.

2 The Backus-Smith Puzzle and Related Literature

The commonly used assumption in open economy macroeconomics that consumers have access to a complete set of state-contingent claims implies that agents can insure themselves against all country-specific risks. As a result, when expressed in a common currency, the *ex ante* marginal utility of income of domestic agents is monotonically related to that of foreign agents.

Backus and Smith (1993) show that if preferences are iso-elastic (additively separable across time and goods), then the risk sharing condition that arises in the presence of state-contingent claims can be expressed as a monotonic relationship between the real exchange rate and the ratio of home to foreign consumption.[3] As a result, a depreciation of the real exchange rate (defined so that a depreciation corresponds to a rise in the real exchange rate) is associated with a relative increase in home consumption.

[3] Specifically, complete state-contingent claims imply: $\frac{Uc(C^*,\zeta^*)}{S_t P_t^*}\kappa = \frac{Uc(C,\zeta)}{P_t}$ where ζ is a shock to preferences and κ is a constant which depends on intial conditions. This link between the home and foreign marginal utilities of income can be re-written as a relationship between the real exchange rate and ratio of marginal utilities of consumption: $\frac{Uc(C^*,\zeta^*)}{Uc(C,\zeta)}\kappa = \frac{S_t P_t^*}{P_t}$. Given iso-elastic preferences, this yields a monotonic relationship between the real exchange rate and the ratio of home to foreign consumption.

Intuitively, if consumption at home increases relative to abroad, the international risk-sharing condition requires the relative price of home consumption to fall in order to maintain equality between the home and foreign marginal utility of income.[4] When PPP is assumed to hold, Backus and Smith (1993) derive an even stronger result, namely that international risk-sharing will equate home and foreign consumption.

Putting their theory to the data, Backus and Smith (1993) find neither a close correlation between home and foreign consumption, nor between relative consumption and the real exchange rate. The latter of these findings is the consumption-real exchange rate anomaly, or the Backus-Smith puzzle.

In table 1, we report on an update of their findings. For periods starting from 1970 until 2002 the correlation between bilateral real exchange rates and relative consumption varies between -.45 and .42. The simple average of correlation coefficients is 0.03.[5]

[TABLE 1 ABOUT HERE]

Table 2 shows the same cross-correlation matrix for a shorter, but common sample period for which data for post unification Germany is available. The simple average of correlation coefficients for this sample is -0.17. The conclusion to be drawn from both table 1 and 2 is that the cross-correlations are small, often negative and in no case close to unity. Comparing country pairs and sample periods suggests that there is no obvious link between

[4] Put differently, when relative consumption rises, it does so because the relative price of home to foreign consumption has fallen. Note that this relationship between the real exchange rate and relative consumption does not hold in the presence of country specific preference shocks.

[5] We use the bilateral quarterly average nominal exchange rate series from the IMF's *IFS*. Real private final consumption expenditure is taken from the OECD Quarterly National Accounts to construct relative consumption. The corresponding deflators are used to construct the consumption based real exchange rate. The series are then logged and H-P filtered and the cross-correlation is calculated. Not all of the data are of the same length. Only for the US, UK and Italy are data available for the full sample (1970 Q1 - 2002 Q2). West German data go from 1970 Q1 until 1994 Q4. French data start in 1978 Q1, Japanese data start in 1980 Q1 and Canadian data start in 1981 Q1. Throughout, we use the longest possible sample.

the type of exchange rate regime and the cross-correlation pattern. This motivates our choice of a model with flexible prices in order to address the anomaly.

[TABLE 2 ABOUT HERE]

In this paper, we follow the suggestions of Backus and Smith (1993). In their influential paper Backus and Smith (1993) write "*One possibility would be to admit demand side shocks in addition to the endowment shocks...*" and "*Other possibilities include (i) wealth effects and ....(iii) incomplete markets*".

Ravn (2001) shows that the data do not support the role of the real exchange rate in explaining cross-country differentials in the marginal utility of consumption, which is an implication of complete markets. Focusing on international risk sharing, Canova and Ravn (1996) as well as Kollmann (1995) both reject the risk sharing proposition across countries.

In addressing the Backus-Smith puzzle, Corsetti, Dedola and Leduc (2002) propose an alternative route to explain the data. Along with frictions in international markets similar to ours, they emphasize goods market frictions in the form of distributive trade using local, non-traded, inputs.[6]

3 The model

The structure of our model closely follows Benigno, P. (2001). The only modification is that we allow for home bias in preferences so that the real exchange rate deviates from PPP.

3.1 Preferences

The world economy is populated by a continuum of agents on the interval $[0, 1]$. The population on the segment $[0, n)$ belongs to the country H (Home), while the segment $[n, 1]$ belongs to F (Foreign). A generic agent j belonging to the world economy is both producer and consumer: a producer

[6] Note instead that in our framework all goods are tradable and the law of one price always holds.

of a single differentiated product and a consumer of all the goods produced in both countries H and F. All goods are traded. Preferences of the generic household j are given by

$$U_t^j = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[U(C_s^j, \xi_{C,s}) - \frac{1}{n} \int_0^n V(y_s^j, \xi_{Y,s}) \right], \qquad (1)$$

where the upper index j denotes a variable that is specific to agent j, E_t denotes the expectation conditional on the information set at date t, while β is the intertemporal discount factor, with $0 < \beta < 1$. Agents obtain utility from consumption and receive disutility from producing goods.[7] We assume that U is an increasing concave function of the index C^j defined as

$$C = \left[v^{\frac{1}{\theta}} C_H^{\frac{\theta-1}{\theta}} + (1-v)^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \ \theta > 0. \qquad (2)$$

