Bargaining over monetary policy in a monetary union and the case for appointing an independent central banker.*
Revised version.

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Abstract

We set up a model of a monetary union where decisions over monetary policy are made through bargaining between two governments with different objectives. They can either choose to directly bargain over monetary policy or to delegate monetary decisions to an independent central banker. In the latter case, the choice of the central banker is obtained by bargaining between the two governments.

We show that, the bargaining power being constant, the delegation of monetary policy to an independent central banker does not necessarily incur a smaller inflation bias nor is systematically welfare improving for any government. It may happen that both governments are better-off when they directly bargain.

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1 Introduction.

In this paper, we prove that the inflation bias in a monetary union may be higher when monetary policy is delegated to an independent central banker than when it is directly exerted by the relevant policymakers, in the presence of bargaining. Moreover, it may happen that both policymakers are better off when they directly bargain over monetary policy decisions. Economic theory gives credit to the advantages of appointing an independent conservative central banker. Rogoff (1985) showed that the delegation of monetary policy to an independent central banker was a means to reduce the inflationary bias due to the time inconsistency problem (Kydland and Prescott, 1977). Delegating monetary policy to an independent central banker means that the latter has his own preferences with respect to monetary policy (which differ from the government's) and chooses monetary policy accordingly, without interference from the government. The government cannot override the central banker, give him instructions nor dismiss him.[1] Following this seminal paper, a large body of literature has grown up over the past years[2] and nowadays it is widely believed that a high level of independence is an efficient institutional device to assure price stability. However, there is still an on-going debate on the empirical validity of Rogoff's claim.[3]

Rogoff's result was obtained in a one-region monetary economy with a unique policymaker. This representation may be oversimplified. Indeed, the nomination of the authority in charge of monetary policy is the result of an intricate process of negotiations, bargaining and compromises between agents whose preferences regarding monetary policy are not identical. Blinder (1997), drawing from his experience as a member of the FOMC, stressed that "monetary policy decisions are made by a committee" and that "committee discussions must aggregate preferences, seek common ground and somehow produce group decision".[4] In many countries, the choice of the central bank committee does not only stem from the executive's preferences but involves as well the legislative body. In the United States, members of the Board of Governors of the Federal Reserve System are chosen by the U.S. president subject to Senate confirmation.[5] The case of the European Monetary Union is particularly interesting. It

[1] This definition is close to the one used in Lohmann (1992) and Eijffinger and Hoeberichts (1998). Both papers distinguish between central bank independence and conservatism. There exist other delegation shemes than "appointing an optimally inflation averse central banker" that we do not consider in the present paper (see the literature on inflation targeting or on performance contracts for instance). Finally, there are other reasons that justify the appointment of an independent central banker, beyond inflation aversion: the existence of political pressures related to the political business cycles, the fear of monetization of public deficits. Here, we do not envisage these cases as they would necessitate other more adequate models.

[2] For a literature review, see for example Gärtner (2000), Persson and Tabellini (1990, 1999), Walsh (1998), Cukierman (1992).

[3] For critical approaches of the literature, see McCallum (1995, 1997), Blinder (1997) and Forder (1998 a,b). Forder, in the latter paper, discusses the nomination process of a central banker. He argues against interpreting Rogoff's results as a fanciful approach to public sector appointments.

[4] see Blinder (1997), p.16.

[5] For a description of the Federal Reserve System, see Havrilesky (1993). Havrilesky stresses

perfectly exemplifies the occurrence of bargaining between political actors for the choice of the central banker: the president of the ECB (and the members of its Executive Board) are chosen by the Council of Ministers and Heads of State, after some tight bargaining between the EMU member states.[6]

Hence, it appears necessary to take into account the negotiations process leading to the nomination of an independent central banker and assess the importance of this process in the conduct of monetary policy in a monetary union, as this may modify the conclusions about the desirability of delegation. This is precisely the purpose of this paper. We address the delegation issue in the case of a two-country monetary union. Given the plural nature of a monetary union, monetary policy matters are solved by a bargaining process between the governments ruling the two countries. We want to relate the properties of monetary policy decided by an independent central banker to the diverging views of political policymakers which are involved in the nomination of the central banker. Moreover, we want to compare the outcome of monetary policy through delegation with the outcome obtained when political policymakers directly bargain over monetary policy. Is it true in this case that delegation always generates a reduced inflationary bias? Is delegation always preferable to direct bargaining over monetary policy? We answer negatively to both questions. Hence, bargaining in a monetary union appears to notably weaken the case for delegating monetary policy to an independent central banker, emphasized by Rogoff (1985).

Few papers have been devoted to the relationship between the sharing of political power and the nomination process of monetary policymakers. Waller (1992, 2000) examines the appointment procedure by which the party in power proposes a candidate for a vacant seat on the central bank committee and the opposite party may veto the proposal. He shows that delegating monetary policy to an independent central banker or policy board reduces the variability induced by policy uncertainty and produces a better outcome than what would be produced by elected leaders. However, our paper differs from Waller's as we do not consider political partisanship nor electoral cycles. Moreover, we do not consider the same appointment process but a Nash bargaining game.[7]

We develop a model of a monetary union with two political policymakers, bargaining and uncertainty. The political policymakers can be thought of as national governments in a multi-national monetary union, like the EMU. We can also think of our monetary union as a two-sector economy. In this case, our policymakers can be thought of as political parties, special interest, agencies or

in his various contributions to the study of American monetary policy the importance of pressures exerted on the monetary policy-makers. In particular, the power of appointment to the FOMC appears to be critical for influencing the course of monetary policy.

[6]The appointment of the first ECB president in May 1998, the Dutch central banker W. Duisenberg, was subject to bargaining and compromises between member states: as some countries supported W. Duisenberg while some others -especially France- supported J.C. Trichet, member states agreed on the appointment of W. Duisenberg with an informal commitment that he would resign after four years and leave the post to J.-C Trichet. J.-C. Trichet succeeding him in November 2003.

[7]Sibert (2003) looks at committees as monetary policymakers. Here we consider a unique central banker.

branches of government, each one acting on behalf of a given economic sector.[8] For the sake of simplicity, we adopt in the paper the first interpretation and we shall refer to national governments.[9] Each government is characterized by a specific loss function, the arguments of which are a national output gap and the union's inflation level. These functions differ according to the weight given to the inflation objective relative to the output one, that is governments differ in terms of "monetary" conservatism, as defined by Rogoff (1985). Moreover, the two countries are hit by idiosyncratic shocks. Hence, the two economies differ because of the differences in the preferences of governments and/or the idiosyncrasies of shocks. We compare two designs for monetary policy. Governments can either choose to delegate the discretionary decision making power to an independent central banker or jointly decide over monetary policy. In the first case, they bargain over the characteristics of the central banker to be selected, i.e. his degree of conservatism; in the other case, they directly bargain over monetary supply. In either case, we assume that a Nash bargaining process takes place.

The paper is organized as follows. The model is developed in the following section. Section 3 is devoted to the study of the bargaining game over the choice of an independent central banker and the properties of its solution. Then, we compare in section 4 the outcome of this delegation game with the outcome of direct bargaining over monetary policy. Section 5 contains a summary of our results and concluding remarks.

2 The model

We consider a two-country economy with two national governments jointly responsible for monetary policy. We follow the usual analytical framework used in the literature devoted to monetary policy games (Barro and Gordon, 1983, Rogoff, 1985). We assume that the inflation rate is directly controlled by the central bank.

In each country (indexed by i), the aggregate supply function is given by:

$$y_{it} = \overline{y} + (\pi_t - \pi_t^e) + \varepsilon_{it}, \qquad i = 1, 2 \tag{1}$$

where y_{it} denotes the national output level in country i in period t, $\overline{y}$ the natural output level, π_t and π_t^e respectively the actual inflation rate and the expected inflation rate in the monetary union and ε_{it} is an i.i.d. supply shock, specific to country i with mean zero and variance σ_i^2. The two shocks are not correlated.[10]

[8] Faust (1996) underlines that the Fed's structure (which emerged in legislation between 1913 and 1935) was a response to conflicts over redistributive powers (in particular, to the diverging views of the farmers and small businessmen on one hand and the financial community on the other hand): the FOMC voting power has been divided between Reserve Bank presidents and politically appointed governors (which implies a sharing of the appointment power). For an historical account of the creation of the Federal Reserve System, see Timberlake (1993).

[9] With minor modifications, however, our model could sustain the second interpretation.

[10] The case of non-zero correlation between shocks could be addressed at the expense of additional complexities.

We denote by $\theta \equiv \sigma_2^2/\sigma_1^2$ the ratio of variances. The two countries are of equal size.

Whatever its institutional determination process, given the linear-quadratic framework we use, money supply is a linear function of the aggregate supply shock:

$$\pi_t = \beta + \alpha\varepsilon_t$$

where β is the inflationary bias of monetary policy and $\varepsilon_t \equiv 1/2\,(\varepsilon_{1t} + \varepsilon_{2t})$.

Each government's objective is to minimize an expected loss function, corresponding to the welfare of residents in its country. The loss function depends on the difference between the actual inflation rate and an inflation target, assumed to be nil for simplicity reasons on the one hand, and the gap between actual output and a target value for output y^* on the other hand. Again for simplicity reasons, the target values for both governments are assumed to be identical. We denote $k \equiv y^* - \overline{y} > 0$.

Government i's (i = 1,2) objective is given by the following quadratic loss function:[11]

$$L_i(y_{it}, \pi_t, \lambda_i) = \frac{1}{2}(y_{it} - y^*)^2 + \frac{\lambda_i}{2}{\pi_t}^2 \tag{2}$$

where λ_i represents the relative weight given to inflation by government i. In the sequel, we shall refer to λ_i as the degree of conservatism of government i. Hence, the two governments' loss functions differ by means of the impact of idiosyncratic shocks and their aversion to inflation, that is by their willingness to fight inflation. We assume without loss of generality that $\lambda_1 > \lambda_2$.

