Econometric Issues in the Analysis of Contagion*

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Abstract

This paper presents a canonical, econometric model of contagion and investigates the conditions under which contagion can be distinguished from inter-dependence. In a two-country (market) set up it is shown that for a range of fundamentals the solution is not unique, and for sufficiently large values of the contagion coefficients it has interesting bifurcation properties with bimodal density functions. The extension of the model to herding behaviour is also briefly discussed. To identify contagion effects in the presence of inter-dependencies the equations for the individual markets or countries must contain country (market) specific forcing variables. This sheds doubt on the general validity of the correlation based tests of contagions recently proposed in the literature which do not involve any country (market) specific fundamentals. Finally, we show that ignoring inter-dependence can introduce an upward bias in the estimate of the contagion coefficient, and using Monte Carlo experiments we further show that this bias could be substantial.

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1 Introduction

Recent episodes of financial crises suggest that they appear in clusters. In the EMS crises in 1992 two countries were forced to leave the exchange rate mechanism and a number of other countries suffered from speculative attacks. In 1994 a number of countries were affected in what came to be known as the "Tequila" crisis. During the Asian crisis in 1997, several Asian economies were struck by severe crises. These episodes have led economists to suggest that financial crises are contagious, that they spread from the originating country to other countries, and that an understanding of the reasons for contagion is essential for forming appropriate economic policies.

The theoretical literature considers a number of reasons for financial crises to appear in clusters. Masson (1999) identifies three categories under which the different theories can be subsumed. First, the theory of "monsoonal effects" suggests that financial crises appear to be contagious because the underlying macroeconomic causes are correlated. Second, financial crises may be transmitted between countries via "spill-overs": a crisis affects another country through external links such as trade. A devaluation in one country exerts pressure on the country's trading partners to devalue. Finally, the theory of "pure contagion" holds that the market jumps from a "good" to a "bad" equilibrium. The jump could be caused by a reversal in a long-standing information cascade. A financial crisis in one country could be considered a signal that, for example, a certain type of economic development strategy is unsustainable. Investors would withdraw their money from countries with apparently similar development strategies and cause a cluster of financial crises.

The first two cases, monsoonal effects and spill-overs, are examples of inter-dependence. Crises resulting from inter-dependence should be largely predictable using macroeconomic fundamentals. If the inter-dependence during non-crises periods is known, the effect of a financial crisis in one country on the likelihood of a crisis in another country can be evaluated. The third case, jumps between equilibria, is what we refer to as contagion in this paper: a largely unpredictable, higher correlation during crises times. This definition of contagion means that a crisis in one country increases the likelihood of a crisis in another country over and above what would be implied by the inter-dependence that prevails between these countries in non-crises times. This definition corresponds to that given by Forbes and Rigobon (2001, 2002).

There are important implications of the distinction between contagion and inter-dependence. Investors need to take a different kind of risk into account for their portfolio choices if markets have a higher correlation after negative shocks. If negative shocks have a much higher correlation across countries than in tranquil times, diversification of portfolios across countries might be less useful than anticipated before a negative shock.

Economic policy-makers need to be aware of the source of contagion when they are evaluating possible policy responses to a crisis. If the cause of a crisis is a random jump between equilibria, i. e. contagion, international institutional lending to prevent contagion could be a highly effective response as it might return the market to the "good" equilibrium. If, in contrast, a crisis spreads to other countries because their fundamentals are correlated or there are spill-overs affecting the economic fundamentals, international institutional lending cannot prevent the crisis unless it is large enough to change the fundamentals.

There is a large body of empirical research regarding the source of contagion of financial crises, recently reviewed by Dornbusch, Park, and Claessen (2000), Pericoli and Sbracia (2002), and Dungey, Fry, Gonzalez-Hermosillo and Martin (2003). The empirical literature on contagion of currency crises has been largely based on the literature on the macroeconomic causes of currency crises. Using a panel data set, Eichengreen, Rose, and Wyplosz (ERW) (1996) used a pooled probit model to explain a binary indicator of currency crises by a set of macroeconomic variables and a dummy variable for contagion. They found that a crisis elsewhere raises the likelihood of a currency crises by about 8% and interpreted this finding as an indication of contagion. Similar studies have also been carried out by Esquivel and Larraín (1998), Kruger, Osakwe, and Page (1998), Stone and Weeks (2001), and Kumar, Moorthy, and Perraudin (2002).

Another set of papers examines contagion of financial markets by testing for higher correlation between markets during crises times (King and Wadhwani (1990), Boyer, Gibson, and Loretan (1999), Loretan and English (2000), Forbes and Rigobon (2002) and Corsetti, Pericoli and Sbracia (2002)). Favero and Giavazzi (2002) found significant contagion dummies in their analysis of interest rate spreads in the ERM. Bae, Karolyi and Stulz (2003) test whether the number of contemporaneous extreme stock market returns across a number of markets in a given region can be explained by three common factors, and find that the average exchange rate in the region, the average interest rate in the region, and the conditional volatility of a regional stock market index are significant.

In this paper we propose a canonical model of contagion and provide a solution in the two country (asset) set up. For a range of fundamentals the solution is not unique and for sufficiently large values of the contagion coefficients has interesting bifurcation properties with bimodal density functions. We briefly discuss extensions of this model to cover herding behaviour.

The problem of identification and estimation of the contagion coefficients are discussed and shown to be an example of the general problem of inference in the non-linear simultaneous equation models. To identify contagion effects in the presence of inter-dependencies the equations for the individual markets or countries must contain country (market) specific forcing variables. Therefore, pure correlation-based tests for contagion cannot be valid. Country specific fundamentals are needed to distinguish contagion from inter-dependence. The correlation based tests of contagions recently proposed in the literature attempts to overcome the identification problem by assuming that the crises periods can be identified (or known *a priori*), and that such episodes are sufficiently prolonged and contiguous so that cross-country (market) correlations during crisis and non-crisis periods can be consistently estimated and compared. These are strong assumptions that are unlikely to hold in practice and their implementation tend to be subject to the sample selection bias. Such correlation analyses, by being *ex post* in nature, are also not very helpful if the focus of the analysis is to develop an early warning system for policy use.

Finally, we show that ignoring the endogeneity of the contagion indicator and/or inter-dependence of the error terms can introduce an upward bias in the estimate of the contagion coefficient, and using Monte Carlo experiments we further show that this bias could be substantial. Our simulations also suggest that the contagion coefficient of 0.54 obtained from pooled probit estimation of ERW's model could be due to neglected inter-dependencies rather than contagion.

2 A Canonical Model of Contagion: A Two Country Framework

Consider the following relations

$$y_{1t} = \boldsymbol{\delta}_1'\mathbf{z}_t + \boldsymbol{\alpha}_1'\mathbf{x}_{1t} + \beta_1 \mathrm{I}(y_{2t} - c_2\sigma_{2,t-1}) + u_{1t} \tag{1}$$

$$y_{2t} = \boldsymbol{\delta}_2'\mathbf{z}_t + \boldsymbol{\alpha}_2'\mathbf{x}_{2t} + \beta_2 \mathrm{I}(y_{1t} - c_1\sigma_{1,t-1}) + u_{2t}, \tag{2}$$

where y_{it} is a performance indicator for country $i = 1, 2$, $t = 1, \ldots, T$, u_{1t} and u_{2t} are serially uncorrelated errors with zero means, conditional variances $\sigma^2_{u1,t-1}$ and $\sigma^2_{u2,t-1}$ and a non-zero correlation coefficient ρ.[1] The regressors, $\mathbf{x}_{it}$, are $k_i \times 1$ country-specific observed factors assumed to be predetermined and distributed independently of u_{jt} for all i and j. Country-specific dynamics can be allowed for by including $y_{i,t-1}, y_{i,t-2}, \ldots$ in $\mathbf{x}_{it}$. $\mathbf{z}_t$ is an $s \times 1$ vector of pre-determined observed common factors, such as international oil prices, or other common features. $\mathrm{I}(A)$ is an indicator function that takes the value of unity if $A > 0$ and zero otherwise

$$\sigma^2_{i,t-1} = \mathrm{Var}\left(y_{it} \mid \Omega_{t-1}\right),$$

Ω_{t-1} is the information available at time $t-1$.[2]

[1] In the analysis of the solution properties of y_{1t} and y_{2t} it is relatively easy to allow for possible time variations in ρ. But such a generalisation could obscure the properties of the correlation between y_{1t} and y_{2t}. As it is shown below $\mathrm{Corr}(y_{1t}, y_{2t})$ could be time varying even if ρ is fixed.

[2] Note that in general $\sigma^2_{ui,t-1} \neq \sigma^2_{i,t-1}$.

Examples of performance indicators include stock market returns used by Forbes and Rigobon (2002) and Corsetti, Pericoli and Sbracia (2002), and the index of "exchange market pressure" employed by Eichengreen, Rose and Wylosz (1996) which is a weighted average of exchange rate depreciation, interest rate differential and international reserves ratios. We are assuming that y_{it} is defined in such a way that a crisis is associated with extreme positive values of y_{it}, and $c_i > 0$.

In this set up inter-dependence is captured through non-zero values of ρ, and is distinguished from contagion effects characterised by non-zero values of β_i.

- It is assumed that contagion takes place only at times of crises, whilst inter-dependence is the result of normal market interactions.[3]
- Country i is said to be in crisis if the performance index, y_{it} rises above a threshold value c_{it}.
- Contagion is said to occur if a crisis in country 2 increases the probability of a crisis in country 1 over and above the usual market interactions, and *vice versa.*
- To test for contagion we first need to establish conditions under which the contagion coefficients, β_i can be identified. Once such conditions are met, a test of contagion in country i can be carried out by testing $\beta_i = 0$ against the one-sided alternatives, $\beta_i > 0$.

The above framework can be readily generalised to deal with both extremes simultaneously,

$$y_{it} = \boldsymbol{\delta}_i' \mathbf{z}_t + \boldsymbol{\alpha}_i' \mathbf{x}_{it} + \beta_{iU} \mathrm{I}(y_{jt} - c_{jU}\sigma_{j,t-1}) + \beta_{iL} \mathrm{I}(-y_{jt} - c_{jL}\sigma_{j,t-1}) + u_{it},$$

for $i = 1, 2$, where β_{iU} and β_{iL} now refer to contagion effects on the upper and the lower tails and $c_{jU}\sigma_{j,t-1}$ and $c_{jL}\sigma_{j,t-1}$ are the associated thresholds with $c_{jU} \geq 0$ and $c_{jL} \geq 0$. It is clear that only one of the indicators can be triggered at a time. In this note we shall focus on the relatively simple case where $\beta_{iL} = 0$, but we conjecture that our approach and arguments can be readily extended to the more general case.

[3] Such phenomena are also frequently encountered in physics and have been studied extensively in the literature on bifurcation and chaos. For example, in the Rayleigh-Bénard convection, heat from the surface of the earth conducts its way to the top of the atmosphere until the rate of heat generation at the surface of the earth gets too high. At this point heat conduction breaks down. The atmosphere develops pairs of convection cells, one rotating left and the other rotating right.

3 Solution and the Possibility of Multiple Equilibria

Setting

$$w_{it} = \boldsymbol{\delta}_i'\mathbf{z}_t + \boldsymbol{\alpha}_i'\mathbf{x}_{it} + u_{it},$$

we re-write (1) and (2) as

$$y_{1t} = w_{1t} + \beta_1 \mathrm{I}(y_{2t} - c_2), \tag{3}$$

$$y_{2t} = w_{2t} + \beta_2 \mathrm{I}(y_{1t} - c_1), \tag{4}$$

where to simplify the notations and without loss of generality we abstract from the (possibly) time varying nature of the thresholds.

This is a system of non-linear and non-differentiable simultaneous equations and has a simple unique solution when either β_1 or β_2 is zero. For example, suppose that $\beta_2 = 0$. Then the solution is given by

$$y_{1t} = w_{1t} + \beta_1 \mathrm{I}(y_{2t} - c_2), \tag{5}$$

$$y_{2t} = w_{2t}. \tag{6}$$

When neither of the contagion coefficients is zero the equation system (3) and (4) can be equivalently written as

$$Y_{1t} = W_{1t} + \mathrm{I}(Y_{2t}), \tag{7}$$

$$Y_{2t} = W_{2t} + \mathrm{I}(Y_{1t}), \tag{8}$$

where

$$Y_{it} = \frac{y_{it} - c_i}{\beta_i},\ W_{it} = \frac{w_{it} - c_i}{\beta_i}. \tag{9}$$

To solve this simplified system we shall consider the following five mutually and exclusive regions in the (W_{1t}, W_{2t}) plane (see also Figure 1):

Region A: $W_{2t} > 0$

Region B: $-1 < W_{2t} \leq 0$ and $W_{1t} > 0$,

Region C: $W_{2t} \leq -1$

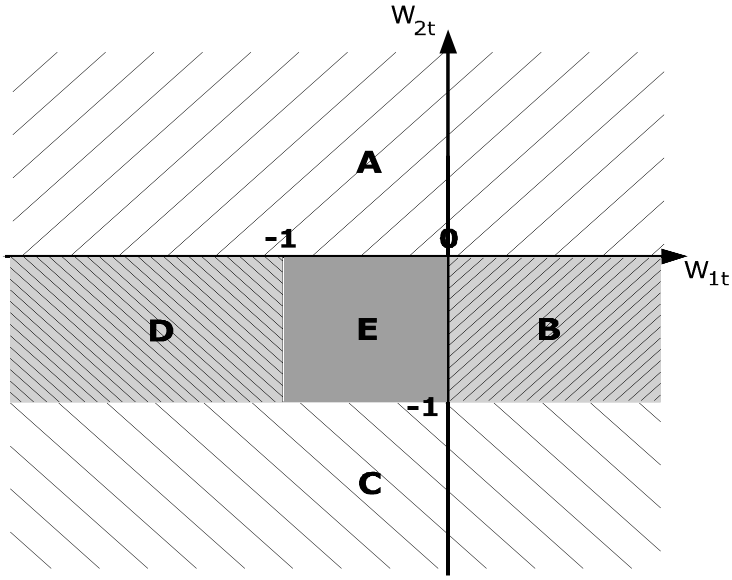
Region D: $-1 < W_{2t} \leq 0$ and $W_{1t} < -1$

Region E: $-1 < W_{2t} \leq 0$ and $-1 < W_{1t} \leq 0$

It is now easily verified that in regions A and B, the solution for Y_{1t} is unique and is given by

$$Y_{1t}^* = 1 + W_{1t}, \tag{10}$$

Figure 1: Regions of W_{1t} and W_{2t}



and, similarly, in regions C and D the solution is unique and is given by

$$Y_{1t}^{*} = W_{1t}. \tag{11}$$

However, in region E the solution is not unique. For example, for $W_{1t} = -1/2$, and $W_{2t} = -1/3$, there are two possible solutions for $\mathbf{Y}_t = (Y_{1t}, Y_{2t})'$ given by

$$\mathbf{Y}_t^a = \begin{pmatrix} -1/2 \\ -1/3 \end{pmatrix} \text{ and } \mathbf{Y}_t^b = \begin{pmatrix} 1/2 \\ 2/3 \end{pmatrix}.$$

Using the index d_t to designate the choice of the solution when $-1 < W_{it} \leq 0$ we have

$$Y_{it}^{*}(d_t) = d_t W_{it} + (1 - d_t)(1 + W_{it}), \text{ for } i = 1, 2, \tag{12}$$

where the "favourable" solution occurs if $d_t = 1$, and the "unfavourable" solution occurs if $d_t = 0$. Notice that in the present set up the crisis (unfavourable outcome) is associated with the upper tail (large positive values). It is clear from Equation (12) that the distribution of $Y_{it}^{*}(d_t)$ is a mean mixture of distributions with d_t as the selection parameter. Hence, $d_t \sim$ Bernoulli(π), where π is the probability of W_{it} being chosen in the mixture.