The corresponding consumption index for foreign residents is given by:

$$C^* = \left[v^{*\frac{1}{\theta}} C_H^{*\frac{\theta-1}{\theta}} + (1-v^*)^{\frac{1}{\theta}} C_F^{*\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \ \theta > 0 \qquad (3)$$

C_H and C_F are the two consumption sub-indexes that refer, respectively, to the consumption of home-produced and foreign-produced goods and θ is the elasticity of intratemporal substitution. v and v^* are parameters that capture the home bias in preferences. When $v > v^*$ home residents put a higher weight than foreign residents on home-produced goods. Our modelling choice is motivated by a desire to keep the framework as canonical as possible. We have

$$C_H = \left[\left(\frac{1}{n} \right)^{\frac{1}{\sigma}} \int_0^n c(z)^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}}, C_F = \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\sigma}} \int_n^1 c(z)^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}} \qquad (4)$$

where $\sigma > 1$ is the elasticity of substitution for goods produced within a country. The consumption-based price index that corresponds to the above specifications of preferences is:

$$P = \left[v P_H^{1-\theta} + (1-v) (P_F)^{1-\theta} \right]^{\frac{1}{1-\theta}}, \theta > 0 \qquad (5)$$

$$P^* = \left[v^* P_H^{*1-\theta} + (1-v^*) (P_F^*)^{1-\theta} \right]^{\frac{1}{1-\theta}}, \theta > 0 \qquad (6)$$

[7] We have assumed that the utility function is separable in these two factors.

where P_H is the price sub-index for home-produced goods expressed in the domestic currency and P_F is the price sub-index for foreign produced goods expressed in the domestic currency.

$$P_H = \left[\left(\frac{1}{n} \right) \int_0^n p(z)^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}}, P_F = \left[\left(\frac{1}{1-n} \right) \int_n^1 p(z)^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}} \quad (7)$$

We assume that prices are set in the producer currency and that the law-of-one-price holds: $p(h)/S = p^*(h)$ and $p(f) = S \cdot p^*(f)$, where S is the nominal exchange rate (the price of F currency in terms of H currency). From this it follows that $P_H = SP_H^*$ and $P_F = SP_F^*$. As long as $\upsilon \neq \upsilon^*$ PPP does not need to hold, i.e. $P \neq SP^*$.

We define the terms of trade T of country F as the ratio of the price of the bundle of goods produced in country F relative to the price of the bundle imported from country H, such that $T = SP_F^*/P_H = P_F/P_H$.[8]

3.2 The real exchange rate

The real exchange rate in our model deviates from PPP because of home bias in preferences. Taking account of the law-of-one-price, we can express the real exchange rate as:

$$RS = \frac{SP^*}{P} = \frac{P_H}{P} \frac{P^*}{P_H^*} \quad (8)$$

which, given the price indices (5) can be expressed as:

$$RS = \left(\frac{\upsilon^* + (1-\upsilon^*)T^{1-\theta}}{\upsilon + (1-\upsilon)T^{1-\theta}} \right)^{\frac{1}{1-\theta}}.$$

When linearized around a deterministic steady state, the real exchange rate becomes proportionate to the terms of trade, where the factor of proportionality is equal to the degree of home bias, $\upsilon - \upsilon^*$.

[8] We use the academic convention that an increase in the terms of trade or the real exchange is defined as a depreciation.

3.3 Demand

Demands for goods are given by the sub-utility function (4), the allocation of demand across each of the goods produced within a given country for consumers H, F is given by

$$\begin{aligned} c(h) &= \frac{1}{n}\left[\frac{p(h)}{P_H}\right]^{-\sigma} C_H,\ c(f) = \frac{1}{1-n}\left[\frac{p(f)}{P_F}\right]^{-\sigma} C_F \\ c^*(h) &= \frac{1}{n}\left[\frac{p^*(h)}{P_H^*}\right]^{-\sigma} C_H^*,\ c^*(f) = \frac{1}{1-n}\left[\frac{p^*(f)}{P_F^*}\right]^{-\sigma} C_F^* \end{aligned} \tag{9}$$

(where h denotes the representative home good and f the representative foreign good) while the consumption aggregator (2) implies that home demands for composite home and foreign goods, C_H and C_F, is given by

$$C_H = v\left[\frac{P_H}{P}\right]^{-\theta} C,\ C_F = (1-v)\left[\frac{P_F}{P}\right]^{-\theta} C. \tag{10}$$

Similar demand functions holds for the foreign consumer

$$C_H^* = v^*\left[\frac{P_H^*}{P^*}\right]^{-\theta} C^*,\ C_F^* = (1-v^*)\left[\frac{P_F^*}{P^*}\right]^{-\theta} C^*. \tag{11}$$

For the individual goods h's total demand is given by

$$\begin{aligned} y^d(h) &= \left[\frac{p(h)}{P_H}\right]^{-\sigma}\left[\frac{P_H}{P}\right]^{-\theta}\left[vC + \frac{v^*(1-n)}{n}\left(\frac{1}{RS}\right)^{-\theta} C^*\right] \\ y^d(f) &= \left[\frac{p(f)}{P_F}\right]^{-\sigma}\left[\frac{P_F}{P}\right]^{-\theta}\left[\frac{(1-v)\,n}{1-n}C + (1-v^*)\left(\frac{1}{RS}\right)^{-\theta} C^*\right] \end{aligned} \tag{12}$$

where we have used the fact that the law-of-one-price holds but PPP does not because of home bias in preferences.

