Saving and Growth with Habit Formation : A Comment

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Carroll, Overland, and Weil (2000) showed that habits can explain why increases in expected future income growth induce consumers to raise their saving rate. The habit formation mechanism they studied follows Ryder and Heal (1973) in assuming that habits are defined in terms of consumption levels. I show that if habits are defined as the present value of past felicity, rather than as the present value of past consumption, the relation between growth and saving is weakened. The reason is that diminishing returns in the felicity of consumption weakens the relation between consumption growth and habit growth. If habits are defined as past felicity, rather than as past consumption, increases in the growth rate of consumption induce smaller increases in the growth rate of habits, and therefore weakens the effect of habit formation on saving.

Keywords : habit formation; relative consumption; growth.
JEL Classification : E21; D91; O40.

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1 Introduction

A growing body of evidence suggests that past experience plays an important role in conditioning choice by forward-looking rational decision makers. In a recent article, Carroll, Overland, and Weil (2000) show that if consumers maximize an intertemporal utility function defined partly over current consumption and partly over a "habit stock" of past consumption, it follows that, for reasonable parameter values, increases in growth cause increases in saving, a prediction that conforms well with the evidence they survey. It is one purpose of this comment to investigate whether these findings are robust to changes in the definition of "habits".

Surprisingly perhaps, the standard theory of habit formation has been developed independently of the theory of well-being, with habits defined in terms of past decisions (consumption units) and not in terms of past experiences (felicity units), and this despite abundant references to the "living standard" and related concepts. However, recent research shows that experienced utility is both measurable and empirically distinct from decision utility[1]. Another purpose of this comment, then, is to analyze a model of consumption decision where habit formation is based on past experiences (welfare), rather than on past decisions (choice).

First, I show that the sign of the relation between growth and saving strongly rests on an assumption of linearity in the habit formation mechanism. Introducing diminishing returns in habit formation weakens the relation between growth and saving, while introducing increasing returns strengthens it. Secondly, I modify the benchmark model, defining habits in terms of felicity, rather than in terms of consumption. Using a calibrated version of the modified model, I show that the balanced growth path of the original model can be replicated by appropriately selecting the strength of habits in the utility function, with some quantitative differences in the short-

[1]See, e.g. Kahneman, Wakker, and Sarin (1997).

run dynamics.

This note is organised as follows. I introduce the model equations, suggest an alternative habit formation mechanism, and reassess the effect of growth and saving in an otherwise identical model.

2 The Benchmark Model

Consider an intertemporal utility function :

$$\int_0^\infty e^{-\theta t}\, U(c_t, h_t) ds, \tag{1}$$

a habit formation mechanism :

$$\dot{h}_t = \rho(c_t - h_t), \tag{2}$$

a budget constraint :

$$\dot{k}_t = (A - \delta)k_t - c_t. \tag{3}$$

The optimal control program consists in maximizing (1) subject to (2) and (3), where c_t is a control variable, and h_t and k_t are two state variables; where c_t is the flow of consumption; k_t is the capital stock, with k_0 given and $\lim_{t\to\infty}(k_t e^{-rt}) = 0$; where h_t is the reference index to which consumption is compared, with h_0 given; θ is the discount rate applied to the future; ρ is the discount rate applied to the past; A is a productivity parameter and δ is the rate of depreciation of the capital stock. The value of *intertemporal utility* is defined in (1) as the discounted value of the entire path of instantaneous utility, where U is the *felicity* function[2], $U(c, h) = (1 - \sigma)^{-1}(c/h^\gamma)^{1-\sigma}$, σ

[2] This felicity function satisfies : $U_c > 0$, $U_{cc} < 0$, $U_h \leq 0$, $U_{hh} \leq 0$ if $\sigma(\gamma - 1) \geq 1$ and $U_{hh} > 0$ if $\sigma(\gamma - 1) < 1$, $U_{ch} \geq 0$, $U_{cc}U_{hh} - (U_{ch})^2 \leq 0$. It is convenient to introduce the parameter $\psi = \gamma(\sigma - 1)$ and to consider $U(c, h) = u(c)h^\psi$, where $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, with $\psi \geq 1$, which satisfies the restriction. Note, incidentally, that this felicity function, introduced by Abel (1990) and used by many authors since, is *not* concave, for $\sigma > 1$ and $\psi \geq 1$. More on this in the appendix.

