The Capital Stock and Equilibrium Unemployment: A New Theoretical Perspective

Sujit Kapadia[1]
Balliol College, Oxford University

March 2004

Abstract

By assuming Cobb-Douglas production technology, many well-known imperfectly competitive macroeconomic models of the labour market (e.g. Layard, Nickell and Jackman, 1991) imply that equilibrium unemployment is independent of the capital stock. This paper introduces a new notion of capacity into the standard framework. Specifically, we adapt the Cobb-Douglas production function so that when the capital-labour ratio drops below a certain threshold, the returns to labour fall while the returns to capital increase. Using this assumption, we show that equilibrium unemployment depends on the capital stock over a certain range. We also briefly discuss the generalisation for an endogenous capital stock.

Keywords: Unemployment, Capital Stock, Investment, Capacity.

JEL Classification Numbers: E22, E23, E24, E25, J64.

Contact Details:

Address: Sujit Kapadia, Balliol College, Oxford, OX1 3BJ, United Kingdom.
E-mail: sujit.kapadia@economics.ox.ac.uk

[1] I am indebted to Jim Malcomson and John Muellbauer for their numerous suggestions and detailed comments on this paper. I would also like to thank Lucy Rees, Kevin Roberts and, particularly, David Vines for their comments on earlier drafts. At various stages, I have benefited from helpful discussions with a number of other people: I am grateful to Andrew Glyn, Terry O'Shaughnessy, Margaret Stevens and Jonathan Thomas for their thoughts, suggestions and advice. Finally, I am grateful to the Bank of England and the ESRC for providing financial support at various stages while this paper was written.

"It takes capital and entrepreneurship to create new firms and jobs."

(Stiglitz, 2002, p. 59)

1. Introduction

1.1 Overview

Although it is intuitively obvious for some economists that employment and hence unemployment[2] should depend on investment and the level of the capital stock, many influential authors (e.g. Layard, Nickell and Jackman, 1991, henceforth LNJ) have argued theoretically that this is not the case. As a result, unemployment is often viewed solely as a labour market phenomenon with the key debate centring on how best to reform labour markets in order to promote greater employment on the existing capital stock.

However, the result that the equilibrium rate of unemployment (sometimes referred to as the NAIRU[3]) is independent of the capital stock hinges on the assumption of Cobb-Douglas production technology. Rowthorn (1999) has shown that assuming CES production instead of Cobb-Douglas production does break down this result. However, although the notion of "capacity" seems as if it should be fundamental to any analysis of the relationship between the capital stock and equilibrium unemployment, the CES production function lacks a clear notion of this concept. In addition, adopting CES production implies that increasing the capital stock can reduce equilibrium unemployment regardless of how much capital firms already have, a result which could be viewed as being slightly unrealistic. Meanwhile, for reasons explained below, in (Cobb-Douglas) putty-clay and putty-semiputty models which do introduce meaningful capacity constraints, changes in the capital stock are not normally able to

[2] In this paper, we follow many authors in assuming that the employment and unemployment rates are related by the identity $u \equiv 1 - e$. Therefore, when we talk about one of the concepts, the reverse statement will always apply to the other concept. Note that this assumption implies that we are ignoring potential changes in inactivity.

[3] Throughout this paper, we will refer to the rate of unemployment that strips away cyclical fluctuations as the *equilibrium* rate of unemployment. We avoid the term NAIRU since its use in the literature is very confused: some authors make a great effort to theoretically distinguish the NAIRU and the natural rate of unemployment (NRU); other authors simply view the NAIRU as the empirical counterpart of the NRU; still others use the terms interchangeably. However, we should note that where the NAIRU is theoretically distinguished from the NRU (e.g. Carlin and Soskice, 1990), the NAIRU is usually defined in a similar way to our "equilibrium" rate of unemployment.

explain *permanent* changes in equilibrium unemployment (though they may be able to explain persistence in unemployment).

In an attempt to offer a more convincing theoretical explanation of the relationship between the capital stock and equilibrium unemployment than currently exists, this paper therefore introduces a new production function in which the notion of capacity is meaningful. Specifically, we adapt the Cobb-Douglas production function so that when the capital-labour ratio drops below a certain threshold, the returns to labour fall discretely while the returns to capital increase discretely by the same amount. We introduce this type of capacity constraint into a standard imperfectly competitive macroeconomic model of unemployment of the type used by LNJ. Assuming that the capital stock is exogenous, we show that if the initial capital stock is within a certain range, increases in its level can permanently reduce equilibrium unemployment. We then illustrate how this result generalises for the case of an endogenous capital stock: in this case, equilibrium unemployment depends on the real user cost of capital over a certain range. In addition, we explain how our model may be adapted so that it is consistent with the stylised fact that the unemployment rate is untrended in the very long-run while the capital-labour ratio has grown steadily since the Industrial Revolution. The policy implications of our results are clear. Policies to promote investment may help to tackle unemployment. By contrast, the current focus on labour market reforms may be overstated.

1.2 Motivation and Discussion of Related Literature

1.2.1 The Prevailing Consensus on Unemployment

Since the early 1970s, unemployment has risen substantially in most developed countries. Having said this, unemployment experiences have been diverse, especially over the past 10-15 years. In particular, there were several “success stories” in the 1990s: the equilibrium rate of unemployment fell substantially in the United Kingdom, the Netherlands, Ireland, Portugal and Denmark. However, unemployment remains a major concern in a number of European countries.

Current thinking (e.g. LNJ; OECD, 1994; Siebert, 1997; Nickell, Nunziata and Ochel, 2002) usually attributes unemployment to labour market inflexibility coupled with an inadequately skilled workforce. In recent years, much research on the topic has therefore focussed on which particular labour market institutions and rigidities are responsible for high levels of unemployment (see, for example, the discussions in Nickell, 1997 and Layard and Nickell, 1999). A related research agenda (e.g. Blanchard and Wolfers, 2000) attempts to explain the evolution of unemployment across countries in terms of the interactions between shocks and institutions. The main argument here is that "flexible" institutions may allow economies to adapt more easily to adverse shocks. Proponents of both of these views often cite the United States as an example of a country where a "flexible" labour market has helped to keep the equilibrium unemployment rate at relatively low levels.

The policy prescriptions for combating unemployment which follow from these lines of research are well-known. They include weakening the power of trade unions, cutting taxes on labour, deregulating labour markets (e.g. reducing employment protection; reducing firing restrictions), spending on active labour market policies (e.g. subsidising education and training opportunities for the unemployed), cutting the value and duration of unemployment benefits and reducing the relative value of minimum wages. Despite the fact that some of these policies potentially have adverse effects on other aspects of welfare, several countries have taken steps to reform their labour markets in these ways. The successes of the United Kingdom and the Netherlands in reducing unemployment are often attributed to their adoption of some of these policies.

However, the evidence linking labour market reforms to lower equilibrium unemployment is controversial. In particular, Ball (1999) raises three issues. Firstly, he argues that the British and Dutch reforms were relatively minor and only moved these countries a very small way towards the highly "flexible" American labour market. More importantly, he argues that many countries, including Belgium, Canada and especially Spain, failed to reduce equilibrium unemployment significantly in spite of pursuing labour market reforms which were at least as large as (and perhaps even bigger than) the British and Dutch reforms. Finally, he argues that conventional explanations are unable to account for the successes of Portugal or Ireland since neither of these countries experienced major changes in their labour market institutions.[4] The comparative success of Portugal in relation to Spain (which has the worst unemployment record in the OECD over the past twenty years) is particularly striking since, as Blanchard and Jimeno (1995) point out, the countries have remarkably similar labour market institutions.

These arguments should not necessarily be viewed as suggesting that labour market reforms are totally unimportant in reducing unemployment. However, they do seem to imply that their importance may be somewhat overstated. In addition, they suggest that we may need to look elsewhere to explain at least part of the falls in equilibrium unemployment rates witnessed in the "success stories" of the 1990s.

1.2.2 Unemployment and the Capital Stock

In particular, it is a striking stylised fact that high levels of investment were prevalent in many of the "success stories" just before their equilibrium unemployment rates started to fall significantly.[5] For example, Portugal and Ireland both experienced investment booms: the former following accession to the European Economic Community in 1986; the latter driven by a very large upswing in foreign direct investment during the mid-1990s. Meanwhile, to a lesser extent, it could be argued that the Netherlands benefited from increased investment in the early 1990s following German reunification. The historical experience of the United Kingdom economy is also interesting. As argued by Kitson and Michie (1996), the British investment record between the 1960s and the mid-1990s was dismal. Over this period, the country's employment record was very poor. By contrast, the United Kingdom's strong investment performance during the mid to late 1990s came at a time when equilibrium unemployment started to fall significantly.

Although there are likely to be some common causal factors which simultaneously drive investment and employment growth, we should note that the reduction in

[4] Glyn (2002) discusses the Irish experience in more detail and compares it to the experience of New Zealand, where the equilibrium unemployment rate hardly changed during the 1990s despite major labour market reforms. Based on the evidence from these two countries, he concludes (p. 16) that "Extensive labour market deregulation is neither a necessary nor a sufficient condition for a radical improvement in employment".

[5] It is also striking that at a broad cross-country level, equilibrium unemployment rose significantly following the worldwide post-1973 investment slowdown.

unemployment in all of these countries has been sustained long after investment rates have fallen. Therefore, despite the usual caveats relating to causation, these wide-ranging experiences do suggest that above trend increases in the capital stock may have some effect in reducing equilibrium unemployment. Moreover, although the collinearity issue means that empirical work in this area needs to be treated with some caution, the few formal studies that have been carried out are quite supportive of this assertion (e.g. Rowthorn, 1995; Arestis and Biefang-Frisancho Mariscal, 2000; Miaouli, 2001; Alexiou and Pitelis, 2003). Despite this, promoting investment is rarely proposed as a policy to tackle unemployment. This seeming contradiction between theory and reality motivates our specific theoretical interest in the relationship between the capital stock and the equilibrium rate of unemployment.[6] However, before discussing our model, we briefly survey the contributions of other authors who have written on this topic. Broadly speaking, these contributions fall into two categories.

1.2.2.1 Capital Scrapping Persistence

A number of authors, including Malinvaud (1980), Soskice and Carlin (1989), Bean (1989, 1994) and Rowthorn (1995), have discussed the potential impact of capacity utilisation on pricing decisions in order to show how falls in the capital stock may generate persistence in unemployment (though without affecting its long-run equilibrium rate). The key assumption of these models is that production technology is putty-clay or putty-semiputty so that ex-post, once capital has been installed, the elasticity of substitution between factors of production is low (or even zero). It is therefore argued that when capacity utilisation is high, firms are likely to increase their price mark-ups in order to choke off excess demand for their products and increase their profit margins. As a result, if an adverse shock erodes the capital stock, then when the shock is reversed, inflation will be generated at lower levels of output and higher levels of unemployment than were previously the case. Therefore, unemployment can only return to its old level once any capital shortfall has been eliminated. Since it may take time for the capital stock to return to its old level, we can see how it might be possible for unemployment to remain persistently high for some time after a shock has been reversed.

[6] As Hahn (1995, p. 52) comments: "One can only be amazed at the neglect of investment and of the capital stock in theories of the natural rate."

However, it is important to note that in these models, the capital stock must always eventually return to its old level. This is because, in the long-run, once the limited factor substitutability constraint has been relaxed, firms will be able to choose their capital and labour inputs optimally. Since a shock which is subsequently reversed does not change anything fundamental which affects this choice, firms will choose their capital and labour inputs in the same way after a shock as they did before it. Therefore, we would always expect unemployment to return eventually to its old level in these models, meaning that they cannot explain changes in the equilibrium rate of unemployment in the long-run.

1.2.2.2 Capital Accumulation and the Equilibrium Rate of Unemployment

Less research has been done on the question of whether changes in the capital stock have direct and *permanent* effects on the equilibrium rate of unemployment. There are two main reasons for this.

Firstly, Bean (1989), LNJ and others have argued that since the unemployment rate is untrended in the very long-run, it cannot be affected by trended variables such as the capital-labour ratio. However, this neglects the possibility that although trend increases in the capital-labour ratio may have no effect on the equilibrium unemployment rate, above trend or below trend increases could still have an impact. In other words, a one-off permanent step change in the absolute *level* of the capital-labour ratio (i.e. relative to its long-run trend growth rate) could potentially permanently affect the equilibrium unemployment rate. So, there is nothing in this argument which rules out the possibility of a temporary investment boom permanently lowering the equilibrium unemployment rate.

Secondly, it has been argued theoretically that the equilibrium rate of unemployment does not depend on the capital stock. This has been shown by LNJ (p. 107) in the competing claims imperfectly competitive macroeconomic model that has (at least in Europe) come to represent the canonical model of equilibrium unemployment. The intuitive explanation of this result is as follows. Increasing the capital stock has two direct effects: it generates employment on the extra capital stock and it increases real wages. However, the real wage increases have a secondary effect: they cause firms to lower their demand for labour, thus reducing employment on the existing capital stock. If the elasticity of substitution is sufficiently high (specifically, if it is equal to one, meaning that production is Cobb-Douglas), then the lost employment on the existing capital stock will exactly cancel out the extra employment on the new capital stock, thus leaving overall employment unchanged. This description clearly illustrates how the LNJ result hinges on the assumption of a Cobb-Douglas production function. The authors acknowledge this fact but then proceed to ignore it in the rest of their analysis, claiming that Cobb-Douglas production technology is "not a bad assumption" (p. 107). However, as we shall argue below, the implication of this assumption that the shares of capital and labour in output are constant may not be appropriate if the capital-labour ratio fluctuates.

Since authors working in this area usually adopt a competing claims framework similar to that used by LNJ, we can therefore see how most of the existing literature has implicitly ignored any potential direct and permanent relationship between the capital stock and equilibrium unemployment. The one major exception is Rowthorn (1999). He argues that the elasticity of substitution between capital and labour is considerably less than one and therefore introduces a CES production function into the LNJ model. Based on this assumption, he shows that increasing the capital stock can theoretically reduce equilibrium unemployment. However, in his model, this conclusion holds regardless of how much capital firms already have. This seems somewhat unrealistic since we would probably not expect investment to have much (or even any) impact on equilibrium unemployment if firms already have a very high capital stock. Moreover, the CES production function does not really contain any notion of capacity and it therefore seems slightly odd to use it for the specific purpose of analysing the relationship between the capital stock and equilibrium unemployment.

1.3 New Theoretical Perspectives

Our model departs from the existing literature by adopting a new production function, which we will refer to as the ***capital constrained (KC) production function***. This production function is simply a slightly more general version of the Cobb-Douglas production function. It essentially has the same functional form. However, in an attempt to offer a more convincing explanation of the relationship between the capital stock and equilibrium unemployment than currently exists, it incorporates a new, meaningful notion of capacity.

We introduce the KC production function into a standard imperfectly competitive macroeconomic model of the type used by LNJ. The bulk of the paper is devoted to solving the model for the case where the capital stock is assumed to be exogenous. In this short-run analysis, we show that changes in the capital stock affect equilibrium unemployment over a certain range. This contrasts with the results of both LNJ and Rowthorn (1999). In addition, by showing that an alternative assumption to CES production can generate the result that equilibrium unemployment depends on the capital stock, our results add further weight to the view of those economists who believe that promoting investment is important in tackling unemployment.

