# Liquidity, Volatility and Growth.

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**Abstract**

This paper studies the relation between average growth and growth volatility. To do so a two period model is built which focuses on how firms choose their debt portfolio maturity. Due to imperfect enforceability problems, we show that contracts financing long term investments are biased towards short-term debt. This can generate maturity mismatches between assets and liabilities and lead to liquidity crises. Then it is shown that the relation between average growth and growth volatility is more likely to be negative in developing countries while it is more likely to be positive in developed economies. We therefore invalidate the idea that volatility is the price for rapid growth in emerging market countries. This framework also allows us to assess the impact of foreign direct investment (FDI) and financial opening (FO). We show that FDI has stabilizing effects in developing economies while FO has destabilizing effects. On the contrary in developed economies FO has stabilizing effects.

## 1. Introduction.

After the last financial crises, many voices rose to explain that these crises were new compared to the preceding ones (Radelet and Sachs [1998]). Indeed the usual concerns and features known to trigger crises (governments unsustainable economic policies such as large fiscal deficits or unrealistic exchange rate (Krugman [1979])) were absent or could not by themselves imply so severe crises (Corsetti, Pesenti and Roubini

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[1999] and Baig and Goldfajn [2002]). Several new explanations have then been brought to the fore front. Two of them have particularly emerged from the debate. According to the first one, "crony capitalism" can help explain recent economic crises (Krugman [1998]). "Crony capitalism" has played a major role in encouraging firms to take inefficient decisions (over investment, excessive risks, etc... ). In other words, it has distorted individual incentives. An illustration of this phenomenon has been the large and possibly excessive levels of short term debt firms have accumulated. Implicit insurance prompted agents to believe that they could benefit from good draws and that someone else (the government) would pay the bill in case of failure or bad draws.

Insert table 1

The second explanation refers to the "Original Sin" hypothesis (Eichengreen and Haussman [1999]). According to it, agents financial positions such as those described in the preceding table are due to firms inability to choose their financial portfolios, firms having no available financial strategy but the risky ones. Although they know ex ante the risks associated with this type of financial instruments, they are somewhat constrained to adopt these "dangerous" financing strategies because this is the only way to get capital from financial markets.

Although both of these explanations may be reasonable and explain the vulnerability of a number of countries to financial crashes, they are still full of gaps and remain fairly ad-hoc in their foundations. The crony capitalism explanation does not say where the implicit insurance scheme or the collusion links between firms managers and politicians come from. They are simply taken as given. There is no positive theory of crony capitalism. In the case of the "Original Sin" explanation, we need to explain why this theory can be relevant for some economies (developing countries) while other countries (developed countries) do not seem to suffer from any "Original Sin" problem. The effects of the "Original Sin" problem seem to disappear along the development process. For instance, the proportion of short term debt in corporate debt portfolios seems to decrease with the level of income per capita in the economy (Demirgüç-Kunt and Maksimovic [1999]).

Insert figure 1

We need to understand how capital accumulation modifies contractual relations between agents to understand the nature of the "Original Sin" problem in developing countries. For these reasons, we try in this paper to use a formal and explicit framework which helps explain why private agents do use risky financial strategies and why and how the riskiness of optimal financial strategies evolves with capital accumulation.

### 1.1. The mechanism of the model.

To answer these questions, we build a model of a decentralized economy and focus on how firms determine the maturity of their debt portfolios. The mechanism is the following. When contracts are imperfectly enforceable, lenders impose on the debt portfolio of borrowers investing in long term activities a bias towards short term debt because entrepreneurs are not able to commit ex ante with full credibility to invest in an illiquid technology with exclusively long term liabilities. From the lender point of view, the problem with long term debt lies in the freedom it leaves to the borrower. In a long term debt contract, there is at least one date between the contracting date and the payment date. Therefore the borrower can do whatever he wants at the interim date even though the lender is not pleased with that decision. In the model, the borrower can decide to liquidate his project interim though liquidation is worse-off strictly speaking. With a capital market this drawback is offset with default: the borrower incurs a loss in the project pay-off but benefits from defaulting on long term loans. Therefore long term debt is not an appropriate instrument to finance long term projects basically because it leaves "too much" freedom to the borrower. Lenders can restrict this freedom by modifying the composition of firms debt portfolios. When debt portfolios contain enough short term debt, lenders have an effective monitoring power: an interim liquidation can be observed **and** can be sanctioned as the lender can ask for short term debt repayments. With a sufficiently large proportion of short term debt, liquidation becomes welfare decreasing and borrowers do not default nor on short nor on long term loans. Short term debt has a monitoring power because it gives the possibility to lenders to inflict welfare costs to borrowers which prevents the latter from liquidating.

However although this mechanism solves a micro incentive problem, it generates a global coordination issue: When borrowers rely too much on short term debt, there can be multiple equilibria. If borrowers have large amounts of short term debts, lenders can refuse to roll-over these debts not because they fear that the borrower is liquidating his project but because they fear that other lenders are doing the same. Since the final return depends positively on the amount of capital still invested interim, this amounts to an increasing returns to scale technology which creates a strategic complementarity between lenders as to their roll-over decision. Although, with the right contracts, borrowers have no incentives to liquidate, they can be forced to do so because of a coordination problem giving rise to inefficient liquidations. The trade-off on which borrowers choose the composition of their debt portfolios is then quite simple. On the one hand if they borrow proportionally more short term debt, the probability that they are confronted to inefficient liquidation increases. This reduces expected profits. On the other hand, a relative increase in the proportion of short term debt enables borrowers to borrow more capital. This increases expected profits. Borrowers choose a low or a high proportion of short term debt depending upon the relative strength of these two effects.

### 1.2. The macroeconomic results.

To derive the macroeconomic consequences of the last mechanism, we focus on the ratio of lenders to borrowers wealth. This ratio has a negative impact on the probability that a run happens but also on the efficiency of the economy since borrowers have access by definition to the most efficient technologies. Combining these two effects, it is shown that average growth and growth volatility are negatively (resp. positively) related in economies where this ratio is low (resp. high). Identifying the former case to that of developing economies and the latter to that of developed economies[1], the model therefore predicts that an increase in average growth is compatible with a decrease (resp. increase) in growth volatility in developing (resp. developed) countries. Moreover we provide empirical evidence which confirms this result. Secondly the model shows that opening the economy to FDI (resp. to financial inflows) increases (resp. decreases) average growth and decreases (resp. increases) growth volatility in developing countries. Opening the economy to FDI (resp. to financial flows) has a negative (resp. positive) impact on the probability that a run on short

[1] Following data from Beck, Demirgüç-Kunt and Levine [1999], there is a positive correlation between the development level and the amount of financial intermediaries assets to GDP which is a proxy for the ratio of lenders to borrowers wealth.

term debt occurs because, the relative capital stock of lenders increases (resp. decreases) which prompts borrowers to take a larger (resp. smaller) proportion of short term debt in their portfolios. Schmukler and Vesperoni [2001] show empirical evidence that financial liberalization is usually followed by a reduction in the maturity of financial contracts.

