“Deep Pockets”, Research and Development Persistence and Economic Growth

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October 10, 2004

Abstract

This paper studies endogenous growth driven by an expanding variety of product where lenders limit credit up to the collateralizable value of existing patents. Due to R&D investment risk, there is a composition effect between innovative firms currently contrained and innovative firms anticipating future contraints (hence accumulating current profit and decreasing current debt). Results are: (i) patent behaviour is lumpy and show some persistence; (ii) the steady state aggregate debt/patent ratio is below the leverage ceiling due to the composition of current versus future financial constraints; (iii) this debt/patent ratio determines a leverage driven steady state growth of the economy.

Keywords: Endogenous Growth, Research and Development, Credit rationing.

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1. Introduction

The financing of investment and the financing of R&D investment in particular may affect economic growth[1], since innovation is viewed as a major factor of growth when monopoly rents provide incentives to entrepreneurs[2]. There is now considerable empirical evidence that variables related to financing constraints such as leverage and/or cash flow availability are correlated with R&D investment in several countries (see recent surveys by Bechetti and Sierra [2001] and Hall [2002][3]). Blundell, Griffith and Van Reenen [1999] provide a "deep pocket" argument stating that: "*A more traditional interpretation of the innovation-market power correlation is that failures in financial markets force firms to rely on their own supra-normal profits to finance the search for innovation. The availability of internal sources of funding ('deep pockets') are useful for all forms of investment, but may be particularly important for R&D*". Causality between cash flow and R&D investment goes both ways in the sense that pre-innovation rents as well as post innovation rents are related to R&D investment (Hall, Mairesse, Branstetter and Crepon [1999]). Patents, innovations and cash-flow are correlated, and patents and innovations show strong history dependence (Geroski, Van Reenen, Walters [2001]). However, most firms are not able to innovate every year so that R&D investment is often lumpy (Geroski, Van Reenen, Walters [1997]), a feature observed to a lesser extent for tangible investment at the plant level (Doms and Dunne [1998]).

Econometric evidence emphasizes the particular importance of adverse selection and moral hazard problems faced by providers of external finance for intangible and R&D investment which are fostered by several factors. R&D investment is riskier than physical investment and keeping private information may generate high returns. Knowledge of the specific research area is required for efficient ex ante selection and ex post control, which involves a costly investment for providers of external finance, such as venture capitalists (Keuschnigg [2004]). Banks often take tangible assets with an efficient second hand market as *ex ante* guarantees for loans. With incomplete debt contracts, collateral may limit credit, but R&D investment is mostly intangible.

This paper studies endogenous growth driven by an expanding variety of products (Romer [1990], Grossmann and Helpman [1991]) where R&D investment opportunities are stochastic and where lenders limit credit up to the collateralizable value of existing

[1]See Greenwood and Jovanovic [1990], Bencivenga and Smith [1991, 1993], King and Levine [1993], Saint-Paul [1993], Bose and Cothren [1996], De la Fuente and Marin [1996], Levine [1997], Blackburn and Hung [1998], Benhabib and Spiegel [2001], Tressel [2003], Keuschnigg [2004].

[2]See Schumpeter [1911], Aghion and Howitt [1992], Romer [1990], Grossman and Helpmann [1991].

[3]e.g. studies by Himmelberg and Petersen [1994], Blundell, Griffith and Van Reenen [1999], Hall, Mairesse, Branstetter and Crepon [1999], Geroski, Van Reenen and Walters [2001], Aghion, Bloom, Blundell, Griffith and Howitt [2002].

patents. Some features of capital market imperfections emphasized in business cycle theory (Kiyotaki and Moore [1997]) are applied to R&D driven growth. Innovative firms which are not currently constrained are expecting to be constrained in the future. Current and past profits finance R&D investment due to a debt ceiling constraint. In particular, profits are accumulated, "digging deep pockets" over periods where the innovator has no profitable ideas for R&D investment. The model departs from Kiyotaki and Moore [1997] small open economy credit cycle model in various ways: it is based on a closed economy with endogenous interest rate, the size of the aggregate capital stock is no longer fixed, but may grow over time, and, finally, expected monopoly rents on existing patents are used as the main collateral, so that they increase the value of collateral, the available amount of loans and economic growth.[4]

The paper serves three purposes: first, to provide a micro-economic underpinning which investigates into details how the dependance of innovations on past innovations increases with innovative rents *relatively more* than in standard expanding variety models of R&D; secondly, to model the joint consequences of R&D investment lumpiness *and* financial constraints on individual firms savings ("deep pockets"), on the aggregate leverage (or debt/patent ratio), and on financially constrained economic growth; and, finally, to model a growing economy where the rate of return of innovation is higher than its user cost, as generally observed, and where the growth of patents is a decreasing function of the interest rate, which is not the case in the standard R&D endogenous growth models.

Policy recommendations are in favour of the enforcement of patent protection and of longer patent duration, which may foster financially constrained growth, first, because they increase the flow of internal finance for innovators, and, secondly, because they increase the collateral ceiling of the stock of external funds. Another institutional policy, which has been much less advocated by economists than by lawyers, is related to improving bankruptcy laws and regulations so that efficient transfers of property rights over the income of patents could be possible for lenders at low cost: this is a way to rise the collateral ceiling constraint and then financially constrained growth.[5] With respect to the effects of more traditional R&D policy over time, an investment tax credit conditional to an effective R&D investment would have an immediate effect on innovation, whereas a decline of corporate income tax will exhibit delayed effects on R&D investment, due to the fact that a proportion of firms do not invest in R&D during the year of the tax downfall and may use these additional funds for future opportunities which may occur far ahead in time.

The paper is organized as follows. The microeconomic behaviors of agents are

[4]The collateral constraint effects on patent growth are likely to be even stronger in "Creative Destruction" models (Aghion and Howitt [1992]), see section 3.3.

[5]PatentRatings is a US service which provides ratings for patents as collateral information. The practice of lenders receiving a collateral assignment of a patent conditional upon the occurrence of continuing default is well known by law firms. For U.S. law, it requires to follow a number of practical tips to limit potential legal risks and legal costs (Schavey Ruff [2003]). See Murphy [2002] for proposals of law improvements for security interests in intellectual property.

described in section 2. Section 3 provides the conditions for steady state aggregate growth. Section 4 concludes the paper with a discussion of the results and related research.

2. The model

2.1. Households

A continuum of wage-earners, distributed on $[0, L]$, maximizes a constant intertemporal elasticity of substitution utility function discounted over an infinite horizon: $U_t = \sum_{\tau=0}^{+\infty} u(c_{t+\tau})\,(1+\rho)^{-\tau}$ with $u(c_t) = (c_t^{1-\sigma} - 1)/(1-\sigma)$ for $\sigma > 0$ and $\sigma \neq 1$ or with $u(c_t) = \ln(c_t)$ for $\sigma = 1$. Consumption at time t is c_t, the rate of time preference is ρ, the discount rate is denoted $\beta_0 = 1/(1+\rho)$, and the elasticity of substitution is $1/\sigma$. Households supply inelastically one unit of labor which is used in the final goods industry and a real wage rate w_t is paid. They have no disutility of labor. They lend to entrepreneurs and earn a rate of return r (the interest factor is denoted $R = 1 + r$) on their individual wealth b_{t-1}^h so that their wealth dynamics is given by $b_t^h = (1+r)b_{t-1}^h + w_t - c_t$. The initial wealth b_0^h is given and identical for all households. Then, optimal consumption growth g_c is given by $1+g_c = c_{t+1}/c_t = C_{t+1}/C_t = (\beta_0 R)^{\frac{1}{\sigma}}$, where $C_t = c_t L$ denotes aggregate consumption. The growth rate of consumption increases with the return on savings and decreases with the rate of time preference and the elasticity of substitution. Optimal consumption is $C_t = C_0(1+g_c)^t$ and U_0 is bounded if $(1+r)^{1-\sigma} < 1 + \rho$ $(g_c < r)$. For $\sigma \geq 1$ this condition is always fulfilled. For $0 < \sigma < 1$, the interest rate has to remain in the following range: $R \in]1+\rho, (1+\rho)^{1/(1-\sigma)}[=]1 + r_{\min}^c, 1 + r_{\max}^c[$.