The target output level y^* is bigger than the natural output level $\overline{y}$ because of distortions on the labor market (by means of some anti-competitive features or restrictions to trade). Hence, the monetary authority is tempted to produce unanticipated inflation so as to reduce the gap between actual output and its target level. In such conditions, an announced policy decision, in particular the optimal one, is (in general) not credible when it is not backed by a commitment technology, and the authority in charge of money supply must adopt a discretionary policy leading to a positive inflation bias (Barro and Gordon, 1983).

We assume that, once appointed, the central banker considers the monetary union's macroeconomic outcome. Then, a candidate to the position of central banker has preferences which depend on inflation and the aggregate output gap $(y_t - y^*)$ where $y_t \equiv 1/2\,(y_{1t} + y_{2t})$. We assume that he has the same target levels as both governments and can only differ because of the weight coefficient given to inflation in his loss function. His loss function is given by:

$$L_{BC}(y_t, \pi_t, \lambda_B) = \frac{1}{2}(y_t - y^*)^2 + \frac{\lambda_B}{2}{\pi_t}^2 \tag{3}$$

where λ_B represents the relative weight given to inflation by this individual, hence his inflation aversion, or equivalently, his degree of conservatism. We

[11] The proper welfare objective is the minimization of the future discounted value of the intertemporal loss function $V = E_0[\sum_{t=1}^{\infty} \delta^{t-1} L_t]$, where δ is the discount factor ($0 \leq \delta \leq 1$). However, since there is no dynamics, the problem of minimizing the intertemporal loss function is equivalent to the static problem of minimizing the expected per-period loss function (2).

shall consider a continuum of candidates depending on this relative weight λ_B, ($\lambda_B \in [0, +\infty]$).

Each government wants to minimize its expected loss function. The degree of conservatism of the central banker is chosen by bargaining and is a function of the degrees of conservatism of both governments λ_1 and λ_2, the (Nash) bargaining ratio γ, and the variances of shocks σ_1^2 and σ_2^2. When γ is equal to 1 (0), government 1 (2) has full power. Hence, $\lambda_B \equiv \Lambda\left(\lambda_1, \lambda_2, \gamma, k, \sigma_1^2, \sigma_2^2\right)$.

We shall introduce the following notations:

- λ_{B1} (λ_{B2}) denotes the degree of conservatism of the central banker chosen by government 1 (2) if it retains all bargaining power. Hence $\lambda_{B1} \equiv \Lambda\left(\lambda_1, \lambda_2, 1, k, \sigma_1^2, \sigma_2^2\right)$ and $\lambda_{B2} \equiv \Lambda\left(\lambda_1, \lambda_2, 0, k, \sigma_1^2, \sigma_2^2\right)$.
- The loss of a given government is always a function of the degree of conservatism of both governments and the bargaining ratio γ, but depends on the institutional setting within which monetary policy is decided. In the case of delegation, $L_1^B\left(\lambda_1, \lambda_2, \gamma, k, \sigma_1^2, \sigma_2^2\right)$ ($L_2^B\left(\lambda_1, \lambda_2, \gamma, k, \sigma_1^2, \sigma_2^2\right)$) represents the loss of government 1 (2). In the case of direct bargaining, $L_1^C\left(\lambda_1, \lambda_2, \gamma, k, \sigma_1^2, \sigma_2^2\right)$ ($L_2^C\left(\lambda_1, \lambda_2, \gamma, k, \sigma_1^2, \sigma_2^2\right)$) represents the loss of government 1 (2).[12]

3 Delegation with bargaining.

In this section, we study the properties of the solution of the delegation game. There are three stages in the game. In the first stage, both governments bargain over the type (the degree of conservatism) of the central banker to be named. Then, private agents rationally form their expectations of the inflation rate and set their wage rate accordingly, before observing the current values of the shocks which hit the economy. Finally, after observing these values, the central banker determines the money supply according to his preferences.

Of particular concern is the influence of the divergence of preferences between the two governments on the type of the chosen central banker, and hence on the outcomes of the game.

3.1 Solution of the delegation game.

Once the central banker is chosen, he is free to select the money supply rule he prefers. It is assumed that there is a deterministic identity link between money supply and the inflation rate, so that we can consider that the central banker chooses the inflation rate.

The central banker's programme is then:

$$\begin{aligned} \min_{\pi_t} L_{BC}(y_t, \pi_t, \lambda_B) &= \frac{1}{2}\left[(\overline{y} + \pi_t - \pi_t^e + \varepsilon_t - y^*)^2 + \lambda_B(\pi_t)^2\right] \\ &\quad \pi_t^e \text{ given.} \end{aligned} \tag{4}$$

[12] The loss of government i depends on the two variances because it is affected by the aggregate inflation rate.

Private agents rationally form their expectations of inflation before observing the current value of the supply shock:

$$\pi_t^e = E_{t-1}(\pi_t) = \frac{k}{\lambda_B}. \tag{5}$$

The resulting inflation obtains:

$$\pi_t = \frac{k}{\lambda_B} - \varepsilon_t \frac{1}{1+\lambda_B}. \tag{6}$$

Aggregate output obtains by substituting equation (6) in (1):

$$y_t = \overline{y} + \varepsilon_t \frac{\lambda_B}{1+\lambda_B}. \tag{7}$$

The solution of this stage leads to the usual consequences of delegation. On average, the inflation rate is not zero. A discretionary monetary policy generates an inflationary bias β_B equal to $\frac{k}{\lambda_B}$, which would be avoided if the monetary authority could follow a rule with precommitment. However, actual output is not equal to its natural level as it also depends on the current value of the shock (cf. (7)). Hence, there exists a trade-off between the fight against inflation and the stabilization of output.

Let us now turn to the choice of the central banker by the two governments. Governments have no information on the values of the supply shocks when they jointly choose the central banker. Their choice results from a Nash bargaining process where γ $(0 \leq \gamma \leq 1)$ denotes the relative bargaining power of government 1.

The central banker's type, or equivalently the λ_B coefficient, is obtained by solving the following programme:

$$\max_{\lambda_B} \{L_1^N - E(L_1^B(\lambda_1, \lambda_2, \gamma, k, \sigma_1^2, \sigma_2^2))\}^{\gamma} \cdot \{L_2^N - E(L_2^B(\lambda_1, \lambda_2, \gamma, k, \sigma_1^2, \sigma_2^2))\}^{1-\gamma} \tag{8}$$

where L_i^N is government i's welfare loss in case of deadlock.[13] Hereafter, for simplicity reasons, we shall assume that $L_1^N = L_2^N = L^N$. To avoid the occurrence of deadlock in the bargaining process, we shall make the assumption that L^N is bigger than the loss obtained by any government in the case of a successful bargaining procedure for any vector of parameters:

$$L^N - E(L_i^B(\lambda_1, \lambda_2, \gamma, k, \sigma_1^2, \sigma_2^2)) > 0 \qquad \forall(\lambda_1, \lambda_2, \gamma, k, \sigma_1^2, \sigma_2^2),\ i = 1, 2. \tag{9}$$

Using equations (2), (6) and (7), we get:

$$\begin{aligned} E(L_i(\lambda_1, \lambda_2, \gamma, k, \sigma_1^2, \sigma_2^2)) &= \frac{1}{8}\frac{\lambda_i + (1+2\lambda_B)^2}{(1+\lambda_B)^2}\sigma_i^2 \\ &\quad + \frac{1}{8}\frac{1+\lambda_i}{(1+\lambda_B)^2}\sigma_j^2 + \frac{1}{2}k^2(1+\frac{\lambda_i}{\lambda_B{}^2}). \end{aligned} \tag{10}$$

[13] For example, a lack of agreement about the choice of the first ECB president may have been harmful (lack of credibility of the ECB and the EMU process, refusal of a country to enter in the EMU...). In the case of a nation state, a lack of agreement about the central banker to be named may reflect a political crisis (freeze of the decisions, political instability,...).

Using logarithms, the first-order condition obtains:

$$
\begin{aligned}
& \gamma \frac{[\frac{\lambda_1}{(\lambda_B)^3} k^2 - \frac{1}{4}\sigma_1^2 \frac{(1-\lambda_1+2\lambda_B)}{(1+\lambda_B)^3} + \frac{1}{4}\sigma_2^2 \frac{(1+\lambda_1)}{(1+\lambda_B)^3}]}{L^N - \frac{1}{8}\sigma_1^2 \frac{\lambda_1+(1+2\lambda_B)^2}{(1+\lambda_B)^2} - \frac{1}{8}\sigma_2^2 \frac{1+\lambda_1}{(1+\lambda_B)^2} - \frac{k^2}{2}(1+\frac{\lambda_1}{\lambda_B{}^2})} \\
& +(1-\gamma) \frac{[\frac{\lambda_2}{(\lambda_B)^3} k^2 + \frac{1}{4}\sigma_1^2 \frac{(1+\lambda_2)}{(1+\lambda_B)^3} - \frac{1}{4}\sigma_2^2 \frac{(1-\lambda_2+2\lambda_B)}{(1+\lambda_B)^3}]}{L^N - \frac{1}{8}\sigma_1^2 \frac{1+\lambda_2}{(1+\lambda_B)^2} - \frac{1}{8}\sigma_2^2 \frac{\lambda_2+(1+2\lambda_B)^2}{(1+\lambda_B)^2} - \frac{k^2}{2}(1+\frac{\lambda_2}{\lambda_B{}^2})} \\
\equiv \quad & \varphi(\lambda_1, \lambda_2, \gamma, \sigma^2, k, \sigma_1^2, \sigma_2^2, L^N) = 0.
\end{aligned}
\tag{11}
$$

The solving of this equation gives us the bargained delegation solution λ_B. It is clear from this equation that the views of the two governments with respect to the choice of the inflation rate may differ, not only because their preferences differ but also because the two variances play opposite roles in the first-order condition. In the following sub-section, we study the impact of exogenous parameters on the choice of the central banker and on the resulting discretionary policy.