**1.3. Related literature.**

Three types of literature are related to the issues studied in this paper. First, liquidity management and its macroeconomic consequences have been studied in the seminal Diamond Dybvig [1983] paper. Panics can happen in the banking sector when the maturity of bank deposits is shorter than that of bank loans due to idiosyncratic liquidity shocks. Banks can then act pools of liquidity to stop these panics. Diamond [1991] is closer to our paper, firstly because it refers to non financial firms and secondly because it studies how financial choices may help elevating informational asymmetries between lenders and borrowers. In Diamond [1991] firms with good prospects are more likely to issue short term debt because their probability of being confronted to liquidity shocks is smaller. Flannery [1986] and Kale and Noe [1990] also consider financial choices as signals on the quality of the projects financed. Rajan [1992] broadens the analysis: he introduces two additional elements, ex ante moral hazard, i.e. how the choices between short and long term debt influences managers efforts to get large returns and the possibility to choose between banking and market finance, the difference lying in the informational advantage of banks on markets. The approach of our paper is different because it is the nature of long term projects (the possibility to liquidate interim) which prevents firms from borrowing only with long term contracts. Here short term debt is not chosen by borrowers but rather more imposed by lenders. A second strand of literature this paper is close to tries to explain micro or macro stylized facts based on corporate financial contracts. Albuquerque and Hopenhayn [2004] study how optimal maturity debt contracts help explain the dynamics of firms development. Rodrik and Velasco [1999] tries to explain why developing countries can rationally accumulate unsustainable amounts of short term debt, the idea being that the long term debt market works with an increasing return to scale technology. If projects are illiquid, then accumulating short term debt increases the prices of long term debt because the premium on long term debt depends positively on the amount of short term debt. This paper is close. However short term debt is not necessarily welfare decreasing as is the case in Rodrik and Velasco [1999]. Finally this paper is related to the literature dealing with the macroeconomic impact of capital market imperfections (Bernanke and Gertler [1989], Greenwood and Jovanovic [1990], Acemoglu and Zilibotti [1997], Kiyotaki and Moore [1997] or Aghion Banerjee and Piketty [1999]) in which a consensus exists on the ideas that first these imperfections generate or exacerbate cycles and fluctuations and second that these effects seem to be more relevant for developing countries.

### 1.4. Road map of the paper.

The paper is organized as follows. The microeconomics of capital markets is established in the next section. In section 3 we apply this micro framework to a macroeconomic model and derive how this model works in its two period version in section 4. The main results are then established in section 5 and conclusion eventually lies in section 6.

## 2. A two period credit market.

### 2.1. A capital market with imperfect enforceability.

When contracts are imperfectly enforceable, there exists a relation between the size of a debt portfolio and the composition (between short term and long term loans) of this debt portfolio. To illustrate it, let us consider:

- a risk neutral borrower-entrepreneur with initial wealth $W$ is granted at date 0 a loan $L$ from a pool of risk neutral investors made a short term loan $\alpha L$ (which must be repaid after one period) and a long term loan $(1-\alpha)L$ (which must be repaid after two periods). The gross risk free interest rate on short (resp. long) term debts is $r_s$ (resp. $r_l$).
- The entrepreneur can invest in a technology (called production technology) with a marginal return 1 at date 1 and $R$ at date 2 ($R > r_l$).

- Contracts are imperfectly enforceable, borrowers can default strategically on their debts. The marginal cost of default is $\tau$.

A borrower therefore pays for his short (resp. long) term debts if and only if his wealth at date $t+1$ (resp. $t+2$) is larger when he pays for his debts than when he defaults on his financial contracts. Short term debts are paid back if and only if $W + L - \alpha r_s L \geq (1-\tau)(W+L)$. Long term debts are paid back[2] if and only if $R(W + L - \alpha r_s L) - (1-\alpha) r_l L \geq (1-\tau) R (W + L - \alpha r_s L)$.

**Proposition 1.** *Noting $\mu = \frac{L}{W}$ the debt equity ratio and $\alpha$ the proportion of short term debt, incentive compatible debt portfolios $(\alpha, \mu)$ write as*

$$\mu \leq \min \left\{ \frac{\tau}{[\alpha r_s - \tau]^+}; \frac{\tau R}{[(1-\alpha) r_l + \alpha \tau R r_s - \tau R]^+} \right\} \tag{2.1}$$

*where $[y]^+ = \max\{y; 0\}$.*

**Proof.** Elementary algebra on the two incentive compatible constraints yields the proposition. ■

As is clear the right hand side of (2.1) can be a non monotonic function of $\alpha$ when $r_l > \tau R r_s$, i.e. if $\tau$ is sufficiently small. We consider this case in what follows[3].

### 2.2. A capital market with imperfect enforceability and interim moral hazard.

Interim moral hazard consists in the possibility for a borrower-entrepreneur to benefit from liquidating his project interim. Such an entrepreneur can claim ex ante to be willing to carry out his project till maturity. But effectively he liquidates his project interim which makes default on long term loans profitable.

#### 2.2.1. Incentives and contracts.

Let us consider the borrower-entrepreneur of the previous paragraph and let us add the following assumptions:

[2] We assume without loss of generality that a borrower who defaults on short term debts cannot carry out till the end the project he has invested in initially.

[3] The condition $r_l > \tau R r_s$ is a necessary condition to generate a trade-off between the quantity of capital an entrepreneur can borrow and the maturity mismatch he accepts between his assets and his liabilities. The case $r_l \leq \tau R r_s$ is therefore uninteresting because trivial.

- At date 0, the entrepreneur can invest in the production technology (with a marginal return 1 at date 1 and $R$ at date 2) or in a storage technology with a marginal return 1 at date 1 and $r$ at date 2.

- At date 1, an entrepreneur who invested at date 0 in the production technology can turn to the storage technology. The marginal cost of default for the production technology is $\tau$ and $\tau'$ for the storage technology.

- There is moral hazard at date 1: $r < R$ and $(1-\tau')\,r > (1-\tau)\,R$.

Applying proposition 1, the incentive compatible contract for an entrepreneur who invests at date 0 in the production technology but whose goal is to invest in the storage technology at date 1 writes as $(\alpha, \mu)$ with $\mu \leq \mu_{liq} = \min\left\{\frac{\tau}{[\alpha r_s - \tau]^+}; \frac{\tau' r}{[(1-\alpha) r_l + \alpha \tau' r r_s - \tau' r]^+}\right\}$ whereas the incentive compatible contract for an entrepreneur whose goal is to carry out his project in the production technology till date 2 writes as $(\alpha, \mu)$ with $\mu \leq \mu_{pro} = \min\left\{\frac{\tau}{[\alpha r_s - \tau]^+}; \frac{\tau R}{[(1-\alpha) r_l + \alpha \tau R r_s - \tau R]^+}\right\}$. As is clear under the assumption $(1-\tau')\,r > (1-\tau)\,R$ we have $\mu_{pro} \geq \mu_{liq}$. An entrepreneur who is offered a contract $\mu_{pro}$ and who turns to the storage technology at date 1 defaults on all his long term debts. Moreover the possibility to write contracts contingent on the technology used cannot solve the issue since technological choices are made ex ante while moral hazard materializes only interim. The following proposition summarizes the design of the contracts $(\alpha, \mu)$ such that an entrepreneur who claims that he will carry out till the end the production technology, will effectively do so.

**Proposition 2.** *Under the assumptions of sections 2.1. and 2.2.1. and noting $\overline{\sigma} = R - (1-\tau')\,r$, incentive compatible contracts $(\alpha, \mu)$ are such that*

$$\mu \leq \min\left\{\frac{\tau}{[\alpha r_s - \tau]^+}; \frac{\overline{\sigma}}{[(1-\alpha)\,r_l + \alpha\overline{\sigma} r_s - \overline{\sigma}]^+}\right\} \tag{2.2}$$

Proof. c.f. appendix.

Three remarks can be made. First, under the assumption of interim moral hazard $\mu_{pro}$ does not belong to the set of incentive compatible contracts verifying (2.2). Interim moral hazard therefore reduces the borrowing capacity of entrepreneurs. Secondly, given the assumptions $(1-\tau')\,r > (1-\tau)\,R$ and $r_l > \tau R r_s$ we have $r_l - \overline{\sigma} r_s > 0$. Then an entrepreneur who wants to invest in the production technology and keep the same level of debt $\mu$ will have to bear a higher proportion of short term debt compared to the situation without interim moral hazard. There is therefore a "bias" towards short term debt compared to the previous situation. This is because short term debt appears as a monitoring device for lenders. They impose this bias to make sure that borrowers do not take advantage of the presence of interim moral hazard.

Insert figure 2

Thirdly and finally, while long term debts can be defaulted on but it is not the case of short term loans. The reason for this is that the interim pay-off is always the same whatever happens as concerns liquidation. If the interim liquidation decision not only modified the final return but also the interim one, default on short term debts would be possible[4].