2.2. Production of the final good

As in other "increasing product variety" models (Romer [1990], Grossman and Helpmann [1991]), the economy has three sectors of production: a final goods sector, whose price is taken as numeraire, an intermediate goods sector, whose output is used in the production of the final good and an R&D sector which discovers blue-prints allowing the creation of new intermediate goods. Producers of the final good operate in perfect competition. The final good Y_t is produced from labor and intermediate inputs, which are fully used up within the period and have to be bought again the next period. Intermediate goods are defined on a set $\{X(i), i \in [0, N_t]\}$. The quantity $X(i)$ is the amount of intermediate good i used in the production process of the final good. The value N_t represents the most recently invented intermediate good, so that the interval $[0, N_t]$ is the variety of intermediate goods available in the economy. Technical progress is described as the invention of new intermediate goods which adds to the range of intermediate goods already invented, and implies an increase of N_t over time.

Then, the constant return to scale production function of the final good is given by:

$$Y_t = AL^{1-\alpha} \int_0^{N_t} X(i)^\alpha di \text{ with } 0 < \alpha < 1. \tag{2.1}$$

The representative producer of final goods buys intermediate goods at given prices p_i. Producers can buy patents from innovators. The subscript t will not be precised when the period considered is not ambiguous. The representative producer of the final good demands a quantity of each intermediate input i, denoted $X(i)$. He maximizes profit taking the production function into account:

$$(X(i), L) \in ArgMax \left(Y - wL - \int_0^{N_t} p_i X(i) di \right), \tag{2.2}$$

which gives the first order conditions that marginal product has to equal the price for each input:

$$\alpha A L^{1-\alpha} X(i)^{\alpha-1} = p_i \text{ for } i \in [0, N_t] \tag{2.3}$$

$$(1-\alpha)\frac{Y}{L} = w. \tag{2.4}$$

The first equation leads to the following demand function for intermediate inputs:

$$X(i) = \left(\frac{\alpha A}{p_i}\right)^{1/(1-\alpha)} L \text{ for } i \in [0, N_t]. \tag{2.5}$$

2.3. Production of intermediate goods

Producers of intermediate goods act as monopolists, selling their goods to the producers of final commodities at a price which adds a mark-up to marginal costs. They have to pay a rent to the innovator for using his blueprints at each date t (an innovative firm has previously discovered N_t blueprints). Production of intermediate goods takes place at constant costs, which is assumed to be equal to the price of final output Y_t. Then the monopoly price is:

$$p_i = \frac{1}{\alpha} > 1. \tag{2.6}$$

The price is identical for all intermediate goods and the monopoly profit per intermediate good sold is:

$$\pi = \left(\frac{1-\alpha}{\alpha}\right) A^{1/(1-\alpha)} \alpha^{2/(1-\alpha)} L. \tag{2.7}$$

2.4. R&D Sector

Every period, a continuum of risk neutral entrepreneurs indexed by j (when necessary) distributed over the interval $[0, 1]$ are engaged in the R&D activity. They maximize the present value V_0 of non-negative consumption given by their dividends $d_t \geq 0$ discounted by the interest factor $R = 1 + r$, where r is the interest rate at which they can borrow or lend:

$$V_0 = \sum_{t=0}^{+\infty} d_t R^{-t} \tag{2.8}$$

They hold an initial endowment of a number of blueprints n_0 and they receive an initial endowment of consumption d_0. Like households, entrepreneurs are able to supply inelastically one unit of human capital ($h_t = 1$) in their own firm and they have no disutility of labor. They face a linearized cost function of R&D investment, $\widetilde{1}_{i_t>0} \cdot \widetilde{1}_{h_t=1} \cdot q \cdot \left(n_{t+1} - \left(1 - \widetilde{\delta}_t\right) n_t\right)$, where q is a technological cost parameter and $\widetilde{1}_{h_t=1}$ is a dichotomous variable equal to zero when the entrepreneur withdraws her specific human capital and else equal to one. Once the R&D activity has started, only the entrepreneur possesses the skills necessary to invent new designs from this input. There is multiplicative uncertainty on R&D investment returns π described by a random variable $\widetilde{1}_{i_t>0}$ known by the entrepreneur at the beginning of the period t. With probability θ ($0 < \theta \leq 1$), the inventor find a number of positive net present value ideas leading to a number of patents during period t ($\widetilde{1}_{i_t>0} = 1$, and such that $\widetilde{1}_{i_t>0}\ \pi = \pi$).[6] She is able to invest an amount i_t in R&D provided the rate of return of R&D investment π is at least equal to the user cost of R&D investment. With probability $1 - \theta$, the inventor has no positive net present value ideas for inventing new intermediate goods during this period ($\widetilde{1}_{i_t>0} = 0$, so that the marginal product of R&D is zero and always lower than the R&D user cost: $\widetilde{1}_{i_t>0}\ \pi = 0$). This captures the empirical observation that R&D investment is often lumpy (Geroski, Van Reenen, Walters [1997]). Finally, the innovative firm faces the threat of obsolescence and/or opposition and litigation due to a "close" prior innovation and/or imitation of a random proportion $\widetilde{\delta}_t$ of their stock of their existing patents ($0 \leq \widetilde{\delta}_t < 1$) which reflects failures in the enforcement of patent protection and which is independently and identically distributed across inventors with $E_{t-1}\ \widetilde{\delta}_t = \delta$ (because there is a large number of inventors, there is no aggregate uncertainty).[7] Then the entrepreneur does not invest in R&D. This leads to the following equation of motion of the stock of blueprints:

[6] See Hall and Ziedonis [2001] for the case of a sector with cumulative innovation (the semi-conductor industry) with an increasing average rate of patents per firm, which followed a strengthening of patent rights.

[7] See Barney for 1996 U.S. patents mortality rates computed over a large sample: patent life expectancy varies from 7.6 years to 18 years.

$$n_{t+1} = \widetilde{1}_{i_t>0} \cdot \widetilde{1}_{h_t=1} \cdot i_t + \left(1 - \widetilde{\delta}_t\right) n_t \tag{2.9}$$

where i_t is the number of new blueprints obtained in a period. An entrepreneur has always the ability to threaten its creditors by withdrawing her human capital input, consume the credit and its interest payments Rb_t (where b_t is the stock of debt), repudiate its debt contract and find other creditors for next periods (Hart and Moore [1994], Kiyotaki and Moore [1997]). Under the assumption that observing $\widetilde{1}_{i_t>0}$ and $\widetilde{1}_{h_t=1}$is too expensive for creditors, new lenders are not able to state ex-post if no investment at last period was due to the lack of investment opportunity or to the withdrawing of specific human capital. Creditors (households) protect themselves by collateralizing the stock of existing blueprints over which the firm has a monopoly rent and take care never to let the size of the debt repayment to exceed the liquidation value of the stock of patent next period after depreciation. The innovative firm receives a rent on each period on each of the blueprints it has discovered on previous periods. The rents received at date t are summed over his stock of blueprints at date $t-1$ and amount to πn_{t-1}. The market value of the blueprint is equal to the discounted flow of profits gained on the production of an intermediate good.[8] Lenders take into account that an expected proportion $\delta = E_t\left(\widetilde{\delta}_t\right)$ of the patents income is lost at each period depending on the enforcement of patent protection, so that the discount rate is increased by a factor δ (they can diversify the obsolescence risk by lending to many entrepreneurs).[9] The credit constraint is then expressed as:

$$Rb_t \leq (1-\mu)\left(\pi n_t + \frac{\pi(1-\delta)n_t}{R} + ...\right) = (1-\mu)\pi n_t \sum_{\tau=0}^{+\infty}\left(\frac{1-\delta}{R}\right)^{\tau} = (1-\mu)\frac{\pi}{r+\delta}Rn_t \tag{2.10}$$

Lenders may loose a proportion $(1-\mu)$, with $0 \leq \mu < 1$, of collateral because of legal or bankruptcy costs related to the change of property rights over the patents.[10] This credit constraint eliminates the incentive for entrepreneurs to withdraw human capital in order to gain the income Rb_t from lenders, so that $\widetilde{1}_{h_t=1}= 1$, in the innovators optimization program. The credit constraint may also be written as a "leverage" or debt/patent ratio x_t bounded by an endogenous ceiling x^c (for homogeneity, the unit cost of R&D investment q multiplies patents):

$$x_t = \frac{b_t}{qn_t} \leq x^c = (1-\mu)\left(\frac{\pi}{q(r+\delta)}\right). \tag{2.11}$$

[8]See Hall, Jaffe and Trajtenberg [2005] for a recent econometric evaluation of the link between market value and the quality of patents.

