

# Theoretical moment restrictions of commodity prices

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## Abstract

This model studies agricultural commodity spot and futures markets. Noise trading, speculation, basis risk and hedging activities are included. Evidence from empirical studies using U.S. rough rice prices shows that the spot and futures prices at maturity do not have to be equal. Therefore, The model allows agents to close none, some or all of their futures position before they realise their production or storage uncertainties. Many researches have derived the variance bounds of stock prices from a relationship between the stock price and a forecast of the stock price- that is, the actual stock price is equal to the forecast of the stock price if the stock market is efficient. As there are many different specifications between the stock market and agricultural commodity futures market, we obtain different variance bounds. Applying a two-period mean-variance approach, we can specify the agents' optimal strategies and the equilibrium prices in the spot and futures markets. From these, we show that there are moment restrictions of prices derived from the relationship between equilibrium spot and futures prices. This derivation is different from the derivation of the variance bounds of stock prices. We find that the variance of commodity spot prices can be smaller than that of commodity futures prices. Also, the covariance between spot and futures prices can lie between the variances of spot and futures prices. This matches what we find in some commodity and financial markets. Finally, we also derive other moment restrictions of intermediate, final commodity prices and noise trading in this paper.

keywords: moment restrictions of commodity prices, mean-variance approach, optimal strategies

Many researches have tried to specify the variance bound of stock prices. Most moment restrictions are derived from a basic relationship between the stock price ( $P$ ) and a forecast of the stock price ( $P^*$ ) derived from a theoretical model - that is, the actual stock price is equal to the forecast of the stock price if the stock market is efficient. Shiller (1981) showed that if (from the market efficiency hypothesis) the actual price is equal to its forecast plus an error term, the variance of the forecast should be less than that of the actual ( $\sigma_{P^*}^2 \leq \sigma_P^2$ ). However, Tirole (1985) and Grossman and Shiller (1981) showed that short-term variances violated this variance bound. Bradford De Long and Becht (1992) also found excess volatility in the German stock market. Kleidon (1986) claimed that the incorrect application of estimation techniques that assume stationarity to nonstationary series caused rejection of the variance bound hypothesis. He derived a conditional variance bound ( $\sigma_{P^*|I_{t-k}}^2 \leq \sigma_{P|I_{t-k}}^2$  where  $I_{t-k}$  denotes the information set at  $t - k$ )<sup>1</sup>. He explained that if changes in current dividends implied changes in expected dividends for the infinite future, the price would change by the present value of these revisions. Since  $P_t^*$  was already calculated using the expost dividend series, changes in current dividends implied no new information and thus no unexpected changes in  $P_t^*$ .

Unlike the other studies, this paper derives the moment restrictions of commodity prices in the spot and futures markets at maturity. As there are many different specifications between the stock market and agricultural commodity futures market, we obtain different variance bounds. We can also derive the moment restrictions of other prices relating to the commodity from our framework too.

We study a commodity that is storable, sold by price-taking producers, and subject to intervention by the government. Unlike other studies, here only the final goods produced from the commodity can be traded internationally instead of the commodity itself due to the difficulty and higher cost of its delivery. Another reason is that the exporting country may lose its competitiveness in the world market because the importing country may replicate production by using the commodity as the input (e.g. rice). Many frameworks assume that the commodity spot and futures prices at maturity are equal. However, often this does not happen in empirically. Another assumption that causes many frameworks (i.e. the models of Anderson and Danthine (1983) and Antoniou (1986)<sup>2</sup>) to end up with the basic relationship between spot and futures prices is that the agents will close all of their futures position at maturity. In fact, the open interest on the last trading day is not always equal to zero. Also, trading volume is not equal or does not tend to zero when there is a significant gap between spot and futures prices at maturity.

For example, in the case of U.S. rough rice futures market, contract months<sup>3</sup> are September, November, January, March, May and July. Normally, there are at least 4-5 contracts traded simultaneously every weekday. The futures contract usually will be issued 12 or 13 months before the spot month. For instance, the futures contract which matures in March 2004 will be issued in March 2003. The monthly futures price is the futures rough rice price quoted in the futures contract with the shortest maturity at the end of the month<sup>4</sup>. The last trading day is the seventh business day preceding the last business day of the delivery month. The last delivery day is the last business day of the delivery month.

The graphs of the daily futures prices of the rough rice futures contract<sup>5</sup> maturing in September 2002 and the defined daily U.S. rough rice cash price<sup>6</sup> are shown in figure 1. The trading volume of this futures contract during 1st February 2000 - 19th September 2000 is shown in figure 2. Apparently, the trading volume in the last trading week tends to be zero (the average trading volume in the last 3 trading days is 0.33 transactions per day) as the futures price tends to be equal to the cash price at the maturity. Figures 3 and 4 show the graphs of the adjusted daily U.S. cash price, the futures prices of the rough rice futures contract maturing in March 2003 and its trading volume in the futures market. For this futures contract, there is non-zero trading volume in the last trading days (the average trading volume in the last 3 trading days is 27.33 transactions per day). This is associated with a non-zero gap between the futures and spot prices at the maturity. From these, we see that spot and futures prices in some markets do not coincide at maturity.

Thus, this framework assumes that the spot and futures prices at the maturity do not have to be equal. This allows the agents to close none, some or all of their futures position before they realise their production or storage uncertainties. Applying a two-period mean-variance approach, we specify the agents' optimal strategies in the spot and futures markets in Section 1.

In Section 2, equilibrium prices are derived from the optimal decisions of the agents. The moment restrictions of prices are derived from the relationship between spot and futures prices in Section 3. We find that the variance of spot prices is smaller than that of futures prices. Also, the covariance between spot and futures prices is smaller than the variance of the futures price but greater than the variance of spot price. This is what we find in some commodity and financial markets. We also find the relationship between

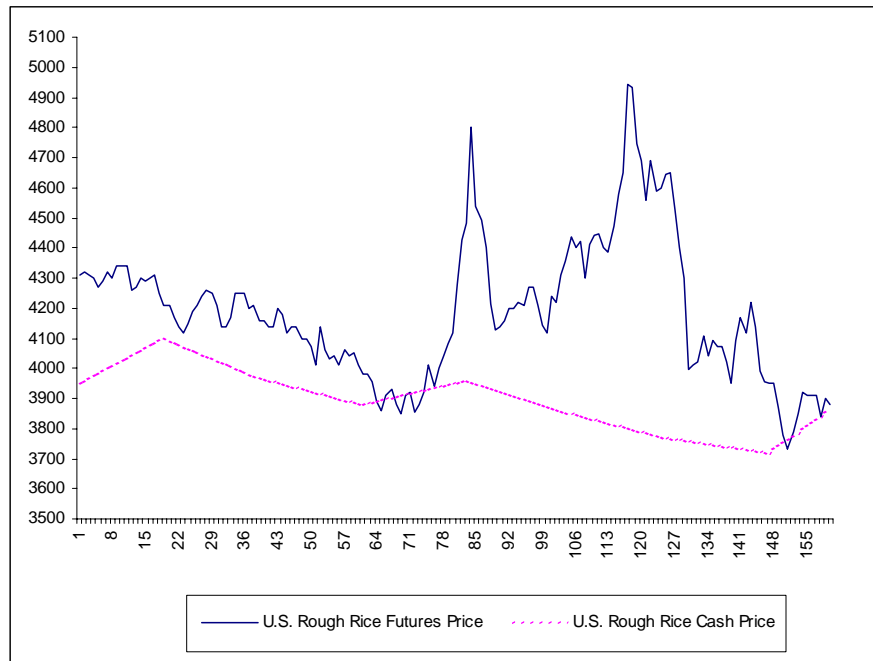


Figure 1: Daily rough rice cash and futures prices of the rough rice futures contract maturing in September 2002