We then consider the case of an endogenous capital stock, explaining intuitively how equilibrium employment depends negatively on the real user cost of capital (and hence on the real interest rate) over a certain range in this long-run analysis. (The formal justification for this result is contained in Chapter 4 of Kapadia, 2003.) That unemployment should be affected adversely by increases in the real interest rate is not a new idea. In particular, a number of potential causal links (of which the effect via capital accumulation is only one) have been discussed by Fitoussi and Phelps (1988) and Phelps (1994). However, in the existing theoretical literature, shifts in the real interest rate are not normally viewed as having a long-run impact on the equilibrium unemployment rate. For example, Blanchard and Wolfers (2000, p. C6) ask themselves whether changes in the real interest rate are likely to have permanent effects on unemployment. They answer that "Theory is largely agnostic here…a plausible answer is that long run effects, if present, are likely to be small". By contrast, our model provides a concrete theoretical explanation of why a change in the real interest rate can permanently affect the equilibrium rate of unemployment.

Finally, we consider the evolution of the equilibrium (un)employment rate over time. In particular, we show that if we introduce labour-augmenting technical progress into our model, it is consistent with the stylised fact that the unemployment rate is untrended in the very long-run while the capital-labour ratio has grown steadily since the Industrial Revolution.

1.4 Structure of the Paper

The remainder of this paper is structured as follows. Section 2 presents and motivates the KC production function, introduces the other basic components of our model and describes the sequence in which decisions are made. Section 3 solves our model for the short-run case where the capital stock is assumed to be exogenous, while section 4 briefly discusses the implications of endogenising the capital stock. Section 5 considers the evolution of the equilibrium employment rate over time in our model. Finally, section 6 concludes.

2. Introducing the Model

The broad approach taken is to adopt an imperfectly competitive macroeconomic model of the type first used by Rowthorn (1977). The product market is characterised by monopolistic competition while wages are the outcome of a bargain between firms and trade unions. Perhaps the most well-known application of this type of model is contained in LNJ and the model presented below is essentially a development of their work. It also has close similarities to the models in Manning (1992) and, to a lesser extent, Blanchard and Kiyotaki (1987).

2.1 The Basic Structure

The economy is closed. We assume that it is composed of F identical imperfectly competitive firms, all of which are assumed to maximise profits in the standard sense. In addition, we assume that all firms are small relative to the aggregate economy. As a result, these firms do not consider the effect of their individual actions on aggregate variables. Finally, in all of what follows, we treat the number of firms as fixed.

2.1.1 The KC Production Function

We assume that the output, Y_i, of firm i is given by:

$$Y_i = A_i \left(\frac{K_i}{N_i C} \right)^b N_i^a K_i^{1-a}$$

$$= A_i \left(\frac{1}{C} \right)^b N_i^{a-b} K_i^{1-a+b} \tag{2.1}$$

where:

$$\begin{aligned} b > 0 \ \text{ for } \ \frac{K_i}{N_i} < C \ \ (\text{"full capacity"}) \\ b = 0 \ \text{ for } \ \frac{K_i}{N_i} > C \ \ (\text{"spare capacity"}) \end{aligned} \tag{2.2}$$

$$0 \leq b < a < 1$$

and where N_i is employment, K_i is capital and A_i (which is assumed to be exogenous) captures the effects of other inputs and technological progress. Meanwhile, C is a constant reflecting the threshold capital-labour ratio at which "full capacity" is reached. Since this threshold relates to the capital-labour ratio, the capacity constraint described is independent of the scale of the firm. Note also that from (2.1), it is clear that a must be greater than b (otherwise the returns to employment would be negative when b was positive) but less than one (otherwise the returns to capital would be negative for b = 0). These restrictions rule out the possibility of increasing returns, a feature which Manning (1990, 1992) has shown may generate multiple equilibria.

The above expressions characterise the capital constrained (KC) production function. This clearly differs from the standard Cobb-Douglas formulation which is used by LNJ. However, we can see that when b = 0 (i.e. when there is "spare capacity"), the KC production function reduces to the Cobb-Douglas case. Since the remaining parts of our model are standard, we can therefore see that if there is always "spare capacity", the result shown by LNJ (p. 107) that capital accumulation does not reduce equilibrium unemployment will also hold in our model. So, we can see that our model encompasses the standard framework.

The interesting case occurs when the capital-labour ratio drops below the threshold capacity constraint at C (i.e. when the firm reaches "full capacity") and b increases discretely from zero to a constant positive value, causing the coefficients on capital and labour in the production function to change discretely. When this happens, it is clear from (2.1) that the returns to labour fall discretely while the returns to capital increase discretely by the same amount.[7] In other words, the share of labour in output (and hence the wage share) falls discretely while the share of capital in output (and hence the profit share) increases discretely.[8] However, there is no discrete change in the level of output (this follows from the fact that $\left(K_i / N_i C\right) = 1$ at the threshold).

To further clarify what happens at the threshold, we sketch the KC production function for a fixed capital stock in (y_i, n_i) space in Figure 1. (Lower case letters denote logs of capital letters, as they will throughout this paper.) The magnitude of the slopes can be derived easily from (2.1).

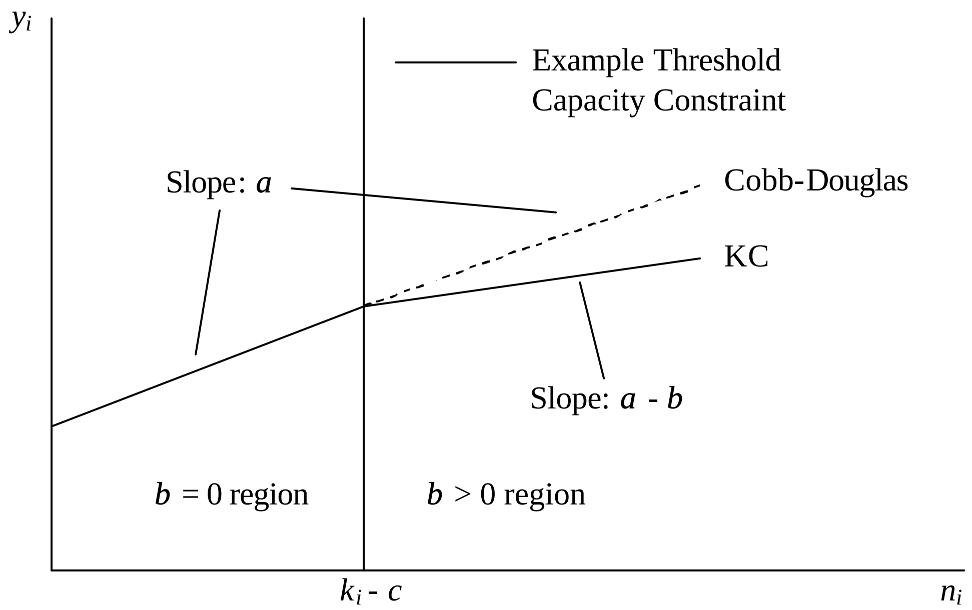

Figure 1

This diagram clearly illustrates the difference between the Cobb-Douglas and KC production functions. In the former case, the production function is linear in log space; in the latter case, the production function is kinked at the threshold capacity constraint, with the reduction in its slope reflecting the discrete fall in the returns to labour at the threshold.

---

[7] If it were the case that a = 1 (a = b), then crossing the threshold would entail moving from a situation resembling Leontief (Cobb-Douglas) production under "spare capacity" to Cobb-Douglas (Leontief) production under "full capacity". In this scenario, we would only have diminishing returns to labour (capital) under "full capacity" ("spare capacity").

[8] Note also that since labour productivity falls discretely at the threshold, it is clear that for a fixed capital stock and real wage, marginal costs must increase discretely as the threshold is crossed.

2.1.2 Motivating the KC Production Function

The assumption of KC production is absolutely critical: it drives *all* of the new results in our model. The key novel aspect of this production function is its notion of "capacity". In the existing literature, the idea of "capacity" is often mentioned but the term is not usually precisely defined. However, it would seem plausible to suggest that "capacity" represents a region where diminishing returns to labour really kick in because workers do not have enough capital to work with.

For example, suppose that we have a production process in which workers would ideally each have their own machine. Suppose also that the number of machines is fixed at five. Now, as the number of workers increases, there may be diminishing returns to labour for standard reasons. However, it seems likely that diminishing returns will be much more severe when we move from five to six workers than when we move from either four to five workers or from six to seven workers. This is because employment of the sixth worker is special in that it results in a switch from a situation where workers can each have their own machine to a situation where they must start to share machines. The fact that workers are forced to share machines also suggests that the returns to capital are likely to increase discretely at this point.

Since the Cobb-Douglas production function is unable to capture these effects, it is unlikely to be appropriate if the capital-labour ratio fluctuates. By contrast, the KC production function can capture these effects. Specifically, the threshold can act as a metaphor for a region where diminishing returns to labour really kick in, because when the capital-labour ratio drops below a certain level and the threshold is crossed, the returns to labour fall sharply. Therefore, when analysing the effect of changes in the capital stock on equilibrium unemployment, the KC production function may be a superior alternative to the Cobb-Douglas production function and any conclusions derived using it may be more realistic (note that since the KC production function nests the Cobb-Douglas production function, they can be no less realistic). Moreover, the KC production function is consistent with the stylised fact that during booms, when we would expect some firms to be capacity constrained, the share of labour in output falls.

2.1.3 The Remaining Components of the Model

Having introduced the KC production function, we briefly describe the remaining components of our model. These are standard and closely follow Manning (1992). We assume that demand for the output of firm *i* is given by:

$$Y_i = \left(\frac{1}{F}\right)\left(\frac{P_i}{P}\right)^{-q} D\left(P, \mathbf{X}\right) \tag{2.3}$$

where P_i is the firm's output price, P is the aggregate price level and D (P, $\mathbf{X}$) is an index of aggregate demand facing the firm with $\mathbf{X}$ being a vector of exogenous variables affecting this. This demand function is derived formally by Blanchard and Kiyotaki (1987, p. 664) using the assumptions (p. 649) that q is greater than one and that households have CES preferences. Note that q (which is technically the elasticity of substitution between goods in household utility) may be taken to represent the degree of product market competitiveness, with an infinite value of q corresponding to perfect competition.

The real profits, Π_i, of firm *i* are given by:

$$\Pi_i = \left(\frac{P_i}{P}\right)Y_i - \left(\frac{W_i}{P}\right)N_i - RK_i \tag{2.4}$$

where R is the real user cost of capital (assumed to be exogenous) and W_i is the nominal wage rate. Substituting (2.3) into (2.4) gives:

$$\Pi_i = \left(\frac{D\left(P, \mathbf{X}\right)}{F}\right)\left(\frac{P_i}{P}\right)^{1-q} - \left(\frac{W_i}{P}\right)N_i - RK_i \tag{2.5}$$

Each firm has a corresponding (risk-neutral) trade union with which it bargains. We assume that the utility of trade union *i* is:

$$U_i\left(\frac{W_i}{P}, N_i\right) = N_i^{\,g}\left(\frac{W_i}{P} - \overline{V}\right)^{1-g} \tag{2.6}$$

where $\overline{V}$ is the alternative wage available to a worker who loses his job with the firm. This is treated as exogenous when we examine the partial equilibrium bargain between an individual firm and its union but is endogenised when we consider the general equilibrium solution. Meanwhile g (which is constrained to lie between zero and one) reflects the relative weighting of employment and relative real wages in the union's utility function. Equation (2.6) is quite a general specification. It covers both the utilitarian (g = ½) and seniority (g = 0) models of Oswald (1982 and 1993 respectively).

Since the nominal wage in each firm is the outcome of a bargain between the firm and its union, we assume that it is chosen to maximise the Nash product:

$$\Omega = \left[U_i\left(\frac{W_i}{P}, N_i\right)\right]^l \left[\Pi_i - \left(-RK_i\right)\right]^{1-l} \tag{2.7}$$

or alternatively, using (2.6):

$$\Omega = \left[N_i^g\left(\frac{W_i}{P} - \overline{V}\right)^{1-g}\right]^l \left(\Pi_i + RK_i\right)^{1-l} \tag{2.8}$$

where l (which is constrained to lie between zero and one) represents the bargaining power of the union. If l = 1, we are effectively just maximising union utility. By contrast, if l = 0, union utility plays no role and we will obtain the competitive outcome for the real wage. This Nash bargaining solution can be derived as the subgame perfect equilibrium of a formal bargaining game (Binmore, Rubinstein and Wolinsky, 1986). Note that since the capital stock is already determined when the Nash bargain takes place (see the timeline below), the fallback level of profits for the firm if it does not reach agreement with the union is equal to $-RK_i$. This relates to the notion of "bygones being bygones" when the bargain takes place. It explains why we use operating profits (i.e. gross of capital costs) rather than total profits in (2.7) and (2.8). Finally, note that by taking logs, we may rewrite (2.8) as:

$$\ln\Omega = l\left[g\ln N_i + \left(1-g\right)\ln\left(\frac{W_i}{P} - \overline{V}\right)\right] + \left(1-l\right)\ln\left(\Pi_i + RK_i\right) \tag{2.9}$$

2.2 Timeline of Decisions

We assume that decisions are made in the order depicted in the diagram below:

K_i	W_i	N_i and P_i
Period t-1	Start of Period t	End of Period t

This is a conventional sequence. The capital stock is determined in the period prior to the determination of wages, employment and prices. This is our justification for treating the capital stock as exogenous in our short-run analysis in section 3. Given the capital stock, the nominal wage is determined according to the Nash bargain (characterised by (2.9)) between the firm and the union at the start of period t. Finally, at the end of period t, the firm chooses employment and the output price to maximise its profits, taking both the capital stock and the nominal wage level as predetermined.

3. Short-Run Analysis: Exogenous Capital Stock

In this section, we solve our model for the case where the capital stock is exogenous. In other words, we only consider decisions made in period t. After giving a brief intuitive discussion of our main results, we concentrate on the partial equilibrium solution at the level of the individual firm. Our analysis discusses both the end of period pricing and employment decisions (this entails deriving the firm's labour demand curve) and start of period wage-setting. We then consider the general equilibrium solution. We show that over a certain range, aggregate equilibrium employment will depend on the level of the capital stock.

3.1 Intuitive Discussion of the Main Results

The nature of the KC production function means that it is effectively composed of two separate Cobb-Douglas "production functions", each with different factor shares, with the relevant one depending on whether the capital-labour ratio is above or below the specified threshold. Therefore, when we solve the short-run model, we obtain a solution for aggregate employment corresponding to each "production function".[9] (We also obtain a corner solution, the details of which are discussed in our formal analysis.) As in the analysis of LNJ, at each of these two solutions, employment does not depend on the capital stock. This is as we would expect from the Cobb-Douglas nature of each of our "production functions". However, the key point is that *the solution at which the economy ends up* will depend on the level of the exogenous capital stock, as this determines the capital-labour ratio and hence the relevant

[9] To see why this must be the case, consider what would happen if we treated our model as two separate models in isolation, with only the coefficients on capital and labour in the production function differing between them.