[9]See Barney [2002] table 3 for the practical effects of Patents life expectancy on U.S. Patents average expected value and Patent Ratings.

[10]For understanding what represents μ in the practice of U.S. law, see Schavey Ruff [2002] presentation of the legal costs for lenders receiving a collateral assignment of a patent.

In the remaining part of the paper, assumption A1 and A2 hold:

Assumption A1: $r > (1-\mu)\frac{\pi}{q} - \delta$. The interest rate has to be sufficiently high so that the debt/patent ceiling is strictly below unity ($x^c < 1$) and the credit constraint may be binding.

Households savings may or may not be collected by a large number of financial intermediaries facing perfect competition (zero profit), whose function is then limited to enforce the collateral constraint for each borrower. If there is no financial intermediaries, it is assumed that households perform this task.

On date t, entrepreneurs consumes at least a strictly positive amount of income $d_m n_{t-1}$ from the profits generated by previous patents πn_{t-1}. This assumption prevents the situation in which firms' owners continually postpone strictly positive consumption leading to a zero utility, that is entrepreneurs indifference between producing or not:

$$d_t \geq d_m n_{t-1} > 0 \ . \tag{2.12}$$

Assumption A2: $0 < d_m < (1-\mu)\pi < \pi$. The upper limit of minimal consumption of entrepreneurs is equal to net patents return net of bankruptcy costs $d_m < (1-\mu)\pi$. Proposition 1 will show that, when that innovators do not consume "too much" of their profits, then, when an innovator does not have a profitable idea, its firm leverage will decrease below the leverage ceiling x^c.

The innovative firm's flow of funds constraint states that dividends should be equal to the profits at date t earned from previously discovered blueprints, to which are added new debt net of interest repayment and substracted the cost investment in R&D:

$$d_t = \pi n_{t-1} + b_t - R b_{t-1} - \widetilde{1}_{i_t>0}\, q\left(n_t - \left(1 - \widetilde{\delta}\right) n_{t-1}\right) \tag{2.13}$$

Appendix one provides the details of the computations of first order conditions of the entrepreneurs utility maximization. The first order condition with respect to debt is:

$$\lambda_t^d = \lambda_t^b + E_t\left(\lambda_{t+1}^d\right) = \lambda_t^b + \sum_{k=1}^{k=T} E_t\left(\lambda_{t+k}^b\right) + E_{t+1}\left(\lambda_{t+T+1}^d\right) \tag{2.14}$$

where λ_t^b is the Lagrange multiplier related to the debt/patent ceiling constraint and where λ_t^d is the Lagrange multiplier of the miniminal consumption level constraint. The minimal consumption constraint is binding ($\lambda_t^d > 0$) when the firms faces currently of credit constraint ($\lambda_t^b > 0$) or when it expects to face a credit constraint in the future ($E_t\left(\lambda_{t+k}^b\right) > 0$ with k a strictly positive integer). The first order condition with respect to the stock of patents is:

$$\frac{\pi}{q} - (r+\delta) = (1-x^c)\frac{\sum_{\tau=0}^{\tau=+\infty}\beta^\tau E_t\left(\lambda_{t+\tau}^d - \lambda_{t+\tau+1}^d\right)}{\sum_{\tau=0}^{\tau=+\infty}\beta^\tau\left(1 + E_t\lambda_{t+\tau+1}^d\right)} + \frac{d_m}{qR}\frac{\sum_{\tau=0}^{\tau=+\infty}\beta^\tau E_t\lambda_{t+\tau+1}^d}{\sum_{\tau=0}^{\tau=+\infty}\beta^\tau\left(1 + E_t\lambda_{t+\tau+1}^d\right)}$$

with $\beta = (1-\theta)(1-\delta)/R$. One may remark that when $\lambda^d_{t+\tau} = 0$ for any integer τ, the corrected discount rate β as well as the probability of investment opportunities θ do not affect the first order conditions. The rate of return and the user cost of R&D investment are not affected by $\theta > 0$ measuring "lumpiness" because the entrepreneur discounts expected incomes from investment opportunities for all future dates in the infinite horizon, when she does not have an opportunity to invest now (see appendix 1). By contrast, when the Lagrange multiplier related to financial constraint binds ($\lambda^d_{t+\tau} > 0$), lumpiness matters. The first order conditions leads to the following proposition:

Proposition 1. Optimal R&D Investment, Saving and Borrowing at the Entrepreneur Level.

In each period, innovating firms can be in one of three regimes, depending on the aggregate equilibrium interest rate r:

(i) A perfect capital market regime is obtained when $\lambda^d_{t+\tau} = 0$ for any integer τ. In this case, the marginal gain of R&D investment is equal to its user cost: $\pi = q(r+\delta)$ so that $r = \frac{\pi}{q} - \delta$, the debt ceiling is never binding and debt policy is indeterminate and does not affect the investment outcome.

Regimes (ii) and (iii) are possible under the condition that the marginal gain of R&D investment is higher than its user costs $\pi > q(r+\delta)$, that is $(1-\mu)\frac{\pi}{q} - \delta < r < \frac{\pi}{q} - \delta$, taking also into account the fact that the leverage ceiling has to be below unity, cf. assumption A1. Parameter conditions for these regimes to occur will be derived later on when determining the aggregate steady state growth rate.

(ii) Currently binding credit constraint regime ($\lambda^d_t > 0$): Innovators having an opportunity to invest at date t will choose a binding debt ceiling and consume no more than its current (private) output of nontradable good ($d_t = d_m n_{t-1}$). Patents are determined by the flow of funds and the debt ceiling ratio leads to an equation of motion of patents for firms investing in R&D on period t which describes how leverage (b_t/n_t) amplifies *the dependance of last period stock of innovation (it amplifies the level of persistence of innovation, when the firm has an opportunity to invest):*

$$b_t = (1-\mu)\left(\frac{\pi}{r+\delta}\right) n_t \tag{2.15}$$

$$n_t = \frac{\left(1 + \frac{\pi - d_m}{q} - \widetilde{\delta}\right) n_{t-1} - R\frac{b_{t-1}}{q}}{1 - (1-\mu)\left(\frac{1}{r+\delta}\right)\frac{\pi}{q}} \tag{2.16}$$

(iii) Anticipated credit constraint regime ($\lambda^d_{t+\tau} > 0$ with at least one strictly positive integer τ): Innovators which have no opportunity for profitable investment now will also consume $d_t = d_m n_{t-1}$ as they expect to find new profitable ideas and to face a financial constraint in the future. Patents are determined by the low of motion. The flow of funds equations determines the law of motion of debt: retained earnings are used to reduce their current debt and they are accumulated for future R&D

investment.

$$n_t = \left(1 - \widetilde{\delta}\right) n_{t-1} \tag{2.17}$$

$$b_t = Rb_{t-1} - (\pi - d_m)\, n_{t-1} \tag{2.18}$$

It is easy to check that assumption A2 guarantees that leverage is below the leverage ceiling in this regime: $x_t < x^c$.

