Model Averaging and Value-at-Risk based Evaluation of Large Multi Asset Volatility Models for Risk Management*

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Abstract

This paper considers the problem of model uncertainty in the case of multi-asset volatility models and argues in favour of the use of Bayesian-type model averaging techniques to deal with the risk of inadvertently using false models in portfolio management. In particular, it is shown that portfolio returns based on an average of Gaussian models with different volatilities is more fat-tailed than an individual Gaussain model with the same average volatility measure. Evaluation of volatility models is also considered and a simple Value-at-Risk (VaR) diagnostic test is proposed for individual as well as 'average' models and its exact and asymptotic properties are established. The model averaging idea and the VaR diagnostic tests are illustrated by an application to portfolios of daily returns based on twenty two of Standard & Poor's 500 industry group indices over the period January 2, 1995 to October 13, 2003, inclusive.

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1 Introduction

Multivariate models of conditional volatility are of crucial importance for optimal asset allocation, risk management, derivative pricing and dynamic hedging. Yet there are few published empirical studies of the performance of multivariate volatility models as applied to portfolios with a relatively large number of assets. Although, many alternative parametric and semi-parametric multivariate volatility models have been advanced in the academic literature, until recently most have been limited in the number of assets that they can handle. In an attempt to provide operationally feasible volatility models for the analysis of portfolios with a large number of assets, many investigators (in financial markets as well as in academia) have focused on highly restricted versions of multivariate generalized autoregressive conditional heteroscedastic (GARCH) specification. These include the conditionally constant correlation (CCC) model of Bollerslev (1990), the Riskmetrics specifications popularized by J.P.Morgan/Reuters (1996), and used predominantly by practitioners, the orthogonal GARCH model of Alexander (2001), and more the dynamic conditional correlation (DCC) advanced by Engle (2002). Multivariate volatility models have also been considered in the stochastic volatility (SV) literature. See Ghysels, Harvey, and Renault (1995) and Shephard (2004) for reviews. However, so far the focus of this literature has been on univariate and multivariate models with a small number of assets, with the notable exceptions of Diebold and Nerlove (1989), Engle, Ng, and Rothschild (1990), King, Sentana, and Wadhwani (1994) and Harvey, Ruiz, and Shephard (1994). These models are similar in structure to a class of factor orthogonal GARCH model discussed below.

The highly restricted nature of the multivariate volatility models could present a high degree of model uncertainty which ought to be recognized at the outset. This is particularly important since due to data limitations and operational considerations it is not possible to subject these models to rigorous statistical testing either. Application of model selection procedures also face additional difficulties when the number of assets is moderately large, and might very well be that no single model choice would be satisfactory in practice and could carry risks that are difficult to asses *a priori*.

In this paper we advocate the use of model averaging in order to minimize the risk associated with model uncertainty. We apply recent developments in model evaluation and model averaging techniques to multi-asset volatility models, in the case where the number of assets under consideration is relatively large. Based on this framework, we develop simple criteria for evaluation of alternative volatility forecasts by examining the Value-at-Risk (VaR) performance of their associated portfolios over an evaluation period. The approach is general and can be applied to strategic asset allocation problems that require volatility forecasts over relatively long periods as well as more traditional VaR problems with horizons ranging from a single day to a month. Our method is relevant both for evaluation of existing portfolios as well as for the construction of efficient portfolios.

The probability forecast combination approach also attempts to avoid the pre-testing problem associated with the standard two-stage procedure where the decision problem is based on a probability model selected as the 'best' from a given set of candidate models according to a suitable criteria. Frequently used selection criteria are Akaike Information Criteria (AIC) and the Schwartz Bayesian Information Criteria (SBC). However, such a two-step procedure is subject to the pre-test (selection) bias problem and tends to under-estimate the uncertainty that surrounds the forecast. Second, usually point forecasts are analyzed and the evaluation of their out-of-sample performance is based on standard metrics such as root mean square forecast errors (RMSFE). However, this approach runs into difficulties when considering volatility models. In fact, volatility is not directly observable and is often proxied by square of daily returns or more recently by the standard error of daily returns using intra-daily observations, known as realized volatility (see, for example, Andersen, Bollerslev, Diebold, and Labys (2003)). In multi-asset contexts the use of standard metrics such as RMSFE is further complicated by the need to select weights to be attached to different types of errors in forecasts of individual asset volatilities and their cross-volatility correlations and choice of such weights is not innocuous in a multivariate framework (see Pesaran and Skouras (2002)).

The use of model averaging techniques in econometrics is not new and dates back to the seminal work of Granger and Newbold (1977) on forecast combination.[1] However, this literature focusses on combining point forecasts and does not address the problem of combining forecast probability distribution functions which is relevant in the risk management literature. The problem is not trivial even in the simplest case of average of normal distributions with different parameters since the average model will not have a normal distribution, except when for the degenerate case of zero-one coefficients.

From a methodological perspective, following Granger and Pesaran (2000b), our approach aims to represent a more unified treatment of the empirical portfolio analysis from a decision-theoretic perspective rather than from a merely statistical one. Granger and Pesaran (2000a) clarify the importance of concentrating on probability forecasts rather than on just event forecasts. Within a risk-management perspective the motivations for doing so appear even stronger since the ultimate goal is not simply to finding the best approximating volatility model but how to best approximate the entire predictive density of asset returns, or at least its tail behaviour.

The remainder of the paper is organised as follows: the decision problem that underlies the VaR analysis and the associated diagnostic tests is set out in Section 2. Section 3 provides a brief outline of the different types of multivariate volatility models considered in the paper. Bayesian and non-Bayesian

[1]For reviews of the forecast combination literature see Clemen (1989), Granger (1989), Diebold and Lopez (1996) and Hendry and Clements (2002).

approaches to model averaging are reviewed and discussed in Section 4. Section 5 introduces a simple Value-at-Risk (VaR) diagnostic test and establishes its distribution. Section 6 provides a detailed empirical analysis using daily returns on twenty two of Standard and Poor's 500 industry indices over the period January 2 1995 to October 13 2003. Section 7 concludes with a summary of the main results and provides suggestions for future research. The mathematical proofs are collected in the Appendix.

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