“production function”. Moreover, aggregate employment is different in each of the two solutions. In particular, it is higher in the solution for which the capital-labour ratio is above the threshold (i.e. when the capital stock is high). This is because, in this solution, the share of labour in output (and hence the wage share) is higher. Therefore less unemployment is required to keep union wage demands in check. Alternatively, we could view the result as stemming from the fact that when the capital stock is high, the productivity of workers is high, meaning that workers are effectively “cheap” compared to the case where labour productivity is low. In other words, marginal costs are lower when the capital stock is high. As a result, firms will employ more workers in this case. Overall then, we can start to see how equilibrium employment will depend positively on the capital stock over a certain range in our model. However, it is clear that if the capital stock is already high, increases in its level will not have any impact on equilibrium employment.

3.2 Partial Equilibrium Labour Demand Curve

We start our formal analysis by deriving the firm’s labour demand curve for a given capital stock and nominal wage level (i.e. we start by considering the firm’s decision at the end of period *t*). To do this, we first eliminate the price term from (2.5) to enable us to maximise the profit function with respect to N_i. Substituting (2.1) into (2.3) and rearranging gives:

$$\left(\frac{P_i}{P}\right)=\left[\left(\frac{D(P,\mathbf{X})}{F}\right)^{-1} A_i \left(\frac{1}{C}\right)^{b} N_i^{a-b} K_i^{1-a+b}\right]^{-\frac{1}{q}} \qquad (3.1)$$

Since (3.1) uniquely determines the firm’s output price for a given level of employment, we can see that by considering the firm’s employment decision at the end of period *t* (as we do below), we are implicitly taking into account their contemporaneous pricing decision. Substituting (3.1) into (2.5) and simplifying gives:

$$\Pi_i=\left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{q}} A_i^{\frac{(q-1)}{q}} \left(\frac{1}{C}\right)^{\frac{b(q-1)}{q}} N_i^{\frac{(a-b)(q-1)}{q}} K_i^{\frac{(1-a+b)(q-1)}{q}} -\left(\frac{W_i}{P}\right)N_i - RK_i \qquad (3.2)$$

which may be rewritten as:

$$\Pi_i=\left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{q}} A_i^{\frac{(q-1)}{q}} \left(\frac{1}{C}\right)^{\frac{b(q-1)}{q}} K_i^{\frac{q-1-qa'}{q}} N_i^{a'} -\left(\frac{W_i}{P}\right)N_i - RK_i \qquad (3.3)$$

where:

$$a' = \frac{(a-b)(q-1)}{q} \tag{3.4}$$

Note that the assumptions on the parameters ($0 \le b < a < 1$ and $q > 1$) imply that $0 < a' < 1$.

For both values of b, subject to the constraints of (2.2), the firm obviously wants to choose employment to maximise its real profits. In other words, the firm wants to choose N_i to maximise (3.3) for the cases $b = 0$ and $b > 0$ subject to the constraints $N_i \le (K_i/C)$ and $N_i \ge (K_i/C)$ respectively.

We proceed by solving these two constrained optimisation problems alongside each other using Kuhn-Tucker theory. If Π_i^0 corresponds to real profits given by (3.3) when $b = 0$, then using (2.2), we can see that the complementary slackness conditions for the $b = 0$ problem are:

$$\frac{d\Pi_i^0}{dN_i} = 0 \ ; \quad N_i \le \frac{K_i}{C} \tag{3.5}$$

$$\frac{d\Pi_i^0}{dN_i} > 0 \ ; \quad N_i = \frac{K_i}{C} \tag{3.6}$$

For the $b > 0$ problem, if Π_i^+ corresponds to real profits given by (3.3) when $b > 0$, the complementary slackness conditions are:

$$\frac{d\Pi_i^+}{dN_i} = 0 \ ; \quad N_i \ge \frac{K_i}{C} \tag{3.7}$$

$$\frac{d\Pi_i^+}{dN_i} < 0 \ ; \quad N_i = \frac{K_i}{C} \tag{3.8}$$

We start by considering the corner solutions (3.6) and (3.8). Employment at both corner solutions is given by:

$$N_i = \frac{K_i}{C} \tag{3.9}$$

Moreover, both corner solutions yield the same level of real profits to the firm. To see this, substitute (3.9) into (3.2) to get:

$$\Pi_i = \left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{q}} A_i^{\frac{(q-1)}{q}} \left(\frac{1}{C}\right)^{\frac{b(q-1)}{q}} \left(\frac{K_i}{C}\right)^{\frac{(a-b)(q-1)}{q}} K_i^{\frac{(1-a+b)(q-1)}{q}} - \left(\frac{W_i}{P}\right)\left(\frac{K_i}{C}\right) - RK_i$$

This simplifies to:

$$\Pi_i = \left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{q}} A_i^{\frac{(q-1)}{q}} K_i^{\frac{(q-1)}{q}} \left(\frac{1}{C}\right)^{\frac{a(q-1)}{q}} - \left(\frac{W_i}{P}\right)\left(\frac{K_i}{C}\right) - RK_i \qquad (3.10)$$

which is independent of b. Since b is the only variable that differs between (3.6) and (3.8), we can see that, for a given wage, profits are the same at both corner solutions. Therefore, in terms of the key variables of interest, the two corner solutions are effectively the same, meaning that we do not need to consider them separately in what follows.

We now consider the interior solutions (3.5) and (3.7), deriving the labour demand curves which we would get if we always had $b = 0$ or $b > 0$ respectively. As all of the firms are small relative to the aggregate economy, we assume that the aggregate price level, P, is fixed when firms maximise their profits (since $\mathbf{X}$ is a vector of exogenous variables, this implies that we can also treat $D(P, \mathbf{X})$ as fixed). Moreover, as our timeline shows, the nominal wage and capital stock are already determined when the firm makes its profit-maximising decision at the end of period t. Finally, recall that F, C and R are all fixed. Therefore, we can proceed by differentiating the profit function given by (3.3) with respect to N_i. The first order condition is:

$$\left(\frac{W_i}{P}\right) = a'\left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{q}} A_i^{\frac{(q-1)}{q}} \left(\frac{1}{C}\right)^{\frac{b(q-1)}{q}} K_i^{\frac{q-1-qa'}{q}} N_i^{a'-1} \qquad (3.11)$$

Since (3.3) is concave in N_i (this follows from the fact that $a' < 1$), this solution is a global maximum. Rearranging (3.11) to make N_i the subject, we get:

$$N_i = \left[a'\left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{q}} A_i^{\frac{(q-1)}{q}} \left(\frac{1}{C}\right)^{\frac{b(q-1)}{q}} K_i^{\frac{q-1-qa'}{q}} \left(\frac{W_i}{P}\right)^{-1}\right]^{\frac{1}{1-a'}} \qquad (3.12)$$

At interior solutions, (3.12) is the firm's labour demand curve: it applies for both $b = 0$ and $b > 0$ and gives the firm's optimal employment choice for a given wage and capital stock. As noted above, it also determines the firm's output price via (3.1). In Appendix A, we show that the firm's real operating profits in terms of the wage at optimal (interior) solutions for employment are given by:

$$\Pi_i + RK = (1-a')\left[(a')^{a'}\left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{q}} A_i^{\frac{q-1}{q}}\left(\frac{1}{C}\right)^{\frac{b(q-1)}{q}} K_i^{\frac{q-1-qa'}{q}}\left(\frac{W_i}{P}\right)^{-a'}\right]^{\frac{1}{1-a'}} \qquad (3.13)$$

We are now able to illustrate the firm's overall labour demand curve by constructing a diagram (Figure 2) in ($w_i - p$, n_i) space. This consists of the firm's labour demand curves for both $b = 0$ and $b > 0$ (sketched using the log version of (3.12)) and an example threshold capacity constraint.

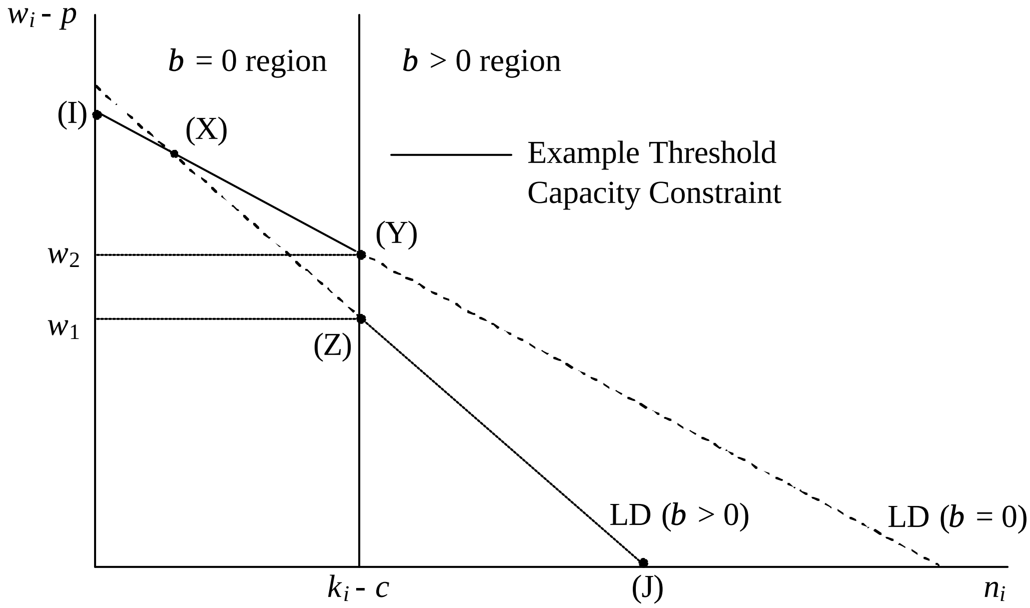

Figure 2

The justification for the relative slopes of the curves is contained in Appendix B. We also show in this appendix that the intersection of these two curves at (X) will always occur at a level of employment below the threshold capacity constraint.

Given the fact that the threshold capacity constraint is to the right of (X), it is immediately clear from Figure 2 that for any given wage and threshold, there is only a single applicable labour demand curve (with an invariant section at the threshold capacity constraint). For wages between w_1 and w_2, neither interior solution is "feasible" (i.e. consistent with (2.2)). As a result, the corner solution (which coincides with the threshold capacity constraint) must be chosen in this wage range. By contrast, outside of this range, the interior solution on the applicable labour demand curve will be chosen by the firm (the corner solution is clearly inferior, regardless of whether b = 0 or b > 0 at it). Specifically, if the wage is greater than w_2, the interior solution corresponding to the b = 0 labour demand curve will be chosen, while if the wage is less than w_1, the interior solution corresponding to the b > 0 labour demand curve will be chosen. Therefore the firm's overall labour demand curve is IYZJ.

Moving out of log form, we can summarise this section as follows. For a given wage, the firm's overall labour demand curve is given by (3.12) with b = 0 for $N_i < (K_i/C)$ (or equivalently $(W_i/P) > (W_i/P)_2$) and by (3.12) with b > 0 for $N_i > (K_i/C)$ (or equivalently $(W_i/P) < (W_i/P)_1$). Meanwhile, for wages satisfying $(W_i/P)_1 \leq (W_i/P) \leq (W_i/P)_2$, it is given by the corner solution (3.9): $N_i = (K_i/C)$.

3.3 Partial Equilibrium Wage-Setting

We now move on to discuss the decisions made at the previous stage in our timeline (i.e. at the start of period t). At this point, the firm and its union bargain over the nominal wage, taking the capital stock as given and assuming that they are too small to affect the aggregate price level. We assume that both parties are also fully aware of the employment and pricing decisions that the firm will make for a given wage at the end of period t and that this information is used when bargaining.

In a crude sense, we may view the wage determination problem as requiring us to choose the nominal wage to maximise the Nash product given by (2.8) (or, in log form, (2.9)) subject to a "budget constraint" which, in this context, is the labour demand curve derived in the previous section. Since this labour demand curve is kinked, we need to consider the possibility of a corner solution at the threshold.

Our approach involves initially searching for the wages that would be set for the b = 0 and b > 0 labour demand curves in the absence of capacity constraints (i.e. without assumption (2.2)). We then determine the wage that will be set at the corner solution. These three solutions for the wage (one for each of the three curves that make up the overall labour demand curve) give us three potential solutions for end of period employment. We then check these employment outcomes for feasibility (i.e.

consistency with (2.2)). Once we have done this, the feasible solutions are compared to see which yields the highest value of the Nash product (2.8). The one that does best will be the one which is chosen.[10]

3.3.1 Interior Solutions

We wish to choose W_i to maximise (2.9) for the interior solution labour demand curve. However, instead of differentiating with respect to W_i, we differentiate with respect to ln W_i. Note that the chain rule implies that this will generate the same solution. Recalling our assumption that the alternative wage, $\bar{V}$, is exogenous in the partial equilibrium analysis, we can see that the first order condition is:

$$l\left[g\frac{d\ln N_i}{d\ln W_i}+(1-g)\frac{d\ln\left[(W_i/P)-\bar{V}\right]}{d\ln W_i}\right]+(1-l)\frac{d\ln(\Pi_i+RK_i)}{d\ln W_i}=0 \qquad (3.14)$$

In Appendix C, we show that at interior solutions this expression reduces to:

$$\frac{W_i}{P}=y\bar{V} \qquad (3.15)$$

where:

$$y=\frac{lg+(1-l)a'}{lg+(1-l)a'-(1-a')l(1-g)} \qquad (3.16)$$

In other words, the wage is simply marked-up over the alternative wage by a factor of y. However, the value of y depends on a' and therefore on b. We also show in Appendix C that y depends positively on b. Therefore, if we let y^0 correspond to the case $b = 0$ in (3.16) and y^+ correspond to the case $b > 0$, then we have $y^+ > y^0$. In other words, if the firm is operating under "full capacity", the mark-up is higher. Intuitively, we may explain this by noting that when $b > 0$ the employment gain from accepting a lower wage is less than the employment gain when $b = 0$. This is because the returns to labour are lower when $b > 0$ and so extra workers are not worth as much to the firm.

[10] At first, this may seem like an unnecessarily lengthy approach. However, we are unable to solve the problem easily by considering the shape of a general indifference curve since the indifference curves, which could be derived by substituting (3.3) into (2.9) and differentiating, clearly depend on a' and will therefore also be kinked at the threshold. As a result of this complexity, it is easier to consider each of the three solution possibilities separately and then compare them, which is what we do in the text.

It is immediately clear from (3.15) that apart from the potential discrete effect on the value of b (via (2.2)), the bargained real wage does not depend on employment. Therefore, if we sketch the wage-setting curve in ($w_i - p$, n_i) space, it will be a horizontal line with its intercept depending on whether $b > 0$ or $b = 0$ (and being higher when $b > 0$). We add these curves onto Figure 2 and remove the example threshold capacity constraint to get Figure 3 below. This is a partial equilibrium picture showing the determination of wages and employment at the level of the individual firm.