Proof. See appendix 1.

In the financially constrained regime, if an entrepreneur has a long history of no opportunity to invest in R&D, he may eventually become a net creditor and earn an interest payment R associated to loans to other entrepreneurs having an opportunity to invest (even before becoming a net creditor, the reward of one unit her savings is indeed the interest R that she avoided to pay by decreasing her debt by one unit). His debt b_t may be negative whereas his stock of patents n_t is always positive and may decline each period due to hazard of obsolescence $\widetilde{\delta}$ (see appendix 3 for a simulation) As described in the following section, the aggregate debt B_t is restricted to be positive, so that both households and entrepreneurs which are net creditor are *all* able to invest their savings in entrepreneurs projects. A positive aggregate debt B_t implies that the proportion of investing entrepreneurs θ is above a given floor. The aggregate adjustment of savings is such that, in the steady state, the lower the number of investing entrepreneurs θ, the higher the overall savings of non-investing entrepreneurs and the lower the growth rate of households savings.

Conversely, when an entrepreneur which has built "deep pockets" over an history of no opportunity to invest in R&D faces an opportunity to invest, she will invest *as much as possible* due to the combination of her linear cost function and the gap between the marginal return of R&D investment and the user cost. This lumpy R&D investment is then only limited by available internal funds and the collateral constraint on the stock of debt. It provides a micro-economic underpinning for Geroski, Van Reenen and Walters [1997] empirical findings. The dependance of the new stock of patents on the former stock of patent (persistence) is then higher when entrepreneurs enjoys a higher markup π rewarding their innovations. This result is found in Blundell, Griffith and Van Reenen [1999] when estimating R&D investment using a reduced form R&D cost function. However, the important point of this micro-economic underpinning is not the numerator of the persistence effect, because this effect is found in standard expanding variety models of endogenous growth (Romer [1990], Grossmann and Helpman [1991]) : $\frac{1+\frac{\pi-d_m}{q}-\widetilde{\delta}}{1-(1-\mu)\left(\frac{1}{r+\delta}\right)\frac{\pi}{q}}$. It is the denominator, which amplifies the overall sensitivity of the persistence effect with respect to the innovative rents π. More precisely, the collateral constraint implies that the higher the gap between the marginal profit of R&D and its user cost $\frac{\pi}{q(r+\delta)}$, (that is: the "deeper" the financial constraint) the higher the persistence of the stock of innovations. In other words, *pre innovation rents* enters the numerator of the right hand side of the patent relationship, provide a *flow of internal funds*, increase patents n_t and its sensitivity to the last period stock of innovations n_{t-1}, in a *backward looking* fashion. In a *forward looking* fashion, *post innovation rents* determine the *stock of external finance* (appearing at the denominator of the right hand side of the patent relationship) and amplifies the persistence effect through a leverage multiplier effect. The distinction between stocks and flows of internal funds and external funds when dealing with financial constraints turns to be particularly important.

3. R&D Persistence and Economic Growth

3.1. Aggregate Patent and Debt Dynamics

Given the optimal investment behavior and credit policy of firms described by proposition 1, one derives the equations of motion for the entrepreneurs aggregate patent and debt. Debt and patents equations are linear in patent and debt in both cases. One can appeal to the law of large number for the hazard of finding profitable ideas and the hazard of obsolescence and aggregate across entrepreneurs to derive the equations of motions of patents and debt without having to keep track of the distribution of the individual entrepreneurs patents and debt (Aggregate patents and debt are denoted by capital letters N_t and B_t). Since the population of entrepreneurs is unity, the equation of motion of the aggregate number of patents is:

$$N_t = (1-\theta)(1-\delta) N_{t-1} + \theta \left(\frac{\left(1 + \frac{\pi - d_m}{q} - \delta\right) N_{t-1} - R \frac{B_{t-1}}{q}}{1 - (1-\mu)\left(\frac{\pi}{q(r+\delta)}\right)} \right). \tag{3.1}$$

The agregation smoothes the lumpiness effects of R&D investment, which is then reflected by the proportion of investing firms θ. The flow of funds equality leads to the equation of motion of aggregate debt:

$$B_t = qN_t - q(1-\delta) N_{t-1} + RB_{t-1} - (\pi - d_m) N_{t-1}. \tag{3.2}$$

There are two aggregate consequences of lumpy investment at the individual level in financially constrained regimes, whereas there is no consequences in the perfect capital market case where the debt/patent ratio is indeterminate. First, the sensitivities of the aggregate stock of patent with respect to the innovative rents and with respect to the components of their user cost decline by a factor θ with respect to the case of investing individual entrepreneurs (see proposition 1). Second, the debt dynamics differs from the patent dynamics because profits are used either to decrease debt temporarily or to finance R&D investment. When $\theta = 1$, all firms do invest, debt is proportional to patents according to the debt ceiling constraint, so that debt dynamics is identical to patents dynamics by a proportionality factor.

The aggregate model is closed by the households consumption (or savings) growth rate $C_t = (\beta_0 R)^{\frac{1}{\sigma}} C_{t-1}$, where the interest rate corrects savings imbalances. As households consumption C_t does not show up in entrepreneurs patents and debt dynamics, it is natural to proceed in two steps to investigate the steady state regimes: first find R&D sector steady state aggregate debt/patent ratio (so that debt and patent grow as the same rate), then find the equilibrium interest rate such that consumption grows at the same rate as patents and debt. Let us proceed to the first step. The equation of motion of patents can be written as a patent growth factor G_N as a function of the debt/patent ratio x_{t-1}:

$$G_N = \frac{N_t}{N_{t-1}} = 1 - \delta + \theta \left(\frac{1 + \frac{\pi - d_m}{q} - \delta - R x_{t-1}}{1 - x^c} - (1 - \delta) \right).$$

The patent growth factor *increases* with the monopoly profits rewarding innovation net of entrepreneurs consumption $\pi - d_m$ and the proportion of investing entrepeneurs θ (as $1/(1 - x^c) > 1$) and *decreases* with the costs of R&D investment variables (debt service $R x_{t-1}$, technological cost q, expected rate of obsolescence δ) and with the transaction cost μ on patent future royalties. Define the profit factor net of innovators consumption and net of depreciation:

$$\Pi_m = 1 + \frac{\pi - d_m}{q} - \delta \tag{3.3}$$

The equation of motion of aggregate debt can be written as a debt growth factor G_B:

$$G_B = \frac{B_t}{B_{t-1}} = R + \frac{q N_t}{B_t} \frac{B_t}{B_{t-1}} - \Pi_m \frac{q N_{t-1}}{B_{t-1}} = \frac{R - \Pi_m \frac{q N_{t-1}}{B_{t-1}}}{1 - \frac{q N_t}{B_t}} = \frac{R - \Pi_m \frac{1}{x_{t-1}}}{1 - \frac{1}{x_t}}. \tag{3.4}$$

This debt growth factor can be written as a patent growth factor G_{NB}:

$$G_{NB} = \frac{x_t}{x_{t-1}} G_B = \frac{N_t}{N_{t-1}} = \frac{\Pi_m - R x_{t-1}}{1 - x_t}. \tag{3.5}$$

The patent growth factor G_{NB} is similar to the one of investing firms except that the leverage during the period is not necessarily maximal. It is equal to the growth factor of internal funds, with wealth net of debt at the denominator and retained earnings at the numerator. Because the equation of motion of debt and patent are valid at any date, the equality $G_{NB} = G_B$ holds at any date, which is an implicit aggregate debt patent dynamics:

$$(1 - \theta)(1 - \delta) + \theta \left(\frac{\Pi_m - R_{t-1} x_{t-1}}{1 - x^c} \right) = \frac{\Pi_m - R_{t-1} x_{t-1}}{1 - x_t}. \tag{3.6}$$