> 1. Carroll, Overland, and Weil (2000) focused on the case where the maximization internalizes the effects of current consumption on future habits. Carroll, Overland, and Weil (1997) also considered the case where habits are an externality in preferences. In both cases, increases in the growth rate lead to increases in the saving rate, for reasonable parameter values. For simplicity I focus throughout on the case where habits are an externality. In this case, the Euler equation associated with the optimal consumption path can be written :

$$\sigma \, \dot{c}_t / c_t = A - \delta - \theta + \gamma(\sigma - 1) \, \dot{h}_t / h_t. \tag{4}$$

In steady state, the growth rate of consumption is equal to $g = \dot{c}_t / c_t = \dot{h}_t / h_t$ $= (A - \delta - \theta)/\Delta$, where $\Delta = \gamma + (1 - \gamma)\sigma$, and the saving rate is equal to s $= (g + \delta)/A$, or :

$$s = \frac{1 + g/\delta}{1 + \theta/\delta + \Delta g/\delta}. \tag{5}$$

Both the numerator and the denominator are increasing in g. The effect of growth on saving can be computed from (5) by differentiating s with respect to g :

$$\frac{ds}{dg} = \frac{1 + \theta/\delta - \Delta}{\delta(1 + \theta/\delta + g\Delta/\delta)^2}. \tag{6}$$

As can be seen in (6) the sign and magnitude of $\frac{ds}{dg}$ depend on the difference between $1 + \theta/\delta$ and Δ, where $\Delta = \gamma + (1 - \gamma)\sigma$ can be interpreted as the inverse of the infinite-horizon elasticity of intertemporal substitution (the long-run response of consumption growth to a permanent change in the rate of interest)[3]. The point emphasised by Carroll, Overland, and Weil (2000) is that Δ can be low for reasonable values of σ as long as γ is not too close to zero (as long as habit formation is strong enough). The intuition is that

[3]In addition, $\frac{d^2 s}{dg^2} < 0$ as $\frac{ds}{dg} > 0$, and vice versa.

habit formation raises the effective elasticity of intertemporal substitution. This may explain why increases in growth induce increases in saving[4].

3 An Alternative Theory of Habit Formation

3.1 Implications of Diminishing Returns in Habit Formation

The preferences described by (1) and (2) originate in Ryder and Heal (1973). The habit stock is defined as the discounted value of past *consumption* levels. This means that a unit increase in present consumption raises the habit stock by one unit while it raises intertemporal utility by the value of felicity. According to (2), if an individual consumes one million units of c_t, at date t, the one millionth unit raises the habit stock as much as the first unit. This is a strong property of the habit formation mechanism. It would seem equally plausible to assume that habits are subject to diminishing marginal returns with respect to increases in past consumption. As it turns out, introducing diminishing returns with respect to increases in past consumption has important implications for the relation between growth and saving. To see why, consider replacing equation (2) by

$$\dot{h}_t = \rho\left(c_t^{\alpha} - h_t\right). \tag{7}$$

Consider a steady state in which $\dot{c}_t/c_t = g$. It follows from (7) that, in steady state, $\dot{h}_t/h_t = \alpha\ \dot{c}_t/c_t$. For a given growth rate of consumption, the presence of diminishing returns in the habit formation mechanism ($\alpha < 1$) reduces the implied growth rate of the habit stock, while increasing returns ($\alpha > 1$) have the opposite effect. With the simple mechanism of equation (7), γ is now in effect replaced by $\alpha\gamma$ in the Euler equation (4). Thus, to maintain the same value of Δ, low values of α (strong diminishing returns) require

[4]A trivial point to note here is that the assumption $U_h \leq 0$ is essential. The reader can check that if the felicity function is instead $U(c,h) = (1-\sigma)^{-1}(ch^{\gamma})^{1-\sigma}$, so that $U_h \geq 0$, habit formation *reduces* the effective elasticity of intertemporal substitution