This is an interesting example where non-uniqueness arises only if the fundamentals (as measured by W_{it}) for both countries (markets) are favourable but weak (in relation to the threshold values). This appears similar to the notion of weak fundamentals used by Sachs, Tornell and Velasco (1996). This result also raises the possibility of policy intervention for ensuring that the "favourable" solution is in fact selected. It is also reasonable to expect that the correlation of Y_{1t} and Y_{2t} would be higher if the unfavourable solution is chosen as compared to the favourable one. Simulation results reported below bear this out.

Collecting the various components of the solution given by (10) to (12) we have

$$\begin{aligned} Y_{1t} = \; & (1+W_{1t})\,\mathrm{I}\,(W_{2t}) && \text{(Region A)} \\ & +(1+W_{1t})\,\mathrm{I}\,(-W_{2t})\;\mathrm{I}(1+W_{2t})\,\mathrm{I}(W_{1t}) && \text{(Region B)} \\ & +W_{1t}\,\mathrm{I}(-1-W_{2t}) && \text{(Region C)} \\ & +W_{1t}\,\mathrm{I}\,(-W_{2t})\;\mathrm{I}(1+W_{2t})\,\mathrm{I}(-1-W_{1t}) && \text{(Region D)} \\ & +Y^{*}_{1t}(d_t)\,\mathrm{I}\,(-W_{2t})\;\mathrm{I}(1+W_{2t}) && \text{(Region E)} \\ & \times \mathrm{I}(-W_{1t})\,\mathrm{I}(1+W_{1t}) \end{aligned} \tag{13}$$

and by symmetry

$$\begin{aligned} Y_{2t} = \; & (1+W_{2t})\,\mathrm{I}\,(W_{1t}) \\ & +(1+W_{2t})\,\mathrm{I}\,(-W_{1t})\;\mathrm{I}(1+W_{1t})I(W_{2t}) \\ & +W_{2t}\,\mathrm{I}(-1-W_{1t}) \\ & +W_{2t}\,\mathrm{I}\,(-W_{1t})\;\mathrm{I}(1+W_{1t})\,\mathrm{I}(-1-W_{2t}) \\ & +Y^{*}_{2t}(d_t)\,\mathrm{I}\,(-W_{1t})\;\mathrm{I}(1+W_{1t})\,\mathrm{I}(-W_{2t})\,\mathrm{I}(1+W_{2t}). \end{aligned} \tag{14}$$

In terms of the original variables we obtain

$$y^{*}_{it} = \beta_i Y^{*}_{it} + c_{it}, \text{ for } i = 1, 2. \tag{15}$$

It is important that the above solution is valid even if $y_{i,t-1}$, $y_{i,t-2}$, are included amongst of the individual-specific regressors, $\mathbf{x}_{it}$. This feature considerably enhance the relevance of the model to the analysis of financial markets that show a mild degree of short term over-shooting.

It is clear that y_{1t} and y_{2t} will be correlated even if w_{1t} and w_{2t} are independently distributed, i.e. for values of $\beta_i > 0$, $\mathrm{Corr}(y_{1t}, y_{2t}) > 0$ even when $\mathrm{Corr}(w_{1t}, w_{2t}) = 0$. For example, consider the simple case where $\beta_2 = 0$ and $\beta_1 > 0$ of Equations (5) and (6) and w_{1t}, w_{2t} are independently distributed. In this case

$$\mathrm{Cov}(y_{1t}, y_{2t}) = \beta_1 \left[1 - \mathrm{F}_2(c_{2t})\right] \left\{\mathrm{E}\left(w_{2t} - c_{2t} \mid w_{2t} > c_{2t}\right) - \mathrm{E}\left(w_{2t} - c_{2t}\right)\right\},$$

and

$$\mathrm{Corr}(y_{1t}, y_{2t}) = \frac{\beta_1 \left[1 - \mathrm{F}_2(c_{2t})\right] \left\{\mathrm{E}\left(w_{2t} - c_{2t} \mid w_{2t} > c_{2t}\right) - \mathrm{E}\left(w_{2t} - c_{2t}\right)\right\}}{\sqrt{\mathrm{Var}(w_{2t}) \left\{\mathrm{Var}(w_{1t}) + \beta_1^2 \mathrm{F}_2(c_{2t}) \left[1 - \mathrm{F}_2(c_{2t})\right]\right\}}},$$

where $F_2(x)$ is the cumulative distribution function of w_{2t}. In the extreme value literature, $E(w_{2t} - c_{2t} \mid w_{2t} > c_{2t})$ is known as the mean excess function of w_{2t}, see for example Embrechts, Klüppelberg and Mikosch (1997). This result provide support for the hypothesis that the degree of the dependence of y_{1t} and y_{2t} is an increasing function of the degree of the fat-tailedness of the w_{2t} process. For $w_{it} \sim N(0,1)$,

$$\text{Corr}(y_{1t}, y_{2t}) = \frac{\beta_1 [1 - \Phi(c_{2t})] \{E(w_{2t} \mid w_{2t} > c_{2t})\}}{\sqrt{1 + \beta_1^2 \Phi(c_{2t}) [1 - \Phi(c_{2t})]}} > 0, \text{ for } \beta_1 > 0, \ c_{2t} > 0.$$

4 Some Numerical Results

Suppose that $c_{it} = 1.64$ (that corresponds to the upper 95% tail of the standard normal), let $\beta_1 = \beta_2 = \beta$, and

$$\begin{pmatrix} w_{1t} \\ w_{2t} \end{pmatrix} \sim N\left(\mathbf{0}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}\right).$$

Using these parameters we can sample the dependent variables and investigate their properties for different values of the contagion coefficient β. The results reported below are based on 30,000 sampled values of y_{1t} and y_{2t}.

Table 1 reports the moments of y_{1t} and the correlation of y_{1t} and y_{2t} under the assumption that only one of the mixture distributions is visited, note however that due to the symmetry of the model the reported moments also apply to y_{2t}. On the left side of the table the results for $\pi = 1$ are reported and on the right side the results for $\pi = 0$.

Table 1: Moments of the distribution of $\mathbf{y}_t$

	$\pi = 1$ $(d_t = 1)$				$\pi = 0$ $(d_t = 0)$			
β	$\bar{y}_1$	$\sigma(y_1)$	Kurt	Corr	$\bar{y}_1$	$\sigma(y_1)$	Kurt	Corr
				$\rho = 0$				
0.5	0.028	1.00	0.08	0.120	0.030	1.01	0.07	0.127
1.0	0.063	1.05	0.43	0.238	0.107	1.11	0.15	0.319
2.0	0.161	1.24	1.96	0.457	0.863	1.69	-1.13	0.706
				$\rho = 0.5$				
0.5	0.065	1.48	0.06	0.602	0.071	1.49	0.03	0.606
1.0	0.154	1.61	0.15	0.677	0.212	1.66	-0.15	0.697
2.0	0.369	1.94	0.19	0.767	0.907	2.18	-1.05	0.816

"Kurt" denotes Kurtosis-3 of the distribution of y_{1t} and "Corr" the correlation between y_{1t} and y_{2t}.

Rather than choosing only one part of the mixture in (15) one can also consider intermediate cases where both parts of the mixture are visited. Below we set $\pi = 0.5$ by sampling $d_t = I(s_t)$ where s_t is the realisation of a random variable characterising the nature of the policy intervention. In a purely random case where $s_t \sim \mathrm{N}(0,1)$ one obtains very pronounced bimodal distributions for y_{it}^*. A clear polar separation of solutions emerge when β is large, as can be seen in Figures 2-3 for $\beta = 2$ and $\rho = 0.8$. More dramatic pictures can be obtained for larger values of β as in Figure 4 and 5. These parameter values are chosen for illustrative purposes and we do not expect to observe such extreme phenomena in practice. For small values of β the polarisation is very slight and cannot be revealed by visual inspection. This can be seen in Figures 6 and 7, which display the results for $\beta = 0.5$ and $\rho = 0.5$.

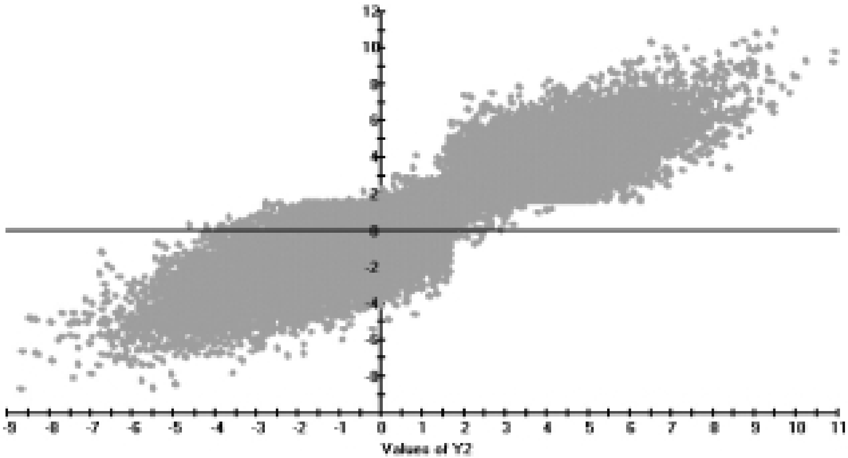


Figure 2: Scatter plot of y_1 on y_2 ($\beta = 2$, $\rho = 0.8$, $\pi = 0.5$)

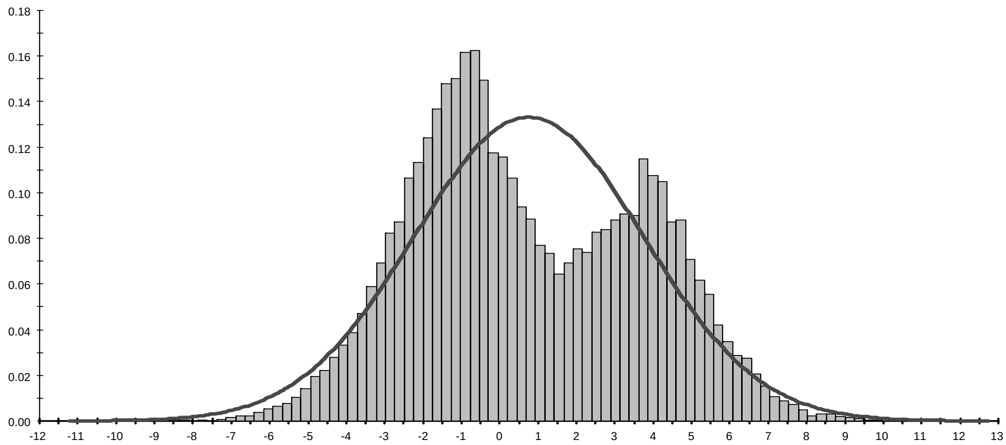


Figure 3: Histogram and normal curve for y_1 ($\beta = 2$, $\rho = 0.8$, $\pi = 0.5$)

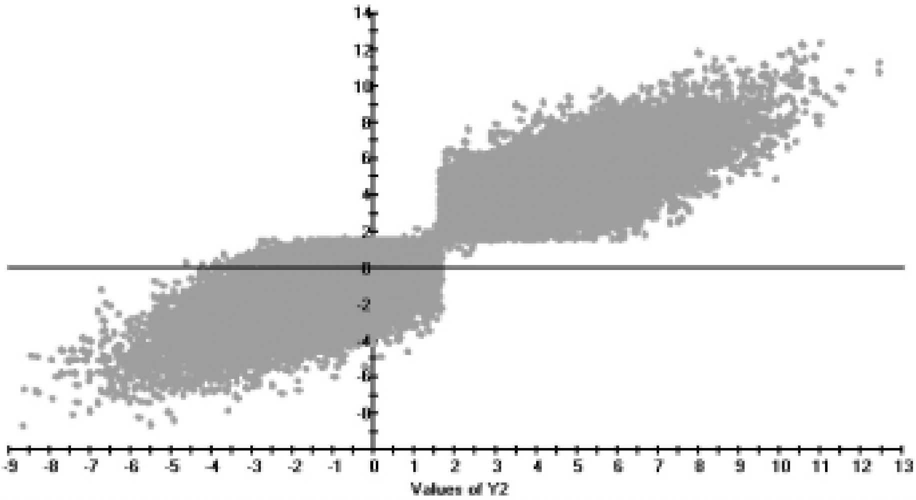


Figure 4: Scatter plot of $y_1$on y_2 ($\beta = 3.5$, $\rho = 0.8$, $\pi = 0.5$)

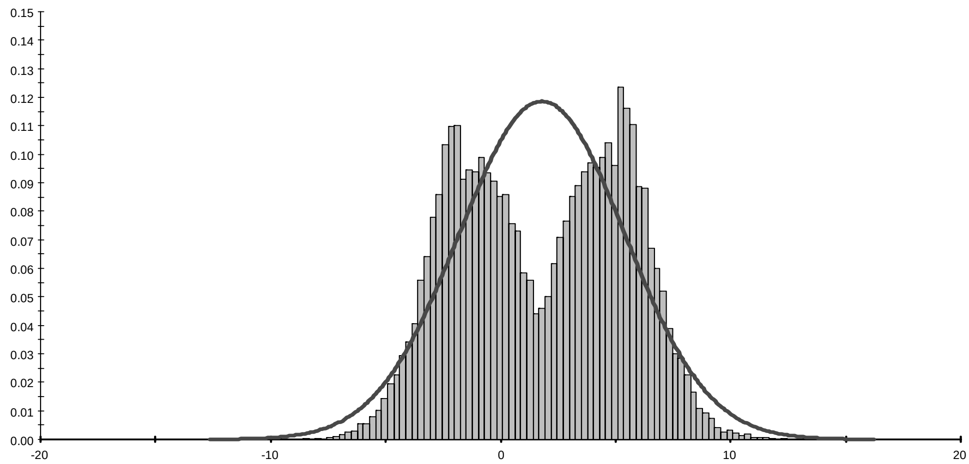


Figure 5: Histogram and normal curve for y_1 ($\beta = 3.5$, $\rho = 0.8$, $\pi = 0.5$)