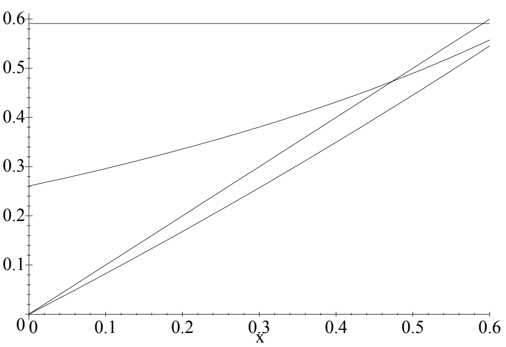
It may be written as an explicit aggregate debt/patent dynamics:

$$x_t = M\left(x_{t-1}\right) = 1 - \frac{1-x^c}{\theta} + \frac{1}{\theta}\frac{\left(1-x^c\right)^2\left(1-\theta\right)\left(1-\delta\right)}{\left(1-x^c\right)\left(1-\theta\right)\left(1-\delta\right)+\theta\Pi_m-\theta R_{t-1}x_{t-1}} \tag{3.7}$$

At the steady state, $x_t = x_{t-1} = x$ for $x \in]0, x^c]$ with $0 \leq x^c < 1$, as aggregate date has to be positive to avoid negative (savings) growth rate equilibria (whereas debt may turn to be negative for some entrepreneurs). Appendix 2 shows that under assumption A3, there is a unique steady state debt/patent ratio strictly positive.

Assumption A3: $\theta > \theta^x_{\min} = \frac{(1-x^c)\frac{\pi-d_m}{q}}{x^c(1-\delta)+\frac{\pi-d_m}{q}}$: The proportion of investing innovators at a given date is above a minimal threshold.

Figure 1 provides a graphical solution for the steady state debt/patent ratio when $r = 3\%, \frac{\pi}{q} = 13\%, \frac{d_m}{q} = 1\%, \delta = 8\%, \mu = 0.5$ for three values of the proportion of entrepreneurs investing in R&D: $\theta = 100\%$, or 30%, or $\theta^x_{\min} \approx 7,4\%$. The horizontal and vertical axis represents the debt/patent ratio.



The curve $y = x$ intersects the function $M(x, \theta = 1)$ which is an horizontal line leading to a steady state debt/patent ratio equal to the debt ceiling as all entrepreneurs invest in R&D; $x^* = x^c = 59\%$. It intersects the increasing curve $M(x, \theta = 0.3)$ for an aggregate steady state debt/patent ratio $x^* = 47\%$ when 30% of entrepreneurs invest in R&D. It intersects the increasing curve $M(x, \theta = \theta^x_{\min})$ for an aggregate steady state debt/patent ratio equal to zero when only 7.4% of entrepreneurs invest in R&D. Solving the steady state debt/patent ratio equation $x = M(x)$ and computing the growth rate of patent for this steady state debt patent ratio $G_N(x^*)$ leads to Proposition 2:

Proposition 2: R&D Sector Financially Constrained Steady State.

- Under assumptions A1 ($x^c < 1$), A2 (upper limit on d_m) and A3 (lower limit on θ) and the condition for a financial constraint, $(1-\mu)\frac{\pi}{q} - \delta < r < \frac{\pi}{q} - \delta$,
- a unique steady state patent and debt growth exist, with a constant strictly positive aggregate debt/patent ratio $0 < x^* \leq x^c$:

$$x^* = \frac{1}{2\theta R}\left\{\theta\left(R+\Pi_m\right) + \left(1-x^c\right)\left[\left(1-\theta\right)\left(1-\delta\right) - R - \sqrt{\Delta}\right]\right\} \text{ with:}$$

$$\Delta = \left\{ \left[(1-\theta)(1-\delta) - R \right](1-x^c) + \theta(\Pi_m - R) \right\}^2 + 4\theta(1-x^c)(\Pi_m - R) R \quad (3.8)$$

This steady state debt/patent ratio rises to the individual debt/patent ceiling if all innovators found profitable ideas and invested ($\theta = 1$) and increases with increases of the endogenous debt/patent ceiling x^c. The steady state aggregate debt/patent ratio increases with the proportion of investing firms θ and decreases with bankruptcy costs μ. It increases with monopoly rents rewarding innovation π and decreases with the unit R&D investment cost q, expected obsolescence rate δ and the marginal cost of debt r.

- (ii) The steady state growth of patents for a given interest rate, is then:

$$G_N(R, x^*) = \frac{1}{2}\left(R + (1-\theta)(1-\delta) + \frac{\theta(\Pi_m - R)}{1 - x^c(R)} + \frac{\sqrt{\Delta(R)}}{1 - x^c(R)} \right) \quad (3.9)$$

As it increases with the aggregate debt/patent ratio, the sensitivities described in (i) are qualitatively similar on the aggregate growth rate of patents. The steady state patent growth rate increases with the proportion of investing firms θ and decreases unambiguously with bankruptcy costs μ. It increases with monopoly rents rewarding innovation π and decreases with the unit R&D investment cost q, expected obsolescence rate δ and the marginal cost of debt r.

Proof. See appendix 2.

Remark: partial derivatives of the aggregate debt/patent ratio with respect to monopoly rents, obsolescence rate and the marginal cost of debt allow the possibility that below different thresholds of the proportion of investing firms, the signs may change because of the composition effect between firms. However, the condition of a positive steady state aggregate debt/patent ratio ($x^* > 0$ and $\theta > \theta^x_{\min}$) is rather strict and eliminated all the cases where slope reversals were found using simulations for standard values for rate of return and interest rates. As a consequence, partial derivatives of the aggregate growth rate of patents with respect to monopoly rents, obsolescence rate and the marginal cost of debt were changing signs only for *highly negative* growth rate values and values of θ below $\theta^x_{\min}$ when computing systematic simulations.

A final remark is that the curve $G_N(R)$ has a unique vertical asymptote defined $r = (1-\mu)\frac{\pi}{q} - \delta$ (for $x^c = 1$) and that it is locally decreasing with the interest rate when the interest rate is higher but closed to the asymptote.

3.2. Steady State Growth Rate of the Economy

Steady state equilibria are given by equality of the growth rate of consumption $G_C(R)$, of new goods $G_N(R, x, \theta)$ and of debt $G_B(R, x)$. This amounts to the equality between

$G_C(R)$ and $G_N(R, x^*, \theta) = G_B(R, x^*)$ which allows to compute the equilibrium real interest rate R^* related to a strictly positive growth rate ($r > \rho$) with bounded utility ($G < R$) under assumption A1 ($x^c < 1$) and the financially constrained condition: $(1-\mu)\frac{\pi}{q} - \delta < r < \frac{\pi}{q} - \delta$, under assumptions A2 (upper limit on d_m) and A3 (lower limit on θ), knowing that when the marginal gain on R&D is not larger than the user costs, a perfect capital market equilibrium occurs for $r = \frac{\pi}{q} - \delta$. That is:

$$H(R) = \frac{\Pi_m - Rx^*(R)}{1 - x^*(R)} - \left(\frac{R}{1+\rho}\right)^{1/\sigma} = 0.$$

As H is a continuous function of the interest rate for $(1-\mu)\frac{\pi}{q} - \delta < r$, there exist at least one strictly positive equilibrium interest rate leading to a financially constrained equilibrium when the two following conditions are fulfilled:

$$H\left(1 + \max\left(\rho, (1-\mu)\frac{\pi}{q} - \delta\right)\right) > 0 > H\left(1 + \frac{\pi}{q} - \delta\right). \tag{3.10}$$

The second inequality (financially constrained equilibrium) leads to:

$$1 + \frac{\pi}{q} - \delta - \frac{\frac{d_m}{q}}{1 - x^*\left(\frac{d_m}{q}\right)} - G_c\left(1 + \frac{\pi}{q} - \delta\right) < 0. \tag{3.11}$$