3.4 Budget constraints and asset markets structure

The asset market structure in the model is relatively standard in the literature. We assume that home individuals are assumed to be able to trade two nominal risk-less bonds denominated in the domestic and foreign currency. These bonds are issued by residents in both countries in order to finance their consumption expenditure. On the other hand, foreign residents can allocate their wealth only in bonds denominated in the foreign currency. Home households face a cost (ie transaction cost) when they take a position in the foreign bond market. This cost depends on the net foreign asset position of the whole economy as in Benigno, P. (2001).[9][10] One way to rationalise this cost is to assume the existence of foreign-owned intermediaries in the foreign asset market who apply a spread over the risk-free rate of interest when borrowing or lending to home agents in foreign currency. This spread depends on the net foreign asset position of the home economy. When the home economy is a net creditor, the rate of interest home agents receive on their foreign currency denominated bond holdings is below that received by foreign agents. Conversely, when the home economy is a net debtor, the rate of interest payable on foreign currency denominated liabilities is above that faced by foreign agents. Domestic firms are assumed to be wholly owned by domestic residents, and profits are distributed equally across households.

Formally the home households' budget constraint is given by:

$$P_t C_t^i + \frac{B_{H,t}^i}{(1+i_t)} + \frac{S_t B_{F,t}^i}{(1+i_t^*)\Theta\left(\frac{S_t B_{F,t}}{P_t}\right)} \leq B_{H,t-1}^i + S_t B_{F,t-1}^i + T_t^i + \frac{\int_0^n p(h) y_t(h) dh}{n} \tag{13}$$

where T^i are lump-sum government transfers, $B_{H,t}^i$ and $B_{F,t}^i$ are the individual's holdings of domestic and foreign nominal risk-less bonds denominated in the local currency. S_t is the nominal exchange rate expressed as units of domestic currency needed to buy one unit of foreign currency. The maximization problem of the home individual consists of maximizing (1) subject to (13) in determining the optimal profile of consumption and bond holdings.

The cost function $\Theta(.)$ drives a wedge between the return on foreign-

[9] Here we follow Benigno, P (2001) in assuming that the cost function $\Theta(.)$ assumes the value of 1 only when the net foreign asset position is at its steady state level, ie $B_{F,t} = \overline{B}$, and is a differentiable decreasing function in the neighbourhood of $\overline{B}$. This cost function is convenient because it allows us to log-linearise our economy properly since in steady state the desired amount of net foreign assets is always a constant $\overline{B}$.

[10] The same stationarity inducing mechanism is also employed in the IMF's Global Economy Model, see Pesenti (2002). See also Benigno and Thoenissen (2002) for an application. Further ways of closing open economy models are discussed in Schmitt-Grohe and Uribe (2001).

currency denominated bonds received by domestic and by foreign residents and insures that domestic holdings of foreign currency denominated bonds return to their steady-state level following a shock, thus making sure that the model is stationary. We assume that profits from intermediation in the foreign bond market are distributed equally among foreign residents (see Benigno, P. 2001). The expression for profits is

$$K = \frac{B_{F,t}}{P_t^* \left(1 + i_t^*\right)} \left[\frac{RS_t}{\phi\left(\frac{S_t B_{F,t}}{P_t}\right)} - 1 \right] \tag{14}$$

where $B_{F,t}$ denotes the domestic holdings of foreign bonds.

Households' equilibrium conditions are described by the following equations:

$$U_C\left(C_t, \xi_{C,t}\right) = (1 + i_t)\beta E_t \left[U_C\left(C_{t+1}, \xi_{C,t+1}\right) \frac{P_t}{P_{t+1}} \right] \tag{15}$$

$$U_C\left(C_t^*, \xi_{C,t}^*\right) = (1 + i_t^*)\beta E_t \left[U_C\left(C_{t+1}^*, \xi_{C,t+1}^*\right) \frac{P_t^*}{P_{t+1}^*} \right] \tag{16}$$

$$U_C\left(C_t, \xi_{C,t}\right) = (1 + i_t^*)\phi\left(\frac{S_t B_{F,t}}{P_t}\right) \beta E_t \left[U_C\left(C_{t+1}, \xi_{C,t+1}\right) \frac{S_{t+1} P_t}{S_t P_{t+1}} \right]. \tag{17}$$

Aggregating the household budget constraints across all households yields:

$$\frac{S_t B_{F,t}}{P_t \left(1 + i_t^*\right) \phi\left(\frac{S_t B_{F,t}}{P_t}\right)} = \frac{S_t B_{F,t-1}}{P_t} + \left(\frac{P_{H,t}}{P_t}\right)^{1-\theta} \left[vC_t + \frac{v^*(1-n)}{n} \left(\frac{1}{RS_t}\right)^{-\theta} C_t^* \right] - C_t. \tag{18}$$

3.5 Price-setting mechanism

We assume that prices are perfectly flexible. Suppliers behave as monopolists in selling their differentiated product, so that prices are a mark-up over marginal costs. From the first order conditions we obtain:

$$(1 - \tau^H) U_C(C_t, \xi_{C,t}) \left(\frac{P_{H,t}}{P_t}\right) = \tag{19}$$

$$\frac{\sigma}{\sigma - 1} V_y \left(\left(\frac{P_{H,t}}{P_t}\right)^{-\theta} \left[vC_t + \frac{v^*(1-n)}{n} \left(\frac{1}{RS_t}\right)^{-\theta} C_t^* \right], \xi_{Y,t} \right),$$

$$(1-\tau^F)U_C(C_t^*,\xi_{C,t}^*)\left(\frac{P_{F,t}}{P_t}\right)\left(\frac{1}{RS_t}\right)= \tag{20}$$
$$\frac{\sigma}{\sigma-1}V_y\left(\left(\frac{P_{F,t}}{P_t}\right)^{-\theta}\left[\frac{(1-\upsilon)\,n}{1-n}C_t+(1-\upsilon^*)\left(\frac{1}{RS_t}\right)^{-\theta}C_t^*\right],\xi_{Y,t}^*\right).$$