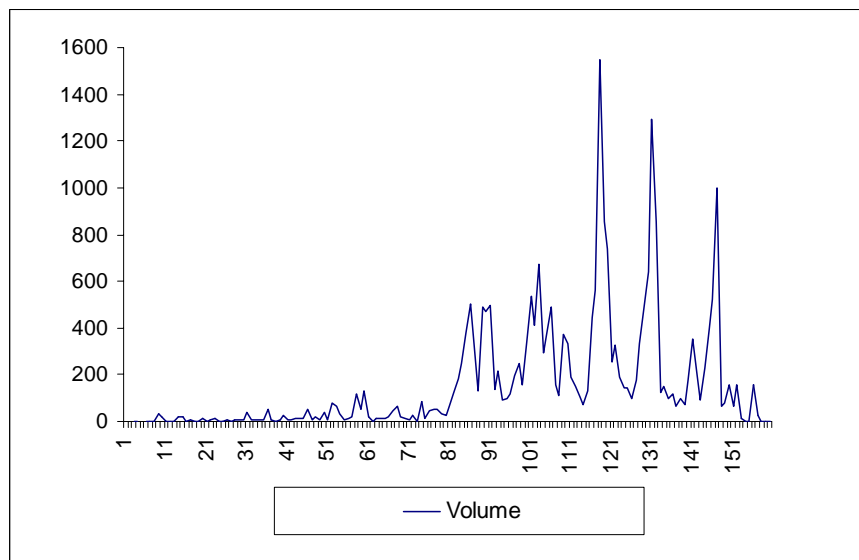


Figure 2: The trading volume of the rough rice futures contract maturing in September 2002

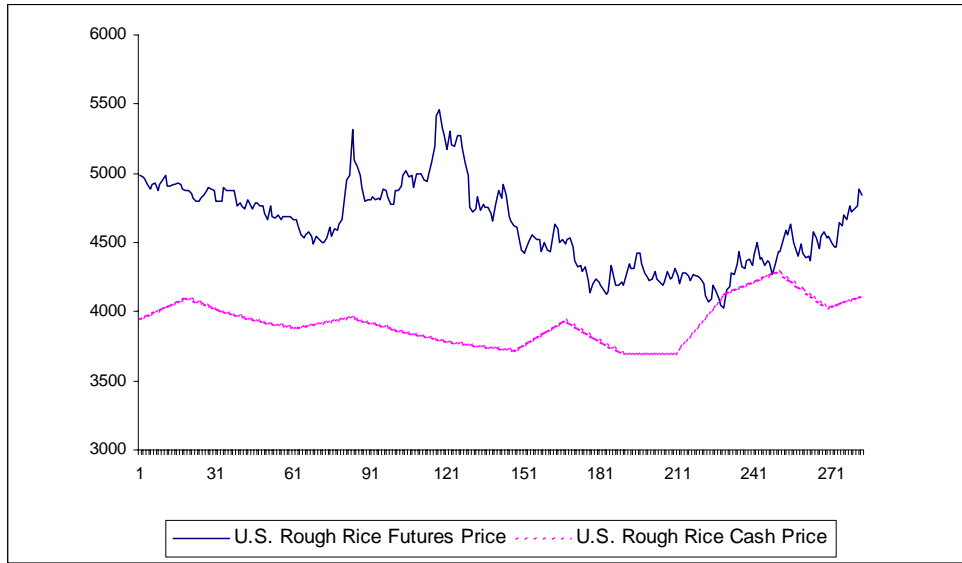


Figure 3: Daily rough rice cash and futures prices of the rough rice futures contract maturing in March 2003

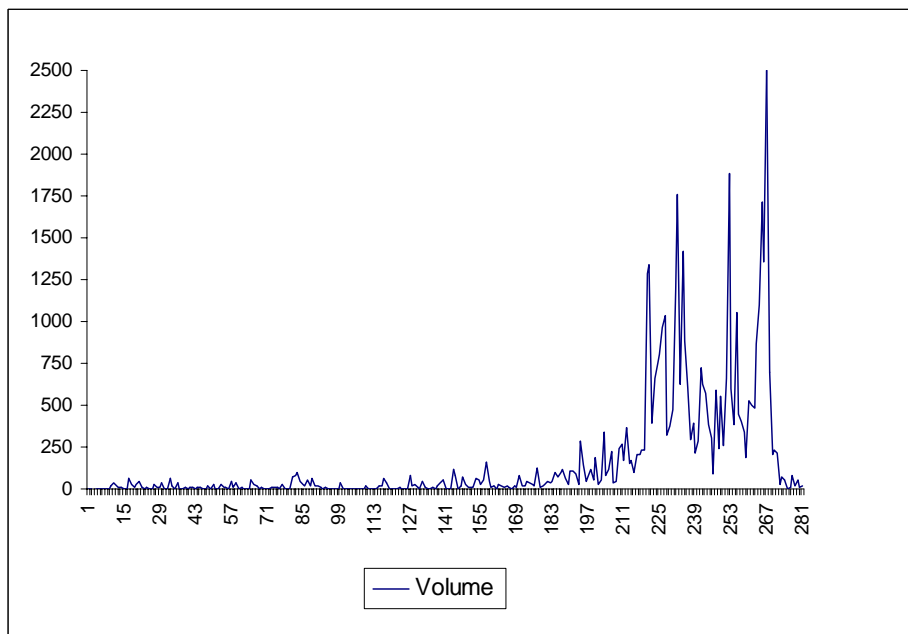


Figure 4: The trading volume of the rough rice futures contract maturing in March 2003

the covariance between intermediate and final commodity prices and the variance of final commodity prices. Finally, the covariance between spot/futures price and noise trading in the futures market is also derived.

## 1 Optimal strategies

There are 4 types of traders in this model: farmers, processors, storage companies and speculators. All traders are assumed to be small traders in both spot and futures markets and to choose their optimal decisions by maximising their expected utility functions ( $V$ ) which are of a mean-variance form. In this framework, there are 2 periods:  $t$  and  $t + 1$ . An assumption is that all agents can hedge or speculate in the futures market by selling or purchasing the futures contract at time  $t$  with a full margin. In period  $t$ , all stochastic variables of time  $t + 1$  are unknown (i.e. spot and futures prices ( $P_{t+1}$  and  $F_{t+1}$ )). The maturity of the futures contract is at  $t + 1$ . Any variables such as the optimal spot and futures positions at time  $t$  chosen by agent  $i$  at time  $t$  depend on his own information set available at time  $t$  ( $I_{it}$ ) as well as on the expectations and variances of variables of period  $t + 1$ ;  $i = f$  for the farmers,  $i = p$  for the processors,  $i = st$  for the storage companies,  $i = s$  for the speculators. Another assumption is that the agents can revise their expectation; thus, there will be some decisions made at time  $t$  and some made at time  $t + 1$ .  $E_{it}$  denotes agent  $i$ 's expectation depending on  $I_{it}$  ( $I_{it} \subset I_{it+1}$ )

In the model,  $X_{it}$  denotes the position of the agent type  $i$  in the primary good at time  $t$  and similarly for  $Y_{it}$  (intermediate good),  $H_{it}$  (final good),  $Y_{it}^f$  (futures contract for  $Y$ ) and  $Y_{it+1}^{fc}$  (the number of futures contract for  $Y$  which is held until the end of time  $t + 1$ ).  $\Pi_i$  is the revenue of agent type  $i$ .  $Var_{it}(\cdot)$  denotes agent  $i$ 's expected variances of the variables depending on  $I_{it}$ .

When the agent  $i$  clears all of his futures position at time  $t + 1$ ,  $Y_{it+1}^{fc} = 0$ . If he holds all of his futures position until the maturity,  $Y_{it+1}^{fc} = Y_{it}^f$ . At the beginning of time  $t + 1$ , the agent chooses the optimal  $Y_{it+1}^{fc}$  when he knows  $P_{t+1}$  and  $F_{t+1}$  but does not know the production and storage uncertainties. There is a dealing cost for closing the futures position,  $C(Y_{it}^f - Y_{it+1}^{fc})^2$ . This allows the model to examine the effects of the basis risk on the agent's decision and also to rule out corner solutions. Agents deliver/take delivery of the intermediate good to/from the futures market after farmers realise their production shock and storage companies realise their storage uncertainty.