$G_c\left(1 + \frac{\pi}{q} - \delta\right)$ corresponds to the perfect capital market endogenous growth equilibrium. To have a financially constrained equilibrium, the marginal gain from R&D investment is higher than its user costs when the minimal consumption level of innovators is sufficiently high:

Assumption A4: A financially constrained steady state growth requires a minimal level on entrepreneurs consumption sufficiently high. This threshold on d_m declines when the transaction cost on the tranfer of property rights over patents income μ is higher and/or when the proportion of investing firms θ is lower and/or when the rate of time preference is lower ρ and/or when the elasticity of substitution σ is lower:

$$\frac{\frac{d_m}{q}}{1 - x^*\left(\frac{d_m}{q}, \mu, \theta\right)} > 1 + \frac{\pi}{q} - \delta - G_c\left(1 + \frac{\pi}{q} - \delta\right) \tag{3.12}$$

The first inequality (strictly positive growth) is necessarily fulfilled when $\rho < (1-\mu)\frac{\pi}{q} - \delta$, because $G_N(r)$ has a unique vertical asymptote defined $r = (1-\mu)\frac{\pi}{q} - \delta$ (corresponding to $x^c = 1$) and it is locally decreasing from infinity with the interest rate when the interest rate is higher and closed to this asymptote.

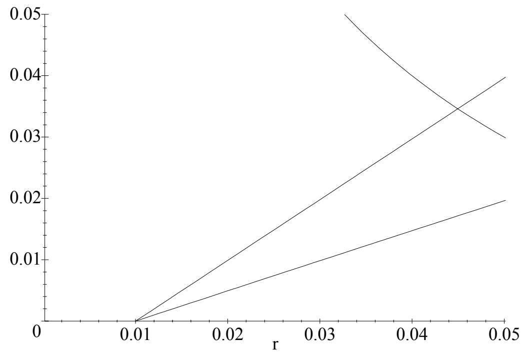
Assumption A5: When $\rho < (1-\mu)\frac{\pi}{q} - \delta$, a *positive* steady state growth rate requires an upper condition on d_m (the consumption of innovators should not be too high, else the growth rate of innovation will be negative due to negative retained earnings):

$$\frac{\frac{\pi}{q} - \frac{d_m}{q} - \delta - \rho x^* \left(\frac{d_m}{q}\right)}{1 - x^* \left(\frac{d_m}{q}\right)} < 0 \tag{3.13}$$

Proposition 3: Steady State Growth Regimes

- Under the above conditions A1 to A5, there exist at least one financially constrained equilibrium with an equilibrium interest rate R^* and a steady state growth rate G^* which is strictly positive and lower than the perfect capital market case. This growth rate decreases with the constant elasticity of substitution σ and the rate of time preference ρ and a rise of entrepreneurs consumption d_m.
- When the condition of higher gains than user costs is not fulfilled (A4), there exist a unique steady state growth rate $G_c\left(\frac{\pi}{q} - \delta\right)$ of endogenous growth which is not financially constrained (provided the standard condition for bounded utility is achieved).

Figure 2 below presents a graphical example. Parameters are set as follows: $\delta = 8\%$, $\frac{\pi}{q} = 13\%$, $\frac{d_m}{q} = 0.1\%$, $\mu = 0.3$, $\theta = 0.8$, $\rho = 1\%$, $\sigma = 1$ or $\sigma = 0.5$.



On figure 2, the horizontal axis represents the interest rate and the vertical axis represents the growth rate. The financially constrained steady state has to be found for the interest rate values: $\max\left(\rho, (1-\mu)\frac{\pi}{q} - \delta\right) = 1\% < r < \frac{\pi}{q} - \delta = 5\%$. The two increasing curves represents two consumption growth curves ($\approx (r-\rho)/\sigma$), the higher one correponding to an intertemporal elasticity of substitution equal to unity $\sigma = 1$, the lower one to $\sigma = 2$. The decreasing curve is a patent growth curve. When the two curves intersects for an interest rate *below* the profit rate net of depreciation: $r < \frac{\pi}{q} - \delta$, there exists a financially constrained steady state growth. Else, the growth rate is given by the intersection of the growth rate of consumption with the vertical curve $r = \frac{\pi}{q} - \delta$ and corresponds to the steady state growth rate in Romer's model. The perfect capital market interest rate is equal to $\frac{\pi}{q} - \delta = 5\%$ and corresponds to a Romer's growth rate $g = \frac{1.05}{1.01} - 1 = 3.96\%$ for the consumption growth curve with

$\sigma = 1$, whereas it corresponds to a Romer's growth rate $g = 1.96\%$ when $\sigma = 2$. When $\sigma = 2$, the growth of consumption and savings is relatively low so that the perfect capital market steady state growth regime prevails. When $\sigma = 1$, the growth of consumption and savings is relatively high and the financially constrained steady state growth regime prevails (cf. condition A4). The financially constrained steady state interest rate is $r = 4.5\%$ and the financially constrained growth rate is given by $g_{FC} = 3.46\%$. In any regimes, $g < r$ so that households and entrepreneurs utilities are bounded.

3.3. Transitory Dynamics and Extensions

In the perfect capital market case or in the financially constrained case without lumpiness effects, there are no transitory dynamics on aggregate variables following a shock on exogenous parameters from the former steady state to the next steady state (the markup reward on innovations π, the marginal cost of innovation q, the expected hazard of obsolescence δ, the proportion of investing firms θ, the transaction cost on the shift of intellectual property on patents to lenders μ, the minimal consumption of entrepreneurs d_m, households' rate of time preference ρ, and the households elasticity of intertemporal substitution σ). Hence, the economy jumps from the old to the new steady state when there is a change of exogenous parameters of the model.

The fact that some firms are not able to invest ($\theta < 1$) and save instead of investing introduces some sand in the aggregate debt dynamics, which is the origin of transitory dynamics. The aggregate model consists of three dynamical equations related to the debt/asset ratio, the growth of patents (or the growth of debt) and households consumption growth. The first equation provides the value of the debt/patent ratio t, which is then used to determine entrepreneurs' aggregate debt on date t. Entrepreneurs aggregate debt has to be equal to households savings on date t, which leads to an equilibrium value of the interest rate r_t (which will be equal to the steady state interest rate only when the debt/patent ratio reaches its steady state value x^*). The aggregate debt/patent dynamics is:

$$x_t = M\left(x_{t-1}, R_{t-1}\right) = 1 - \frac{1-x^c}{\theta} + \frac{1}{\theta}\frac{\left(1-x^c\right)^2\left(1-\theta\right)\left(1-\delta\right)}{\left(1-x^c\right)\left(1-\theta\right)\left(1-\delta\right) + \theta\Pi_m - \theta R_{t-1}x_{t-1}} \tag{3.14}$$

For the relevant values of the debt/patent ratio ($0 < x_t \leq x^c$), there is a unique steady state $x^*\left(R_{t-1}\right)$ for a given interest rate (denoted R_{t-1}), The slope of the function M ($0 < M'\left(x_t\right) < 1$) ensures regular convergence with to this steady state, without any cyclical, chaotic or indeterminate patterns (cf. figure 1). For example, assume a shock on exogenous parameters leading to a new and higher steady state value of the debt/patent ratio x^{**}, to a new steady state interest rate r^{**} and a new steady state growth rate g^{**}. The convergence will be, for example, such that $x_0^* < x_t < x_{t+1} \leq x^{**}$. Then the interest R_t is derived recursively, knowing that the growth rate of aggregate debt of entrepreneurs has to match the growth rate rate of households savings (identical to the growth rate of households consumption) at each period t, so that:

$$\left(\frac{R_t}{1+\rho}\right)^{\frac{1}{\sigma}} = \frac{\Pi_m - R_{t-1}x_{t-1}}{1-x_t} = \frac{\Pi_m - R_{t-1}x_{t-1}}{1-M\left(x_{t-1}, R_{t-1}\right)} \tag{3.15}$$

Knowing R_t, one proceeds to the next step $x_{t+1} = M\left(x_t, R_t\right)$.[11] Knowing R_t, the growth factor on date t is found using $G_t = \left(\frac{R_t}{1+\rho}\right)^{\frac{1}{\sigma}}$. When the interest rate rises, so does the economy growth rate, as in other convergence models based on Ramsey's savings behaviour. However, those dynamics present a remarkable feature. There may be interest rate transitory dynamics whereas the marginal product of R&D investment *does not change* over time. Because of financial constraints, the observed dynamics of the interest rate and of the marginal productivity is no longer predicted to be identical. This is not the case in most of the literature on GDP per head convergence of nations, which are based on perfect capital markets, where the user cost equals the marginal product of capital (for example, the standard Solow type convergence model).