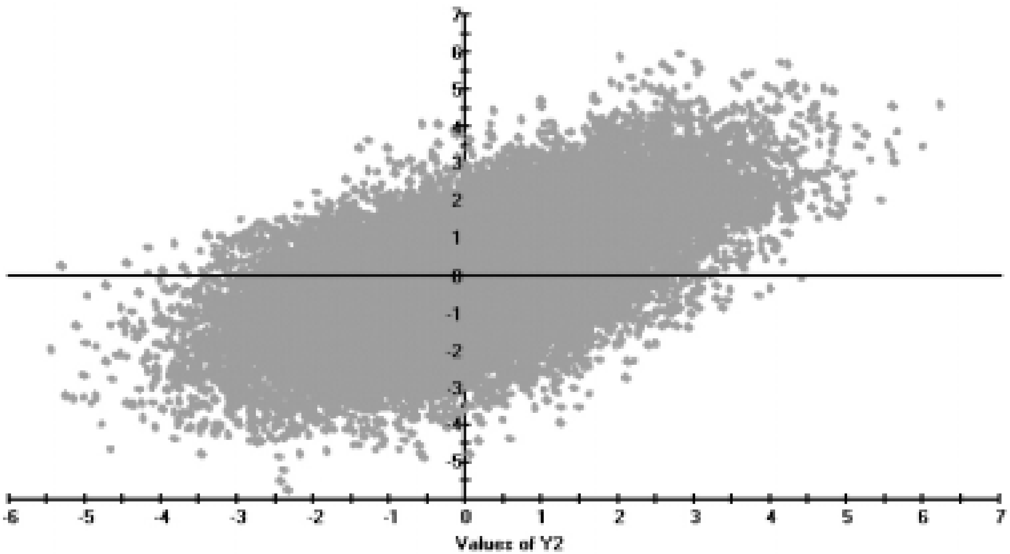


Figure 6: Scatter plot of $y_1$on y_2 ($\beta = 0.5$, $\rho = 0.8$, $\pi = 0.5$)

Figure 7: Histogram and normal curve for y_1 ($\beta = 0.5$, $\rho = 0.8$, $\pi = 0.5$)

5 Herding and Contagion

The literature on herding places important emphasis on the discounting of private (or individual-specific) information at times of crisis. Combining a herding effect with contagion the system of equations (1) and (2) can be generalised as

$$\begin{aligned} y_{1t} &= \boldsymbol{\delta}_1' \mathbf{z}_t + \boldsymbol{\alpha}_1' \mathbf{x}_{1t} \left[1 - \varphi_1 \mathrm{I}(y_{2t} - c_2 \sigma_{2,t-1})\right] + \beta_1 \mathrm{I}(y_{2t} - c_2 \sigma_{2,t-1}) + u_{1t}, \\ y_{2t} &= \boldsymbol{\delta}_2' \mathbf{z}_t + \boldsymbol{\alpha}_2' \mathbf{x}_{2t} \left[1 - \varphi_2 \mathrm{I}(y_{1t} - c_1 \sigma_{1,t-1})\right] + \beta_2 \mathrm{I}(y_{1t} - c_1 \sigma_{1,t-1}) + u_{2t}, \end{aligned}$$

where φ_i can be viewed as a herding coefficient if the individual-specific variables $\mathbf{x}_{1t}$ and $\mathbf{x}_{2t}$ are independently distributed. It would be reasonable to expect that $1 \geq \varphi_i \geq 0$, so that herding has a discounting, non-perverse effect on private information. In the case where $\mathbf{x}_{1t}$ and $\mathbf{x}_{2t}$ are correlated, the "herding factors", $-\varphi_i \mathrm{I}(y_{jt} - c_j \sigma_{j,t-1})$ $i, j = 1, 2$, must be applied to the non-correlated components of $\mathbf{x}_{1t}$ and $\mathbf{x}_{2t}$. Under this set up private information is discounted only in crisis periods. Dynamics can be introduced in the model by allowing $\mathbf{x}_{it}$ to contain lagged values, $y_{i,t-1}, y_{i,t-2}, \ldots$.

The model might also be written as

$$y_{it} = \boldsymbol{\delta}_i' \mathbf{z}_t + \boldsymbol{\alpha}_i' \mathbf{x}_{it} + \left[\beta_i - \varphi_i \boldsymbol{\alpha}_i' \mathbf{x}_{it}\right] \mathrm{I}(y_{jt} - c_j \sigma_{j,t-1}) + u_{it}, \text{ for } i, j = 1, 2,$$

and solved/analysed as before by treating β_i in the previous set up as a time varying coefficient, $\beta_{it} = \beta_i - \varphi_i \boldsymbol{\alpha}_i' \mathbf{x}_{it}$. In the case where $\mathbf{x}_{1t}$ and $\mathbf{x}_{2t}$ are independently distributed it is easy to show that the covariance of y_{1t} and y_{2t} does not increase as a result of herding, so long as contagion effects are not operating ($\beta_i = 0$). In fact herding (as defined here) reduces correlations as it raises volatilities without increasing covariances. This is in contrast to the case of contagion which is generally associated with a rise in correlations. Therefore, there is some potential in joint consideration of herding and contagion. But a detailed discussion would be beyond the scope of the present paper.

6 Identification and Estimation of the Contagion Coefficients

The system of equations (1) and (2) represent a two-equation non-linear simultaneous equation model which has been studied extensively in the econometric literature as summarised by Amemiya (1985), for example. The above equation systems whilst non-linear in the endogenous variables, $\mathbf{y}_t = (y_{1t}, y_{2t})'$, are linear in the parameters for known threshold values, c_1 and c_2. This somewhat simplifies the identification and estimation problems. In what follows we focus on this relatively simple case by assuming that c_1 and c_2 are known and that the variances $\sigma_{i,t-1}$ are time invariant and can be absorbed in c_i. The non-uniqueness of the solution is not by itself an impediment to identification and/or consistent estimation of the unknown parameters. However, efficient estimation of the parameters, for example by the maximum likelihood method, requires that the solutions (13) and (14) are augmented with an additional process that specifies the distribution of d_t, the solution indicator. As in the case of simultaneous equation models, it is possible to consistently estimate the parameters of a single equation in a system without necessarily having to fully specify the system of equations. An additional equation for d_t, is not essential for the consistent estimation of the contagion coefficients β_i, for example. But the identification problem becomes much more complicated and poses new challenges if the focus of the analysis is also on the identification of the d_t process itself. The resolution of this problem poses new challenges and is beyond the scope of the present paper. Hence, our focus will be on identification and consistent estimation of the contagion coefficients.

6.1 Inconsistency of the OLS Estimators

Consider the Ordinary Least Squares (OLS) regressions of y_{it} on $\mathbf{z}_t$, $\mathbf{x}_{i,t}$, $\mathrm{I}(y_{jt} - c_j)$, for $i, j = 1, 2$ and for simplicity suppose that the two equations only contain one country-specific regressor each and assume that these regressors (x_{1t}, x_2) are strictly exogenous and stationary, distributed independently of the errors, u_{1t} and u_{2t} :

$$y_{1t} = \alpha_1 x_{1t} + \beta_1 \mathrm{I}(y_{2t} - c_2) + u_{1t}, \tag{16}$$

$$y_{2t} = \alpha_2 x_{2t} + \beta_2 \mathrm{I}(y_{1t} - c_1) + u_{2t}, \tag{17}$$

where

$$\begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix} | x_{1t}, x_{2t} \sim N \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{u1}^2 & \rho\sigma_{u1}\sigma_{u2} \\ \rho\sigma_{u1}\sigma_{u2} & \sigma_{u2}^2 \end{pmatrix} \right].$$

Suppose also that probability of crisis occurring in either of the two countries are neither zero nor unity, namely[4]

$$T^{-1} \sum_{t=1}^{T} \mathrm{I}(y_{jt} - c_j) \to \pi_j, \text{ where } 1 > \pi_j > 0. \tag{18}$$

[4] This is not a primitive assumption and is made here for convenience. The crisis probabilities, π_j, $j = 1, 2$ depend in a complicated manner on the parameters of the model and the probability distribution functions of the forcing variables, x_{jt}, and the disturbances, u_{jt}, for $j = 1, 2$. These probabilities can be computed numerically using the solutions given by (13) and (14).

We also have

$$T^{-1}\sum_{t=1}^{T}x_{jt}^{2} \rightarrow \sigma_{xj}^{2}>0, \tag{19}$$

$$T^{-1}\sum_{t=1}^{T}x_{jt}u_{it} \rightarrow 0, \text{ for } i,j=1,2. \tag{20}$$

The OLS estimator of β_1 is given by

$$\hat{\beta}_1=\left(\mathbf{d}_2'\mathbf{M}_1\mathbf{d}_2\right)^{-1}\mathbf{d}_2'\mathbf{M}_1\mathbf{y}_1,$$

where $\mathbf{d}_2=(\mathrm{I}(y_{21}-c_2),\mathrm{I}(y_{22}-c_2),\ldots,\mathrm{I}(y_{2T}-c_2))'$, $\mathbf{M}_1=\mathbf{I}_T-\mathbf{x}_1(\mathbf{x}_1'\mathbf{x}_1)^{-1}\mathbf{x}_1'$, $\mathbf{x}_1=(x_{11},x_{12},\ldots,x_{1T})'$, and $\mathbf{y}_1=(y_{11},y_{12},\ldots,y_{1T})'$. But

$$T^{-1}\left(\mathbf{d}_2'\mathbf{M}_1\mathbf{d}_2\right)=T^{-1}\sum_{t=1}^{T}\mathrm{I}(y_{2t}-c_2)-\frac{\left[T^{-1}\sum_{t=1}^{T}\mathrm{I}(y_{2t}-c_2)x_{1t}\right]^2}{T^{-1}\sum_{t=1}^{T}x_{1t}^2},$$

and $T^{-1}\left(\mathbf{d}_2'\mathbf{M}_1\mathbf{d}_2\right)$ tends to a non-zero constant, $\omega_{22}>0$. This is easily seen in the simple case where $x_{1t}=1$ for all t. In this case $T^{-1}\left(\mathbf{d}_2'\mathbf{M}_1\mathbf{d}_2\right)$ converges to $\pi_2(1-\pi_2)>0$. Hence

$$\operatorname*{plim}_{T\to\infty}\left(\hat{\beta}_1\right)=\beta_1+\frac{\operatorname*{plim}_{T\to\infty}\left(\frac{\mathbf{d}_2'\mathbf{M}_1\mathbf{u}_1}{T}\right)}{\omega_{22}}.$$

where $\mathbf{u}_1=(u_{11},u_{12},\ldots,u_{1T})'$. Also under our assumptions (see in particular (19) and (20))

$$\begin{aligned}\operatorname*{plim}_{T\to\infty}\left(\frac{\mathbf{d}_2'\mathbf{M}_1\mathbf{u}_1}{T}\right) &= \operatorname*{plim}_{T\to\infty}\left(\frac{\mathbf{d}_2'\mathbf{u}_1}{T}\right)-\frac{\operatorname*{plim}_{T\to\infty}\left(\frac{\mathbf{d}_2'\mathbf{x}_1}{T}\right)\operatorname*{plim}_{T\to\infty}\left(\frac{\mathbf{x}_1'\mathbf{u}_1}{T}\right)}{\sigma_{x1}^2}\\ &= \mathrm{E}\left[u_{1t}\mathrm{I}(y_{2t}-c_2)\right],\end{aligned}$$

and

$$\operatorname*{plim}_{T\to\infty}\left(\hat{\beta}_1\right)=\beta_1+\frac{\mathrm{E}\left[u_{1t}\mathrm{I}(y_{2t}-c_2)\right]}{\omega_{22}}.$$

In general, $\mathrm{E}\left[u_{1t}\mathrm{I}(y_{2t}-c_2)\right]\neq 0$, and the OLS estimator of β_1 is inconsistent. The sign and the magnitude of the inconsistency of $\hat{\beta}_1$ depends on β_2 and ρ. The OLS estimator of β_1 is consistent only if $\beta_2=\rho=0$, namely if the contagion model is recursive (trinagular) and there are no interdependencies through the errors. To see this consider the relatively simple case where $\beta_2=0$, and note that under normally distributed errors we have

$$u_{1t}=\rho\left(\frac{\sigma_{u1}}{\sigma_{u2}}\right)u_{2t}+v_t, \tag{21}$$

where u_{2t} and v_t are independently distributed. Note also that v_t is distributed independently of x_{1t} and x_{2t} and has a zero mean. In this case

$$\begin{aligned}\mathrm{E}\left[u_{1t}\mathrm{I}(y_{2t}-c_2)\right] &= \mathrm{E}\left[u_{1t}\mathrm{I}(\alpha_2 x_{2t}+u_{2t}-c_2)\right] \\ &= \rho\left(\frac{\sigma_{u1}}{\sigma_{u2}}\right)\mathrm{E}\left[u_{2t}\mathrm{I}(\alpha_2 x_{2t}+u_{2t}-c_2)\right]+\mathrm{E}\left[v_t\mathrm{I}(\alpha_2 x_{2t}+u_{2t}-c_2)\right].\end{aligned}$$

Since v_t is distributed independently of x_{2t} and u_{2t}, then conditional on x_{2t} and u_{2t}

$$\mathrm{E}\left[v_t\mathrm{I}(\alpha_2 x_{2t}+u_{2t}-c_2)\left|u_{2t},x_{2t}\right.\right]=\mathrm{I}(\alpha_2 x_{2t}+u_{2t}-c_2)\mathrm{E}\left(v_t\left|u_{2t},x_{2t}\right.\right)=0,$$

and

$$\mathrm{E}\left[u_{1t}\mathrm{I}(y_{2t}-c_2)\right]=\rho\left(\frac{\sigma_{u1}}{\sigma_{u2}}\right)\mathrm{E}\left[u_{2t}\mathrm{I}(\alpha_2 x_{2t}+u_{2t}-c_2)\right].$$

The following lemma shows that when $\rho>0$, and $\beta_2=0$, then $\mathrm{E}[u_{2t}\mathrm{I}(y_{2t}-c_2)]>0$, and $\hat{\beta}_1$ will be a consistent estimator of β_1 if and only if $\rho=0$. The direction of the bias is upward when $\rho>0$, and downward if $\rho<0$.