3.6 Equilibrium fluctuations

In what follows we describe the evolution of the system around the well defined deterministic steady state described in the appendix. The system represents deviations from steady state under the assumptions that prices are perfectly flexible and the economy is subject to demand (ξ_C,ξ_C^*) and supply shocks (ξ_Y,ξ_Y^*). Log-linearizing the price setting equations (19) and (20)[11] yields:

$$-\rho\widetilde{C}_t+\xi_{C,t}+(\upsilon-1)(1+\eta\theta)\widehat{T}=\eta s\widetilde{C}_t+\eta(1-s)\widetilde{C}_t^*+\theta\eta(1-s)\widetilde{RS}_t-\overline{Y}_t \tag{21}$$

$$-\rho\widetilde{C}_t^*+\ \xi_{C^*,t}^*+\upsilon(1+\eta\theta)\widetilde{T}_t=\left(1+\theta\eta\frac{(1-\upsilon^*)(1-s)\overline{\gamma}}{\upsilon^*(1-\upsilon)+(1-\upsilon^*)(1-s)\overline{\gamma}}\right)\widetilde{RS}_t \tag{22}$$
$$-\overline{Y}_t^*+\frac{\eta\left(\upsilon^*(1-\upsilon)\widetilde{C}_t+(1-\upsilon^*)(1-s)\overline{\gamma}\widetilde{C}_t^*\right)}{\upsilon^*(1-\upsilon)+(1-\upsilon^*)(1-s)\overline{\gamma}}$$

where we have redefined $\xi_{C,t}=\frac{U_{C\xi}}{U_C}\xi_{C,t}$ and $\overline{Y}_t\equiv-\frac{V_{y\xi}}{U_C}\xi_{Y,t}$. The log-linearization of the current account equations yields:

$$\beta\widetilde{b}_t\,(1+\overline{a}\delta)=\widetilde{b}_{t-1}+\overline{a}(\beta i_t^*+\upsilon\Delta S_t)+(1-\theta)\,\overline{\gamma}(\upsilon-1)\widetilde{T}_t+ \tag{23}$$
$$(\upsilon-1)\widetilde{C}_t+(1-s)\overline{\gamma}\widetilde{C}_t^*+\theta(1-s)\overline{\gamma}\widetilde{RS}_t$$

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[11] Where we have defined $\frac{Y}{C}\equiv\gamma=1+\overline{a}(\beta-1)$. Now assuming utility function with constant elasticity we obtain: $\rho=-\frac{U_{CC}\overline{C}}{U_C}$; $\eta=\frac{V_{yy}\overline{Y}}{U_C}$; so that $\frac{V_{yy}\overline{C}}{U_C}=\frac{\eta}{\gamma}$ and $\frac{V_{yy}\overline{C}^*}{U_C}=\frac{V_{yy}\overline{Y}}{U_C}\frac{C}{Y}\frac{\overline{C}^*}{\overline{C}}=\eta\frac{n}{\upsilon^*(1-n)}(1-s)$. We can similarly write $\eta=\frac{V_{yy}\overline{Y}^*}{U_C}$ and we have that $\frac{V_{yy}\overline{C}^*}{U_C}=\eta\frac{(1-s)\gamma}{\upsilon^*(1-\upsilon)+(1-\upsilon^*)(1-s)\gamma}$ and $\frac{V_{yy}\overline{C}}{U_C}=\eta\frac{1}{\upsilon^*(1-\upsilon)+(1-\upsilon^*)(1-s)\gamma}\frac{\upsilon^*(1-n)}{n}$ where we have defined $s\equiv\upsilon/\gamma$ so that $(1-s)=(\gamma-\upsilon)/\gamma$.

Note also that from the definition of the price index we obtain $\frac{\widetilde{P}_{H,t}}{P_t}=(\upsilon-1)\,\widetilde{T}_t$ and $\frac{\widetilde{P}_{F,t}}{P_t}=\upsilon\widetilde{T}_t$ where $T\equiv\frac{P_F}{P_H}$.

where $\overline{a} \equiv \frac{\overline{b}}{\overline{C}}$ and $\widetilde{b}_t = \frac{\left(\frac{S_t B_{F,t}}{P_t} - \overline{b}\right)}{\overline{C}}$ and $\delta = -\phi'(\overline{b})\overline{C}$. From our steady state equations we have that $\frac{\overline{Y}}{\overline{C}} = (\beta - 1)\,\overline{a} + 1 \equiv \overline{\gamma}$ and $1 - s = (\overline{\gamma} - \upsilon)/\overline{\gamma}$ and $s = \upsilon/\overline{\gamma}$.