Finally, let us consider two simple extensions of the model. First, one may consider a finite duration T of patents rights protection (around 18 years in the U.S.), which decreases the collateral ceiling for creditors: $(1-\mu)\pi n_t \sum_{\tau=0}^{T}\left(\frac{1-\delta}{R}\right)^{\tau}$. A similar pattern emerges with creative destruction models, where the expected duration is endogenous. This indeed fosters the persistance effects at the individual level (cf. the denominator of the persistance coefficient, section 2), and affects the aggregate growth rate and the interest rate in a similar way than a rise of the transaction cost on property rights over patents royalties for lenders (μ). This means that financial and collateral constraints are likely to exhibit even stronger effects in "destructive creation" R&D growth models (Aghion and Howitt [1992]) than in expanding variety R&D growth models. Institutional policies improving the enforcement of patent rights and increasing the duration of patents foster financially constrained R&D growth.

Secondly, one may consider an endogenous probability of having a positive net present value investment opportunity. When the random shock affecting the marginal product of innovation ($\varepsilon_t \pi$) is not confined to two states of nature ($\varepsilon_t = 0$ with probability $1-\theta$, $\varepsilon_t = 1$ with probability θ) but distributed over a large number of states of nature according to a cumulative distribution function $F\left(\varepsilon_t\right)$, (with an expectation equal to unity, $E\left(\varepsilon_t\right) = 1$ so that $\pi = E_{t-1}\left(\varepsilon_t \pi\right)$ in the model) then the probability of a positive net present value R&D project is given by: $\theta = Prob\left(\varepsilon_t \pi \geq q\left(r+\delta\right)\right) = 1 - F\left(\frac{q(r+\delta)}{\pi}\right)$. The probability of investing now increases with the marginal benefit

[11] For the sake of simplicity, it is assumed here that the forward looking debt ceiling variable x^c discounts patents royalties at the final steady state interest rate r^{**} after the initial shock. This means that the debt ceiling is lower than when discounting the future royalties of existing patents by the future sequence of transitory interest rate (a second order effect which increases marginally the speed of convergence).

of R&D and decreases with its user cost. Then, the sensitivity of the growth rate of patents with respect to the interest rate increases (the slope of the patent growth curve increases on the diagram 2) with respect to the case where the probability θ is constant. This extension may lead to relatively lower steady state financially constrained growth rate and interest rate.

4. Conclusion

This paper describes an endogenous growth model with lenders limiting credit up to collateralizable value of existing patents and with a composition between innovative firms facing a probability to find a positive net present value R&D investment opportunity or not each period. The combination of financially constrained growth, taking into account the distinction between stocks and flows of both internal and external finance, and of R&D investment lumpiness provided the following results

First, at the entrepreneur level, financial constraints and lumpiness leads to a specific entrepreneurs saving behaviour where they build "deep pockets", by anticipation of financial constraint when facing a future lumpy R&D investment opportunity. When this lumpy investment opportunity occurs, the dependance of the persistence of R&D investment on the markup rewarding innovations is amplified by the debt/patent collateral constraint (a specification which matches an empirical model of R&D investment estimated by Blundell, Griffith and Van Reenen [1999]).

Secondly, there are specific consequences on macro-economic growth of R&D investment lumpiness at the entrepeneur level which emerge when combined with financial constraints and which do not show up in the perfect capital market case. The reason is that entrepreneurs' savings by anticipation of financially constrained lumpy R&D investments affect differently the growth rate of aggregate debt and the growth rate of aggregate patents. The aggregation of entrepreneurs behaviour, some of them saving, some of them investing, determines a steady state endogenous aggregate leverage (or debt/patent ratio) below the leverage ceiling.

Thirdly, the probability of R&D investment opportunity increases the financially constrained steady state growth rate of the economy, which is not the case in the perfect capital market case. This financially constrained steady state arise under a general condition (labeled A4) when the entrepreneurs consumption is "high" and limit their savings and their investment, when the households growth rate of savings is relatively high, when the transfer of property rights over patents royalties to creditors is poor, when the enforcement of patents protection is poor and/or the hazard of obsolescence of patents are high, and when the probability of finding net present value R&D investment opportunity for entrepreneurs is low. This financially constrained steady state has two characteristics. First, the marginal benefit of R&D investment exceeds its marginal cost (a fact observed empirically, although the measurement of the marginal cost is particularly difficult). Secondly, the growth of patent is a decreasing function of interest rate, which is not the case in standard expanding variety growth models with perfect capital markets.

Finally, firms savings by anticipation of a lumpy R&D investment may cause a sluggish macroeconomic adjustment to a new steady state, following shocks on exogenous characteristics of a financially constrained economy. Those transitory dynamics on the debt, patent, consumption and interest rates may occur without any transitory dynamics on the R&D marginal return, since the equality linking the interest rate with the marginal return on R&D does not hold in a financially constrained regime.

Economic policy targeting growth in a financially constrained regime may promote the enforcement of patent protection and changes in bankruptcy law decreasing the cost of the tranfer of property rights over patents royalties to creditors. Lumpy R&D investment would be more directly supported by investment tax credit conditional to an effective R&D investment instead of a non targeted decrease of the corporate income tax.

Direction of future research may consider structural estimations of R&D individual, sectorial and aggregate behaviour based on this model.

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5. Appendix 1

The Lagrangian of the entrepreneur program is:

$$
\begin{aligned}
(n_t, b_t) &\in Arg \max E_0 \sum_{t=1}^{+\infty} R^{-t} L_t && (5.1)\\
\text{with } L_t &= d_t + \lambda_t^b \left((1-\mu) \frac{\pi}{r+\delta} n_t - b_t \right) + \lambda_t^d \left(d_t - d_m n_{t-1} \right) && (5.2)
\end{aligned}
$$

where λ_t^b is the Lagrange multiplier related to the debt ceiling constraint, λ_t^d is the Lagrange multiplier related to the minimal consumption constraint, and with consumption d_t given by the flow of funds constraint:

$$d_t = \pi n_{t-1} + b_t - \widetilde{1}_{i_t>0}\, q \left(n_t - \left(1 - \widetilde{\delta} \right) n_{t-1} \right) - R b_{t-1} + d_m n_{t-1} \tag{5.3}$$

The Euler equation on debt b_t is, for any date t:

$$1 + \lambda_t^d - \lambda_t^b + R^{-1} E_t \left(1 + \lambda_{t+1}^d \right) (-R) = 0 \Rightarrow \lambda_t^d = \lambda_t^b + E_t \left(\lambda_{t+1}^d \right) \tag{5.4}$$

