Taylor Rules in Practice: How The Central Bank Can Intercept Sunspot Expectations

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Abstract

This paper derives new results on the effects of employing Taylor rules in economies that are subject to real-market imperfections such as production externalities. It suggests that rules which should be avoided (chosen) in perfect markets environments do in fact ensure (yield) unique (multiple) rational expectations solutions in alternative settings. Therefore, exact knowledge on the degree of market imperfection is pivotal for robust policy advice.

*I would like to thank Michael Burda for helpful comments. Remaining errors are, of course, my own. I would also like to thank the UCLA Economics Department for its hospitality. This paper was written while Weder was a *DFG Heisenberg Fellow*. *Keywords*: Indeterminacy, Increasing returns to scale, Taylor Rules, Cash-in-advance economies. *JEL classification*: **E32**, **E52**.

1 Introduction

The Taylor (1993) rule provides a good description of how many central banks attempt to set interest rates in order to achieve stable prices while avoiding large fluctuations in output and employment. There is increasing evidence, however, that Taylor rules can be a source of economic instability by themselves. For example, Benhabib et al. (2001) demonstrate that steering under such policy may introduce real indeterminacy in an otherwise determinate economy. These sunspot equilibria imply that cycles arise simply because agents believe in their existence. As a consequence, the Taylor rule debate frequently advises the monetary authority to assign aggressive backward-looking principles in which interest rates respond to predetermined variables, in particular to inflation (see for example Calmstrom and Fuerst, 1999).

The present paper qualifies this assertion by suggesting that before spelling out concrete policy rules, the monetary authority must first be *au courant* with the specific environment of the economy. In particular, it demonstrates that the presence of mild forms of market imperfections – arising from production externalities, for example – may have fundamental consequences on how monetary policy should be conducted.

The motivation for the current research stems from the insights of recently formulated dynamic general equilibrium models with sunspot equilibria and self-fulfilling prophecies.[1] In these models the possibility of a continuum of equilibria is the consequence of empirically plausible market imperfections – therefore, sunspot equilibria are more than theoretical curiosities. However, these models remain in the confines of real (i.e. non-monetary) economies. The current paper augments these models by introducing money in order to examine the effects of monetary policy on indeterminacy as well assess to assess monetary policy recommendations in suboptimal economies.

Indeterminacy economies are always Pareto-inefficient because they are linked to some form of market imperfection. Their presence calls for three possible general policy strategies: (i) policies that promote optimism, (ii) policies that bump the economy towards better equilibria or (iii) policies that eliminate coordination failures altogether. The current essay concerns itself primarily with point (iii). In particular, it stresses the pertinence of

[1] A non-exhaustive list includes Benhabib and Nishimura (1998), Christiano and Harrison (1999), Farmer and Guo (1994), Perli (1998), Schmitt-Grohé and Uribe (1997), Weder (1998, 2001), and Wen (1998).

Taylor rules in suboptimal environments. I will inspect monetary policies in economies that display endogenous fluctuations independently of policy and will offer strategies to eliminate sunspots' aptitude.[2]

The main findings can be stated as follows: by responding sufficiently to output movements in setting the nominal interest rate, the monetary authority can stabilize the (sunspot-driven) economy. The reasoning is that the nominal interest rate operates like an inflation tax. By the central bank fighting output fluctuations, sunspot blips will be dimmed: when the central bank builds up the costs of buoyant expectations these will no longer be sustainable in the first place. The result is related to Guo and Lansing (1998) and Christiano and Harrison (1999) who establish that non-fundamental equilibria stemming from increasing returns can be removed by progressive taxation. The tax mechanism operates by intercepting the effects of high increasing returns to scale – quite similar to the Taylor design that is discussed here.

I also find that Taylor rules work quite differently depending on the fundamentals of the economy. In fact, it appears to be the case that rules which should be avoided (chosen) in perfect markets environments do in fact yield unique (multiple) rational expectations solutions in alternative settings. For example, backward-looking policy settings that ensure unique rational expectations in cases of constant returns to scale often are the ones connected to determinacy at moderate imperfections and *vice versa*. Consequently, the central bank should have a clear picture of market imperfections before setting policy rules. Unfortunately, existing empirical studies do not provide an unambiguous answer on the extent of the imperfections.

The paper is primarily concerned with backward-looking rules, however, it will also demonstrate that forward-looking, current-looking and hybrid rules imply similar results.

The argument which is developed in the current paper is framed within a fully specified environment which has been shown by Calmstrom and Fuerst (2000) to have fundamentally different policy implications than (*ad hoc*) New-IS-LM-frameworks.[3] My study differs from theirs in two important aspects, however. First, their production technology is constant returns to scale. By contrast, I allow empirically plausible production externalities which lead to increasing returns. Second, in their model the central bank's

[2] The framework I will draw on is Wen (1998) which is currently the most successful attempt in terms of obtaining sunspot equilibria at small increasing returns and in generating realistic business cycles. See also Benhabib and Wen (2002).

[3] King (2000) is a good review of the New IS-LM model.

nominal interest rate target responds to inflation only whereas the current paper considers a Taylor formula in which the rate is increased or decreased according to what is happening to both real GDP and inflation.

The paper is organized as follows. The next section presents the model economy. Section 3 discusses the connection of monetary policy, market imperfections and sunspot equilibria. Section 4 discusses implications and concludes.

2 The economy

The physical setup of the economy's real part is a standard real business cycle model augmented by production complementarities. Currency is introduced by imposing restrictions on the timing of exchanges.

2.1 Preferences and technologies

The representative household seeks at time $t = 0$ to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t (\ln c_t - \eta l_t) \qquad 0 < \beta < 1,\ \eta > 0$$

where β, c_t and l_t are the discount factor, consumption and labor during t.[4] The household rents labor and capital services to firms. All markets are perfectly competitive. The household's budget constraint can be stated as

$$M_{t+1} + P_t k_{t+1} = M_t + \Pi_t + P_t w_t l_t + P_t(r_t + 1 - \delta_t)k_t - P_t c_t + N_t(R_t - 1)$$

where P_t is the price level, M_t are the cash balances at the beginning of t, w_t is the real wage and r_t is the real rental rate of capital, k_t. The variable u_t denotes the degree of capital utilization. The depreciation rate of installed capital, δ_t, is increasing in utilization

$$\delta_t = \frac{1}{\theta} u_t^{\theta} \qquad \theta > 1.$$

N_t stands for one-period bank deposits which pay a short-term nominal interest given by R_t. Π_t is the profit flow from firms and intermediaries. A positive

[4] Indivisible labor is standard in real business cycle modelling.

value is assigned to the inconvertible currency by assuming that during the shopping session the household is subject to the cash-in-advance-restriction

$$M_t + P_t w_t l_t - N_t \geq P_t c_t,$$

that is, consumption expenditures must be covered by money holdings and by wage payments.