3.2 Properties of the solution

Rogoff exploited the idea that for strategic reasons, in general no one wants to appoint herself as central banker but prefers to appoint a more conservative candidate, as the delegation allows to manipulate the anticipations of private agents. The same feature is at work here. But, in addition, the fact that the chosen central banker will evenly consider both parts of the monetary union also affects one's view on the "adequate" central banker to choose. Taking into account of possible divergences between governments' preferences leads us to the following proposition:

Proposition 1 *i/ The conservatism degree of the central banker chosen by bargaining is a convex combination of the conservatism degrees of the central bankers desired by each government without bargaining.*

ii/ Whatever the set of parameters, the conservatism degree of the chosen central banker is an increasing function of the conservatism degree of either government, an increasing function of the bargaining ratio γ and an ambigous function of the variance ratio $\theta \equiv \sigma_2^2/\sigma_1^2$.

iii/ The inflation bias β_B is a decreasing function of the bargaining ratio γ and an ambiguous function of the variance ratio θ.

P roof. See Appendix A. ■

The first part of Proposition 1 characterizes the compromise reached by governments over the selection of the central banker through their bargaining process. The farther apart they are, the farther are their best preferred central bankers. The one actually selected will be a compromise between these two types of bankers wished by either government (λ_{B1} and λ_{B2}): the larger the distance between λ_{B1} and λ_{B2}, the farther from the types of bankers preferred by either government is λ_B. As this distance is an increasing function of governments' conservatism degrees, the farther apart their preferences, the farther from the types of bankers wished by either government is λ_B.

The second part of Proposition 1 (Proposition 1-ii/) addresses the issue of the sensitivity of the chosen degree of conservatism to the characteristics of both governments. It makes explicit the strategic relationship between governments.

In particular, Proposition 1-ii/ claims that for small values of γ, the higher θ, the less conservative the appointed central banker. Whereas for high values of γ, the relation is reversed (the higher θ, the more conservative the appointed central banker). For small values of γ, government 2 has high influence on the choice of the central banker. Remember that it gives a larger relative weight to the stabilization objective and suffers from relatively high output variability relative to country 1. Hence, the higher θ, the bigger the loss generated by the occurrence of shocks in country 2, the less conservative the central banker whished by government 2 and finally appointed. Government 1 has a higher aversion towards inflation and the occurrence of relatively large shocks in country 2 will imply a looser monetary policy than the one the central banker would implement if he only considers country 1's economy. In order to fight this tendency, the higher θ, the more conservative the central banker supported by government 1.

Proposition 1-ii/ also claims that an increase in conservatism of any government, whatever its degree of conservatism relative to the other government, leads to a more conservative central banker. This is fairly in accordance with what was expected. But this gives some insights on new issues that are worth future investigation.

In the present paper, we assume that governments do not disguise their preference toward inflation and do not cheat. Proposition 1-ii/ suggests that there are ample reasons for cheating on one's true aversion toward inflation. Consider for example the most conservative government. Given the degree of conservatism announced by the other government, it has every reason to claim a higher aversion toward inflation. By so doing, the selected banker will be more conservative than if it had revealed its true degree of conservatism and will become closer to λ_{B1}, the conservatism degree wished by government 1. The same is true for the less inflation averse government: it too has an interest in claiming a lower degree of conservatism. In other words, cheating is an attempt to overcome the obligation to compromise through bargaining. Remark that there is a lower limit to the announced degree of aversion toward inflation for the less inflation averse government, as it cannot claim more than being indifferent toward inflation. On the contrary, there is no limit to the announced degree of aversion toward inflation for the most conservative government. This government has then a strategic advantage over the less inflation averse government. Since the latter can at most announce that it is indifferent toward inflation, the most conservative government can claim such a high dislike of inflation that the chosen central banker is in accordance with its true goal. Hence, in such a setting, it is likely that there is an ingrained bias toward conservatism. It is then of real importance that revelation mechanisms are implemented so as to induce governments to reveal their true preferences.

The last part of Proposition 1 is consistent with bargaining theory and merely states that the inflation bias (or equivalently the degree of conservatism) of the appointed central banker decreases (increases) with the power of the most conservative government.

4 Comparing delegation and discretion with bargaining.

We now move to the comparison of outcomes of delegation and discretion (i.e. direct bargaining over monetary decision) in a monetary union. The first step is to assess the course of discretionary monetary policy when governments directly bargain over the inflation rate.

4.1 Discretion with bargaining.

We consider that governments do not delegate monetary policy to a central banker chosen by bargaining but directly bargain over the inflation rate to enforce. In such a setting, the stages of the game are as follows. Before observing the current values of shocks, private agents rationally form their expectation of the inflation rate and fix their wage rate. Then governments, after observing the current values of shocks, decide over the money supply by following a Nash bargaining process.

In this framework, the bargaining programme is the following:

$$\max_{\pi} [L^N - (L_1^C(\lambda_1, \lambda_2, \gamma, k, \sigma_1^2, \sigma_2^2))]^{\gamma} \cdot [L^N - (L_2^C(\lambda_1, \lambda_2, \gamma, k, \sigma_1^2, \sigma_2^2))]^{1-\gamma}$$
$$\pi_t^e \text{ given.}$$

Using equations (1), (2) and logarithms, we get:

$$\begin{aligned} \max_{\pi_t} \gamma \log[L^N - (\frac{1}{2}(\overline{y} + \pi_t - \pi_t^e + \varepsilon_{1t} - y^*)^2 + \frac{\lambda_1}{2}(\pi_t)^2)] \\ +(1-\gamma) \log[L^N - (\frac{1}{2}(\overline{y} + \pi_t - \pi_t^e + \varepsilon_{2t} - y^*)^2 + \frac{\lambda_2}{2}(\pi_t)^2)]. \end{aligned} \tag{12}$$

Private agents rationally form their expectations:

$$\pi_t^e = E_{t-1}\pi_t. \tag{13}$$

The first-order condition is given by:

$$\gamma \frac{-(\pi_t - \pi_t^e + \varepsilon_{1t} - k) - \lambda_1 \pi_t}{L^N - \frac{1}{2}(\pi_t - \pi_t^e + \varepsilon_{1t} - k)^2 - \frac{\lambda_1}{2}{\pi_t}^2} + (1-\gamma) \frac{-(\pi_t - \pi_t^e + \varepsilon_{2t} - k) - \lambda_2 \pi_t}{L^N - \frac{1}{2}(\pi_t - \pi_t^e + \varepsilon_{2t} - k)^2 - \frac{\lambda_2}{2}{\pi_t}^2} = 0. \tag{14}$$

The Nash equilibrium is determined by the system of equations formed by equations (14) and (13).

Let β_C denote the inflationary bias in the case of direct bargaining over the inflation rate. It is then possible to determine the inflationary bias in the case of direct bargaining, by using (14) in the absence of shock. In this case, the expected inflation is equal to β_C. From (14) and (13), one gets:

$$\gamma \frac{k - \lambda_1 \beta_C}{L^N - \frac{1}{2}k^2 - \frac{\lambda_1}{2}(\beta_C)^2} + (1-\gamma) \frac{k - \lambda_2 \beta_C}{L^N - \frac{1}{2}k^2 - \frac{\lambda_2}{2}(\beta_C)^2} = 0 \tag{15}$$

$$\Leftrightarrow \psi(\beta_C, \lambda_1, \lambda_2, \gamma) = 0 \tag{16}$$

The properties of this solution are detailed in the following:

Proposition 2 *The inflation bias β_C belongs to the interval $\left[\frac{k}{\lambda_1}, \frac{k}{\lambda_2}\right]$ and is a decreasing function of the bargaining power ratio γ.*

P roof. See appendix B. ■

This proposition is consistent with standard bargaining theory. The bargained solution is bounded by the two solutions wished by the two governments. The more power a government enjoys, the more able it is to draw the bargained solution towards its desired one.

4.2 Comparison with the delegation outcome.

We are now able to compare the outcomes of both institutional settings, in particular with respect to the inflationary bias. Comparing inflation biases under the two schemes for monetary policy is the object of the following proposition:

Proposition 3 *There exist values of the set of parameters, when γ is closed to 0, such that $\lambda_{B2} < \lambda_2$, and therefore $\beta_B > \beta_C$.*

P roof. See appendix C. ■

This result is at variance with Rogoff's classical result: it proves that the nomination of an independent central banker does not always lead to a lower inflation bias when it happens in a monetary union with asymmetric shocks and when the nomination is obtained through bargaining between governments with different objectives. Clearly, the trade-offs involved in the case of multiple bargaining governments are more complex than when a single government freely chooses an independent central banker.

This can be explained as follows, using a simple example. Consider the case when government 2 has almost all power about monetary policy, either when it directly controls the union's money supply or when it appoints the central banker (γ close to 0). Suppose in addition that the variance of the shock affecting country 2 is very large relative to the variance of the shock affecting country 1 (θ large). Government 2 is much in favor of stabilizing output as it suffers from relatively high variability relative to country 1. It knows that the chosen central banker will adopt a broad view on the functioning of the monetary union, looking at the aggregate output, which is less variable than country 2's. In order to achieve its goal, government 2 has an interest in appointing a low inflation-averse central banker. It may even have an interest in appointing a central banker who is less conservative than itself (the central banker is not bounded from below by λ_2). By so doing, it counters the balanced view of the independent central banker. We refer to this case as the appointment of a "lax" central banker. This leads to a high inflation bias. This inflation bias k/λ_B, as a result, depends on the relative variances of shocks and is higher than the inflationary bias in the case of direct bargaining. In the latter case, the highest possible value for the inflation bias is k/λ_2 (the inflation bias when government 2 has the whole bargaining power).

