# Macroeconomic evolution after a shock: the role for financial intermediation.

D. Vinogradov<sup>1</sup>

This version: June 2003

<sup>&</sup>lt;sup>1</sup>Alfred-Weber-Institute, University of Heidelberg (mailing address: Chair of Economic Theory I, AWI, Grabengasse 14, 69117, Germany) and the Higher School of Economics, Moscow. E-mail: dvinogra@ix.urz.uni-heidelberg.de. I am deeply grateful to Jürgen Eichberger, Hans Gersbach, Marina Doroshenko and Ani Guerd-jikova for fruitful discussions and helpful comments. All remaining errors are of mine.

# Macroeconomic evolution after a shock: the role for financial intermediation

#### Abstract

An overlapping generations model with two production factors and two types of agents is considered in presence of financial intermediation. The research focuses at the analysis of the consequences of a suddain production shock on a financial intermediation capacities and consequently on the economy as a whole. The model exhibits a property of the "chain reaction" when a single macroeconomic shock can lead to the exhaustion of credit resources and the bankruptcy of the whole banking system. To maintain the capability of the system to recover a regulatory intervention is needed even in presence of the state guarantees on agents' deposits in the banks (workout incentives). Comparison with a pure market economy shows, that a system with properly regulated intermediation provides intertemporal smoothing of shocks, and the social losses induced by the shock are below those in the market economy.

**Keywords:** Financial intermediation, overlapping generations, general equilibrium, regulation, intertemporal smoothing

**JEL Classification:** D50, G21, G28, E44, E53, O16

# 1 Introduction

A huge literature in "modern" financial intermediation grew up since Gurley and Shaw (1960), and one of the most complete recent reviews in financial intermediation theory is the one by Gorton and Winton (2003). Financial intermediaries, defined as firms that borrow from consumers/savers and lend to firms that need resources for investment, and mostly referred to as banks (but not limited to them), are often argued to be promoters of economic efficiency. This is explained among others through funds channeling, transaction costs and risk reduction, and information production by banks. However the question of the role the banks play in the economy and the structure of the financial system (bank-dominated or market-dominated) is not yet closed (see e.g. Allen and Gale, 2000, and Bolton, 2002). Market dominated systems are claimed to be more efficient in information provision and better in capital provision for entrepreneurs. This paper attempts to continue the comparison of bank-dominated and market-dominated macroeconomies, studying their evolution after a sharp exogenous shock. Exogenous and policy shocks belong to the sources of financial crises (see e.g. Sachs, 1998, who distinguishes between four triggering mechanisms of financial, exchange and banking crises: exogenous shock, policy shock, exhaustion of borrowing limits and a self-fulfilling panics; and Mishkin, 2000, for analysis of sources of financial crises).

This short introduction does not aim at the discussion of possible ways to study crises mechanisms and preventive measures. Some reviews of the theoretical developments in the field of banking regulation, dedicated to avoid crises and/or minimise their economic consequences can be found in Dewatripont and Tirole (1994), Bhattacharia, Boot and Thakor (1998). Empirical findings show however, that theoretically optimal regulation not necessarily leads to avoidance of distortions, so e.g. Demirgüc-Kunt and Detragiache (2000) question whether deposit insurance increases banking system stability; Barth, Caprio and Levine (2000) rise the same question with respect to regulation and ownership. Further empirical evidence on bank insolvencies and crises can be found e.g. in Caprio and Klingebiel (1996) or Arteta and Eichengreen (2000). These findings show that there is a need for a solid macroeconomic theory of financial intermediation, to give the possibility of studying the problem in its complexity, behind the framework that is nor-

<sup>&</sup>lt;sup>1</sup>In contrast to earlier economic view on financial intermediation as a normal business.

mally imposed in microeconomic theory of banking.

Bernanke and Gertler (1987) embed the banking sector into a stylised general equilibrium framework to show, that banks matter to real activity mainly because they provide the only available conduit between the savers and investment projects, which require intensive evaluiation and auditing. They show also the usefulness of such models for understanding of financial crises, disintermediation, banking regulation and certain types of monetary policy. A general equilibrium model with financial intermediation in static form is as well developed by Bencivenga and Smith (1991), who suggest a simple theoretical framing for study of various financial regulations of the financial system in the context of macroeconomic growth. We follow the same approach in embedding financial intermediation in the macroeconomy in this paper.

In order to study the effects of regulatory interventions, we construct a dynamical setting similar to Gersbach and Wenzelburger (2002, a and b), who also study the evolution of the intermediated macroeconomy in presence of shocks.

Gersbach and Wenzelburger assume that the economy is subject to small repeated stochastic shocks. Allen and Gale (1997) small stochastic shocks through stochastic dividends paid on stocks. At the same time they start their paper with an example of sharp exogenous shocks such as an Oil Shock in 1970-s. Such exogenous shocks are undeviersifiable risks, and the concept of intertemporal smoothing, suggested by Allen and Gale, is developed further in current paper. We focus however rather on a single sharp disturbance, taking an example of production shock.

The paper is organised as follows. In Section 2 we discuss the decisions of the agents in each generation and construct therefore an Overlapping Generations framework<sup>2</sup>. In the same section we introduce the concept of macroeconomic shock and discuss three degrees of shock, which are relevant for the analysis. In Section 3 the general equilibrium is defined, and its existence and uniqueness proved for the economy described above. Section 4 intro-

<sup>&</sup>lt;sup>2</sup>The question of equivalence between long-term oriented Overlapping Generations (OLG) and short-term oriented Infinitely Lived Representative Agent (ILRA) models is not discussed in this paper, although it can be expected that the results obtained for a simlpified OLG will also be valid for quickly developing short-term processes, e.g. under the assumption of the myopia in agents behaviour. Some discussion on both approaches and similarity of the results can be found e.g. in Lovo (2000), Aiyagari (1987 and 1992), Huo (1987) and Woodford (1986) so that we do not focus on this matter further.

duces regulation into the model and highlights the principal differences of the regulatory economy in comparison to the unregulated case. In Sections 5 and 6 we study the evolution of the economy after a macroeconomic shock in presence and absence of intermediation; we show how regulative intervention can prevent from crisis development in banking sector and what are the differences in intermediated evolution and that without intermediation. The paper concludes with an overview of results and possible policy implications.

# 2 The Model

## 2.1 Agents and decisions

Consider the economy with population living for two periods and devided into two groups: workers and entrepreneurs. The whole population is distributed at the interval [0,1], where the subinterval [0,p) belongs to workers, leaving [p,1] for entrepreneurs, so that p is the share of workers in each generation.<sup>3</sup> The entrepreneurs can only run their firms as they acquire some experience in the first period of their lives, so that the whole generation only works when young. The young generation works, consumes and saves. The old generation consumes (if workers) or produces and consumes (if entrepreneurs). All agents are therefore endowed with one unit of labour in the first period, and the entrepreneurs possess equal entrepreneural skills, which they apply in the second period of their lives, so that the heterogeneity appears neither among workers nor among entrepreneurs.

All the agents of generation t have identical intertemporal utility function  $u(c_t, c_{t+1})$ , which satisfies standard assumptions on monotonicity and concavity. Time-index denote the beginning of the period, so that generation t is born at the moment t when the period t begins, is young till the moment t+1, is old in the period t+1, and dies in the moment t+2, which ends the period t+1.

All agents when young solve intertemporal utility maximisation problem, which determines their consumption  $c_t$  and savings  $e_t$  in period t. The po-

<sup>&</sup>lt;sup>3</sup>We assume here for simplicity that the share of workers in each generation is fixed. This seems not to be crucial, and the same results can be obtained with the market–side-switching as in Gersbach-Wenzelburger (2002 a and b). Some preliminary explorations of the author show that the market side switching does not change the results of current paper qualitatively.

tential entrepreneurs have perfect knowledge about the wage rate  $w_{t+1}$  and interest rates  $r_t^d$  and  $r_t^c$  on deposits and credits correspondingly.

Optimisation problem of the workers is given by

$$\max_{c_{t}, c_{t+1}} u(c_{t}, c_{t+1})$$

$$c_{t} = w_{t} - e_{t}^{D}$$

$$c_{t+1} = (1 + r_{t}^{d}) e_{t}^{D}$$
(1)

This problem determines the savings function of workers (depositors)  $e_t^D \begin{pmatrix} r_t^d, w_t \\ + \end{pmatrix}$ .

Potential entrepreneurs solve their respective utility maximisation problem

$$\max_{c_{t}, c_{t+1}} u(c_{t}, c_{t+1})$$

$$c_{t} = w_{t} - e_{t}^{E}$$

$$c_{t+1} = \prod_{t+1}^{e} (e_{t}^{E}, w_{t+1}, r_{t}^{c})$$
(2)

where  $\Pi_{t+1}^e$  denote their expected profit in the period t+1 after starting production (running the firms). We assume therefore that  $\Pi_{t+1}^e \geq \left(1 + r_t^d\right) e_t^E$ , otherwise the problem of switching from entrepreneurs to depositors arises (see e.g. Gersbach and Wenzelburger, 2002). The amount of expected profit  $\Pi_{t+1}^e$  generally depends on macroeconomic shock and is discussed below.

Entrepreneurs run firms when old, and the production technology of these firms is given by  $f(k_{t+1}, l_{t+1})$ , where  $k_{t+1}$  = physical capital for production in period t+1, and  $l_{t+1}$  = the amount of labour used for production in the period t+1. All entrepreneurs have access to the same production technology (so that there is no heterogeneity among entrepreneurs), and maximise their expected profits, defining thus both  $k_{t+1}$  and  $l_{t+1}$ . In period t potential entrepreneurs apply for credits to finance their investment  $I_t$ , needed to acquire capital stock  $k_{t+1}$ , so that  $k_{t+1} = e_t^E + I_t$ .

The economy is assumed to suffer from macroeconomic shock  $q_{t+1}$ , which influences the actual output of entrepreneurs in period t+1, so that the profit, obtained by the entrepreneurs in this period is

 $\Pi_{t+1} = \max \left[ q_{t+1} f(k_{t+1}, l_{t+1}) - (1 + r_t^c) \left( k_{t+1} - e_t^E \right) - w_{t+1} l_{t+1}, 0 \right].$ The sheet sequence to the following distributions.

The shock parameter  $q_{t+1}$  has the following distribution:

$$q_{t+1} = \left\{ \begin{array}{cc} 1 & \text{if } t \neq \tau \\ q^* < 1 & \text{if } t = \tau \end{array} \right\}$$
 (3)

so that the probability  $P(q_{t+1} = 1) = 1 - \varepsilon$ , and  $P(q_{t+1} = q^*) = \varepsilon$  with  $\varepsilon \to 0$ .