Lemma 1 *Suppose $\beta_2=0$, and conditional on x_{2t}, u_{2t} is normally distributed, then $\mathrm{E}\left[u_{2t}\mathrm{I}(y_{2t}-c_2)\right]>0$ if $\rho>0$.*

Proof. Under $\beta_2=0$, $u_{2t}\mathrm{I}(y_{2t}-c_2)=u_{2t}\mathrm{I}(\alpha_2 x_{2t}+u_{2t}-c_2)=u_{2t}^*$, where

$$u_{2t}^*=\begin{cases}u_{2t} & \text{if } u_{2t}>c_2-\alpha_2 x_{2t},\\ 0 & \text{otherwise.}\end{cases}$$

Conditional on x_{2t}, noting that by assumption x_{2t}, and u_{2t} are independently distributed we have,

$$\mathrm{E}\left(u_{2t}^*\left|x_{2t}\right.\right)=\Pr(u_{2t}>c_2-\alpha x_{2t}\left|x_{2t}\right.)\mathrm{E}\left(u_{2t}|u_{2t}>c_2-\alpha_2 x_{2t}\ ,\ x_{2t}\right).$$

But

$$\mathrm{E}\left(u_{2t}|u_{2t}>c_2-\alpha_2 x_{2t},\ x_{2t}\right)=\frac{\sigma_{u2}\phi\left(\frac{c_2-\alpha_2 x_{2t}}{\sigma_{u2}}\right)}{\Pr(u_{2t}>c_2-\alpha_2 x_{2t},\ x_{2t})}.$$

and hence

$$\mathrm{E}\left(u_{2t}^*\left|x_{2t}\right.\right)=\sigma_{u2}\phi\left(\frac{c_2-\alpha_2 x_{2t}}{\sigma_{u2}}\right),$$

Since $\phi\left(\frac{c_2-\alpha_2 x_{2t}}{\sigma_{u2}}\right)>0$ for all values of x_{2t}, we also have:

$$\mathrm{E}\left(u_{2t}^*\right)=\mathrm{E}\left[u_{2t}\mathrm{I}(y_{2t}-c_2)\right]>0.$$

■

Consider now the general case where $\rho > 0$ and $\beta_2 > 0$, and note that in this case (using (21)) we have

$$\mathrm{E}\left[u_{1t}\mathrm{I}(y_{2t}-c_2)\right] = \rho\left(\frac{\sigma_{u1}}{\sigma_{u2}}\right)\mathrm{E}\left[u_{2t}\mathrm{I}(Y_{2t})\right] + \mathrm{E}\left[\varepsilon_{1t}\mathrm{I}(Y_{2t})\right], \tag{22}$$

where Y_{2t} is given by the solution (14), which takes either the value of W_{2t} or $1+W_{2t}$. The probability of whether the solution is W_{2t} or $1+W_{2t}$ depends, in a complicated manner, on the probability of W_{1t} and W_{2t} falling in the regions A,B,C, D, and E, and the probability of a particular solution being selected if W_{1t} and W_{2t} fall in region E. In the Appendix we give results from Monte Carlo experiments, which show that the expectation is positive for a wide range of values of $\beta_1, \beta_2, \alpha_1, \alpha_2$, and ρ. Therefore, unless $\beta_2 = \rho = 0$, the OLS estimator of β_1 will be inconsistent. The large sample bias will be upward when $\rho > 0$ and $\beta_1 > 0$.

6.2 Consistent Estimation of the Contagion Coefficients

Consistent estimation of β_i can be achieved by instrumental variable techniques assuming there exists pre-determined variables specific to country i that are correlated with $\mathrm{I}(y_{it}-c_i)$ and uncorrelated with the errors u_{it}.

If there are no country-specific regressors, namely if $\boldsymbol{\alpha}_1 = \boldsymbol{\alpha}_2 = 0$, the contagion coefficients, β_i, are not identified. In this case

$$\begin{aligned} y_{1t} &= \boldsymbol{\delta}_1'\mathbf{z}_t + \beta_1\mathrm{I}(y_{2t}-c_2) + u_{1t}, \\ y_{2t} &= \boldsymbol{\delta}_2'\mathbf{z}_t + \beta_2\mathrm{I}(y_{1t}-c_1) + u_{2t}, \end{aligned}$$

and the observed common drivers, $\mathbf{z}_t$, cannot be used as instruments for the crisis indicators. In this case pooling of the country equations will not help either, even if the slope homogeneity assumption is imposed (namely if $\boldsymbol{\delta}_1 = \boldsymbol{\delta}_2$, and $\beta_1 = \beta_2$).

If, however, country (market) specific regressors exist, i.e. $\boldsymbol{\alpha}_i \neq 0$, $i = 1, 2$, the following instrumental variables estimator can be used. Suppose that c_1 and c_2 are known and the observations $\mathbf{y}_t$, $\mathbf{w}_t = (\mathbf{z}_t', \mathbf{x}_{1t}', \mathbf{x}_{2t}')'$, $t = 1, 2, \ldots, T$ are given and that the following conditions are met.

(i)
$$\frac{\sum_{t=1}^{T}\mathbf{w}_t\mathbf{w}_t'}{T} \xrightarrow{p} \boldsymbol{\Sigma}_{ww},$$
where $\boldsymbol{\Sigma}_{ww}$ is a (non-stochastic) positive definite matrix.

(ii) Let $\mathbf{h}_{1t} = (\mathbf{z}_t', \mathbf{x}_{1t}', \mathrm{I}(y_{2t}-c_2))'$, and $\mathbf{h}_{2t} = (\mathbf{z}_t', \mathbf{x}_{2t}', \mathrm{I}(y_{1t}-c_1))'$, and
$$\frac{\sum_{t=1}^{T}\mathbf{w}_t\mathbf{h}_{i,t}'}{T} \xrightarrow{p} \mathbf{Q}_i,$$
where $\mathbf{Q}_i$ $i = 1, 2$ are full column rank matrices and the convergence to $\mathbf{Q}_i$ is uniform.

Then the IV estimator of $\boldsymbol{\theta}_i = (\boldsymbol{\delta}_i', \boldsymbol{\alpha}_i', \beta_i)'$, defined by

$$\hat{\boldsymbol{\theta}}_i = \left(\hat{\mathbf{Q}}_i' \hat{\boldsymbol{\Sigma}}_{ww}^{-1} \hat{\mathbf{Q}}_i\right)^{-1} \hat{\mathbf{Q}}_i' \hat{\boldsymbol{\Sigma}}_{ww}^{-1} \hat{\mathbf{q}}_i$$

where

$$\hat{\mathbf{Q}}_i = \frac{\sum_{t=1}^T \mathbf{w}_t \mathbf{h}_{i,t}'}{T}, \hat{\boldsymbol{\Sigma}}_{ww} = \frac{\sum_{t=1}^T \mathbf{w}_t \mathbf{w}_t'}{T}, \hat{\mathbf{q}}_i = \frac{\sum_{t=1}^T \mathbf{w}_t y_{it}}{T},$$

is consistent for $\boldsymbol{\theta}_i$ as $T \to \infty$.

The validity of these conditions need to be checked in the case of the particular model under consideration. For example, suppose the model of interest is given by (16) and (17), and that the conditions (18) to (20) hold, and $T^{-1} \sum_{t=1}^T x_{2t} x_{1t}$ tends to a finite limit as $T \to \infty$. Let

$$\operatorname*{plim}_{T \to \infty} \begin{pmatrix} T^{-1} \sum_{t=1}^T x_{1t}^2 & T^{-1} \sum_{t=1}^T x_{1t} \mathrm{I}(y_{2t} - c_2) \\ T^{-1} \sum_{t=1}^T x_{2t} x_{1t} & T^{-1} \sum_{t=1}^T x_{2t} \mathrm{I}(y_{2t} - c_2) \end{pmatrix} = \mathbf{V}_1.$$

Then α_1 and β_1 can be identified if $\mathbf{V}_1$ has a full rank. This rank condition can be investigated using the solutions (13) and (14). Although, the exact form of $\mathbf{V}_1$ depends on the way the indeterminacy of the solution is resolved in periods where $-1 < W_{it} = (\alpha_i x_{it} + u_{it} - c_i)/\beta_i \leq 0$, for $i = 1, 2$, it would nevertheless be possible to check if $\mathbf{V}_1$ is full rank without a full specification of the d_t process. For example, it suffices to postulate that d_t follows a general Bernoulli process with a probability that varies with the state variables, x_{it}, $i = 1, 2$. In the case where x_{it} and u_{it} are strictly stationary, in view of (13) and (14), it follows that y_{it}, $i = 1, 2$ are also strictly stationary, and

$$T^{-1} \sum_{t=1}^T x_{1t} \mathrm{I}(y_{2t} - c_2) \xrightarrow{p} \mathrm{E}\left[x_{1t} \mathrm{I}(y_{2t} - c_2)\right],$$

$$T^{-1} \sum_{t=1}^T x_{2t} \mathrm{I}(y_{2t} - c_2) \xrightarrow{p} \mathrm{E}\left[x_{2t} \mathrm{I}(y_{2t} - c_2)\right].$$

These results, in conjunction with the solution (13) and (14) allow us to establish the rank of $\mathbf{V}_1$ without an exact knowledge of the d_t process.

7 Correlation Based Tests of Contagion

In a number of papers by Boyer, Gibson, and Loretan (1999), Loretan and English (2000), Forbes and Rigobon (2002) and Corsetti, Pericoli and Sbracia (2002) attempts have been made to identify contagion effects from pair-wise correlation of stock market returns by testing whether correlation is significantly higher during crises times compared to normal periods. The main difference between the studies is in how the correlation coefficient is adjusted for the higher volatility in crises periods. The studies require *a priori* specification of the crises periods. The data employed are daily return observations and do not consider global or country-specific variables in their analysis.

In terms of our set up the basic model underlying this approach can be written as (following the approach of Corsetti et al.)

$$y_{1t} = \alpha_1 + \beta_1 \mathrm{I}(y_{2t} - c_{2t}) + u_{1t},$$
$$y_{2t} = \alpha_2 + \beta_2 \mathrm{I}(y_{1t} - c_{1t}) + u_{2t},$$

where c_t is *gleaned from the data*, and the inter-dependence across the two countries is characterised using the single factor specification

$$u_{it} = \gamma_i \; f_t + \varepsilon_{it}, \tag{23}$$

where f_t is the unobserved common factor, and ε_{it}, $i = 1, 2$ are idiosyncratic shocks:

$$f_t \sim iid(0, 1),$$
$$\varepsilon_{it} \sim iid(0, \sigma_i^2).$$

f_t and ε_{it} are also assumed to be independently distributed. For the two-country set up the single factor model is algebraically equivalent to assuming u_{1t} and u_{2t} are correlated with the correlation coefficient

$$\rho = \frac{\gamma_1 \gamma_2}{\sqrt{\sigma_1^2 + \gamma_1^2}\sqrt{\sigma_2^2 + \gamma_2^2}}.$$

Under this set up there exist no valid instruments with which to identify the contagion coefficient from the inter-dependence coefficient ρ. The identification problem is overcome in this literature by assuming that the crises periods are known *a priori*, and are sufficiently prolonged and continuous so that correlation of y_{1t} and y_{2t} during crisis and non-crisis periods can be consistently estimated and compared.

Therefore, this approach is problematic on three counts.

1. The endogeneity problem discussed in the previous section is circumvented by separating crises periods from non-crises periods. Since crisis periods are identified *ex post*, after passing through the observations, the endogeneity bias is re-introduced, however, in form of a sample selection bias.

2. Multi-country, multi-assets (markets) generalisations of the correlation/covariance approach will require existence of much longer periods of continuous crisis for the estimation and testing strategy to be meaningful. Such data sets are unlikely to exist since by their very nature crisis periods are relatively short.

3. The analysis can not be used in forecasting and is of limited scope in a structural understanding of the crises and the factors behind their occurrence.

8 Contagion in a Multi-Country Setting

Consider now a sample of N countries observed over the period $t = 1, 2, \ldots, T$, some or all of which could be subject to a crisis at least at some times over the sample period. A generalisation of (1) and (2) to the case of $N > 2$ can be written as

$$y_{it} = \boldsymbol{\delta}_i'\mathbf{z}_t + \boldsymbol{\alpha}_i'\mathbf{x}_{it} + \beta_i \sum_{j=1}^{N} w_{ij}\mathrm{I}(y_{jt} - c_j\sigma_{j,t-1}) + u_{it},\ i = 1, 2 \ldots, N,$$

where the weights $w_{ij} \geq 0$ are such that $\sum_{j=1}^{N} w_{ij} = 1$, and $w_{ii} = 0$, for all i. The theoretical literature on contagion can often be cast in terms of this general formulation. For example, Allen and Gale (2000) consider a theoretical model of financial contagion where bank failures spread from one region to another under different market structures. They set $N = 4$ and consider three types of market structures, namely "complete", "incomplete", and "disconnected incomplete". In terms of our set up these correspond to different weighting schemes as defined by the following patterns

$$\mathbf{W}_{Complete} = (w_{ij}) = \begin{pmatrix} 0 & 1/3 & 1/3 & 1/3 \\ 1/3 & 0 & 1/3 & 1/3 \\ 1/3 & 1/3 & 0 & 1/3 \\ 1/3 & 1/3 & 1/3 & 0 \end{pmatrix},$$

$$\mathbf{W}_{Incomplete} = (w_{ij}) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix},$$

and

$$\mathbf{W}_{Disconnected} = (w_{ij}) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

Notice also that the incomplete structures pre-suppose the existence of certain ordering of the regions, although no particular ordering of the regions is required under the complete market structure. Under the disconnected incomplete structure the $N = 4$ problem reduces to two separate $N = 2$ problems and their solutions do not pose any new difficulties. The incomplete market pattern can be reduced to the following generalisation of (7) and (8)

$$\begin{aligned} Y_{1t} &= W_{1t} + \mathrm{I}(Y_{2t}), \\ Y_{2t} &= W_{2t} + \mathrm{I}(Y_{3t}), \\ Y_{3t} &= W_{3t} + \mathrm{I}(Y_{4t}), \\ Y_{4t} &= W_{4t} + \mathrm{I}(Y_{1t}), \end{aligned}$$

where as before

$$Y_{it} = \frac{y_{it} - c_i \sigma_{i,t-1}}{\beta_i},\ W_{it} = \frac{\boldsymbol{\delta}_i' \mathbf{z}_t + \boldsymbol{\alpha}_i' \mathbf{x}_{it} + u_{it} - c_i \sigma_{i,t-1}}{\beta_i},\ i = 1, 2, 3, 4. \quad (24)$$

The solution in this case can be obtained along similar lines followed for the simple case of $N = 2$, although at the expense of much greater details. As before there will also be multiple solutions. For example, in the case where $W_{it} = 0$, two solutions are possible, namely $Y_{it}^a = 0$ and $Y_{it}^b = 1$. A complete characterisation of the solutions for all possible values of W_{it} will be beyond the scope of the present paper.

However, some interesting results can be obtained under the complete market structure. In this case (for a general N) we have

$$y_{it} = \boldsymbol{\alpha}' \mathbf{x}_{it} + \beta \left(\frac{\sum_{j=1, j \neq i}^{N} \mathrm{I}(y_{jt} - c_j)}{N - 1} \right) + \gamma f_t + \varepsilon_{it},\ i = 1, 2 \ldots, N, \quad (25)$$

where for simplicity we have omitted the common observed effects ($\mathbf{z}_t$), assumed all the coefficients are homogeneous and have charachterised the inter-dependence of the errors using the single factor structure given by (23). Define the crisis indicator $\kappa_{it} = \mathrm{I}(y_{it} - c_i)$. Then,

$$\frac{\sum_{j=1, j \neq i}^{N} \mathrm{I}(y_{jt} - c_j)}{N - 1} = \left(\frac{N}{N - 1} \right) \bar{\kappa}_t - \frac{1}{N - 1} \kappa_{it},$$

where $\bar{\kappa}_t = N^{-1} \sum_{i=1}^{N} \kappa_{it}$. Averaging (25) over $t = 1, 2, \ldots, T$, we have[5]

$$\bar{y}_t = \boldsymbol{\alpha}' \bar{\mathbf{x}}_{t-1} + \beta \bar{\kappa}_t + \gamma f_t + \bar{\varepsilon}_t.$$

Using this result in (25) to eliminate the unobserved common effect, $f_{t,}$, we have

$$y_{it} = \boldsymbol{\alpha}' \mathbf{x}_{it} + \beta \left[\left(\frac{N}{N - 1} \right) \bar{\kappa}_t - \frac{1}{N - 1} \kappa_{it} \right] + \left(\bar{y}_t - \boldsymbol{\alpha}' \bar{\mathbf{x}}_t - \beta \bar{\kappa}_t - \bar{\varepsilon}_t \right) + \varepsilon_{it},$$
$$i = 1, 2 \ldots, N.$$

[5] See Pesaran (2002) for a general discussion of the analysis of cross-sectional dependence in large panels.