Turning now to the crucial issue of the welfare property of delegation, Rogoff proved that delegation is always to be preferred to discretion, because of the reduction in the inflationary bias, even though a drawback of nominating a more conservative central banker than the ruling government is that it lessens the stabilization properties of monetary policy.[14] However, in the presence of bargaining in a monetary union, we cannot extend Rogoff's claim as we are able to establish the following:

Proposition 4 *There exist values of the exogenous parameters* $(\lambda_1, \lambda_2, \gamma, k, \sigma_1^2, \sigma_2^2)$ *such that either government 1 or government 2 prefers discretion with direct bargaining over delegation.*

P roof. See appendix D. ■

This proposition states that it is not systematically true that delegation of monetary policy to an independent central banker chosen by bargaining in a monetary union is always preferred to direct bargaining by all parties involved in the decision: there are cases where at least one party prefers direct bargaining. This can be explained as follows.

Consider government 2. Suppose that it has (almost) all bargaining power, but has a very inflation-averse partner having shocks with low variance. When government 2 directly exerts monetary policy, it only takes into consideration its own situation. But in the case of delegation, as the independent central banker adopts a balanced view, this may lead government 2 to choose a "lax" delegate, implying an inflation bias which may be much higher than the inflation bias under direct bargaining (as seen in Proposition 3). On the whole, although government 2 can choose the central banker it prefers, it will be worse off in the case of delegation, as it will have a higher inflation and a less efficient output stabilization.

Government 1 may also prefers direct bargaining. Suppose that it has (almost) no bargaining power, is very averse to inflation but faces a low inflation-averse government in country 2. Then, it knows that in the case of delegation, government 2 will efficiently push for a lax central banker, definitely less inflation-averse than itself (from Proposition 3). As it very strongly dislikes inflation, it prefers to let government 2 directly control monetary policy. This explains the proposition.

Then an immediate corollary follows from this proposition:

Corollary 5 *There exist values of the exogenous parameters* $(\lambda_1, \lambda_2, \gamma, k, \sigma_1^2, \sigma_2^2)$

[14] However, it is known that the Rogoff delegation scheme is not optimal. On this point, see Lockwood, Miller and Zhang (1998) and Svensson (1997).

such that both governments prefer discretion with direct bargaining over delegation.

P roof. See Appendix E. ■

This corollary sheds new light on the constitutional design of a monetary union: it states that it may happen that countries forming a monetary union may be better off by not establishing an independent central bank, but by directly bargaining over monetary policy. To be more explicit, suppose that the creation of a monetary union involves a two-stage game. In the first stage, which is a-temporal, that is before the actual working of the economy, both governments must decide over the institutions of the monetary union to be created: namely, they choose non-cooperatively between direct bargaining and delegation with bargaining over the selection of the central banker. In the second stage, the economy is working through an infinite sequence of periods, and is properly described by the model we set up in section 2. In any stage, each government is concerned with the welfare of the representative agent of its country. Then, the above corollary states that direct bargaining can be the unique equilibrium of this two-stage game. In other words, there are circumstances where governments choose not to create an independent central bank for the union. The case for an independent central bank is therefore not overwhelming in a monetary union.

4.3 Simulations and special cases

For the sake of analytical tractability, the previous propositions were established using extreme cases for values of some parameters. In particular, the bargaining power ratio γ was assumed to be equal or close to zero. Is the result that the inflation bias is bigger under delegation than under discretion in the presence of bargaining robust enough? To answer this question, we resort to simulations.[15]

A first set of simulations proves that the result that the inflation bias is larger under delegation is robust. Table 1 gives us the intervals for values of γ such that the inflation bias is lower under direct bargaining than under delegation to an independent central banker, for various values of θ. The values for the other parameters are the following: $L^N = 10^5, k = 1, \sigma_1^2 = 1, \lambda_1 = 2, \lambda_2 = 0.5$. Some telling graphics representing the inflation biases in both regimes in function of the bargaining power γ are given in Appendix F.

θ	1	5	10	20	30	40	50	100	150	200
γ	none	none	none	none	none	[0,0.34]	[0,0.41]	[0,0.57]	[0,0.68]	[0,0.79]

Table 1

Two facts are noticeable:

[15] Given the high non-linearity of the optimization programmes, it is impossible to solve them analytically. Our simulations were achieved using *Mathematica*. The complete set of simulations may be found on : http://eurequa.univ-paris1.fr/membres/kempf/english/cv.htm. Here we highlight some telling results.

1. The upper limit of the interval is increasing in σ_2^2, that is in θ. The higher the country 2's variance, the higher the upper threshold value for γ. This is not inconsistent with what we obtained in Proposition 3.

2. More importantly, the superiority of the inflation bias β_B over β_C happens for not extreme values of θ and γ. For example, when θ is above (or equal to) 40, the interval includes positive values of γ. This interval increases with θ. On the whole, the range of parameter values for which this happens is rather large.

These two facts perfectly match our explanations given to Proposition 3.

Simulations allow to point to another interesting stylized fact. For other sets of parameters, the interval for values of γ such that β_B is superior to β_C may have a non null lower bound. The result of a lower inflation bias in the case of direct bargaining is obtained for intermediate values of θ. Table 2 gives such intervals obtained by simulations for the following values of the parameters: $L^N = 10^5$, $k = 1$, $\sigma_1^2 = 1$, $\lambda_1 = 5$, $\lambda_2 = 0.2$. Some telling graphics representing the inflation biases in both regimes in function of the bargaining power γ are given in Appendix F.

θ	1	5	10	20	30	40	50	100	150	200
γ	none	none	none	none	[0.1,0.41]	[0.06,0.47]	[0.05,0.51]	[0.01,0.65]	[0,0.77]	[0,0.9]

Table 2

The difference with the previous set of simulations comes from the higher discrepancy in inflation aversion between the two governments. When γ is low and close to zero, the inflation bias is lower under delegation in the case of intermediate values of θ: in this situation, even when it retains (almost) all power, government 2 is not willing to name a central banker less inflation averse than itself as the gains in terms of stabilization are not that high (the variance of shocks in country 2 is not large). On the other hand, when γ is close enough to 1, government 1 has (almost) all power. As it is highly inflation averse, it chooses a more inflation averse central banker than itself and the inflation bias is then clearly lower than in the case of discretion. For intermediate values of γ, the inflation bias is however higher when delegating monetary policy. Consider the lower bound of the interval for values of γ such that β_B is superior to β_C: with γ close to zero, government 2 appoints a central banker who is more conservative than itself. However, if there is a high discrepancy in inflation aversion between both governments, the central banker may be less conservative than government 1. Suppose that γ increases. Then, government 1 has more bargaining power. In the direct bargaining regime, it will use this higher bargaining power to implement a tighter monetary policy, which reduces the inflation bias β_C. In the delegation regime, it will use its higher bargaining power to appoint a more conservative central banker. However, it is not sufficient to counterbalance the fact that the central banker adopt an even view, looking at the aggregate output. Then, the inflation bias β_B is reduced but it decreases less strongly than when government 1 directly bargains over monetary policy. On the whole, the inflation bias is lower in the case of direct bargaining for a range of values of γ (which becomes larger when θ increases), until government 1 has enough bargaining power to appoint a sufficiently conservative central banker (more conservative than itself).

Finally, the result still obtains when the two variances are equal, admittedly for high values of variances (see Appendix F). However, equal variances of shock do not mean that shocks are symmetrical when shocks are not correlated. The asymmetry between both countries results from not correlated shocks with high extend.

In brief, according to our simulations, the relevance of Proposition 3 seems to be rather large. The superiority of an independent central banker over discretion in delivering a lower inflation bias is not to be taken for granted in a monetary union.

5 Conclusion.

In this paper, we studied the issue of delegating monetary policy to a central banker in the framework of a standard macro model of a monetary union, when there are two governments involved in monetary policy matters. These two governments bargain over monetary decisions. In particular they may either directly and jointly decide over monetary policy or select an independent central banker.

First, we established several properties of the solution of the delegation game, in particular the impact of bargaining and shocks asymmetries (captured by a ratio of the shocks variances). Second, we compared the delegation scheme in the presence of bargaining with the alternative scheme of direct bargaining. We showed that delegating monetary policy to an independent central banker does not necessarily generate a lower inflationary bias nor is always welfare improving for any government. When forming a monetary union, under some external circumstances, it may happen that both governments prefer a union without an independent central banker but with direct bargaining. Hence, the systematic superiority of delegation over discretion obtained by Rogoff in the case of a single policymaker dealing with a single shock does not hold anymore in the case of a monetary union when a plurality of governments jointly controlling monetary policy is introduced and when these governments bargain over monetary matters.

These theoretical results, obtained in an overly simplified macro model, cast an interesting light over the set-up of EMU. Economic theory has been influential in the design of EMU's institutions. It was decided in the Maastricht Treaty that the European System of Central Banks be (fairly) independent. EMU is typically a case where bargaining matters: member countries have to bargain over the selection of the committee in charge of monetary policy. Obviously, our results do not *a priori* justify this decision without qualifications. They suggest that the superiority of the chosen scheme depends on the various structural elements of the economy, and in particular on the inflation aversions and relative bargaining powers of member countries. True, our model is not fully in accordance with EMU's complex institutions. In the EMU, bargaining is taking place both for the choice of the members of the Executive Board of the European Central Bank itself, but also within the Governing Council, where delegates of member countries (the governors of the national central banks) sit. Still, it suggests that the positive and normative properties of the chosen institutional framework are far from clear and that a close attention to the consequences of bargaining is central to such understanding.

Moreover, our results may explain why the transition phase has been carefully designed and planned. Germany was particularly reluctant to renounce to the Deutsche Mark and share a common currency with other countries Germans considered as more lenient toward inflation. Before EMU, Germany could independently decide about monetary policy. Despite their sovereignty over monetary policy, the other European countries had to more or less stringently follow the German monetary leadership. In the early nineties, following German reunification, they had to bear high nominal and real interest rates, set to accommodate German macroeconomic needs only. Once EMU enforced, this would radically change: having a common monetary policy, Germany feared to be in an unfavorable position and to accept compromises with countries characterized by low inflation aversion. Even the high degree of independence of the European Central Bank[16] was not enough to alleviate these fears. The above propositions explain why these fears could have been justified. Germany could well have been placed in the position of government 1 in our model: a very conservative government having to compromise in a weak bargaining position with partners weakly committed to fight inflation. This could well lead to a high inflation rate, much higher than what was considered as acceptable by the German authorities and people. It was then crucial for Germany to get firm assurances from her partners about their conversion to high aversion toward inflation and therefore be sure that the independence of the European Central Bank would indeed lead to a low enough inflation bias. This explains both the strong provisions of the transition toward EMU, included in the Maastricht Treaty and the important role played by the head of the Bundesbank in the early design of European monetary policy. Germany asked for guarantees from her would-be partners, in terms of their commitment toward fighting inflation and the reduction of preference divergences over inflation.