### 1.1 Farmer

Farmers can buy grain to plant at time  $t$ . They sell their output in the spot market or deliver it to the futures market at time  $t + 1$ . At the beginning of period  $t + 1$ , they know  $P_{t+1}$  and  $F_{t+1}$  and choose how many contracts they would like to buy back from the futures market,  $(Y_{ft}^f - Y_{ft+1}^{fc})$ . Their production shock ( $\epsilon_{t+1}$ ) is realised just before delivering (selling) their output to the futures (spot) market.

	t	t+1	t+1	
	$r_t, F_t, i_{t+1}$	$P_{t+1}, F_{t+1}$	$\epsilon_{t+1}$	
Spot	buy $X_{ft}$	sell $E_t(Y_{ft+1}) - Y_{ft+1}^{fc}$	produce $Y_{ft+1}$	deliver $Y_{ft+1} - Y_{ft+1}^{fc}$
Futures	short $Y_{ft}^f$	buy $Y_{ft}^f - Y_{ft+1}^{fc}$ back		deliver $Y_{ft+1}^{fc}$

Let  $Y_{ft+1} = f(X_{ft}, \epsilon_{t+1})$  and  $r_t$  be the competitive price of seeds  $X_{ft}$  at time  $t$ ; the production of  $Y_{ft+1}$  takes one period from  $t$  to  $t + 1$ . At time  $t$ , the farmer faces uncertainty of the future spot price, when the crop is harvested and sold, and production uncertainty with  $E_{ft}(\epsilon_{t+1}) = 0$  and  $Var_{ft}(\epsilon_{t+1}) = \sigma_\epsilon^2$ .  $\theta X_{ft}^2$  is the production cost excluding the cost of seeds. His profit function is

$$\begin{aligned} \Pi_f = & -r_t X_{ft} - \theta X_{ft}^2 + Y_{ft}^f F_t + \rho \{ P_{t+1} (f(X_{ft}, \epsilon_{t+1}) - Y_{ft+1}^{fc}) \\ & - (Y_{ft}^f - Y_{ft+1}^{fc}) F_{t+1} - C(Y_{ft}^f - Y_{ft+1}^{fc})^2 \} \end{aligned} \quad (1)$$

where  $\rho$  is the discount rate assumed to be equal to  $1/(1 + i_{t+1})$ ;  $i_{t+1}$  is the interest rate at time  $t + 1$  and perfectly foreseen at time  $t$ . For simplicity, we assume that the production has constant returns to scale and additive production shock,  $f(X_{ft}, \epsilon_{t+1}) = bX_{ft} + \epsilon_{t+1}$ .

## 1.2 Processor

Processors can buy the intermediate good from the spot market or take delivery from the futures market at time  $t + 1$  to produce the final goods within the same period. They can close some or all of their futures position at the beginning of period  $t + 1$  (by the last trading day in the maturity month). They can sell their final goods in either the domestic or foreign spot market. Unlike other frameworks, processors can only import or export the final good processed from the commodity. Assume that there is only a futures market for the intermediate good. For example, the Chicago Board of Trade (CBOT) trades rough rice futures contracts, not milled rice futures contracts. Following the empirical trading, processors precommit with importers (exporters) to export (import) final goods at a fixed price  $Q_{mt+1}$  at time  $t$  because international trading normally needs more time for preparation and delivery and has a higher cost than domestic trading.  $Q_{mt+1}$  represents the foreign price of the final good at time  $t + 1$ , which is known at time  $t$ . Their production shock is realised after they choose the optimal amounts of final good to produce and to sell in the domestic and foreign markets.

	t	t+1		
	$Q_{mt+1}, F_t, i_{t+1}$	$P_{t+1}, F_{t+1}$	$\nu_{t+1}$	
Spot		buy $Y_{pt+1}$	produce $H_{pt+1}$	sell $H_{pt+1} - Y_{pmt+1}$
Forward	contracts for $Y_{pmt+1}$			deliver $Y_{pmt+1}$
Futures	long $Y_{pt}^f$	sell $Y_{pt}^f - Y_{pt+1}^{fc}$	take the delivery $Y_{pt+1}^{fc}$	

The processor's profit function is

$$\begin{aligned} \Pi_p = & -Y_{pt}^f F_t + \rho \{ Q_{t+1} (H_{pt+1} - Y_{pmt+1}) - P_{t+1} Y_{pt+1} + Q_{mt+1} Y_{pmt+1} (1 + \tau) \\ & - \alpha (Y_{pt+1} + Y_{pt+1}^{fc})^2 + (Y_{pt}^f - Y_{pt+1}^{fc}) F_{t+1} - C(Y_{pt}^f - Y_{pt+1}^{fc})^2 \}. \end{aligned} \quad (2)$$

where the production function of  $H_{pt+1}$  is  $a(Y_{pt+1} + Y_{pt+1}^{fc}) + \nu_{t+1}$  with the assumption that there is a constant returns to scale production with an additive random shock ( $\nu_{t+1}$ ).  $E_t(\nu_{t+1}) = 0$  and  $Var_t(\nu_{t+1}) = \sigma_\nu^2$ .  $Y_{pmt+1}$  is the net export of final good. A positive (negative)  $\tau$  is the import tariff rate (the export premium).  $\alpha(Y_{pt+1} + Y_{pt+1}^{fc})^2$  is the production cost of other inputs which depends on the amount of input and  $0 < \alpha < 1$ .

### 1.3 Storage companies

In the spot market, the storage company's manager can buy the intermediate commodity at time  $t$ , store it until time  $t+1$  with the storage cost  $cY_{st,t}^2 + u_{t+1}$  where  $0 < c < 1$ , and then sell his stock at time  $t + 1$ . He also can trade the futures contract at time  $t$ , close some or all of his futures position and deliver or take the delivery of the intermediate commodity for the contract held until the maturity at time  $t + 1$ ; this depends on  $P_{t+1}$  and  $F_{t+1}$ .

	t	t+1	t+1
	$P_t, F_t, i_{t+1}$	$P_{t+1}, F_{t+1}$	$u_{t+1}$
Spot	buy $Y_{st,t}$	sell $Y_{st,t} - Y_{st,t+1}^{fc}$	deliver $Y_{st,t} - Y_{st,t+1}^{fc}$
Futures	short $Y_{st,t}^f$	buy $Y_{st,t}^f - Y_{st,t+1}^{fc}$ back	deliver $Y_{st,t+1}^{fc}$

The company has to choose the optimal strategy by maximising its profit function which is

$$\begin{aligned} \Pi_{st} = & -P_t Y_{st,t} + Y_{st,t}^f F_t + \rho \{ (Y_{st,t} - Y_{st,t+1}^{fc}) P_{t+1} - (Y_{st,t}^f - Y_{st,t+1}^{fc}) F_{t+1} \\ & - C(Y_{st,t}^f - Y_{st,t+1}^{fc})^2 - cY_{st,t}^2 - u_{t+1} \}. \end{aligned} \quad (3)$$

### 1.4 Speculator

Speculators can hold a futures position at time  $t$  and close their futures position by the last trading day of the contract.

	t	t+1	t+1
	$F_t, i_{t+1}$	$F_{t+1}$	
Futures	long $Y_{st}^f$	sell $Y_{st}^f$ back	

As the speculator makes a decision at time  $t$ , he has to maximise his expected utility function at time  $t$  which depends on his profit:

$$\Pi_s = -Y_{st}^f F_t + \rho \{ Y_{st}^f (F_{t+1}) - C Y_{st}^{f2} \}. \quad (4)$$

### 1.5 Summary

Each agent chooses his/her optimal strategy by maximising his/her expected utility based on a mean-variance approach as shown in appendix A. If  $P_{t+1} > F_{t+1}$ , the farmers and storage companies will buy  $Y_{it}^f - Y_{it+1}^{fc*}$  back at time  $t + 1$  so that they can sell this amount of output in the spot market. If  $P_{t+1} < F_{t+1}$ , the processors will sell  $Y_{it}^f - Y_{it+1}^{fc*}$  back at time  $t+1$  so that they can make a profit in the futures market. One reason why  $Y_{it+1}^{fc*} = Y_{it}^f$  even though  $P_{t+1} \neq F_{t+1}$  could be that there may be a very high dealing cost ( $C \rightarrow \infty$ ).