Our Euler equations come from the first order conditions of the home and foreign consumers. The arbitrage condition for home and foreign bonds yields our uncovered interest rate parity condition.

$$\rho E_t \widetilde{C}_{t+1} - E_t \xi_{C,t+1} = \rho \widetilde{C}_t - \xi_{C,t} + \widetilde{i}_t - (1 - \upsilon) E_t \Delta S_{t+1} \tag{24}$$

$$\rho E_t \widetilde{C}^*_{t+1} - E_t \xi^*_{C,t+1} = \rho \widetilde{C}^*_t - \xi^*_{C,t} + \widetilde{i}^*_t + \upsilon^* E_t \Delta S_{t+1} \tag{25}$$

$$E_t \Delta \widetilde{S}_{t+1} = \widetilde{i}_t - \widetilde{i}^*_t + \delta \widetilde{b}_t \tag{26}$$

The system is closed with the law of motion for the terms of trade and with the relationship between the real exchange rate and the terms of trade.

$$\widetilde{T}_t = \widetilde{T}_{t-1} + \Delta S_t \tag{27}$$

$$\widetilde{RS}_t = (\upsilon - \upsilon^*)\widetilde{T}_t \tag{28}$$

4 Relative consumption, the real exchange rate and the current account

Before proceeding to analyse the moments generated by our calibrated model economy, this section outlines the main mechanism behind our results. There are two mechanisms in our model that cause the cross-correlation between the real exchange rate and relative consumption to deviate from unity. The first is the presence of demand shocks. The intuition is straightforward: a positive shock to home demand raises relative consumption. Output, being demand determined, rises, but not as much as demand. The resulting excess demand is eliminated by a rise in the relative price of the domestic agent's consumption bundle. Because we have assumed home bias in preferences, PPP will not hold, causing the real exchange rate to appreciate (RS falls).

This mechanism is not new and holds even in a complete markets framework as the one analysed by Backus and Smith (1993). In that set up, the presence of state-contingent claims equates the real exchange rate to the ratio of marginal utilities of consumption: $RS = \kappa \frac{U_C(C^*,\xi^*)}{U_c(C,\xi)}$. Here, the presence of asymmetric demand shocks breaks the monotonic relationship between the real exchange rate and relative consumption.

The second mechanism is related to the dynamics of the current account. When analysing the moments of our calibrated model we show that under supply side shocks the cross-correlation is only equal to unity when a shock does not result in the accumulation of foreign-currency denominated bonds. When agents accumulate or decumulate bonds in response to shocks, the resulting wealth effect drives a 'wedge' between the dynamics of the real exchange rate and relative consumption. The larger is the response of the current account, the greater will be this 'wedge' and the smaller the cross-correlation.

We can illustrate these two mechanisms by subtracting equation (25) from (24) and using equations (26),(27) and (28) such that

$$\rho \left(\mathrm{E}_t \widetilde{C}^R_{t+1} - \widetilde{C}^R_t \right) = \mathrm{E}_t \widetilde{RS}_{t+1} - \widetilde{RS}_t - \delta \widetilde{b}_t - \xi^R_{C,t}. \qquad (29)$$

where $\widetilde{C}^R_t = \widetilde{C}_t - \widetilde{C}^*_t$. Equation (29) shows that in an incomplete markets model, the real exchange rate and relative consumption are related in terms of expected first differences. This equation shows that the cross-correlation between relative consumption and the real exchange rate (in expected first difference terms) is equal to unity only in the absence of demand shocks ($\xi^R_{C,t} = 0$) and when domestic holdings of foreign currency-denominated bonds remain at their equilibrium level ($\widetilde{b}_t = 0, \forall t$). We use this equation to analyse the cross-correlations arising from different calibrations reported in table 1.[12]

[12] Equation (29) is only used to illustrate the intuition. The cross-correlation between $\widetilde{C}^R_t$ and $\widetilde{RS}_t$ is not the same as the cross-correlation between $\left(\mathrm{E}_t \widetilde{C}^R_{t+1} - \widetilde{C}^R_t \right)$ and $\left(E_t \widetilde{RS}_{t+1} - \widetilde{RS}_t \right)$.

5 Calibration

Our calibration is generic; we assume that both economies are of equal size, but do not attempt to match the salient features of a particular economy. We start with a very simple baseline calibration in which the coefficient of relative risk aversion, $\rho = -\frac{U_{CC}\overline{C}}{U_C} = 1$; the intratemporal elasticity of substitution between home and foreign-produced traded goods, $\theta = 1$ and the steady-state net foreign asset position relative to steady-state consumption, $\overline{a} = 0$. In our model with deviations from PPP, this combination of parameters combined with only supply shocks replicates the allocation that would arise under market completeness (in Benigno, P. (2001) where PPP is assumed to hold, the only requirement would be to have $\theta = 1$ and no asymmetric demand shocks). We start with this calibration to illustrate the Backus-Smith puzzle. We follow Chari, Kehoe and McGrattan (2002) in setting the elasticity of labour supply for a given marginal utility equal to 1/2 such that $\eta = \frac{V_{yy}\overline{Y}}{U_C} = 2$. We follow Benigno, P. (2001) in choosing a 10 basis point spread of the domestic interest rate on foreign assets over the foreign rate, such that $\delta = 0.001$. The real exchange rate deviates from PPP because of home bias. Our baseline calibration assumes a very modest degree of home bias, specifically we assume that $\upsilon = 0.6$ and $\upsilon^* = 0.5$.