Hence

$$y_{it} - \bar{y}_t = \boldsymbol{\alpha}'\left(\mathbf{x}_{it} - \bar{\mathbf{x}}_t\right) - \beta\left(\frac{\kappa_{it} - \bar{\kappa}_t}{N-1}\right) + (\varepsilon_{it} - \bar{\varepsilon}_t).$$

In the case where N is sufficiently large, the second term converges to zero and β cannot be identified, although a consistent estimator of $\boldsymbol{\alpha}$ can be obtained from an OLS regression of $y_{it} - \bar{y}_t$ on $(\mathbf{x}_{it} - \bar{\mathbf{x}}_t)$. Allowing for parameter heterogeneity does not resolve this problem. For N fixed as $T \rightarrow \infty$, the condition for identification of β is similar to the two-country case discussed in Section 6 above.

9 Panel Estimates of Contagion

Eichengreen, Rose and Wyplosz (1996, 1997), Esquivel and Larrain (1998), Kruger, Osakwe and Page (1998), Kumar, Moorthy and Perraudin (2002) and Stone and Weeks (2001) attempt to estimate and test for contagion effects using panel data. The econometric approach taken in these papers is based on binary choice models with linear index functions

$$y_{it} = \alpha_{0i} + \boldsymbol{\alpha}'\mathbf{x}_{it} + \varepsilon_{it}, \quad \text{for } i = 1, 2, \ldots, N, \ t = 1, 2, \ldots, T, \qquad (26)$$

where y_{it} is a latent variable observed qualitatively through a univariate binary response indicator, $\kappa_{it} = \mathrm{I}(y_{it})$, the currency crisis indicator. $\mathbf{x}_{it}$ is a $k \times 1$ vector of observed macroeconomic and political variables, $\boldsymbol{\alpha}$ is a $k \times 1$ vector of unknown coefficients and ε_{it} is an idiosyncratic error assumed to be serially uncorrelated for each i, and *iid* normal distributed across i with mean zero, a unit variance. Except for the paper by Esquivel and Larrain (1998), who use a random effects probit model, the literature assumes that $\alpha_{0i} = \alpha_0$.

Contagion is addressed by including a dummy variable, $\mathcal{C}_{it}$, in model (26),

$$y_{it} = \alpha_{0i} + \beta\mathcal{C}_{it} + \boldsymbol{\alpha}'\mathbf{x}_{it} + \varepsilon_{it},$$

where

$$\mathcal{C}_{it} = \mathrm{I}\left(\sum_{j=1, j\neq i}^{N} \kappa_{jt}\right). \qquad (27)$$

Under this formulation the crisis indicator, $\mathcal{C}_{it}$, takes the value of unity if any one of the $N-1$ remaining countries find themselves in a crisis state. This formulation is quite similar to that discussed above and is subject to similar identification and estimation issues. Due to the non-linear nature of this formulation, in order to assess the impact of the endogeneity on the parameter estimates in the probit model of (26) we conduct a Monte Carlo experiment using the data of Eichengreen et al. (1996). Details of the data are given in the Appendix 12.

9.1 Experimental Design

The Monte Carlo experiments are based on the following data generating process (DGP):

$$y_{it}^r = \alpha_0 + \boldsymbol{\alpha}'\mathbf{x}_{it}^r + u_{it}^r,$$

where $i = 1, 2, \ldots, N$, $t = 1, 2, \ldots, T$, $r = 1, 2, \ldots, R$, r refers to the replication number in the Monte Carlo experiments, R is the total number of replications, $\boldsymbol{\alpha}$ is a $k \times 1$ vector of parameters, $\mathbf{x}_{it}^r$ is a $k \times 1$ vector of simulated exogenous variables. Under this DGP, $\boldsymbol{\beta} = \mathbf{0}$ and all other coefficients are assumed to be identical across i.

The estimation of α_0 and $\boldsymbol{\alpha}$ under a probit specification only makes use of $\kappa_{it}^r = \mathrm{I}(y_{it}^r)$ and, hence, without loss of generality the variance of the error term, u_{it}^r, may be set equal to unity. To allow for correlation across the errors of different cross section units we adopt the following standardised one-factor structure

$$u_{it}^r = \frac{\gamma_i f_t^r + \varepsilon_{it}^r}{\sqrt{1+\gamma_i^2}}$$

where γ_i is a scalar, $f_t^r \sim \mathrm{iidN}(0,1)$, and $\varepsilon_{it}^r \sim \mathrm{iidN}(0,1)$. Under these assumptions we have $\mathrm{E}(u_{it}^r) = 0$ and $\mathrm{Var}(u_{it}^r) = 1$. The pairwise correlation coefficient of the errors is given by

$$\mathrm{Corr}\left(u_{it}^r, u_{jt}^r\right) = \frac{\gamma_i \gamma_j}{\sqrt{\left(1+\gamma_i^2\right)\left(1+\gamma_j^2\right)}}.$$

Regarding values of $y_{it}^r > 0$ as crisis, in all our experiments we fix α_0 such that the fraction of observations, π, with $y_{it}^r > 0$ is non-zero but relatively small, namely $\pi = 5\%$. For this purpose, assuming that the regressors are normally distributed we have $\boldsymbol{\alpha}'\mathbf{x}_{it} + u_{it} \sim \mathrm{iidN}(0, 1+\boldsymbol{\alpha}'\Sigma_x\boldsymbol{\alpha})$ and therefore

$$\Pr\left(y_{it}^r > 0\right) = \Pr\left(\boldsymbol{\alpha}'\mathbf{x}_{it}^r + u_{it}^r > -\alpha_0\right) = 1 - \Phi\left(\frac{-\alpha_0}{\sqrt{1+\boldsymbol{\alpha}'\boldsymbol{\Sigma}_x\boldsymbol{\alpha}}}\right) = \pi.$$

Hence, we set

$$\alpha_0 = -\left(1+\boldsymbol{\alpha}'\boldsymbol{\Sigma}_x\boldsymbol{\alpha}\right)^{1/2}\ \Phi^{-1}(1-\pi). \tag{28}$$

This is an important choice in the Monte Carlo experiment because the contagion dummy becomes a vector of ones if the proportion of crises periods is too high and then the right hand side variables are perfectly collinear as they contain an intercept and the contagion dummy. On the other hand, data sets without crises are meaningless for the concept of contagion and the estimation of a probit model is not possible.

For each replication a contagion dummy, $\mathcal{C}_{it}^r$, is constructed as

$$\mathcal{C}_{it}^r = \mathrm{I}\left(\sum_{j=1, j\neq i}^{N} \kappa_{jt}^r\right).$$

For the probit estimation only the binary indicator $\kappa_{it}^r = \mathrm{I}(y_{it}^r)$ is observed. The probability of $\kappa_{it}^r = 1$ is modelled as

$$\Pr(\kappa_{it}^r = 1) = \Phi(\alpha_0 + \beta \mathcal{C}_{it}^r + \boldsymbol{\alpha}' \mathbf{x}_{it}^r),$$

and for the linear OLS regression the assumed model is

$$y_{it}^r = \alpha_0 + \beta \mathcal{C}_{it}^r + \boldsymbol{\alpha}' \mathbf{x}_{it}^r + e_{it}^r,$$

where $e_{it}^r \sim \mathrm{iid}(0, \sigma_e^2)$. The parameters of the probit model (in particular the contagion coefficient, β) are computed by the maximum likelihood method.

In a first set of Monte Carlo experiments, we generate $\mathbf{x}_{it}^r \sim \mathrm{iid}(\mathbf{0}, \boldsymbol{\Sigma}_x)$ for two values of k, namely $k = 1$ and $k = 2$. We fix $\boldsymbol{\Sigma}_x$ implicitly by generating the regressors with the following common factor structure

$$x_{it}^r = \frac{1}{\sqrt{1 + \phi_i}} (q_{it}^r + \phi_i h_t^r),$$

where $q_{it}^r \sim \mathrm{iidN}(0,1)$, and $h_t^r \sim \mathrm{iidN}(0,1)$. To ensure that the regressors are distributed independently of the errors, h_t^r and f_t^r are taken to be independent draws. Finally, without loss of generality we set $\boldsymbol{\alpha} = \boldsymbol{\iota}_k$, a $k \times 1$ vector of ones. Note that under $\phi_i = 0$, $\boldsymbol{\Sigma}_x = \mathbf{I}_k$, and using (28) we have $\alpha_0 = 1.96 (\sqrt{1+k})$ for $\pi = 0.025$, and $\alpha_0 = 1.64 (\sqrt{1+k})$ for $\pi = 0.05$. In the case where $\phi_i > 0$, $\boldsymbol{\Sigma}_x$ will have typical off diagonal elements $\sigma_{ij} = \phi_i \phi_j / (\sqrt{1+\phi_i}\sqrt{1+\phi_j})$, and α_0 follows from (28).

Note that, while we appreciate that parameter heterogeneity may be important in applications, we abstract from it in the Monte Carlo experiment for simplicity. Intercept heterogeneity could be introduced via a random effects probit model or a conditional logit model, see Hsiao (2003).

In a second set of Monte Carlo experiments the exogenous regressors of Eichengreen et al. (1996, ERW) are used and taken as given across all the replications. Under the null of no contagion β is set equal to zero and the other parameters, $(\alpha_0, \boldsymbol{\alpha})$, are set equal to the estimates of the pooled probit model computed using the ERW data. These estimates are given in Table 2.

Table 2: Probit model with ERW data

variable	$(\hat{\alpha}_0, \hat{\boldsymbol{\alpha}})$	$\lvert t \rvert$-value
Intercept $(\hat{\alpha}_0)$	-1.886	10.751
Capital controls	-0.134	0.717
Government victory	-0.060	1.141
Government loss	-0.332	0.787
Credit growth	0.016	1.880
Inflation	0.065	3.584
Output growth	0.020	0.732
Employment growth	0.043	1.007
Unemployment rate	0.073	3.010
Budget position	0.042	2.042
Current account	-0.024	1.072

Total number of observations = 645

Hence, a vector $\mathbf{y}^r$ is generated as

$$y_{it}^r = \hat{\alpha}_0 + \hat{\boldsymbol{\alpha}}'\mathbf{x}_{it} + u_{it}^r$$

The specification of the error term and the estimation are as in the case of artificial data.

9.2 Results of the Monte Carlo Experiments

9.2.1 Results for the Simulated Regressors

Tables 3–8 give the results for the Monte Carlo experiments with artificially generated regressors. Tables 3 and 4 report the results for $k = 1$, Tables 5 and 6 for $k = 2$ with orthogonal regressors, and Tables 7 and 8, report the results for $k = 2$ where the regressors are correlated with $\phi_i = 0.5$, $\forall i$. The first of each pair of tables uses only a discretised dependent variable and estimates a probit model, while the second uses the continuous dependent variable and estimates the model by OLS.

It can be seen that throughout all experiments the bias increases with the size of the correlation of the error term across i. For small and even medium sample sizes the estimate of β is quite imprecise in the probit model even under $\gamma = 0$. However, the OLS estimates of the contagion effects (β) under error inter-dependence $(\rho = \gamma^2/(1+\gamma^2) \neq 0)$ is positive in all the experiments, confirming the upward bias derived theoretically in the context of our simple two-country canonical model.

The last panel of each table gives the rejection probability for the hypothesis of no contagion, i. e. $\beta = 0$. It can be seen that the rejection probability rises as inter-dependence increases. With $\gamma = 1$, that is with error correlation 0.5, $N = 100$ and $T = 100$ the hypothesis of no contagion is always rejected in all models. However, even mild inter-dependence leads to high rejection rates. In the OLS estimation with $k = 1$, $\gamma = 0.4$, which implies correlation of 0.14, and $N = T = 50$ the hypothesis of no contagion is rejected in 97.3% of cases. It can also be seen that the probit model has poor size, over-rejecting the null in all cases.

A further interesting result is that even for homogeneous γ the precision of the estimates does not improve equally with increasing N and T. It is clear that for all the experiments the root mean square errors are systematically lower with T larger than N for a given value of the product $N \times T$. For example in Table 3, for $\gamma = 1$, the RMSE is 1.192 for $T = 50$, $N = 100$, and 1.055 for $T = 100$, $N = 50$. The intuition behind this is the way the contagion variable is constructed, which means that the information contents of increasing N and T are not the same. Recall that the contagion variable is 1 for all i if there are at least two crises in the period, and hence the effect of increasing N will be limited.

9.2.2 Results Based on the ERW Regressors

The pooled OLS and pooled probit results for this case are summarised in Table 9. Both sets of results clearly show an upward bias in the estimates of the contagion coefficient for non-zero values of γ, with the bias increasing steadily with γ. The bias could be substantial even for moderate degrees of cross dependence. For example, for $\gamma = 0.4$ (which corresponds to a pairwise cross correlation coefficient of around 0.14) the pooled panel estimate of β is 0.27 as compared to its true value of zero. This result holds both under homogeneous and heterogeneous $\gamma's$, and estimation procedures.

The null hypothesis of $\beta = 0$ is also rejected well in excess of the nominal 5% level for all non-zero values of γ. The pooled probit estimates also exhibit a substantial degree of over-rejection (12.3% as compared to 5%) even under $\gamma = 0$. The degree of over-rejection of the pooled OLS estimates (7.2%) is much less pronounced, although still significantly different from 5% considering that the experiments are based on 2000 replications.

In view of these results it is reasonable to conclude that the estimate of the contagion coefficient of 0.54 that one obtains from pooled probit estimation using the ERW data could be wholly or partly due to neglected inter-dependencies of the equation errors across different countries.

10 Conclusion

In this paper we have developed a canonical model of contagion. Using this model, we have considered the issue of identification and consistent estimation of contagion coefficients. We show that in the presence of error inter-dependencies contagion effects cannot be consistently estimated without country-specific fundamentals. This clearly highlights some of the pitfalls that surround the empirical studies of currency crises and financial contagions that are extant in the literature. Correlation analysis that looks for significant shifts in correlation coefficients across the crisis and tranquil periods are usually based on high frequency data (daily or weekly) for which there are no observations on country specific fundamentals. In the case of such data sets identification of contagion is achieved by making strong *a priori* assumptions concerning sample splits into “crisis” and “no-crisis” periods. Invariably, this also involves the identification of the source country in which the crisis is purported to have begun.