### 2.2.2. Short term debts roll-over.

Let us finally consider the borrower-entrepreneur of the previous paragraph and add a final assumption:

- Lenders can observe entrepreneurs decision interim (at date $t+1$) **and then** decide on that basis how to behave as to short term debt repayment.

We have shown in the previous paragraph that the introduction of interim moral hazard generates a "bias" towards short term debt because for the same debt equity ratio $\mu$, entrepreneurs have to rely relatively more on short term contracts. Since liquidating a long term project is observable, lenders can decide to ask for short term contracts repayments or accept short term contracts prorogation on a basis contingent to the liquidation decision. If an entrepreneur decides to carry out till the end his project in the production technology then it is incentive compatible for lenders to reduce the proportion of short term debt whereas if an entrepreneur decides to liquidate his project then lenders have to ask for full repayments. Since entrepreneurs

[4] The assumptions we have considered are compatible with the intuition that long term contracts always embed more risks than short term ones.

can anticipate this type of behavior, no entrepreneur will try to liquidate his investment, basically because such an option becomes worthless if lenders accept to prorogate some or all short term contracts when the entrepreneur does not liquidate his project. The following proposition then gives the precise conditions on how short term debt roll-over or short term debt prorogation is realized:

**Proposition 3.** *Under the assumptions of sections 2.1. and 2.2., it is incentive compatible for lenders to exchange a debt portfolio* $(\alpha, \mu)$ *against a portfolio* $(\beta, \mu)$ *if and only if*

$$\beta \geq \frac{1}{r_{l,s} - \tau R r_s} \left[ \alpha r_{l,s} + (1-\alpha) r_l - \tau R \frac{1+\mu}{\mu} \right]^+ \tag{2.3}$$

*where* $r_{l,s}$ *the gross interest rate on rolled-over short term debts and* $r_l$ *the gross interest rate on long term debts.*

Proof. c.f. appendix.

We have therefore established three results in this part.

- The maturity structure of a debt portfolio has an influence on the size of this portfolio.
- When borrowers can deviate from the project they invest in initially, lenders bias debt portfolios towards short-term debt.
- There is room for short term debt roll-over when short term debt repayments happen after the deviation decision has been observed.

Let us now introduce this capital market framework in a macroeconomic model in order to shed some light on the aggregate consequences of the structure of financial contracts. In particular we determine how borrowers choose their portfolios and the conditions under which short term debts are rolled-over or not.

## 3. Hypotheses and description of the model.

### 3.1. Agents and technologies.

We consider a single good economy with two types of risk neutral agents, entrepreneurs and workers. There is a continuum of unit mass of each type of agent. All agents live for two periods and generations are non overlapping. The utility of an agent born at time $t$ is $U_{t+2} = E_t \left[ B_{t+2}^{\gamma} C_{t+2}^{1-\gamma} \right]$. Consumption at time $t+2$ is represented by $C_{t+2}$ and $B_{t+2}$ is the bequest made at date $t+2$ by an agent born at time $t$ to his off-spring born at time $t+2$ and $\gamma$ is a parameter $(0 < \gamma < 1)$. All agents have access at any time to a storage technology. This technology is liquid: cash flows follow investments after one period. This technology uses capital and produces capital, $y_{t+1} = rk_t$ with $r > 1$.

Entrepreneurs have access to a production technology which uses capital and labor (supplied by workers) to produce capital. The production function writes as $y_{t+2} = (k_t^*)^{\varepsilon} (A_t l_t)^{1-\varepsilon}$ with $A_t = \overline{A}^{\frac{1}{1-\varepsilon}} \frac{k_t^*}{l_t}$ and $k_t^* = \min \{k_t, k_{t+1}\}$. This is an AK technology with two major differences. First the relevant capital stock for output is the capital stock invested initially if and only if it is the capital stock still in the project at date $t+1$. This assumption captures the idea that entrepreneurs can extract capital from their project at the interim date $t+1$ to meet any possible need but such extractions are costly in the sense that they reduce final output. To extract capital entrepreneurs can use a liquidation technology. An entrepreneur who liquidates $k$ units of capital at date $t+1$ gets $k$ units of capital at date $t+1$. The second major difference lies in the illiquidity of the production technology. A project in which too much capital is liquidated interim does not produce any output: $\overline{A} = A\mathbf{1}\left[k_t^* \geq (1-\eta)\, k_t\right]$ with $A$ a strictly positive number and $\mathbf{1}[x]$ is equal to 1 if $x$ is true and 0 otherwise. Entrepreneurs can liquidate interim at most a proportion $\eta$ of the capital stock initially invested without "destroying" their projects. If they go beyond the proportion $\eta$, then the remaining project is worthless: its total factor productivity is zero. The labor market being competitive, the wage rate and the marginal return to capital when projects are carried out till maturity respectively write as $w = (1-\varepsilon)\, Ak_t^*$ and $R = \varepsilon A$.

In this economy there is a capital market similar to that of section 2 on which entrepreneurs can borrow from workers. There is imperfect enforceability, borrowers can default strategically. There are two types of financial contracts, a short and a long term debt contract. Defaulting on one's contracts imposes to pay a cost on final output ($\tau$ for the production technology, $\tau'$ for the storage technology). The production technology is subject to *interim* moral hazard. It is more efficient if entrepreneurs decide to pay for their debts but it is less efficient if entrepreneurs decide to default on their contracts: $R > r^2$ and $(1-\tau)\,R < (1-\tau')\,r$. This means that entrepreneurs would better liquidate their long term projects and invest in the storage technology. The two available technologies are exclusive, entrepreneurs cannot not carry out at the same time projects in both technologies. Agents types and technological choices are all observable.

### 3.2. Timing of the model.

At the beginning date (date $t$), agents make technological (liquid or illiquid technologies) and financial (short or long term debt) choices. At the *interim* date (date $t+1$), short term debts are partially or fully rolled-over, illiquid projects may be liquidated. At the final date (date $t+2$), the returns on the different projects are realized according to what happened at the previous date. Long term and rolled-over short term debts are paid back, agents consume part of their end-of-life wealth and bequeath the other part to their unique off-spring.

Insert figure 3

## 4. The two periods static model.

### 4.1. Optimal debt portfolios without interim moral hazard.

When there is no interim moral hazard between lenders and borrowers, then the ex ante observation of technological choices enables lenders to write down the corresponding set of incentive compatible contracts. In the case an entrepreneur chooses the production technology, then his expected profit writes as

$$E_t\pi_{t+2} = \left[(1+\mu-\alpha\mu r_s)\,R - (1-\alpha)\,\mu r_l\right] w_t$$

where $w_t$ is the initial wealth of the entrepreneur. His program consists in

$$\max_{\alpha,\mu} \mu. \left[R - r_l - \alpha \left(r_s R - r_l\right)\right] \tag{4.1}$$
$$\text{s.t.} \begin{cases} \alpha \mu r_s \leq \eta \left(1+\mu\right) \\ \mu \leq \min \left\{ \frac{\tau}{\left[\alpha r_s - \tau\right]^{+}} ; \frac{\tau R}{\left[(1-\alpha) r_l + \alpha \tau R r_s - \tau R\right]^{+}} \right\} \end{cases}$$

**Proposition 1.** *When there is no interim moral hazard, entrepreneurs choose assets and liabilities with identical maturities.*

**Proof.** With simple algebra, it can be shown that (4.1) is a always a decreasing function of $\alpha$. Therefore entrepreneurs choose the largest amount of capital they can borrow that is compatible with exclusively long term liabilities. The optimal debt portfolio therefore does not contain short term debts, the optimal debt equity ratio is $\mu_{fb} = \frac{\tau R}{r_l - \tau R}$ and expected profits are $\pi_{fb} = (1-\tau) \frac{R r_l}{r_l - \tau R}$. ■

#### 4.2. Optimal debt portfolios with interim moral hazard.

Let us consider an entrepreneur whose initial wealth in normalized to one, who invests in the production technology with a debt portfolio whose size is $\mu$ and contains $\alpha\mu$ short term debts. Given the results of sections 2.1.-2.2., such an entrepreneur can be confronted to two different situations. Lenders can ask him to pay for $\beta \mu r_s$ or $\alpha \mu r_s$ as short term debt repayments with $\beta \leq \alpha$.