6 Cross-correlations

We solve the log-linearized systems of equations (21) - (28) using the Reds/Solve algorithm of King and Watson (1998). This algorithm provides a general solution of the form:

$$\begin{aligned} y_t &= Dk_t + Fx_t \\ k_{t+1} &= Gk_t + Hx_t \end{aligned} \tag{30}$$

where y_t is a vector of 'jump variables', k_t is a vector of predetermined or state variables, $\widetilde{b}_t$ and $\widetilde{T}_t$ in our case and x_t is a vector containing the supply and demand shocks in the two countries. Given the variance-covariance matrix of the shocks, we use the ACM/VCV algorithms of King and Watson to derive the variance-covariance matrix of the jump and predetermined variables of the model. Specifically, table 2 reports on the cross-correlation between the (log-linearized) real exchange rate and relative consumption (log-linearized) conditional on domestic supply shocks (column 3) and domestic demand shocks (column 4).[13] We make no specific assumptions regarding the variance or covariance of shocks hitting the model economy, instead we assume that shocks to supply and preferences are white noise.

Table 2 shows the cross-correlations between the real exchange rate and relative consumption for supply and demand shocks hitting the home economy. We start with our baseline calibration outlined above, changing only those parameters given in the second column.

[TABLE 3 ABOUT HERE]

As outlined above, table 3 shows that in our simple incomplete markets model the cross-correlation between the real exchange rate and relative consumption is, in general, not unity.

Our first observation from table 3 is that in our simple model, demand shocks result in a negative cross-correlations between the real exchange rate and relative consumption. The size of the cross-correlation depends on the elasticity of output, $1/\eta$. The more elastic is the supply of goods, the smaller is the resulting excess demand and thus the smaller is the real appreciation relative to the increase in relative consumption. Hence for a given demand shock, the cross-correlation becomes larger (less negative) the smaller is η. This can be seen by comparing rows three and eight.

Our baseline calibration illustrates the role of the current account. It is well known that under Cobb-Douglas preferences, movement in the terms of trade have a risk sharing role so that the complete markets allocation can be achieved without explicitly modelling the asset market. This has been shown by Cole and Obstfeld (1991) and Corsetti and Pesenti (2001) among others. Benigno, P. (2001) shows that this result only holds in the absence of asymmetric demand shocks. Our baseline calibration suggests that if PPP does not hold, we require a unitary intertemporal elasticity of substitution

[13] $Corr(\widetilde{RS}, \widetilde{C} - \widetilde{C}^{*}) = \frac{Cov(\widetilde{RS}, \widetilde{C} - \widetilde{C}^{*})}{\sqrt{Var(\widetilde{RS}) \times Var(\widetilde{C} - \widetilde{C}^{*})}}$

($\rho = 1$) as well as a unitary intratemporal elasticity of substitution to replicate the complete markets allocation, but only for asymmetric supply side shocks. In this calibration, changes in the real exchange rate and in relative consumption offset each other, leaving the current account unchanged. As a result, the cross-correlation generated by the model is 1.0 for supply shocks.

Rows two and three illustrate that for supply side shocks, the cross-correlation deviates from unity as the calibration moves away from Cobb-Douglas preferences, in terms of both the intra [2] and intertemporal elasticity of substitution [3]. In choosing the values of θ and ρ in rows two and three, we follow the calibration of Chari, Kehoe and McGrattan (2002). Intuitively, the more substitutable are home and foreign-produced consumption goods (the larger is θ) the less the terms of trade (and thus the real exchange rate) respond to supply side shocks. In the extreme case, where there is no specialization in trade, or where θ is very large, the terms of trade do not respond to supply side shocks, which would result in a zero cross-correlation. In terms of equation (29), an intra-temporal elasticity of substitution different from unity introduces current account dynamics that break the otherwise monotonic relationship between the real exchange rate and relative consumption.

An increase in ρ lowers the elasticity of marginal utility with respect to consumption, and in the case of the iso-elastic utility function (1) raises the coefficient of relative risk aversion. The more risk averse are individuals, the more they aim to smooth consumption across states of nature. Thus the greater is ρ the lower is the volatility of relative consumption. Equation (23) shows that if the response of the real exchange rate to a shock is larger than the response of relative consumption, so that changes in the relative consumption do not fully offset changes in the real exchange rate, the current account will, *ceteris paribus,* deviate from zero. The dynamics of the current account break the link between the real exchange rate and relative consumption as equation (29) illustrates.

Whereas the cross-correlations in rows two and three are less than unity, they are still far higher than those observed in the data. In row 4, we combine the changes to the baseline calibration made in rows 2 and 3 to reproduce the calibration in Chari, Kehoe and McGrattan (2002). For this set of parameters, we find that our simple model generates cross-correlations of around 0.4 and -0.4 for white noise supply and demand shocks, respectively. This result illustrates that a simple incomplete markets model can, for a reasonable calibration, generate cross-correlations that are close to those found in the data.

A key parameter in our model is the steady-state level of net foreign assets relative to steady-state consumption, $\overline{a}$. In order to calibrate this parameter, we refer to values compatible with Lane and Milesi-Ferretti (2001). Rows five and six report cross-correlations for a calibration which is the same as the baseline calibration, save for the values of ρ and $\overline{a}$. We find that even for supply side shocks, our model generates cross-correlations close to those in the data if $\overline{a}$ is different from zero. Non-zero net foreign assets add a further term to the current account equation of the model. The current account, equation (23), is not just affected by changes in relative consumption and the real exchange rate, but also by a direct term capturing the changes in the rate of return on holdings of foreign bonds $\overline{a}(\beta i_t^* + v\Delta S_t)$. Except in the special case where $\theta = \rho = 1$, this additional term adds to the volatility of the current account, further undermining the positive relationship between the real exchange rate and relative consumption.