If the shock  $q^* < 1$  happens, i.e. in period  $t+1 = \tau+1$ , profit of the entrepreneurs is  $\Pi_{t+1} = \max \left[ q^* f\left(k_{t+1}, l_{t+1}\right) - \left(1 + r_t^c\right) \left(k_{t+1} - e_t\right) - w_{t+1} l_{t+1}, 0 \right]$ . In case their revenues are not high enough to cover the expenditures, there is a priority of payments, so that firstly the wages are paid to the workers, and the creditors (banks) receive the rest. Repayments to workers are hence given by  $\widehat{W}_{t+1} = \min \left[ w_{t+1} l_{t+1}, q^* f\left(k_{t+1}, l_{t+1}\right) \right]$  and the repayments to the banks are  $\widehat{B}_{t+1} = \min \left[ \left(1 + r_t^c\right) \left(k_{t+1} - e_t\right), q^* f\left(k_{t+1}, l_{t+1}\right) - \widehat{W}_{t+1} \right]$ .

Expected profit of the entrepreneurs is hence given by

$$\Pi_{t+1}^{e} = (1 - \varepsilon) \,\Pi_{t+1}^{q=1} + \varepsilon \Pi_{t+1}^{q=q^*} \tag{4}$$

Due to the probability distribution of q, expected profit of the entrepreneurs  $\Pi_{t+1}^e \to f(k_{t+1}, l_{t+1}) - (1 + r_t^c)(k_{t+1} - e_t) - w_{t+1}l_{t+1}$ . Profit maximisation problem under positive prices  $r_t^c$  and  $w_{t+1}$  implies then

$$f(k_{t+1}, l_{t+1}) - (1 + r_t^c)(k_{t+1} - e_t) - w_{t+1}l_{t+1} \to \max_{k_{t+1}, l_{t+1}}$$
(5)

and the solution of this problem for any level of savings  $e_t$  appears as

$$k\left(r_t^c, w_{t+1}\right) : \frac{\partial k}{\partial r_t^c} < 0, \frac{\partial k}{\partial w_{t+1}} < 0 \tag{6}$$

$$l\left(r_t^c, w_{t+1}\right) : \frac{\partial l}{\partial r_t^c} < 0, \frac{\partial l}{\partial w_{t+1}} < 0 \tag{7}$$

Note that the solution of profit maximisation problem does not depend on the savings level as stated in the following proposition:

**Proposition 1** Decisions of the entrepreneurs on their optimal savings and optimal production plan are separated.

### Proof.

For notational needs we will distinguish between two types of optimisation problems:

(I) Entrepreneurs decide first on the optimal amount of capital and labour needed for optimal production in sense of profit maximisation. They consider therefore their savings level as exogenousely given. After that they decide upon the optimal savings level, given optimal production plan. (II) Entrepreneurs decide simultaneously, how much capital and labour they need for the production, and how much savings they need to make. In this case they consider savings level endogenously.

Consider first the solution (6-7) of the profit maximisation problem (5) above, which is made independently on the savings decision (optimisation problem of the type I). The first-order conditions (FOC) for this problem are

$$f_k - (1+r) = 0 (8)$$

$$f_l - w = 0$$

with  $f_k$  and  $f_l$  = first deirvatives of the production function with respect to capital and labour correspondingly.

We will show now that the solution (6-7) delivers optimum to the simultaneous profit-savings optimisation problem (II) as well. FOC for the profit maximisation problem (II) would be:

$$f_k - (1+r) + (1+r)\frac{\partial e}{\partial k} = 0$$

$$f_l - w + (1+r)\frac{\partial e}{\partial l} = 0$$
(9)

so that for the proof it suffices to show that the soulution of problem (I) is at the same time a solution of the problem (II).

In the settings of problem (I) FOC for the utility maximisation (2) is:

$$-u_1 + (1+r)u_2 = 0$$

with  $u_1$  and  $u_2$  = derivatives of the utility function with respect to its first and second arguments correspondingly. Differentiating this equation with respect to capital and using the constraints from (2) gives the derivative  $\frac{\partial e}{\partial k}$ :

$$\frac{\partial e}{\partial k} = (f_k - (1+r)) \frac{u_{12} - (1+r)u_{22}}{u_{11} - 2(1+r)u_{12} + (1+r)^2 u_{22}}$$

Substituting this into FOC for capital (9) for the problem (II) obtain

$$\left[\frac{u_{11} - (1+r)u_{12}}{u_{11} - 2(1+r)u_{12} + (1+r)^2 u_{22}}\right] (f_k - (1+r)) = 0$$

The term in square brackets in this equation is always positive due to the properties of the utility function, so that the only possibility for optimum is

that the second bracket is zero, which coincides with the FOC (8) for the problem (I). Similarly for labour. Hence the independently made optimal choice coincides with optimal simultaneous choice, i.e. the solution of Problem (I) delivers optimum to the problem (II) and decisions upon optimal production and optimal savings can be separated.

The intuition behind this proposition is that the optimisation of the production involve the choice of the optimal amount of capital, and not that of the pure credits, which are capital minus savings of the entrepreneurs. Another intuition is that entrepreneurs could have made their savings in a form of deposits, which would bring the same interest rate as that on credits. In this case the problem is equivalent, but it makes clear, that the savings and production decisions are separated.

Given this optimal choice of production factors, utility maximisation problem of the entrepreneurs determines their savings level:

$$\max_{e_{t}} u\left(c_{t}, c_{t+1}\right)$$

$$c_{t} = w_{t} - e_{t}^{E}$$

$$c_{t+1} = \Pi_{t+1} = f\left(k_{t+1}, l_{t+1}\right) - \left(1 + r_{t}^{c}\right) \left(k_{t+1} - e_{t}^{E}\right) - w_{t+1}l_{t+1}$$
with  $(k_{t+1}, l_{t+1}) = \arg\max\Pi_{t+1}$ 

so that savings of the entrepreneurs are given by the function

$$e_t^E = e\left(w_t, w_{t+1}, r_t^c\right) \tag{11}$$

Properties of the solutions of optimisation problems are taken as standard results of microeconomic analysis. $^4$ 

# 2.2 Degrees of shock

Given the priorities of repayments, decribed above, we can distinguish between three degrees of shock:

<sup>&</sup>lt;sup>4</sup>To prove these properties it suffices to find respective derivatives of the implicit savings- and factor demand functions, given by the first-order conditions. Of course, standard assumption on impossibility of corner solutions applies. For the comprehensive study of demand functions properties see e.g. Barten and Böhm (1982) and Nadiri (1982).

- 1. Insignificant shock, which leaves entrepreneurs able to meet their obligations both in wages and credit repayments. In this case  $q^* \in [\overline{q}, 1]$  where  $\overline{q}$  provides  $\overline{q}f(k_{t+1}, l_{t+1}) = (1 + r_t^c)(k_{t+1} e_t) + w_{t+1}l_{t+1}$ . This is the lowest level of disturbance, which still makes possible for the entrepreneurs to pay out their debts  $(k_{t+1} e_t)$  and interest on them, as well as to completely pay out workers' wage, although the profit obtained by the entrepreneurs is zero. Wage repayments are  $\widehat{W}_{t+1} = w_{t+1}l_{t+1}$  and debt repayments  $\widehat{B}_{t+1} = (1 + r_t^c)(k_{t+1} e_t)$ .
- 2. Significant shock, which still allows entrepreneurs to meet their obligations with respect to wages, but does not allow to completely repay debts and interest on them.  $q^* \in [\underline{q}, \overline{q})$  where  $\overline{q}$  is the lowest insignificant disturbance as above, and  $\underline{q}$  provides  $\underline{q}f(k_{t+1}, l_{t+1}) = w_{t+1}l_{t+1}$ . The worst consequence of this type of shock is that the creditors (in our model banks, which are defined below) receive nothing  $(q^* = \underline{q})$  but all wages are still fully paid. Repayments to workers from each entrepreneur are hence  $\widehat{W}_{t+1} = w_{t+1}l_{t+1}$ , and debt repayments are  $\widehat{B}_{t+1} = q^* f(k_{t+1t}, l_{t+1}) w_{t+1}l_{t+1}$
- 3. Extreme shock  $q^* \in [0, \underline{q})$ , which makes entrepreneurs not only unable to meet their obligations with respect to debt, bur also influences the level of wage repayments to workers. Debt repayments are in this case zero  $\widehat{B}_{t+1} = 0$ , and the wage repayments are  $\widehat{W}_{t+1} = q^* f(k_{t+1}, l_{t+1})$

#### 2.3 Intermediation and interactions

We consider first the economy with financial intermediation. The reduction to the case without financial intermediation is straightforward and made further in the text.

Financial intermediation is presented in the economy through banks, which collect savings from workers in form of deposits, and offer credits to entrepreneurs to finance their demand for credits, given by the excess of the optimal capital level for their production technology over the accumulated entrepreneural savings (which play the role of own funds of the entrepreneurs).

Collection of deposits  $D_t$  starts in period t, when the workers of generation t create their savings  $e_t^D$ . In the end of period t entrepreneurs apply for credits

 $C_t$  to start their business (run firms). Payouts of the entrepreneurs to banks take place within the period t+1 and amount to  $\widehat{B}_{t+1}$ .

There is no other storage technology in the economy so that the value of deposits made with the banks is equal to the value of aggregate savings of the workers:  $D_t = pe_t^D$ . In period t+1 banks have to repay  $p\left(1+r_t^d\right)e_t^D$  to depositors.

Entrepreneurs require credits from banks in amount needed to finance their investment demand and we assume that there is no credit rationing so that no credit application is rejected, and  $C_t = I_t = k_{t+1} - e_t^E$ . Investment (in production technology) takes place at the end of period t.<sup>5</sup> Within period t+1 all entrepreneurs repay to the banks  $(1-p) \hat{B}_{t+1}$  as defined above.

If in period t+1 total repayments from entrepreneurs to banks (duefroms) do not cover the total due-to of the banks, the banks experience deficits  $d_{t+1} = (1-p)\,\hat{B}_{t+1} - p\,\left(1+r_t^d\right)e_t^D < 0$ , and the banks start next period with this non-zero deficit. If in some period would hold  $d_{t+1} > 0$ , we call it banks' surplus.

The sequence of events is presented in Fig. 1

Banks operate in a competitive environment so that neither depoist rates  $r_t^d$  nor credit rates  $r_t^c$  differ among banks and we omit any indexes corresponding to individual banks, taking interest rates as uniform in the market. Moreover, in this competitive environment either banks set credit and deposit interest rates equal (see e.g. Gersbach, 2003, for discussion for the case with market-side switching<sup>6</sup>) or take them equal as given by the market, so that in pure competitive environment  $r_t^d = r_t^c = r_t$  (see e.g. Baltensperger and Jordan, 1997)

**Proposition 2** As soon as there are no deficits in banking system competitive interest rate on deposits coincide with credit interest rate,  $r_t^c = r_t^d = r_t^*$   $\forall t$ 

<sup>&</sup>lt;sup>5</sup>It can be viewed also as though entrepreneurs of generation t create their production facilities along the period t, investing in total  $k_{t+1}$ , so that the investment process ends at the end of the period t, which means that all credits are allocated by the banks in period t under the interest rate  $r_t^c$ .