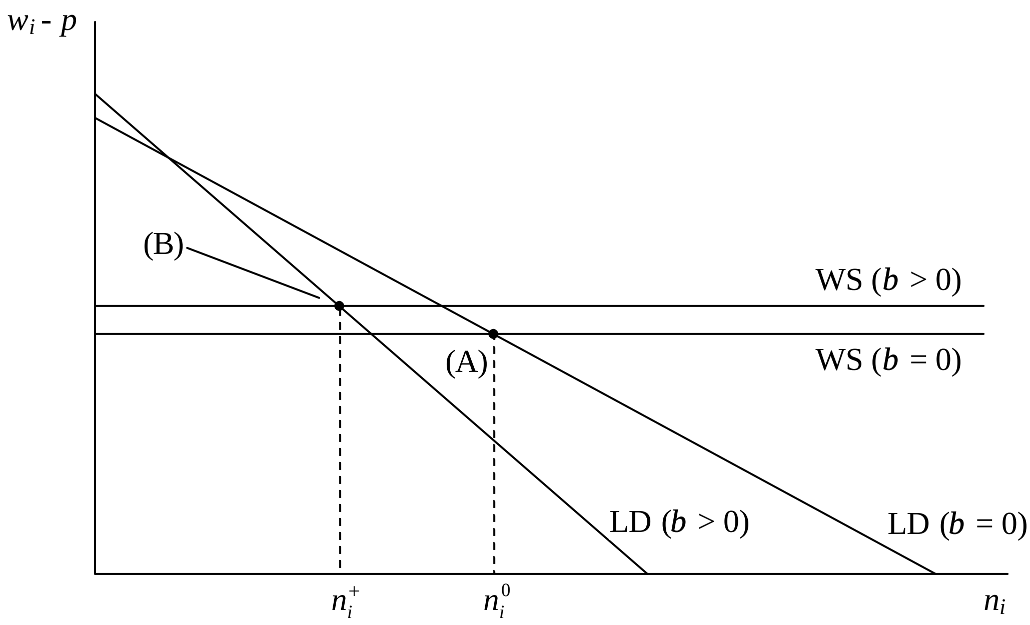

Figure 3

As is clear from the diagram, there are two equilibria: one corresponding to $b = 0$ (call this equilibrium (A)) and one corresponding to $b > 0$ (call this equilibrium (B)). (A) corresponds to a higher level of employment but a lower wage. In other words, if N_i^0 is the employment level at (A) (when $b = 0$) and N_i^+ is the employment level at (B) (when $b > 0$), then we have $N_i^+ < N_i^0$. We can derive analytical expressions for N_i^0 and N_i^+ by substituting (3.15) into (3.12) and taking $b = 0$ or $b > 0$ as appropriate. This gives:

$$N_i^0 = \left[\frac{a(q-1)}{q} \left(\frac{D(P, \mathbf{X})}{F} \right)^{\frac{1}{q}} A_i^{\frac{(q-1)}{q}} K_i^{\frac{(1-a)(q-1)}{q}} \left(y^{\,0} \overline{V} \right)^{-1} \right]^{\frac{q}{q-a(q-1)}} \tag{3.17}$$

$$N_i^{+}=\left[(a')\left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{q}}A_i^{\frac{(q-1)}{q}}\left(\frac{1}{C}\right)^{\frac{b(q-1)}{q}}K_i^{\frac{q-1-qa'}{q}}\left(y+\overline{V}\right)^{-1}\right]^{\frac{1}{1-a'}} \tag{3.18}$$

For b = 0 and b > 0 respectively, (3.17) and (3.18) are the interior solutions for period t employment. They also determine the firm's output price via (3.1). However, we must bear in mind that they might not be feasible due to the threshold capacity constraint. In addition, it may turn out that a corner solution does better than the interior solutions. It is to these issues that we now turn.

3.3.2 Corner Solution

The corner solution is given by (3.9). From the end of section 3.2, we know that the firm will only choose this solution at the end of period t if the real wage satisfies the constraint $(W_i/P)_1 \le (W_i/P) \le (W_i/P)_2$. When choosing ln W_i to maximise (2.9) for the corner solution labour demand curve, this implies that the complementary slackness conditions are:

$$\frac{d\ln\Omega}{d\ln W_i}=0\ ;\quad \left(\frac{W_i}{P}\right)_1\le\left(\frac{W_i}{P}\right)\le\left(\frac{W_i}{P}\right)_2 \tag{3.19}$$

$$\frac{d\ln\Omega}{d\ln W_i}<0\ ;\quad \left(\frac{W_i}{P}\right)=\left(\frac{W_i}{P}\right)_1 \tag{3.20}$$

$$\frac{d\ln\Omega}{d\ln W_i}>0\ ;\quad \left(\frac{W_i}{P}\right)=\left(\frac{W_i}{P}\right)_2 \tag{3.21}$$

We start by considering (3.19). In Appendix C, we show that the wage equation associated with the first order condition in this case is:

$$\left(\frac{W_i}{P}\right)=\frac{l(1-g)\left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{q}}A_i^{\frac{(q-1)}{q}}K_i^{-\frac{1}{q}}\left(\frac{1}{C}\right)^{\frac{a(q-1)}{q}-1}+(1-l)\overline{V}}{1-lg} \tag{3.22}$$

To see whether this solution satisfies the corresponding complementary slackness condition, we need to determine the values of $(W_i/P)_1$ and $(W_i/P)_2$. From Figure 2, we can see that these are given by the real wage at the intersection of the threshold capacity constraint and the labour demand curves for b > 0 and b = 0 respectively.

Recalling the fact that a' is defined by (3.4), the $b > 0$ (inverse) labour demand curve (3.11) may be written as:

$$\left(\frac{W_i}{P}\right)=a'\left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{q}} A_i^{\frac{(q-1)}{q}} \left(\frac{1}{C}\right)^{\frac{b(q-1)}{q}} K_i^{\frac{(1-a+b)(q-1)}{q}} N_i^{\frac{(a-b)(q-1)}{q}-1} \qquad (3.23)$$

Substituting (3.9) into (3.23) and simplifying gives $\left(W_i/P\right)_1$:

$$\left(\frac{W_i}{P}\right)_1 = a'\left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{q}} A_i^{\frac{(q-1)}{q}} \left(\frac{1}{C}\right)^{\frac{a(q-1)}{q}-1} K_i^{\frac{-1}{q}} \qquad (3.24)$$

The $b = 0$ (inverse) labour demand curve follows from substituting $b = 0$ into (3.23):

$$\left(\frac{W_i}{P}\right)=\left[\frac{a(q-1)}{q}\right]\left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{q}} A_i^{\frac{(q-1)}{q}} K_i^{\frac{(1-a)(q-1)}{q}} N_i^{\frac{a(q-1)}{q}-1} \qquad (3.25)$$

Substituting (3.9) into (3.25) and simplifying gives $\left(W_i/P\right)_2$:

$$\left(\frac{W_i}{P}\right)_2 = \left[\frac{a(q-1)}{q}\right]\left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{q}} A_i^{\frac{(q-1)}{q}} \left(\frac{1}{C}\right)^{\frac{a(q-1)}{q}-1} K_i^{\frac{-1}{q}} \qquad (3.26)$$

Comparisons between (3.22), (3.24) and (3.26) can now be made to see whether the complementary slackness condition in (3.19) is satisfied. If it is, the real wage will be given by (3.22); if not, it will be given by (3.24) (if (3.20) is satisfied) or (3.26) (if (3.21) is satisfied). Comparing (3.22) and (3.26), we can see that since $\left[(1-l)\bar{V}\right]/(1-lg)>0$, we will have $\left(W_i/P\right)>\left(W_i/P\right)_2$ (thus violating (3.19)) if:

$$\frac{l(1-g)\left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{q}} A_i^{\frac{(q-1)}{q}} K_i^{-\frac{1}{q}} \left(\frac{1}{C}\right)^{\frac{a(q-1)}{q}-1}}{1-lg} > \left[\frac{a(q-1)}{q}\right]\left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{q}} A_i^{\frac{(q-1)}{q}} \left(\frac{1}{C}\right)^{\frac{a(q-1)}{q}-1} K_i^{\frac{-1}{q}}$$

Cancelling terms (which are all positive) and then rearranging gives:

$$l > \frac{a(q-1)}{q(1-g)+ga(q-1)} \qquad (3.27)$$

Therefore, if condition (3.27) is satisfied, (3.19) will be violated but (3.21) will be satisfied and the corner solution real wage will be given by (3.26) (meaning that the overall equilibrium will be at the intersection of the threshold and the $b = 0$ labour demand curve). We can see that (3.27) will be satisfied if the union's bargaining power (l) is sufficiently high (this is as we would expect: increasing wages for a fixed employment level reduces profits[11] and is therefore bad for the firm but good for the union). It is also more likely to be satisfied if g is low (i.e. if the union attaches a relatively low weight to employment in its utility function).

In what follows, we are going to assume that (3.27) is *always* satisfied. This may be justified on the grounds that the whole imperfectly competitive approach to the determination of unemployment is only interesting if the union has a reasonable amount of bargaining power (i.e. l is significantly greater than zero) and cares to a certain degree about relative real wages (i.e. g is significantly less than one). (As noted in section 2, if the union had no bargaining power or if it only cared about employment, we would obtain the perfectly competitive outcome for the real wage.) We also make this assumption because it illustrates all of the points which we wish to make and simplifies the analysis. Although we could consider the other cases, we would not gain any further major insights: the details would be different but the overall conclusion about equilibrium employment depending on the capital stock over some range would be unaffected.

3.3.3 Checking for Feasibility and Comparing Solutions

We are now able to check all of our solutions for feasibility and then compare the feasible solutions to see which one yields the highest value of the Nash product (2.8) (or, in log form, (2.9)) and will therefore be chosen. There are three potential solutions to consider. These are the corner solution just discussed and the two interior solutions discussed in section 3.3.1: (A), for which $b = 0$, employment is N_i^0 and the real wage is $y^0\overline{V}$; and (B), for which $b > 0$, employment is N_i^+ and the real wage is $y^+\overline{V}$. Although the corner solution is always feasible (this almost follows by definition - since it coincides with the threshold capacity constraint, it cannot violate (2.2)), the interior solutions might not be feasible.

In fact, we can immediately show that it is impossible for both (A) and (B) to be feasible at the same time. To see this, suppose that (B) is feasible. As $b > 0$ at (B), it is clear from (2.2) that this requires $N_i^+ > \left(K_i/C\right)$. Since we know that employment is

[11] Assuming that output remains constant (i.e. there are no efficiency wage effects).

higher at (A) than at (B), this implies that $N_i^0 > \left(K_i / C\right)$. But, since b = 0 at (A), this is not consistent with (A) being feasible. Hence the result follows.

However, which (if either) of the interior solutions is feasible will depend on the threshold employment level at which b becomes positive. Recalling the fact that C is fixed, we can see from (2.2) that this depends directly on the level of the (exogenous) capital stock. Therefore, we now vary the capital stock in order to check for feasibility in different scenarios. We have three cases to consider (the type of line used in Figure 4 below is described in brackets):

(i) “Spare Capacity” (dots)

$$K_i = K_i^H \quad \text{where} \;\; N_i^+ < N_i^0 < \frac{K_i^H}{C} \tag{3.28}$$

(ii) “Moderately” Capacity Constrained (long dashes)

$$K_i = K_i^M \quad \text{where} \;\; N_i^+ < \frac{K_i^M}{C} < N_i^0 \tag{3.29}$$

(iii) “Severely” Capacity Constrained (dots and dashes)

$$K_i = K_i^L \quad \text{where} \;\; \frac{K_i^L}{C} < N_i^+ < N_i^0 \tag{3.30}$$

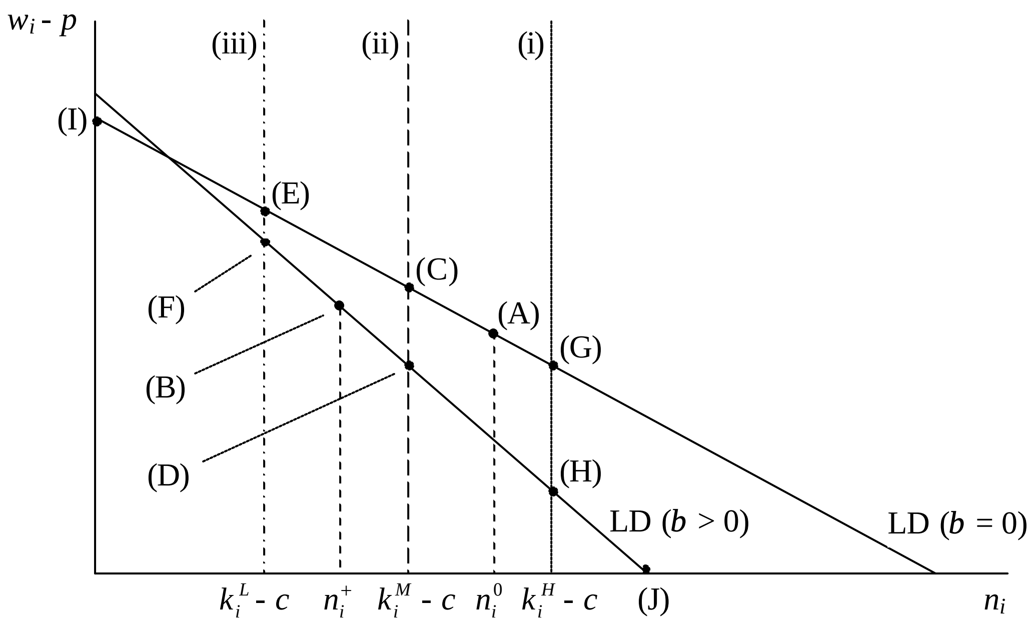

Figure 4

The log forms of example thresholds representing each of these three cases are added onto a simplified version of Figure 3 to give Figure 4.[12]

As before, the interior solutions are at (A) and (B). The corner solution varies according to the threshold. Provided condition (3.27) is satisfied, we know from the previous subsection that the corner solution will be at the intersection of the threshold capacity constraint line and the $b = 0$ labour demand curve. Therefore, in terms of Figure 4, it will be at (G) in case (i), at (C) in case (ii) and at (E) in case (iii).

We start by discussing case (ii) since it is the easiest one to consider. Since $b = 0$ at levels of employment lower than the threshold (i.e. to the left of the threshold) and since $b > 0$ at levels of employment higher than the threshold, the overall labour demand "constraint" is given by ICDJ. In this scenario, it is clear both from Figure 4 and from (2.2) that neither (A) nor (B) is feasible. The only feasible solution is the corner solution at (C). Therefore this will be chosen, meaning that employment will be given by $N_i = K_i^M / C$.

In case (i), the overall labour demand curve is IGHJ. As a result (A) is feasible but (B) is infeasible. Therefore, we must compare (A) with some arbitrary corner solution represented by (G) to see which yields a higher value of the Nash product (2.8). Using a revealed preference argument, it is immediately clear that (A) will be preferred. Since (A) and (G) are both on the $b = 0$ labour demand curve, wages generating both of these solutions could have been chosen when we solved the $b = 0$ wage determination problem (recall that this problem maximised the Nash product). However, since the wage corresponding to (A) (i.e. $y^0\overline{V}$) was chosen then, it will continue to be chosen in this scenario at the start of period *t*. As a result, the firm will choose a level of employment equal to N_i^0 at the end of period *t*.