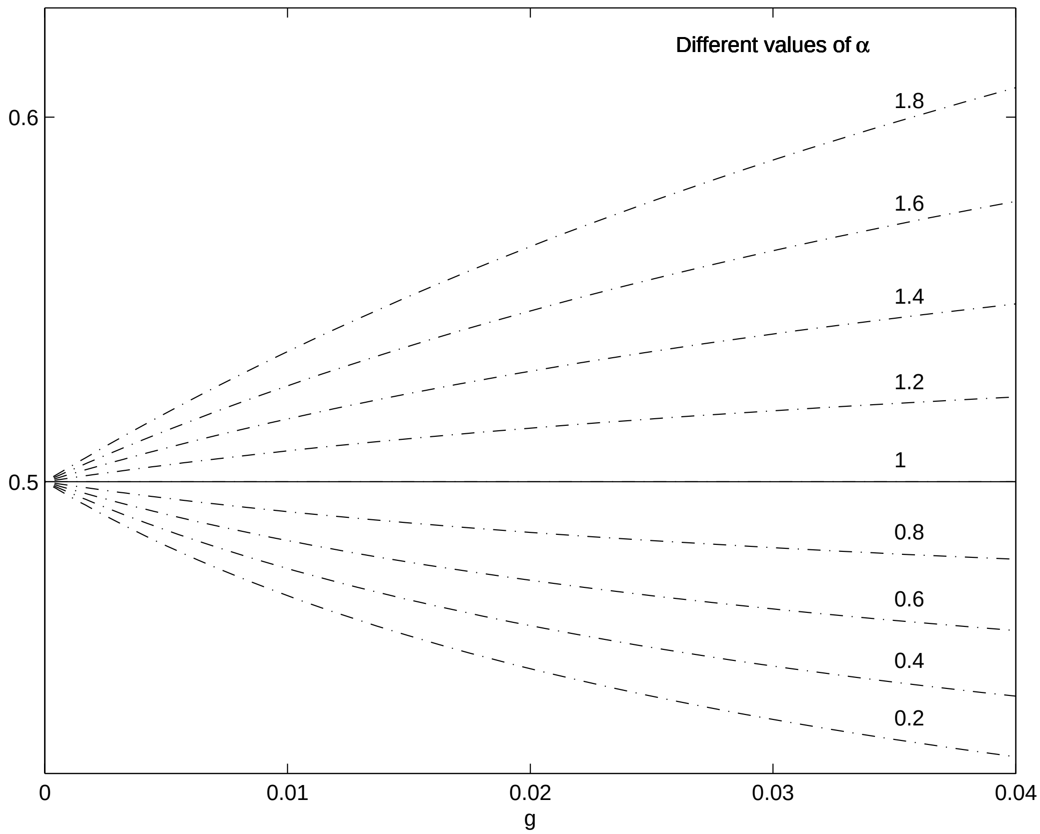


Fig. 1: **Diminishing Versus Increasing Returns in Habit Formation**
The plots display the long-run relation between growth and saving for different values of α. The growth rate g is on the horizontal axis, and the saving rate s is on the vertical axis. The continuous line corresponds to the benchmark case where habit formation is subject to constant returns (Ryder and Heal, 1973; Carroll, Overland, and Weil, 1997 and 2000). Benchmark simulation : $\rho = 0.2$, $\theta = \delta = 0.05$, $\sigma = 3$, $\gamma = 1/2$. The relation turns from positive to negative for $\alpha < 1$.

high values of γ (strong habits in preferences). Values of α greater than 1 reinforce an existing positive relation, while values of α less than 1 weaken it. The following condition is sufficient to turn the relation from positive to negative :

$$\alpha < \frac{\sigma - 1 - \theta/\delta}{\gamma(\sigma - 1)}. \tag{8}$$

Figure 1 displays the long-run relation between growth and saving for different values of α, keeping γ constant, for benchmark parameter values. On a balanced growth path, reducing the degree of returns in the habit formation mechanism is akin to reducing the importance of habits in the felicity function (reducing α is akin to reducing γ).

In what follows I show that the kind of diminishing returns present in equation (7) follow from a plausible definition of habit formation. I then re-assess the relation between growth and saving.

3.2 From Decision-Based to Experience-Based Habit Formation

An alternative to (2) is to define a habit stock in terms of past *felicity* levels. The felicity function U defines a mapping from an effective consumption level to an experienced felicity level. According to this alternative definition, developing a consumption habit means that past experienced felicity levels are "remembered" (consciously or not) as felicity levels, rather than as consumption levels. Let h_t denote the habit stock defined in units of consumption, and let z_t denote the habit stock defined in units of felicity. I suggest the following habit formation mechanism :

$$\dot{z}_t = \rho \left(U(c_t, z_t) - z_t \right). \tag{9}$$

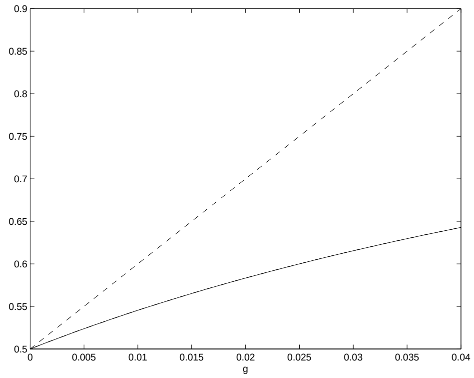
It is convenient to introduce the felicity function $U(c, z) = u(c)(-z)^{-\psi}$, where $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, with $\sigma > 1$ and $\psi \geq 0$, implying $u(c) < 0$ and $z < 0$. The presence of $-z$ and of $-\psi$ ensures that the felicity function has standard curvature properties[5]. There are several other ways in which the habit formation could be defined, yet (9) is a natural alternative to (2). If habits are defined as the present value of past felicity, the relation between saving and growth predicted by the linear growth model is significantly weakened, as the next section shows.