## 2 Equilibrium Solutions

To simplify the solution, there are 2 assumptions. Firstly, the expectations, variances and covariances are heterogeneous by type of agents<sup>7</sup>. With this assumption, define

$$\Gamma_i = Var_{it,t}(P_{t+1}) - Cov_{it,t}(P_{t+1}, F_{t+1}) / CVar_{it,t}(P_{t+1})$$

and

$$\Theta_i = A_i \text{Var}_{it}(P_{t+1})$$

where  $i = f, st$ . Let

$$\Gamma_p = \frac{\text{Cov}(F_{t+1}, Q_{t+1})\text{Cov}_{pt}(P_{t+1}, Q_{t+1}) - \text{Var}_{pt}(Q_{t+1})\text{Cov}_{pt}(P_{t+1}, F_{t+1})}{C [\text{Var}_{pt}(Q_{t+1})\text{Var}_{pt}(P_{t+1}) - \text{Cov}_{pt}^2(P_{t+1}, Q_{t+1})]} + \frac{1}{C}$$

and

$$\Theta_p = A_p \left[ \text{Var}_{pt}(P_{t+1}) - \frac{\text{Cov}_{pt}^2(P_{t+1}, Q_{t+1})}{\text{Var}_{pt}(Q_{t+1})} \right].$$

Also,

$$\Theta_s = A_s \text{Var}_{st}(F_{t+1}).$$

Secondly, assume that information variables and prices are approximately jointly normal so that conditional variances and covariances of prices are constant over time. In some cases,  $r_t$  is the price of seeds which is the intermediate commodity spot price,  $P_t$ .  $n_p$  denotes the number of processors,  $n_f$  denotes the number of farmers,  $n_{st}$  denotes the number of storage companies, and  $n_s$  denotes the number of speculators. From

$$n_f Y_{ft}^{f*} + n_{st} Y_{st,t}^{f*} = n_p Y_{pt}^{f*} + n_s Y_{st}^{f*} + \xi_t, \quad (5)$$

the equilibrium futures price at time  $t$  is

$$\begin{aligned} F_t = & \frac{1}{\left[ \sum_{i=f,p,st} \frac{n_i}{\rho \Theta_i} + n_f \frac{b^2}{\theta} + \frac{n_{st}}{c} + \frac{n_s}{\rho[C+\Theta_s]} \right]} \left\{ \sum_{i=f,p,st} n_i \Gamma_i E_{it}(F_{t+1}) + n_f \frac{P_t b}{\theta} + \frac{n_{st} P_t}{c} \right. \\ & - \sum_{i=f,p,st} n_i \left( \Gamma_i - \frac{1}{\Theta_i} \right) E_{it}(P_{t+1}) [(1+\tau)Q_{mt+1} - E_{pt}(Q_{t+1})a] + \frac{n_s E_{st}(F_{t+1})}{[C+\Theta_s]} \\ & + n_p \left[ \frac{\text{Cov}_{pt}(Q_{t+1}, P_{t+1})}{[\text{Var}_{pt}(Q_{t+1})\text{Var}_{pt}(P_{t+1}) - \text{Cov}_{pt}^2(P_{t+1}, Q_{t+1})]} \right] \\ & + n_p \left[ \frac{a E_{pt}(Q_{t+1})}{\alpha} - \frac{E_{pt}(P_{t+1})}{\alpha} \right] + 2n_f \frac{\text{Cov}_{ft}(P_{t+1}, \epsilon_{t+1})}{\text{Var}_{pt}(P_{t+1})} E_{ft}(P_{t+1}) + 2\xi_t \\ & \left. + \frac{2n_p \text{Cov}_{pt}(Q_{t+1}, \nu_{t+1})\text{Cov}_{pt}(P_{t+1}, Q_{t+1})}{[\text{Var}_{pt}(Q_{t+1})\text{Var}_{pt}(P_{t+1}) - \text{Cov}_{pt}^2(P_{t+1}, Q_{t+1})]} E_{pt}(Q_{t+1}) \right\} \end{aligned} \quad (6)$$

From the agents' optimal decisions made at  $t+1$ , the equilibrium spot and futures price at time  $t+1$  can be obtained from

$$n_p Y_{pdt+1}^* = n_f \left[ f(X_{ft}^*) + \epsilon_{t+1} - Y_{ft+1}^{fc*} \right] + n_{st} \left[ Y_{st,t}^* - Y_{st,t+1}^{fc*} \right] \quad (7)$$

and

$$n_f (Y_{ft}^{f*} - Y_{ft+1}^{fc*}) + n_{st} (Y_{st,t}^{f*} - Y_{st,t+1}^{fc*}) = n_p (Y_{pt}^{f*} - Y_{pt+1}^{fc*}) + n_s (Y_{st}^{f*} - Y_{st+1}^{fc*}) + \xi_{t+1} \quad (8)$$

where  $\xi_t$  is noise trading in the futures market at time  $t$  with zero mean and constant variance. The left-hand side of both conditions represents the demand in the spot and futures markets while the right-hand side represents the supply in the spot and futures markets. From these two market clearing conditions, the relationship between the spot and futures prices at maturity is

$$F_{t+1} = P_{t+1} - \frac{C n_s}{\sum_{i=f,p,st} n_i} \frac{E_{st}(F_{t+1}) - \frac{F_t}{\rho}}{[C + \Theta_s]} - \frac{2C}{\sum_{i=f,p,st} n_i} \xi_{t+1} \quad (9)$$



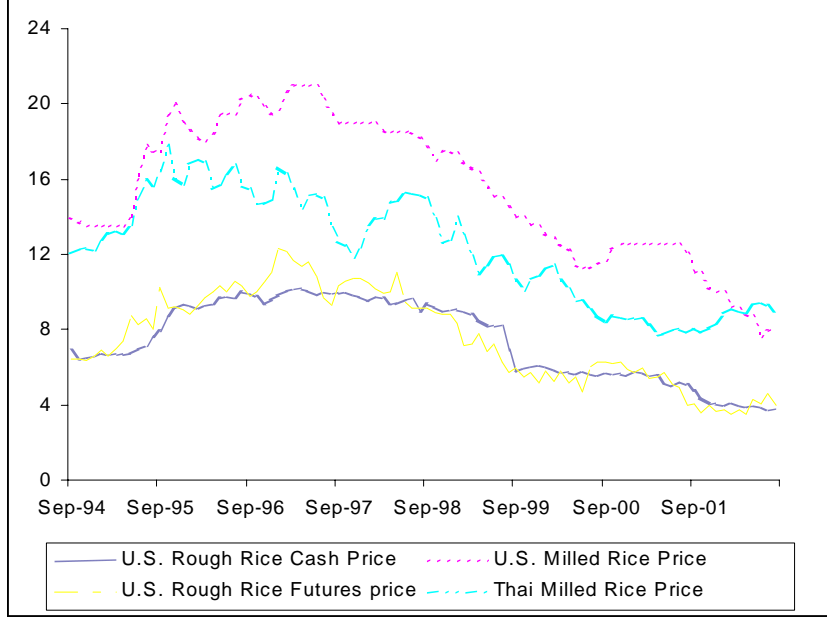


Figure 5: Monthly U.S. rough rice cash and futures prices and monthly U.S. and Thai milled rice prices.