##### 4.2.1. The safe financing strategy.

When lenders ask the entrepreneur to pay for $\alpha \mu r_s$ the entrepreneur is still able to carry out his project in the production technology if and only if $\alpha \mu r_s \leq \eta (1+\mu)$. Then it is incentive compatible for lenders to ask only for $\beta \mu r_s$ as short term debt repayments since the entrepreneur is always able to continue his long term project and has no incentive to deviate. The expected profit of that entrepreneur[5] then writes as

$$E_t w_{t+2} = \left[\left(1+\mu-\beta\mu r_s\right)R - \left(1-\beta\right)\mu r_l\right] w_t$$

and his program consists in

$$P_1 : \quad \begin{array}{c} \max\limits_{\mu;\alpha} \mu.\left[R - r_l - \beta.\left(Rr_s - r_l\right)\right] \\ \text{s.c.} \quad \left\{ \begin{array}{c} \alpha\mu r_s \leq \eta\left(1+\mu\right) \\ \mu \leq \min\left\{\frac{\tau}{\left[\alpha r_s - \tau\right]^+}; \frac{\overline{\sigma}}{\left[(1-\alpha) r_l^* + \alpha\overline{\sigma} r_s - \overline{\sigma}\right]^+}\right\} \end{array} \right. \end{array}$$

The solution to this problem $(\alpha_1;\mu_1)$ is reached for $\mu_1 = \mu_{fb}$ and $\alpha_1 = \alpha_{sb} = \frac{\tau R - \overline{\sigma}}{r_l - \overline{\sigma} r_s}\frac{r_l}{\tau R}$. However $(\alpha_1, \mu_1)$ must be such that $\alpha_1\mu_1 r_s \leq \eta\left(1+\mu_1\right)$. Therefore this optimum is possible if and only if

$$\eta\left(\frac{r_l}{r_s} - \overline{\sigma}\right) \geq \tau R - \overline{\sigma} \tag{4.2}$$

This inequality means that when the long term technology is not "too illiquid" then entrepreneurs are able to reach the "no interim moral hazard" optimum. Put differently, when (4.2) is verified, there is no contradiction between maximizing firms profits and supplying incentives to forbid liquidations. In this case entrepreneurs expected profits write as $\pi_1 = \pi_{fb}$. On the contrary when (4.2) is not verified, the technological constraint $\alpha_1\mu_1 r_s \leq \eta\left(1+\mu_1\right)$ is binding and the optimal debt portfolio[6] then writes as

$$\begin{aligned} \alpha_1 &= \frac{\eta r_l}{\eta r_l + (1-\eta)\overline{\sigma} r_s} \\ \mu_1 &= \frac{(1-\eta)\overline{\sigma} + \eta\frac{r_l}{r_s}}{r_l - (1-\eta)\overline{\sigma} - \eta\frac{r_l}{r_s}} \end{aligned}$$

[5] This expression is valid under the assumption that the market for short term debt roll-over is perfectly competitive. This will be the case through out the paper. This assumption implies in particular that the interest rate on rolled-over short term debt is identical to the interest rate on long term debt.

[6] To determine $\alpha_1$ and $\mu_1$ in this case, we need to solve for the system $\mu = \mu\left(\alpha, \overline{\sigma}\right)$ and $\alpha\mu r_s = \eta\left(1+\mu\right)$.

As is clear, the case where (4.2) holds is not interesting since there is no trade-off between individual incentives and firms profits. If (4.2) holds entrepreneurs are able to reach their "first best" profits $\pi_{fb}$ and $(\alpha_1, \mu_1)$ is always the optimal debt portfolio. Therefore in what follows we suppose that (4.2) does not hold. Entrepreneurs profits[7] then write as $\pi_1 = (1 + \mu_1) R - \mu_1 r_l$.

#### 4.2.2. The risky financing strategy.

When lenders ask the entrepreneur to pay for $\alpha\mu r_s$ then the entrepreneur is able to carry out his project in the production technology if and only if $\alpha\mu r_s \leq \eta (1 + \mu)$ while when lenders ask the entrepreneur to pay only for $\beta\mu r_s$ then the entrepreneur is able to carry out his project in the production technology if and only if $\beta\mu r_s \leq \eta (1 + \mu)$. Therefore when

$$\mu\beta r_s \leq (1 + \mu) \eta < \alpha\mu r_s \tag{4.3}$$

there are multiple equilibria: on the one hand the roll-over decision of lenders at the interim date determines whether entrepreneurs are able or not to carry out their long term projects till maturity while on the other hand the capacity of entrepreneurs to carry out their long term projects till maturity determines whether lenders decide to roll-over their short term contracts or not.

Let us note $p$ the probability that lenders decide to ask for full repayment of short term debts. This means that lenders ask entrepreneurs with a probability $p$ to pay for $\alpha\mu r_s$ and with a probability $1 - p$ to pay for $\beta\mu r_s$. Then entrepreneurs' expected profit writes as

$$E_t w_{t+2} = [(1 - p) [(1 + \mu - \beta\mu r_s) R - (1 - \beta) \mu r_l] + p [(1 + \mu - \alpha\mu r_s) r - (1 - \alpha) \mu r_l]] w_t$$

[7] On can verify that when (4.2) holds, $\min \beta (\alpha_1, \mu_1) = 0$. This means that all short term debts entrepreneurs contract are rolled-over.

Therefore the program[8] of the entrepreneur writes as

$$P_2: \quad \begin{array}{l} \max\limits_{\alpha,\mu} \mu.\left[R_p - r_l - \beta\left(r_s R - r_l\right)(1-p)\right] \\ \text{s.t.} \quad \begin{cases} \mu \leq \min\left\{\frac{\tau}{[\alpha r_s - \tau]^+}; \frac{\tau r}{[(1-\alpha) r_l + \alpha \tau r r_s - \tau r]^+}\right\} \\ \beta \mu r_s \leq (1+\mu)\,\eta < \alpha \mu r_s \end{cases} \end{array}$$

where $R_p = (1-p)\,R + pr$. The solution then writes as $\mu_2 = \mu_{fb}$ and $\alpha_2 = \alpha_{sb}$. Therefore firms optimal expected profits write as

$$\pi_2(p) = (1+\mu_2)\,R_p - \mu_2 r_l$$

Let us note strategy $i$ the solution to program $P_i$. Then we have the following proposition.

**Proposition 2.** *When (4.2) holds, entrepreneurs always choose strategy* 1. *When (4.2) does not hold, entrepreneurs choose strategy* 1 *if* $p > q$ *and stategy 2 if* $p < q$ *with* $q = \frac{\mu_2 - \mu_1}{1+\mu_2}\frac{R-r_l}{R-r}$.

Proof. Comparing $\pi_1$ and $\pi_2$ yields the proposition.

When the production technology is sufficiently illiquid, i.e. (4.2) is not verified, then entrepreneurs simply take financial decisions according to the risk of liquidation they anticipate. If an entrepreneur anticipates a low probability of roll-over, i.e. a high probability that a run will occur, on his short term liabilities, then he finances his investment with the maximum proportion of short term debt compatible with no run on his short term liabilities. In the other case where the roll-over probability is high then entrepreneurs choose the proportion of short term debt that guarantees a full roll-over in case of prorogation.

We now raise the question of how sustainable the situation of asset-liability maturity mismatch can be in a macroeconomic framework. The following section tries to answer this question.