4 Revisiting Saving and Growth

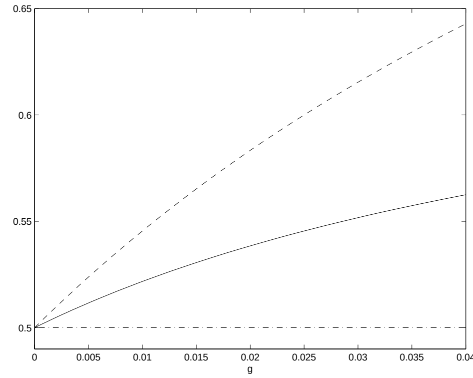
The modified optimal control program consists in maximizing (1) subject to (3) and (9). The only change is in the habit formation mechanism. While in (2) the habit formation mechanism is defined in terms of consumption levels, in (9) it is defined in terms of felicity levels. In steady state, the growth rate of consumption is $g = (A - \delta - \theta)/\Delta$, the saving rate is $s = (g + \delta)/A$, and the effect of growth on saving is again given by equation (6). It is straightforward to find that $\Delta^c = \sigma - \psi$ if habits are defined in terms of consumption and the felicity function is $U(c, h) = u(c)h^{\psi}$ (the superscript c in Δ^c stands for "consumption"), and $\Delta^f = (\sigma + \psi)/(1 + \psi)$ if habits are defined in terms of felicity and the felicity function is $U(c, z) = u(c)(-z)^{-\psi}$ (the superscript f in Δ^f stands for "felicity").

Figure 2 summarizes some numerical results with the benchmark parameter values used in Carroll, Overland, and Weil (2000). Three models are compared : the benchmark model with no habits, the model with "consumption habits", and the model with "felicity habits". Figure 2 depicts the relation between growth and saving for different values of σ and a given value of ψ, that is for a fixed weight of the reference index in preferences.

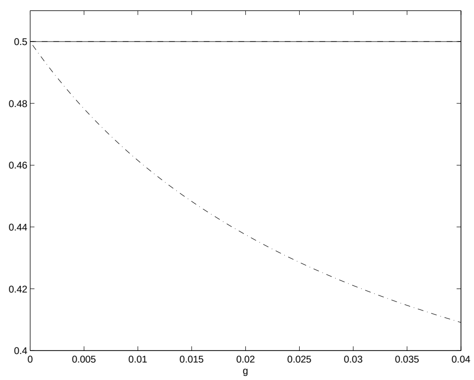
[5]This felicity function satisfies : $U_c > 0$, $U_{cc} < 0$, $U_z \leq 0$, $U_{zz} \leq 0$, $U_{cz} \geq 0$, $U_{cc}U_{zz} - (U_{cz})^2 \geq 0$. This felicity function is concave for any $\sigma > 1$ and $\psi \geq 0$. More on this in the appendix. Affine transformations of the felicity function $U(c, z)$ can be accommodated by appropriately redefining the initial value of habits z_0. However, for simplicity, we assume $U(c, z) < 0$, for all (c, z), implying that the case $\sigma = 1$ does *not*, here, tend to logarithmic felicity.



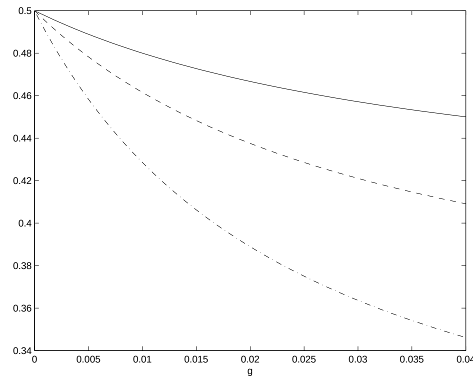
(a) $\sigma = 1$



(b) $\sigma = 2$



(c) $\sigma = 3$



(d) $\sigma = 4$

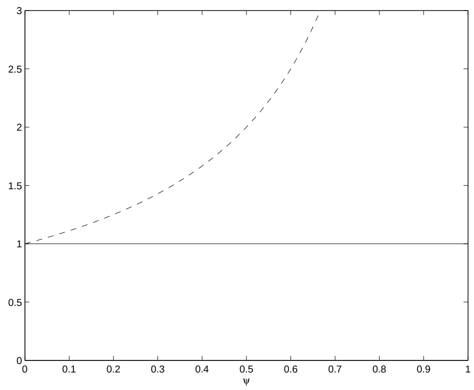
Fig. 2: **The Relation Between Growth and Saving** The plots display the long-run relation between growth and saving in three different models. The horizontal axis has the growth rate g, and the vertical axis has the saving rate s. The dashed-and-dotted line corresponds to the benchmark case of no habits. The dashed line corresponds to the case where habits h are defined in units of consumption, with $U(c,h) = u(c)h^{\psi}$. The continuous line corresponds to the case where habits z are defined in units of felicity, with $U(c,z) = u(c)(-z)^{-\psi}$, and $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$. The relation between growth and saving is given by $s = (g+\delta)/(\delta+\theta+g\Delta)$, with $\Delta^c = \sigma - \psi$ in the first case, and $\Delta^f = (\sigma+\psi)/(1+\psi)$ in the second case. Benchmark simulation : $\rho = 0.2$, $\theta = \delta = 0.05$, $\psi = 1$. With $\theta/\delta = 1$ and $\psi = 1$, the highest value of σ for which there is a positive relation is 3. (the value of γ follows from $\gamma = \psi/(\sigma-1)$) – similar results obtain if γ, rather than ψ, is fixed