In row seven, we raise the degree of home bias relative to our baseline calibration. We find that the (absolute value of the) cross-correlation increases along with the degree of home bias. Home bias in consumption introduces an additional degree of asymmetry into the model. The more asymmetric the model, the more relative consumption responds to country specific shocks. We find that the volatility of the consumption differential relative to that of the real exchange rate increases with the degree of home bias. For shocks where the real exchange rate and relative consumption tend to move in the same (opposite) direction, this increases (reduces) the cross-correlation.

6.1 Sensitivity analysis

In this section, we examine the cross-correlation between the real exchange rate and relative consumption over a broader parameter range. In particular, we focus on supply shocks and the role of the intertemporal elasticity of substitution, ρ, the intratemporal elasticity of substitution ,θ, and the steady-state ratio of foreign-currency denominated debt to consumption in the home country, $\overline{a}$.

Figure 1 plots the cross-correlation for various values of ρ and $\overline{a}$. Apart from these two parameters, the calibration corresponds to our baseline scenario. The graph shows that as long as both θ and ρ are equal to one, the correlation is equal to unity, independent of the level for steady state net foreign assets. We also find that, for this calibration, the largest correlations correspond to a zero net foreign asset position. The correlation declines for non-zero values of $\overline{a}$. The larger is ρ, the more sensitive is the correlation to changes in the level of steady state net foreign assets.

[FIGURE 1 ABOUT HERE]

Figure 2 plots the cross-correlation for various values of θ and $\overline{a}$. In addition to varying these two parameters, we depart from the baseline calibration by setting $\rho = 5$ as suggested by Chari, Kehoe and McGrattan. Given this calibration, we find that the correlation is significantly below one for most of the parameter range. Again, our model can replicate the Backus-Smith puzzle, but only for a limited range of combinations of θ and $\overline{a}$. The level of steady-state net foreign assets that yields the highest cross-correlation varies with the elasticity of substitution between home and foreign-produced traded goods.

[FIGURE 2 ABOUT HERE]

Our sensitivity analysis suggests that in our model a cross-correlation between the real exchange rate and relative consumption close to unity, as associated with the Backus-Smith puzzle, arises only under very specific parameter combinations. In general, our model generates cross-correlations significantly lower than unity, and for reasonable calibrations can replicate cross-correlations observed in the data even for supply side shocks.

7 Conclusion and policy relevance

This paper addresses the Backus-Smith puzzle regarding the absence of a link in the cross-correlation between relative consumption and real exchange rates in a simple dynamic general equilibrium open economy model. Following Backus and Smith (1993), we show that a very simple form of market incompleteness combined with wealth effects is sufficient in generating the observed cross-correlations. From a policy point of view, our paper flags two important issues. First, we show that for most calibrations the joint response of the real exchange rate and the relative consumption depends on the source of shocks hitting the economy. A positive co-movement can be observed if shocks hitting the economy originate from the supply side, whereas a negative co-movement results mainly from demand side shocks. Second, our results highlight the importance of wealth effects for the transmission mechanism of shocks, as well as the role of the net foreign asset position in determining such wealth effects. We show that by allowing for non-zero steady-state net foreign asset positions, we introduce a further element of volatility to the current account, which in our model amplifies the (temporary) wealth effect on consumption and the real exchange rate. We also show that the sign of the net foreign asset position matters for the response of consumption and the real exchange rate following supply side shocks, and thus for policy.

Future work might usefully extend our framework to a sticky price setting with endogenous monetary policy, to check the robustness of our findings along that dimension.

A Steady-state equations

In the steady state, we normalize $P_H = P_F$, such that $\frac{P_H}{P} = \frac{P_F}{P} = RS = 1$. Applying this normalization to (19) and (20) yields:

$$U_C(\overline{C},0) = \frac{\sigma}{\sigma-1}V_y\left(\left[v\overline{C} + \frac{v^*(1-n)}{n}\overline{C}^*\right],0\right) \tag{31}$$

$$U_C(\overline{C}^*,0) = \frac{\sigma}{\sigma-1}V_y\left(\left[\frac{(1-v)\,n}{1-n}\overline{C} + (1-v^*)\overline{C}^*\right],0\right). \tag{32}$$

From the aggregate budget constraint, we obtain the steady-state link between consumption and bond holdings:

$$\beta\overline{b} = \overline{b} + \left[v\overline{C} + \frac{v^*(1-n)}{n}\overline{C}^*\right] - \overline{C} \tag{33}$$

where $b = \frac{SB_F}{P}$.

This is the steady state around which we log-linearize. We use this steady state to derive three steady-state ratios used in the log-linearization:

All output is consumed:

$$\begin{aligned} \overline{Y} &= v\overline{C} + \frac{v^*(1-n)}{n}\overline{C}^* \\ \overline{Y}^* &= \frac{(1-v)\,n}{1-n}\overline{C} + (1-v^*)\overline{C}^* \end{aligned}$$

The ratio of foreign to home consumption is determined by the steady state aggregate budget constraint:

$$\frac{\overline{C}^*}{\overline{C}} = \frac{n}{v^*(1-n)}\left[1 - v + \overline{a}(\beta - 1)\right]$$

Which shows that $\frac{\overline{C}^*}{\overline{C}}$ decreases with the steady-state net foreign asset position of the home economy, $\overline{a} = \overline{b}/\overline{C}$. We can use these three relationships to define the following two ratios:

$$\begin{aligned} \frac{\overline{Y}}{\overline{C}} &= 1 + \overline{a}(\beta - 1) \equiv \overline{\gamma} \\ \frac{\overline{Y}^*}{\overline{C}^*} &= \frac{v^*(1-v) + (1-v^*)(1-s)\overline{\gamma}}{(1-s)\overline{\gamma}} \end{aligned}$$