Finally, in case (iii), the overall labour demand curve is IEFJ, meaning that (B) is now feasible but (A) is infeasible. Therefore, this time, we must compare (B) with some

[12] Since the level of the capital stock affects the position of the labour demand curves, we should technically draw three different diagrams to represent the three different cases. However, since we are only ever considering one of the thresholds at any given time in the analysis below, we can abstract from this point and do so in order to illustrate the differences between the cases more clearly.

arbitrary corner solution represented by (E). Unfortunately, it is not as easy to make comparisons in this case as (E) was not available when (B) was chosen in the b > 0 wage determination problem. Indeed, we are unable to compare the two solutions analytically (this is because of the $\ln\left[(W_i/P)-\bar{V}\right]$ term in (2.9) which creates problems). However, we can still make two points (since these are all we need for our main results, an analytical comparison would not add much).

Firstly, we can say that the *maximum* level of employment in case (iii) is N_i^+ (i.e. the low equilibrium employment level corresponding to (B)). This is because if employment at the (arbitrary) corner solution (E) were greater than this, then the threshold would be to the right of (B) and we would be in case (ii).

Secondly, as shown in Appendix D, the outcome of the choice between the interior solution (B) and the (arbitrary) corner solution (E) is ambiguous: in terms of the Nash product, sometimes we will have:

$$(B) \succ (E) \Rightarrow N_i = N_i^+ \tag{3.31}$$

and sometimes we will have:

$$(E) \succ (B) \Rightarrow N_i = \frac{K_i^L}{C} \tag{3.32}$$

The actual outcome will depend on both the level of the (exogenous) capital stock (through its impact on the position of the threshold) and the parameter values.

By continuity, this result implies that for given parameters, there will be a particular level of the capital stock in case (iii) for which (B) and some arbitrary corner solution represented by (E) are indifferent. Let us define this particular level of the capital stock as K_i^1 (or k_i^1 in log form). By continuity with respect to case (ii), it is clear that levels of the capital stock just above K_i^1 will lead to the arbitrary corner solution (E) being chosen while levels of the capital stock just below K_i^1 will lead to (B) being chosen.

We can now describe what happens as the (exogenous) capital stock falls in the vicinity of K_i^1. When the capital stock is just greater than K_i^1, the solution will be at an arbitrary corner solution like (E) and the corresponding employment level is less than N_i^+ (which can be obtained at (B)). As the capital stock falls below K_i^1, the solution switches to (B) and employment therefore increases discretely. (Note that this implies that the *minimum* level of employment in case (iii) is K_i^1/C.)

This rise in employment for a fall in the capital stock in the vicinity of K_i^1 seems quite surprising at first, especially since everywhere else in the short-run model, a rising capital stock either increases employment or leaves it unchanged. The result is essentially driven by the simplifying assumption of a two-regime model which results in switching at a specific point. Intuitively, we may explain what happens as follows. The assumption that condition (3.27) holds effectively means that we are assuming that the union is in a relatively strong position when bargaining. As a result, the bargainers (i.e. the union and the firm) generally prefer to be on the $b = 0$ labour demand curve rather than the $b > 0$ labour demand curve if possible. This is because the returns to labour and hence the overall wage share are greater when $b = 0$ than when $b > 0$. (The fact that the profit share is lower when $b = 0$ than when $b > 0$ is not weighted too heavily during bargaining because of the relative weakness of the firm.) Therefore, as the threshold moves to the left within case (iii)" (due to falls in the exogenous capital stock), the bargainers try to maintain the value of b at zero even though they must sacrifice some employment just to achieve this. Initially, they are happy to do this since the cost is small but the gain is relatively large. However, as the capital stock falls and the threshold continues to move to the left, the cost in terms of lost employment increases rapidly. Eventually, when the capital stock reaches K_i^1, the costs become so great that the bargainers give up on their attempts to keep b equal to zero and accept the fact that it will take a positive value. This causes a discrete fall in the wage share (and the wage level) but results in a discrete increase in employment because the threshold constraint is released.

3.3.4 Summary of the Partial Equilibrium Solution

Provided that condition (3.27) is satisfied, if there is "spare capacity", a wage of $y^{\,0}\overline{V}$ will be set at the start of period t and, as a result, employment will be N_i^0. Meanwhile, if the firm is "severely" capacity constrained, the maximum level of employment will be N_i^+, which we know is less than N_i^0, and the minimum level of employment will be K_i^1/C. Finally, if the firm is "moderately" capacity constrained, we will be at the corner solution and employment will be K_i/C, which depends directly on the level of the exogenous capital stock. Overall then, we have a partial equilibrium range of employment between K_i^1/C and N_i^0 in this short-run model. The exact employment outcome within this range will depend on the level of the exogenous capital stock. Moreover, we can see that, with a minor exception in the vicinity of K_i^1, increases in the capital stock increase employment over this range.

3.4 General Equilibrium Solution

We now analyse our model in a general equilibrium context. In aggregate, since all firms are identical, we have:

$$Y_i = \frac{Y}{F};\quad N_i = \frac{N}{F};\quad K_i = \frac{K}{F};\quad A_i = A;\quad P_i = P;\quad W_i = W \tag{3.33}$$

where the absence of a subscript denotes an economy-wide variable.

3.4.1 Labour Demand Curve

We start by using the relationships in (3.33) to derive the aggregate labour demand curve. At the corner solution, this immediately follows from (3.9):

$$N = \frac{K}{C} \tag{3.34}$$

At the interior solutions, we show in Appendix E that it is given by:

$$\left(\frac{N}{K}\right) = \left[A\left(\frac{1}{C}\right)^{b}\right]^{\frac{1}{1-a+b}} \left[\frac{qW}{(a-b)(q-1)P}\right]^{\frac{-1}{1-a+b}} \tag{3.35}$$

If we set $b = 0$ and $A = 1$, (3.35) reduces to:

$$\left(\frac{N}{K}\right) = \left[\frac{qW}{a\,(q-1)\,P}\right]^{\frac{-1}{1-a}} \tag{3.36}$$

This is identical to equation (14) on page 105 of LNJ.[13] So, if b always equals zero in our model (i.e. there is always "spare capacity"), then we are back to standard theory and all of the results shown by LNJ continue to apply.

3.4.2 Wage-setting

We now consider wage-setting - this will enable us to derive expressions for aggregate employment and unemployment in the (short-run) model. Here, the key difference between the partial and general equilibrium cases is that in the general equilibrium case, we can no longer treat the alternative wage, $\overline{V}$, as exogenous. Instead, following standard convention (and implicitly assuming that workers are risk-neutral), we model it by:

$$\overline{V} = \frac{N}{L}\left(\frac{W}{P}\right) + \left(1 - \frac{N}{L}\right)\left(\frac{B}{P}\right) \tag{3.37}$$

where L is the size of the labour force, implying that:

$$\frac{N}{L} = e \tag{3.38}$$

is the (endogenous) probability of being employed while

$$\left(1 - \frac{N}{L}\right) = u \tag{3.39}$$

is the probability of being unemployed and receiving the nominal unemployment benefit, B. Note that by our definition, we have assumed that the employment rate, e, and the unemployment rate, u, are related by the identity $u \equiv 1 - e$.

At the interior solutions, the partial equilibrium wage is given by (3.15). Substituting (3.37) into this, aggregating using (3.33) and rearranging gives:

$$\frac{W}{P} = y\left(\frac{L - N}{L - y\, N}\right)\left(\frac{B}{P}\right) \tag{3.40}$$

Following LNJ (p. 107), we assume that the government indexes benefits to wages so that the benefit replacement ratio

$$\frac{(B/P)}{(W/P)} = \frac{B}{W} = b \quad \text{where} \quad 0 < b < 1 \tag{3.41}$$

[13] Their W denotes the real wage while ours denotes the nominal wage; their k is elsewhere defined by $k = (q - 1)/q$

is fixed. Aggregate equilibrium employment at the interior solutions then follows directly from (3.40) and (3.41):

$$\frac{1}{b}=y\left(\frac{L-N}{L-yN}\right)$$

$$N=\frac{L(1-by)}{y(1-b)} \tag{3.42}$$

Since aggregate employment cannot be negative, (3.42) is only valid if $by<1$. If this condition is violated, then N = 0. Intuitively, this means that if both the mark-up and the benefit replacement ratio are very high, then, as we might expect, we will have no employment.

Substituting (3.42) into (3.35), we can derive an expression for the aggregate real wage at the interior solutions:

$$\left(\frac{W}{P}\right)=A\left(\frac{1}{C}\right)^{b}\left[\frac{Ky(1-b)}{L(1-by)}\right]^{1-a+b}\left[\frac{(a-b)(q-1)}{q}\right] \tag{3.43}$$

We can also use (3.38), (3.39) and (3.42) to derive expressions for the aggregate employment and unemployment rates:

$$e=\frac{1-by}{y(1-b)} \quad \text{for } by<1;\ \ e=0 \text{ otherwise} \tag{3.44}$$

$$u=\frac{y-1}{y(1-b)} \quad \text{for } by<1;\ \ u=1 \text{ otherwise} \tag{3.45}$$

Obviously all of these expressions depend on the value of y. Returning to (3.42), we can see that:

$$\frac{dN}{dy}=\frac{y(1-b)(-bL)-L(1-by)(1-b)}{y^{2}(1-b)^{2}}=\frac{-L}{y^{2}(1-b)}<0 \ \text{ since } b<1 \tag{3.46}$$

As shown in Appendix C, y depends positively on b. Therefore, it follows from (3.46) that if b takes a constant positive value, aggregate equilibrium employment will be lower than if b = 0. So, a rise in b (which could be induced by a fall in the exogenous capital stock) will lead to a fall in aggregate equilibrium employment. In other words, if, as defined above, we let y^{0} correspond to the case b = 0 in (3.16) and y^{+} correspond to the case b > 0, then from (3.42):

$$N^0 = \frac{L\left(1-by^0\right)}{y^0\left(1-b\right)} \text{ and } N^+ = \frac{L\left(1-by^+\right)}{y^+\left(1-b\right)}$$

with $N^0 > N^+$.

As in the analysis of LNJ, it is clear from these expressions that aggregate equilibrium employment is independent of the level of the capital stock within each regime. However, its actual level does depend on the capital stock since this determines the regime which applies. Specifically, if the (exogenous) capital stock is sufficiently high (i.e. condition (3.28) is satisfied), the partial equilibrium solution will be at (A) and aggregate employment will be N^0. By contrast, if the capital stock is very low (i.e. condition (3.30) is satisfied), then we know from our discussion above that the partial equilibrium solution will either be at (B) or at the arbitrary corner solution (E) and the *maximum* level of aggregate employment will be N^+, which is lower than N^0. (More specifically, if $K < K^1$, then employment will definitely be N^+.) Therefore, over a certain range, increases in the (exogenous) capital stock may lower aggregate equilibrium unemployment and we have broken down the LNJ result (p. 107) that "unemployment…is independent of capital accumulation". This follows solely from using the alternative KC production function presented above: indeed it is interesting to note how sensitive the LNJ result is to such a small change of assumption.

We may illustrate our result even more starkly by considering the corner solution. In this case, assuming that condition (3.27) holds, the partial equilibrium wage is given by (3.26):

$$\left(\frac{W_i}{P}\right) = \left[\frac{a(q-1)}{q}\right]\left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{q}} A_i^{\frac{q-1}{q}} K_i^{\frac{-1}{q}} \left(\frac{1}{C}\right)^{\frac{a(q-1)}{q}-1} \tag{3.26}$$

If this wage is set in all firms, then all firms will choose the corner solution for employment given by (3.9). However, since (3.26) is independent of the alternative wage, $\overline{V}$, the employment choices of individual firms will have no bearing on the aggregate real wage. As a result, the aggregate real wage at the corner solution will simply be given by the aggregate version of (3.26):

$$\left(\frac{W}{P}\right) = \left[\frac{a(q-1)}{q}\right]\left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{q}} A^{\frac{q-1}{q}} \left(\frac{K}{F}\right)^{\frac{-1}{q}} \left(\frac{1}{C}\right)^{\frac{a(q-1)}{q}-1} \tag{3.47}$$

Meanwhile, economy-wide employment will be given by aggregating (3.9) which gives (3.34): $N = K/C$. Therefore, we can see that in this case, aggregate equilibrium employment depends directly and continuously on the (exogenous) capital stock. In Appendix E, we eliminate $D\,(P,\,\mathbf{X})$ from (3.47) to derive an expression for the corner solution aggregate real wage solely in terms of the exogenous variables and parameters:

$$\left(\frac{W}{P}\right) = \left[\frac{a\,(q-1)}{q}\right] A \left(\frac{1}{C}\right)^{a-1} \tag{3.48}$$

3.4.3 Summary of the General Equilibrium Solution

Letting K^1 be the aggregate version of K_i^1 (the point at which (B) and the arbitrary corner solution (E) are indifferent and where regime-switching occurs), we may summarise the equilibrium outcomes for aggregate employment in the short-run model as follows:

$$N = \begin{cases} N^+ = \dfrac{L\left(1-by^+\right)}{y^+\left(1-b\right)} \\ \dfrac{K}{C} \\ N^0 = \dfrac{L\left(1-by^0\right)}{y^0\left(1-b\right)} \end{cases} \quad \text{for} \quad \begin{cases} K < K^1 \quad \text{where} \quad K^1 < N^+C \\ K^1 \le K \le N^0C \\ K > N^0C \end{cases} \tag{3.49}$$

From this, we can see that the summary at the end of the partial equilibrium section continues to apply in the general equilibrium case. In particular, in equilibrium, aggregate employment may lie anywhere in the range $\left[K^1/C,\, N^0\right]$. The exact employment outcome within this range (and hence the equilibrium employment and unemployment rates) will depend on the level of the (exogenous) aggregate capital stock. If the capital stock is high, employment is likely to be high; if it is low, employment is likely to be low. However, outside of the range $\left[K^1,\, N^0C\right]$, changes in the capital stock will not affect employment. In particular, if the initial capital stock is fairly high, increases in its level will not be able to increase employment. This contrasts with the results derived by both LNJ using Cobb-Douglas production technology and Rowthorn (1999) using CES production technology.

4. Endogenising the Capital Stock

The results presented above were derived under the assumption that the capital stock was exogenous. It is clearly interesting to consider how endogenising the capital stock might affect these short-run results. In chapter 4 of Kapadia (2003), we show that even in the long-run solution of our model, there is no guarantee that aggregate employment will always be at the high equilibrium level. Instead, with the capital stock endogenous, employment will be affected by the real user cost of capital (and hence by the real interest rate) over a certain range.