The value chosen is $\psi = 1$, implying[6][7] that $\Delta^c = \Delta^f$ for $\sigma = 3$. For low values of σ, the relation between growth and saving is positive for all three models. The model with no habits (dashed-and-dotted line) predicts the weakest positive relation and the strongest negative relation, while the model with consumption habits (dashed line) predicts the strongest positive relation, and the model with felicity habits (continuous line) predicts the weakest negative relation. For low values of σ the model with felicity habits (continuous line) is closer to the model with no habits than to the model with consumption habits (in fact, with $\psi = \sigma = 1$ the model with felicity habits and the model with no habits predict the same relation between growth and saving, and this is why the two lines coincide in quadrant (a) of Figure 2). For higher values of σ, the positive relation weakens and eventually turns negative. This happens first for the model with no habits. There is a range of values of σ such that the two models with habits predict a positive relation while the model with no habits predicts a negative relation (with the benchmark parameter values used here this is the case with $\sigma \in [2, 3]$). This confirms that habit formation can help explain a positive relation between growth and saving for parameter values that would otherwise yield a negative relation. As σ is raised further, all three models predict a negative relation. However, the model with felicity habits predicts a weaker negative relation than either the model with consumption habits or the model with no habits.

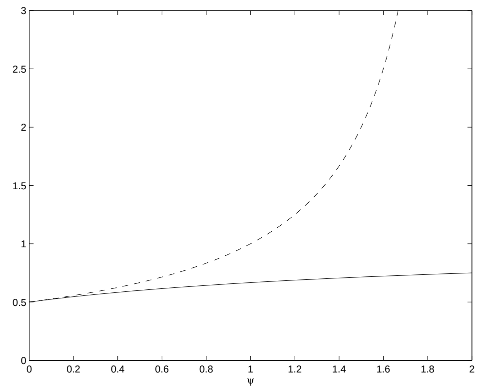
The models with habit formation – whether consumption habits or felicity habits – yield a positive relation between growth and saving for parameter values that would otherwise yield a negative relation. For a given importance of habits in preferences (the same value of ψ and γ fixed in both models), the relation between growth and saving is weaker in the model with felicity habits than in the model with consumption habits – a weaker positive relation for

[6] $\sigma = 3$ is the solution of $\sigma - \psi = (\sigma + \psi)/(1 + \psi)$ for $\psi = 1$.

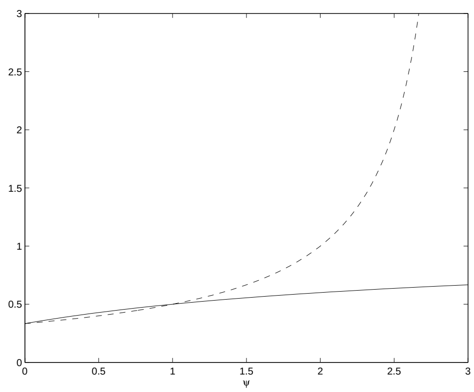
[7] The same type of results would obtain if instead γ was fixed, say $\gamma = 1$. If Δ is fixed, both models with habit formation obviously predict the same relation between growth and saving.