The present model could be enlarged in several ways. First, we have assumed that governments' preferences only differ in the degree of aversion toward inflation. This could be modified. We could consider differences between governments in target values both for inflation and output. This would lead to ex post antagonism between countries. Second, we assumed away any moral hazard

[16] Legally, the statutes of the European System of Central Banks (ESCB) and the European Central Bank (ECB) mandate that no member of their decision-making bodies is to seek or take instructions from the Community institutions and bodies, the governments of the Member States or any other body. The Eurosystem may not grant loans to Community body or national government entities. The ECB has its own budget and its capital has been subscribed and paid up by the national central banks. The members of the ESCB's decision-making bodies have long terms of office and cannot be dismissed (except for serious misconduct or inability to perform their duties).

problem by assuming that governments truthfully report their real objective functions. In reality, there is scope in such a setting for strategic misrepresentation by any government of its true aversion relative to inflation. Our results point to the temptation for any government to disguise its true objectives and to attempt to bias the collective decision in a direction more favorable to its actual objectives. In particular, our analysis points to a structural advantage for the more inflation-averse parties, which would lead to an unduly restrictive monetary policy stance. This insight has to be confirmed by more elaborate analyses. Clearly informational asymmetries are a major element in understanding the consequences of bargaining.

Finally, other schemes justifying the independence of a central bank have been offered in the literature, besides the simple scheme of appointing a central banker which turns out to be conservative. Whether these schemes are still supporting the independence of the central bank when bargaining is involved is an interesting question.[17]

These various extensions are left to future research.

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A Proof of Proposition 1

A.1 Proof of Proposition 1-i/

The proof of proposition 1-i/ is done in three steps.

A) λ_{Bi} *is an increasing function of* λ_i. *Consequently, since* $\lambda_1 > \lambda_2$, *it implies* $\lambda_{B1} > \lambda_{B2}$.

If government i could choose alone the central banker, it would choose a central banker who would minimize its loss function :

$$\underset{\lambda_{Bi}}{Min} E[\frac{1}{2}(\overline{y} + \pi_t - \pi_t^e + \varepsilon_{it} - y^*)^2 + \frac{\lambda_i}{2}\pi_t^2] \tag{17}$$

$$s.t.\ \pi_t \quad = \quad \frac{k}{\lambda_{Bi}} - \frac{1}{1+\lambda_{Bi}}\ \varepsilon_t. \tag{18}$$

The first-order condition corresponding to this programme is:

$$-\frac{\lambda_i}{(\lambda_{Bi})^3}\ k^2 + \frac{1}{4}{\sigma_i}^2 \frac{(1-\lambda_i+2\lambda_{Bi})}{(1+\lambda_{Bi})^3} - \frac{1}{4}{\sigma_j}^2 \frac{(1+\lambda_i)}{(1+\lambda_{Bi})^3} \quad = \quad 0 \Leftrightarrow \tag{19}$$

$$\xi(\lambda_{Bi}, \lambda_i,\ k, \sigma_i^2, \sigma_j^2) \quad = \quad 0. \tag{20}$$

The variation of λ_{Bi} as a function of λ_i can be studied by applying the implicit function theorem to function $\xi(\lambda_{Bi}, \lambda_i,\ k, \sigma_i^2, \sigma_j^2)$. One then gets $\frac{d\lambda_{Bi}}{d\lambda_i} = -\frac{\xi_{\lambda_i}}{\xi_{\lambda_B}}$, where ξ_x is the derivative (19) relative to variable x. The sign of $\frac{d\lambda_{Bi}}{d\lambda_i}$ is the sign of $-\frac{\xi_{\lambda_i}}{\xi_{\lambda_B}}$. For λ_{Bi} to be a minimum, it is necessary that $\xi_{\lambda_{Bi}}$ be positive. The sign of $\frac{d\lambda_{Bi}}{d\lambda_i}$ is then opposite to the sign of ξ_{λ_i}.

$$\xi_{\lambda_i} = -\frac{1}{(\lambda_{Bi})^3}\ k^2 - \frac{1}{4}{\sigma_i}^2 \frac{1}{(1+\lambda_{Bi})^3} - \frac{1}{4}{\sigma_j}^2 \frac{1}{(1+\lambda_{Bi})^3} < 0.$$

$\frac{d\lambda_{Bi}}{d\lambda_i}$ is therefore positive.

B) λ_B *is a continuous increasing function of* γ.

The first-order condition of the bargaining programme (8) is given by equation (11). From the implicit function theorem applied to $\varphi(\lambda_1, \lambda_2, \gamma, k, \sigma_i^2, \sigma_j^2, L^N)$, $\frac{d\lambda_B}{d\gamma} = -\frac{\varphi_\gamma}{\varphi_{\lambda_B}}$, where φ_x is the derivative of (11) relative to variable x. The sign of $\frac{d\lambda_B}{d\gamma}$ is the sign of $-\frac{\varphi_\gamma}{\varphi_{\lambda_B}}$. For λ_B to be a maximum, it is necessary that φ_{λ_B} be negative. The sign of $\frac{d\lambda_B}{d\gamma}$ is therefore the sign of φ_γ defined as:

$$\begin{aligned}\varphi_\gamma \quad = \quad & \frac{\frac{\lambda_1\ k^2(1+\lambda_B)^3 - \frac{1}{4}\sigma_1^2(1-\lambda_1+2\lambda_B)(\lambda_B)^3 + \frac{1}{4}\sigma_2^2(1+\lambda_1)(\lambda_B)^3}{(\lambda_B)^3(1+\lambda_B)^3}}{L^N - \frac{1}{8}\sigma_1^2\frac{\lambda_1+(1+2\lambda_B)^2}{(1+\lambda_B)^2} - \frac{1}{8}\sigma_2^2\frac{1+\lambda_1}{(1+\lambda_B)^2} - \frac{k^2}{2}(1+\frac{\lambda_1}{{\lambda_B}^2})} \\ & - \frac{\frac{\lambda_2\ k^2(1+\lambda_B)^3 + \frac{1}{4}\sigma_1^2(1+\lambda_2)(\lambda_B)^3 - \frac{1}{4}\sigma_2^2(1-\lambda_2+2\lambda_B)(\lambda_B)^3}{(\lambda_B)^3(1+\lambda_B)^3}}{L^N - \frac{1}{8}\sigma_1^2\frac{1+\lambda_2}{(1+\lambda_B)^2} - \frac{1}{8}\sigma_2^2\frac{\lambda_2+(1+2\lambda_B)^2}{(1+\lambda_B)^2} - \frac{k^2}{2}(1+\frac{\lambda_2}{{\lambda_B}^2})}.\end{aligned} \tag{21}$$

Remark that $\varphi_\gamma = f_1/g_1 - f_2/g_2$ where f_1 is the numerator of the first fraction, g_1 its denominator, f_2 the numerator of the second fraction and g_2 its denominator. According to our assumption on L^N, $g_1 > 0$ and $g_2 > 0$. If $(1-\lambda_2+2\lambda_B) < 0$, then $f_2 > 0$. But $(1-\lambda_2+2\lambda_B) < 0$ implies $(1-\lambda_1+2\lambda_B) < 0$ as $\lambda_1 > \lambda_2$. Then, $f_1 > 0$. However, one cannot simultaneously get $f_2 > 0$ and $f_1 > 0$. Hence $(1 - \lambda_2 + 2\lambda_B)$ is positive, and the sign of f_2 is ambiguous. Similarly, since the sign of $(1 - \lambda_1 + 2\lambda_B)$ is ambiguous, so is the sign of f_1.

Let us show that f_2 is negative and f_1 is positive. Assume that $f_2 > 0$. Then :

$$k^2 > \frac{\frac{1}{4}\sigma_2^2(1-\lambda_2+2\lambda_B)(\lambda_B)^3 - \frac{1}{4}\sigma_1^2(1+\lambda_2)(\lambda_B)^3}{\lambda_2(1+\lambda_B)^3} \tag{22}$$

This implies for f_1 :

$$f_1 > \frac{1}{4}\frac{\sigma_2^2\ \frac{\lambda_1}{\lambda_2}\ (1-\lambda_2+2\lambda_B) - \ \sigma_1^2\ \frac{\lambda_1}{\lambda_2}\ (1+\lambda_2) - \sigma_1^2(1-\lambda_1+2\lambda_B) + \sigma_2^2(1+\lambda_1)}{(1+\lambda_B)^3} \equiv \frac{N_1}{D_1}.$$

Each term in N_1 is known to be positive, except $-\ \sigma_1^2\ \frac{\lambda_1}{\lambda_2}\ (1+\lambda_2)$ which is negative and $-\sigma_1^2(1-\lambda_1+2\lambda_B)$ which is ambiguous.

If $\sigma_2^2 > \sigma_1^2$:

$$\frac{N_1}{D_1} > \frac{1}{2}\frac{\sigma_2^2\ \lambda_B(\frac{\lambda_1}{\lambda_2}-1)}{(1+\lambda_B)^3}$$

This implies (as $\lambda_1 > \lambda_2$) :

$$f_1 > \frac{1}{2}\frac{\sigma_2^2\ \lambda_B(\frac{\lambda_1}{\lambda_2}-1)}{(1+\lambda_B)^3} > 0$$

But one cannot simultaneously get $f_1 > 0$ and $f_2 > 0$ as the LHS term in equation (11) cannot be zero. Therefore, f_2 has to be negative. Then, because of (11), f_1 cannot be negative as one cannot simultaneously get $f_1 < 0$ and $f_2 < 0$. Hence, $f_2 < 0$ and $f_1 > 0$. Finally, $\varphi_\gamma = f_1/g_1 - f_2/g_2 > 0$. Consequently, λ_B is an increasing function of γ.