Since the derivation of these results is quite long and since the results do not really change the essence of our short-run conclusions in any case, we do not present the formal argument here. However, the intuition behind the results is fairly easy to see. If the real user cost of capital is relatively high, it is clear that the firm will choose a low capital stock, meaning that the capital-labour ratio will be below its threshold and the low equilibrium employment outcome (which is associated with the low wage share and high profit share) will result. It is not quite so obvious that the firm will choose a high capital stock (which generates the high employment equilibrium) if the real user cost of capital is low. This is because, if it does this, the capital-labour ratio will be above its threshold, meaning that the profit share will take its lower value. Nevertheless, if the real user cost of capital is sufficiently low, it is clear that the firm will indeed choose a high capital stock. This is because the direct loss from deliberately choosing a lower capital stock just to maintain a high profit share must eventually outweigh any potential gain. To see this, consider what happens in the limit as the real user cost of capital approaches zero. In this scenario, the firm can earn infinite profits by choosing an infinite capital stock. It is clearly not going to choose a finite capital stock just to increase its profit share.

From all of this, we can therefore see how if the capital stock is endogenous in our model, equilibrium employment (and hence unemployment) will depend on the real user cost of capital (and therefore on the real interest rate) over a certain range. Specifically, if the real interest rate is high, equilibrium unemployment is likely to be high. By contrast, if the real interest rate is low, equilibrium unemployment is likely to be low.[14] Moreover, if investment (and hence the aggregate capital stock) is affected by factors other than the real user cost of capital (e.g. by taxes, the level of current and expected future profitability, the level of demand, corporate governance structure, or the ease of access to credit), then these factors will also have an impact on long-run equilibrium unemployment. The policy implications of all this are fairly obvious and will be discussed in our conclusion.

5. The Equilibrium Employment Rate Over Time

In this section, we consider the evolution of the equilibrium employment rate over time in our model. In particular, we adapt an argument developed by Rowthorn (1999, pp. 421-423) to show how our model can be made consistent with the stylised fact, mentioned by many authors (e.g. Bean (1989); LNJ), that the (un)employment rate is untrended in the very long-run while the capital-labour ratio has grown steadily since the Industrial Revolution.

As our model currently stands, there is no mechanism which generates a constantly growing capital-labour ratio over time. Moreover, we cannot simply postulate that the capital-labour ratio grows at some exogenous rate, since the model would then imply that the economy would always eventually end up with a sufficiently high capital-labour ratio to ensure that the high aggregate equilibrium employment rate results.

However, suppose that all technical progress is labour-augmenting. (Our results would not be affected if we used the weaker assumption that technical progress has a labour-augmenting bias – see Rowthorn (1999)). In this case, to maintain the status quo of our model (i.e. to keep the relative threshold unchanged), the capital-labour ratio must grow at the same rate as labour productivity. Intuitively, this is because if labour productivity increases, each unit of labour "requires" a greater level of capital to work on for the returns to labour not to fall.

To see this more formally, let us adapt the threshold capacity constraint (2.2) to be:

[14] Note that changes in the real interest rate may partially be driven by changes in both output and inflation volatility. This then raises the interesting point that changes in volatility could potentially affect equilibrium unemployment. In other words, the long-run steady state could be influenced by the degree of short-term volatility, thus breaking down the dichotomy between the short-run and the long-run which is sometimes assumed in macroeconomic models.

$$b > 0 \text{ for } \frac{K_i}{\Lambda_N N_i} < C \text{ ("full capacity")}$$
$$b = 0 \text{ for } \frac{K_i}{\Lambda_N N_i} > C \text{ ("spare capacity")} \quad (5.1)$$

where Λ_N is an index of the productive efficiency of labour (assumed to be the same across all firms). Labour-augmenting technical progress is indicated by an increase in Λ_N. The rate of growth of Λ_N is assumed to be exogenous.

With this new condition, the employment level at which the threshold is crossed is:

$$N_i = \frac{K_i}{\Lambda_N C} \quad (5.2)$$

It is clear that if Λ_N and (K_i/N_i) grow at the same rate, then the model we have developed will apply in every time period (i.e. the status quo of our model will always apply). By contrast, if (K_i/N_i) grows more slowly than Λ_N, then the threshold will be hit at lower and lower levels of employment as time progresses and it will become increasingly hard to sustain the high aggregate equilibrium employment rate.

Now suppose that it is indeed the case that Λ_N and (K_i/N_i) grow at the same rate on the trend equilibrium path. This assumption has been justified by Rowthorn (1999), but may also be justified by appealing to the Solow growth model (consider what happens on the balanced growth path). In this case, only above trend or below trend increases in the capital-labour ratio can affect the equilibrium rate of employment, with above trend increases having a positive impact and below trend increases having a negative impact. Therefore, our model is able to simultaneously generate a constantly growing capital-labour ratio and an untrended (un)employment rate.

Moreover, any policy change which results in a shift in the real user cost of capital, *R*, will affect the absolute level of the capital-labour ratio and will therefore (over a certain range) have a one-off (i.e. level) effect on employment. However, it will have no effect on the long-run growth rate of the capital-labour ratio since this is determined by the rate of growth of Λ_N. Therefore, we can see that in our model, it is really the real user cost of capital (and hence the real interest rate) which determines the equilibrium employment rate (though the channel is through a one-off change in the level of the capital stock). Since the real interest rate is untrended over time, our model therefore does not succumb to the argument of Bean (1989), LNJ and others that the (un)employment rate cannot be related to trended variables in the very long-run. Finally, we should also note that the other determinants of investment, which we cited above as potentially being able to affect the equilibrium unemployment rate, are all untrended over time as well.

6. Conclusion

6.1 Summary of the Paper and its Main Results

The main objective of this paper was to theoretically investigate the relationship between the capital stock and the equilibrium rate of unemployment. Our approach involved using a new production function, referred to as the KC production function. This was designed to incorporate meaningful capacity effects not captured by either the Cobb-Douglas or CES production functions. We introduced the KC production function into an otherwise standard imperfectly competitive macroeconomic model of unemployment. Solving our short-run model, we showed that equilibrium unemployment depends on the level of the capital stock over a certain range. We also explained intuitively how endogenising the capital stock in our model implies that it is the real user cost of capital which affects equilibrium unemployment instead. Finally, we showed how our model could be adapted to make it consistent with the stylised fact that the unemployment rate is untrended in the very long-run while the capital-labour ratio has grown steadily since the Industrial Revolution.

6.2 Possible Extensions

In this paper, we assumed that all firms were identical. However, with heterogeneous firms, it is possible for some firms to be operating under "spare capacity" while other firms are operating under "full capacity". Considering the implications of this on aggregate outcomes could be a possible extension. However, intuitively it does not seem that our broad conclusions would be affected: even with heterogeneous firms, provided that the real user cost of capital were sufficiently low (high), we would still expect most firms to be operating under “spare capacity” (“full capacity”), thus continuing to generate the high (low) aggregate equilibrium employment rate.

We also only searched for the equilibrium solutions for employment and other variables. Developing the model in a dynamic context to consider what might happen during the transition to equilibrium following a shock is clearly another possible extension.

For example, we could assume both that adjustment to a new equilibrium capital stock following a change in the real interest rate takes time rather than being instantaneous and that capital can be scrapped more quickly than it can be accumulated. By similar mechanisms to those analysed in the capital scrapping persistence literature cited and discussed in section 1.2.2.1, introducing these assumptions could generate the potential for persistence in unemployment following a rise in the real interest rate which is then reversed.

Perhaps a more interesting alternative would be to assume, in the spirit of Keynes, that the investment rate (and hence the level of the aggregate capital stock) is influenced by the level of aggregate demand via its impact on current and hence expected future profitability. This then opens up the possibility of very interesting hysteresis mechanism. Suppose that equilibrium employment is initially at its low level and there is a positive demand shock (it does not matter whether this shock is intentional or unintentional). In the standard imperfect competition framework, this will cause inflation to rise. This is unsustainable in the long-run and policies to reduce demand would presumably eventually be introduced. However, if higher demand induces greater investment, then the resulting increased aggregate capital stock may cause the equilibrium level of employment to rise. This effect could potentially address the problem of rising inflation, since employment would no longer be above its (new) equilibrium level. If so, the economy will be left in a new equilibrium with a higher level of both capital and employment. This hysteresis mechanism, which is clearly symmetric, illustrates how it might be possible for changes in demand to permanently affect equilibrium unemployment in our model.

Obviously, the idea presented above is merely a conjecture. Formal analysis of the problem would probably be quite complicated since we would need to consider the adjustment processes of both the capital stock and inflation. Nevertheless, using a similar framework, Sawyer (2002) has shown that it is definitely a theoretical possibility. Having said this, in the context of our model, it is probable that the above mechanism would only apply under certain circumstances. Firstly (for the positive hysteresis channel), the economy would probably need to have a very low capital stock to start with. Secondly, the capital stock would need to increase before inflation picked up significantly. This is probably more likely to happen during gradual demand expansions as these do not cause inflation to rise so quickly. Perhaps more critically, it is also probably more likely to occur if the expansion is export-led or investment-led. Indeed, with an investment-led expansion, the economy may get into a virtuous cycle, benefiting from a two way linkage between investment and demand. By contrast, a consumption-led expansion is much more likely to run into high inflation (and, in an open economy, balance of payments problems) before extra capacity comes on stream. Finally, this mechanism is more likely to be successful if there are increasing returns to scale over some range. If this is the case, investment which increases the scale of operations may boost productivity through Verdoorn effects. This may help to lower the costs of firms, possibly reducing their incentive to raise prices following the demand expansion and hence reducing the possibility of inflation.

6.3 Policy Implications

The policy implications of our results are clear. They are fairly similar to those discussed by Rowthorn (1995, 1999). In particular, to tackle unemployment, promoting investment may often be a superior alternative to pursuing labour market reforms. The case for this is made even stronger when we consider that high levels of investment are generally seen as beneficial to the economy as a whole, while some labour market reforms are associated with adverse effects on other aspects of welfare.

There is much debate concerning how best to encourage investment (see, for example, the discussion in Bond and Jenkinson, 2000). However, possible policies include making the tax regime more favourable for investment, encouraging savings, and trying to encourage equity market investors to have longer time horizons (e.g. by reducing capital gains tax on long-term equity holdings). Meanwhile, if there is scope for positive hysteresis along the lines discussed above, then gradual demand expansions (especially if they are export-led or investment-led) may also be an effective way of increasing investment. Finally, it has been argued (e.g. McKibbin and Vines, 2000) that increases in real interest rates were caused by the large fiscal deficits and restrictive monetary policy associated with both “Reaganomics” and German reunification. Since high real interest rates are bad for investment, this suggests that it might be beneficial for countries to pursue the reverse of this combination of policies: namely a reduction in fiscal deficits coupled with an expansionary monetary policy.[15] Obviously, this is quite a sweeping statement which neglects both the probable desirability of expansionary fiscal policy during recessions and the fact that restrictive monetary policy may sometimes be necessary to counter inflation. Nevertheless, it sheds interesting light on the large tax cuts recently made by the Bush administration in the United States: the associated fiscal deficits could cause world real interest rates to rise, something which may possibly have adverse effects on equilibrium unemployment rates. It also suggests that the current hawkish attitude of the European Central Bank coupled with the rising fiscal deficits in many Eurozone countries may mean that the outlook for employment in parts of Europe is not particularly good.

[15] Solow (2000) also proposes this policy mix for tackling unemployment in some European countries.

Appendix A

We wish to show how (3.13) is derived. From (3.12):

$$N_i^{a'} = \left[a' \left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{q}} A_i^{\frac{(q-1)}{q}} \left(\frac{1}{C}\right)^{\frac{b(q-1)}{q}} K_i^{\frac{q-1-qa'}{q}} \left(\frac{W_i}{P}\right)^{-1}\right]^{\frac{a'}{1-a'}} \tag{A.1}$$

Substituting (3.12) and (A.1) into (3.3) and dropping the arguments of $D(P,\, \mathbf{X})$ gives:

$$\Pi_i = \left(\frac{D}{F}\right)^{\frac{1}{q}} A_i^{\frac{(q-1)}{q}} \left(\frac{1}{C}\right)^{\frac{b(q-1)}{q}} K_i^{\frac{q-1-qa'}{q}} (a')^{\frac{a'}{(1-a')}} \left(\frac{D}{F}\right)^{\frac{a'}{q(1-a')}} A_i^{\frac{a'(q-1)}{q(1-a')}} \left(\frac{1}{C}\right)^{\frac{a'b(q-1)}{q(1-a')}} K_i^{\frac{a'(q-1-qa')}{q(1-a')}} \left(\frac{W_i}{P}\right)^{\frac{-a'}{(1-a')}}$$

$$-\left(\frac{W_i}{P}\right)(a')^{\frac{1}{(1-a')}} \left(\frac{D}{F}\right)^{\frac{1}{q(1-a')}} A_i^{\frac{(q-1)}{q(1-a')}} \left(\frac{1}{C}\right)^{\frac{b(q-1)}{q(1-a')}} K_i^{\frac{q-1-qa'}{q(1-a')}} \left(\frac{W_i}{P}\right)^{\frac{-1}{(1-a')}} - RK_i$$

$$= (a')^{\frac{a'}{1-a'}} \left(\frac{D}{F}\right)^{\frac{1}{q(1-a')}} A_i^{\frac{q-1}{q(1-a')}} \left(\frac{1}{C}\right)^{\frac{b(q-1)}{q(1-a')}} K_i^{\frac{q-1-qa'}{q(1-a')}} \left(\frac{W_i}{P}\right)^{\frac{-a'}{1-a'}}$$

$$-(a')^{\frac{1}{1-a'}} \left(\frac{D}{F}\right)^{\frac{1}{q(1-a')}} A_i^{\frac{q-1}{q(1-a')}} \left(\frac{1}{C}\right)^{\frac{b(q-1)}{q(1-a')}} K_i^{\frac{q-1-qa'}{q(1-a')}} \left(\frac{W_i}{P}\right)^{\frac{-a'}{1-a'}} - RK_i$$

Therefore, as required:

$$\Pi_i + RK_i = (1-a')\left[(a')^{a'} \left(\frac{D}{F}\right)^{\frac{1}{q}} A_i^{\frac{q-1}{q}} \left(\frac{1}{C}\right)^{\frac{b(q-1)}{q}} K_i^{\frac{q-1-qa'}{q}} \left(\frac{W_i}{P}\right)^{-a'}\right]^{\frac{1}{1-a'}} \tag{3.13}$$

Appendix B

Relative Slopes of the Labour Demand Curves

We wish to justify the relative slopes of the labour demand curves in Figure 2. These curves are sketched using the log version of (3.12) for the cases $b = 0$ and $b > 0$:

$$n_i = \frac{1}{(1-a')}\left[\ln(a') + \frac{1}{q}(d-f) + \frac{(q-1)}{q}a_i - \frac{b(q-1)}{q}c + \frac{(q-1-qa')}{q}k_i - (w_i - p)\right] \tag{B.1}$$

Differentiating (B.1) with respect to w_i and then inverting gives:

$$\frac{dw_i}{dn_i} = -(1 - a') \tag{B.2}$$

Since a', defined by

$$a' = \frac{(a-b)(q-1)}{q} \tag{3.4}$$

is less than one (recall that $0 \le b < a < 1$ and $q > 1$), both curves will be downward sloping. Moreover, substituting (3.4) into (B.2) gives:

$$\frac{dw_i}{dn_i} = -\left[1 - \frac{a(q-1)}{q} + \frac{b(q-1)}{q}\right] \tag{B.3}$$

From (B.3), we can clearly see that the $b > 0$ labour demand curve will be more steeply downward sloping than the $b = 0$ labour demand curve. This is as depicted in Figure 2.