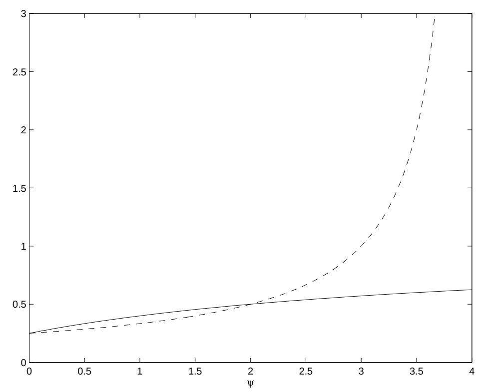
(a) $\sigma = 1$



(b) $\sigma = 2$



(c) $\sigma = 3$



(d) $\sigma = 4$

Fig. 3: **Elasticity of Intertemporal Substitution (Δ^{-1})** The plots display the elasticity of intertemporal substitution, Δ^{-1} (vertical axis), against the weight of the habit stock in the felicity function, ψ (horizontal axis), where $\psi \in [0, \sigma]$. The dashed line corresponds to the case where habits h are defined in units of consumption, with $U(c, h)$ $= u(c)h^{\psi}$. The continuous line corresponds to the case where habits z are defined in units of felicity, with $U(c, z) = u(c)(-z)^{-\psi}$, with in both cases $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$. Note that with consumption habits ψ satisfies some restrictions, (i) $\psi < \sigma$ (otherwise $\Delta^{-1} < 0$); (ii) $\psi \geq 1$ implies $U_{hh} \leq 0$. The key point is that in economies with strong habits (high ψ), the elasticity is smaller in the case of felicity habits than in the case of consumption habits : $(\Delta^f)^{-1} < (\Delta^c)^{-1}$. Benchmark simulation : $\rho = 0.2$, $\theta = \delta = 0.05$; A chosen so that $g =$ 2%.

lower values of σ; and a weaker negative relation for higher values of σ. The reason for these differences between the two specifications of habits can be traced to the infinite-horizon elasticity of intertemporal substitution Δ^{-1}. Equation (6) shows that higher values of Δ (lower values of the elasticity Δ^{-1}) tilt the balance in favour of a negative relation between growth and saving. The presence of Δ (squared) in the denominator of equation (6) explains why higher values of Δ weaken the relation between growth and saving, whether the relation is positive or negative.

Figure 3 plots the elasticity Δ^{-1}, for both specifications of habits, for different values of ψ and σ. For a given value of σ, the elasticity Δ^{-1} rises as the importance of habits ψ rises; for higher values of σ, the effect of habit formation ψ on the elasticity Δ^{-1} is weaker (as expected). The point here is that the increase in the value of the elasticity is less steep with felicity habits than with consumption habits. In particular, in economies with strong habits (high ψ) the elasticity is lower in the case of felicity habits. This may be seen by studying the limiting behaviour of the elasticity Δ^{-1} as ψ is raised to its highest admissible value. In the case of consumption habits, ψ is bounded above by σ, and $\lim_{\psi\to\sigma}(\Delta^c)^{-1} = \infty$. In the case of felicity habits, ψ is unbounded above, and $\lim_{\psi\to\infty}(\Delta^f)^{-1} = 1$. In the limit, therefore, $(\Delta^f)^{-1} < (\Delta^c)^{-1}$. The figures clearly show that if habit formation is strong, the elasticity is lower if habits are defined in terms of felicity than if they are defined in terms of consumption. It follows that the long-run relation between growth and saving is weaker if habits are defined in units of felicity rather than in units of consumption.

It is also instructive to compare the transitional dynamics of the models, with consumption habits and with felicity habits. Carroll, Overland, and Weil (2000) show that habit formation can lead to a positive short-run response of saving to an increase in growth even if there is no long-run positive correlation between growth and saving. This is still true with felicity habits. Consider an exogenous destruction of the capital stock. Figure 4 depicts