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TABLES 1, 2 & 3

Table 1: The correlation between real exchange rates and relative consumption - Source: OECD and IMF

	US	W. Ger	France	Italy	Canada	Japan
UK	-0.4504	0.2210	0.0201	0.2041	0.0781	0.3266
US		-0.1309	-0.2016	-0.3232	-0.1936	0.3945
W. Ger			0.3688	-0.1	0.0363	0.1134
France				-0.2457	0.063	0.2275
Italy					-0.1557	0.0054
Canada						0.4208

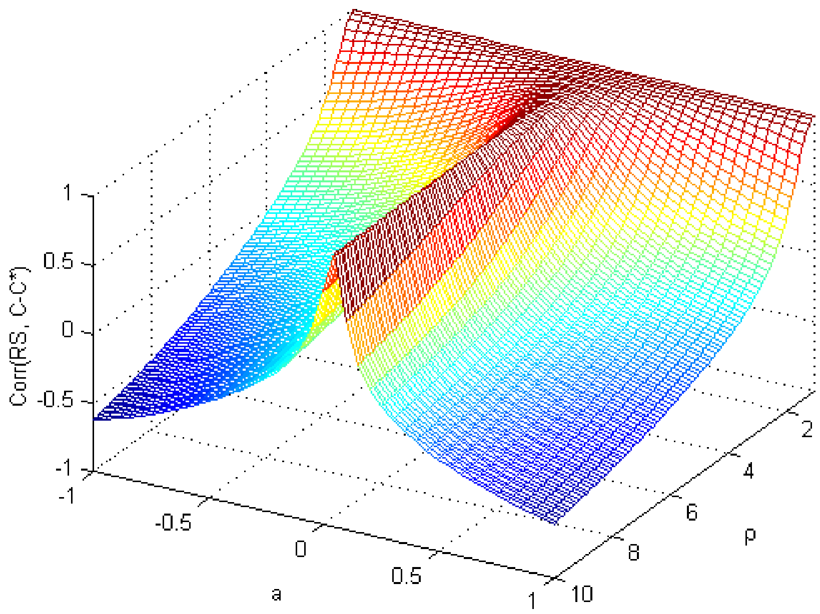
Table 2: The correlation between real exchange rates and relative consumption - 1991 Q1: 2002 Q2. Source: OECD and IMF

	US	Germany	France	Italy	Canada	Japan
UK	0.0942	-0.3022	-0.2977	-0.0159	0.1961	0.4705
US		-0.2900	-0.4487	-0.5350	-0.4420	0.3134
Germany			0.0444	-0.6454	-0.4313	0.3468
France				-0.6680	-0.5412	0.2659
Italy					-0.5538	-0.1207
Canada						0.0684

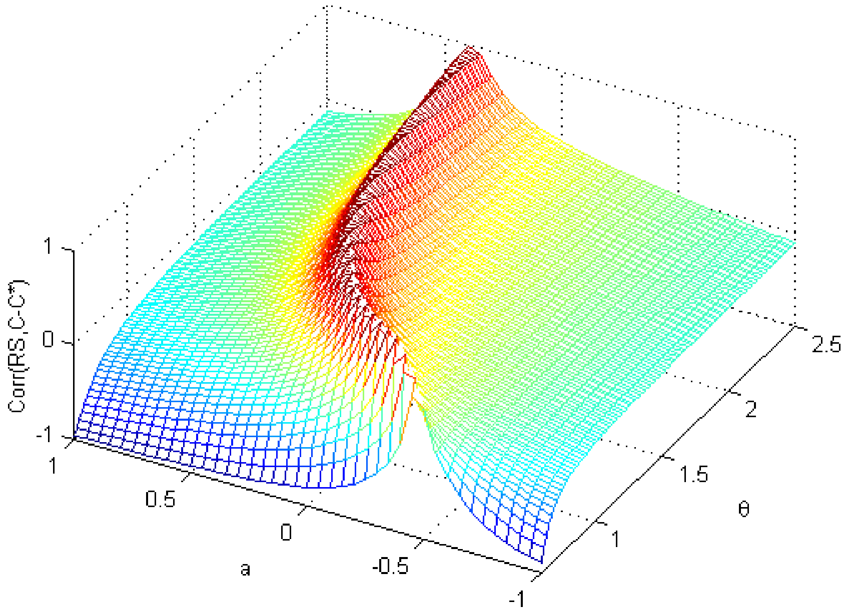
Table 3: Relative consumption-real exchange rate cross-correlations

		Supply shocks	Demand shocks
[1]	Baseline calibration	1.0	-0.797
[2]	$\theta = 1.5$	0.852	-0.826
[3]	$\rho = 5$	0.898	-0.309
[4]	$\rho = 5$, $\theta = 1.5$	0.425	-0.408
[5]	$\rho = 5$, $\overline{a} = 0.5$	-0.017	-0.981
[6]	$\rho = 5$, $\overline{a} = -0.5$	0.088	-0.287
[7]	$\rho = 5$, $v^{*} = 0.4$	0.918	-0.566
[8]	$\rho = 5$, $\eta = 1/2$	0.915	-0.082

FIGURES 1 & 2



The correlation between the real exchange rate and relative consumption for various values ρ and $\overline{a}$.



The correlation between the real exchange rate and relative consumption for various values θ and $\overline{a}$.