<sup>&</sup>lt;sup>6</sup>As noted before, the framework of the current paper can be extended for the case of heteregeneous agents, with entrepreneurs deciding upon applying for the credits for investing or depositing with the banks, depending on their (entrepreneurs') respective quality.

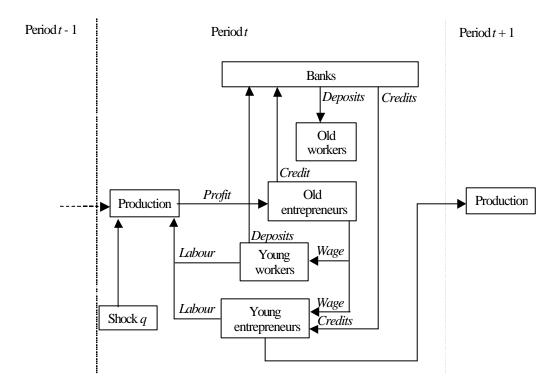


Figure 1: Sequence of events in the intermediated economy

#### Proof.

Consider the profit function of each bank:

$$\Pi^{Bank} = \frac{(1-p)}{N} (1+r_t^c) \left( k_{t+1} - e_t^E \right) - \frac{p}{N} \left( 1 + r_t^d \right) e_t^D$$

where N denotes the total number of banks in the banking system. We assume therefore that deposits and credits are distributed among banks equally. If N is sufficiently big to provide competitive outcome, i.e. zero profit of each individual bank, then

$$\frac{(1-p)}{N} (1+r_t^c) \left( k_{t+1} - e_t^E \right) - \frac{p}{N} (1+r_t^c) e_t^D = 0$$

The shock in the system is completely unpredictable and its probability is close to zero, so that the banks do not make any additional reserves, and all the deposits acquired are used for crediting the firms. In this case balance sheet constraint of each individual bank implies

$$\frac{(1-p)}{N}\left(k_{t+1} - e_t^E\right) = \frac{p}{N}e_t^D$$

Substituting this last equation into the zero profit condition, obtain

$$1 + r_t^c = 1 + r_t^d$$

which proves the statement

Banks are owned by all agents in the economy in equal shares, and the ownership is transferred to another periods (future generations) through bequests, hence stock market for banks' shares is not considered. Dividend payments could shift intertemporal budget constraints in the utility maximisation problems solved by the agents, but the proposition above shows, that banks operate with zero profit, so that no dividend appear, and they are not presented in optimisation problems (1) and (2) of the agents above.<sup>7</sup>

<sup>&</sup>lt;sup>7</sup>This point deserve some further discussion. Indeed, if we assume, that the capital of the banks is negligeably small, all the agents will receive infinitely small dividend payments, so that they do not influence consumption-savings decisions. Put another way we could also assume that the banks belong only to depositors, and resemble in this sense mutual funds, so that there is no capital, which would differ from the deposits. Important moment is that the (possible) dividends are payed out only in case the banks work with surplus, so that  $d_t > 0$ , otherwise any possible banks profit is used to cover deficits. It seems that this assumption (on negligeability of the dividends) is excessive, and the qualitative results do not change even if dividend payments are introduced into optimisation programs, although this generates some external effects and somewhat complicates the presentation.

# 3 Markets and equilibrium

The model so far describes two markets: for capital and for labour. We define the equilibrium in the economy in any period t as a price vector  $\left\{r_t^{d*}, r_t^{c*}, w_{t+1}^*\right\}$  and an allocation  $\left\{k_{t+1}^*, l_{t+1}^*, e_t^{E*}, e_t^{D*}\right\}$  under parameters  $\left\{w_t, d_t\right\}$ , which provide that:

- 1. Capital market is cleared, i.e. aggregate demand for capital equals aggregate supply of it:  $(1-p) k (r_t^c, w_{t+1}) = pe_t^D + d_t + (1-p) e_t^E$ . In other words, aggregate demand for credits equals its aggregate supply:  $(1-p) (k_{t+1} e_t^E) = pe_t^D + d_t$
- 2. Labour market is cleared, i.e. aggregate demand for labour, which is created by old entrepreneurs, is met with the aggregate supply of labour, created by old workers and the whole young generation (both young workers and young entrepreneurs, who have not yet started their businesses):  $(1-p) l(r_t^c, w_{t+1}) = 1.8$
- 3. No agents have incentives to make any changes in their choices.

We discuss therefore a temporary equilibrium, since the equilibrium is a constellation of interest rates  $r_t^{d*}$  and  $r_t^{c*}$ , which are established in period t to be paid out in period t+1, and wage  $w_{t+1}^*$ , which level is determined in period t to be paid out in period t+1. In this context wage level  $w_t$  is actually a parameter for the period t, since is taken as given from the previous period or from initial condition. Given equilibrium interest rates and wages, as defined above, the agents can decide in period t about the amount of capital and labour, needed for production in period t+1, and on the optimal amount of savings to be made in period t, which gives equilibrium allocation  $\left\{k_{t+1}^*, l_{t+1}^*, e_t^{E*}, e_t^{D*}\right\}$ .

There exist implicitly also the market for goods and the market for deposits in this economy. They have however no influence on the equilibrium definition above due to following. The market for goods is in equilibrium by construction: the economy is described in real terms, so that the budget constraints guarantee equilibrium in the market for goods; in each period t output in the economy is  $Y_t = (1-p) q_t f(k_t, l_t)$  and is distributed for consumption of the old generation and consumption and savings of the

<sup>&</sup>lt;sup>8</sup>If the labour is elastically supplied by the young generation, right-hand side of this equation will be a function of wage level  $w_{t+1}$ .

young generation. Consumption of the old workers  $c_t^{OW}$  equals their aggregate wage-earnings plus earnings from deposits, placed in the banks in the previous period, along with that the consumption of young generation  $(c_t^{YW} + c_t^{YE})$  equals their wage-earnings minus their savings:<sup>9</sup>

$$c_t^{OW} + c_t^{YW} + c_t^{YE} = (1 - p)\widehat{W}_t + p\left(1 + r_{t-1}^d\right)e_{t-1}^D - pe_t^D - (1 - p)e_{t-1}^E$$
 (12)

consumption of the old entrepreneurs  $c_t^{OE}$  is given by their aggregate profit  $\Pi_t$ 

$$c_t^{OE} = (1 - p) \max \left[ q_t f(k_t, l_t) - \left( 1 + r_{t-1}^c \right) (k_t - e_{t-1}) - w_t l_t, 0 \right]$$
 (13)

Recall that  $\widehat{W}_{t} = \min \left[ w_{t} l_{t}, q_{t} f\left(k_{t}, l_{t}\right) \right]$  and

 $\widehat{B}_{t} = \min \left[ \left( 1 + r_{t-1}^{c} \right) \left( k_{t} - e_{t-1}^{E} \right), q_{t} f\left( k_{t}, l_{t} \right) - \widehat{W}_{t} \right]$ Aggregate consumption  $C_{t} = c_{t}^{OW} + c_{t}^{YW} + c_{t}^{YE} + c_{t}^{OE}$  plus aggregate savings  $S_{t} = pe_{t}^{D} + (1 - p)e_{t-1}^{E}$  in the economy in period t is hence given by

$$C_{t} + S_{t} = (1 - p) \min \left[ w_{t} l_{t}, q_{t} f\left(k_{t}, l_{t}\right) \right] + p \left(1 + r_{t-1}^{d}\right) e_{t-1}^{D}$$

$$+ (1 - p) \max \left[ q_{t} f\left(k_{t}, l_{t}\right) - \left(1 + r_{t-1}^{c}\right) \left(k_{t} - e_{t-1}^{E}\right) - w_{t} l_{t}, 0 \right]$$

$$= (1 - p) q_{t} f\left(k_{t}, l_{t}\right) + p \left(1 + r_{t-1}^{d}\right) e_{t-1}^{D} - (1 - p) \hat{B}_{t}$$

$$= Y_{t} - d_{t}$$

$$(14)$$

If  $d_t = 0$  then the goods market is clearly in equilibrium. If  $d_t < 0$  then the current output is not sufficient to cover consumption and savings needs of the population, but the excess demand is covered by banks, which use for this purpose newly accumulated deposits, and the goods market is still in equilibrium.

The deposits market is in equilibrium by the assumption that banks do not ration deposit contracts, and acquire therefore all the savings brought by workers, so that neither excess supply of deposits nor excess demand for deposits is possible as soon as  $d_t > \underline{d_t} = -pe_t^D$ . In this extreme case newly accumulated deposits are not sufficient to cover banks' deficits, and the whole banking system is bankrupt, so that the situation is out of our framework. In other words the borrowing capacities of the economy are exhausted.

<sup>&</sup>lt;sup>9</sup>Here the first term denotes wage-repayments from all the entrepreneurs to all workers, the second term - interest repayments from banks to old workers, the third term aggregate savings of young workers, and the fourth term - aggregate savings of the young entrepreneurs.

**Proposition 3** The general equilbrium in the economy with financial intermediaries, given by definition above, exists and is unique for any period t > 0.

#### Proof.

Existence

Consider first labour market (LM). LM-Equilibrium condition is

$$l(r_t^c, w_{t+1}) = \frac{1}{1-p} \tag{15}$$

Function  $l(r_t^c, w_{t+1})$  positive since the production function fullfill the requirements needed to avoid corner solutions. Morover the labour demand function is defined for positive prices, again due to the profit-maximisation problem. This provides, that for any p and for any given positive  $r_t^c$  there exist such  $w_{t+1}$ , that the equation (15) holds and the labour market equilibrium exists for any interest rate. Moreover, the function  $l(r_t^c, w_{t+1})$  is monotonically decreasing in both interest rate and wage level  $(\frac{\partial l}{\partial r_t^c} < 0, \frac{\partial l}{\partial w_{t+1}} < 0)$ . Implicit function theorem provides that the equation (15) defines a unique function  $r_t^c(w_{t+1})$ , and moreover  $\frac{\partial r_t^c}{\partial w_{t+1}} < 0$ . This means, that for any given interest rate, established in the credit market, there will always exist only one equilibrium wage level in the labour market.

Consider now equilibrium in Credit Market (CM):

$$(1-p) k_{t+1} = p e_t^D + (1-p) e_t^E + d_t$$
(16)

Note that the condition of bancruptcy avoidance  $d_t > \underline{d_t} = -pe_t^D$  along with nonnegativity of entrepreneural savings  $e_t^E$  provides that the right-hand side of the equation is positive. Left-hand side is positive due to the absence of corner solutions in profit-maximisation problem (as in case of labour demand). Both savings functions are positive again due to the choice of utility functions, which provide no corner solutions. For any positive  $w_{t+1}$  equation (16) will deliver at least one positive interest rate value  $r_t^c$ , which solves the equation and provides equilibrium in the market for credits.