where

$$\Theta_s = A_s \text{Var}_{st}(F_{t+1}).$$

and

$$\begin{aligned}
P_{t+1} = & aQ_{t+1} + \alpha \left[ \frac{\text{Cov}_{pt}(Q_{t+1}, P_{t+1}) [E_{pt}(Q_{t+1})a - (1 + \tau)Q_{mt+1}]}{[\text{Var}_{pt}(Q_{t+1})\text{Var}_{pt}(P_{t+1}) - \text{Cov}_{pt}^2(P_{t+1}, Q_{t+1})]} \right] \\
& - 2 \frac{\alpha \text{Cov}_{pt}(Q_{t+1}, \nu_{t+1}) \text{Cov}_{pt}(Q_{t+1}, P_{t+1}) E_{pt}(Q_{t+1})}{[\text{Var}_{pt}(Q_{t+1})\text{Var}_{pt}(P_{t+1}) - \text{Cov}_{pt}^2(P_{t+1}, Q_{t+1})]} \\
& + [E_{pt}(P_{t+1}) - aE_{pt}(Q_{t+1})] + \frac{\alpha}{n_p} \sum_{i=f,p,st} n_i \frac{F_t}{\rho \Theta_i} - \frac{n_s \alpha E_{st}(F_{t+1}) - \frac{F_t}{\rho}}{[C + \Theta_s]} \\
& - 2 \frac{n_f \alpha \text{Cov}_{ft}(P_{t+1}, \epsilon_{t+1})}{n_p \text{Var}_{pt}(P_{t+1})} E_{ft}(P_{t+1}) - \frac{\alpha}{n_p} \sum_{i=f,p,st} n_i \Gamma_i E_{it}(F_{t+1}) \\
& + \frac{\alpha}{n_p} \sum_{i=f,p,st} n_i (\Gamma_i - \frac{1}{\Theta_i}) E_{it}(P_{t+1}) - 2 \frac{\alpha n_f}{n_p} \epsilon_{t+1} - \frac{2\alpha}{n_p} \xi_{t+1} \tag{10}
\end{aligned}$$

For the empirical study, we use the monthly sequence of spot and futures prices of U.S. rough rice, spot prices of U.S. and Thai milled rice<sup>8</sup>. There are 96 observations covering the period from September 1994 to August 2002. The prices are quoted as U.S. Dollars per 100 cwt<sup>9</sup>. The graphs in figure 5 are the graphs of the monthly U.S. spot rough rice prices ( $P_t$ ), the U.S. and Thai milled rice prices ( $Q_t$  and  $Q_{mt}$ ) and the futures rough rice prices ( $F_t$ ).

Using the Vector Autoregressive (VAR) - Co-integration method, we obtain the normalised co-integrating vector of the equilibrium spot price as follows.

$$P_t = \underset{(0.664)}{-1.012} + \underset{(0.224)}{0.455} F_t - \underset{(0.358)}{0.320} Q_{t-1} + \underset{(0.067)}{0.162} Q_{mt}$$

The values in parentheses are the estimated standard errors. Next, the hypotheses that the coefficient of the futures price is equal to 1 and that the constant term and the coefficient of milled rice prices are equal to 0 are tested.

Table 4: The result of LR test<sup>1</sup>

$H_o$	$\chi^2_{(1)}$	prob.
$a_2 = 0$	0.7353	0.3912
$a_2 = 0, a_1 = 1$	5.6044	0.0607
$a_2 = 0, a_1 = 1, a_0 = 0$	5.6398	0.1305
$a_2 = 0, a_1 = 1, a_0 = 0, a_3 = 0$	11.001	0.0266

The result shows that the constant term and the coefficient of the previous U.S. milled rice price are insignificant whereas the coefficient of Thai milled rice price is significant at 0.05 significance level. Also, the coefficient of the futures price is insignificantly different from 1 at 0.05 significance level. Thus, this result also supports that the spot price depends on not only the futures price but also the final good price.

### 3 Moment Restrictions

Whereas earlier work has derived the variance bound from the relationship between the current price and the forecast from the previous period (the efficient market hypothesis), we derive the moment restrictions from the equilibrium prices at maturity. Define

$$\begin{aligned}
p_{t+1} = & \alpha \left[ \frac{Cov_{pt}(Q_{t+1}, P_{t+1}) [E_{pt}(Q_{t+1})a - (1 + \tau)Q_{mt+1}]}{[Var_{pt}(Q_{t+1})Var_{pt}(P_{t+1}) - Cov_{pt}^2(P_{t+1}, Q_{t+1})]} \right] + \frac{\alpha}{n_p} \sum_{i=f,p,st} n_i \frac{F_t}{\rho \Theta_i} \\
& - 2 \frac{\alpha Cov_{pt}(Q_{t+1}, \nu_{t+1}) Cov_{pt}(Q_{t+1}, P_{t+1}) E_{pt}(Q_{t+1})}{[Var_{pt}(Q_{t+1})Var_{pt}(P_{t+1}) - Cov_{pt}^2(P_{t+1}, Q_{t+1})]} - \frac{n_s \alpha}{n_p} \frac{E_{st}(F_{t+1}) - \frac{F_t}{\rho}}{[C + \Theta_s]} \\
& + [E_{pt}(P_{t+1}) - aE_{pt}(Q_{t+1})] - 2 \frac{n_f \alpha}{n_p} \frac{Cov_{ft}(P_{t+1}, \epsilon_{t+1})}{Var_{pt}(P_{t+1})} E_{ft}(P_{t+1}) \\
& - \frac{\alpha}{n_p} \sum_{i=f,p,st} n_i \Gamma_i E_{it}(F_{t+1}) + \frac{\alpha}{n_p} \sum_{i=f,p,st} n_i \left( \Gamma_i - \frac{1}{\Theta_i} \right) E_{it}(P_{t+1}).
\end{aligned}$$

Because  $Q_{mt+1}$  and the conditional expectations and variances of variables of period  $t+1$  are known at time  $t$ ,  $p_{t+1}$  is non-random at time  $t$  ( $E_t(p_{t+1}) = p_{t+1}$ ). We can rewrite equation (10) as

$$P_{t+1} = p_{t+1} + aQ_{t+1} - 2 \frac{\alpha n_f}{n_p} \epsilon_{t+1} - \frac{2\alpha}{n_p} \xi_{t+1} \quad (11)$$

and thus

$$E_t(P_{t+1}) = E_t(p_{t+1}) + aE_t(Q_{t+1}). \quad (12)$$

As shown in appendix B, we can derive the following moment restrictions for this model .

1.

$$Cov_t(P_{t+1}, \epsilon_{t+1}) = E_t(P_{t+1}\epsilon_{t+1}) - E_t(P_{t+1})E_t(\epsilon_{t+1}) = -2\alpha \frac{n_f}{n_p} Var_t(\epsilon_{t+1})$$

2.

$$Cov_t(P_{t+1}, Q_{t+1}) = E_t(P_{t+1}Q_{t+1}) - E_t(P_{t+1})E_t(Q_{t+1}) = aVar_t(Q_{t+1})$$

---

<sup>1</sup>  $P_t = a_o + a_1 F_t + a_2 Q_{t-1} + a_3 Q_{mt}$

Since  $0 < a < 1$ , the correlation between  $P_{t+1}$  and  $Q_{t+1}$  should be less than 1.