Output is produced by a large number of competitive firms with identical technologies. The economy as a whole is affected by organizational synergies that cause the output of an individual firm to be higher if all other firms in the economy are producing more. The term A_t stands for these aggregate externalities. The production complementarities are taken as given for the individual optimizer and they cannot be priced or traded. Deviations from constant returns to scale are measured by γ.[5] A firm of type i has technology

$$y_{i,t} = A_t^{\gamma}(u_t k_{i,t})^{\alpha} l_{i,t}^{1-\alpha} \qquad A_t = (u_t k_t)^{\alpha} l_t^{1-\alpha} \quad \text{and} \quad 0 < \alpha < 1.$$

Here, k_t (l_t) denotes – by way of normalization – the economy-wide average capital (labor) input. Lastly, firms must acquire cash borrowed at the short term rate from an intermediate sector to finance their wage bills.

2.2 Intermediaries and central banking

The monetary branch of the economy comes in two parts: the intermediary sector and the central bank. The perfectly competitive intermediary has two sources of cash – deposits by households and lump-sum central bank-issued currency, $M_{t+1}^s - M_t^s$. The loan constraint is

$$N_t + M_{t+1}^s - M_t^s \geq P_t w_t l_t.$$

Most central banks implement monetary policy by controlling a short-term nominal interest rate. Accordingly, it has become standard to represent monetary policy in terms of commitment to a rule for the nominal rate of interest. In the present paper, the monetary authority sets the short-run

[5] See Benhabib and Farmer (1994) and others for an alternative (and in reduced-form equivalent) formulation that incorporates internal increasing returns at the intermediate-firm level in an imperfectly competitive market structure without free entry. In that case, the parameter γ would (also) relate to the monopoly markup.

nominal interest rate based on what is happening to both real GDP and inflation. The frequently applied backward-looking rule is given by

$$R_{t+1} = R\left(\frac{\pi_t}{\pi}\right)^{\tau}\left(\frac{y_t}{y}\right)^{\omega} \qquad \tau \geq 0 \quad \omega \geq 0$$

or in linearized from

$$\widehat{R}_{t+1} = \tau\widehat{\pi}_t + \omega\widehat{y}_t$$

in which the variables appear as percentage deviations from their stationary states R, π and y. We denote rules with $\tau < 1$ ($\tau > 1$) as passive (aggressive) since the nominal interest rate moves less (more) than one-for-one with inflation. If $\tau = 1$, the nominal rate of interest responds one-for-one to changes in actual inflation and the policy is neutral. The term $\widehat{y}_t$ is interpretable as the output gap and ω refers to the weight given to deviations of real GDP from a normal level. Since the general equilibrium setting imposes a money demand relationship (that is, the cash-in-advance setup), the interest rate rule implies that the money supply is endogenous.

2.3 Dynamics and calibration

In what follows, I restrict the analysis to a symmetric equilibrium in which $u_{i,t} = u_t$, $k_{i,t} = k_t$ and $h_{i,t} = h_t$. It follows that the aggregate production technology becomes

$$y_t = (u_t k_t)^{\alpha(1+\gamma)} l_t^{(1-\alpha)(1+\gamma)}$$

which exhibits increasing returns to scale. The economy's dynamics are given by

$$\begin{aligned}
\eta c_t &= \frac{w_t}{R_t} = \frac{\alpha y_t l_t^{-1}}{R_t} && (1)\\
u_t^{\theta} &= \alpha\frac{y_t}{k_t} && (2)\\
E_t\frac{\beta}{c_{t+1}\pi_{t+1}} &= \frac{1}{c_t R_t} && (3)\\
\frac{1}{c_t R_t} &= E_t\frac{1}{c_{t+1}R_{t+1}}\beta[\alpha\frac{y_{t+1}}{k_{t+1}} + 1 - \frac{1}{\theta}u_{t+1}^{\theta}] && (4)\\
c_t + k_{t+1} &= y_t + (1 - \frac{1}{\theta}u_t^{\theta})k_t. && (5)
\end{aligned}$$

Equation (1) describes the leisure-consumption trade-off and (2) pins down the optimal utilization rate of capital. Equations (3) and (4) are the usual Fisher and Euler conditions. (5) repeats the intertemporal constraint. No closed-form solution exists, thus the model must be approximated. In log-linearized form, the dynamics boil down to (the Appendix discerns details)

$$\begin{bmatrix} E_t\widehat{c}_{t+1} \\ \widehat{R}_{t+1} \\ \widehat{k}_{t+1} \\ E_t\widehat{R}_{t+2} \end{bmatrix} = \mathbf{M} \begin{bmatrix} \widehat{c}_t \\ \widehat{R}_t \\ \widehat{k}_t \\ \widehat{R}_{t+1} \end{bmatrix}.$$

The dynamical system contains two non-predetermined (or jump) variables: $E_t\widehat{c}_{t+1}$ and $E_t\widehat{R}_{t+2}$. Therefore, indeterminacy requires that at most one eigenvalue of the 4×4-matrix $\mathbf{M}$ is outside the unit circle. Two eigenvalues larger than one (and two smaller than one) imply determinacy. If, say, only one eigenvalue is outside the unit circle, then there are multiple rational expectations solutions which take on the form

$$\begin{bmatrix} \widehat{R}_{t+1} \\ \widehat{k}_{t+1} \\ E_t\widehat{R}_{t+2} \end{bmatrix} = \widetilde{\mathbf{M}} \begin{bmatrix} \widehat{R}_t \\ \widehat{k}_t \\ \widehat{R}_{t+1} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \zeta_{t+1} \end{bmatrix}$$

with ζ_{t+1} being an arbitrary random variable with $E_t\zeta_{t+1} = 0$.