[8] In this case it is incentive compatible for entrepreneurs to pay for their long term debts even in the case where they are compelled to liquidate their long term project. If we considered the case in which entrepreneurs pay for their long term debts if and only if they are able to carry out till maturity their long term project then it can be easily shown that the latter situation is always dominated by the former because the entrepreneur has to pay for default costs $\tau' r$ while there are no benefits as to the optimal debt portfolio (which size is still equal to $\mu_2$) or as to long term interest rates (which are priced at $\frac{r_l}{1-p}$ since the repayment probability is then equal to $1-p$).

### 4.3. Runs on short term debt and the equilibrium of the capital market.

To answer the question of whether the amount of short term debt accumulated in the economy is sustainable or not, we define what is a run on short term liabilities and how lenders coordinate to run or not.

**Definition 1.** *In a run on short term debt, lenders ask borrowers to pay for all short term debts whose repayment may change projects returns. The ex ante probability that a run happens is the ratio of the amount of short term debts subject to run to the amount of capital available for potential refinancing.*

This definition first implies that lenders never run on projects financed with debt portfolios $(\alpha_1, \mu_1)$. Runs on short term debt are possible if and only if there are projects financed with portfolios $(\alpha_2, \mu_2)$. Secondly if we note $w_e$ the entrepreneurs wealth, $w_l$ the lenders wealth, $\nu$ the proportion of entrepreneurs who play strategy 2 and $\delta = \frac{w_l}{w_e}$, then the amount of short term debts subject to run and the amount of potential refinancing respectively write as

$$\nu r_s (\alpha_2 - \beta_2) \mu_2 w_e$$

$$r [\delta - (1-\nu) \mu_1 (1-\beta_1) - \nu \mu_2 (1-\beta_2)] w_e$$

We still have to determine $\nu$, i.e. the type of equilibrium (pure or mixed strategy) which appears. The following proposition gives the precise conditions on the type of equilibrium which emerges.

**Proposition 3.** *The equilibrium of the capital market always exists and is always unique. The probability $p$ that a run on short term debt happens and the share $\nu$ of entrepreneurs who adopt the risky strategy are given by*

$$\{p(\delta), \nu(\delta)\} = \begin{cases} \left\{\frac{\alpha_2 \mu_2}{\delta - \mu_2}; 1\right\} \ \textit{if}\ \delta \geq \mu_2 + \frac{\alpha_2 \mu_2}{q} \\ \left\{q; \frac{\delta - \mu_1}{\left(\frac{\alpha_2 + q}{q}\right)\mu_2 - \mu_1}\right\} \ \textit{if}\ \mu_1 < \delta \leq \mu_2 + \frac{\alpha_2 \mu_2}{q} \\ \{0; 0\} \ \textit{if}\ \delta \leq \mu_1 \end{cases}$$

Proof. c.f. appendix.

There are three types of possible equilibria. First there can be a pure strategy equilibrium where all entrepreneurs choose strategy 2 and the ex ante probability that a run happens is $\frac{\alpha_2\mu_2}{\delta-\mu_2}$. Second there can be a mixed strategy equilibrium where a proportion $\nu = \frac{\delta-\mu_1}{\left(\frac{\alpha_2+q}{q}\right)\mu_2-\mu_1}$ of entrepreneurs choose strategy 2. Then the probability that a run happens is $q$. Thirdly there can be a pure strategy equilibrium where all entrepreneurs borrow $\delta$ per unit of own capital and the probability of a run on short term debt is zero.

## 5. Growth and macro-economic fluctuations.

### 5.1. The theoretical framework.

The average growth rate of the economy and the standard deviation of the growth rate can be computed as functions of the wealth distribution $\delta$. Given the last proposition, the following expressions can be obtained.

**Proposition 1.** *If $\delta > \mu_1$ the average gross growth rate of the economy $g_t$ and the variance of the gross growth rate respectively write as*

$$Eg_t = \frac{1}{1+\delta}\left[(1+\mu_2)\left[p(\delta)r + (1-p(\delta))A\right] + (\delta-\mu_2)r^2 - (\mu_2-\mu_1)(1-\nu(\delta))\left(r^2-r\right)\frac{A-R}{R-r}\right]$$

$$\operatorname{var}(g_t) = p(\delta)(1-p(\delta))\left(\frac{\nu(\delta)}{1+\delta}(1+\mu_2)(A-r)\right)^2$$

*If $\delta \leq \mu_1$ then $Eg_t = A$ and* $\operatorname{var}(g_t) = 0$.

Proof: c.f. annexes.

These expressions can be interpreted as follows. The expected growth rate is the sum of two terms: $\frac{(\delta-\mu_2)r^2+(1+\mu_2)(A-(A-r)p(\delta))}{1+\delta}$ is the pure strategy equilibrium expected growth rate and $(1-\nu(\delta))\frac{(\mu_2-\mu_1)\left(r^2-r\right)}{1+\delta}\frac{A-R}{R-r}$ is the growth loss induced by the mixed strategy equilibrium. This loss is due to the fact that the threshold probability $q$ is to low from a social point of view: a social planner who takes into account all the added value of the project $A$ and not only the capital share $R$ would choose the risky financing strategy for a larger short term debt run probability. As to the growth rate variance it depends only upon the volatility of investments made in the production technology and financed with risky debt portfolios. At this stage, it is possible to study the variation of the expected growth rate $Eg_t$ against the volatility of the growth rate var $(g_t)$. To these end we establish the following proposition.

**Proposition 2.** *In the mixed strategy equilibrium, expected growth decreases with $\delta$ and growth volatility increases with $\delta$. In the pure strategy equilibrium case, expected growth increases with $\delta$ if and only if $\delta < \mu_2 + z_1$ and growth volatility increases with $\delta$ if and only if $\delta < \mu_2 + z_2$.*

Proof. c.f. appendix.

In the pure strategy equilibrium case, an increase in $\delta$ has two effects. First it increases the proportion of the macroeconomic capital stock invested in the liquid technology which decreases the expected growth rate because the liquid technology has a lower return. Second an increase in $\delta$ reduces the probability that a run on short term debt occurs because the refinancing possibilities of lenders are larger and this increases the expected growth rate. The proposition says that the first effect dominates for large values of $\delta$ while the second effect dominates for low values of $\delta$. Growth volatility is also non monotonic w.r.t. $\delta$ because an increase the probability that a run on short term debt happens can increase or decrease growth volatility. In the mixed strategy equilibrium case, an increase in $\delta$ also has two effects. First as previously it increases the proportion of the macroeconomic capital stock invested in the liquid technology which decreases the expected growth rate because the liquid technology is less efficient. Second an increase in $\delta$ increases the proportion of entrepreneurs who choose the risky financial strategy which increases the expected growth rate. However the first effect always dominates the second one. Growth volatility on the contrary is only influenced by the second effect. Therefore when $\delta$ is low ($\delta \leq \mu_2 + \frac{\alpha_2}{q}\mu_2$), the economy experiences mixed strategies equilibria and the correlation between growth volatility and average growth is negative. On the contrary when $\delta$ is large ($\delta \geq \mu_2 + \max\{z_1, z_2\}$), the economy experiences pure strategy equilibria and the correlation between growth volatility and average growth is positive.

Insert figure 4

### 5.2. Empirical evidence.

In order to test the validity of the growth volatility predictions of the model, we use data from two sources: The Penn world tables and the World Bank financial structure and economic development database. From the first source we get data on GDP. We use the GDP per capita in PPP as a measure of output per capita. We compute the growth rate of this variable and the mean and the standard deviation of the GDP per capita growth rate. From the financial structure and economic development database, we measure $\delta$ (the ratio of the financial sector to the non financial sector assets) with two proxies: the amount of liquid liabilities to GDP or alternatively the sum of financial intermediaries (central bank, deposit money banks and other financial institutions) assets to GDP. The model predicts that growth volatility is negatively related to average growth in countries where the financial sector assets are relatively small but positively related to growth in countries where the financial sector assets are relatively large. To test empirically this prediction, we estimate the volatility of the GDP per capita growth rate as a function of the average GDP per capita growth rate, a proxy for $\delta$ and an interaction term between these to last variables. To confirm the model, we need that the coefficient of the average GDP per capita growth rate be negative while that of the interaction term be positive. Finally in line with previous empirical volatility studies we introduce a "catch-up" effect through the level of the GDP per capita growth rate which is meant to capture that more developed economies are always less volatile. The econometric results follow.