Multi-country panel analyses of the type carried out by ERW do contain country specific fundamentals and could in principle be used to shed light on the issue of contagion versus inter-dependence. However, panel data studies are typically carried out assuming that contagion indices are exogenous and that errors across countries/markets are independently distributed, and as we have shown this could introduce a substantial upward bias in the estimates of the contagion coefficients. A simultaneous estimation of inter-dependence and contagion effects are required. The canonical model presented in this paper could be viewed as a first step towards such an objective.

11 Appendix: Simulation of $\mathbf{E}[u_{2t}I(y_{1t}-c_1)]$

Table A reports the simulated values of $\mathrm{E}\left[u_{2t}\mathrm{I}(y_{1t}-c_1)\right]$ using $\sum_{t=1}^{T}\left[u_{2t}\mathrm{I}(y_{1t}-c_1)\right]/T$ with $T=2,000,000$. The data are generated from the reduced form of the model given by Equations (13) and (14) with $k=1$, $x_{it},u_{it}\sim \mathrm{iid}\,\mathrm{N}(0,1)$, $\Pr(d_t=1)=0.50$, and $c_i=1.64$. It can be seen that only for $\rho=\beta=0$ the simulated value is zero. Similar results are also obtained for other choices of the solution indicator, d_t, namely $d_t=0$, or $d_t=1$.

Table A: Simulated Values of $\sum_{t=1}^{T} [u_{2t}\mathrm{I}(y_{1t} - c_1)] / T$

ρ	β	α				
		−4	−1	0	1	4
−0.99	−4	−0.095	−0.143	−0.103	−0.142	−0.096
	−1	−0.092	−0.143	−0.103	−0.142	−0.092
	0	−0.089	−0.142	−0.103	−0.143	−0.088
	1	−0.082	−0.140	−0.103	−0.140	−0.083
	4	−0.054	−0.037	−0.024	−0.039	−0.053
−0.50	−4	−0.056	−0.077	−0.052	−0.077	−0.056
	−1	−0.050	−0.075	−0.052	−0.075	−0.050
	0	−0.045	−0.072	−0.052	−0.072	−0.044
	1	−0.036	−0.051	−0.040	−0.051	−0.037
	4	−0.011	0.030	0.023	0.031	−0.011
0.00	−4	−0.018	−0.017	−0.005	−0.017	−0.017
	−1	−0.007	−0.010	−0.004	−0.010	−0.008
	0	0.000	0.000	−0.000	0.000	−0.000
	1	0.008	0.040	0.045	0.041	0.008
	4	0.032	0.089	0.060	0.090	0.032
0.50	−4	0.022	0.035	0.026	0.035	0.021
	−1	0.036	0.053	0.036	0.052	0.036
	0	0.045	0.072	0.052	0.072	0.045
	1	0.055	0.128	0.135	0.128	0.055
	4	0.075	0.134	0.082	0.134	0.074
0.99	−4	0.060	0.078	0.010	0.078	0.060
	−1	0.078	0.112	0.047	0.112	0.079
	0	0.089	0.142	0.103	0.142	0.089
	1	0.099	0.208	0.213	0.207	0.099
	4	0.114	0.165	0.070	0.166	0.115

The results are from data generated according to Equations (13) and (14),with $k = 1$, $x_{it}, u_{it} \sim \text{iid}\,\mathrm{N}(0,1)$, $\Pr(d_t = 1) = 0.5$, $c_i = 1.64$, and $T = 2{,}000{,}000$.

12 Data Appendix

The data set used by Eichengreen et al. (1996, 1997) is available on the internet at

`http://haas.berkeley.edu/~arose/RecRes.htm`

along with a *Stata* log file. The description of the data is identical in Eichengreen et al. (1996, pp. 477–478) and (1997, pp. 23–25).

According to Eichengreen et al. (1997, p. 23) "[t]he data set is quarterly, spanning 1959 through 1993 for twenty industrial countries." The countries are the USA, UK, Austria, Belgium, Denmark, France, Italy, Netherlands, Norway, Sweden, Switzerland, Canada, Japan, Finland, Greece, Ireland, Portugal, Spain, Australia and Germany as the centre country. "Most of the variables are transformed into differential percentage changes by taking differences between domestic and German annualised fourth-differences of natural logarithms and multiplying by a hundred." (Eichengreen et al. 1997, p. 23).

The variables are: Total non-gold international reserves (IMF IFS line 1ld), exchange rate with US dollar (rf), money market rates (60b) or where unavailable discount rates (60), exports and imports (70 and 71), the current account (80) and the central governments budget position (80) both as percentages of nominal GDP (99a), long term bond yields (61), nominal stock market index (62), domestic credit (32), M1 (34), M2 (35 + M1), CPI (64), real GDP (99a.r), and relative unit labour cost (reu). Further from the OECD's Main Economic Indicators employment and unemployment, and Eichengreen et al. construct "indicators of government electoral victories and defeats, using Keesing's *Record of World Events* and the World Banks' *Political Handbook of the World*." (Eichengreen et al. 1997, p. 24)

Eichengreen et al. use the following definition of the exchange-rate market pressure index

$$EMP_{it} = \lambda_1 \% \Delta e_{it} + \lambda_2 \% \Delta (r_{it} - r_{Gt}) - \lambda_3 (\% \Delta f_{it} - \% \Delta f_{Gt}), \qquad (29)$$

where e_{it} is the exchange rate to the US Dollar, r_{it} the interest rate, and f_{it} the international reserves of country i. Subscript G indicates variables for Germany, which is taken as the center country. Eichengreen et al. (1997, pp.23–24) say that they "weight the components so as to equalize the volatility of the three components". This is accomplished by setting $\lambda_i = 1/\sigma_i$, where σ_i is the standard deviation of component i. For this data set $\sigma_1 = 0.243$, $\sigma_2 = 0.037$, and $\sigma_3 = 0.0047$.

The crisis index is the calculated as

$$y_{it} = \begin{cases} 1 & EMP_{it} > \mu_{EMP} + 1.5\sigma_{EMP} \\ 0 & \text{otherwise} \end{cases}$$

where μ_{EMP} is the mean and σ_{EMP} is the standard deviation of the exchange rate market pressure index.

The credit growth, the inflation rate, the output growth and the current account are calculated as

$$dx_{it} = 100 * \ln(x_{it}/x_{it-4}) - \ln(x_{Gt}/x_{Gt-4}), \qquad (30)$$

where x_{it} is the variable for country i and Germany, G. The relative unemployment rate is $dx_{it} = x_{it} - x_{Gt}$. The relative budget position is defined as $db_{it} = b_{it}/y_{it} - b_{Gt}/y_{Gt}$, where b_{it} is the nominal government budget of country i, y_{it} is the GDP of country i and Germany, G. The dummies for capital controls, government electoral victory and government electoral loss are not transformed. The other variables mentioned above are not used.

"To avoid counting the same crisis more than once, we exclude the later observation(s) when two (or more) crises occur in successive quarters." (Eichengreen et al. 1997, p.22) Country by country excluding time periods with missing data results in 645 observations for 17 countries with 56 crises observations. The countries are the USA, the UK, Austria, Belgium, Denmark, France, Germany, Italy, the Netherlands, Norway, Canada, Japan, Finland, Greece, Ireland, Portugal, Spain, and Australia.

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Table 3a: Bias and RMSE of the Contagion Coefficient in a Probit Model ($k = 1$)

	$T =$	20			50				100			
γ	$N = 10$	20	50	100	10	20	50	100	10	20	50	100
						Bias						
						homogeneous γ						
0	−1.947	−0.602	−0.021	0.118	−1.093	−0.081	−0.017	−0.045	−0.335	−0.024	−0.004	−0.017
0.2	−1.911	−0.445	0.046	0.206	−0.877	0.017	0.095	0.122	−0.176	0.053	0.094	0.130
0.4	−1.449	−0.179	0.338	0.435	−0.419	0.224	0.338	0.423	0.065	0.264	0.332	0.433
0.6	−0.877	0.134	0.576	0.711	−0.130	0.478	0.596	0.703	0.340	0.500	0.597	0.715
0.8	−0.661	0.445	0.841	0.995	0.258	0.715	0.840	0.950	0.593	0.725	0.837	0.950
1	−0.198	0.774	1.054	1.196	0.514	0.914	1.039	1.160	0.819	0.924	1.036	1.150
						heterogeneous $\gamma_i \sim \mathrm{U}\left(\frac{1}{2}\gamma, \frac{3}{2}\gamma\right)$						
0.2	−1.808	−0.411	0.067	0.216	−0.830	0.030	0.091	0.118	−0.171	0.055	0.092	0.126
0.4	−1.454	−0.158	0.289	0.407	−0.479	0.213	0.322	0.397	0.017	0.247	0.323	0.400
0.6	−0.874	0.156	0.554	0.658	−0.155	0.470	0.552	0.648	0.334	0.463	0.558	0.655
0.8	−0.663	0.352	0.766	0.884	0.205	0.681	0.771	0.871	0.558	0.678	0.769	0.863
1	−0.308	0.584	0.978	1.087	0.432	0.858	0.952	1.055	0.787	0.861	0.956	1.049
						RMSE						
						homogeneous γ						
0	2.989	1.692	0.427	0.603	2.185	0.581	0.250	0.346	1.184	0.235	0.169	0.240
0.2	2.979	1.598	0.433	0.584	2.031	0.473	0.260	0.337	1.025	0.243	0.194	0.251
0.4	2.786	1.527	0.566	0.711	1.709	0.514	0.418	0.502	0.857	0.354	0.374	0.471
0.6	2.654	1.495	0.773	0.908	1.616	0.635	0.652	0.748	0.818	0.568	0.624	0.736
0.8	2.606	1.566	1.029	1.147	1.460	0.830	0.886	0.985	0.888	0.770	0.860	0.966
1	2.670	1.767	1.211	1.350	1.516	1.032	1.082	1.192	0.980	0.965	1.055	1.166
						heterogeneous $\gamma_i \sim \mathrm{U}\left(\frac{1}{2}\gamma, \frac{3}{2}\gamma\right)$						
0.2	2.940	1.557	0.419	0.583	1.986	0.454	0.255	0.342	0.998	0.239	0.191	0.247
0.4	2.776	1.442	0.538	0.645	1.753	0.513	0.406	0.482	0.966	0.351	0.368	0.440
0.6	2.634	1.508	0.715	0.851	1.612	0.632	0.615	0.698	0.777	0.526	0.590	0.680
0.8	2.659	1.767	0.997	1.034	1.497	0.812	0.821	0.910	0.842	0.726	0.794	0.882
1	2.692	1.700	1.128	1.264	1.473	0.959	0.998	1.091	0.954	0.901	0.977	1.067

See footnote to Table 3b.

Table 3b: Size and Power of the Contagion Test in a Probit Model ($k = 1$)

	$T =$	20			50				100			
γ	$N = 10$	20	50	100	10	20	50	100	10	20	50	100
	Rejection probability											
	homogeneous γ											
0	0.080	0.095	0.129	0.131	0.103	0.108	0.128	0.144	0.104	0.110	0.136	0.170
0.2	0.099	0.132	0.164	0.231	0.133	0.167	0.246	0.300	0.150	0.196	0.304	0.387
0.4	0.156	0.239	0.397	0.467	0.223	0.353	0.624	0.742	0.299	0.515	0.821	0.908
0.6	0.234	0.390	0.623	0.743	0.363	0.621	0.906	0.962	0.545	0.835	0.991	0.999
0.8	0.300	0.532	0.790	0.900	0.524	0.806	0.980	0.998	0.730	0.962	0.999	1.000
1	0.397	0.631	0.876	0.946	0.640	0.904	0.996	1.000	0.848	0.988	1.000	1.000
	heterogeneous $\gamma_i \sim \mathrm{U}\big(\frac{1}{2}\gamma, \frac{3}{2}\gamma\big)$											
0.2	0.111	0.133	0.178	0.246	0.125	0.169	0.237	0.304	0.137	0.192	0.300	0.379
0.4	0.152	0.243	0.362	0.455	0.211	0.357	0.615	0.712	0.293	0.475	0.802	0.882
0.6	0.257	0.362	0.603	0.721	0.356	0.616	0.872	0.943	0.518	0.795	0.985	0.996
0.8	0.285	0.490	0.752	0.864	0.510	0.790	0.972	0.994	0.700	0.942	1.000	1.000
1	0.369	0.589	0.858	0.921	0.613	0.869	0.993	0.999	0.821	0.985	1.000	1.000

Data are generated from $y_{it}^r = \alpha_0 + \boldsymbol{\alpha}'\mathbf{x}_{it}^r + u_{it}^r$, where $\mathbf{x}_{it}^r = \frac{1}{\sqrt{1+\phi_i}}(q_{it}^r + \phi_i \mathbf{h}_t^r)$, $\mathbf{h}_t^r, q_{it}^r \sim iid\,\mathrm{N}(0,1)$, $\boldsymbol{\alpha}$ is a vector of ones, and $\alpha_0 = -1.96\sqrt{1+\boldsymbol{\alpha}'\Sigma\boldsymbol{\alpha}}$. $u_{it} = \frac{1}{\sqrt{1+\gamma_i^2}}(\gamma_i f_t^r + \varepsilon_{it}^r)$, where $\gamma_i \sim \mathrm{U}\big(\frac{1}{2}\gamma, \frac{3}{2}\gamma\big)$, $f_t^r, \varepsilon_{it}^r \sim iid\,\mathrm{N}(0,1)$, where $\mathrm{U}(a,b)$ denotes the Uniform distribution with lower limit a and upper limit b. The probit estimations use a discretised dependent variable, $\kappa_{it}^r = \mathrm{I}(y_{it}^r)$. For the estimations, a spurious contagion dummy was added and the common factor was ignored. The results in the table are for the contagion coefficient, $\hat{\beta}$. Part a of the table reports the bias of the coefficient of the contagion coefficient, i. e. $\sum_{r=1}^{R}(\hat{\beta}^{(r)} - \beta^0)/R$, and the root mean square error, $(\sum_{r=1}^{R}(\hat{\beta}^{(r)} - \beta^0)^2/R)^{1/2}$, where the true value $\beta^0 = 0$ in the DGP and $r = 1, 2, \ldots, R$ with $R = 2000$ is the number of replications. Part b reports the one-sided rejection probability, which is defined as the probability that the t-value is larger than the 95% critical value (1.645), where the rejection probability under $\gamma = 0$ is the size and under $\gamma \neq 0$ the power.