Intersection of the Labour Demand Curves

We wish to show that the point of intersection (X) of the $b = 0$ and $b > 0$ labour demand curves will always occur at a level of employment below the threshold capacity constraint. We start by noting that the inverse labour demand curve is given by:

$$\left(\frac{W_i}{P}\right) = a'\left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{q}} A_i^{\frac{(q-1)}{q}} \left(\frac{1}{C}\right)^{\frac{b(q-1)}{q}} K_i^{\frac{q-1-qa'}{q}} N_i^{a'-1} \tag{3.11}$$

which may be rewritten as:

$$\left(\frac{W_i}{P}\right) = \left[\frac{(a-b)(q-1)}{q}\right]\left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{q}} A_i^{\frac{(q-1)}{q}} \left(\frac{1}{C}\right)^{\frac{b(q-1)}{q}} K_i^{\frac{(1-a+b)(q-1)}{q}} N_i^{\frac{(a-b)(q-1)}{q}-1} \tag{B.4}$$

If $b > 0$, the inverse labour demand curve is simply (B.4). Meanwhile, when $b = 0$, the inverse labour demand curve is given by setting $b = 0$ in (B.4):

$$\left(\frac{W_i}{P}\right) = \frac{a(q-1)}{q}\left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{q}} A_i^{\frac{(q-1)}{q}} K_i^{\frac{(1-a)(q-1)}{q}} N_i^{\frac{a(q-1)}{q}-1} \tag{B.5}$$

If we solve (B.4) and (B.5) as a pair of simultaneous equations, we will obtain the point of intersection of the $b = 0$ and $b > 0$ labour demand curves. Therefore, we set the right-hand sides of these two equations equal to each other. Doing this, cancelling several terms and simplifying gives:

$$\left(a-b\right)\left(\frac{1}{C}\right)^{\frac{b(q-1)}{q}} K_i^{\frac{(1-a+b)(q-1)}{q}} N_i^{X\,\frac{(a-b)(q-1)}{q}-1} = a\, K_i^{\frac{(1-a)(q-1)}{q}} N_i^{X\,\frac{a(q-1)}{q}-1}$$

$$\left(a-b\right)\left(\frac{1}{C}\right)^{\frac{b(q-1)}{q}} K_i^{\frac{b(q-1)}{q}} N_i^{X\,\frac{-b(q-1)}{q}} = a$$

$$\left(\frac{K_i}{N_i^X C}\right)^{\frac{b(q-1)}{q}} = \frac{a}{a-b}$$

$$N_i^X = \left(\frac{K_i}{C}\right)\left(\frac{a-b}{a}\right)^{\frac{q}{b(q-1)}} \tag{B.6}$$

In (B.6), N_i^X is the level of employment at the intersection of the two curves. Since $0 \le b < a < 1$, we have $\left[\left(a-b\right)/a\right] < 1$. Therefore:

$$N_i^X < \frac{K_i}{C} \tag{B.7}$$

This establishes the result.

Appendix C

Derivations Associated with Section 3.3.1

We first wish to show how (3.15) is derived. From the main text, the first order condition is:

$$l\left[g\,\frac{d\ln N_i}{d\ln W_i} + \left(1-g\right)\frac{d\ln\left[\left(W_i/P\right)-\overline{V}\right]}{d\ln W_i}\right] + \left(1-l\right)\frac{d\ln\left(\Pi_i + RK_i\right)}{d\ln W_i} = 0 \tag{3.14}$$

At interior solutions, log labour demand is given by (B.1) and real operating profits are given by (3.13). Taking logs of (3.13), we get:

$$\begin{aligned}\ln\left(\Pi_i + RK_i\right) = \ln\left(1-a'\right) + \frac{1}{1-a'}\Bigg[& a'\ln\left(a'\right) + \frac{1}{q}\left(d-f\right) + \frac{q-1}{q}a_i \\ & - \frac{b\left(q-1\right)}{q}c + \frac{q-1-qa'}{q}k_i - a'\left(w_i - p\right)\Bigg]\end{aligned} \tag{C.1}$$

From (B.1):

$$\frac{d\ln N_i}{d\ln W_i} = \frac{dn_i}{dw_i} = \frac{-1}{1-a'} \tag{C.2}$$

From (C.1):

$$\frac{d\ln\left(\Pi_i + RK_i\right)}{d\ln W_i} = \frac{d\ln\left(\Pi_i + RK_i\right)}{dw_i} = \frac{-a'}{1-a'} \qquad \text{(C.3)}$$

To calculate, $\frac{d\ln\left[\left(W_i/P\right)-\overline{V}\right]}{d\ln W_i}$, we set:

$$s = \ln W_i \Rightarrow W_i = e^s \qquad \text{(C.4)}$$

Now, using the chain rule:

$$\frac{d\ln\left[\left(W_i/P\right)-\overline{V}\right]}{d\ln W_i} = \frac{d\ln\left[\left(W_i/P\right)-\overline{V}\right]}{ds}\frac{ds}{d\ln W_i} = \frac{d\ln\left[\left(W_i/P\right)-\overline{V}\right]}{ds} \qquad \text{(C.5)}$$

since $\frac{ds}{d\ln W_i} = \frac{d\ln W_i}{d\ln W_i} = 1$.

Therefore, using (C.4) and (C.5), we can see that:

$$\frac{d\ln\left[\left(W_i/P\right)-\overline{V}\right]}{d\ln W_i} = \frac{d\ln\left[\left(e^s/P\right)-\overline{V}\right]}{ds} = \frac{\left(e^s/P\right)}{\left[\left(e^s/P\right)-\overline{V}\right]} \qquad \text{(C.6)}$$

Substituting (C.4) into (C.6), we get:

$$\frac{d\ln\left[\left(W_i/P\right)-\overline{V}\right]}{d\ln W_i} = \frac{\left(W_i/P\right)}{\left[\left(W_i/P\right)-\overline{V}\right]} \qquad \text{(C.7)}$$

We now return to the first order condition. Substituting (C.2), (C.3) and (C.7) into (3.14), we get:

$$l\left[\frac{-g}{1-a'} + \frac{\left(1-g\right)\left(W_i/P\right)}{\left[\left(W_i/P\right)-\overline{V}\right]}\right] + \left(1-l\right)\left(\frac{-a'}{1-a'}\right) = 0 \qquad \text{(C.8)}$$

Rearranging to make the real wage the subject of (C.8) gives:

$$\frac{(1-g)(W_i/P)}{\left[(W_i/P)-\overline{V}\right]}=\frac{g}{1-a'}+\frac{(1-l)}{l}\left(\frac{a'}{1-a'}\right)$$

$$\frac{(W_i/P)}{(W_i/P)-\overline{V}}=\frac{lg+(1-l)a'}{l(1-a')(1-g)}$$

$$1-\frac{\overline{V}}{(W_i/P)}=\frac{l(1-a')(1-g)}{lg+(1-l)a'}$$

$$\frac{\overline{V}}{(W_i/P)}=\frac{lg+(1-l)a'-l(1-a')(1-g)}{lg+(1-l)a'}$$

$$\frac{W_i}{P}=\frac{lg+(1-l)a'}{lg+(1-l)a'+-(1-a')\,l\,(1-g)}\overline{V}$$

$$\frac{W_i}{P}=y\overline{V} \tag{3.15}$$

where:

$$y=\frac{lg+(1-l)a'}{lg+(1-l)a'-(1-a')\,l\,(1-g)} \tag{3.16}$$

which is what we have in the main text.

We also wish to determine how the value of y depends on a' and therefore on b. From the main text, we have:

$$a'=\frac{(a-b)(q-1)}{q} \tag{3.4}$$

Differentiating (3.16) with respect to a' using the quotient rule gives:

$$\frac{dy}{da'}=\frac{\left[lg+(1-l)a'-(1-a')\,l\,(1-g)\right](1-l)-\left[lg+(1-l)a'\right]\left[(1-l)+l(1-g)\right]}{\left[lg+(1-l)a'-(1-a')\,l\,(1-g)\right]^2}$$

$$=\frac{\left[-(1-a')\,l\,(1-g)\right](1-l)-\left[lg+(1-l)a'\right]\left[l(1-g)\right]}{\left[lg+(1-l)a'-(1-a')\,l\,(1-g)\right]^2}$$

$$=\frac{a'\,l\,(1-g)(1-l)-l(1-g)(1-l)-l^2g(1-g)-a'\,l\,(1-g)(1-l)}{\left[lg+(1-l)a'-(1-a')\,l\,(1-g)\right]^2}$$

$$=\frac{-(1-g)\left[l(1-l)+l^2g\right]}{\left[lg+(1-l)a'-(1-a')\,l\,(1-g)\right]^2}$$

Since g and l are both constrained to lie between zero and one, this implies that:

$$\frac{dy}{da'} < 0 \quad \text{(C.9)}$$

Moreover, since $q > 1$, it is clear from (3.4) that:

$$\frac{da'}{db} = -\frac{(q-1)}{q} < 0 \quad \text{(C.10)}$$

Putting (C.9) and (C.10) together gives:

$$\frac{dy}{db} > 0 \quad \text{(C.11)}$$

Therefore y depends positively on b. In other words, the higher the value of b, the higher the mark-up over the alternative wage.

Derivations Associated with Section 3.3.2

We wish to show how (3.22) is derived. Setting $\frac{d \ln \Omega}{d \ln W_i} = 0$ gives the same first order condition as above:

$$l\left[g\frac{d \ln N_i}{d \ln W_i} + (1-g)\frac{d \ln\left[(W_i/P) - \bar{V}\right]}{d \ln W_i}\right] + (1-l)\frac{d \ln(\Pi_i + RK_i)}{d \ln W_i} = 0 \quad \text{(3.14)}$$

At the corner solution, employment is independent of the real wage and is given by (3.9). Therefore:

$$\frac{d \ln N_i}{d \ln W_i} = 0 \quad \text{(C.12)}$$

Real profits at the corner solution are given by (3.10):

$$\Pi_i = \left(\frac{D(P, \mathbf{X})}{F}\right)^{\frac{1}{q}} A_i^{\frac{(q-1)}{q}} K_i^{\frac{(q-1)}{q}} \left(\frac{1}{C}\right)^{\frac{a(q-1)}{q}} - \left(\frac{W_i}{P}\right)\left(\frac{K_i}{C}\right) - RK_i \quad \text{(3.10)}$$

Rearranging and taking logs:

$$\ln(\Pi_i + RK_i) = \ln\left[\left(\frac{D(P, \mathbf{X})}{F}\right)^{\frac{1}{q}} (A_i K_i)^{\frac{(q-1)}{q}} \left(\frac{1}{C}\right)^{\frac{a(q-1)}{q}} - \left(\frac{W_i}{P}\right)\left(\frac{K_i}{C}\right)\right] \quad \text{(C.13)}$$

Differentiating (C.13) with respect to ln W_i gives:

$$\frac{d\ln\left(\Pi_i + RK_i\right)}{d\ln W_i} = \frac{d\ln\left[\left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{q}}\left(A_iK_i\right)^{\frac{(q-1)}{q}}\left(\frac{1}{C}\right)^{\frac{a(q-1)}{q}} - \left(\frac{W_i}{P}\right)\left(\frac{K_i}{C}\right)\right]}{d\ln W_i}$$

We use the substitutions in (C.4) to proceed:

$$\frac{d\ln\left(\Pi_i + RK_i\right)}{d\ln W_i} = \frac{d\ln\left[\left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{q}}\left(A_iK_i\right)^{\frac{(q-1)}{q}}\left(\frac{1}{C}\right)^{\frac{a(q-1)}{q}} - \left(\frac{e^s}{P}\right)\left(\frac{K_i}{C}\right)\right]}{ds}$$

$$= \frac{-\left(\frac{e^s}{P}\right)\left(\frac{K_i}{C}\right)}{\left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{q}}\left(A_iK_i\right)^{\frac{(q-1)}{q}}\left(\frac{1}{C}\right)^{\frac{a(q-1)}{q}} - \left(\frac{e^s}{P}\right)\left(\frac{K_i}{C}\right)}$$

$$= \frac{-\left(\frac{W_i}{P}\right)\left(\frac{K_i}{C}\right)}{\left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{q}}\left(A_iK_i\right)^{\frac{(q-1)}{q}}\left(\frac{1}{C}\right)^{\frac{a(q-1)}{q}} - \left(\frac{W_i}{P}\right)\left(\frac{K_i}{C}\right)} \qquad \text{(C.14)}$$

Finally, as for the interior solutions:

$$\frac{d\ln\left[(W_i/P) - \overline{V}\right]}{d\ln W_i} = \frac{(W_i/P)}{\left[(W_i/P) - \overline{V}\right]} \qquad \text{(C.7)}$$

We can now return to the first order condition. Substituting (C.12), (C.14) and (C.7) into (3.14), we get:

$$\frac{l(1-g)(W_i/P)}{\left[(W_i/P) - \overline{V}\right]} - \frac{(1-l)\left(\frac{W_i}{P}\right)\left(\frac{K_i}{C}\right)}{\left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{q}}\left(A_iK_i\right)^{\frac{(q-1)}{q}}\left(\frac{1}{C}\right)^{\frac{a(q-1)}{q}} - \left(\frac{W_i}{P}\right)\left(\frac{K_i}{C}\right)} = 0$$

$$(W_i/P)\left[\frac{l(1-g)}{\left[(W_i/P) - \overline{V}\right]} - \frac{(1-l)\left(\frac{K_i}{C}\right)}{\left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{q}}\left(A_iK_i\right)^{\frac{(q-1)}{q}}\left(\frac{1}{C}\right)^{\frac{a(q-1)}{q}} - \left(\frac{W_i}{P}\right)\left(\frac{K_i}{C}\right)}\right] = 0$$

$$\Rightarrow \lambda(1-\gamma)\left[\left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{\theta}}(A_iK_i)^{\frac{(\theta-1)}{\theta}}\left(\frac{1}{C}\right)^{\frac{\alpha(\theta-1)}{\theta}}-\left(\frac{W_i}{P}\right)\left(\frac{K_i}{C}\right)\right]-(1-\lambda)\left(\frac{K_i}{C}\right)\left(\frac{W_i}{P}-\overline{V}\right)=0$$

Rearranging:

$$\left(\frac{W_i}{P}\right)\left(\frac{K_i}{C}\right)\left[\lambda(1-\gamma)+(1-\lambda)\right]=\lambda(1-\gamma)\left[\left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{\theta}}(A_iK_i)^{\frac{(\theta-1)}{\theta}}\left(\frac{1}{C}\right)^{\frac{\alpha(\theta-1)}{\theta}}\right]+(1-\lambda)\left(\frac{K_i}{C}\right)\overline{V}$$

$$\left(\frac{W_i}{P}\right)(1-\lambda\gamma)=\lambda(1-\gamma)\left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{\theta}}A_i^{\frac{(\theta-1)}{\theta}}K_i^{-\frac{1}{\theta}}\left(\frac{1}{C}\right)^{\frac{\alpha(\theta-1)}{\theta}-1}+(1-\lambda)\overline{V}$$