Table 2. Dependent variable: standard deviation of GDP per capita growth.

| Estimation | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $g_{i,t}$ | -0,25 | -0,23 | -0,38 | -0,37 | -0,11 | -0,18 | -0,24 | -0,31 | -0,00 | -0,04 | -0,04 | -0,07 |
| $ll_{i,t}$ ($\times 100$) | | | -0,42 | -0,32 | | | -0,43 | -0,27 | | | -0,11 | -0,02 |
| $g_{i,t} * ll_{i,t}$ | | | 0,44 | 0,39 | | | 0,28 | 0,33 | | | 0,10 | 0,11 |
| $\log y_{i,t}$ ($\times 100$) | | -0,31 | | -0,19 | | -0,16 | | -0,18 | | -0,06 | | -0,08 |
| Adj. R-square | 0,83 | 0,87 | 0,62 | 0,63 | 0,34 | 0,34 | 0,61 | 0,46 | 0,12 | 0,15 | 0,13 | 0,17 |

Table 3. Dependent variable: standard deviation of GDP per capita growth.

| Estimation | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $g_{i,t}$ | -0,45 | -0,39 | -0,64 | -0,59 | -0,27 | -0,35 | -0,30 | -0,52 | -0,02 | -0,08 | -0,08 | -0,13 |
| $fia_{i,t}$ ($\times 100$) | | | -0,23 | -0,32 | | | -0,28 | -0,22 | | | -0,24 | -0,18 |
| $g_{i,t} * fia_{i,t}$ | | | 0,40 | 0,35 | | | 0,16 | 0,24 | | | 0,05 | 0,08 |
| $\log y_{i,t}$ ($\times 100$) | | -0,31 | | -0,04 | | -0,27 | | -0,31 | | -0,12 | | -0,09 |
| Adj. R-square | 0,87 | 0,66 | 0,81 | 0,62 | 0,72 | 0,61 | 0,56 | 0,74 | 0,12 | 0,24 | 0,29 | 0,32 |

Note: In Table 2 and 3, estimations 1-4 contain individual and time effects, estimations 5-8 contain fixed effects only and estimations 9-12 contain time effects only. In table 2, the sample includes 71 countries, 4 periods and 261 observations. In table 3, the sample includes 39 countries 4 periods and 136 observations. Both samples are unbalanced. Each time period covers 10 years from 1961 to 2000 on which the mean and standard deviation of the GDP per capita growth rate are computed. All equations have been estimated with an intercept and under the assumption of heteroscedastic residuals. The GDP per capita growth rate during the relevant period in country $i$ is $g_{i,t}$. The amount of liquid liabilities to GDP is $ll_{i,t}$, the ratio of financial intermediaries assets to GDP is $fia_{i,t}$ and $y_{i,t}$ is the level of GDP per capita in PPP. Begining of period values have been considered for these last three variables. Rows on which a ($\times 100$) has been added after the variable name indicates that the coefficient reported is one hundred times the estimated parameter in the regression. All reported coefficients are significant at the 1% level apart from those in small characters which are not significant at the 5% level. The adjusted R square reported is weighted.

These estimations give us four results. First the simple correlation between the size of financial intermediaries (measured by $ll$ or $fia$) and volatility is always negative. Second the correlation between the development level (measured by the log of GDP per capita) and volatility is also always negative. Thirdly the simple correlation between growth and volatility is also always negative. Finally the interaction term between growth and financial intermediaries assets always has a positive influence on volatility. Therefore the econometric results confirm the predictions of the model as to the growth volatility relationship: it is negative in economies where financial intermediaries have a low level of assets relatively to the rest of the economy while it is positive in economies where financial intermediaries have a high level of assets relatively to the rest of the economy. These estimations also show that an increase in financial intermediaries assets relatively to the rest of the economy reduces volatility, every thing else equal, if and only if average growth is sufficiently low. In other words in economies with large average growth rates, financial development is likely to increase growth volatility.

### 5.3. Foreign Direct Investment and financial capital flows.

Since growth volatility and average growth are functions of the ratio of lenders to borrowers wealths $\delta = w_l/w_e$. any economic policy which modifies this ratio influences growth and fluctuations. For example foreign direct investments inflows have a positive effect on $\delta$ while financial capital inflows have a negative effect on $\delta$. Given the results we obtained in the last proposition, FDI has a stabilizing role, i.e. decreases volatility, in economies where the amount of financial intermediaries assets is small relatively to the rest of the economy. Aghion, Bachetta and Banerjee [1998] obtain a somewhat similar result. On the contrary financial capital inflows have a destabilizing role in those economies. This is because an increase in the relative size of the financial sector prompts firms to borrow relatively more in short maturities. This increases the difference between assets and liabilities maturity and increases the growth volatility. Schmukler and Vesperoni [2003] identify this mechanism empirically. They show that financial liberalization contributes, every thing else equal, to shortening the average maturity of firms debts.

Insert figure 5

## 6. Conclusion.

In this paper we have shown that macroeconomic fluctuations in the form of liquidity crises can emerge endogenously when financial contracts are imperfectly enforceable. Imperfect enforceability creates a bias towards short term debt because lenders use this financial instrument to overcome the possibility borrowers have to default strategically. However this bias generates maturity mismatches between assets and liabilities which can lead to global liquidity crises when projects are illiquid. Based on this microeconomic mechanism, we have obtained some theoretical results as concerns the correlation between growth volatility and average growth showing that it is positive in economies where lenders are relatively well-endowed but negative in economies where are relatively ill-endowed. Moreover some empirical evidence has been brought which seems to confirm this view. This gives a new insight to the growth volatility debate showing that neither polar conception is likely to be coherent with the data.

## 7. Appendix.

### 7.1. Tables and figures.

| | Pakistan | Thailand | Argentina | Korea | Mexico | India | Brazil | Malaysia | S. Afri. |
|---|---|---|---|---|---|---|---|---|---|
| D-E ratio | 0,999 | 0,915 | 0,328 | 2,485 | 0,342 | 0,546 | 0,934 | 0,114 | 0,079 |
| C. ratio | 0,993 | 1,143 | 1,195 | 1,078 | 1,303 | 1,438 | 1,275 | 1,296 | 1,441 |
| Q. ratio | 0,51 | 0,697 | 0,747 | 0,773 | 0,89 | 0,904 | 0,911 | 0,913 | 0,937 |
| | Hong Kong | Philippines | Venezuela | China | Peru | Colombia | Taiwan | Sri Lanka | U.S.A. |
| D-E ratio | 0,42 | 0,239 | 0,411 | 0,553 | 0,1 | 0,467 | 0,195 | 0,277 | 0,16 |
| C ratio | 1,352 | 1,37 | 1,559 | 1,321 | 2,396 | 1,684 | 1,587 | 1,555 | 2,097 |
| Q ratio | 0,947 | 0,961 | 0,964 | 0,968 | 0,975 | 0,979 | 1,037 | 1,087 | 1,385 |

Table 1 : Aggregate financial indicators (median) for non financial firms[9].

[9] Source : Claessens, Djankov et Nenova [2000]. D-E ratio refers to the debt equity ratio which is the ratio of total debt to the market value of the firm. C. ratio refers to the current ratio which represents the ratio between assets and liabilities with a maturity below one year. Q. ratio refers to the quick ratio is the ratio of current assets less inventories to current liabilities. Data are for 1995-1996.