Table 4a: Bias and RMSE of the Contagion Coefficient in an OLS Model ($k = 1$)

	$T =$	20				50				100		
γ	$N = 10$	20	50	100	10	20	50	100	10	20	50	100
						Bias						
						homogeneous γ						
0	−0.006	−0.005	−0.005	−0.001	−0.003	−0.001	−0.003	−0.012	−0.001	0.001	0.001	−0.005
0.2	0.065	0.076	0.092	0.146	0.068	0.076	0.102	0.139	0.073	0.079	0.102	0.143
0.4	0.246	0.273	0.322	0.402	0.248	0.268	0.322	0.395	0.252	0.267	0.326	0.403
0.6	0.470	0.479	0.543	0.630	0.465	0.485	0.546	0.626	0.464	0.491	0.547	0.625
0.8	0.667	0.682	0.732	0.805	0.667	0.685	0.735	0.804	0.665	0.684	0.735	0.802
1	0.849	0.853	0.884	0.939	0.833	0.853	0.885	0.937	0.838	0.851	0.885	0.936
						heterogeneous $\gamma_i \sim \mathrm{U}\left(\frac{1}{2}\gamma, \frac{3}{2}\gamma\right)$						
0.2	0.066	0.075	0.096	0.135	0.067	0.079	0.102	0.131	0.068	0.077	0.101	0.140
0.4	0.238	0.263	0.299	0.378	0.236	0.257	0.310	0.385	0.241	0.258	0.311	0.385
0.6	0.450	0.459	0.521	0.578	0.436	0.468	0.514	0.592	0.445	0.459	0.519	0.591
0.8	0.632	0.642	0.685	0.752	0.626	0.643	0.688	0.755	0.634	0.646	0.688	0.752
1	0.793	0.783	0.819	0.881	0.773	0.789	0.832	0.880	0.782	0.793	0.830	0.882
						RMSE						
						homogeneous γ						
0	0.194	0.114	0.083	0.087	0.120	0.070	0.049	0.071	0.083	0.049	0.034	0.044
0.2	0.223	0.161	0.150	0.211	0.150	0.116	0.125	0.174	0.118	0.100	0.114	0.158
0.4	0.366	0.338	0.364	0.453	0.301	0.294	0.339	0.414	0.278	0.280	0.335	0.412
0.6	0.566	0.542	0.580	0.669	0.507	0.508	0.561	0.640	0.484	0.502	0.554	0.632
0.8	0.752	0.739	0.768	0.841	0.706	0.706	0.750	0.816	0.683	0.695	0.742	0.809
1	0.936	0.906	0.922	0.973	0.867	0.874	0.899	0.950	0.855	0.862	0.892	0.942
						heterogeneous $\gamma_i \sim \mathrm{U}\left(\frac{1}{2}\gamma, \frac{3}{2}\gamma\right)$						
0.2	0.229	0.160	0.152	0.205	0.151	0.118	0.126	0.166	0.118	0.099	0.113	0.156
0.4	0.367	0.329	0.346	0.432	0.292	0.284	0.328	0.405	0.269	0.273	0.320	0.394
0.6	0.555	0.521	0.562	0.623	0.484	0.494	0.530	0.607	0.469	0.473	0.527	0.599
0.8	0.726	0.701	0.727	0.788	0.670	0.667	0.704	0.770	0.655	0.660	0.697	0.759
1	0.886	0.840	0.858	0.918	0.812	0.812	0.848	0.894	0.803	0.805	0.838	0.889

See footnote of Table 4b.

Table 4b: Size and Power of the Contagion Test in an OLS Model ($k = 1$)

	$T =$	20			50				100			
γ	$N = 10$	20	50	100	10	20	50	100	10	20	50	100
	Rejection probability											
	homogeneous γ											
0	0.053	0.051	0.060	0.072	0.053	0.057	0.053	0.061	0.060	0.064	0.065	0.053
0.2	0.129	0.237	0.438	0.575	0.176	0.356	0.665	0.733	0.260	0.523	0.841	0.894
0.4	0.423	0.682	0.892	0.919	0.639	0.910	0.993	0.991	0.850	0.989	1.000	1.000
0.6	0.718	0.891	0.984	0.988	0.912	0.995	1.000	1.000	0.993	1.000	1.000	1.000
0.8	0.862	0.965	0.997	0.999	0.980	0.999	1.000	1.000	0.999	1.000	1.000	1.000
1	0.925	0.986	0.999	1.000	0.992	0.999	1.000	1.000	1.000	1.000	1.000	1.000
	heterogeneous $\gamma_i \sim \mathrm{U}\left(\frac{1}{2}\gamma, \frac{3}{2}\gamma\right)$											
0.2	0.139	0.228	0.434	0.547	0.187	0.373	0.663	0.707	0.268	0.500	0.840	0.892
0.4	0.402	0.670	0.872	0.898	0.609	0.895	0.990	0.988	0.823	0.982	0.999	1.000
0.6	0.692	0.878	0.976	0.979	0.899	0.993	0.999	1.000	0.985	1.000	1.000	1.000
0.8	0.829	0.952	0.992	0.995	0.966	0.998	1.000	1.000	0.999	1.000	1.000	1.000
1	0.901	0.980	0.999	0.999	0.988	0.999	1.000	1.000	1.000	1.000	1.000	1.000

Data are generated from $y_{it}^r = \alpha_0 + \boldsymbol{\alpha}'\mathbf{x}_{it}^r + u_{it}^r$, where $\mathbf{x}_{it}^r = \frac{1}{\sqrt{1+\phi_i}}(q_{it}^r + \phi_i \mathbf{h}_t^r)$, $\mathbf{h}_t^r, q_{it}^r \sim iid\,\mathrm{N}(0,1)$, $\boldsymbol{\alpha}$ is a vector of ones, and $\alpha_0 = -1.96\sqrt{1+\boldsymbol{\alpha}'\Sigma\boldsymbol{\alpha}}$. $u_{it} = \frac{1}{\sqrt{1+\gamma_i^2}}(\gamma_i f_t^r + \varepsilon_{it}^r)$, where $\gamma_i \sim \mathrm{U}\left(\frac{1}{2}\gamma, \frac{3}{2}\gamma\right)$, $f_t^r, \varepsilon_{it}^r \sim iid\,\mathrm{N}(0,1)$. The OLS estimations uses the continuous dependent variable, y_{it}^r. For the estimations, a spurious contagion dummy was added and the common factor was ignored. The results in the table are for the contagion coefficient, $\hat{\beta}$. Part a reports the bias of the coefficient of the contagion coefficient, i. e. $\sum_{r=1}^{R}(\hat{\beta}^{(r)} - \beta^0)/R$, and the root mean square error, $(\sum_{r=1}^{R}(\hat{\beta}^{(r)} - \beta^0)^2/R)^{1/2}$, where the true value $\beta^0 = 0$ in the DGP and $r = 1, 2, \ldots, R$ with $R = 2000$ is the number of replications. Part b reports the one-sided rejection probability , which is defined as the probability that the t-value is larger than the 95% critical value, where the rejection probability under $\gamma = 0$ is the size and under $\gamma \neq 0$ the power.

Table 5a: Bias and RMSE of the Contagion Coefficient in a Probit Model ($k = 2$, $\phi = 0$)

	$T =$	20			50				100			
γ	$N = 10$	20	50	100	10	20	50	100	10	20	50	100
						Bias						
						homogeneous γ						
0	−1.996	−0.608	−0.029	0.136	−1.108	−0.071	−0.011	0.076	−0.316	−0.015	−0.011	0.014
0.2	−1.912	−0.467	0.053	0.196	−0.913	0.001	0.074	0.163	−0.236	0.046	0.088	0.121
0.4	−1.453	−0.232	0.299	0.434	−0.629	0.170	0.282	0.380	−0.069	0.208	0.290	0.377
0.6	−1.181	0.026	0.545	0.675	−0.376	0.393	0.524	0.633	0.224	0.420	0.514	0.646
0.8	−0.742	0.370	0.751	0.891	−0.038	0.618	0.745	0.878	0.463	0.617	0.731	0.883
1	−0.425	0.584	0.964	1.079	0.359	0.777	0.935	1.065	0.661	0.793	0.927	1.077
						heterogeneous $\gamma_i \sim \mathrm{U}\left(\frac{1}{2}\gamma, \frac{3}{2}\gamma\right)$						
0.2	−1.914	−0.471	0.051	0.224	−0.951	−0.004	0.075	0.170	−0.217	0.042	0.081	0.132
0.4	−1.564	−0.279	0.256	0.399	−0.659	0.166	0.273	0.370	−0.038	0.194	0.272	0.368
0.6	−1.094	0.049	0.495	0.624	−0.334	0.367	0.502	0.598	0.212	0.402	0.496	0.607
0.8	−0.733	0.248	0.691	0.835	−0.097	0.573	0.688	0.812	0.413	0.581	0.683	0.804
1	−0.563	0.438	0.899	1.003	0.155	0.730	0.850	0.993	0.585	0.743	0.852	0.981
						RMSE						
						homogeneous γ						
0	3.165	1.822	0.466	0.684	2.297	0.563	0.261	0.291	1.161	0.262	0.183	0.218
0.2	3.166	1.712	0.485	0.649	2.151	0.556	0.281	0.336	1.149	0.266	0.205	0.253
0.4	2.924	1.565	0.586	0.762	1.964	0.527	0.395	0.485	1.087	0.333	0.343	0.428
0.6	2.871	1.573	0.743	0.883	1.850	0.608	0.593	0.698	0.909	0.502	0.549	0.677
0.8	2.750	1.643	0.950	1.066	1.660	0.799	0.802	0.925	0.873	0.673	0.759	0.905
1	2.715	1.778	1.162	1.244	1.580	0.913	0.989	1.105	0.928	0.849	0.952	1.096
						heterogeneous $\gamma_i \sim \mathrm{U}\left(\frac{1}{2}\gamma, \frac{3}{2}\gamma\right)$						
0.2	3.159	1.683	0.468	0.647	2.164	0.590	0.275	0.340	1.130	0.263	0.201	0.260
0.4	2.960	1.635	0.537	0.710	1.974	0.517	0.384	0.476	1.015	0.345	0.332	0.424
0.6	2.835	1.536	0.711	0.868	1.818	0.581	0.580	0.666	0.888	0.486	0.534	0.641
0.8	2.697	1.614	0.873	1.040	1.765	0.761	0.752	0.865	0.930	0.644	0.713	0.830
1	2.651	1.730	1.093	1.146	1.688	0.875	0.906	1.038	0.956	0.798	0.880	1.003

See footnote to Table 3b

Table 5b: Size and Power of the Contagion Test. in a Probit Model ($k = 2$, $\phi = 0$)

	$T =$	20			50				100			
γ	$N = 10$	20	50	100	10	20	50	100	10	20	50	100
	Rejection probability											
	homogeneous γ											
0	0.078	0.102	0.110	0.136	0.087	0.116	0.123	0.173	0.097	0.112	0.114	0.151
0.2	0.086	0.122	0.159	0.188	0.112	0.129	0.207	0.266	0.142	0.177	0.272	0.312
0.4	0.134	0.186	0.321	0.404	0.179	0.279	0.474	0.596	0.230	0.368	0.678	0.804
0.6	0.182	0.307	0.529	0.639	0.264	0.483	0.801	0.887	0.399	0.670	0.955	0.988
0.8	0.249	0.431	0.685	0.807	0.361	0.690	0.929	0.984	0.575	0.873	0.995	1.000
1	0.309	0.503	0.786	0.884	0.515	0.779	0.978	0.995	0.705	0.943	0.999	1.000
	heterogeneous $\gamma_i \sim \mathrm{U}\left(\frac{1}{2}\gamma, \frac{3}{2}\gamma\right)$											
0.2	0.086	0.123	0.156	0.205	0.117	0.142	0.197	0.294	0.121	0.163	0.251	0.336
0.4	0.121	0.182	0.286	0.385	0.177	0.254	0.455	0.601	0.228	0.351	0.641	0.764
0.6	0.183	0.277	0.487	0.599	0.269	0.450	0.767	0.865	0.374	0.663	0.932	0.973
0.8	0.245	0.395	0.647	0.760	0.363	0.659	0.910	0.966	0.562	0.851	0.986	0.998
1	0.295	0.458	0.754	0.872	0.458	0.756	0.967	0.993	0.655	0.940	0.998	0.999

See footnote of Table 3b.

Table 6a: Bias and RMSE of the Contagion Coefficient in an OLS Model ($k = 2$, $\phi = 0$)

	$T =$	20			50				100			
γ	$N = 10$	20	50	100	10	20	50	100	10	20	50	100
					Bias							
					homogeneous γ							
0	−0.006	−0.002	−0.010	−0.000	−0.002	0.002	−0.004	0.002	0.001	−0.002	−0.001	−0.001
0.2	0.062	0.061	0.082	0.126	0.058	0.066	0.084	0.124	0.062	0.065	0.087	0.124
0.4	0.212	0.221	0.275	0.371	0.204	0.223	0.277	0.368	0.206	0.226	0.282	0.365
0.6	0.391	0.417	0.488	0.592	0.383	0.414	0.484	0.585	0.385	0.416	0.482	0.588
0.8	0.578	0.604	0.664	0.764	0.554	0.592	0.661	0.752	0.561	0.592	0.661	0.763
1	0.729	0.742	0.797	0.881	0.712	0.731	0.802	0.886	0.706	0.736	0.803	0.892
					heterogeneous $\gamma_i \sim \mathrm{U}\left(\frac{1}{2}\gamma, \frac{3}{2}\gamma\right)$							
0.2	0.055	0.065	0.081	0.124	0.061	0.066	0.083	0.122	0.059	0.064	0.086	0.124
0.4	0.207	0.212	0.264	0.348	0.198	0.212	0.267	0.347	0.200	0.216	0.268	0.354
0.6	0.370	0.394	0.448	0.558	0.359	0.390	0.469	0.554	0.364	0.396	0.461	0.556
0.8	0.540	0.550	0.617	0.712	0.521	0.563	0.616	0.712	0.530	0.552	0.622	0.715
1	0.675	0.675	0.754	0.841	0.657	0.688	0.754	0.842	0.671	0.690	0.750	0.842
					RMSE							
					homogeneous γ							
0	0.182	0.110	0.081	0.086	0.118	0.068	0.048	0.052	0.081	0.048	0.032	0.040
0.2	0.216	0.151	0.148	0.206	0.146	0.109	0.111	0.160	0.111	0.089	0.101	0.143
0.4	0.335	0.293	0.328	0.433	0.267	0.252	0.297	0.392	0.234	0.242	0.292	0.377
0.6	0.498	0.481	0.533	0.643	0.432	0.440	0.501	0.605	0.410	0.429	0.491	0.598
0.8	0.679	0.667	0.711	0.809	0.600	0.617	0.679	0.770	0.582	0.605	0.669	0.772
1	0.819	0.806	0.842	0.927	0.752	0.756	0.820	0.903	0.725	0.749	0.812	0.901
					heterogeneous $\gamma_i \sim \mathrm{U}\left(\frac{1}{2}\gamma, \frac{3}{2}\gamma\right)$							
0.2	0.215	0.158	0.147	0.206	0.148	0.110	0.110	0.156	0.108	0.086	0.100	0.143
0.4	0.332	0.288	0.318	0.414	0.262	0.245	0.289	0.374	0.235	0.233	0.280	0.367
0.6	0.484	0.462	0.498	0.616	0.412	0.418	0.487	0.575	0.392	0.412	0.471	0.566
0.8	0.645	0.615	0.663	0.763	0.571	0.592	0.636	0.732	0.554	0.566	0.631	0.725
1	0.775	0.741	0.801	0.888	0.705	0.714	0.773	0.860	0.694	0.703	0.759	0.851

See footnote to Table 4b

Table 6b: Size and Power of the Contagion Test in an OLS Model ($k = 2$, $\phi = 0$)

	$T =$	20				50				100		
γ	$N = 10$	20	50	100	10	20	50	100	10	20	50	100
						Rejection probability						
						homogeneous γ						
0	0.053	0.049	0.046	0.070	0.052	0.060	0.056	0.066	0.056	0.057	0.053	0.071
0.2	0.139	0.204	0.391	0.510	0.163	0.313	0.569	0.685	0.218	0.427	0.764	0.832
0.4	0.370	0.584	0.828	0.888	0.543	0.830	0.973	0.980	0.746	0.960	0.998	0.998
0.6	0.628	0.853	0.962	0.972	0.836	0.979	0.999	0.999	0.962	1.000	1.000	1.000
0.8	0.802	0.933	0.988	0.993	0.946	0.999	1.000	1.000	0.996	1.000	1.000	1.000
1	0.886	0.964	0.997	0.998	0.990	0.999	1.000	1.000	0.999	1.000	1.000	1.000
						heterogeneous $\gamma_i \sim \mathrm{U}\left(\frac{1}{2}\gamma, \frac{3}{2}\gamma\right)$						
0.2	0.127	0.219	0.389	0.507	0.160	0.305	0.561	0.683	0.204	0.424	0.746	0.840
0.4	0.354	0.574	0.808	0.861	0.521	0.798	0.963	0.971	0.714	0.954	0.997	0.998
0.6	0.604	0.828	0.938	0.960	0.807	0.967	0.998	0.999	0.949	0.998	1.000	1.000
0.8	0.770	0.913	0.987	0.983	0.918	0.996	1.000	1.000	0.992	1.000	1.000	1.000
1	0.857	0.952	0.994	0.997	0.965	0.999	1.000	1.000	0.998	1.000	1.000	1.000

See footnote of Table 4b.