Therefore, as required:

$$\left(\frac{W_i}{P}\right)=\frac{\lambda(1-\gamma)\left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{\theta}}A_i^{\frac{(\theta-1)}{\theta}}K_i^{-\frac{1}{\theta}}\left(\frac{1}{C}\right)^{\frac{\alpha(\theta-1)}{\theta}-1}+(1-\lambda)\overline{V}}{1-\lambda\gamma} \qquad (3.22)$$

Appendix D

To show that both (3.31) and (3.32) are possible, we adapt Figure 4 by removing the example case (i) and case (ii) thresholds and adding on two more example case (iii) thresholds to obtain Figure 5 below:

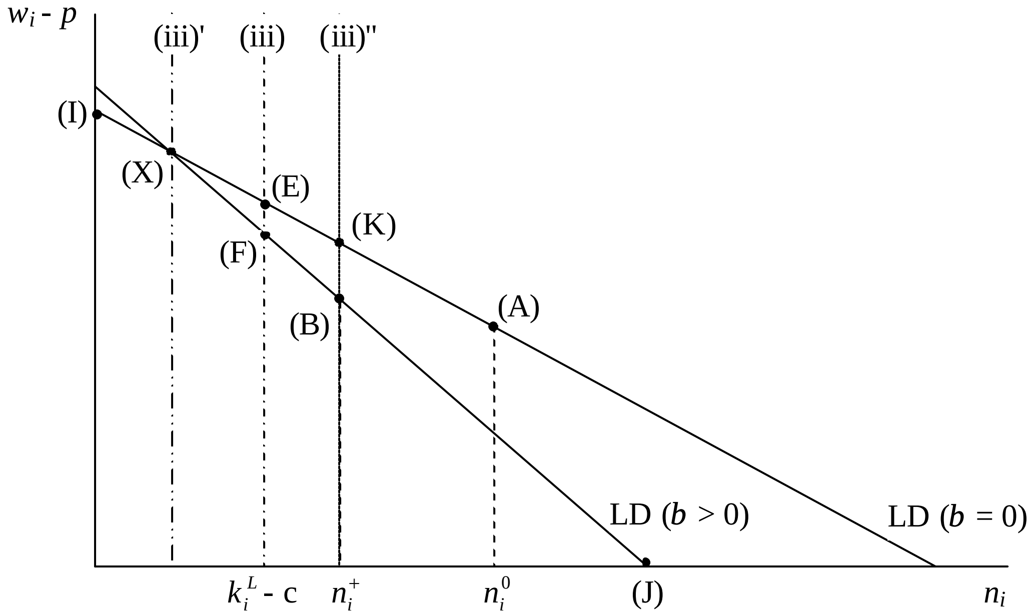

Figure 5

As discussed previously, with the original example case (iii) threshold, the choice is between (B) and (E). Now consider the two new example case (iii) thresholds: (iii)' and (iii)". These are at the extremes of possible case (iii) thresholds: the threshold cannot be to the left of (iii)' because this would violate the fact, proved in Appendix B, that the threshold is always to the right of the intersection at (X); the threshold cannot be to the right of (iii)" because we would then be in case (ii). As for all case (iii) thresholds, the choice at the start of period t is between the wage generating the feasible interior solution at (B) and the wage generating the relevant corner solution. For the (iii)' threshold, the corner solution is at (X). However, by a similar revealed preference argument to the one used in the main text for case (i) thresholds, we know that (B) must be preferred to (X). Therefore, in this situation, the interior solution will be chosen. Meanwhile, for the (iii)" threshold, the relevant corner solution is at (K). In this situation, provided condition (3.27) is satisfied, (K) will be chosen. This follows directly from the argument in section 3.3.2 where we showed that, given employment is at the threshold (which it must be if the threshold is (iii)"), the solution will be given by the intersection of the threshold and the $b = 0$ labour demand curve. Overall then, we can see that within case (iii), there exist some thresholds (e.g. (iii)') which generate the interior solution at (B) and other thresholds (e.g. (iii)") which generate the arbitrary corner solution (E). Therefore, we can see that depending on the level of the exogenous capital stock, sometimes (3.31) is satisfied and sometimes (3.32) is satisfied.

Appendix E

We wish to show how (3.35) is derived. The relationships in (3.33) may be substituted into (3.11) to obtain:

$$\left(\frac{W}{P}\right) = a'\left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{q}} A^{\frac{(q-1)}{q}} \left(\frac{1}{C}\right)^{\frac{b(q-1)}{q}} \left(\frac{K}{F}\right)^{\frac{(1-a+b)(q-1)}{q}} \left(\frac{N}{F}\right)^{\frac{(a-b)(q-1)-q}{q}} \tag{E.1}$$

Aggregating (2.1) gives:

$$\frac{Y}{F} = A\left(\frac{1}{C}\right)^{b} \left(\frac{N}{F}\right)^{a-b} \left(\frac{K}{F}\right)^{1-a+b} \tag{E.2}$$

while aggregating (2.3) gives:

$$\frac{Y}{F} = \frac{D(P,\mathbf{X})}{F} \tag{E.3}$$

From (E.2) and (E.3), we have:

$$\frac{D(P,\mathbf{X})}{F} = A\left(\frac{1}{C}\right)^{b}\left(\frac{N}{F}\right)^{a-b}\left(\frac{K}{F}\right)^{1-a+b}$$

$$\left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{q}} = \left[A\left(\frac{1}{C}\right)^{b}\left(\frac{N}{F}\right)^{a-b}\left(\frac{K}{F}\right)^{1-a+b}\right]^{\frac{1}{q}} \tag{E.4}$$

which may be substituted into (E.1) to get:

$$\left(\frac{W}{P}\right) = a'\left[A\left(\frac{1}{C}\right)^{b}\left(\frac{N}{F}\right)^{a-b}\left(\frac{K}{F}\right)^{1-a+b}\right]^{\frac{1}{q}} A^{\frac{(q-1)}{q}}\left(\frac{1}{C}\right)^{\frac{b(q-1)}{q}}\left(\frac{K}{F}\right)^{\frac{(1-a+b)(q-1)}{q}}\left(\frac{N}{F}\right)^{\frac{(a-b)(q-1)-q}{q}}$$

$$= a'A\left(\frac{1}{C}\right)^{b}\left(\frac{N}{F}\right)^{a-b-1}\left(\frac{K}{F}\right)^{1-a+b}$$

Therefore, using (3.4):

$$\left(\frac{W}{P}\right) = A\left(\frac{1}{C}\right)^{b}\left(\frac{K}{N}\right)^{1-a+b}\left[\frac{(a-b)(q-1)}{q}\right] \tag{E.5}$$

Inverting (E.5) gives the aggregate labour demand curve at interior solutions for both $b = 0$ and $b > 0$:

$$\left(\frac{N}{K}\right) = \left[A\left(\frac{1}{C}\right)^{b}\right]^{\frac{1}{1-a+b}}\left[\frac{qW}{(a-b)(q-1)P}\right]^{\frac{-1}{1-a+b}} \tag{3.35}$$

This is what we have in the main text.

To derive (3.48), we use (3.34) and (E.4) to eliminate $D(P, \mathbf{X})$ from (3.47):

$$\left(\frac{W}{P}\right) = \left[\frac{a(q-1)}{q}\right]\left[A\left(\frac{1}{C}\right)^{b}\left(\frac{N}{F}\right)^{a-b}\left(\frac{K}{F}\right)^{1-a+b}\right]^{\frac{1}{q}} A^{\frac{q-1}{q}}\left(\frac{K}{F}\right)^{\frac{-1}{q}}\left(\frac{1}{C}\right)^{\frac{a(q-1)}{q}-1}$$

$$= \left[\frac{a(q-1)}{q}\right]\left[A\left(\frac{1}{C}\right)^{b}\left(\frac{K}{FC}\right)^{a-b}\left(\frac{K}{F}\right)^{1-a+b}\right]^{\frac{1}{q}} A^{\frac{q-1}{q}}\left(\frac{K}{F}\right)^{\frac{-1}{q}}\left(\frac{1}{C}\right)^{\frac{a(q-1)}{q}-1}$$

$$= \left[\frac{a(q-1)}{q}\right]\left[A\left(\frac{1}{C}\right)^{a}\left(\frac{K}{F}\right)\right]^{\frac{1}{q}} A^{\frac{q-1}{q}}\left(\frac{K}{F}\right)^{\frac{-1}{q}}\left(\frac{1}{C}\right)^{\frac{a(q-1)}{q}-1}$$

$$= \left[\frac{a(q-1)}{q}\right]A\left(\frac{1}{C}\right)^{a-1} \tag{3.48}$$

which is what we have in the main text.

References

Alexiou, C. and Pitelis, C. (2003). “On Capital Shortages and European Unemployment: A Panel Data Investigation.” *Journal of Post-Keynesian Economics*, 25(4), 613-631.

Arestis, P. and Biefang-Frisancho Mariscal, I. (2000). “Capital Stock, Unemployment and Wages in the UK and Germany.” *Scottish Journal of Political Economy*, 47(5), 487-503.

Ball, L. (1999). “Aggregate Demand and Long-Run Unemployment.” *Brookings Papers on Economic Activity*, (2), 189-251.

Bean, C. R. (1989). “Capital Shortages and Persistent Unemployment.” *Economic Policy*, 8, 12-53.

Bean, C. R. (1994). “European Unemployment: A Survey.” *Journal of Economic Literature*, 32 (June), 573-619.

Binmore, K., Rubinstein, A. and Wolinsky, A. (1986). “The Nash Bargaining Solution in Economic Modeling.” *RAND Journal of Economics*, 17(2), 176-188.

Blanchard, O. and Jimeno, J. F. (1995). “Structural Unemployment: Spain versus Portugal.” *American Economic Review*, 85(2), 212-18.

Blanchard, O. and Kiyotaki, N. (1987). “Monopolistic Competition and the Effects of Aggregate Demand.” *American Economic Review*, 77(4), 647-666.

Blanchard, O. and Wolfers, J. (2000). “The Role of Shocks and Institutions in the Rise of European Unemployment: The Aggregate Evidence.” *Economic Journal*, 110 (March), C1-C33.

Bond, S. R. and Jenkinson, T. J. (2000). "Investment Performance and Policy." In Jenkinson, T. J. (ed.), *Readings in Macroeconomics*, Second Edition. Oxford University Press, Oxford.

Carlin, W. and Soskice, D. (1990). *Macroeconomics and the Wage Bargain: A Modern Approach to Employment, Inflation and the Exchange Rate.* Oxford University Press, Oxford.

Fitoussi, J.-P., and Phelps, E. S. (1988). *The Slump in Europe*. Basil Blackwell, Oxford.

Glyn, A. (2002). "Labour Market Success and Labour Market Reform: Lessons from Ireland and New Zealand." Unpublished paper, Oxford University. Available at http://www.economics.ox.ac.uk/Members/andrew.glyn/IrelandandNZ4.pdf

Hahn, F. (1995). "Theoretical Reflections on the 'Natural Rate of Unemployment'." In Cross, R. (ed.), *The Natural Rate of Unemployment: Reflections on 25 Years of the Hypothesis*. Cambridge University Press, Cambridge.

Kapadia, S. (2003). "Investment, Capacity Constraints, Hysteresis and Long-Run Unemployment." M.Phil. Thesis, University of Oxford. Available at http://www.economics.ox.ac.uk/students/sujit.kapadia/mphilthesis.pdf

Kitson, M. and Michie, J. (1996). "Britain's Industrial Performance since 1960: Underinvestment and Relative Decline." *Economic Journal*, 106 (January), 196-212.

Layard, R., Nickell, S. J. and Jackman, R. (1991). *Unemployment: Macroeconomic Performance and the Labour Market*. Oxford University Press, Oxford.

Malinvaud, E. (1980). *Profitability and Unemployment*. Cambridge University Press, Cambridge.

Manning, A. (1990). "Imperfect Competition, Multiple Equilibria and Unemployment Policy." *Economic Journal*, 100 (Conference Papers), 151-162.

Manning, A. (1992). "Multiple Equilibria in the British Labour Market." *European Economic Review*, 36, 1333-1365.

McKibbin, W. J. and Vines, D. (2000). "Modelling Reality: The Need for both Inter-Temporal Optimization and Stickiness in Models for Policy-Making." *Oxford Review of Economic Policy*, 16(4), 106-137.

Miaouli, N. (2001). "Employment and Capital Accumulation in Unionised Labour Markets: Evidence from five South-European Countries." *International Review of Applied Economics*, 15(1), 5-29.

Nickell, S. J. (1997). "Unemployment and Labor Market Rigidities: Europe versus North America." *Journal of Economic Perspectives*, 11(3), 55-74.

Nickell, S. J. and Layard, R. (1999). "Labor Market Institutions and Economic Performance." In Ashenfelter, O. C. and Card, D. (eds.), *Handbook of Labor Economics*, vol. 3C. North Holland, Amsterdam.

Nickell, S. J., Nunziata, L. and Ochel, W. (2002). "Unemployment in the OECD since the 1960s. What do we know?" Unpublished paper, Bank of England. Available at http://www.nuff.ox.ac.uk/users/nunziata/publications_files/nicknun02.pdf

OECD (1994). *The OECD Jobs Study*. OECD, Paris.

Oswald, A. J. (1982). "The Microeconomic Theory of the Trade Union." *Economic Journal*, 92 (September), 576-595.

Oswald, A. J. (1993). "Efficient Contracts are on the Labour Demand Curve: Theory and Facts." *Labour Economics*, 1, 85-113.

Phelps, E. S. (1994). *Structural Slumps: The Modern Equilibrium Theory of Unemployment, Interest and Assets*. Harvard University Press, Cambridge MA.

Rowthorn, R. E. (1977). “Conflict, Inflation and Money.” *Cambridge Journal of Economics*, 1, 215-39.

Rowthorn, R. E. (1995). “Capital Formation and Unemployment.” *Oxford Review of Economic Policy*, 11(1), 26-39.

Rowthorn, R. E. (1999). “Unemployment, Wage-Bargaining and Capital-Labour Substitution.” *Cambridge Journal of Economics*, 23, 413-425.

Sawyer, M. (2002). “The NAIRU, Aggregate Demand and Investment.” *Metroeconomica*, 53(1), 66-94.

Siebert, H. (1997). “Labor Market Rigidities: At the Root of Unemployment in Europe.” *Journal of Economic Perspectives*, 11(3), 37-54.

Solow, R. M. (2000). “Unemployment in the United States and in Europe – A Contrast and the Reasons.” *Ifo Studien*, 46, 1-12.

Soskice, D. and Carlin, W. (1989). “Medium-Run Keynesianism: Hysteresis and Capital Scrapping.” In Davidson, P. and Kregel, J. (eds.), *Macroeconomic Problems and Policies of Income Distribution*. Edward Elgar, Aldershot.

Stiglitz, J. E. (2002). *Globalization and its Discontents*. Allen Lane, London.