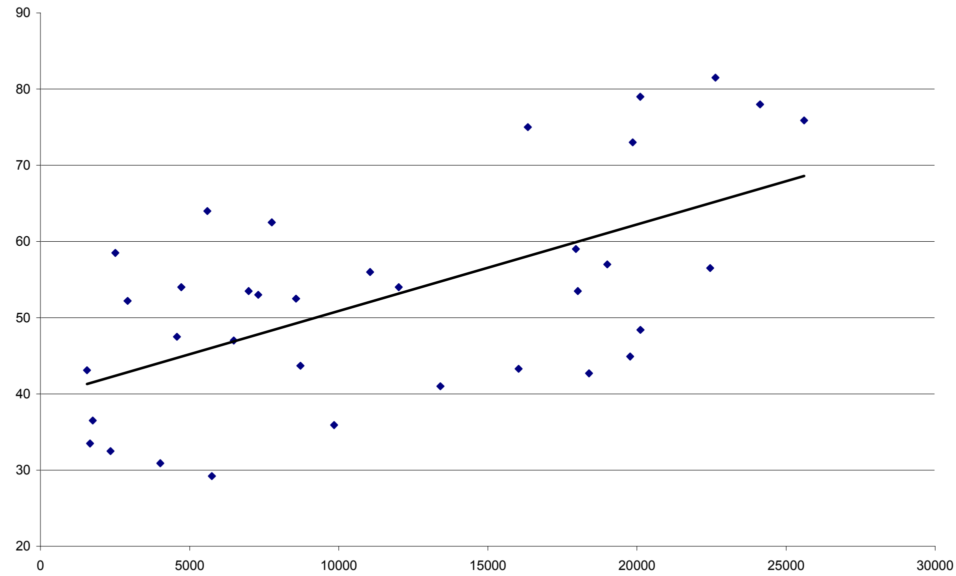


Figure 1 : PPP income per capita vs. proportion of long term debt [10].

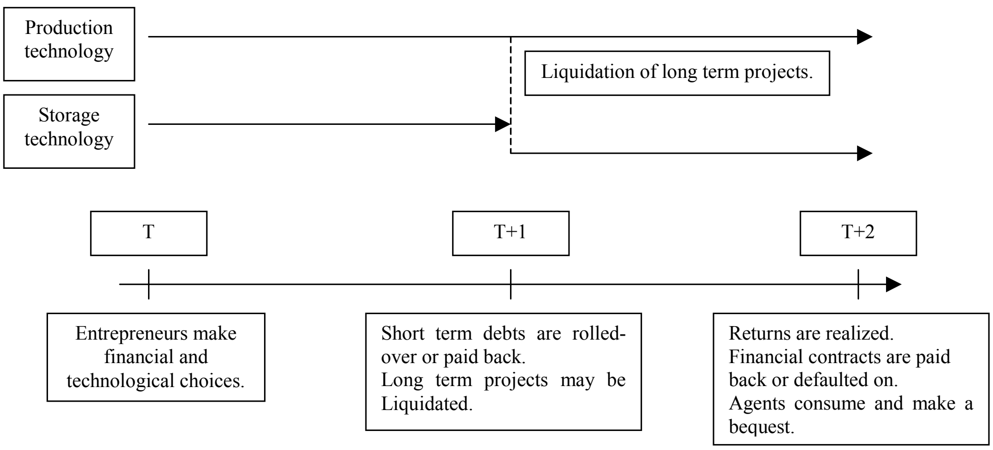


Figure 3 : Timing of the model.

[10] Source : Claessens, Djankov and Lang [1998] and Penn World Tables 6.1. Each point represent a the median debt portfolio for non financial firms in a given country. The income per capita in 1988 in PPP is on the x-axis and the median proportion of long term debt for non financial firms for 1988-1996 is on the y-axis.

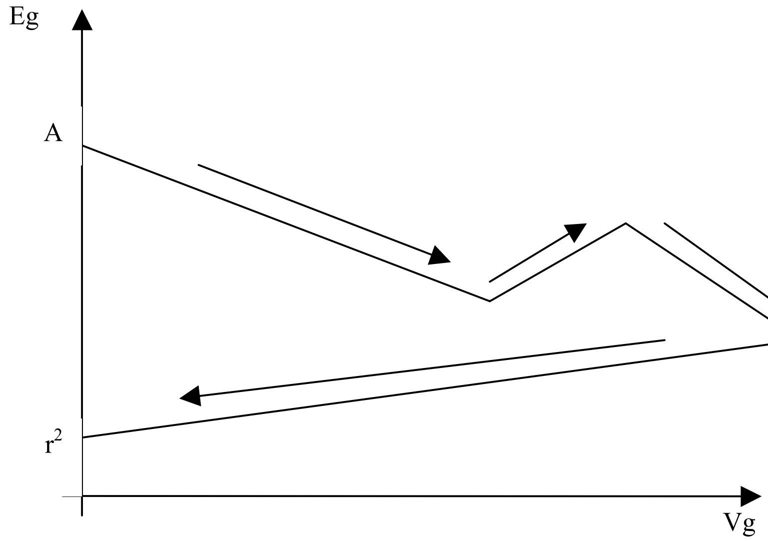


Figure 4: Growth Mean Volatility Diagram[11].

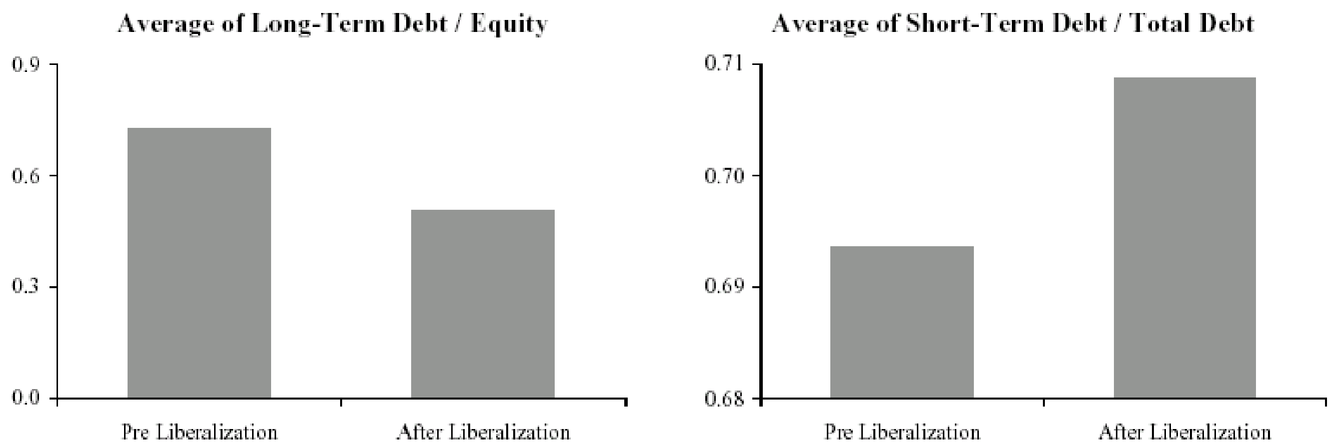


Figure 5: Financial Liberalization, debt and maturity. Source: Schmukler and Vesperoni [2003]

### 7.2. Incentive compatible contracts.

Let us consider a contract $(\alpha, \mu)$. This contract must be such that entrepreneurs are better-off when they pay for their long term debts than when they default. When an entrepreneur decides to pay for his long

[11] Arrows indicate the effect of a positive change in $\delta$ on average growth and growth volatility.