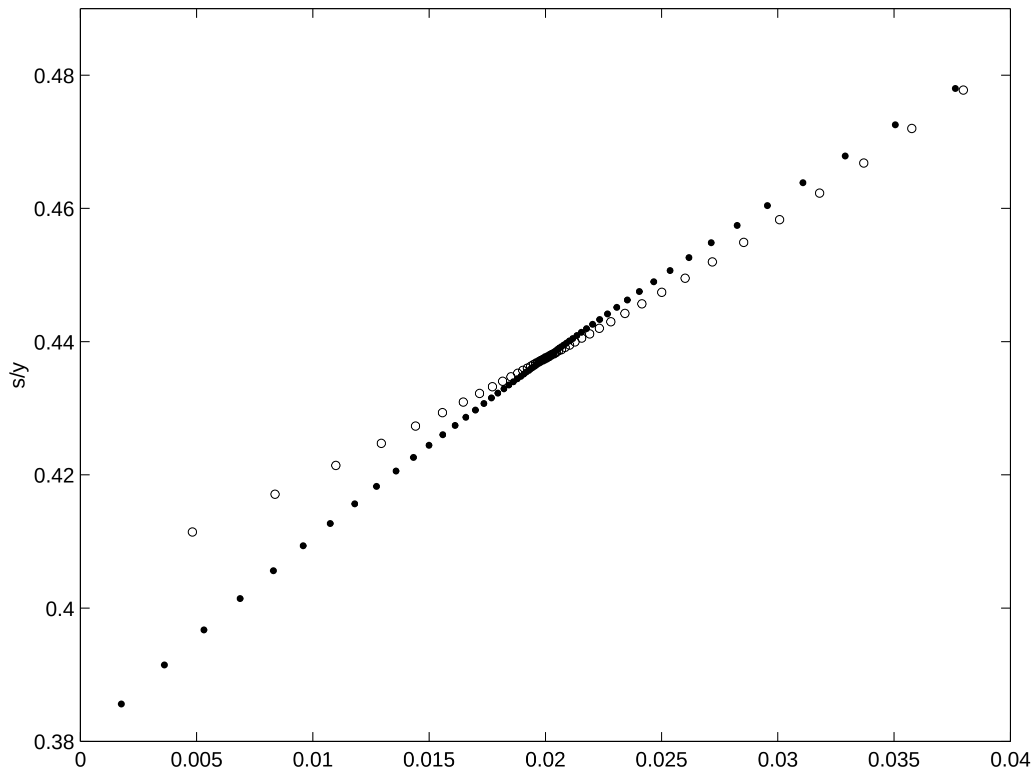


Fig. 4: **Transitional Dynamics** The plot compares the transitional dynamics of the model with consumption habits (black circles) and the model with felicity habits (white circles), following an exogenous destruction of the capital stock. Both models yield a positive short-run relation between the growth rate of consumption and the saving rate. The immediate effect of a destruction in the capital stock is to reduce the saving rate, in order to maintain consumption or felicity (as the case may be) relative to its habitual level. The reduction in the saving rate is higher with consumption habits than with felicity habits. Both models are calibrated on the growth rate and on the saving rate. The parameter values are such that the long-run relation between growth and saving is negative ($ds/dg \simeq -1.95$). Benchmark simulation : $\rho = 0.2$, $\theta = \delta = 0.05$; $\sigma = 7$ and $\psi = 2$, so that $\Delta = 3$; A chosen so that $g = 2\%$.

the transitional dynamics in terms of two variables of interest, consumption growth g_t and the saving rate s_t. The importance of habits in the felicity function is set, in each simulation, so as to keep Δ constant (and equal to 3). The immediate effect of a destruction in the capital stock is to reduce the saving rate. The reduction in the saving rate needed to maintain felicity near its desired level is higher with consumption habits than with felicity habits. The short-run relation between growth and saving is positive in both models, steeper for consumption habits than for felicity habits.

5 Concluding Comments

Carroll, Overland, and Weil (2000) showed that habits can explain why increases in expected future income growth induce consumers to raise their saving rate. The habit formation mechanism they studied follows Ryder and Heal (1973) in assuming that habits are defined in terms of consumption levels. I show that if habits are defined as the present value of past felicity, rather than as the present value of past consumption, the relation between growth and saving is weakened. The reason is that increases in the growth rate of consumption induce smaller increases in the growth rate of habits. This follows from diminishing returns in the felicity of consumption. The effect of diminishing returns on consumption works via two channels. On the one hand, diminishing returns make consumers more willing to postpone consumption in response to an increase in interest rates. This willingness to postpone consumption is weakened by the presence of habits. On the other hand, diminishing returns weaken the effect of consumption on habits. The second channel is absent if habits are measured in units of past consumption rather than in units of past felicity.

While the comments of this paper focus on the relation between growth and saving in an aggregative model, the framework itself is more general and could be applied to a wide range of issues. I have chosen to follow very closely the framework of Carroll, Overland, and Weil (2000). Future studies may find it instructive to depart further. There is no reason why an index of "customary living standard" should not include, for instance, a measure of health, the value of leisure, or the quality of infrastructure. An important empirical question remains. Do habits develop from the ends (felicity) or from the means (consumption) ? A better understanding of the physiology and psychology of habit formation is needed before we can answer this question with any confidence.

Appendix

A Definition of the Habit Stock

Let the habit stock be defined as the discounted value of past felicity :

$$z_t = \rho \int_{-\infty}^{t} e^{\rho(s-t)} U(c_s, z_s) ds, \tag{A-1}$$

with U(c, z) $= u(c)(-z)^{-\psi}$; where $\rho > 0$, $u(c_t) < 0$, $z_t < 0$, and $\psi \geq 0$. Differentiating (A-1) with respect to t yields :

$$\dot{z}_t = \rho(u(c_t)(-z_t)^{-\psi} - z_t) \tag{A-2}$$

The following change of variable $y_t = (-z_s)^{1-\psi}$ in (A-2) yields :

$$\dot{y}_t = \rho(1+\psi)(u(c_t) - y_t) \tag{A-3}$$

Integrating (A-3) with respect to t, and imposing $\lim_{t\to-\infty}(y_t e^{\rho(1+\psi)t}) = 0$, yields :

$$y_t = \rho(1+\psi) \int_{-\infty}^{t} e^{\rho(1+\psi)(s-t)} u(c_s) ds \tag{A-4}$$

Reverting the change of variable in (A-4) yields :

$$z_t = \left(\rho(1+\psi) \int_{-\infty}^{t} e^{\rho(1+\psi)(s-t)} u(c_s) ds\right)^{\frac{1}{1+\psi}} \tag{A-5}$$

Equations (A-1) and (A-5) are equivalent definitions of the stock of felicity habits. Any affine transformation of $U(c, z)$ can be subsumed into the present model by rescaling the initial value of the stock of habits.

Curvature of the Felicity Functions

Let $\sigma > 1$ and $\psi \geq 0$. The felicity function used by Abel (1990) and Carroll, Overland and Weil (1997, 2000) has the following properties :

$$
\begin{array}{lcll}
U & = & c^{1-\sigma}h^{\psi}/(1-\sigma) & < 0 \\
U_c & = & c^{-\sigma}h^{\psi} & > 0 \\
U_h & = & \psi c^{1-\sigma}h^{\psi-1}/(1-\sigma) & \leq 0 \\
U_{cc} & = & -\sigma c^{-\sigma-1}h^{\psi} & < 0 \\
U_{hh} & = & \psi(\psi-1)c^{1-\sigma}h^{\psi-2}/(1-\sigma) & \lesseqqgtr 0 \quad \text{as } \psi \gtreqqless 1 \\
U_{ch} & = & \psi c^{-\sigma}h^{\psi-1} & \geq 0 \\
U_{cc}U_{hh}-(U_{ch})^2 & = & \psi(\sigma-\psi)c^{-2\sigma}h^{2(\psi-1)}/(1-\sigma) & \gtreqqless 0 \quad \text{as } \psi \lesseqqgtr \sigma
\end{array}
$$

The felicity function used in this paper has the following properties :

$$
\begin{array}{lcll}
U & = & c^{1-\sigma}(-z)^{-\psi}/(1-\sigma) & < 0 \\
U_c & = & c^{-\sigma}(-z)^{-\psi} & > 0 \\
U_z & = & \psi c^{1-\sigma}(-z)^{-\psi-1}/(1-\sigma) & \leq 0 \\
U_{cc} & = & -\sigma c^{-\sigma-1}(-z)^{-\psi} & < 0 \\
U_{zz} & = & \psi(\psi+1)c^{1-\sigma}(-z)^{-\psi-2}/(1-\sigma) & \leq 0 \\
U_{cz} & = & \psi c^{-\sigma}(-z)^{-\psi-1} & \geq 0 \\
U_{cc}U_{zz}-(U_{cz})^2 & = & -\psi(\sigma+\psi)c^{-2\sigma}(-z)^{-2(\psi+1)}/(1-\sigma) & \geq 0
\end{array}
$$

The felicity function $U(c,h) = c^{1-\sigma}h^{\psi}/(1-\sigma)$ is *not* jointly concave in (c,h) if $\psi < \sigma$, as assumed. It is concave in both c and h separately only if $\psi \geq 1$. It follows that it is not possible to study the model's behaviour as $\psi \to 0$ without violating concavity in h. The felicity function $U = c^{1-\sigma}(-z)^{-\psi}/(1-\sigma)$, on the other hand, is jointly concave in (c,h) for any positive value of ψ. It is thus possible to study the model's behaviour as $\psi \to 0$.

On page 342, Carroll, Overland and Weil (1997) write a parameter restriction that they claim ensures concavity "in both arguments". First, note that the restriction ensures *partial* concavity with respect to h, but not *global* concavity (the function is *not* jointly concave). Secondly, the restriction they state contains a typo : they write $\sigma \geq (\gamma-1)^{-1}$ whereas in fact the restriction is $\gamma \geq (\sigma-1)^{-1}$ (this follows from $\psi = \gamma(\sigma-1)$ and $\psi \geq 1$). Thirdly, concavity in the objective function is a sufficient – but not necessary – condition for optimality; the condition $U_{cc}U_{zz} - (U_{cz})^2 \geq 0$, which ensures concavity of the objective function, is explicitly stated in Ryder and Heal (1973). Finally, note that Carroll, Overland and Weil (1997) impose $\gamma \in [0,1]$. In the model with consumption habits this restriction naturally follows from $\gamma = (\sigma - \Delta)/(\sigma - 1)$ and $\Delta \in [1, \sigma]$. In the model with felicity habits, by contrast, this restriction need not apply, since $\gamma = (\sigma - \Delta)/(\sigma - 1)(\Delta - 1)$; however, note that $\Delta \in [2, \sigma]$ implies $\gamma \in [0, 1]$.

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