With probability $1-\theta$, the Bernoulli random variable $\widetilde{1}_{i_t>0}$is equal to zero, and the entrepreneur faces the constraint $n_t = \left(1 - \widetilde{\delta}_{t-1} \right) n_{t-1}$, so that he decides n_t only when, with probability θ, the Bernoulli random variable $\widetilde{1}_{i_t>0}$is equal to unity. Then the variable $(1-\delta)^{\tau} n_t$ is present with probability $(1-\theta)^{\tau}\theta$ in the R&D cost function and in the debt ceiling constraint in the discounted element of the Lagrangian $R^{-t-\tau} L_{t+\tau}$ for any integer τ. The first order condition with respect to the stock of patents n_t is then:

$$\left(1 + \lambda_t^d \right) q = \lambda_t^b q x^c + \frac{1}{R} \sum_{\tau=0}^{\tau=+\infty} \beta^{\tau} \left\{ \left(1 + E_t \lambda_{t+1+\tau}^d \right) \left(\pi + \theta q (1-\delta) \right) \right\} + \frac{d_m}{R} \sum_{\tau=0}^{\tau=+\infty} \beta^{\tau} E_t \lambda_{t+1+\tau}^d \tag{5.5}$$

with $\beta = (1-\theta)(1-\delta)/R$ and assuming that $E_t \left(\lambda_{t+1+\tau}^d\, \widetilde{\delta}_j \right) = E_t \left(\lambda_{t+1+\tau}^d \right) E_t \left(\widetilde{\delta}_j \right)$ for any integer τ and j. Using the first order condition for debt and the equality $\left(R / \sum_{\tau=0}^{\tau=+\infty} \beta^{\tau} \right) - \theta (1-\delta) = r + \delta$, this equation can be written as:

$$\frac{\pi}{q} - (r+\delta) = (1-x^c) \frac{\sum_{\tau=0}^{\tau=+\infty} \beta^{\tau} E_t \left(\lambda_{t+\tau}^d - \lambda_{t+\tau+1}^d \right)}{\sum_{\tau=0}^{\tau=+\infty} \beta^{\tau} \left(1 + E_t \lambda_{t+\tau+1}^d \right)} + \frac{d_m}{Rq} \frac{\sum_{\tau=0}^{\tau=+\infty} \beta^{\tau} E_t \lambda_{t+\tau+1}^d}{\sum_{\tau=0}^{\tau=+\infty} \beta^{\tau} \left(1 + E_t \lambda_{t+\tau+1}^d \right)} \tag{5.6}$$

The rate of return is equal to the user cost when $\lambda_{t+\tau}^d = 0$ for any integer τ. Suppose now that $\lambda_t^d > 0$, so that the consumption is kept at its minimal level $d_m n_{t-1}$

and the rate of return is higher than the user cost according to the above equation. Then, in the case where the firm has an opportunity to invest $\widetilde{1}_{i_t>0}= 1$, maximizing utility amounts to maximize n_t. The flow of funds constraints shows that this is obtained when the debt ceiling is binding so that $\lambda_t^b > 0$. Else, in the case where the firm has no opportunity to invest $\widetilde{1}_{i_t>0}= 1$, the following constraint is binding $n_t = \left(1-\widetilde{\delta}_{t-1}\right) n_{t-1}$ so that the flow of funds constraint determines debt and the debt/patent ratio:

$$x_t = \frac{R}{q(1-\delta)}x_{t-1} - \frac{\pi - d_m}{q(1-\delta)}$$

The debt/patent ratio decreases $x_t < x_{t-1} \leq x^c$ when $d_m < \mu\pi$. Else a firm financially constrained previously would keep its debt/patent ratio at the ceiling level when it does not have an R&D investment opportunity.

6. Appendix 2

This appendix deals with the existence and unicity of the steady state debt/patent ratio x given by the following quadratic equation (we use the implicit dynamics for the debt/patent ratio provided by $G_{NB} = G_N$):

$$N(x) = R + \frac{1 + \frac{\pi - d_m}{q} - \delta - R}{1-x} - (1-\theta)(1-\delta) - \theta\left(\frac{1 + \frac{\pi - d_m}{q} - \delta - Rx}{1-x^c}\right) = 0$$

The function $N(x)$ is continuous on the interval $[0, x^c]$ and strictly increasing:

$$\frac{\partial N(x)}{\partial x} = \frac{1 + \frac{\pi - d_m}{q} - \delta - R}{(1-x)^2} + \theta\frac{R}{1-x^c} > 0$$

A unique solution exist for a positive steady state debt/patent ratio $0 < x^* \leq x^c < 1$ under the conditions $N(0) < 0$ and $N(x^c) > 0$. First, $N(x^c) > 0$ is always fulfilled as long as $\theta \leq 1$:

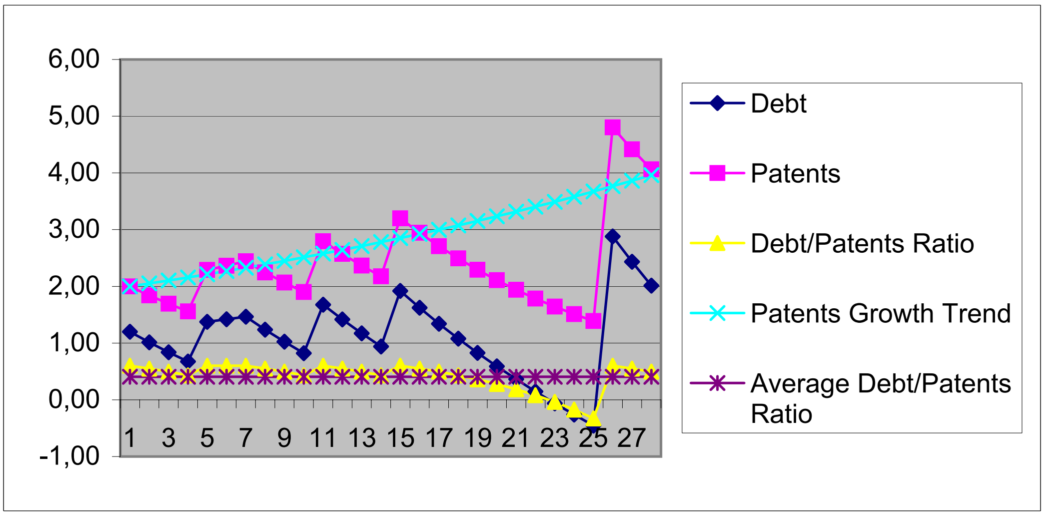
$$N(x^c) = R + \frac{1 + \frac{\pi - d_m}{q} - \delta - R}{1-x^c} - (1-\theta)(1-\delta) - \theta\frac{1 + \frac{\pi - d_m}{q} - \delta - x^c R}{1-x^c} \geq 0$$

Second, $N(0) < 0$ leads to condition A3 on θ:

$$\begin{aligned} N(0) &= 1 + \frac{\pi - d_m}{q} - \delta - (1-\theta)(1-\delta) - \theta\frac{1 + \frac{\pi - d_m}{q} - \delta}{1-x^c} < 0 \\ &\Rightarrow \theta > \theta_{\min}^x = \frac{\frac{\pi - d_m}{q}(1-x^c)}{x^c(1-\delta) + \frac{\pi - d_m}{q}} > 0 \text{ (A3)} \end{aligned}$$

For the steady state debt/patent ratio to be strictly positive, the proportion of firms who invest should be at least over $\theta^{x}_{\min} < 1$. Identical results are obtained using the explicit function $x_{t+1} = M(x_t)$ when looking for solutions of the fixed point equality: $M(x) - x = 0$.

Appendix 3 and Figure 3: Patents and Debt Dynamics at the Innovative Firm Level
Parameter values identical to those of figure 2 with an interest rate of 4.5%,
except for 28 draws of the random possibility of Net Present Value R&D Investment corresponding to a low average probability of investing equal to 25%.



The firm turns to be a net creditor after eight periods, an event with a probability 0.10 (for θ=0.25 at the power 8).
Doms and Dunne (1998) evaluation for θ related to tangible investment is 0.7
For such a value, the probability that a firm turns to be a net creditor is 0.0001 when other parameters remain unchanged.
Due to non-investing periods, the average debt/patents ratio is below the debt/ceiling.
There is an increasing Patent Growth trend besides random R&D Investment.