Table 7a: Bias and RMSE of the Contagion Coeffient in a Probit Model ($k=2$, $\phi=0.5$)

	$T=$	20				50				100		
γ	$N=10$	20	50	100	10	20	50	100	10	20	50	100
						Bias						
						homogeneous γ						
0	−1.231	−0.234	−0.023	−0.007	−0.324	−0.027	−0.010	−0.006	−0.072	−0.015	−0.001	−0.014
0.2	−1.123	−0.148	0.048	0.107	−0.258	0.030	0.066	0.080	0.007	0.0499	0.072	0.097
0.4	−0.868	0.059	0.247	0.334	−0.006	0.196	0.264	0.314	0.159	0.217	0.263	0.321
0.6	−0.450	0.235	0.513	0.578	0.223	0.396	0.490	0.566	0.344	0.409	0.495	0.574
0.8	−0.105	0.525	0.718	0.840	0.361	0.593	0.693	0.794	0.525	0.597	0.700	0.799
1	0.162	0.705	0.931	1.061	0.570	0.762	0.884	1.004	0.684	0.772	0.875	0.984
						heterogeneous $\gamma_i \sim \mathrm{U}\left(\frac{1}{2}\gamma, \frac{3}{2}\gamma\right)$						
0.2	−1.110	−0.141	0.060	0.123	−0.269	0.028	0.068	0.067	0.009	0.052	0.078	0.091
0.4	−0.782	−0.014	0.268	0.304	−0.030	0.184	0.251	0.293	0.144	0.213	0.254	0.306
0.6	−0.495	0.228	0.470	0.529	0.169	0.378	0.475	0.532	0.337	0.400	0.457	0.517
0.8	−0.205	0.429	0.635	0.771	0.382	0.549	0.640	0.726	0.511	0.564	0.644	0.729
1	0.135	0.616	0.850	0.966	0.494	0.724	0.800	0.893	0.663	0.704	0.802	0.898
						RMSE						
						homogeneous γ						
0	2.613	1.137	0.487	0.648	1.261	0.327	0.253	0.291	0.548	0.219	0.175	0.200
0.2	2.588	1.139	0.515	0.687	1.193	0.332	0.280	0.293	0.433	0.223	0.186	0.212
0.4	2.529	1.202	0.583	0.702	1.026	0.388	0.376	0.418	0.422	0.318	0.322	0.375
0.6	2.309	1.169	0.810	0.887	0.971	0.536	0.563	0.633	0.563	0.476	0.533	0.605
0.8	2.215	1.375	0.960	1.088	1.049	0.705	0.756	0.846	0.631	0.647	0.729	0.825
1	2.295	1.528	1.210	1.305	1.101	0.878	0.942	1.051	0.798	0.816	0.901	1.006
						heterogeneous $\gamma_i \sim \mathrm{U}\left(\frac{1}{2}\gamma, \frac{3}{2}\gamma\right)$						
0.2	2.605	1.084	0.480	0.715	1.198	0.331	0.263	0.292	0.435	0.225	0.193	0.213
0.4	2.507	1.220	0.610	0.721	1.089	0.392	0.365	0.402	0.430	0.315	0.313	0.360
0.6	2.327	1.125	0.764	0.816	1.037	0.544	0.555	0.604	0.543	0.468	0.496	0.557
0.8	2.321	1.318	0.912	1.049	1.024	0.664	0.710	0.782	0.667	0.619	0.677	0.756
1	2.372	1.505	1.096	1.246	1.165	0.843	0.865	0.943	0.781	0.756	0.833	0.923

See footnote to Table 3b

Table 7b: Size and Power of the Contagion Test in a Probit Model ($k = 2$, $\phi = 0.5$)

	$T =$	20			50				100			
γ	$N = 10$	20	50	100	10	20	50	100	10	20	50	100
	Rejection probability											
	homogeneous γ											
0	0.071	0.082	0.094	0.082	0.103	0.099	0.103	0.128	0.105	0.096	0.118	0.113
0.2	0.080	0.109	0.139	0.150	0.118	0.140	0.192	0.201	0.150	0.166	0.220	0.274
0.4	0.128	0.201	0.275	0.336	0.212	0.292	0.456	0.547	0.260	0.422	0.631	0.737
0.6	0.192	0.302	0.491	0.545	0.347	0.518	0.772	0.852	0.488	0.732	0.929	0.979
0.8	0.264	0.450	0.653	0.733	0.452	0.706	0.916	0.973	0.668	0.911	0.993	0.999
1	0.348	0.527	0.746	0.846	0.587	0.824	0.974	0.992	0.812	0.968	1.000	1.000
	heterogeneous $\gamma_i \sim \mathrm{U}\left(\frac{1}{2}\gamma, \frac{3}{2}\gamma\right)$											
0.2	0.089	0.125	0.148	0.145	0.118	0.136	0.169	0.192	0.151	0.162	0.235	0.271
0.4	0.139	0.198	0.282	0.298	0.216	0.291	0.441	0.494	0.254	0.431	0.615	0.716
0.6	0.199	0.292	0.465	0.523	0.329	0.504	0.748	0.820	0.488	0.720	0.906	0.948
0.8	0.271	0.397	0.584	0.687	0.465	0.678	0.872	0.950	0.670	0.883	0.989	0.995
1	0.330	0.496	0.710	0.799	0.549	0.803	0.944	0.988	0.773	0.951	0.998	1.000

See footnote of Table 3b.

Table 8a: Bias and RMSE of the Contagion Coefficient in an OLS Model ($k = 2$, $\phi = 0.5$)

	$T =$	20				50				100		
γ	$N = 10$	20	50	100	10	20	50	100	10	20	50	100
						Bias						
						homogeneous γ						
0	−0.006	−0.005	−0.003	−0.005	0.001	0.002	−0.001	−0.002	0.001	−0.003	−0.001	−0.002
0.2	0.045	0.059	0.068	0.081	0.057	0.062	0.071	0.084	0.057	0.062	0.072	0.083
0.4	0.193	0.215	0.242	0.279	0.206	0.222	0.247	0.275	0.208	0.218	0.244	0.279
0.6	0.393	0.405	0.438	0.471	0.391	0.399	0.443	0.480	0.392	0.405	0.440	0.481
0.8	0.575	0.573	0.618	0.650	0.556	0.584	0.607	0.651	0.558	0.579	0.614	0.650
1	0.725	0.707	0.753	0.799	0.711	0.724	0.751	0.786	0.707	0.720	0.745	0.782
						heterogeneous $\gamma_i \sim \mathrm{U}\left(\frac{1}{2}\gamma, \frac{3}{2}\gamma\right)$						
0.2	0.052	0.059	0.069	0.081	0.054	0.061	0.069	0.080	0.058	0.061	0.072	0.082
0.4	0.198	0.212	0.237	0.256	0.206	0.208	0.230	0.263	0.202	0.210	0.235	0.262
0.6	0.378	0.374	0.411	0.437	0.364	0.383	0.416	0.446	0.368	0.387	0.412	0.446
0.8	0.542	0.521	0.558	0.604	0.536	0.540	0.575	0.599	0.524	0.541	0.568	0.605
1	0.667	0.667	0.684	0.723	0.653	0.670	0.694	0.728	0.667	0.670	0.699	0.732
						RMSE						
						homogeneous γ						
0	0.183	0.114	0.076	0.069	0.114	0.068	0.044	0.040	0.081	0.050	0.032	0.028
0.2	0.213	0.156	0.137	0.158	0.143	0.109	0.102	0.115	0.108	0.087	0.088	0.099
0.4	0.314	0.290	0.298	0.344	0.261	0.252	0.270	0.300	0.238	0.234	0.256	0.291
0.6	0.499	0.473	0.492	0.529	0.434	0.426	0.463	0.502	0.415	0.419	0.450	0.491
0.8	0.672	0.640	0.671	0.704	0.598	0.609	0.627	0.671	0.577	0.592	0.624	0.660
1	0.806	0.774	0.803	0.846	0.747	0.750	0.769	0.805	0.725	0.732	0.757	0.791
						heterogeneous $\gamma_i \sim \mathrm{U}\left(\frac{1}{2}\gamma, \frac{3}{2}\gamma\right)$						
0.2	0.218	0.158	0.136	0.156	0.140	0.108	0.098	0.112	0.110	0.087	0.088	0.098
0.4	0.320	0.291	0.297	0.327	0.265	0.242	0.256	0.288	0.234	0.227	0.247	0.275
0.6	0.483	0.448	0.462	0.497	0.413	0.415	0.436	0.469	0.394	0.403	0.423	0.458
0.8	0.648	0.594	0.612	0.660	0.580	0.567	0.596	0.620	0.548	0.555	0.579	0.615
1	0.767	0.739	0.739	0.776	0.699	0.700	0.716	0.748	0.690	0.686	0.709	0.743

See footnote of Table 4b

Table 8b: Size and Power of the Contagion Test in an OLS Model ($k = 2$, $\phi = 0.5$)

	$T =$	20			50				100			
γ	$N = 10$	20	50	100	10	20	50	100	10	20	50	100
	Rejection probability											
	homogeneous γ											
0	0.047	0.050	0.054	0.060	0.061	0.056	0.047	0.041	0.061	0.051	0.060	0.049
0.2	0.120	0.194	0.353	0.460	0.171	0.287	0.505	0.617	0.224	0.402	0.678	0.765
0.4	0.340	0.569	0.772	0.827	0.560	0.836	0.952	0.964	0.764	0.948	0.993	0.997
0.6	0.633	0.829	0.928	0.948	0.868	0.978	0.995	0.998	0.978	0.997	1.000	1.000
0.8	0.807	0.923	0.973	0.979	0.951	0.997	1.000	1.000	0.997	1.000	1.000	1.000
1	0.895	0.952	0.988	0.994	0.988	0.999	1.000	1.000	0.999	1.000	1.000	1.000
	heterogeneous $\gamma_i \sim \mathrm{U}\left(\frac{1}{2}\gamma, \frac{3}{2}\gamma\right)$											
0.2	0.120	0.208	0.345	0.459	0.153	0.294	0.486	0.604	0.223	0.399	0.675	0.761
0.4	0.336	0.564	0.754	0.788	0.551	0.782	0.921	0.954	0.738	0.936	0.993	0.997
0.6	0.615	0.792	0.920	0.935	0.823	0.961	0.997	0.996	0.959	0.998	1.000	1.000
0.8	0.765	0.895	0.970	0.972	0.947	0.994	1.000	1.000	0.994	1.000	1.000	1.000
1	0.853	0.941	0.984	0.991	0.972	0.997	0.999	1.000	0.999	1.000	1.000	1.000

See footnote of Table 4b.

Table 9: Bias, RMSE, Size and Power for the Coefficient of a Spurious Contagion Index (ERW Data)

	Probit			OLS		
γ	Bias	RMSE	$[t > c]$	Bias	RMSE	$[t > c]$
			homogeneous $\gamma_i = \gamma$			
0	−0.012	0.245	0.123	−0.005	0.095	0.072
0.2	0.074	0.245	0.215	0.079	0.128	0.297
0.4	0.270	0.362	0.518	0.278	0.300	0.902
0.6	0.521	0.580	0.849	0.506	0.522	0.999
0.8	0.773	0.818	0.981	0.713	0.725	1.000
1	0.995	1.034	0.996	0.884	0.894	1.000
			heterogeneous $\gamma_i \sim \mathrm{U}\big(\frac{1}{2}\gamma, \frac{3}{2}\gamma\big)$			
0.2	0.069	0.247	0.212	0.079	0.127	0.289
0.4	0.282	0.375	0.535	0.276	0.300	0.887
0.6	0.528	0.588	0.858	0.492	0.510	0.996
0.8	0.774	0.822	0.977	0.696	0.711	1.000
1	0.998	1.042	0.996	0.863	0.875	1.000

Data are generated from $y_{it}^r = \boldsymbol{\alpha}'\mathbf{x}_{it} + \varepsilon_{it}^r$, where $\mathbf{x}_{it}$ are the data of ERW and $\boldsymbol{\alpha}$ the respective probit estimates of the parameters. $\varepsilon_{it}^r = \gamma_i^r f_t^r + u_{it}^r$, where $\gamma_i^r \sim \mathrm{U}\big(\frac{1}{2}\gamma, \frac{3}{2}\gamma\big)$, $f_t^r, u_{it}^r \sim iid\ \mathrm{N}(0,1)$. The probit estimations use a discretised dependent variable, $\kappa_{it}^r = \mathrm{I}(y_{it}^r)$, and the OLS estimations the continuous dependent variable, y_{it}^r. For the estimations, a spurious contagion dummy was added and the common factor was ignored. The results in the table are for the contagion coefficient, $\hat{\beta}$. Reported are the bias of the coefficient of the contagion coefficient, i. e. $\sum_{r=1}^{R}(\hat{\beta}^{(r)} - \beta^0)/R$, the root mean square error, $(\sum_{r=1}^{R}(\hat{\beta}^{(r)} - \beta^0)^2/R)^{1/2}$, where the true value $\beta^0 = 0$ in the DGP and $r = 1, 2, \ldots, R$ with $R = 2000$ is the number of replications. Finally, the one-sided rejection probability denoted $[t > c]$ is reported, which is defined as the probability that the t-value is larger than the 95% critical value (1.645), where the rejection probability under $\gamma = 0$ is the size and under $\gamma \neq 0$ the power.