3.

$$Cov_t(P_{t+1}, \xi_{t+1}) = E_t(P_{t+1}\xi_{t+1}) - E_t(P_{t+1})E_t(\xi_{t+1}) = -\frac{2\alpha}{n_p}\sigma_\xi^2$$

4.

$$Cov_t(F_{t+1}, \xi_{t+1}) = E_t(F_{t+1}\xi_{t+1}) - E_t(F_{t+1})E_t(\xi_{t+1}) = -2 \left[ \frac{C}{\sum_{i=f,p,st} n_i} + \frac{\alpha}{n_p} \right] \sigma_\xi^2$$

5.

$$Var_t(F_{t+1}) = Var_t(P_{t+1}) + 4 \frac{C}{\sum_{i=f,p,st} n_i} \left[ \frac{C}{\sum_{i=f,p,st} n_i} + \frac{2\alpha}{n_p} \right] \sigma_\xi^2$$

6.

$$\begin{aligned} Var_t(F_{t+1}) &= E_t(F_{t+1}^2) - E_t(F_{t+1})^2 \\ &= Cov_t(P_{t+1}, F_{t+1}) + 4 \frac{C}{\sum_{i=f,p,st} n_i} \left[ \frac{C}{\sum_{i=f,p,st} n_i} + \frac{\alpha}{n_p} \right] \sigma_\xi^2 \end{aligned}$$

With moment restrictions 5 and 6, we can find that the correlation between the spot and futures prices at time  $t+1$  is equal to 1.

In short, we derive the moment restrictions of commodity prices from the equilibrium in two related markets which are spot and futures markets in this framework. This derivation is different from the derivation of Shiller (1981) and Kleidon (1986). They derived the moment restrictions of actual price and its forecast from the market efficiency hypothesis. We find that the variance of spot prices can be smaller than that of futures prices. In addition, the covariance between spot and futures prices can lie between the variances of spot and futures prices. This depends on whether noise trading can affect the futures price. Also, we find that the long-run correlation between commodity spot and futures prices is equal to 1.

## 4 Conclusion

From the empirical results, both actual data and the co-integrating vector support the assumptions that the spot and futures price are not equal at the maturity and that agents are allowed to trade again in the maturity month (before the last trading day). They also support the theoretical relationship between the equilibrium spot and futures prices at maturity. There are significant coefficients of milled rice prices and the coefficient of the futures price is insignificantly different from 1 in the long run relationship. Thus, we should be able to derive the moment restrictions from the theoretical relationship between the spot and futures prices obtained from this framework. By contrast, Shiller (1981) and Kleidon (1986) derived the moment restrictions of actual price and its forecast from the market efficiency hypothesis. Unlike their finding, we find that the variance of commodity spot prices is smaller than that of commodity futures prices. Also, the covariance between spot and futures prices is smaller than the variance of the futures price but greater than the variance of spot price. These moment restrictions result from the effect of noise trading on the futures

price. Moreover, we find that the correlation between intermediate and final commodity prices should lie between 0 and 1. We also derive the covariance between spot/futures price and noise trading in the futures market from the expressions for equilibrium spot and futures prices. Finally, we find that the long-run correlation between commodity spot and futures prices should be equal to 1.

## Appendix A

We assume that the agent choose his/her optimal strategy by maximising his/her expected utility based on mean-variance approach. The optimal strategy for each agent is shown below.

- Farmers

Assume that the production has constant returns to scale and additive production shock,  $f(X_{ft}, \epsilon_{t+1}) = bX_{ft} + \epsilon_{t+1}$ . The expected utility of a farmer is

$$\begin{aligned} V_f = & \underset{Y_{ft}^f, X_{ft}}{Max} [U_{ft}(-r_t X_{ft} - \theta X_{ft}^2 + Y_{ft}^f F_t) + E_{ft} \rho \underset{Y_{ft+1}^{fc}}{Max} \{E_{ft+1} U_{ft+1}(P_{t+1} \epsilon_{t+1}) \\ & + P_{t+1}(f(X_{ft}, \epsilon_{t+1}) - Y_{ft+1}^{fc}) - (Y_{ft}^f - Y_{ft+1}^{fc}) F_{t+1} - C(Y_{ft}^f - Y_{ft+1}^{fc})^2\}]. \end{aligned} \quad (13)$$

Consequently, the optimal input level,  $X_{ft}^*$  which the farmer uses to produce  $Y_{ft}^*$  is

$$X_{ft}^* = \frac{F_t - r_t/b}{2\theta/b},$$

and the optimal number of futures contracts which he sells at time  $t$  is

$$\begin{aligned} Y_{ft}^{f*} = & \frac{1}{2} \left[ \frac{Cov_{ft}(P_{t+1}, F_{t+1}) - Var_{ft}(P_{t+1})}{CVar_{ft}(P_{t+1})} \right] [E_{ft}(F_{t+1}) - E_{ft}(P_{t+1})] \\ & + \frac{\frac{F_t}{\rho} - E_{ft}(P_{t+1})}{2A_f Var_{ft}(P_{t+1})} + bX_{ft}^* + \frac{Cov_{ft}(P_{t+1}, \epsilon_{t+1})}{Var_{ft}(P_{t+1})} E_{ft}(P_{t+1}). \end{aligned}$$

- Processors

Again, we assume that the production has constant returns to scale and additive production shock,  $H_{pt+1} = a(Y_{pt+1} + Y_{pt+1}^{fc}) + \nu_{t+1}$ . The expected utility of a processor is

$$\begin{aligned} V_p = & \underset{Y_{pt}^f, Y_{pmt+1}}{Max} E_{pt} [U_{pt}(-Y_{pt}^f F_t) + \rho \underset{Y_{pt+1}^{fc}, Y_{pmt+1}}{Max} \{E_{pt+1} U_{pt+1}(Q_{t+1}(H_{pt+1} - Y_{pmt+1}) - P_{t+1} Y_{pmt+1}) \\ & + Q_{mt+1} Y_{pmt+1} (1 + \tau) - \alpha(Y_{pt+1} + Y_{pt+1}^{fc})^2 + (Y_{pt}^f - Y_{pt+1}^{fc}) F_{t+1} - C(Y_{pt}^f - Y_{pt+1}^{fc})^2\}]. \end{aligned} \quad (14)$$

$E_t(\nu_{t+1}) = 0$  and  $Var_t(\nu_{t+1}) = \sigma_\nu^2$ . The optimal net export of final goods is

$$\begin{aligned} Y_{pmt+1}^* = & \frac{a}{2\alpha} [aE_{pt}(Q_{t+1}) - E_{pt}(P_{t+1})] - [E_{pt}(F_{t+1}) - E_{pt}(P_{t+1})] \\ & \frac{[Cov_{pt}(F_{t+1}, Q_{t+1}) Var_{pt}(P_{t+1}) - Cov_{pt}(Q_{t+1}, P_{t+1}) Cov_{pt}(P_{t+1}, F_{t+1})]}{2 [Var_{pt}(Q_{t+1}) Var_{pt}(P_{t+1}) - Cov_{pt}^2(P_{t+1}, Q_{t+1})]} \\ & + \frac{[E_{pt}(P_{t+1}) - \frac{F_t}{\rho}] [Cov_{pt}(Q_{t+1}, P_{t+1})] + Var_{pt}(P_{t+1}) [Q_{mt+1}(1 + \tau) - E_{pt}(Q_{t+1})]}{2A_p [Var_{pt}(Q_{t+1}) Var_{pt}(P_{t+1}) - Cov_{pt}^2(P_{t+1}, Q_{t+1})]} \\ & + \frac{Cov_{pt}(Q_{t+1}, \nu_{t+1}) Var_{pt}(P_{t+1}) E_{pt}(Q_{t+1})}{[Var_{pt}(Q_{t+1}) Var_{pt}(P_{t+1}) - Cov_{pt}^2(P_{t+1}, Q_{t+1})]} \end{aligned}$$