term debts, he carries out his project in the production technology till maturity and his final profit is equal to

$$\pi_{pro} = (1 + \mu - \alpha \mu r_s) R - (1 - \alpha) \mu r_l$$

On the contrary if he decides to default then he turns to the storage technology interim and his final profit is equal to

$$\pi_{liq} = (1 + \mu - \alpha \mu r_s) (1 - \tau') r$$

Contracts which ensure that entrepreneurs pay for their long term liabilities need that $\pi_{pro} \geq \pi_{liq}$ which simplifies as $\mu \leq \frac{\overline{\sigma}}{[(1-\alpha)r_l + \alpha r_s \overline{\sigma} - \overline{\sigma}]^+}$ with $\overline{\sigma} = R - (1 - \tau') r$. The incentive constraint which ensures that entrepreneurs pay for their short term debt writes as $\mu \leq \frac{\tau}{[\alpha r_s - \tau]^+}$. Then incentive compatible contracts $(\alpha, \mu)$ write as

$$\mu \leq \min \left\{ \frac{\tau}{[\alpha r_s - \tau]^+}; \frac{\overline{\sigma}}{[(1 - \alpha) r_l^* + \alpha r_s \overline{\sigma} - \overline{\sigma}]^+} \right\}$$

**7.3. Incentive compatible short term debt roll-over.**

Let us consider the case of an entrepreneur who carries out a project in the production technology with a debt portfolio $(\alpha, \mu)$. Then it is incentive compatible to exchange this portfolio against a portfolio $(\beta, \mu)$ if and only if

$$R (W + L - \beta r_s L) - (\alpha - \beta) r_{l,s} L - (1 - \alpha) r_l L \geq (1 - \tau) R ((W + L) - \beta r_s L)$$

If we note $\mu = \frac{L}{W}$, and under the assumption that $r_{l,s} > \tau R r_s$ this last expression can be simplified as

$$\beta \geq \frac{1}{r_{l,s} - \tau R r_s} \left[ \alpha r_{l,s} + (1 - \alpha) r_l - \tau R \frac{1 + \mu}{\mu} \right]^+$$

In this case the entrepreneurs debt portfolio $(\alpha, \mu)$ becomes $(\beta, \mu)$.

### 7.4. Equilibrium of the capital market.

To determine the probability of a run on short term debt at the equilibrium, we need to write down the probability that is generated by entrepreneurs best response functions. Entrepreneurs best response functions write as

$$(\alpha^*, \mu^*) = \begin{cases} (\alpha_1, \mu_1) \text{ if } p > q \\ (\alpha_2, \mu_2) \text{ if } p < q \end{cases}$$

Given this function the resulting probability $\Gamma$ that emerges from entrepreneurs choices writes as

$$\Gamma(p) = \begin{cases} \frac{\alpha_2 \mu_2}{[\delta - \mu_2]^+} \text{ if } p < q \\ 0 \text{ if } p > q \end{cases}$$

Equilibria can then be identified with fixed points of the function $\Gamma(p)$. Since it is a non-increasing function of $p$, there is at most one fixed point an thereby one equilibrium. If $\frac{\alpha_2 \mu_2}{[\delta - \mu_2]^+} < q$ then there is a unique fixed point for $p = \frac{\alpha_2 \mu_2}{\delta - \mu_2}$. It is a pure strategy equilibrium where all entrepreneurs choose contracts $(\alpha_2, \mu_2)$ $(\nu = 1)$. This case is possible if and only if $\delta \geq \mu_2 + \frac{\alpha_2}{q}\mu_2$. On the contrary if $\frac{\alpha_2 \mu_2}{[\delta - \mu_2]^+} > q$ then $\Gamma$ has no fixed point and we look for mixed strategies equilibria. Given the definitions adopted as to how financial contracts determine the probability of a run on short term debts, a mixed strategies equilibrium is a proportion $\nu$ which solves the equation $q = \frac{\nu \alpha_2 \mu_2}{[\delta - \nu\mu_2 - (1-\nu)\mu_1]^+}$. Given that the right hand side is a continuous strictly increasing function in $\nu$ on $\left[0, \frac{\alpha_2 \mu_2}{\delta - \mu_2}\right]$ there is a unique solution to this equation.

$$\nu = \frac{\delta - \mu_1}{\frac{\alpha_2}{q}\mu_2 + \mu_2 - \mu_1}$$

This last case is possible if and only if $\mu_1 < \delta \leq \left(1 + \frac{\alpha_2}{q}\right)\mu_2$. Finally when $\delta \leq \mu_1$ entrepreneurs cannot collectively borrow nor $\mu_1$ nor $\mu_2$. The economy is short of financial capital. Then all entrepreneurs borrow

$\delta$ per unit of own capital and the probability that a run occurs is zero.

### 7.5. Expected growth and growth variance expressions.

The gross growth rate of the economy writes as

$$g_s = \frac{\nu\left(1+\mu_2\right) w_e A_s + (1-\nu)\left(1+\mu_1\right) w_e A + \left[w_l - \nu\mu_2 w_e - (1-\nu)\,\mu_1 w_e\right] r^2}{w_l + w_e}$$

where $A_s = r$ with a probability $p$ and $A_s = A$ with a probability $1-p$. This expression is valid if and only if $w_l - \mu_1 w_e > 0$. Then the average gross growth rate is equal to

$$Eg = \frac{w_e}{w_l + w_e}\left[\nu\left(1+\mu_2\right)\left[pr + (1-p)\,A\right] + (1-\nu)\left(1+\mu_1\right) A + \left[\frac{w_l}{w_e} - \nu\mu_2 - (1-\nu)\,\mu_1\right] r^2\right]$$

and the growth rate variance is equal to

$$Eg = p\,(1-p)\left(\frac{w_e}{w_l + w_e}\right)^2 \left[\nu\left(1+\mu_2\right)(A-r)\right]^2$$

### 7.6. Volatility and expected growth variations.

In the mixed strategy equilibrium we have

$$\frac{\partial\,\mathrm{E}\,g_t}{\partial\delta} = -\frac{\left(1+\mu_1\right)(A-r)}{\left(\frac{\alpha_2}{q}\mu_2 + \mu_2 - \mu_1\right)(1+\delta)^2}\left[\frac{\alpha_2}{q}\frac{A-r^2}{A-r}\mu_2 + \left(\mu_2-\mu_1\right)\frac{R-r^2}{R-r}\right]$$

This quantity is always negative: expected growth decreases with $\delta$. In the pure strategy equilibrium we have

$$\frac{\partial\,\mathrm{E}\,g_t}{\partial\delta} = \frac{\left(1+\mu_2\right)}{(1+\delta)^2}(A-r)\left[\left[\frac{1+\delta}{\delta-\mu_2} + 1\right] p\,(\delta) - \frac{A-r^2}{A-r}\right]$$

It is positive if and only if $\frac{A-r}{A-r^2} p(\delta)\left[1+\frac{\delta+1}{\delta-\mu}\right]>1$ which simplifies as $\delta-\mu_2<z_1$ with

$$z_1=\frac{A-r}{A-r^2}\alpha_2\mu_2+\sqrt{\left[\frac{A-r}{A-r^2}\alpha_2\mu_2\right]^2+\frac{A-r}{A-r^2}(1+\mu_2)\alpha_2\mu_2}$$

As to the variance of the gross growth rate, in the mixed strategy equilibrium we have

$$\frac{\partial\operatorname{var}(g_t)}{\partial\delta}=2\frac{\nu(\delta)}{(1+\delta)^3}q(1-q)((1+\mu_2)(A-r))^2\frac{1+\mu_1}{\mu_2-\mu_1+\frac{\alpha_2\mu_2}{q}}$$

which is always positive. In the pure strategy equilibrium we have

$$\frac{\partial\operatorname{var}(g_t)}{\partial\delta}=\frac{p(\delta)}{(1+\delta)^3}\left[\left[\frac{1+\delta}{\delta-\mu_2}+1\right](2p(\delta)-1)-1\right]((1+\mu_2)(A-r))^2$$

It is positive if and only if $[2p(\delta)-1]\left[1+\frac{\delta+1}{\delta-\mu}\right]>1$ which simplifies as $\delta-\mu_2<z_2$

$$z_2=\frac{1+\mu_2-4\alpha_2\mu_2}{6}+\sqrt{\left[\frac{1+\mu_2-4\alpha_2\mu_2}{6}\right]^2+\frac{2}{3}(1+\mu_2)\alpha_2\mu_2}$$

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