Optimal choice of the entrepreneurs implies  $\frac{\partial k}{\partial r_t^c} < 0$ ,  $\frac{\partial k}{\partial w_{t+1}} < 0$ ,  $\frac{\partial e_t^E}{\partial w_{t+1}} > 0$  and  $\frac{\partial e_t^E}{\partial r_{t+1}^c} > 0$ . Optimal choice of the workers implies  $\frac{\partial e_t^D}{\partial r_{t+1}^d} > 0$ . Note that  $r_t^d = r_t^c = r_t$  in equilibria. Sum and difference of differentiable functions are differentiable, hence equation (16) also gives implicitly a differentiable function  $r_t(w_{t+1})$ , which is unique.

Combining  $r_t^{CM}(w_{t+1})$  defined in the credit market, with  $r_t^{LM}(w_{t+1})$ , defined in the labour market, we obtain equilibrium interest rate and wage level. To find the equilibrium let us chose any arbitrary positive  $w_{t+1}^0$  and find correspondent equilibrium interest rate in the credit market  $r_t^0$ . If this pair of interest rate and wage level delivers equilibrium to the labour market, the general equilibrium is constructed. Assume however, that equilibrium wage level  $w_{t+1}^1$ , determined by the labour market for given interest rate  $r_t^0$  differs from  $w_{t+1}^0$ , and for certainty  $w_{t+1}^0 > w_{t+1}^1$  (the other case is similar). Labour market is hence in disequilibrium, and in this case particularly exhibits excess supply of labour. Market mechanisms will imply that the wage level decreases in direction of  $w_{t+1}^1$ . With smaller wage level however the credit market will generate another equilibrium interest rate. Indeed, the demand for credits  $(1-p)(k_{t+1}-e_t^E)$  increases, so that the new equilibrium interest rate increases to  $r_t^1 > r_t^0$ . This increase in interest rate provokes further decrease in wage level as determined by the labour market. This disequilbrium process stops as soon as any point  $(r_t^*, w_{t+1}^*)$  is reached, which provides simultaneously equilibrium in both markets.

Uniqueness

Assume that there are several equilibria, and chose the one with smallest  $w_{t+1} = \underline{w}$ . Consider now the gap  $r_t^{CM}(w_{t+1}) - r_t^{LM}(w_{t+1})$ , which is zero in the equilibrium chosen. This gap increases as  $w_{t+1}$  increases (see Appendix for proof):

$$\frac{\partial r_t^{CM}}{\partial w_{t+1}} - \frac{\partial r_t^{LM}}{\partial w_{t+1}} > 0 \tag{17}$$

This provides the gap  $r_t^{CM}(w_{t+1}) - r_t^{LM}(w_{t+1})$  to be positive for any  $w_{t+1} > \underline{w}$ . This means there are no equilibria with  $w_{t+1} > \underline{w}$ . There are also no equilibria with  $w_{t+1} < \underline{w}$ , because of the choice of  $\underline{w}$ . This proves the uniqueness of the equilibrium point  $(w^* = \underline{w}, r^* = r^{LM}(\underline{w}) = r^{CM}(\underline{w}))$ 

Note that the proposition above allows representing the general equilibrium in terms of two lines in the  $(w_{t+1}, r_t)$ -plane: LM depicting possible equilibria in labour market and CM depicting possible equilibria in credit market. Lemmas in Appendix prove that

$$\left\{ \frac{\partial r_t}{\partial w_{t+1}} \right\}^{LM} < \left\{ \frac{\partial r_t}{\partial w_{t+1}} \right\}^{CM} < 0$$
(18)

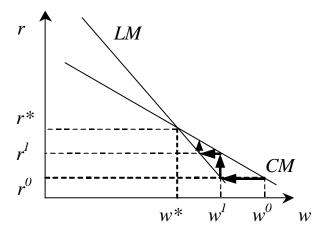


Figure 2: General equilibrium in the model sets up similarly to ISLM approach

Graphical illustration of the establishment of equilibrium in the model is given in Fig. 2

Note also, that changes in deficits level influence only the CM-line, and do not influence the LM-line, althouh general equilibrium would differ for different values of  $d_t$ . Deterioration of banks' deficits ( $d_t$  falls below zero) increases equilibrium interest rate as defined by credit market for any wage level  $w_{t+1}$  so that CM-line shifts upwards in ( $w_{t+1}$ ,  $r_t$ )-plane:

$$\frac{\partial r_t^{CM}}{\partial d_t} < 0 \tag{19}$$

This means, that equilibrium interest rate and the equilibrium wage level are determined by the movement of general equilibrium point along the LM-line so that the equilibrium interest rate increases and the equilibrium wage level decreases:

$$\frac{\partial r_t^*}{\partial d_t} < 0; \frac{\partial w_{t+1}^*}{\partial d_t} > 0 \tag{20}$$

The effect on the general equilibrium of banks deficits falling down below zero level is shown in Fig. 3.

The system of equilibrium conditions (16-15) together with definition of deficits gives a dynamical system with respect to w and d:  $\Phi(w_t, d_t) \rightarrow$ 

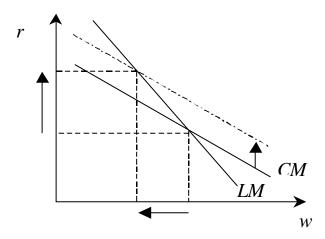


Figure 3: Effect of the change in banks deficits  $(d \downarrow, i.e.$  banks experience larger lack of funds) on general equilibrium.

 $(w_{t+1}, d_{t+1})$  with  $\Phi(w_t, d_t)$  given by:

$$\Phi(w_{t}, d_{t}) \rightarrow (w_{t+1}, d_{t+1}) :$$

$$\begin{cases}
(1 - p) l (r_{t}, w_{t+1}) = 1 \\
(1 - p) \left(k_{t+1} (r_{t}, w_{t+1}) - e_{t}^{E} (w_{t}, w_{t+1}, r_{t})\right) = p e_{t}^{D} (w_{t}, w_{t+1}, r_{t}) + d_{t} \\
d_{t+1} = (1 - p) \widehat{B}_{t+1} - p (1 + r_{t}) e_{t}^{D} (w_{t}, w_{t+1}, r_{t})
\end{cases}$$
with  $\widehat{B}_{t+1} = \min \left[ (1 + r_{t}) \left(k_{t+1} - e_{t}^{E}\right), q_{t+1} f (k_{t+1}, l_{t+1}) - \widehat{W}_{t+1} \right]$ 
and  $\widehat{W}_{t+1} = \min \left[ w_{t+1} l_{t+1}, q_{t+1} f (k_{t+1}, l_{t+1}) \right]$ 

**Proposition 4** Dynamical system (21) has a steady state (stationary) with d = 0 and  $w = \hat{w}$ , which does not depend on intitial conditions.

#### Proof.

First, note that a steady state presumes no external shocks, so that in steady state holds  $q_t = 1$ . Given this, the map for deficits in (21) turns to

$$d_{t+1} = (1-p)(1+r_t)(k_{t+1} - e_t^E) - p(1+r_t)e_t^D$$

Credit market equilibrium for the period t implies

$$(1-p)\left(k_t - e_t^E\right) = pe_t^D + d_t$$

Combining these two equations we obtain

$$d_{t+1} = (1+r_t)d_t$$

which, under positive interest rates is true if and only if

$$d_{t+1} = d_t = 0 = d$$

The first part of the statement is hence proved. Now consider the modified map  $\Phi_0(w_t) \equiv \Phi(w_t, 0)$ :

$$\Phi_{0}(w_{t}) \rightarrow w_{t+1}:$$

$$\begin{cases}
(1-p) l(r_{t}, w_{t+1}) = 1 \\
(1-p) (k_{t+1}(r_{t}, w_{t+1}) - e_{t}^{E}(w_{t}, w_{t+1}, r_{t})) = pe_{t}^{D}(w_{t}, w_{t+1}, r_{t})
\end{cases}$$
(22)

To prove the existence of stationary it suffices to prove the existence of a solution to

$$\Phi_0(w_t) = w_t$$

Substituting  $w_t$  instead of  $w_{t+1}$  (or  $w_{t+1}$  instead of  $w_t$ , which is equivalent in sense of stationarity) into (22) reduces the problem to the case of market equilibrium above. Indeed, labour market equilibrium depends only on the wage rate  $w_{t+1}$ , so that the slope of LM-line stays unchanged. The slope of the CM-line is still flatter than that of the LM-line (see Appendix), so that the analysis of equilibrium above applies also here, which proves existence (and at the same time uniqueness) of a point  $(\widehat{w}, r(\widehat{w}))$ , which provides  $\Phi_0(\widehat{w}) = \widehat{w}$ . By construction, steady state does not depend on the initial conditions.

# 3.1 Regulation and bailouts

As a regulation measure we consider deposit rate ceiling  $r^{dreg}$ . In this case either credit rates are below the ceiling, so that no bank is interested in setting the deposit rate above the credit rate, and hence the situation coincide with unregulated economy with  $r^c = r^d = r^*$ ; or credit rate (the interest rate determined by the credit market) is above the ceiling, which means that the deposit rate is fixed:

$$r^d = \min\left(r^{dreg}, r^c\right) \tag{23}$$

Fixing deposit rate makes deposit supply depending only on wage level; the deposit market is still in equilibrium (all deposits supplied are acquired by banks), that does not change the conclusion about the existence and uniqueness of general equilibrium.

Equilibrium condition for labour market is not disturbed by introducing the regulation since depends only on credit interest rate:

$$(1-p)l(r_t^c, w_{t+1}) = 1 (24)$$

Equilibrium condition in credit market simplifies to

$$(1-p)\left(k\left(w_{t+1}, r_{t}^{c}\right) - e^{E}\left(w_{t}, w_{t+1}, r_{t}^{c}\right)\right) = pe^{D}\left(w_{t}, r^{dreg}\right) + d_{t}$$
 (25)

so that the right-hand side does not depend on the credit interest rate anymore. Obviously equilibrium credit rate is negatively related with regulated deposit rate, since left-hand side negatively depends on the credit rate:  $\frac{\partial r^c}{\partial r^{dreg}} < 0$ . This is easily explained by the fact, that decreasing deposit rate leads to less deposits placed with the banks, and therefore to lower supply of credits, which become more expensive in order to hold the equilibrium. In other words, setting the regulated deposit rate at the level below that of the unregulated equilibrium, increases credit interest rate and hence makes banks' marge positive (since in unregulated case the rates are equal). Graphically this corresponds to the shift of CM-line upwards as soon as the regulated deposit interest rate is set below the equilibrium level.

The regulated case coincides with the unregulated one as soon as p=0 and therefore relationship  $\left\{\frac{\partial r_t}{\partial w_{t+1}}\right\}_{p=0}^{LM}$  still holds and the proof of the uniqueness of equilibrium is still valid, as well as the analysis of the equilibrium with the help of CM-LM diagram introduced above.