and the optimal futures position at time  $t$  is

$$\begin{aligned}
Y_{pt}^{f*} = & \left[ \frac{Cov(F_{t+1}, Q_{t+1})Cov_{pt}(P_{t+1}, Q_{t+1}) - Var_{pt}(Q_{t+1})Cov_{pt}(P_{t+1}, F_{t+1})}{[Var_{pt}(Q_{t+1})Var_{pt}(P_{t+1}) - Cov_{pt}^2(P_{t+1}, Q_{t+1})]} + 1 \right] \\
& \frac{E_{pt}(F_{t+1}) - E_{pt}(P_{t+1})}{2C} + \frac{Cov_{pt}(Q_{t+1}, \nu_{t+1})Cov_{pt}(P_{t+1}, Q_{t+1})E_{pt}(Q_{t+1})}{[Var_{pt}(Q_{t+1})Var_{pt}(P_{t+1}) - Cov_{pt}^2(P_{t+1}, Q_{t+1})]} \\
& + \frac{[Q_{mt+1}(1 + \tau) - E_{pt}(Q_{t+1})]Cov_{pt}(P_{t+1}, Q_{t+1})}{2A_p [Var_{pt}(Q_{t+1})Var_{pt}(P_{t+1}) - Cov_{pt}^2(P_{t+1}, Q_{t+1})]} + \frac{aE_{pt}(Q_{t+1}) - E_{pt}(P_{t+1})}{2\alpha} \\
& + \frac{\left[ E_{pt}(P_{t+1}) - \frac{F_t}{\rho} \right] Var_{pt}(Q_{t+1})}{2A_p [Var_{pt}(Q_{t+1})Var_{pt}(P_{t+1}) - Cov_{pt}^2(P_{t+1}, Q_{t+1})]}.
\end{aligned}$$

As can be seen from the equations above, the optimal futures position is related to the optimal net export of the final good. However, this futures position only partially hedges the risk from international trading. The optimal  $Y_{pt+1}^{fc}$  and  $Y_{pt+1}$  are

$$Y_{pt+1}^{fc*} = Y_{pt}^{f*} + \frac{P_{t+1} - F_{t+1}}{2C}, \text{ and } Y_{pt+1}^* = \frac{aQ_{t+1} - P_{t+1}}{2\alpha} - Y_{pt}^{fc*}.$$

Obviously, the optimal futures position at time  $t + 1$  depends on the futures position at time  $t$  and the difference between  $P_{t+1}$  and  $F_{t+1}$  on the last trading day. In addition, his optimal input depends on the output and input prices as well as the amount of input bought from the futures market at time  $t + 1$ .

- Storage companies

The manager has to maximise the expected utility function which is

$$\begin{aligned}
V_{st} = & \underset{Y_{st,t}^f, Y_{st,t}}{Max} [U_{ft}(-P_t Y_{st,t} + Y_{st,t}^f F_t) + E_{st} \rho \underset{Y_{st,t+1}^{fc}}{Max} \{E_{st,t+1} U_{t+1} ((Y_{st,t} - Y_{st,t+1}^{fc}) P_{t+1} \\
& - (Y_{st,t}^f - Y_{st,t+1}^{fc}) F_{t+1} - C(Y_{st,t}^f - Y_{st,t+1}^{fc})^2 - cY_{st,t}^2 - u_{t+1})\}].
\end{aligned} \tag{15}$$

His optimal stock is

$$Y_{st,t}^* = \frac{F_t - P_t}{2c\rho}.$$

Also, the optimal number of futures contracts held at time  $t$  and  $t + 1$  are

$$\begin{aligned}
Y_{st,t}^{f*} = & \frac{1}{2} \left[ \frac{Cov_{st,t}(P_{t+1}, F_{t+1}) - Var_{st,t}(P_{t+1})}{CVar_{st,t}(P_{t+1})} \right] [E_{st,t}(F_{t+1}) - E_{st,t}(P_{t+1})] \\
& + \frac{1}{2} \left[ \frac{\frac{F_t}{\rho} - E_{st,t}(P_{t+1})}{A_{st} Var_{st,t}(P_{t+1})} \right] + Y_{st,t}^*, \text{ and} \\
Y_{st,t+1}^{fc*} = & Y_{st,t}^f + \frac{F_{t+1} - P_{t+1}}{2C}.
\end{aligned}$$

- Speculators

As the speculator makes a decision at time  $t$ , he has to maximise his expected utility function at time  $t$  which is

$$V_s = \underset{Y_{st}^f}{Max} [U_{st}(-Y_{st}^f F_t) + \rho E_{st} U_{st+1} (Y_{st}^f F_{t+1} - CY_{st}^{f2})] \tag{16}$$

and obtain

$$Y_{st}^{f*} = \frac{E_{st}(F_{t+1}) - \frac{F_t}{\rho}}{2[C + A_s Var_{st}(F_{t+1})]}.$$

Higher  $C$  and volatility of futures prices can reduce his long position.

## Appendix B

### 1. The moment restriction of the spot price and the production shock

From equations (11) and (12),

$$\begin{aligned} Cov_t(P_{t+1}, \epsilon_{t+1}) &= E_t(P_{t+1}\epsilon_{t+1}) - E_t(P_{t+1})E_t(\epsilon_{t+1}) \\ &= E_t(p_{t+1})E_t(\epsilon_{t+1}) + aE_t(Q_{t+1}\epsilon_{t+1}) - 2\alpha\frac{n_f}{n_p}E_t(\epsilon_{t+1}^2) - \frac{2\alpha}{n_p}E_t(\xi_{t+1}\epsilon_{t+1}) \\ &\quad - E_t(p_{t+1})E_t(\epsilon_{t+1}) - aE_t(Q_{t+1})E_t(\epsilon_{t+1}) \\ &= -2\alpha\frac{n_f}{n_p}Var_t(\epsilon_{t+1}) \end{aligned} \tag{17}$$

due to

$$E_t(\epsilon_{t+1}^2) - [E_t(\epsilon_{t+1})]^2 = Var_t(\epsilon_{t+1})$$

and

$$E_t(\xi_{t+1}\epsilon_{t+1}) = E_t(\xi_{t+1})E_t(\epsilon_{t+1}) = 0.$$

This is because  $\xi_{t+1}$  and  $\epsilon_{t+1}$  are independent and  $E_t(\epsilon_{t+1}) = 0$ .

### 2. The moment restriction of intermediate and final commodity prices

Again, from equations (11) and (12),

$$\begin{aligned} Cov_t(P_{t+1}, Q_{t+1}) &= E_t(P_{t+1}Q_{t+1}) - E_t(P_{t+1})E_t(Q_{t+1}) \\ &= E_t(p_{t+1})E_t(Q_{t+1}) + aE_t(Q_{t+1}^2) - 2\alpha\frac{n_f}{n_p}E_t(Q_{t+1}\epsilon_{t+1}) \\ &\quad - \frac{2\alpha}{n_p}E_t(\xi_{t+1}Q_{t+1}) - E_t(p_{t+1})E_t(Q_{t+1}) - aE_t(Q_{t+1})^2, \\ &= aVar_t(Q_{t+1}) \end{aligned} \tag{18}$$

where

$$E_t(Q_{t+1}\epsilon_{t+1}) - E_t(Q_{t+1})E_t(\epsilon_{t+1}) = 0$$

and

$$E_t(\xi_{t+1}Q_{t+1}) = E_t(\xi_{t+1})E_t(Q_{t+1}) = 0.$$

This is because  $Q_{t+1}$  is independent of  $\xi_{t+1}$  and  $\epsilon_{t+1}$ .

### 3. The moment restriction of the spot price and noise trading

From equations (11) and (12),

$$\begin{aligned}
Cov_t(P_{t+1}, \xi_{t+1}) &= E_t(P_{t+1}\xi_{t+1}) - E_t(P_{t+1})E_t(\xi_{t+1}) \\
&= E_t(p_{t+1})E_t(\xi_{t+1}) + aE_t(Q_{t+1}\xi_{t+1}) - 2\alpha\frac{n_f}{n_p}E_t(\xi_{t+1}\epsilon_{t+1}) \\
&\quad - \frac{2\alpha}{n_p}E_t(\xi_{t+1}^2) - E_t(p_{t+1})E_t(\xi_{t+1}) - aE_t(Q_{t+1})E_t(\xi_{t+1}), \\
&= -\frac{2\alpha}{n_p}Var_t(\xi_{t+1})
\end{aligned} \tag{19}$$

where

$$E_t(\xi_{t+1}^2) - [E_t(\xi_{t+1})]^2 = Var_t(\xi_{t+1})$$

The noise trading is assumed to have a constant variance  $\sigma_\xi^2$ .

### 4. The moment restriction of futures prices and noise trading

From equation (9), we obtain

$$E_t(F_{t+1}) = E_t(P_{t+1}) - \frac{Cn_s}{\sum_{i=f,p,st} n_i} \frac{E_t(F_{t+1}) - \frac{F_t}{\rho}}{[C + \Theta_s]} \tag{20}$$

The covariance between futures prices and noise trading is derived as follows.

$$\begin{aligned}
Cov_t(F_{t+1}, \xi_{t+1}) &= E_t(F_{t+1}\xi_{t+1}) - E_t(F_{t+1})E_t(\xi_{t+1}) \\
&= E_t(P_{t+1}\xi_{t+1}) - E_t(\xi_{t+1})\frac{Cn_s}{\sum_{i=f,p,st} n_i} \frac{E_t(F_{t+1}) - \frac{F_t}{\rho}}{[C + \Theta_s]} - 2\frac{C}{\sum_{i=f,p,st} n_i} E_t(\xi_{t+1}^2) \\
&\quad - E_t(\xi_{t+1})E_t(P_{t+1}) + E_t(\xi_{t+1})\frac{Cn_s}{\sum_{i=f,p,st} n_i} \frac{E_t(F_{t+1}) - \frac{F_t}{\rho}}{[C + \Theta_s]} \\
&= Cov_t(P_{t+1}, \xi_{t+1}) - 2\frac{C}{\sum_{i=f,p,st} n_i} Var_t(\xi_{t+1}) \\
&= -2\frac{\alpha}{n_p} Var_t(\xi_{t+1}) - 2\frac{C}{\sum_{i=f,p,st} n_i} Var_t(\xi_{t+1}) \\
&= -2 \left[ \frac{C}{\sum_{i=f,p,st} n_i} + \frac{\alpha}{n_p} \right] \sigma_\xi^2
\end{aligned} \tag{21}$$

5. The relationship between the variances of spot and futures prices

From equations (9) and (20),

$$\begin{aligned}
Var_t(F_{t+1}) &= E_t(F_{t+1}^2) - E_t(F_{t+1})^2 \\
&= E_t \left[ \left( P_{t+1} - \frac{Cn_s}{\sum_{i=f,p,st} n_i} \frac{E_{st}(F_{t+1}) - \frac{F_t}{\rho}}{[C + \Theta_s]} - \frac{2C}{\sum_{i=f,p,st} n_i} \xi_{t+1} \right)^2 \right] \\
&\quad - \left( E_t(P_{t+1}) - \frac{Cn_s}{\sum_{i=f,p,st} n_i} \frac{E_t(F_{t+1}) - \frac{F_t}{\rho}}{[C + \Theta_s]} \right)^2 \\
&= Var_t(P_{t+1}) + 4 \left[ \frac{C}{\sum_{i=f,p,st} n_i} \right]^2 Var_t(\xi_{t+1}) - 4 \left[ \frac{C}{\sum_{i=f,p,st} n_i} \right] Cov_t(P_{t+1}, \xi_{t+1}) \\
&= Var_t(P_{t+1}) + 4 \left[ \frac{C}{\sum_{i=f,p,st} n_i} \right]^2 Var_t(\xi_{t+1}) + 8 \left[ \frac{C\alpha}{n_p \sum_{i=f,p,st} n_i} \right] Var_t(\xi_{t+1}) \\
&= Var_t(P_{t+1}) + 4 \frac{C}{\sum_{i=f,p,st} n_i} \left[ \frac{C}{\sum_{i=f,p,st} n_i} + \frac{2\alpha}{n_p} \right] \sigma_\xi^2 \tag{22}
\end{aligned}$$

6. The relationship between the variance of futures price and the covariance between the spot and futures prices

From equations (9) and (20),

$$\begin{aligned}
Var_t(F_{t+1}) &= E_t(F_{t+1}^2) - E_t(F_{t+1})^2 \\
&= E_t(F_{t+1}P_{t+1}) - E_t(F_{t+1}) \frac{Cn_s}{\sum_{i=f,p,st} n_i} \frac{E_t(F_{t+1}) - \frac{F_t}{\rho}}{[C + \Theta_s]} - 2 \frac{C}{\sum_{i=f,p,st} n_i} E_t(F_{t+1}\xi_{t+1}) \\
&\quad - E_t(F_{t+1})E_t(P_{t+1}) + E_t(F_{t+1}) \frac{Cn_s}{\sum_{i=f,p,st} n_i} \frac{E_t(F_{t+1}) - \frac{F_t}{\rho}}{[C + \Theta_s]} \\
&= Cov_t(P_{t+1}, F_{t+1}) + 4 \frac{C}{\sum_{i=f,p,st} n_i} \left[ \frac{C}{\sum_{i=f,p,st} n_i} + \frac{\alpha}{n_p} \right] Var_t(\xi_{t+1}) \\
&= Cov_t(P_{t+1}, F_{t+1}) + 4 \frac{C}{\sum_{i=f,p,st} n_i} \left[ \frac{C}{\sum_{i=f,p,st} n_i} + \frac{\alpha}{n_p} \right] \sigma_\xi^2 \tag{23}
\end{aligned}$$

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## Notes

<sup>1</sup>If the dividend process is

$$d_t = \rho d_{t-1} + \eta_t$$

where  $\eta_t$  is independently and identically distributed (i.i.d.)  $(0, \sigma_\eta^2)$ .

$$P_t^* = ad_t$$

Then

$$\sigma_{P^*|I_{t-k}}^2 = \sigma_\eta^2 a \frac{1 - \rho^{2k}}{1 - \rho^2}$$

and

$$\sigma_{P|I_{t-k}}^2 = \sigma_{P^*|I_{t-k}}^2 + \sigma_\eta^2 a^2 \frac{(1+r)^2}{\rho^2 r(2+r)}$$

where  $r$  is an assumed constant discount rate from

$$P_t^* = \sum_{\tau=1}^{\infty} \frac{E\{d_{t+\tau} | I_t\}}{(1+r)^\tau}.$$

<sup>2</sup>Antonioni applied a mean-variance approach to study the optimal strategies of 5 types of agents: farmers (producers of the intermediate good), processors (producers of final goods), storage companies, the intermediate good exporter and speculator. The traders made all decisions at time  $t$ . He found that the futures position had both speculative and hedging components.

<sup>3</sup>Contract months are the maturity months of futures contracts. Thus, the contract month is the spot month.

<sup>4</sup>For example, the futures prices in January and February 2003 are the prices quoted for the rough rice of the futures contract maturing in March 2003. Then, the futures prices in March and April 2003 are the prices quoted for the rough rice of the futures contract maturing in May 2003.

<sup>5</sup>Data source: [www.cbot.com](http://www.cbot.com)

<sup>6</sup>The daily cash price in the graph is obtained by adjusting the monthly cash price.

<sup>7</sup>Glosten, L. R. and Milgrom, P. R. (1985) and Basili, M. and Fontini, F. (2001) show that trade will not take place if the agents have common beliefs. However, the agents, who have or need underlying assets and face the uncertainty, may trade in the market although they have common beliefs.

<sup>8</sup>Data sources: [www.ers.usda.gov](http://www.ers.usda.gov). and [www.futurestrading.chart.com](http://www.futurestrading.chart.com).

<sup>9</sup>the rough rice which is traded in the Chicago Board of Trade (CBOT) is U.S. rough rice no.2 or better long grain rough rice with a total milling yield of not less than 65% including head rice of not less than 48%.

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