C*)* Obviously, $\lambda_B = \lambda_{B1}$ *if* $\gamma = 1$; $\lambda_B = \lambda_{B2}$ *if* $\gamma = 0$.

Finally, because of these three steps, λ_B is a convex combination of λ_{B1} and λ_{B2}, depending on γ.

A.2 Proof of Proposition 1-ii/

A) According to step B in appendix A.1, λ_B is a continuous increasing function of γ, for any vector $\left(L^N, k, \theta, \sigma_1^2, \sigma_2^2, \lambda_1, \lambda_2\right)$.

B) For any vector $\left(L^N, k, \theta, \sigma_1^2, \sigma_2^2, \gamma\right)$, we now prove that λ_B is an increasing function of λ_1 and λ_2.

From the implicit function theorem applied to $\varphi(\lambda_1, \lambda_2, \gamma, k, {\sigma_i}^2, {\sigma_j}^2, L^N)$, $\frac{d\lambda_B}{d\lambda_1} = -\frac{\varphi_{\lambda_1}}{\varphi_{\lambda_B}}$, where φ_x is the derivative of (11) relative to variable x. The sign of $\frac{d\lambda_B}{d\lambda_1}$ is the sign of $-\frac{\varphi_{\lambda_1}}{\varphi_{\lambda_B}}$. For λ_B to be a maximum, it is necessary that φ_{λ_B} be negative. The sign of $\frac{d\lambda_B}{d\lambda_1}$ is therefore the sign of φ_{λ_1} defined as:

$$
\begin{aligned}
\varphi_{\lambda_1} \;=\;& \gamma\;\cdot[L^N-\frac{1}{8}\sigma_1^2\tfrac{\lambda_1+(1+2\lambda_B)^2}{(1+\lambda_B)^2}-\frac{1}{8}\sigma_2^2\tfrac{1+\lambda_1}{(1+\lambda_B)^2}-\frac{k^2}{2}(1+\tfrac{\lambda_1}{\lambda_B{}^2})]^{-2}\cdot\\
&\{[\tfrac{k^2}{(\lambda_B)^3}\;+\frac{1}{4}\sigma_1^2\tfrac{1}{(1+\lambda_B)^3}+\frac{1}{4}\sigma_2^2\tfrac{1}{(1+\lambda_B)^3}][L^N-\frac{1}{8}\sigma_1^2\tfrac{\lambda_1+(1+2\lambda_B)^2}{(1+\lambda_B)^2}-\frac{1}{8}\sigma_2^2\tfrac{1+\lambda_1}{(1+\lambda_B)^2}-\frac{k^2}{2}(1+\tfrac{\lambda_1}{\lambda_B{}^2})]\\
&-[-\frac{1}{8}\sigma_1^2\tfrac{1}{(1+\lambda_B)^2}-\frac{1}{8}\sigma_2^2\tfrac{1}{(1+\lambda_B)^2}-\frac{k^2}{2}\tfrac{1}{\lambda_B{}^2}][\tfrac{\lambda_1}{(\lambda_B)^3}\;k^2-\frac{1}{4}\sigma_1^2\tfrac{(1-\lambda_1+2\lambda_B)}{(1+\lambda_B)^3}+\frac{1}{4}\sigma_2^2\tfrac{(1+\lambda_1)}{(1+\lambda_B)^3}]\}\\
>\;& 0
\end{aligned}
$$

as $\frac{\lambda_1}{(\lambda_B)^3}\;k^2-\frac{1}{4}\sigma_1^2\frac{(1-\lambda_1+2\lambda_B)}{(1+\lambda_B)^3}+\frac{1}{4}\sigma_2^2\frac{(1+\lambda_1)}{(1+\lambda_B)^3}>0$ (see appendix A.1, step B). Thus, $\frac{d\lambda_B}{d\lambda_1}>0$.

From the implicit function theorem applied to $\varphi(\lambda_1,\lambda_2,\gamma,k,{\sigma_i}^2,{\sigma_j}^2,L^N)$, $\frac{d\lambda_B}{d\lambda_2}=-\frac{\varphi_{\lambda_2}}{\varphi_{\lambda_B}}$, where φ_x is the derivative of (11) relative to variable x. The sign of $\frac{d\lambda_B}{d\lambda_2}$ is the sign of $-\frac{\varphi_{\lambda_2}}{\varphi_{\lambda_B}}$. For λ_B to be a maximum, it is necessary that φ_{λ_B} be negative. The sign of $\frac{d\lambda_B}{d\lambda_2}$ is therefore the sign of φ_{λ_2} defined as:

$$
\begin{aligned}
\varphi_{\lambda_2} \;=\;& (1-\gamma)\;\cdot[L^N-\frac{1}{8}\sigma_1^2\tfrac{1+\lambda_2}{(1+\lambda_B)^2}-\frac{1}{8}\sigma_2^2\tfrac{\lambda_2+(1+2\lambda_B)^2}{(1+\lambda_B)^2}-\frac{k^2}{2}(1+\tfrac{\lambda_2}{\lambda_B{}^2})]^{-2}\cdot\\
&\{[\tfrac{k^2}{(\lambda_B)^3}\;+\frac{1}{4}\sigma_1^2\tfrac{1}{(1+\lambda_B)^3}+\frac{1}{4}\sigma_2^2\tfrac{1}{(1+\lambda_B)^3}][L^N-\frac{1}{8}\sigma_1^2\tfrac{1+\lambda_2}{(1+\lambda_B)^2}-\frac{1}{8}\sigma_2^2\tfrac{\lambda_2+(1+2\lambda_B)^2}{(1+\lambda_B)^2}-\frac{k^2}{2}(1+\tfrac{\lambda_2}{\lambda_B{}^2})]\\
&-[-\frac{1}{8}\sigma_1^2\tfrac{1}{(1+\lambda_B)^2}-\frac{1}{8}\sigma_2^2\tfrac{1}{(1+\lambda_B)^2}-\frac{k^2}{2}\tfrac{1}{\lambda_B{}^2}][\tfrac{\lambda_2}{(\lambda_B)^3}\;k^2+\frac{1}{4}\sigma_1^2\tfrac{(1+\lambda_2)}{(1+\lambda_B)^3}-\frac{1}{4}\sigma_2^2\tfrac{(1-\lambda_2+2\lambda_B)}{(1+\lambda_B)^3}]\}
\end{aligned}
$$

The term in brace brackets is ambiguous (see appendix A.1, step B). However, we can show that φ_{λ_2} is positive if L^N is sufficiently large (which is what we assume). Consider for example that $L^N\geq 2L_2$. Then:

$$
\begin{aligned}
\varphi_{\lambda_2}\;\geq\;& (1-\gamma)\;\cdot[L^N-\frac{1}{8}\sigma_1^2\tfrac{1+\lambda_2}{(1+\lambda_B)^2}-\frac{1}{8}\sigma_2^2\tfrac{\lambda_2+(1+2\lambda_B)^2}{(1+\lambda_B)^2}-\frac{k^2}{2}(1+\tfrac{\lambda_2}{\lambda_B{}^2})]^{-2}\cdot\\
&\{\frac{1}{2}\tfrac{k^4(\lambda_B^2+2\lambda_2)}{(\lambda_B)^5}\;+\frac{1}{8}\tfrac{k^2}{(\lambda_B)^3}\;\sigma_1^2\tfrac{(\lambda_B^3+4\lambda_2\lambda_B+2\lambda_B+1)}{(1+\lambda_B)^3}\\
&+\frac{1}{8}\tfrac{k^2}{(\lambda_B)^3}\;\sigma_2^2\tfrac{(5\lambda_B^3+6\lambda_B^2+4\lambda_B(\lambda_2+1)+2\lambda_2+1)}{(1+\lambda_B)^3}+\frac{1}{16}(\sigma_1^2)^2\tfrac{(1+\lambda_2)}{(1+\lambda_B)^5}\\
&+\frac{1}{16}(\sigma_2^2)^2\tfrac{(2\lambda_B{}^2+\lambda_B+\lambda_2)}{(1+\lambda_B)^5}+\frac{1}{16}\sigma_1^2\sigma_2^2\tfrac{(2\lambda_B^2+\lambda_B+2\lambda_2+1)}{(1+\lambda_B)^5}\}
\end{aligned}
$$

All the terms inside the brace brackets are positive. Thus $\varphi_{\lambda_2}>0$ and $\frac{d\lambda_B}{d\lambda_2}>0$.