So far it has been also assumed, that the deposits in banks are fully guaranteed, i.e. there exists a bailout scheme for the case of bankruptcy of the banking system. We will assume, that the regulator povides banks with a subvention, needed to avoid the bankruptcy. The amount of this subvention  $S_t$  reduces banks' deficits  $d_t$  to the level  $\hat{d}_t = d_t + S_t$  so that  $pe_t^D + \hat{d}_t > 0$ .

# 4 Pure market economy and its evolution

The economy without intermediation (market dominated economy) exhibits the same equilibrium as in the case of intermediated economy with zero deficits in banks. Indeed, in the pure market economy aggregate demand

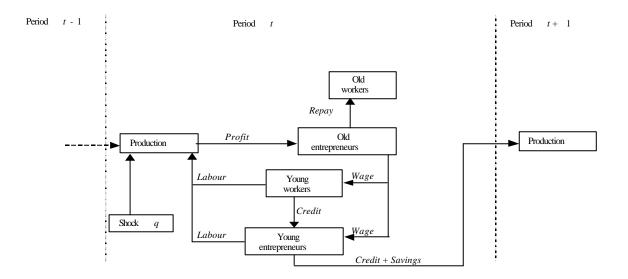


Figure 4: Agents and their interactions in a pure market economy without financial intermediation

for credits  $(1-p) \left(k_{t+1} - e_t^E\right)$  equals its aggregate supply  $pe_t^D$ , and since intermediaries are not presented here, there is no place for deficits in market equilibrium. The equilibrium in labour market does not depend on intermediation, and is the same as in intermediated case. Interest rate on savings in this economy is equal to interest rate on credits  $r_t^d = r_t^c = r_t$ , since there is only one market for them. Agents and interactions in pure market economy are presented in Fig. 4

Temporary equilibrium in the pure market economy in any period t is hence defined as a price vector  $\{r_t^*, w_{t+1}^*\}$  and an allocation  $\{k_{t+1}^*, l_{t+1}^*, e_t^{E*}, e_t^{D*}\}$  under parameters  $\{w_t, d_t\}$ , which provide that:

- 1. Credit market is cleared:  $(1-p)\left(k_{t+1}-e_t^E\right)=pe_t^D$
- 2. Labour market is cleared:  $(1-p) l(r_t, w_{t+1}) = 1$ .
- 3. No agents have incentives to make any changes in their choices.

For pure market economy is also valid:

**Proposition 5** The general equilbrium in pure market economy, given by definition above, exists and is unique for any period t > 0.

#### Proof.

The proof of the proposition repeats the proof of the existence and uniqueness of the equilibrium for intermediated economy. It has only to be noted, that  $d_t$  represents now (in consistency with its definition) the difference between actual and expected credit repayments from old entrepreneurs to old workers. Another point to be noted is that now there is no deposit market, and only credit market is considered, where savers (workers) and investors (entrepreneurs) meet each other.

Without shocks the equilibrium in pure market economy reaches its steady state as in the case of intermediated economy. The evolution of the market economy after a shock appears as follows.

If the shock  $q = q^* < 1$  occurs, entrepreneurs pay  $\widehat{W}_t = \min \left[ w_t l_t, q^* f\left(k_t, l_t\right) \right]$  to young workers as wage repayments and

 $\widehat{B}_t = \min \left[ (1 + r_{t-1}) \left( k_t - e_{t-1}^E \right), q^* f(k_t, l_t) - \widehat{W}_t \right]$  to old workers in repayment of debts. These changes result in following:

- 1. Old workers, who acted as creditors, obtain less than expected, and their consumption in this period is consequently less than needed to achieve planned level of utility. In extreme case, when  $\widehat{W}_t = q^* f(k_t, l_t)$ , the old workers receive nothing, so that their consumption is zero.
- 2. Young workers either receive their wages fully (if  $w_t l_t \leq q^* f(k_t, l_t)$ , i.e. the shock is either insignificant or just significant) or in a reduced amount (if  $q^* f(k_t, l_t) < w_t l_t$ , i.e. the shock is extreme). In the case of full wage repayment savings decisions of a new generation repeat those of the old generation one period before, so that the equilibrium persists. In the case of smaller wage repayments the amount of new savings  $e^D\left(\frac{\widehat{W}_t}{l_t}, r_t\right)$  decreases, which leads to higher equilibrium interest rate in the credit market. In the CM-LM diagram this corresponds to a shift of CM-line upwards, and new temporary equilibrium is characterised by higher interest rate and lower wage level.
- 3. After the shock the system gradually recovers to a steady state, which coincides with the one for intermediated economy above, and hence

depends neither on the initial conditions, nor on the degree of shock. Therefore the shock"echoes" in the next few periods, where wage level is below the stationary, and the interest rate is higher than stationary.

# 5 Evolution of the intermediated economy

#### 5.1 Before the shock

If  $t < \tau$ , the shock parameter q is always q = 1. The economy sets in the steady state equilibrium described above with zero deficits in the banks. This equilibrium is stationary until the shock occurs.

#### 5.2 After the shock

Consider now the evolution of the economy after the shock  $q = q^* < 1$ , i.e. for  $t \ge \tau$ .

- 1. If the shock is insignificant in terms  $q^* \in [\overline{q}, 1]$  with  $\overline{q}$  defined above, entrepreneurs are able to meet their obligations with respect to wages and debt repayments in period  $\tau$ . Next period starts at the same equilibrium as before the shock, so that the stationary persists. There is no need to analyse this case further.
- 2. If the shock is significant, i.e.  $q^* \in [q, \overline{q})$ , the entrepreneurs are able to cover their obligations with respect to wages, but are not able to pay their indebtedness to banks in full. Banks experience in this case deficits  $d_{\tau+1} = (1-p) \left(q^* f\left(k_{\tau}, l_{\tau}\right) w_{\tau} l_{\tau}\right) p \left(1 + r_{\tau}^d\right) e_{\tau}^D < 0$ , with which they start the period  $\tau + 1$ . Change in deficits (d falls from  $d_{\tau} = 0$  to some negative level  $d_{\tau+1}$ ) causes CM-line to shift upwards (for any wage level credit market will clear with the higher interest rate), so that the change in general equilibrium is given by the LM-line, and the equilibrium wage level  $w_{\tau+2}$  is lower than  $w_{\tau+1} = w_{\tau}$ ; along with that equilibrium interest rate increases from  $r_{\tau}$  to  $r_{\tau+1}$ . No new production shocks and no changes in the behaviour of the agents occur. The dynamics of the deficits in banks is given by

$$d_{\tau+n+1} = (1-p)\left(1 + r_{\tau+n}^c\right)\left(k_{\tau+n+1} - e_{\tau+n}^E\right) - p\left(1 + r_{\tau+n}^d\right)e_{\tau+n}^D \tag{26}$$

Since credit market is in equilibrium (16), and since  $r_{\tau+n-1}^c = r_{\tau+n-1}^d = r_{\tau+n-1}^*$ , we obtain

$$d_{\tau+n+1} = (1 + r_{\tau+n}^*) d_{\tau+n} \tag{27}$$

which is valid for any  $n \in N$ . As soon as  $r_{\tau+n}^* > 0$  deficits in banks deteriorate untill banking system is bankrupt, i.e.  $d_{\tau+n+p} = \underline{d_{\tau+n+p}} = -pe_{\tau+n+p}^D$ .

3. The case of an extreme shock  $q^* \in [0,\underline{q})$  differs from the above in that the initial change in deficits is bigger (since banks receive nothing from the entrepreneurs). Since newly acquired deposits  $e_{\tau+1}^D$  are smaller than  $e_{\tau}^D$  due to  $\widehat{W}_{\tau} < (1-p) w_{\tau}l_{\tau} = (1-p) w_{\tau-1}l_{\tau-1}$  (actual wage earnings of the workers are less than expected, and hence their savings decrease), the deficit level in banks at the beginning of period  $\tau+1$  is

$$d_{\tau+1} = -p\left(1 + r_{\tau}^{d}\right)e_{\tau}^{D} < -pe_{\tau+1}^{D} = \underline{d_{\tau+1}}$$
(28)

and the banking system is bankrupt already in the next period after the shock, because it can not fully repay its obligations to previous depositors.

4. As soon as the banking system is bankrupt, the regulator can intervene with a subvention to banks. To improve the situation it is necessary to turn defecits level in banks to zero, otherweise any negative level of deficits will again deteriorate in time. Intervention in moment of bankruptcy is therefore more expensive since it requires to subsidise in amount of  $S = d_{\tau} \prod_{k=1}^{n} (1 + r_{\tau+k})$  in case of significant shock or  $S = pe_{\tau+1}^{D}$  in case of the extreme shock. The cheapest intervention would therefore be subsidising banks immediately after the shock in amount of  $S = p \left(1 + r_{\tau}^{d}\right) e_{\tau}^{D} - (1 - p) \left(q^{*} f\left(k_{\tau}, l_{\tau}\right) - w_{\tau} l_{\tau}\right)$ , which covers exactly the deficit in banking system that appears immediately after the shock.

# 5.3 Regulated dynamics

If  $r^{dreg} > r^c$ , deposit interest rate will be equal to credit interest rate  $r^c = r^d = r^*$  so that regulated dynamics repeats the case of unregulated. After the

shock however credit interest rate increases (due to deterioration in deficits), as shown for the case of unregulated dynamics, and at some point will hold  $r^{dreg} < r^c$  so that the deposit rate is fixed by the regulation<sup>10</sup>.

Assume that this happens at  $t = \tau$ , so that  $r_{\tau+k}^d = r^{dreg} < r_{\tau+k}^c \ (k \in N)$ . The dynamics of the deficits in banks is then given by

$$d_{\tau+k+1} = (1-p)\left(1 + r_{\tau+k}^c\right)\left(k_{\tau+k+1} - e_{\tau+k}^E\right) - p\left(1 + r^{dreg}\right)e_{\tau+k}^D$$
 (29)

Due to credit market equilibrium condition (16) this implies

$$d_{\tau+k+1} = (1 + r_{\tau+k}^c) d_{\tau+k} + p \left( r_{\tau+k}^c - r^{dreg} \right) e_{\tau+k}^D$$
 (30)

To provide reduction in deficits, i.e.  $d_{\tau+k+1} > d_{\tau+k}$ , it is necessary that

$$r^{dreg} < r^{c}_{\tau+k} \left( 1 + \frac{d_{\tau+k}}{pe^{D}_{\tau+k}} \right) < r^{c}_{\tau+k}$$
 (31)

The simplest way to meet this criterion is to set  $r^{dreg} = 0$ . This sharp measure is however needed only in case the system falls down close to bank-ruptcy, i.e. when  $(-d_{\tau+k}) \to pe_{\tau+k}^D$ . In all other cases this criterion can be fulfilled with strictly positive values of ceiling.

**Proposition 6** Economy with appropriately chosen (in sence of (31)) deposit rate ceiling and bailout possibility recovers after the shock to a steady state with  $d_{\tau+k} = 0$ ,  $\forall k \in \mathbb{N}$ .

#### Proof.

The proof of the proposition follows from the discussion above. It should be just noticed that as soon the deficits achieve their zero level, the system returns to its development path in absence of shocks, for which the existence of steady state is proved above. If the shock is extreme, a subvention is needed to return deficits to the level above the critical one, and the system recovers as well.

<sup>&</sup>lt;sup>10</sup>Of course if the deposit rate ceiling is not set too high:  $r^{dreg} < r^{crit}$  where  $r^{crit} = \arg \left[ d_t = -pe^D\left(\cdot, r^{crit}\right) \right]$ , i.e. critical value of the market interest rate, when the banking system is bankrupt, and no further development occurs.

# 6 Comparison of the two systems

As the analysis above shows, macroeconomic evolution after a sharp macroeconomic shock depends on the arrangement of financial system and on the degree of shock. The dynamics of both types of systems is presented in diagrams (Figures)<sup>11</sup>. In case of significant shock it can be seen, that banking deficits do not disappear immediately, but last in time. Along with that banking deficits are not automatically social losses, they are eliminated in future periods due to the positive banking marge with the ceiling set on the steady state level. No other changes occur in market economy, however the decrease in wage level, and consequent decrease in savings are to be observed in intermediated economy. Initial rise in entrepreneural savings is due to the increase in credit rate immediately after the shock. Diagrams that represent extreme shock, exhibit that the extreme shock leads to the exhaustion of resources in intermediated economy, so that a subvention from the regulator is needed. Smaller deficits in banking system, compared with the defaults in market economy are due to the subsidies paid to the banking sector (the subvention reduces amount of deficits). The evolution of other variables is similar in both sytems, although intermediated economy needs somewhat longer time to recover to the steady state. It should not be forgotten, that the losses due to default are fully borne in market economy by the creditors, when however in the intermediated economy social losses are measured only by the amount of subvention and are not distributed among the agents of several generations.

In the following comparison we define social losses as explicit losses in consumption of all the agents, and assume that the shock  $q^*$  happens in moment t after the system settled in steady state.

# 6.1 Insignificant shock

In both systems only one group of agents suffers from the shock: namely, entrepreneurs of generation t-1 suffer from lower consumption when old.

Illustrations are generated by a simulation with utility function  $u(c_t, c_{t+1}) = (c_t)^{\frac{1}{2}}(c_{t+1})^{\frac{1}{2}}$  and production function  $f(k, l) = k^{\frac{1}{3}}l^{\frac{1}{3}}$ , which generate capital and labour demands  $k = \frac{1}{27(1+r_t)^2w_{t+1}}$ ,  $l = \frac{1}{27(1+r_t)(w_{t+1})^2}$ , savings of workers  $e^D = \frac{w_t}{2}$  and savings of entrepreneurs  $e^E = \frac{w_t}{2} - \frac{1}{54(1+r_t)^2w_{t+1}}$ . Deposit rate ceiling fixed on the steady state level.

Social losses are

$$\Delta_{t} = (1 - p) \left( c_{t}^{E} - q^{*} f\left(k_{t}, l_{t}\right) \right)$$
Recall that  $c_{t}^{E} = f\left(k_{t}, l_{t}\right) - \left(1 + r_{t-1}^{c}\right) \left(k_{t} - e_{t-1}^{E}\right) - w_{t} l_{t}$  so that
$$\Delta_{t} = (1 - p) \left( (1 - q^{*}) f\left(k_{t}, l_{t}\right) - \left(1 + r_{t-1}^{c}\right) \left(k_{t} - e_{t-1}^{E}\right) - w_{t} l_{t} \right)$$

## 6.2 Significant shock

Recall that significant shock is defined as  $q^* \in [\underline{q}, \overline{q})$  where  $\overline{q}$  provides  $\overline{q}f(k_t, l_t) = (1 + r_{t-1})(k_t - e_{t-1}^E) + w_t l_t$ , and  $\underline{q}$  provides  $\underline{q}f(k_t, l_t) = w_t l_t$ .

1. Market economy. Entrepreneurs of generation t-1 are bankrupts, and their consumption in this period is zero, so that their losses equal to planned consumption  $f(k_t, l_t) - (1 + r_{t-1}) \left(k_t - e_{t-1}^E\right) - w_t l_t$ . Creditors (old workers) of generation t-1 suffer from insufficient loan repayments  $(1 + r_{t-1}) e_{t-1}^D - q^* f(k_t, l_t)$ . Young generation receives their wage repayments in full and does not suffer from the shock. Total social losses are in this case

$$\Delta_{t}^{M} = (1-p) \left( f(k_{t}, l_{t}) - (1+r_{t-1}) \left( k_{t} - e_{t-1}^{E} \right) - w_{t} l_{t} \right)$$
$$+ p \left( (1+r_{t-1}) e_{t-1}^{D} - q^{*} f(k_{t}, l_{t}) \right)$$

2. **Intermediated economy**. Entrepreneurs of generation t-1 are bankrupts and their consumption falls to zero. Banks experience deficits, but manage to repay all debts to depositors, so that no losses on the side of depositors occur. Social losses are

$$\Delta_t^I = (1 - p) \left( f(k_t, l_t) - (1 + r_{t-1}) \left( k_t - e_{t-1}^E \right) - w_t l_t \right)$$

There are indirect losses associated with the increase in credit interest rate and decrease both in deposit interest rate and wage level, so that few next generations can not enjoy from consumption at the steady state level. But no other direct social losses occur becaus of the shock. With due regulation economy recovers to the stationary state.

Comparing both cases, receive:

$$\Delta_t^M - \Delta_t^I = p\left((1 + r_{t-1})e_{t-1}^D - q^*f(k_t, l_t)\right) > 0$$

#### 6.3 Extreme shock

Extreme shock is defined as  $q^* \in [0, \underline{q})$  where  $\underline{q}$  provides  $\underline{q}f(k_t, l_t) = w_t l_t$ .

1. Market economy. Entrepreneurs of generation t-1 are again bankrupts with zero consumption. Creditors (old workers) of generation t-1 receive zero loan repayments and their individual losses amount to  $(1+r_{t-1}) e_{t-1}^D$ . Young generation receives reduced wage repayments and their losses are  $w_t l_t - q^* f(k_t, l_t)$ . Total social losses are in this case

$$\Delta_t^M = (1-p) \left( f(k_t, l_t) - (1+r_{t-1}) \left( k_t - e_{t-1}^E \right) - w_t l_t \right) + p \left( 1 + r_{t-1} \right) e_{t-1}^D + w_t l_t - q^* f(k_t, l_t)$$

2. Intermediated economy. As in case of market economy, entrepreneurs of generation t-1 are bankrupts, and the joung generation recieves reduced wage repayments. Along with that banking system is bankrupt, since banks receive zero loan repayments, so that it can seem to produce the same result as in case of market economy. But recall that to recover banking system it suffices to turn deficits  $d_t$  back to the level above  $\underline{d} = -pe_t^D$  so that a subvention S from the Regulator in this amount would solve the problem of bankruptcy and social losses would amount to

$$\Delta_t^I = (1-p) \left( f(k_t, l_t) - (1+r_{t-1}) \left( k_t - e_{t-1}^E \right) - w_t l_t \right) + S + w_t l_t - q^* f(k_t, l_t)$$

So that the difference in social losses is

$$\Delta_t^M - \Delta_t^I = p(1 + r_{t-1})e_{t-1}^D - S$$

The amount of the necessary subvention has to prevail the difference between deficits occured after the shock and the critical level of deficits

$$S > \underline{d} - d_t =$$
  
=  $-pe_t^D + (1-p)(1+r_{t-1})(k_t - e_{t-1}^E)$ 

Substituting this expression into the equation above yields

$$\Delta_t^M - \Delta_t^I < p(1 + r_{t-1}) e_{t-1}^D + p e_t^D - (1 - p)(1 + r_{t-1}) (k_t - e_{t-1}^E)$$

Recall now that before the shock the economy was in steady state with  $d_{t-1} = 0$ . Using credit market equilibrium condition for the previous period (16) gives

$$\Delta_t^M - \Delta_t^I$$

Remember that actual wage repayments fall due to the extreme shock. Along with that imposed deposit rate ceiling makes decreases the return on deposits. Both effects lead to a reduction in deposits inflow, but not necessarily to exhaustion of them. Inequality (32) tells us, that the social costs in market economy would be lower as in intermediated one only if the new inflow of deposits is zero. The only precondition for that is that the shock is so extreme that no production occurs, which leads to zero wage repayments. This case can be excluded, since is equivalent to the distruction of all productive resources. In all other cases there exists such amount of subvention S, that

$$0 < \Delta_t^M - \Delta_t^I < pe_t^D$$

This proves following proposition:

**Proposition 7** In case of negative macroeconomic shock intermediated economy incures less social losses as the market economy. Equal social losses arise only in the case of insignificant shock.

#### Proof.

See above

# 6.4 Notion on degrees of shock

The shock in the model was defined by the shock parameter  $q \in [0, 1]$ , and is measured as output after the shock as percentage of the output in steady state. This does not however mean, that the shock of  $q^*\%$  is automatically significant or extreme. the severity of the shock depends on the current prices, namely on wage and interest rate level, which occur in steady state, preceding the shock. As it can be seen from the definition of the shock limits  $\underline{q}$  und  $\overline{q}$ , they are uniquely determined by the price system in steady state. So that the same shock of  $q^*\%$  can appear to be significant in one

price system, insignificant in another price system, and extreme in the third price system.

Lower limit of the significant shock is

$$\overline{q} = \frac{(1 + r_{t-1}) \left(k_t - e_{t-1}^E\right) + w_t l_t}{f(k_t, l_t)}$$

and the upper limit for the extreme shock is

$$\underline{q} = \frac{w_t l_t}{f\left(k_t, l_t\right)}$$

In general, the more efficient is the production technology, the lower are both ratios, which decreases the probabilities of both significant and extreme shocks and increases the propability that the shock is insignificant. This small qualitative remark can be another fact in favour of establishing bank-oriented financial systems in emerging economies, due to their poorer technological development. On the contrary, in developed economies the probability of severe shocks is lower, and advantages of the banking system in intertemporal smoothing of exogenous negative shocks are less noticeable.

# 7 Conclusion

The dynamical model in this paper incorporates both labour and capital markets and discribes the role of financial intermediation in the evolution of the economy after a negative exogenous macroeconomic shock. General equilibrium analysis is supported with diagrammatical illustration resembling that of famous ISLM-model, and allows concluding on the changes in both markets after the shock in general case without searching for equilibrium explicitly. As it follows from the analysis, deposit insurance (bailout guarantees) itself can not prevent from macroeconomic collaples, which corresponds to empirical findings. This result is valid even if deposit insurance (bailout guarantees) cannot change risk incentives of the banks (since there is no possibility for shifts in investment decisions of the banks in our model). The model provides a tool for analysis of the regulatory interventions, and to study their efficiency in recovering the system after a shock. It is in particular shown, that imposing deposit rate ceiling is a necessary comlement to baioluts scheme, which helps an intermediated economy to recover. It is

important that the deposit rate ceiling is not too high, otherwise it has no influence on the recovery capabilities of the banking system.

A classification of shocks, presented in paper gives shows, that the same shock (as percentage of GDP) can differently affect the macroeconomy. Obviously economies with higher level of technological development are better "insured" agains shocks, and the advantages of intertemporal risk smoothing are not so relevant for them. On the contrary, economies with lower level of technological development face greater probability of the severe shock, and can exploit intertemporal smoothing provided by banking system. In this case social costs of shock are lower, than in case of market-based economies.

The model expicitely considers just one type f shock, namely negative production shock. The seem however to be valid for the case of other shocks, which can drive to deficits in banking system. Such examples as Oil Shock in 1970-s and Default of Russian government in 1998 would lead to the same conclusions in the framework of analysis above. The main feature of the model holds the same: banks face difficulties due to insufficient returns from their investments, and recovery or bankruptcy chances depend only on the ability of banks to derive profit from their investment operations.

# 8 References

- 1. Aiyagari R.S. (1987) "Optimality and monetary equilibria in stationary overlapping generations models with long-lived agents: growth versus discounting", *Journal of Economic Theory*, 43, pp.292-313.
- 2. Aiyagari R.S. (1992) "Co-existence of a representative agent type equilibrium with a non-representative agent type equilibrium", *Journal of Economic Theory*, 57, pp.230-238.
- 3. Allen, F., D. Gale (1997) "Financial markets, intermediaries and intertemporal smoothing", *Journal of Political Economy*, Vol. 105 No 3, pp.523-546
- 4. Allen, F., D. Gale (2000) "Comparing financial systems", MIT Press, Cambridge, MA
- Arteta C., B. Eichengreen (2000) "Banking crises in emerging markets: presumptions and evidence", University of Berkeley, Center for International and Development Economics Research (CIDER), Working Paper C00-115
- Baltensperger, E. and T. J. Jordan (1997) "Seigniorage, banking, and the optimal quantity of money", *Journal of Banking and Finance* 21, pp.781-796.
- Barten A., V. Böhm (1982) "Consumer Theory" in Handbook of Mathenatical Economics, vol. II, eds. K. Arrow and M. Intrilligator, North Holland
- Barth J.R., G. Caprio Jr., R. Levine (2000) "Banking systems around the globe: do regulation and ownership affect performance and stability?" World Bank Working Papers, WP2325
- 9. Bencivenga V., B. Smith (1991) "Financial intermediation and endogenous growth", *Review of Economic Studies* 58, pp. 195-209
- 10. Bernanke, B., M. Gertler (1987) "Banking and Macroeconomic Equilibrium," in *New Approaches to Monetary Economics*, ed. W. Barnett and K. Singleton, New York, Cambridge University Press

- 11. Bhattacharia S., A. Boot, A. Thakor (1998) "The economics of bank regulation", *The Journal of Money, Credit and banking*, Vol 30 No 4 p.745
- 12. Bolton P., (2002) "Banking in Emerging Markets" Journal of Financial Intermediation 11, pp. 362-365
- 13. Caprio G., D. Klingebiel (1996) "Bank insolvencies: cross-country experience", World Bank Working Papers, WP1620
- 14. Demirgüc-Kunt A., E. Detragiache, (2000) "Does deposit insurance increase banking system stability? An empirical investigation", Proceedings of the 8th Econometric Society World Congress, Seattle, Washington, Contributed Paper No 1751
- 15. Dewatripont M. and J. Tirole (1994) "The prudential regulation of banks", MIT Press
- 16. Diamond D., P. Dybvig (1983) "Bank runs, deposit insurance and liquidity", Journal of Political Economy 91, pp. 401-419
- 17. Gersbach H. (2003) "Double-sided Bertrand competition in financial intermediation with market side switching", University of Heidelberg, Working paper
- 18. Gersbach, H, J. Wenzelburger (2002a), "Banking and the Macroeconomy", University of Heidelberg, Discussion paper No343.
- 19. Gersbach, H, J. Wenzelburger (2002b), "The workout of banking crises: a macroeconomic perspective", University of Bielefeld, Discussion paper No490.
- 20. Gersbach, H, J. Wenzelburger (2003), "Banking Crises and Recoveries", University of Heidelberg, mimeo.
- 21. Gorton G., A. Winton (2003) "Financial intermediation" in *Handbook* of the Economics and Finance, eds. G. Constantanides, M. Harris and R. Stulz, Amsterdam, North Holland Elsevier
- 22. Gurley, J.G., E.S. Shaw (1960) "Money in a theory of finance", Brookings

- 23. Huo T.M.(1987) "Observational equivalence of the overlapping generations and the cash-in-advance economies", *Economics Letters* 25, pp. 9-13.
- Lovo S. (2000) "Infinitely lived representative agent exchange economy with myopia", IDEI, Université de Sceinces Sociales de Toulouse, Discussion paper
- 25. Mishkin F. (2001) "Financial Policies and the Prevention of Financial Crises in Emerging Market Countries", NBER Working Paper w8087
- Nadiri, M. Ishaq (1982) "Producers Theory" in Handbook of Mathenatical Economics, vol. II, eds. K. Arrow and M. Intrilligator, North Holland
- Rochet, J.-C., X. Vives, (2002) "Coordination Failures and the Lender of Last Resort: Was Bagehot Right After All?", Hamburg Institute of International Economics, Discussion Paper 184.
- 28. Sachs J., (1998) "Alternative approaches to financial crises in emerging markets" in *Capital flows and financial crises*, ed. Miles Kahler, Manchester University Press
- 29. Thießen U., (2000), "Banking crises in transition countries. Theory and empirical evidence: the case of Russia", German Institute for Economic Research (DIW, Deutsches Institut für Wirtschaftsforschung), Discussion paper 193
- 30. Woodford M. (1986) "Stationary Sunspot equilibria in a finance constraint economy", *Journal of Economic Theory*, 40, pp.128-137.

# 9 Appendix

**Lemma 1** Equilibrium gap  $r_t^{CM}(w_{t+1}) - r_t^{LM}(w_{t+1})$  increases in  $w_{t+1}$ 

**Proof.** The slope of the equilibrium line in the credit market can be found through implicit differentiation:

$$\frac{\partial r_t}{\partial w_{t+1}} = \frac{(1-p)\left(\frac{\partial k}{\partial w_{t+1}} - \frac{\partial e_t^E}{\partial w_{t+1}}\right)}{p\frac{\partial e_t^D}{\partial r_t^d} - (1-p)\left(\frac{\partial k}{\partial r_t^c} - \frac{\partial e_t^E}{\partial r_{t+1}^c}\right)}$$
(33)

The denominator in this fraction is always positive. Since (33) is valid for any values of the parameter p, we can check, whether it is positive or negative for its upper and lower limits:

$$\inf \left\{ \frac{\partial r_t}{\partial w_{t+1}} \right\}^{CM} = \left\{ \frac{\partial r_t}{\partial w_{t+1}} \right\}_{p=0}^{CM} = -\frac{\frac{\partial k}{\partial w_{t+1}} - \frac{\partial e_t^E}{\partial w_{t+1}}}{\frac{\partial k}{\partial r_s^c} - \frac{\partial e_t^E}{\partial r_s^c}} < 0 \tag{34}$$

$$\sup \left\{ \frac{\partial r_t}{\partial w_{t+1}} \right\}^{CM} = \left\{ \frac{\partial r_t}{\partial w_{t+1}} \right\}_{n=1}^{CM} = 0$$
 (35)

Furthemore, for each set of functions k,  $e^E$  and  $e^D$  the slope of the CM-line for any given  $w_{t+1}$  gradually increases in p as p changes from 0 to 1:

$$\frac{\partial}{\partial p} \left\{ \frac{\partial r_t}{\partial w_{t+1}} \right\}^{CM} = -\frac{\frac{\partial e_t^D}{\partial r_t^d} \frac{\partial k}{\partial w_{t+1}} - \frac{\partial e_t^D}{\partial w_{t+1}} \frac{\partial k}{\partial r_t^c}}{\left[ p \frac{\partial e_t^D}{\partial r_t^d} - (1-p) \left( \frac{\partial k}{\partial r_t^c} - \frac{\partial e_t^D}{\partial r_{t+1}^c} \right) \right]^2} > 0$$
(36)

This ensures that in any point  $w_{t+1}$  the derivative  $\frac{\partial r_t}{\partial w_{t+1}}$  is always bounded by (34) from below and by (35) from above. Since  $\left\{\frac{\partial r_t}{\partial w_{t+1}}\right\}^{LM} < \left\{\frac{\partial r_t}{\partial w_{t+1}}\right\}^{CM}_{p=0}$  (for proof see the Lemma 2 below), the slope of the LM-line is smaller than the smallest possible slope of the CM-line in any point  $w_{t+1}$ , so that the gap  $r_t^{CM} - r_t^{LM}$  increases in  $w_{t+1}$ . Due to arbitrary choice of the functions family  $k, e^E$  and  $e^D$  and arbitrary choice of w the Lemma holds in general case.

<sup>&</sup>lt;sup>12</sup>It is sufficient to consider the derivative of this gap.

**Lemma 2** The slopes of the LM- and CM-lines are related with

$$\left\{ \frac{\partial r_t}{\partial w_{t+1}} \right\}^{LM} < \inf \left\{ \frac{\partial r_t}{\partial w_{t+1}} \right\}^{CM}$$
(37)

**Proof.** The slope of the LM-line is given by

$$\left\{ \frac{\partial r_t}{\partial w_{t+1}} \right\}^{LM} = -\frac{l_w}{l_r} \tag{38}$$

where  $l_w = \frac{\partial l}{\partial w_{t+1}}$  and  $l_r = \frac{\partial l}{\partial r_t}$ The slope of the CM-line is given by

$$\left\{ \frac{\partial r_t}{\partial w_{t+1}} \right\}^{CM} = \frac{\left(1 - p\right) \left(\frac{\partial k}{\partial w_{t+1}} - \frac{\partial e_t^E}{\partial w_{t+1}}\right)}{p \frac{\partial e_t^D}{\partial r_t^d} - \left(1 - p\right) \left(\frac{\partial k}{\partial r_t^c} - \frac{\partial e_t^E}{\partial r_{t+1}^c}\right)}$$
(39)

Replacing for p=0, and comparing with the previous inequality, obtain, that we need to prove that

$$-\frac{l_w}{l_r} < -\frac{k_w - e_w^E}{k_r - e_r^E} \tag{40}$$

where  $k_w, e_w^E, k_r, e_r^E$  denote wage- and interest rate- derivatives of the respective functions.

This last condition is fulfilled as soon as

$$l_w k_r - l_w e_r > k_w l_r - e_w l_r \tag{41}$$

Consider the properties of factor demands<sup>13</sup>:

$$l_w = \frac{f_{kk}}{f_{kk}f_{ll} - (f_{kl})^2} < 0 (42)$$

$$k_w = l_r = \frac{-f_{kl}}{f_{kk}f_{ll} - (f_{kl})^2} < 0 (43)$$

The denominator  $f_{kk}f_{ll} - (f_{kl})^2$  has to be positive, otherwise factor demands would positively depend on the factor prices, which contradicts to a standard assumption in microeconomics about factor demands decreasing in factor prices.

$$k_r = \frac{f_{ll}}{f_{kk} f_{ll} - (f_{kl})^2} < 0 (44)$$

Hence

$$k_w l_r - l_w k_r = \frac{(f_{lk})^2 - f_{kk} f_{ll}}{\left[f_{kk} f_{ll} - (f_{kl})^2\right]^2} < 0$$
(45)

Combining this with  $e_w < 0$  and  $e_r > 0$ , we obtain

$$-l_w e_r > k_w l_r - l_w k_r - e_w l_r \tag{46}$$

since left-hand side is positive and the right-hand side is negative, which proves 41 and consequently the statement of the lemma.

**Lemma 3** The slopes of the LM- and CM-lines in steady state problem (22) are related with

$$\left\{ \frac{\partial r}{\partial w} \right\}^{LM} < \left\{ \frac{\partial r}{\partial w} \right\}^{CM} \tag{47}$$

**Proof.** In steady state  $r_{t+1} = r_t$  and  $w_{t+1} = w_t$ , so the indexes in the equation above are omitted. The slope of the LM-line stays unchanged due to the fact, that the labour demand function does not depend on the wage level of "current" period. Equilibrium in Credit Market includes however savings functions of the agents, which depend on the "current" wage level as well. So, any change in steady state would produce two-sided effect on the LM-line: first through reaction of the savings decision on change in current wage, and second through reaction of all other functions on change in the "future" wage. To distinguish between these two effects we attribute again respective indexes to the correspondent variables. The slope of the LM-line is hence given by

$$\left\{\frac{\partial r}{\partial w}\right\}^{CM} = \frac{(1-p)\left(\frac{\partial k}{\partial w_{t+1}} - \frac{\partial e^E}{\partial w_t} - \frac{\partial e^E}{\partial w_{t+1}}\right) + p\frac{\partial e^D}{\partial w_t}}{p\frac{\partial e^D}{\partial r} - (1-p)\left(\frac{\partial k}{\partial r} - \frac{\partial e^E}{\partial r}\right)}$$

Here  $\frac{\partial e_t^D}{\partial w_t}$  and  $\frac{\partial e_t^E}{\partial w_t}$  denote first derivatives of the savings function of depositors and entrepreneurs with respect to his/her wage in first period of life.

Similarly  $\frac{\partial e_t^E}{\partial w_{t+1}}$  denotes the first derivative of the same agents with respect to the wage level in second period of life. Note that  $\frac{\partial e_t^E}{\partial w_t} = \frac{\partial e_t^D}{\partial w_t}$  due to the identical utility functions and identical budget constraints for the first period of agents' lives. Rearranging the nominator in the expression above, and setting p = 0, we obtain the lowest possible slope for CM-line

$$\inf \left\{ \frac{\partial r}{\partial w} \right\}^{CM} = -\frac{\frac{\partial k}{\partial w_{t+1}} - \frac{\partial e_t^E}{\partial w_{t+1}}}{\frac{\partial k}{\partial r_t^C} - \frac{\partial e_t^E}{\partial r_t^C}} < 0 \tag{48}$$

as in previous Lemma, which proves the statement.

38

#### **Figures**

- 1.Insignificant shock is not presented since does not disturb the economy
- 2. Significant shock (Figures 5 8)
- 3. Extreme shock (Figures 9 12)

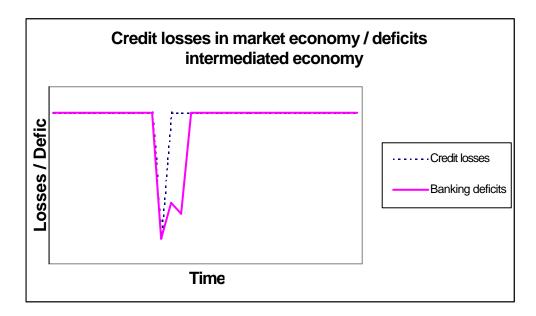


Figure 5: A significant shock generates credit defaults, which lead to the deficits in banking system. For the market economy theses defaults are explicit losses, however for the intermediated economy they are not transmitted to the population directly. Instead, defaults are accumulated in banking system as deficits, which are gradually eliminated due to positive banking marge. When however this marge is not sufficiently big, some temporary deterioration in deficits is possible, which is also to be seen in the diagram, and does not mean that the system looses its ability to recover.

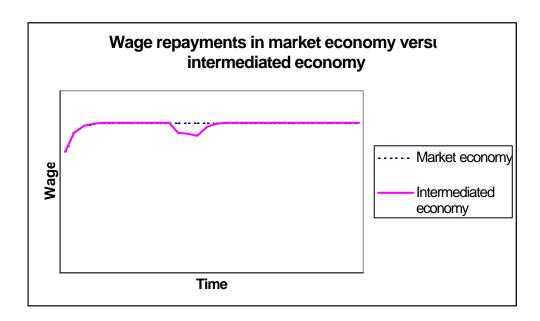


Figure 6: Market economy does not suffer from the reduction in wage level after a significant shock. The intermediated economy however exhiits some temporary fall in the equilibrium wage level.

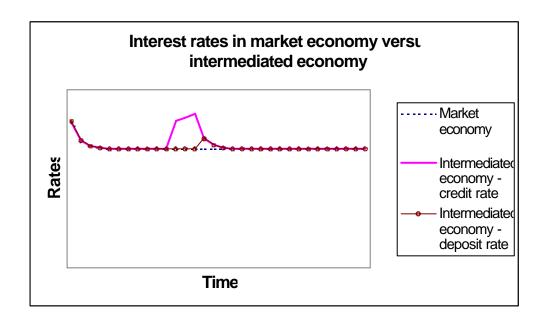


Figure 7: Again, market economy does not suffer from changes in interest rate, but the intermediated economy exhibits rise in credit interest rate. The deposit rate is fixed by the regulation (ceiling) at the steady state level, which produces positive banking marge. This marge is the key element for recovery.

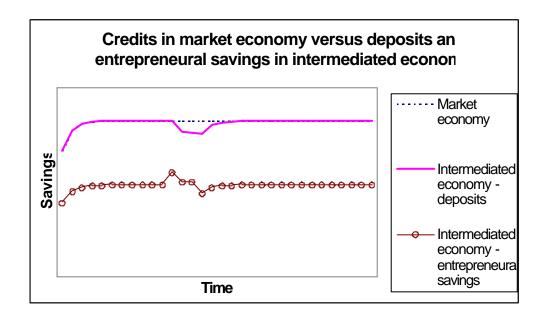


Figure 8: No changes appear in credit capacities of the market economy. instead, intermediated economy suffers some periods after the shock from lower deposits level, which is however compensated by higher entrepreneural savings in the periods where credit rate is high. As soon as the deficits level reaches zero, interest rates on credits and deposits are equal, and the system recovers to the steady state.

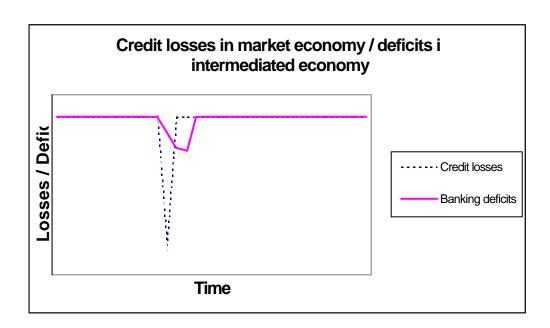


Figure 9: In case of extreme shock deficits in banking system seem to be smaller than the repayment defaults in market economy. Tjis is however not the fact, since after the extreme shock a subvention ist needed to lift the deficits to the level above the critical. The picture hence represents the amount of this subvention as the difference between defaults (market economy) and actual deficits (intermediated economy).

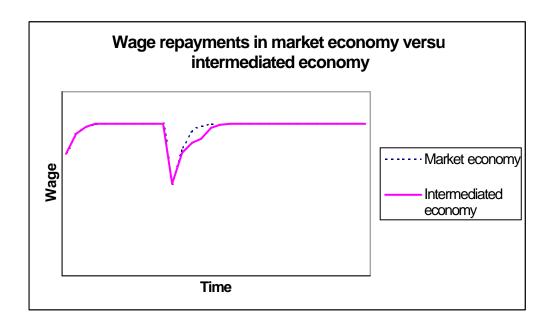


Figure 10: Extreme shock influences capability of entrepreneurs to repay wages in both economies. This leads to equal initial abrupt change in actual wage income. After that both economies recover to the steady state, however the recovery of intermediated economy lasts longer due to the nonzero deficits in banking system and consequently lower equilibrium wages, compared to the market case.

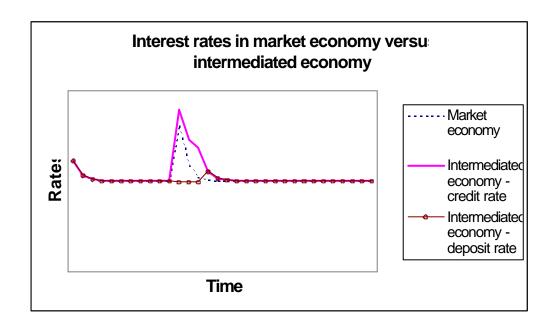


Figure 11: After extreme shock interest rates rise both in market and intermediated economies. Deposit rate in intermediated economy is fixed at the steady state level due to the ceiling. This provides positive banking marge, which reduces the deficits and leads to recovery. As soon as deficits are zero, there is no more need in rehulatory ceiling, so that the deposit rate increases to the credit one, after which the economy glides to the steady steate.

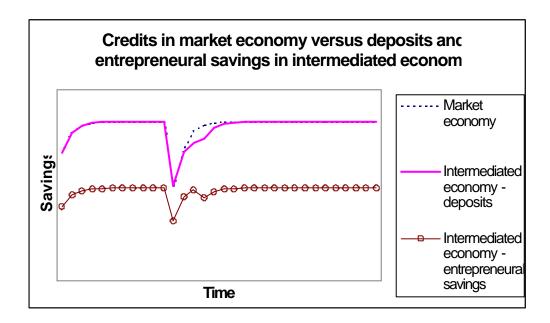


Figure 12: Credit capacities of the working population fall down after the shock due to insufficient wage repayments from entrepreneurs. After that both economies recover, however again the recovery of the intermediated economy lasts longer due to gradual elimination of deficits. Two phases of recovery are to be seen: first the recovery to "regulated steady state", where the deposit interest rate is fixed by the ceiling, and second after the system has no deficits, it recovers to competitive steady state.