I will now assign parameter values and demonstrate the empirical plausibility of sunspot equilibria. The time unit is taken to be a quarter of a year. The calibration is based on empirical observations on post-war U.S. data. The capital share, α, is set equal to 30 percent, the discount factor, β, is chosen to be 0.99 and the steady state rate of capital depreciation, δ, is 2.5 percent. The parameter θ can then be derived from steady state conditions

$$\theta = \frac{1/\beta - 1 + \delta}{\delta}.$$

When abstracting from the monetary side, the calibration implies minimum increasing returns to scale needed for indeterminacy that amount to 1.10368175 which is reasonable value. For example, Caballero and Lyons (1992) obtain increasing returns estimates in the order of 1.26 to 1.56. Baxter and King (1991) find returns to scale of 1.53, however, combined with a standard error of 0.56. Burnside, Eichenbaum and Rebelo (1995) report a point estimate of 0.98. Again their standard error of 0.34 is large. Basu and Fernald (1997) also find close to constant returns, however, the imparted estimation-uncertainty is significant again.

The reasoning for the multiplicity is as follows. Equations (2) and (5) entail the reduced form-output

$$y_t = \text{const} * k_t^{\frac{\alpha(1+\gamma)(\theta-1)}{\theta-\alpha(1+\gamma)}} l_t^{\frac{(1-\alpha)(1+\gamma)\theta}{\theta-\alpha(1+\gamma)}}.$$

Thus, the effective labor-output elasticity is larger than unity for[6]

$$\gamma > \frac{\alpha(\theta-1)}{\alpha+(1-\alpha)\theta} > 0.$$

Accordingly, the reduced-form labor demand curve is upward sloping at mild increasing returns.[7]

Now, how do sunspot equilibria come about? Suppose agents suddenly have pessimistic expectations and expect lower future consumption. The permanent income motive will reduce today's consumption as well. Then, the static-first order condition (1) implies that the labor supply schedule moves out (left-hand-side expressions are marginal utilities). However, given the upward sloping equilibrium labor demand curve, employment and investment will actually both fall today. As a consequence, the future capital stock, output and consumption will all be low and the initially pessimistic expectations self-fulfilled. The sunspot movement must be stationary by definition. In the model, decreasing capital utilization costs will ultimately bring the contraction to a halt and revert the cycle's direction. This is similar to the accelerator-multiplier effect in *ad hoc* models.

3 How should monetary policy be conducted?

This section discusses the effects of two versions of the Taylor rule on the qualitative dynamics of the artificial economy. It opens by assuming that the central bank sets nominal interest rates after having observed (past) inflation and output.

[6] If the depreciation costs are high ($\theta \to \infty$) and accordingly capital utilization is set constant by agents, the condition reduces to the same as the one found by Harrison and Weder (2002).

[7] Upward sloping labor demand arises for $\gamma > 0.094488189$, thus the condition is not a sufficient one for indeterminacy. When formulating the model in continuous time, the two tresholds converge.

3.1 Indeterminacy zones with backward-looking rules

I will start combing for parametric indeterminacy zones by considering a constant returns to scale technology ($\gamma = 0$) which will help in understanding other cases. When I set $\omega = 0$, the four eigenvalues of $\mathbf{M}$ are[8]

$$\left\{ \frac{1-\beta(1-\alpha)(1-\delta)}{\alpha\beta}, \frac{\alpha}{1-\beta(1-\alpha)(1-\delta)}, \tau, 0 \right\}.$$

The eigenvalues are the same as those reported in Calmstrom and Fuerst (2000) despite the absence of variable capital utilization and depreciation in their model. The eigenvalue-expressions expose the policy's direct impact on dynamics: policy-induced indeterminacy can be avoided simply by aggressively responding to past inflation ($\tau > 1$) whereas passive responses lead to multiplicity ($\tau < 1$).[9] The reasoning for the occurrence of sunspot equilibria can be understood as follows: suppose that current inflation increases by one percent, which given an aggressive policy, implies that the $t+1$ nominal rate goes up by more than one percent. As a consequence, the inflation tax depresses future consumption and lifts current consumption – the real rate goes down. This, however, is only possible when the rate of inflation, π_{t+1}, increases. As a result, the central bank's target R_{t+2} rises by even more and policy induces an unsustainable explosive inflation-pattern. The initial rise in inflation is not supported and consequently the sunspot cycle is stopped. Clearly, if the bank follows a passive policy, the chain of events remains stationary.

Now, suppose that the central bank reacts in parts to past movements in output, $\omega > 0$. Dynamics can be derived analytically within a simplified version without capital and in which output is produced with the linear technology

$$y_t = Al_t.$$

The model reduces to the scalar equation

$$E_t\widehat{R}_{t+2} = (\tau - \omega)\widehat{R}_{t+1}.$$

[8] It is reasonable to assume that $0 < \delta < 1$.

[9] The first (second) eigenvalue is outside (inside) the unit circle. Unfortunately, by introducing $\omega \neq 0$ or $\gamma \neq 0$, the respective expressions become "not very practical" and I will therefore derive some results numerically.

Indeterminacy arises for

$$\omega - 1 < \tau < \omega + 1. \tag{6}$$

The right-hand-side inequality in (6) repeats the afore-mentioned result. Namely that active inflation-fighting policies eliminate sunspot equilibria. More generally, the central bank must simultaneously select both policy parameters as there exists an intermediate range of τ values that generates indeterminacy for given ω. For example, letting $\omega = 1.5$, the bank's output-coupled response must fall outside $\tau \in (1/2, 2.5)$ otherwise policy-induced cycles crop up.

Figure 1 shows regions of indeterminacy when including endogenous capital accumulation while varying the two Taylor-parameters with τ measured on the horizontal axis and ω measured on the vertical axis. In this more general case, the left-hand-side boundary in (6) becomes a nonlinear function of the other model parameters. Moreover, it moves up into regions that are not relevant in the sense of observed monetary policies.[10] Therefore, Figure 1 plots only the lower boundary. It again calls for the central bank to not be too aggressive in curbing output fluctuations as this would create endogenous cycles by itself.

The indeterminacy arises as follows. Suppose that current inflation heats up. Accordingly, R_{t+1} rises. The inflation tax raises current consumption, yet the labor supply curve shifts inward which, since the capital stock is given, lowers current output. The initial rise in inflation is not supported as long as the output-related interest rate reaction is weak: there are two effects acting on the interest rate

$$\widehat{R}_{t+1} = \tau \underset{(\uparrow)}{\widehat{\pi}_t} + \omega \underset{(\downarrow)}{\widehat{y}_t}.$$

If ω is small, then equilibrium sequences are explosive and sunspot equilibria are not possible. However, larger ω-weights imply that the interest rate movement is checked and stationary equilibrium sunspot-driven sequences materialize.

Next, I turn to cases in which the economy is imperfect and subject to real indeterminacy stemming from production externalities. The question

[10] For example, if τ is set at 1.5, then $\omega > 56.84582$ is needed to transform the economy back into one of determinacy (at $\tau = 1$ we require $\omega > 45.47665$ and at $\tau = 1/2$ $\omega > 34.10749$).

I ask is: can the central bank stamp out sunspot equilibria by choosing an appropriate Taylor-design? I will again begin by assuming that $\omega = 0$. This is the frequently considered case (for example Calmstrom and Fuerst, 2000, and others, who, however, assume constant returns to scale). Acting passively, $\tau < 1$, leaves the qualitative dynamics unaffected (Figure 2). At the neutral position $\tau = 1$, the minimum increasing returns are $\gamma^{\min} = 0.103708061$, which is essentially the number that arises in the nonmonetary version. At $\tau = 1.5$, the minimum degree of scale economies becomes $\gamma = 0.103716838$, thus the quantitative effect of policy is imperceptible. When the central bank reacts to inflationary movements, indeterminacy arising from production externalities cannot be eliminated.

I therefore shift attention to cases involving $\omega > 0$. Figure 1's qualitative results remain unchanged. To see this, consider the case of increasing returns low enough to not generate externalities-affiliated indeterminacy, say $\gamma = 0.05$. Figure 3 indicates that sunspot equilibria are linked to policy in very much the same way as under constant returns. However, the vertical borderline between determinacy and indeterminacy shifts down slightly: it places a slightly more stringent cap on ω.

Let us now step up γ to 0.11 such that production complementarities induce sunspot equilibria in environments without money. The specific value draws on Benhabib and Wen (2002) who propose it by observing that the model's cycle frequency matches that of U.S. output. Furthermore, the magnitude of scale economies falls into the scope of the studies mentioned in Section 2.3.

Figure 4 pins down the policy advice under the assumption of $\gamma^{\min}$. The only way for the central bank to tackle endogenous cycles is now by working against output movements. If $\omega = 0$, indeterminacy always occurs. Furthermore, Figure 4 shows that for any given τ, the output-related response must be sufficiently large to obtain unique solutions. Moreover, even when ω takes on strictly positive values, the specific choice of τ may imply every of the three possible regimes (indeterminacy, determinacy or source – in which the latter region may imply endogenous and deterministic cycles on its own).

The result is reminiscent of Guo and Lansing (1998) and Christiano and Harrison (1998) who find that progressive tax systems effectively eliminate indeterminacy by taxing away increasing returns. The Taylor policy suggested here fabricates a similar device. If ω is sufficiently large, then output fluctuations that arise from believing in them simply become too costly. Let us walk through a sunspot sequence that is stopped by the central bank for further understanding of the result. Suppose that agents embellish optimistic expectations without any real cause. By projecting high future consumption, they will ratchet up today's consumption expenditures. The high increasing returns will increase today's employment and output as a result of the upward sloping equilibrium labor demand curve. Now, if the output response of the central bank is strong enough, then R_{t+1} will increase and the initially sanguine expectations will be intercepted by an inflation tax that reduces future consumption. The initially optimistic expectations are not fulfilled and the sunspot cycle is broken. To conclude, the discussion of backward-looking rules suggests that central banks should be very careful about the specific economic environment when setting policy rules. I will next turn to forward-looking rules.

3.2 Forward looking rules?

It may be suspected that the result is dependent on assuming a backward-looking policy rule. Yet, the general picture that Taylor rules settings should consider technology does not change when the central bank pays attention to expected values such as in

$$\widehat{R}_t = \tau E_t \widehat{\pi}_{t+1} + \omega E_t \widehat{y}_{t+1} \qquad \tau \geq 0 \quad \omega \geq 0.$$

The Appendix outlines the solution of the complete model which boils down to

$$\begin{bmatrix} E_t \widehat{c}_{t+1} \\ E_t \widehat{R}_{t+1} \\ \widehat{k}_{t+1} \end{bmatrix} = \mathbf{J} \begin{bmatrix} \widehat{c}_t \\ \widehat{R}_t \\ \widehat{k}_t \end{bmatrix}.$$

Beginning with the case of $\omega = \gamma = 0$, the three eigenvalues of $\mathbf{J}$ are

$$\left\{ \frac{1-\beta(1-\alpha)(1-\delta)}{\alpha\beta}, \frac{\alpha}{1-\beta(1-\alpha)(1-\delta)}, \frac{1}{\tau} \right\}$$

which reverses the results obtained under backward-looking rules: all policy responses $\tau > 1$ are precluded, so it is necessary for policy to be passive in Taylor's sense. Let us turn to cases involving $\omega > 0$. Resorting again to a linear production technology without capital leads to the simplified model

$$E_t \widehat{R}_{t+2} = \frac{1}{\tau - \omega} \widehat{R}_{t+1}$$

from which more general Taylor formulas can be discussed analytically: determinacy requires

$$\tau > \omega - 1 \qquad \text{for} \qquad \tau < \omega$$

or

$$\tau < \omega + 1 \qquad \text{for} \qquad \tau > \omega.$$

Now there are two zones of indeterminacy. Again, if $\omega = 0$, the monetary authority should be passive when responding to expected inflation. If $\tau = 0$, the monetary authority should be passive when responding to expected output. This insight essentially carries over when weights are given to both inflation and real GDP. Weights given to both variables should be similar and extreme (in the sense of aggressive) responses will likely drive the economy into sunspot districts.

Let us now consider the more general case with capital accumulation. As long as the central bank sets $\tau < 1$, indeterminacy does not arise with constant returns as in Calmstrom and Fuerst (2002).[11] The result carries over to $\gamma < \gamma^{\min}$. However, once increasing returns are stronger it becomes even harder for the central bank to prevent technology-induced indeterminacy than in the backward-looking case. Figure 5 identifies $\tau - \omega-$combinations that achieve the goal and it demonstrates the difficulty in finding the right mix.[12] As was the case with backward-looking policies, $\omega = 0$, indeterminacy always commences. Suppose that production externalities are $\gamma = 0.11$. Thus, suppose that the central bank sets

$$\tau = 1.5 \qquad \text{and} \qquad \omega = 0.2$$

then sunspot cycles are ostracized (they are not if $\tau < 1$). However, if the scales-estimate that the central bank is acting on turns out to be incorrect, the policy may be chosen wrongly: at slightly smaller externalities, say at $\gamma = 0.10$, indeterminacy commences.[13]

[11] There exists the possibilty for indeterminacy for very large ω-values, numerically exceeding 26.

[12] The area below (above) the determinacy region involves two (three) eigenvalues inside the unit root.

[13] There is an alternative way to see how forward looking rules do not contribute in obtaining robust advise. For example, $\tau = 1.5$ and $\omega = 0.5$ designs indeterminacy for $\gamma < 0.1005439$ and $\gamma > 0.102707401$. Yet the small (but empirically plausible) intermediate measure of increasing returns $0.1005439 < \gamma < 0.102707401$ implies uniqueness.

The lesson from this Section is parallel to the backward-looking case. When set appropriately, Taylor rules may automatically stabilize the economy. However, the presence of increasing returns significantly changes concrete policy proposals.

3.3 A case against current looking rules and a (partial) case for them

Let us next consider current looking rules of the form

$$\widehat{R}_t = \tau \widehat{\pi}_t + \omega \widehat{y}_t \qquad \tau \geq 0 \quad \omega \geq 0.$$

Assuming away capital, there are no unique equilibria since the model reduces to

$$\left(1 - \frac{1}{\tau} - \frac{\omega}{\tau}\right) E_t \widehat{R}_{t+1} = 0.$$

Assuming $\omega = 0$, indeterminacy carries over when capital is included and production is constant returns since the dynamical system produces the three eigenvalues

$$\left\{ \frac{1 + \tau(1 + (1-\alpha)(1-\delta)\beta) + \beta(1-\alpha)\delta}{\alpha\beta\tau}, \frac{\alpha}{1 - \beta(1-\alpha)(1-\delta)}, 0 \right\}$$

of which the last two are always inside the unit circle. Since the underlying dynamical system involves two jump variables, the economy is always subject to indeterminacy. Numerical simulations suggest that allowing a rule in which the nominal rate responds to inflation and output delivers the same result. Therefore, the upshot from this appears to be a clear case against using current looking rules.

However, the picture changes when $\gamma > \gamma^{\min}$. Figure 6 plots determinacy regions for $\gamma = 0.11$. Now there exists an area in which applying a current looking Taylor rule can eliminate sunspot equilibria. In a nutshell, the central bank must react sufficiently to both output and inflation. Thus, current looking rules may be a vehicle to eliminate sunspot fluctuations, yet, the central bank must be confident that increasing returns exist otherwise these Taylor settings always lead to indeterminacy.

3.4 A note on nominal interest rate smoothing

Empirical studies on the Taylor rule generally include the lagged interest rate. Therefore, let us consider Taylor-type rules such as

$$\widehat{R}_t = \rho \widehat{R}_{t-1} + \tau E_t \widehat{\pi}_{t+1} + \omega \widehat{y}_t$$

in which the parameter ρ stands for interest rate smoothing (the formulation applies Clarida, Gali and Gertler's, 1998, baseline case). Generally, the estimates find a high degree of inertia displayed by central bank policy rules and slightly greater than one-for-one increases in the nominal rate in response to inflation. Furthermore, the response to the output gap is mostly found to be small for the U.S. post-1980 period. Thus, let us consider the case $\omega = 0$ and constant returns to scale. The following four eigenvalues depict the dynamics

$$\left\{ \frac{1-\beta(1-\alpha)(1-\delta)}{\alpha\beta}, \frac{\alpha}{1-\beta(1-\alpha)(1-\delta)}, \frac{1-\sqrt{1-4\rho\tau}}{2\tau}, \frac{1+\sqrt{1-4\rho\tau}}{2\tau} \right\}$$

(the dynamical system involves two jump variables). The first two eigenvalues split around the unit circle. If $\rho = 0$, the third eigenvalue is zero and the policy should be passive; of course, this argument simply repeats a finding on forward-looking rules (Section 3.2.). Now by making ρ positive, the last two eigenvalues become complex at

$$\rho' = \frac{1}{4\tau}$$

and cross the unit circle at

$$\rho'' = 1 - \tau$$

depending on

$$1 \gtrless 4\tau(1-\tau)$$

which happens first. In any case, we see from the ρ''-condition, that setting purely on a passive rule does no longer guarantee determinacy since, *ceteris paribus*, increasing values of ρ pushes the economy out of the determinacy region. Empirical evidence points to policies such as $\rho \approx 0.85$ and $\tau \approx 1.1$, thus the analysis suggests that the central bank should avoid these policies for not creating endogenous fluctuations.[14] The central bank can dodge indeterminacy by aggressively acting upon output fluctuations: $\rho = 0.85$, $\tau = 1.1$ and $\omega > 2.5237$ yield unique equilibria. However, this large ω-value is at variance with empirically observed responses by the *Federal Reserve*, the *Bundesbank* and other central banks. Moreover, it can be shown that any such aggressive ω-policy is without qualitative effects on indeterminacy if $\gamma = 0.11$ – repeating the central bank's dilemma as outlined above.

Rotemberg and Woodford (1999), Giannoni and Woodford (2002) and Benhabib et al. (2003) have suggested that backward-looking Taylor rules' performances can be improved by adding lagged values of the nominal interest rate. In particular, they find that the rule

$$\widehat{R}_{t+1} = \rho \widehat{R}_t + \tau \widehat{\pi}_t \qquad\qquad \rho > 1$$

delivers unique equilibria. Let us again consider constant returns to scale and capital accumulation (the above mentioned papers abstract from capital). The dynamics are characterized by the four eigenvalues

$$\left\{ \frac{1-\beta(1-\alpha)(1-\delta)}{\alpha\beta}, \frac{\alpha}{1-\beta(1-\alpha)(1-\delta)}, \tau+\rho, 0 \right\}.$$

The parameters τ and ρ enter in complementary fashion and determinacy simply requires that $\tau+\rho > 1$ (the underlying dynamical system has two jump variables). Thus, $\rho > 1$ is a sufficient condition for ruling out indeterminacy. However, the advice given by Rotemberg and Woodford (1999), Giannoni and Woodford (2002) and Benhabib et al. (2003) is no longer valid once market imperfections are present: no (τ, ρ)-constellation delivers determinacy at $\gamma = 0.11$. Once again, the presence of market imperfections has nontrivial effects on the design of monetary policy. Setting the interest rate based on what is happening to real GDP may slash sunspot forces, however: the (ρ, τ, ω)-triplet $(0.85, 1.1, 0.95)$ constitutes a policy that delivers determinacy at real market imperfections.

4 Implications and conclusion

The recent literature on Taylor rules has suggested that the monetary authority should adopt aggressive, backward-looking rules. However, there are

[14] See for example Walsh (2002).

limitations to any generalized proposals once market imperfections are a possibility. Then, the interest rate's response to output may turn out material to actively rule out indeterminacy arising from production externalities and it is therefore instrumental for the central bank to take into account the technological regime before tuning policy parameters. Recall Figures 1 and 4 on backward-looking Taylor formulas. Choosing rules to the Southeast decreases the likelihood that the central bank may induce indeterminacy in constant returns environments. However, it is exactly that area that should be avoided by the central bank when increasing returns to scale are present. In fact, the (numerical) analysis suggests that there appears to be no $\tau-\omega-$combinations that yield determinacy in both settings. For example, Taylor (1993) recommended that τ and ω be set equal to 1.5 and 1/2 in his original study and, in fact, that very policy implies the bank found a successful code of stabilizing sunspot fluctuations with a backward-looking rule at $\gamma = 0.11$. However, if increasing returns are slightly smaller (at $\gamma = 0.10$), then the economy skids into a regime under the clout of sunspots. Alternative Taylor rule formulations (forward-looking, current-looking and hybrid) involve parallel results.

In summary, essential information on how monetary design should be framed in practice must be inferred from empirical estimates of market imperfections. Unfortunately, the existing work on this issue does not offer a clear cut answer – the measurement of the degree of increasing returns is simply too imprecise – which given my results poses a dilemma for the central bank. Cole and Ohanian (1999) suggest a basic problem for this ambiguity: insufficient variations in factor inputs. They conclude that currently available methods are not adequate to return estimates of scale economies such that we can eventually draw a conclusive diagnosis against or in favor of models with indeterminacy such as those summarized in footnote 1. I conclude that estimates currently available are also not adequate to square conflicting Taylor policy proposals.

References

[1] Basu, Susanto, John Fernald (1997): "Returns to Scale in U.S. Production: Estimates and Implications", *Journal of Political Economy* **105**, 249-283.

[2] Baxter, Marianne and Robert G. King (1991): "Productive Externalities and Business Cycles", mimeographed.

[3] Benhabib, Jess and Kazuo Nishimura (1998): "Indeterminacy and Sunspots with Constant Returns to Scale", *Journal of Economic Theory* **81**, 58-96.

[4] Benhabib, Jess, Stephanie Schmitt-Grohé and Martín Uribe (2001): "Monetary Policy and Multiple Equilibria", *American Economic Review* **91**, 167-186.

[5] Benhabib, Jess, Stephanie Schmitt-Grohé and Martín Uribe (2003): "Backward-Looking Interest-Rate Rules, Interest Rate Smoothing and Macroeconomic Instability", mimeographed.

[6] Benhabib, Jess, and Yi Wen (2002): "Indeterminacy, Aggregate Demand, and the Real Business Cycle", mimeographed.

[7] Burnside, Craig, Martin Eichenbaum and Sergio Rebelo (1995): "Capital Utilization and Returns to Scale", in: Bernanke, B., Rotemberg J. (Eds.) NBER Macroeconomics Annual 1995, MIT Press, Cambridge, 67-100.

[8] Caballero, Ricardo and Richard Lyons (1992): "External Effects in U.S. Productivity", *Journal of Monetary Economics* **29**, 209-226.

[9] Clarida, Richard, Jordi Galí and Mark Gertler (1998): "Monetary Policy Rules in Practice: Some International Evidence", *European Economic Review* **42**, 1033-1067.

[10] Carlstrom, Charles T. and Timothy S. Fuerst (1999): "Forecasts and sunspots: looking back for a better future", *Federal Reserve Bank of Cleveland Economic Commentary*, 1999.

[11] Carlstrom, Charles T. and Timothy S. Fuerst (2000): "Forward-Looking Versus Backward-Looking Taylor Rules", Federal Reserve Bank of Cleveland Working Paper #00-09.

[12] Christiano, Lawrence J. and Sharon G. Harrison (1999): "Chaos, Sunspots and Automatic Stabilizers", *Journal of Monetary Economics* **44**, 3-31.

[13] Cole, Harold L. and Lee E. Ohanian (1999): "Aggregate Returns to Scale: Why Measurement is Imprecise", *Federal Reserve Bank of Minneapolis Quarterly Review* **23**, 19-28.

[14] Giannoni, Marc and Michael Woodford (2002): "Optimal Interest Rate Rules: II. Applications", mimeographed.

[15] Guo, Jang-Ting and Kevin J. Lansing (1998): "Indeterminacy and Stabilization Policy", *Journal of Economic Theory* **82**, 481-490.

[16] Farmer, Roger E. A. and Jang-Ting Guo (1994): "Real Business Cycles and the Animal Spirits Hypothesis", *Journal of Economic Theory* **63**, 42-73.

[17] Harrison, Sharon G. and Mark Weder (2002): "Tracing Externalities as Sources of Indeterminacy", *Journal of Economic Dynamics and Control* **26**, 851-867.

[18] King, Robert G. (2000): "The New IS-LM Model: Language, Logic and Limits", *Federal Reserve Bank of Richmond Economic Quarterly*, 45-103

[19] Perli, Roberto (1998): "Indeterminacy, Non-Market Production and the Business Cycle: a Calibrated Analysis", *Journal of Monetary Economics* **41**, 105-125.

[20] Rotemberg, Julio and Michael Woodford (1999): "Interest- Rules in an Estimated Sticky-Price Model", in John B. Taylor (ed.) *Monetary Policy Rules*, University of Chicago Press, Chicago.

[21] Schmitt-Grohé, Stephanie and Martin Uribe (1997): "Balanced Budget Rules, Distortionary Taxes, and Aggregate Instability", *Journal of Political Economy* **105**, 976-1000.

[22] Taylor, John B. (1993): "Discretion versus Policy Rules in Practice", *Carnegie-Rochester Conference Series on Public Policy* **39**, 195-214.

[23] Walsh, Carl E. (2002): "Labor Market Search, Sticky Prices, and Interest Rate Policies", mimeographed.

[24] Weder, Mark (1998): "Fickle Consumers, Durable Goods and Business Cycles", *Journal of Economic Theory* **81**, 37-57.

[25] Weder, Mark (2001): "Indeterminacy in a Small Open Economy Ramsey Growth Model", *Journal of Economic Theory* **98**, 339-356.

[26] Wen, Yi (1998): "Capacity Utilization under Increasing Returns to Scale", *Journal of Economic Theory* **81**, 7-36.

5 Appendix

The unique steady state is given by

$$\begin{aligned}
\eta\frac{c}{y} &= \frac{\alpha}{lR} \\
u^{\theta} &= \alpha\frac{y}{k} \\
\theta\delta &= 1/\beta - 1 + \delta \\
\beta R &= \pi \\
1 &= \beta[\alpha\frac{y}{k} + 1 - \delta] \\
\delta &= \frac{y}{k} - \frac{c}{k}.
\end{aligned}$$

Calibrating η, α, δ, β, l determines c/y, u, y/k, θ, R and π. Yet, not all these are needed in the approximated version of the model with is the collection of seven equations

$$\widehat{y}_t = \alpha(1+\gamma)\widehat{u}_t + \alpha(1+\gamma)\widehat{k}_t + (1-\alpha)(1+\gamma)\widehat{l}_t$$

$$\widehat{l}_t = \widehat{y}_t - \widehat{c}_t + \widehat{R}_t$$

$$\theta\widehat{u}_t = \widehat{y}_t - \widehat{k}_t$$

$$\begin{gathered}
-\widehat{c}_t - \widehat{R}_t \\
= -E_t(\widehat{c}_{t+1} + \widehat{R}_{t+1}) + \alpha\beta\frac{y}{k}\left[E_t\widehat{y}_{t+1} - \widehat{k}_{t+1}\right] - \beta\delta E_t\theta\widehat{u}_{t+1}
\end{gathered}$$

$$\widehat{k}_{t+1} = (1-\delta)\widehat{k}_t - \delta\theta\widehat{u}_t + \frac{y}{k}\widehat{y}_t - \frac{c}{k}\widehat{c}_t$$

$$\widehat{R}_t + \widehat{c}_t = E_t\widehat{\pi}_{t+1} + E_t\widehat{c}_{t+1}$$

The backward-oriented rule

$$\widehat{R}_{t+1} = \tau\widehat{\pi}_t + \omega\widehat{y}_t$$

together with the Fisher-equation implies

$$\widehat{R}_t + \widehat{c}_t = \frac{1}{\tau}\widehat{R}_{t+2} - \frac{\omega}{\tau}\widehat{y}_{t+1} + E_t\widehat{c}_{t+1}.$$

The linear model can then be reduced to (from the first three equations)

$$\begin{bmatrix} \widehat{l}_t \\ \widehat{y}_t \\ \widehat{u}_t \end{bmatrix} = \mathbf{R} \begin{bmatrix} \widehat{c}_t \\ \widehat{R}_t \\ \widehat{k}_t \\ \widehat{R}_{t+1} \end{bmatrix}$$

and

$$\mathbf{M}_1 \begin{bmatrix} E_t\widehat{c}_{t+1} \\ \widehat{R}_{t+1} \\ \widehat{k}_{t+1} \\ E_t\widehat{R}_{t+2} \end{bmatrix} + \mathbf{M}_2 \begin{bmatrix} E_t\widehat{l}_{t+1} \\ E_t\widehat{y}_{t+1} \\ E_t\widehat{u}_{t+1} \end{bmatrix} = \mathbf{M}_3 \begin{bmatrix} \widehat{c}_t \\ \widehat{R}_t \\ \widehat{k}_t \\ \widehat{R}_{t+1} \end{bmatrix} + \mathbf{M}_4 \begin{bmatrix} \widehat{l}_t \\ \widehat{y}_t \\ \widehat{u}_t \end{bmatrix}$$

Eliminating the "$\widehat{l}$-vectors" yields

$$\begin{bmatrix} E_t\widehat{c}_{t+1} \\ \widehat{R}_{t+1} \\ \widehat{k}_{t+1} \\ E_t\widehat{R}_{t+2} \end{bmatrix} = \mathbf{M} \begin{bmatrix} \widehat{c}_t \\ \widehat{R}_t \\ \widehat{k}_t \\ \widehat{R}_{t+1} \end{bmatrix}$$

where

$$\mathbf{M} \equiv [\mathbf{M}_1 + \mathbf{M}_2\mathbf{R}]^{-1} [\mathbf{M}_3 + \mathbf{M}_4\mathbf{R}]$$

and $\mathbf{M}$ is 4×4. Forward-looking rules

$$\widehat{R}_t = \tau E_t\widehat{\pi}_{t+1} + \omega E_t\widehat{y}_{t+1}$$

imply the Fisher-equation

$$\widehat{R}_t + \widehat{c}_t = \frac{1}{\tau}\widehat{R}_t - \frac{\omega}{\tau}\widehat{y}_{t+1} + E_t\widehat{c}_{t+1}.$$

Dynamics are given by

$$\begin{bmatrix} E_t\widehat{c}_{t+1} \\ E_t\widehat{R}_{t+1} \\ \widehat{k}_{t+1} \end{bmatrix} = \mathbf{J} \begin{bmatrix} \widehat{c}_t \\ \widehat{R}_t \\ \widehat{k}_t \end{bmatrix}.$$

The matrix $\mathbf{J}$ is 3×3. A current looking rule

$$\widehat{R}_t = \tau\widehat{\pi}_t + \omega\widehat{y}_t$$

implies the Fisher-equation

$$\widehat{R}_t + \widehat{c}_t = \frac{1}{\tau}\widehat{R}_{t+1} - \frac{\omega}{\tau}\widehat{y}_{t+1} + E_t\widehat{c}_{t+1}$$

and the model dynamics reduce to

$$\begin{bmatrix} E_t\widehat{c}_{t+1} \\ E_t\widehat{R}_{t+1} \\ \widehat{k}_{t+1} \end{bmatrix} = \mathbf{W} \begin{bmatrix} \widehat{c}_t \\ \widehat{R}_t \\ \widehat{k}_t \end{bmatrix}.$$

$\mathbf{W}$ is 3×3. Finally, the hybrid rule

$$\widehat{R}_t = \rho\widehat{R}_{t-1} + \tau E_t\widehat{\pi}_{t+1} + \omega\widehat{y}_t$$

implies

$$\begin{bmatrix} E_t\widehat{c}_{t+1} \\ E_t\widehat{R}_{t+1} \\ \widehat{k}_{t+1} \\ \widehat{R}_t \end{bmatrix} = \mathbf{P} \begin{bmatrix} \widehat{c}_t \\ \widehat{R}_t \\ \widehat{k}_t \\ \widehat{R}_{t-1} \end{bmatrix}.$$

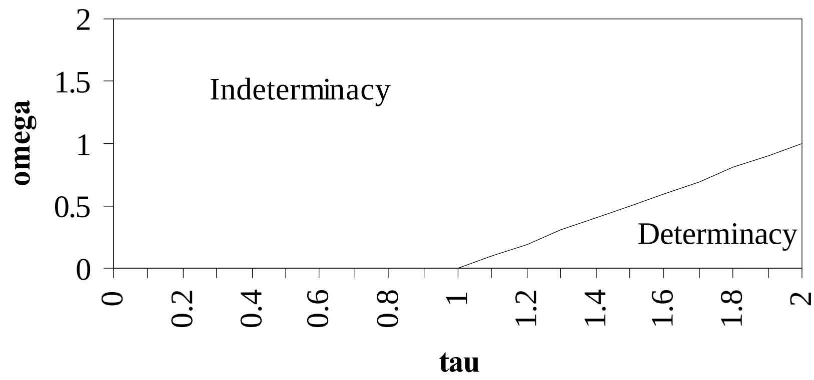

Figure 1: Regions of indeterminacy: the CRS-case

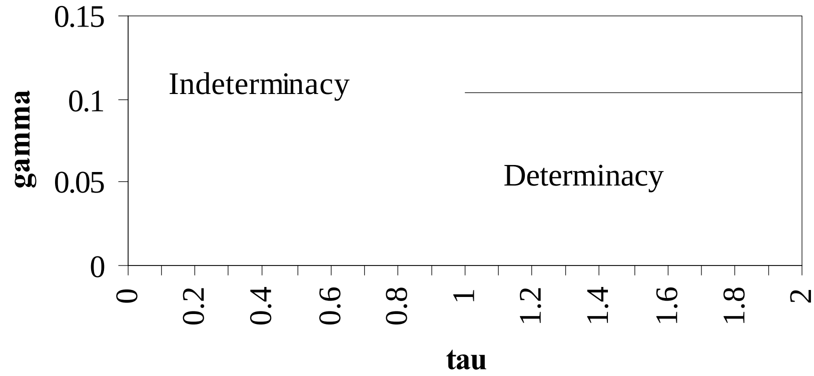

Figure 2: Regions of indeterminacy: minimum IRS

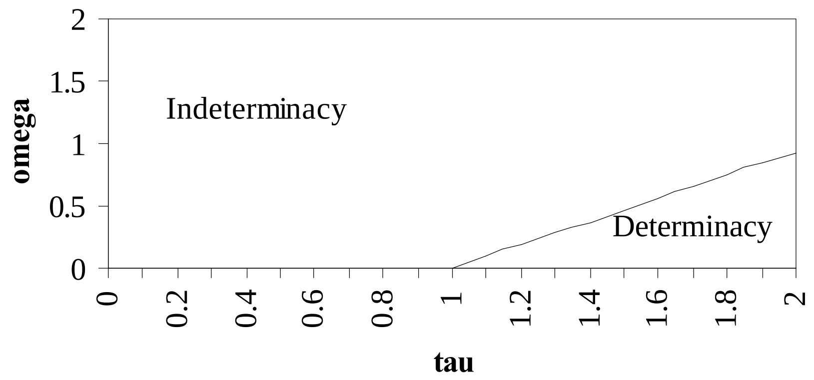

Figure 3: Regions of indeterminacy: the IRS-case I

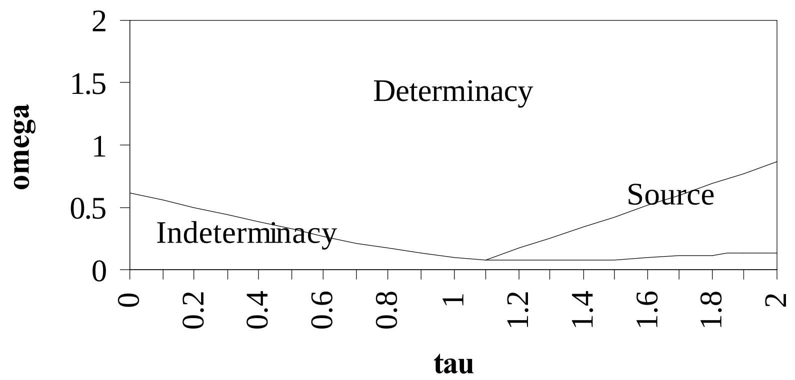

Figure 4: Regions of indeterminacy: the IRS-case II

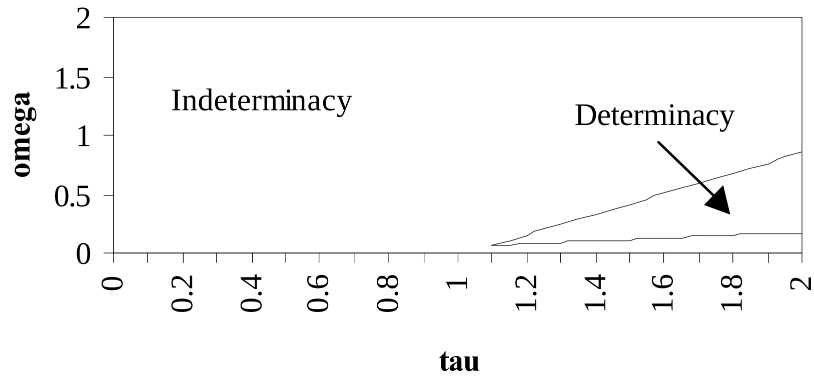

Figure 5: Regions of indeterminacy: forward-looking rules

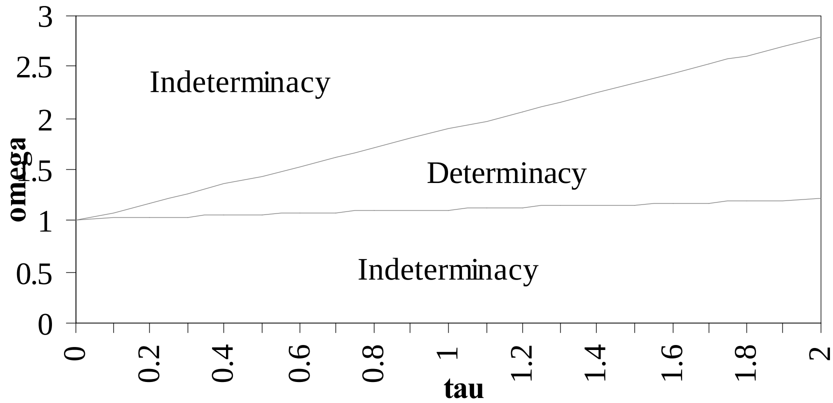

Figure 6: Indeterminacy regions: current-looking rules