C) For any vector $\left(L,k,\theta,\lambda_1^2,\lambda_2^2,\gamma,\sigma_1^2\right)$, we prove that λ_B is an ambiguous function of $\theta\equiv\sigma_2^2/\sigma_1^2$. θ captures the asymmetry of the shocks of both countries. The first-order condition of the bargaining programme (8), given by equation (11), may be written in function of θ:

$$\gamma \frac{\frac{\lambda_1}{(\lambda_B)^3}\frac{k^2}{\sigma_1^2} - \frac{1}{4}\frac{(1-\lambda_1+2\lambda_B)}{(1+\lambda_B)^3} + \frac{1}{4}\theta\frac{(1+\lambda_1)}{(1+\lambda_B)^3}}{\frac{L^N}{\sigma_1^2} - \frac{1}{8}\frac{\lambda_1+(1+2\lambda_B)^2}{(1+\lambda_B)^2} - \frac{1}{8}\theta\frac{1+\lambda_1}{(1+\lambda_B)^2} - \frac{k^2}{2\frac{1}{4}\sigma_1^2}(1+\frac{\lambda_1}{\lambda_B{}^2})}$$

$$+(1-\gamma)\frac{\frac{\lambda_2}{(\lambda_B)^3}\frac{k^2}{\sigma_1^2} + \frac{1}{4}\frac{(1+\lambda_2)}{(1+\lambda_B)^3} - \frac{1}{4}\theta\frac{(1-\lambda_2+2\lambda_B)}{(1+\lambda_B)^3}}{\frac{L^N}{\sigma_1^2} - \frac{1}{8}\frac{1+\lambda_2}{(1+\lambda_B)^2} - \frac{1}{8}\theta\frac{\lambda_2+(1+2\lambda_B)^2}{(1+\lambda_B)^2} - \frac{k^2}{2\sigma_1^2}(1+\frac{\lambda_2}{\lambda_B{}^2})} = 0.$$

which is equivalent to:

$$\varphi(\lambda_B, \lambda_1, \lambda_2, \gamma, \theta, L^N, k, \sigma_1^2) = 0$$

From the implicit function theorem applied to $\varphi(\lambda_B, \lambda_1, \lambda_2, \gamma, \theta, L^N, k, \sigma_1^2)$, $\frac{d\lambda_B}{d\theta} = -\frac{\varphi_\theta}{\varphi_{\lambda_B}}$, where φ_x is the derivative of function $\varphi(\lambda_B, \lambda_1, \lambda_2, \gamma, \theta, L^N, k, \sigma_1^2)$ relative to variable x. The sign of $\frac{d\lambda_B}{d\theta}$ is the sign of $-\frac{\varphi_\theta}{\varphi_{\lambda_B}}$. For λ_B to be a maximum, it is necessary that φ_{λ_B} be negative. The sign of $\frac{d\lambda_B}{d\theta}$ is therefore the sign of φ_θ defined as:

$$\varphi_\theta = \gamma \ \cdot [A]^{-2} \cdot B + (1-\gamma) \cdot [C]^{-2} \cdot D$$

where $A \equiv [\frac{L^N}{\sigma_1^2} - \frac{1}{8}\frac{\lambda_1+(1+2\lambda_B)^2}{(1+\lambda_B)^2} - \frac{1}{8}\theta\frac{1+\lambda_1}{(1+\lambda_B)^2} - \frac{k^2}{2\frac{1}{4}\sigma_1^2}(1+\frac{\lambda_1}{\lambda_B{}^2})] > 0$,

$B \equiv [\frac{1}{4}\frac{1+\lambda_1}{(1+\lambda_B)^3}][\frac{L^N}{\sigma_1^2} - \frac{1}{8}\frac{\lambda_1+(1+2\lambda_B)^2}{(1+\lambda_B)^2} - \frac{1}{8}\theta\frac{1+\lambda_1}{(1+\lambda_B)^2} - \frac{k^2}{2\frac{1}{4}\sigma_1^2}(1+\frac{\lambda_1}{\lambda_B{}^2})]$ $-[-\frac{1}{8}\frac{1+\lambda_1}{(1+\lambda_B)^2}][\frac{k^2}{\sigma_1^2}\frac{\lambda_1}{(\lambda_B)^3} - \frac{1}{4}\frac{(1-\lambda_1+2\lambda_B)}{(1+\lambda_B)^3} + \frac{1}{4}\theta\frac{(1+\lambda_1)}{(1+\lambda_B)^3}] > 0$,

$C \equiv [\frac{L^N}{\sigma_1^2} - \frac{1}{8}\frac{1+\lambda_2}{(1+\lambda_B)^2} - \frac{1}{8}\theta\frac{\lambda_2+(1+2\lambda_B)^2}{(1+\lambda_B)^2} - \frac{k^2}{2\sigma_1^2}(1+\frac{\lambda_2}{\lambda_B{}^2})] > 0$,

$D \equiv [-\frac{1}{4}\frac{(1-\lambda_2+2\lambda_B)}{(1+\lambda_B)^3}][\frac{L^N}{\sigma_1^2} - \frac{1}{8}\frac{1+\lambda_2}{(1+\lambda_B)^2} - \frac{1}{8}\theta\frac{\lambda_2+(1+2\lambda_B)^2}{(1+\lambda_B)^2} - \frac{k^2}{2\sigma_1^2}(1+\frac{\lambda_2}{\lambda_B{}^2})]$ $-[-\frac{1}{8}\frac{\lambda_2+(1+2\lambda_B)^2}{(1+\lambda_B)^2}][\frac{\lambda_2}{(\lambda_B)^3}\frac{k^2}{\sigma_1^2} + \frac{1}{4}\frac{(1+\lambda_2)}{(1+\lambda_B)^3} - \frac{1}{4}\theta\frac{(1-\lambda_2+2\lambda_B)}{(1+\lambda_B)^3}] < 0$.

Then, the sign of φ_θ is ambiguous and depends on the value of γ. If $\gamma = 0$, $\varphi_\theta < 0$. If $\gamma = 1$, $\varphi_\theta > 0$. There exists a value $\gamma^* \in [0,1]$ such as:

$$\varphi_\theta < 0, \ \forall\gamma < \gamma^* \text{ and } \varphi_\theta > 0, \ \forall\gamma > \gamma^*.$$

A.3 Proof of Proposition 1-iii/

We now study the impact of γ over the inflationary bias in case of delegation β_B. This bias is equal to $\frac{k}{\lambda_B}$, λ_B being defined by (11). One gets then:

$$\frac{d\beta_B}{d\gamma} = \frac{\partial\beta_B}{\partial\lambda_B}\frac{d\lambda_B}{d\gamma}.$$

It is known that $\dfrac{\partial\beta_B}{\partial\lambda_B} = \dfrac{-k}{(\lambda_B)^2}$ and $\dfrac{d\lambda_B}{d\gamma} > 0$ according to appendix A1 - step B. Hence: $\frac{d\beta_B}{d\gamma} < 0$.

As λ_B is comprised between λ_{B1} and λ_{B2}, the minimal and maximal values of the inflationary bias when delegating are the following ones: if $\gamma = 1$, $\beta_B = \frac{k}{\lambda_{B1}}$ and if $\gamma = 0$, $\beta_B = \frac{k}{\lambda_{B2}}$. This implies:

$$\frac{k}{\lambda_{B1}} \leq \frac{k}{\lambda_B} \leq \frac{k}{\lambda_{B2}}.$$

Hence, the inflationary bias with delegation is a decreasing function of the bargaining power coefficient, having a minimal value $\frac{k}{\lambda_{B2}}$, when $\gamma = 0$, and a maximal value $\frac{k}{\lambda_{B1}}$, when $\gamma = 1$.

B Proof of Proposition 2.

One cannot get $\beta_C > \frac{k}{\lambda_2}$ as it implies $k - \lambda_2 \beta_C < 0$ and $k - \lambda_1 \beta_C < 0$, which is impossible since the LHS term in the first-order condition (14) must be zero. Similarly, one cannot get $\beta_C < \frac{k}{\lambda_1}$ as it implies $k - \lambda_2 \beta_C > 0$ and $k - \lambda_1 \beta_C > 0$. Hence, $\beta_C < \frac{k}{\lambda_2}$, i.e. $k - \lambda_2 \beta_C > 0$ and $\beta_C > \frac{k}{\lambda_1}$, or $k - \lambda_1 \beta_C < 0$. Applying the implicit function theorem to $\psi(\beta_C, \lambda_1, \lambda_2, \gamma)$ (cf. (16)), it can be shown that the inflationary bias with direct bargaining is a decreasing function of γ too. Indeed, one then gets:

$$\frac{d\beta_C}{d\gamma} = -\frac{\psi_\gamma}{\psi_{\beta^C}}.$$

ψ_{β^c} is negative since β_C is a maximum. The sign of $\frac{d\beta_C}{d\gamma}$ is therefore the same as the sign of ψ_γ which is equal to:

$$\psi_\gamma = \frac{k - \lambda_1 \beta_C}{L^N - \frac{1}{2}k^2 - \frac{\lambda_1}{2}(\beta_C)^2} - \frac{k - \lambda_2 \beta_C}{L^N - \frac{1}{2}k^2 - \frac{\lambda_2}{2}(\beta_C)^2}.$$

As $k - \lambda_2 \beta_C > 0$ and $k - \lambda_1 \beta_C < 0$, this implies that: $\psi_\gamma < 0$. Hence $\frac{d\beta_C}{d\gamma} < 0$. From (15), the minimal and maximal values of β_C on [0,1] can easily be computed. If $\gamma = 1$, $\beta_C = \frac{k}{\lambda_1}$. If $\gamma = 0$, $\beta_C = \frac{k}{\lambda_2}$. Then, $\frac{k}{\lambda_1} < \beta_C < \frac{k}{\lambda_2}$.

The inflationary bias with direct bargaining is a decreasing function of γ on [0,1], with a minimal value $\frac{k}{\lambda_2}$ when $\gamma = 0$ and a maximal value $\frac{k}{\lambda_1}$ when $\gamma = 1$.

C Proof of proposition 3.

Consider the case where $\gamma = 0$. Then, the programme leading to the nomination of the Union's central banker becomes:

$$\max_{\lambda_B} \log \left[L_N - \left(\frac{1}{8} \frac{\lambda_2 + (1 + 2\lambda_B)^2}{(1 + \lambda_B)^2} \sigma_2^2 + \frac{1}{8} \frac{1 + \lambda_2}{(1 + \lambda_B)^2} \sigma_1^2 + \frac{1}{2} k^2 (1 + \frac{\lambda_2}{\lambda_B{}^2}) \right) \right] \tag{23}$$

and the first-order condition is:

$$0 = \frac{\lambda_2}{(\lambda_B)^3} k^2 + \frac{1}{4} \sigma_1^2 \frac{(1 + \lambda_2)}{(1 + \lambda_B)^3} - \frac{1}{4} \sigma_2^2 \frac{(1 - \lambda_2 + 2\lambda_B)}{(1 + \lambda_B)^3}. \tag{24}$$

We have to prove that for some values of parameters, the central banker is less conservative than government 2 (i.e. $\lambda_B < \lambda_2$). We consider the case where $\sigma_1^2 = k = \lambda_2 = 1$. Then, $\theta = \sigma_2^2$ and (24) can be written as:

$$0 = \frac{1}{(\lambda_B)^3} + \frac{1}{2}\frac{1}{(1+\lambda_B)^3} - \frac{1}{2}\theta\frac{\lambda_B}{(1+\lambda_B)^3} \equiv \Omega(\theta, \lambda_B).$$

Ω_θ is clearly negative and Ω_{λ_B} is also negative (as λ_B is a maximum). Using the implicit function theorem as in step A, in the proof of Prop.1-i., λ_B is a decreasing function of θ. For $\theta = 17$, we get $\lambda_B = 1$ as it is the sole positive solution. Hence, for higher values of θ, λ_B is inferior to $1 = \lambda_2$. By continuity, this is true for values of γ close enough to 0.

D Proof of proposition 4.

A - The case for government 2.

Consider a given vector of parameters $(L^N, \lambda_1, \lambda_2, \gamma, k, \sigma_1^2, \sigma_2^2)$. We have to prove that for some values of this vector, Government 2 is not better-off with the negotiated assignment of an independent central banker than with a direct bargaining over discretionary monetary policy. Clearly, we have to reason on loss averages.[18] We look for some values of parameters such that:

$$E\left(L_2^B\left((L^N, \lambda_1, \lambda_2, \gamma, k, \sigma_1^2, \sigma_2^2)\right)\right) - E\left(L_2\left(L^N, \lambda_1, \lambda_2, \gamma, k, \sigma_1^2, \sigma_2^2\right)\right) > 0. \quad (25)$$

First, we consider the case where $\gamma = 0$. In this case, $\lambda_B = \lambda_{B2}$. We have to prove that, for some values of the parameters, government 2 is better off with discretion than with delegation of monetary policy to a central banker chosen by bargaining, when it retains all bargaining power. Hence, it suffices to prove that for some values of $\left(\lambda_1, \lambda_2, k, \sigma_1^2, \sigma_2^2\right)$:

$$E\left(L_2^B\left((\lambda_1, \lambda_2, 0, k, \sigma_1^2, \sigma_2^2)\right)\right) - E\left(L_2^C\left(\lambda_1, \lambda_2, 0, k, \sigma_1^2, \sigma_2^2\right)\right) > 0 \quad (26)$$

with:

$$\begin{aligned} E(L_2^B(\lambda_1, \lambda_2, 0, k, \sigma_1^2, \sigma_2^2)) &= \frac{1}{2}k^2\left(1 + \frac{\lambda_2}{\lambda_{B2}{}^2}\right) + \frac{1}{8}\frac{1+\lambda_2}{(1+\lambda_{B2})^2}\sigma_1^2 \quad (27) \\ &+ \frac{1}{8}\frac{\lambda_2 + (1+2\lambda_{B2})^2}{(1+\lambda_{B2})^2}\sigma_2^2 \end{aligned}$$

and:

$$E\left(L_2^C\left(\lambda_1, \lambda_2, 0, k, \sigma_1^2, \sigma_2^2\right)\right) = \frac{1}{2}k^2\left(\frac{\lambda_2+1}{\lambda_2}\right) + \frac{1}{2}\sigma_2^2\frac{\lambda_2}{1+\lambda_2}. \quad (28)$$

[18] At some periods, for a high enough shock, it may happen that both governments would have preferred to use discretionary powers at the expense of the inflation bias. This is not enough to legitimate the choice of direct bargaining for any period.

Hence, (26) obtains:

$$\begin{aligned}
& E\left(L_2^B\left((\lambda_1,\lambda_2,0,k,\sigma_1^2,\sigma_2^2)\right)\right)-E\left(L_2^C\left(\lambda_1,\lambda_2,0,k,\sigma_1^2,\sigma_2^2\right)\right)\\
= & \frac{1}{2}k^2\lambda_2\left(\frac{1}{\lambda_{B2}^2}-\frac{1}{\lambda_2^2}\right)+\frac{1}{8\left(1+\lambda_{B2}\right)^2}\sigma_1^2\left[\frac{\left(1+4\lambda_{B2}^2+4\lambda_{B2}\left(1-\lambda_2\right)+\lambda_2^2-2\lambda_2\right)\theta-\left(1+\lambda_2\right)^2}{\left(1+\lambda_2\right)}\right]
\end{aligned}$$

Given Prop. 1-ii and Prop. 3, this difference is positive for $\lambda_2 = 1/2$ and θ sufficiently large, so that $\lambda_{B2} < \lambda_2$.

By a continuity argument, this is true for values of γ close enough to 0.

B- The case for government 1.

We look for some values of parameters such that:

$$E\left(L_1^B\left(L^N,\lambda_1,\lambda_2,\gamma,k,\sigma_1^2,\sigma_2^2\right)\right)-E\left(L_1^C(L^N,\lambda_1,\lambda_2,\gamma,k\sigma_1^2,\sigma_2^2)\right)>0. \quad (29)$$

Consider the case where $\gamma = 0$. Then:

$$\begin{aligned}
& E\left(L_2^B\left((\lambda_1,\lambda_2,0,k,\sigma_1^2,\sigma_2^2)\right)\right)-E\left(L_2^C\left(\lambda_1,\lambda_2,0,k,\sigma_1^2,\sigma_2^2\right)\right)\\
= & \frac{1}{2}k^2\lambda_1\left(\frac{1}{\lambda_{B2}^2}-\frac{1}{\lambda_2^2}\right)+\frac{\lambda_1-3-4\lambda_{B2}}{8\left(1+\lambda_{B2}\right)^2}\sigma_1^2+\frac{1}{8}\left(\frac{1}{\left(1+\lambda_{B2}\right)^2}-\frac{1}{\left(1+\lambda_2\right)^2}\right)\left(1+\lambda_1\right)\sigma_2^2\\
= & A+B\sigma_1^2+C\sigma_2^2. \qquad (30)
\end{aligned}$$

$A > 0$ and $C > 0$ if $\lambda_{B2} < \lambda_2$, and $B > 0$ if $\lambda_1 > 3+4\lambda_{B2}$. In the case when $\lambda_2 = 1/2$, $\lambda_1 > 3+4\lambda_2 = 4.5$ and θ sufficiently large such that $\lambda_{B2} < \lambda_2$ imply that (30) is positive.

By a continuity argument, this is true for values of γ close enough to 0.

E Proof of Corollary.

Immediate from Proposition 4, with γ close to zero, λ_1 sufficiently large and λ_2 sufficiently small, θ sufficiently large.

F Simulations

Appendix F shows some telling graphics representing the inflation biases in both regimes in function of the bargaining power γ.

A/ First set of simulations (figure 1 : graphics 1 to 3):

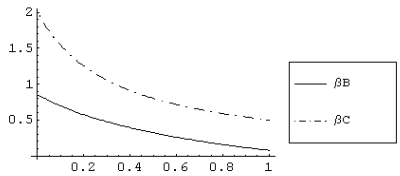
We consider the following values of parameters: $L^N = 10^5, k = 1, \sigma_1^2 = 1, \lambda_1 = 2, \lambda_2 = 0.5$.

B/ Second set of simulations (figure 2 : graphics 4 to 6):

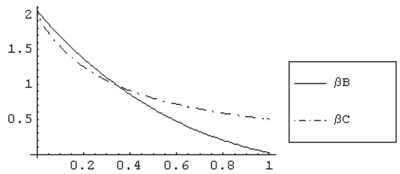
We consider the following values of parameters: $L^N = 10^5, k = 1, \sigma_1^2 = 1, \lambda_1 = 5, \lambda_2 = 0.2$.

C/ Third set of simulations (figure 3 : graphics 7 to 9). We impose $\theta = 1$

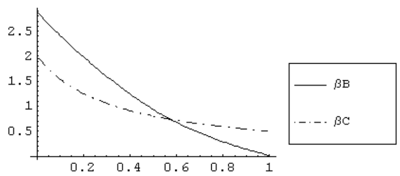
We consider the following values of parameters: $L^N = 10^5, k = 1, \lambda_1 = 2, \lambda_2 = 0.5$



graphic 1 : q = 5

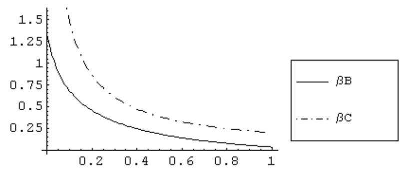


graphic 2 : q = 40

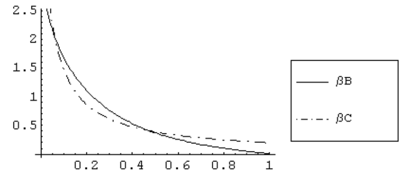


graphic 3 : q = 100

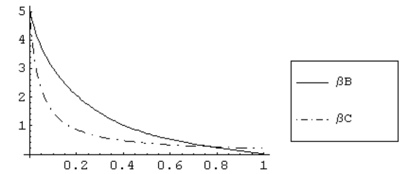
Figure 1:



graphic 4 : q = 5

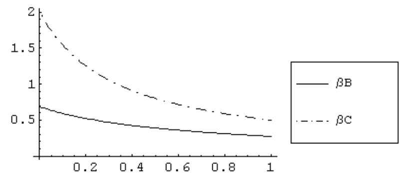


graphic 5 : q = 40

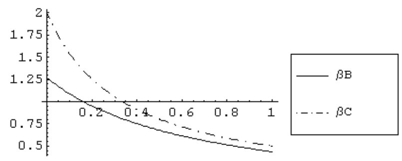


graphic 6 : q = 150

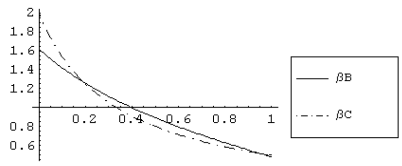
Figure 2:



graphic 7 : $s_1{}^2 = s_2{}^2 = 5$



graphic 8 : $s_1{}^2 = s_2{}^2 = 40$



graphic 9 : $s_1{}^2 = s_2{}^2 = 150$

Figure 3: