Optimal research in financial markets with heterogeneous private information: a rational expectations model*

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Abstract

This paper investigates prices and endogenous research decision for financial assets. In rational expectations models with public information, higher order beliefs make the investors to overweight the public information relative to underlying fundamentals. The extent of this mispricing is higher if the of private signals is relatively high. The model presented in this paper extends this setting by incorporating the research cost decision and essentially endogenises the variance of private signals of short-lived investors obtain in each period. It turns out that investors will be less willing to research in periods when there is an alternative with high return available. Furthermore, the optimal research decision will depend on the time left until the maturity of asset. This explains in a rational setting why long lived assets like stocks may be traded based on the public information rather than research on fundamentals.

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1 Introduction

This paper aims to address some financial markets imperfections in a rational setting. It incorporates the need for heterogeneous market participants to pay attention to the average expectations of the market in addition to their private information. The model investigates the optimal research effort and its implications on asset prices. In particular, it illustrates the dependence of the research effort on the maturity of assets and the impact of alternative returns.

The view among investors seems to be that they would not necessarily "go against market sentiment" based on their internal fundamental research only. A paper by Menkhoff (1998) presents results of a survey among the participants of the foreign exchange market in Germany. The respondents were asked to evaluate the relative importance of fundamentals, technical analysis and monitoring order flows in their trading decisions. The importance of fundamental research was evaluated to be around 45 per cent. The survey also shows that investors consider psychological influences and importance of opinion leaders relevant to the market.

Allen, Morris and Shin (2004) investigate the impact of average beliefs and existence of public signals in a dynamic rational expectations model with a single risky asset. In their model, short-lived investors have noisy public information and heterogeneous private information about the fundamental value of the asset (that is, the liquidation value at the final period). Taking into account the higher order beliefs (that is, beliefs about other investors beliefs), asset prices will be systematically biased towards the public signal. This is because the public signal still contains information about the fundamental value, so all investors take it into account when forming their expectations. While noise in private signals averages out at the aggregate level, noise in public information does not. This bias will be larger the higher the variance of private signals. Furthermore, the bias will be larger the more time is left until investors receive the liquidation value.

This paper adopts largely the setting of Allen, Morris and Shin (2004). The additions are the existence of an alternative asset and the endogenous research cost decision. Investors can do fundamental research on the true value of a risky asset. There are three things to note. First, no analyst would be capable of knowing the true value of a risky asset without an error, and different investors are heterogeneous in their estimates about the true value. Second, having an individual view about the true value is costly.

A financial market institution must have a research department and they can choose the size and research effort in it. Although it is not explicitly modelled in this paper, the research cost can also be thought as investors losing time by trying to find out more about the true value of asset. Instead they can put their effort on investing on the spot. Last, as argued before, it is not rational for investors to make their investment decision based on their fundamental research only. This is because they need to take into account their expectations about other market participants' expectations. The extent of how much they rely on their fundamental research compared to the public signal depends on the variance of their private signal. Therefore, the optimal research cost effort can be modelled as a choice of optimal variance of the private signal. The paper investigates the determinants of optimal research costs.

One question that this paper aims to address, is how the optimal research cost changes over time and how it will depend on the maturity of the asset. For example, it may be argued that fundamental research, especially on long lasting assets like stocks, has a reputation of being more of a marketing tool than an important determinant of trading decisions in the market. It is interesting to analyse whether it can be rational to spend less resources on fundamental research of such assets. If this is the case then such assets are more likely to be mispriced.

The second question relates to alternative returns. One can expect that if there is a less risky alternative asset available, and if such asset offers a high return, investors may choose to do less research on the risky asset. For example, if we think of the risky asset being a portfolio of assets in emerging markets, extraordinarily good returns in developed countries assets may reduce the incentives to research emerging markets assets. If we think of investors behaviour during the Asian crisis vs. the Turkish or Argentine crises, investors seem to react more heavily on the negative news in the first rather than in the latter, across all emerging markets. Indeed, if we compare the returns in developed countries asset markets during these periods, we see that during the Asian crisis the MSCI Europe and North American equity indices offered average annualised monthly returns of 32.6 and 21.9 per cent respectively (in USD prices). On the contrary, during the Turkish crisis these returns were -14.5 and -10.2 per cent, while during the Argentine crisis they were -4.7 and -5.8 per cent respectively. Another way of looking at it may be comparing the research on stocks depending on the returns of commercial bonds, or research on commercial papers depending on returns of government bonds etc. It is also interesting to analyse how research decisions would depend on permanently or temporarily higher alternative asset returns.

The paper presents a rational expectations model where a continuum of risk averse investors can invest in a risky asset and an alternative asset. The risky asset has a single liquidation value in $T+1$ and is traded in periods 1 to T by short-lived rational agents. Investors are heterogeneous in the trading stage, because they receive different noisy private signals. Investors are homogeneous in their research cost decisions, because these will be made before each trading period and before receiving individual private signals.

The paper is organized as follows. Section 2 reviews the related literature. Section 3 presents the model for the general T period case, which can only be solved numerically. It also analyses the more tractable case of one trading period. Section 4 concludes.

2 Related Literature

Calvo and Mendoza (1999) focus on herding behaviour due to information costs. They analyse portfolio choice with both fixed costs and manager's different marginal costs from over or under-performing the market return. They analyse the incentives to research a particular asset instead of believing a rumour about it. In contrast to the current paper, their assumptions are about asset returns and the model lacks market clearing. They also assume short selling constraints and focus on the negative impact of globalisation. The latter is due to the fact that if investors have more assets available, they have lower incentives to spend resources in order to reduce portfolio variance, by finding out more about one particular asset.

Barberis, Shleifer and Vishny (1998) aim to explain the empirical evidence of under-reaction of stock prices on news about companies' earnings and overreaction on series of good or bad news. They base it on evidence from psychology, and they have a model where the representative investor prices assets incorrectly. Namely, the investor assumes that earnings follow one of two regimes, a trend stationary or a mean reverting process, while earnings are actually a random walk. Therefore this model has a component of irrationality in investors beliefs. Allen, Morris and Shin (2004) explain overreaction of stock prices to noisy public information by heterogeneous private information of investors and higher order beliefs, in a fully rational setting. Their model has a similar setup to this paper, but does not analyse what the investors' incentives to collect private information depend on. Bacchetta and van Wincoop (2004) also analyse the impact of higher order expectations on asset prices. They find that an additional term in standard asset pricing formulation (the higher order wedge) disconnects the price from the present value of future pay-offs. They show that this effect can be quantitatively significant.

The aforementioned models are based on the earlier rational expectations literature. Grossman (1976), Hellwig (1980), Diamond and Verrecchia (1981) present static models and derive the rational equilibrium, when investors can infer information from prices. Grossman (1976) points out how prices must contain a noise component, otherwise they would be fully revealing and, as long as private information gathering has a positive cost, no-one would have an incentive to collect it. Admati (1985) extends these models to a static multiple asset model. Singleton (1986), Grundy and McNichols (1989), He and Wang (1995) extend the rational expectations models into a dynamic context. Compared to Allen, Morris and Shin (2004) these models do not emphasize the bias in asset prices caused by the higher order beliefs.

3 The model

Investors can invest in a risky asset and in an alternative asset. The alternative asset provides a known risk-free return $r_t \geq 1$, on each trading period t. The risky asset has liquidation value θ at date $T+1$, and is traded on periods 1 to T. Investors do not know the liquidation value, but they can obtain information about it from public signals, private signals and prices. The characteristics of these signals will be described later in this section.

The model uses the assumption of overlapping generations of traders. There are two reasons for this. Firstly, it emphasizes the behaviour of investors who care about short-term price movements in addition to the fundamental value of asset. As pointed out by Allen, Morris and Shin (2004), investors would care about short term price movements, if they wanted to smooth their consumption. Furthermore, if we think of an asset management firm, the short horizon seems plausible. Traders working there are professionals having to present the performance of the fund to their clients with some regularity and the short term prices of assets in their portfolio matter. Secondly, this assumption simplifies the model by allowing investors to be homogeneous in their research decision. It has to be pointed out that this assumption excludes investors possible gain from their private fundamental research done in earlier periods. As shown later, the research done in early periods matters, only to the extent that it is revealed in the prices of later periods.

There is a continuum of rational investors normalized in the interval $[0,1]$, endowed with w units of funds. These investors buy assets at date t and consume at date $t+1$. A risk averse investor i, who is trading at date t has the mean-variance utility function

$$U_{i,t} = E(c_{i,t+1}) - \frac{\tau}{2} V(c_{i,t+1}), \tag{1}$$

where the risk aversion is measured by the constant τ.

The budget constraint for investor i trading on period t is

$$c_{i,t+1} = h_{i,t}(P_{t+1} - r_t P_t) + r_t w - r_t \kappa(\beta_{i,t}),$$

where $c_{i,t+1}$ is his consumption at $t+1$, $h_{i,t}$ is his demand for the risky asset and $\kappa(\beta_{i,t})$ is the research cost function, which will be specified later. The price of the risky asset at time t is denoted by P_t. Note also that $P_{T+1} = \theta$.

In addition to rational traders, there are noise traders in each period, modelled as a noisy net supply of the risky asset $s_t \sim \mathcal{N}[0, 1/\delta]$. Supply shocks are assumed to be uncorrelated along time and across both public and private signals of investors in every time period. The market clearing condition is $H_t = s_t$ for every period t, where H_t is the aggregate demand.

There is some initial public information about the risky assets. The liquidation value is drawn from the prior distribution $\theta \sim \mathcal{N}[y, 1/\alpha]$. The normality assumptions allow straightforward updating of investors' beliefs and are standard in the literature.

Investors trading on period t can obtain a costly and noisy private signal about the fundamental value of the asset, $x_{i,t} = \theta + \varepsilon_{i,t}$, $\varepsilon_{i,t} \sim \mathcal{N}[0, 1/\beta_{i,t}]$. It is assumed that $\varepsilon_{i,t}$ is uncorrelated across investors, along time and with the supply shocks and the noise in public signal.

Investors can choose the variance of their private signal $(1/\beta_{i,t})$, if they spend $\kappa(\beta_{i,t})$ on research. The research cost function should capture the idea that having a more precise view about the fundamental value must be more expensive, and knowing the true value with certainty is too expensive. This implies the following conditions: $\kappa'(\beta) > 0$, $\kappa(\beta) \to \infty$ as $\beta \to \infty$ and $\kappa(0) = 0$.

The sequence of events is the following:

1) The fundamental value θ is drawn by nature at date 0.

2) The first generation of investors simultaneously choose their research cost for the risky asset, before receiving the private signals and observing the price. The information set about fundamentals available to each of them is the same: $I_{-1} = \{y\}$.

3) These investors trade and the period 1 market clears. The information available for their trading decision now contains the private signals and the price signal. So, $I_{i,1} = \{y, x_{i,1}, P_1\}$. In the following period, they will receive income corresponding to the price of each asset, consume and die without revealing their private signals to the following generations.

4) The generations 2 to T will make their research and trading decisions in a similar manner. The only difference is that they will obtain information about the fundamental also from historical prices. Hence, the information set available for their research decision is $I_{-t} = \{y, P_1, \ldots, P_{t-1}\}$. Their trading decisions will be based on the information set $I_{i,t} = \{I_{-t}, x_{i,t}, P_t\}$. Apart from the generation trading at T, the consumption of investors will depend on prices at $t+1$ rather than on the fundamental value θ.

5) At $T+1$ the risky asset will be liquidated and generation T will obtain θ.

This setting implies that investors are heterogeneous in their trading decisions, while being homogeneous in their research decision.

3.1 Solving the model

Investors of each generation face a two stage decision problem. The model is solved by first deriving investors' demand and equilibrium prices, taking the research decisions as given. Given the solution from that stage, the optimal research costs can be derived conditional on the available information.

From the utility function (1), we can find the demand for the risky asset of investor i,

$$h_{i,n,t} = \frac{E_i[P_{t+1}|I_{i,t}] - r_t P_t}{\tau V_i(P_{t+1}|I_{i,t})}. \tag{2}$$

3.1.1 Equilibrium prices

Assume that the asset price follows a linear rule on each period, $P_t = \eta_t(\lambda_t y_t + \mu_t \theta - s_t)$. The term y_t is the public signal about the asset at period t. It includes the initial public signal y and the information from prices in trading periods 1 to $t-1$. The coefficients η_t, λ_t, μ_t can be found by the method of undetermined coefficients from the market clearing condition. (See Appendix A for the derivation.)

Rearranging the pricing equation, investors can observe a price signal A_t in period t, such that $A_t \equiv (P_t/\eta_t - \lambda_t y_t)/\mu_t = \theta - s_t/\mu_t$, and $A_t|\theta \sim \mathcal{N}[\theta, \mu_t^{-2}\delta^{-1}]$. The updated distribution of θ conditional on prices will also be normal. It will be the public signal for the investors trading in period $t+1$: $\theta|A_t \sim \mathcal{N}[y_{t+1}, \alpha_{t+1}^{-1}]$. The public signal evolves over time as

$$y_{t+1} = \frac{\alpha_t y_t + \mu_t^2 \delta A_t}{\alpha_{t+1}}, \tag{3}$$

$$\alpha_{t+1} = \alpha_t + \mu_t^2 \delta. \tag{4}$$

The first generation investors cannot observe historical prices, so

$$\alpha_1 = \alpha \text{ and } y_1 = y. \tag{5}$$

As investors trading in period t will also obtain a private signal $x_{i,t}$, they will believe the fundamental to be drawn from

$$\theta|I_{i,t} \sim N\left(\frac{y_t\alpha_t + \mu_t^2\delta A_t + \beta_{i,t}x_{i,t}}{\alpha_t + \mu_t^2\delta + \beta_{i,t}}, \frac{1}{\alpha_t + \mu_t^2\delta + \beta_{i,t}}\right).$$

In equilibrium, all investors of the same generation will choose the same research cost, that is, $\beta_{i,t} = \beta_t$ for all i. This is because, by assumption, investors are identical in their preferences and base their research decision on the same information I_{-t}. Therefore, in a symmetric equilibrium, all investors would have chosen the same optimal research cost, taking everyone else's research decision as given.

Aggregating the demand of all investors averages out the noise in the private signals. However, it does not average out the noise in the public signal.

Proposition 1 *The prices in period t will be determined by coefficients z_t and $z_{s,t}$, so that*

$$P_t = \frac{(1-z_t)y_t + z_t\theta - z_{s,t}s_t}{R_t}, \tag{6}$$

where

$$z_t = \frac{\mu_t^2\delta(\alpha_t + \mu_t^2\delta + \beta_t) + z_{t+1}\beta_t\alpha_t}{(\alpha_t + \mu_t^2\delta + \beta_t)(\alpha_t + \mu_t^2\delta)}, \tag{7}$$

$$\begin{aligned} z_{s,t} &= \frac{\mu_t\delta(\alpha_t + \mu_t^2\delta + (1 - z_{t+1})\beta_t)}{(\alpha_t + \mu_t^2\delta + \beta_t)(\alpha_t + \mu_t^2\delta)} + \\ &\quad + \frac{\tau z_{t+1}^2}{(\alpha_t + \mu_t^2\delta + \beta_t)R_t} + \frac{\tau z_{s,t+1}^2}{\delta R_t}. \end{aligned} \tag{8}$$

$$R_t \equiv \prod_{k=t}^{T+1} r_k$$

This is similar to Allen, Morris and Shin (2004) with the addition of the discount factor. Integrating out the noisy supply, asset prices will be a weighted average of the true value and the public signal for the relevant period. (For a proof of these expressions, see Appendix A.)

On period $T + 1$ the risky asset will be liquidated at the fundamental value. Also, there will be no more opportunities to invest in the risk-free asset. So, the terminal conditions will be

$$z_{T+1} = 1,\ z_{s,T+1} = 0 \text{ and } r_{T+1} = 1. \tag{9}$$

Finally, comparing the pricing equation (6) with the linear pricing rule assumed initially, it is clear that

$$\mu_t = \frac{z_t}{z_{s,t}}. \tag{10}$$

The dynamic system (3)-(5) and (7)-(10) solves for the values of z_t and $z_{s,t}$, given the values of β_1 to β_T.

3.1.2 The research decision

The optimal research cost decision is equivalent to choosing $\beta_{i,t}$ taking other investor's $\beta_{j,t}$, $j \neq i$ as given. Assume the research cost function is $\kappa(\beta_{i,t}) = K\beta_{i,t}^2/4$, where the constant K measures how expensive the research is. Replacing the optimal demand into the utility function, simplifying and taking expectations implies that the optimal research decision solves

$$\max_{\beta_{i,t}} E_i[U_{i,t}|I_{-t}] = \frac{E_i[(E_i[P_{t+1}|I_{i,t}] - r_t P_t)^2 | I_{-t}]}{2\tau V_i(P_{t+1}|I_{i,t})} - \frac{1}{4} r_t K \beta_{i,t}^2.$$

By doing research, investors can gain a reduction in the variance of their estimated next period price. They can also gain a better view about risky return opportunities. Given that investors are allowed to short sell, they can gain equally from either an increase or a decrease in the next period price.

From the pricing equation (6) the variance of the following period price is

$$V_i(P_{t+1}|I_{i,t}) = \frac{1}{R_{t+1}^2}\left(\frac{z_{t+1}^2}{\alpha_t + \mu_t^2\delta + \beta_{i,t}} + \frac{z_{s,t+1}^2}{\delta}\right).$$

It is obvious from the equation above that the variance of expected price should be lower if investor i decides to research a lot ($\beta_{i,t}$ is high) and if the weight on the fundamental value (z_{t+1}) in the next period pricing equation is high.

Investors face additional sources of uncertainty about the expected return of the risky asset at the research decision stage, compared to the trading stage. They do not know the realization of their private signal, as well as the price signal they can observe at t. By doing research, they can influence the variance of the private signal. The only information they have about the fundamental value is the public signal y_t. Noting that $E_i[x_{i,t}|I_{-t}] = E_i[A_t|I_{-t}] = E_i[\theta|I_{-t}] = y_t$, it is easy to show that $E_i[(E_i[P_{t+1}|I_{i,t}] - r_t P_t)|I_{n,-t}] = 0$. Thus, the expected squared return from the risky asset equals its variance. Derivation of the variance term is presented in the Appendix B.

When deriving the first order conditions, the terms z_t, $z_{s,t}$, and μ_t are taken as given. These do depend on the market β_t, but an individual investor is too small to influence it. After taking the first order conditions, we can impose a symmetric equilibrium where $\beta_{i,t} = \beta_t$. Replacing in the expressions for z_t and $z_{s,t}$ and simplifying, the optimal β_t is shown to be

$$\begin{aligned} K r_t \beta_t &= \frac{z_{t+1}\left(2\delta\mu_t R_{t+1} + \tau z_{t+1}\right)}{\delta(\alpha_{t+1}+\beta_t)^2 R_{t+1}^2} + \\ &+ \frac{z_{t+1}^2 \delta[\alpha_{t+1}(z_{s,t+1}^2 \alpha_{t+1} + z_{t+1}^2 \delta) - \beta_t^2 z_{s,t+1}]}{\tau(\alpha_{t+1}+\beta_t)^2 (z_{s,t+1}^2(\alpha_{t+1}+\beta_t) + z_{t+1}^2\delta)^2}. \end{aligned} \tag{11}$$

The dynamic system of equations (10), (11) can be solved with respect to $\{\beta_1, \ldots, \beta_T, \mu_1, \ldots, \mu_T\}$ numerically (using also (4), (5), (7),(8) and (9)). The numerical results are presented in Appendix D and analysed in Section 3.3.

3.2 One trading period example

Before discussing the numerical results for the multiple trading period case, consider the one trading period model. For $T = 1$ the solution of the model simplifies substantially. In such case, $z_2 = 1$ and $z_{s,2} = 0$, the value of μ_1 can be found from (7),(8) and (10) to be $\mu_1 = \beta_1/\tau$. The intuition behind this is the following. If the variance of the private signals is higher, prices will be more informative, for a given level of noise in supply. Prices will be more informative also if risk aversion is lower, since rational investors will invest more in risky assets.

Replacing z_2, $z_{s,2}$, μ_1 and knowing that $\alpha_2 = \alpha_1 + \mu_1^2\delta$, with $\alpha_1 = \alpha$, the equation for the optimal research cost (11) simplifies to

$$K r_1 \beta_1 = \frac{\tau^2 + \alpha\delta + \frac{\beta_1^2}{\tau^2}\delta^2 + 2\delta\beta_1}{\tau\delta\left(\alpha + \beta_1 + \frac{\beta_1^2}{\tau^2}\delta\right)^2}. \tag{12}$$

In the one trading period case, the sensitivity of β_1 with respect to the parameters of the model can be shown analytically. The implicit derivatives of β_1 are reported in the Appendix C.

Investors will choose less precise private signals, if research is expensive (K is high). They will also research less if the information available for free is more accurate. This is the case, when the variance of the public signal is low, or when the variance of supply is low, causing price signals to be more revealing. The dependence of the research decision on risk aversion is more ambiguous. There are three forces affecting it in different directions. First, a higher risk aversion makes the investors to care more about reducing the variance, which they can achieve this by research. Second, more risk averse investors are less willing to invest in risky assets in the first place, and therefore have less incentives to research instead of investing in the risk-free asset. Finally, higher risk aversion makes the demand of other rational investors in the risky assets lower, making prices less informative, which again increases the incentives to research. In very high and very low levels of risk aversion, the optimal research cost is increasing in risk aversion.

A higher risk-free return reduces the optimal research cost. This confirms the hypothesis that investors are less willing to research if there are good alternatives available. It is assumed that knowing the return of such alternative does not to require research effort. Therefore, instead of spending resources and trying to find out more about assets in the risky category, they invest more to the less risky alternative.

The weight on the true value in the pricing equation, z_1, simplifies to

$$z_1 = \frac{\frac{\beta_1^2}{\tau^2}\delta + \beta_1}{\alpha + \frac{\beta_1^2}{\tau^2}\delta + \beta_1},$$

which is clearly decreasing in β_1. This means that prices are expected to be closer to the common public signal, if less research is chosen. Hence, if investors choose to research less, a "good" risky asset is likely to be underpriced and a "bad" asset overpriced. An asset being "good" ("bad") in this context is an asset with true value above (below) y.

Consider the following example illustrating the impact of alternative return. Assume $y = 1$, $\alpha = 2.5$, $\tau = 6$, $\delta = 2.5$, and $K = 0.05$. The high alternative return is $r_1 = 1.2$ and the low return $r_1 = 1$. The value for K is chosen such that it gives β_1 around 2.5, for the low r_1 case. These values aim to match those in Bacchetta and Wincoop (2004). Integrating out the noise in the supply, it should hold that $r_1 P_1 = \theta$. However, the information structure makes the risky asset price biased towards the public signal; that is, as argued before, $r_1 P_1 = (1 - z_1)y + z_1\theta$ for $s_1 = 0$. Assume that an asset has true liquidation value $\theta = 1.2$ and the mean of the public signal $y = 1$. If the alternative return is low, then $r_1 P_1 = 1.11$, which is lower than the one corresponding to the true value. High alternative return reduces incentives to research and makes the prices even more biased towards the public mean and $r_1 P_1 = 1.10$.

3.3 Numerical results

Since the T trading periods case is too complicated to analyse analytically, an example with $T = 7$ is presented. The baseline case has the same parameter values as the previous subsection, with the exception that K is chosen such that β_4 is close to 2.5. This can be achieved by $K = 0.005$. In the baseline case the alternative interest rates are constant at 1. The arbitrary form of the research cost function assumed, and the fact that the alternative return is modelled as risk-free, imply that the numerical results should be interpreted with caution. The objective is to present the direction of impact of various parameters in the model.

The results of the numerical solution are presented in Appendix D. Similarly to the one trading period case, the research costs in all periods are increasing in higher noise in supply and public signal. Both, higher variance of supply and public signal make investors to spend more on research in order to compensate for less information in costlessly available signals. The high variance in public signal also sets the expected prices closer to the fundamental. It is clear that if research is more expensive (higher K), investors choose a higher variance for their private signals in all periods.

As in the one trading period example, the impact of the risk aversion on optimal research in some period t is ambiguous. In the numerical example presented, it reduces research in later periods, while increasing it in earlier periods. However, it is interesting to notice that it makes the weight on the supply noise term in the pricing equation (6) lower in all periods. It also makes prices closer to the fundamental in all periods, due to the higher participation of rational investors in the market.

In the following subsections we concentrate on the main questions of the paper: the development of research incentives over time and the impact of alternative returns on the research decision.

3.3.1 Development of research incentives over time and its impact on investors willingness to separate assets in a category.

As shown already in Allen, Morris and Shin (2004), the existence of public and heterogeneous private signals makes asset prices to react slowly to changes in the underlying fundamental. The numerical results show that allowing investors to make their research decision endogenously makes asset prices even more similar in earlier periods. That is due to earlier periods investors lower incentives to research; reasons for this will be analysed shortly. (The numerical results are presented in the Appendix D.)

Figure 1 provides an illustration of this effect. The dashed line represents the expected price of the asset, if it was traded according to the fundamental. The dotted line shows the asset price development over time, due to the information structure only (similarly to Allan, Morris and Shin, 2004). This is if the research cost is assumed constant over time (at their period $T-1$ level in endogenous research cost case). Finally, the solid line presents prices when the research decision is taken endogenously. The alternative return is assumed to be constant at $r_t = 1$, for all t.

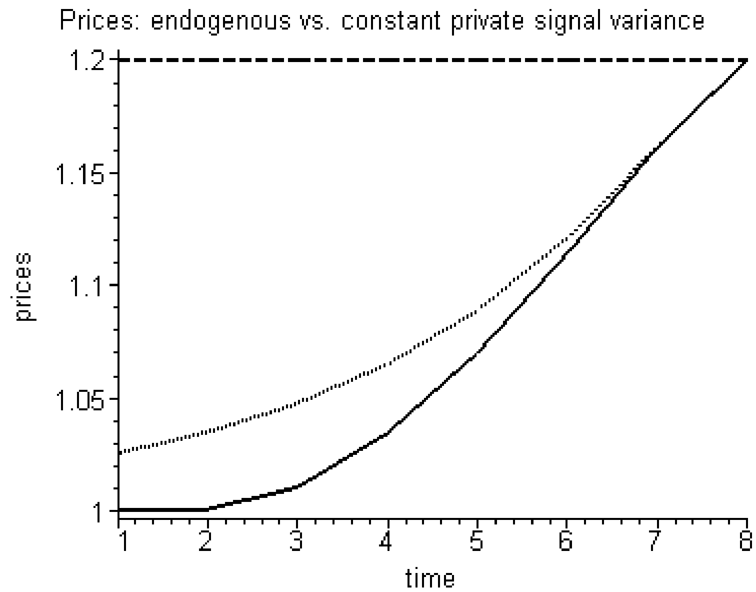

Figure 1

We can see that even if research effort is constant over time, the more long-lived assets would be priced more towards the public signal mean. Endogenous research decision makes this mispricing even stronger, because in earlier trading periods investors have lower incentives to research.

Thinking of long-lasting assets like stocks in the light of these findings rationalises the investors' unwillingness to trade such assets based on the information from internal fundamental research. It is not rational to "go against the market" following internal fundamental research. It is also not optimal to spend a lot of resources on fundamental research of such assets, since imprecise public information can make asset prices to be persistently far from the fundamental.

Furthermore, fundamental research may be more useful as a tool for influencing the public information, which can justify the reputation of such research as a marketing tool. However, analysing this question in depth is out of the scope of this paper.

The reasons for lower incentives for research in earlier periods are the following. As we can see in the Appendix D, the weight on the true value (z_t) in the pricing equation (6) is increasing over time time. So, given that period t traders care about the next period price rather than the true value, earlier periods investors can find out about a smaller component of next period's expected price by doing fundamental research. In addition to that, the coefficient on noise in supply ($z_{s,t}$) is decreasing over time. Therefore, earlier periods investors also face higher uncertainty due to noise trading and they cannot reduce this uncertainty by fundamental research. These two forces reduce earlier periods investors' marginal gains from research.

However, there is an opposite force affecting the incentive to research; the availability of price history. As long as at least some research is done in earlier periods, the variance of the public signals (evolving according to (4) and (5)) decreases over time, because historical prices carry information about the fundamental. As shown before, a lower variance of the public signal reduces investors' willingness to spend resources on research. Therefore this force lowers the incentive to research for investors, who have longer price history in their information set i.e. the later period ones. Nevertheless, this force does not appear to be strong enough to make the later period investors to research as little as earlier period ones.

The result of less research in earlier periods seems robust. Constant or decreasing research cost over time could be the solution only if the alternative interest rates would be less than 1 in earlier periods. Such alternative returns are not feasible, given that there is always at least cash available with nominal return of 1.

Returning to the impact of a longer price history, we can notice another feature. If two assets have the same time to maturity, while one of them has a longer price history available, the prices of the latter will not necessarily be closer to the fundamental. This is despite of the fact that for the latter asset there is more information available. Table 1 illustrates by this comparing the optimal research costs (proportional to β_T) and price of a risky for $T = 1$ case

Time	β_7^{7per}	β_1^{1per}	P_7^{7per}	P_1^{1per}
Baseline	6.851	6.948	1.161	1.161
$\alpha = 1.5$	7.171	7.298	1.176	1.176
$\delta = 1.5$	8.199	8.261	1.163	1.163
$\tau = 3$	5.375	5.797	1.174	1.172
$K = 0.0075$	5.848	5.904	1.154	1.154

The baseline values are assumed to be the same as in Appendix D.
Fundamental value of the risky assets is $\theta = 1.2$.

Table 1: The impact of existence of price history on optimal variance and prices

and those in the 7th trading period in $T = 7$ case. In both cases, there is one period remaining until maturity. It appears that the prices (integrating out the noise in supply) for assets with longer price history are not necessarily closer to the fundamental. This is because in such case investors will choose to research less, obtain more information from historical prices and in effect free-ride on the research done in earlier periods.

3.3.2 Impact of alternative returns on incentives to research

We will now consider the impact of alternative returns in a dynamic setting. As we can see from the tables in Appendix D, a temporary increase of the alternative return reduces research on that period, and makes investors invest more in the alternative. In the periods following the temporary increase, investors will choose to research a bit more compared to the baseline case. This is because they have to compensate for the worse price signals from the earlier trading periods.

Using the example of Section 1, during the Asian crisis investors might have put less research effort in emerging markets because of lower incentives to research. This may have been because they could invest easily to less risky alternative assets and reacted on the general negative public signal due to crisis in some countries. This is consistent with the model predictions, if we consider the alternative being assets in developed countries.

A permanently higher alternative return reduces research in all periods. A temporarily higher return in some period also reduces research in all periods preceding it. The latter is due to the assumption that alternative returns in each period are known to the investors. In such case, earlier period investors know that their consumption will depend on prices that are expected to be further from the fundamental. This in turn, is due to the later periods investors' higher reluctance to invest into research.

4 Conclusions

This paper presented a model where investors make optimally their trading decisions according to both public and private information. Adding the endogenous research decision to a model in the style of Allen, Morris and Shin (2004) leads to following conclusions.

First, as already shown by the above mentioned paper, the existence of a public signal makes asset prices biased towards it. This is because all investors take into account their expectations about other investors' expectations and everyone observes the public signal. Furthermore, this bias will be larger if investors trade assets that have more time left to maturity. Allowing investors to choose the variance of their private signal increases this bias in earlier periods even more. Namely, investors that trade with more long-lived assets can gain less from their private fundamental research, because of the higher uncertainty of returns from risky assets caused by noise trading in following periods. Furthermore, given that investors care more about the price that they will get for their assets in the market when exiting, rather than the underlying fundamentals, their incentive to research is lower the earlier they trade compared to the maturity date of their assets. This justifies the lack of relevance of internal fundamental research in the research decisions about long-lived assets, like stocks. It provides scope for the prices of such assets to be persistently far from the ones corresponding to fundamentals only.

Second, for a fixed time to maturity, a longer price history does not make asset prices necessarily closer to the ones corresponding to fundamentals. This is because investors, who can obtain more information from historical prices, will have an incentive to research less and free-ride on the research effort made by the earlier trading period investors.

Finally, temporarily higher alternative returns reduce investors' incentives to research. Instead, they prefer to save some of the research cost and invest more into the alternative asset. If the alternative returns were to fall after that, then later periods' investors would be more willing to research, in order to compensate for the worse signals coming from asset prices. Hence, in the circumstances where there are some assets in the global market offering unusually high returns with low risk, then the riskier assets are likely to be researched very little.

The paper could be extended in multiple directions. The short lived traders assumption overlooks the possible gain investors can obtain from researching in earlier periods. Namely, if they were trading also in the following period, they could gain from their own research done in earlier periods. On the other hand, the model presented overlooks another opposite force, coming from a possible feedback from prices to the fundamental. For example, a company that has its stock underpriced, may find it harder to get loans to finance its projects and its fundamental value may be damaged. If there were a possibility of the fundamental changing due to prices, investors' incentives to spend on research in early periods, should be lower again.

Furthermore, the assumptions about the information structure could be extended as well. There could be incentives for investors to try to manipulate the public signals. However, this would require a much more complicated setting, in which one would need to consider the resulting agency problems.

Finally, this model is too stylized to allow for an assessment of the quantitative significance of these effects. This is because of the simplifying assumptions regarding the functional form of research cost, and the assumption that alternative asset is risk-free. Hence, it would be interesting to develop a framework that would allow for empirical investigation of the magnitude of the effects of the various parameters in the model.

A Derivation of the equilibrium price equation

Consider an investor of generation T. His demand for the risky asset is
$h_{i,T} = (E_i[\theta|I_{i,T}] - r_t P_T)/\tau V_i(\theta|I_{i,T})$.
Replacing in the conditional expectations and variance, we can rewrite this as
$h_{i,T} = \frac{1}{\tau}(y_T\alpha_T + \mu_T^2\delta A_T + \beta_{i,T}x_{i,T} - r_T P_T(\alpha_T + \mu_T^2\delta + \beta_{i,T}))$.
Aggregating this across investors, and noting that in the symmetric equilibrium $\beta_{i,T} = \beta_T$, the aggregate demand will be
$H_T = \frac{1}{\tau}(y_T\alpha_T + \mu_T^2\delta A_T + \beta_T\theta - r_T P_T(\alpha_T + \mu_T^2\delta + \beta_T))$.
Equating supply and demand, replacing in $A_T = \theta - H_T/\mu_T$ and rearranging will make the equilibrium price
$P_T = \dfrac{\tau + \mu_T\delta}{r_T(\alpha_T + \mu_T^2\delta + \beta_T)}\left(\dfrac{\alpha_T}{\tau + \mu_T\delta}y_T + \dfrac{\beta_T + \mu_T^2\delta}{\tau + \mu_T\delta}\theta - s_t\right)$.
Equating the initially assumed coefficients with the results of market clearing, will give
$\eta_T = \dfrac{\tau(\tau^2 + \delta\beta_T)}{r_T(\alpha_T + \mu_T^2\delta + \beta_T)}$, $\lambda_T = \dfrac{\alpha_t\tau}{\tau^2 + \delta\beta_T}$, $\mu_T = \dfrac{\beta_T}{\tau}$.
The equilibrium price can also be expressed as
$P_T = \dfrac{1 - z_T}{r_T}y_T + \dfrac{z_T}{r_T}\theta - \dfrac{z_{s,T}}{r_T}s_T$, where $z_T = \dfrac{\beta_T + \mu_T^2\delta}{\alpha_t + \mu_t^2\delta + \beta_t}$, $z_{s,T} = \dfrac{\tau + \mu_T\delta}{\alpha_t + \mu_t^2\delta + \beta_t}$.

Next, we show that for any trading period t, where in the next trading period equilibrium price will be
$P_{t+1} = \frac{1-z_{t+1}}{R_{t+1}}y_{t+1} + \frac{z_{t+1}}{R_{t+1}}\theta - \frac{z_{s,t+1}}{R_{t+1}}s_{t+1}$, where $R_t = \prod_{k=t}^{T+1} r_k$
we can find z_t and $z_{s,t}$ such that we can form a similar pricing equation.

Investors trading at t will know the value of y_{t+1} as it incorporates the price signals for periods 1 to t. The expected value and variance of the next period price of asset are
$E_i[P_{t+1}|I_{i,t}] = \frac{1-z_{t+1}}{R_{t+1}}y_{t+1} + \frac{z_{t+1}}{R_{t+1}}E_i[\theta|I_{i,t}]$,
$V_i[P_{t+1}|I_{i,t}] = \frac{z_{t+1}^2}{R_{t+1}^2}V_i[\theta|I_{i,t}] + \frac{z_{s,t+1}^2/\delta}{R_{t+1}^2}$.
Replacing in the conditional expectations, variance and the public signal and

aggregating it across investors, result in the aggregate demand equation

$$
\begin{aligned}
H_T &= \frac{1}{\tau}\frac{R_{t+1}(1-z_{t+1})\frac{\alpha_t y_t+\mu_t^2\delta\theta-\mu_t\delta s_t}{\alpha_t+\mu_t^2\delta}}{z_{t+1}^2/(\alpha_t+\mu_t^2\delta+\beta_t)+z_{s,t+1}^2/\delta}+ \\
&\quad +\frac{z_{t+1}\frac{\alpha_t y_t+\beta_t\theta+\mu_t^2\delta\theta-\mu_t\delta s_t}{\alpha_t+\mu_t^2\delta+\beta_t}-r_tP_t}{z_{t+1}^2/(\alpha_t+\mu_t^2\delta+\beta_t)+z_{s,t+1}^2/\delta}.
\end{aligned}
$$

Equating this to supply and rearranging, there will be z_t and z_s (equations (7) and (8) respectively), such that the pricing equation (6) can be formed. The coefficients η_t, λ_t and μ_t solve

$$
\eta_t=\frac{z_{s,t+1}}{R_{t+1}},\qquad \lambda_t=\frac{(1-z_{t+1})}{z_{s,t+1}},\qquad \mu_t=\frac{z_{t+1}}{z_{s,t+1}}.
$$

Explicit expressions for η_t, λ_t and μ_t cannot be derived for $t<T$.

It is easy to see that replacing in the terminal conditions $z_{T+1}=1$, $z_{s,T+1}=0$ and $r_{T+1}=1$, (7) and (8) also simplify to the expressions z_T and $z_{s,T}$ derived.

B Derivation of the expected squared return of risky asset at the research decision stage

Using the pricing equation (6) and conditional expectations of the true value, the expected return of the risky asset for investor i can be expressed as

$$
\begin{aligned}
E_i[P_{t+1}|I_{i,t}]-r_tP_t &= \tfrac{1}{R_{t+1}}\left[\tfrac{(1-z_{t+1})(\alpha_t y_t+\mu_t^2\delta A_t)}{\alpha_t+\mu_t^2\delta}+\tfrac{z_{t+1}(\alpha_t y_t+\beta_{i,t}x_{i,t}+\mu_t^2\delta A_t)}{\alpha_t+\beta_{i,t}+\mu_t^2\delta}\right]+ \\
&\quad -\tfrac{r_t}{R_{t+1}}\left[(1-z_t)y_t+z_t\theta-z_ts_t\right].
\end{aligned}
$$

The public signal is $A_t=\theta-s_t/\mu_t$ and the private signal $x_{i,t}=\theta+\varepsilon_{i,t}$. Replacing these above and rearranging, $E_i[P_{t+1}|I_{i,t}]-r_tP_t$ is

$$
\begin{aligned}
\tfrac{1}{R_{t+1}}\quad &\Bigg[\left(\tfrac{(1-z_{t+1})\mu_t^2\delta}{\alpha_t+\mu_t^2\delta}+\tfrac{z_{t+1}(\beta_{i,t}+\mu_t^2\delta)}{\alpha_t+\beta_{i,t}+\mu_t^2\delta}-z_t\right)\theta \\
&+\left(-\tfrac{(1-z_{t+1})\mu_t\delta}{\alpha_t+\mu_t^2\delta}-\tfrac{z_{t+1}\mu_t\delta}{\alpha_t+\beta_{i,t}+\mu_t^2\delta}+z_{s,t}\right)s_t \\
&+\tfrac{z_{t+1}\beta_{i,t}}{\alpha_t+\beta_{i,t}+\mu_t^2\delta}\varepsilon_{i,t}+\text{a non stochastic term wrt. } I_{-t}\Bigg].
\end{aligned}
$$

Since $V_i[\varepsilon_{i,t}|I_{-t}] = 1/\beta_{i,t}$, $V_i[s_t|I_{-t}] = 1/\delta$ and $V_i[\theta|I_{-t}] = 1/\alpha_t$, the variance of the expected return, $V_i[E_i[P_{t+1}|I_{i,t}] - r_t P_t|I_{-t}]$ is

$$\frac{1}{R_{t+1}^2} \left[\left(\frac{(1-z_{t+1})\mu_t^2\delta}{\alpha_t+\mu_t^2\delta} + \frac{z_{t+1}(\beta_{i,t}+\mu_t^2\delta)}{\alpha_t+\beta_{i,t}+\mu_t^2\delta} - z_t \right)^2 \frac{1}{\alpha_t} + \left(\frac{(1-z_{t+1})\mu_t\delta}{\alpha_t+\mu_t^2\delta} + \frac{z_{t+1}\mu_t\delta}{\alpha_t+\beta_{i,t}+\mu_t^2\delta} - z_{s,t} \right)^2 \frac{1}{\delta} + \frac{z_{t+1}^2\beta_{i,t}}{(\alpha_t+\beta_{i,t}+\mu_t^2\delta)^2} \right].$$

C The dependence of optimal precision of the private signal on the parameters of the model in one trading period case

Assume that $\alpha, \beta_t, \delta, \tau, K, r_t > 0$, and denote
$Q \equiv \tau^6\alpha + 3\beta_1\tau^6 + 3\delta^3\beta_1^4 + 9\delta\beta_1^2\tau^4 + 9\delta^2\beta_1^3\tau^2 +$
$+\tau^4\delta\alpha^2 + 3\delta\beta_1\tau^4\alpha + 4\delta^2\beta_1^2\tau^2\alpha > 0$
The implicit derivatives from (12), of optimal precision $\beta_1$with respect to the parameters of the model, are

$$\frac{d\beta_1}{dr_1} = -\frac{K\delta\beta_1^2(\alpha\tau^2 + \beta_1\tau^2 + \beta_1^2\delta)^3}{\tau Q} < 0$$

$$\frac{d\beta_1}{dK} = -\frac{r_1\delta\beta_1^2(\alpha\tau^2 + \beta_1\tau^2 + \beta_1^2\delta)^3}{\tau Q} < 0$$

$$\frac{d\beta_1}{d\alpha} = -\frac{\beta_1\tau^2(\delta^2\beta_1^2 + 2\tau^4 + \tau^2\delta\alpha + 3\tau^2\delta\beta_1)}{Q} < 0$$

$$\frac{d\beta_1}{d\delta} = -\frac{\beta_1(\delta^3\beta_1^4 + \beta_1\tau^6 + \delta^2\beta_1^2\tau^2\alpha + \tau^6\alpha + 3\delta\beta_1^2\tau^4 + 3\delta^2\beta_1^3\tau^2)}{\delta Q} < 0$$

$$\frac{d\beta_1}{d\tau} = \frac{\beta_1((\alpha + \beta_1)\tau^6 - \delta(\alpha^2 + 3\alpha\beta_1 - 3\beta_1^2)\tau^4 + 3\delta^2\beta_1^2\tau^2 + 3\delta^3\beta_1^4))}{\tau Q}$$

The direction of impact from risk aversion to optimal precision of private signal is ambiguous, depending on the parameters of the model. β_1 will be increasing in τ for small values of τ, may be decreasing in τ in a small area in the middle, and is increasing in τ again in high values of τ.

D Numerical results for the $T = 7$ model

Baseline parameters: $y = 1$, $\alpha = 2.5$, $\tau = 6$, $\delta = 2.5$, $K = 0.005$ and $r_t = 1$, for all t. In order for the results to be more intuitive, the supply is integrated out; in fact s_t is assumed to be 0 for all t. The table reports prices as $P'_t = \prod_{k=t}^{8} r_k P_t$ ($P_8 = \theta$), to be more easily comparable for different levels of r_t.

Table D1. Optimal precision of private signal: β_t (research cost is proportional β_t).

Time	1	2	3	4	5	6	7
Baseline	.001	.187	1.106	2.434	3.957	5.569	6.851
$\alpha = 1.5$	1.692	2.583	3.505	4.448	5.461	6.478	7.171
$\delta = 1.5$	.093	.854	2.125	3.569	5.112	6.777	8.199
$\tau = 3$	.012	.444	1.648	3.013	4.057	4.705	5.375
$K = 0.0075$	.000	.010	.394	1.490	2.906	4.487	5.848
$r_6 = 1.2$	.000	.035	.610	1.810	3.294	5.150	6.859
$r_1, .., r_7 = 1.2$	.000	.000	.061	0.836	2.443	4.433	6.367

Table D2. Weight on the true value in the pricing equation (z_t).

Time	1	2	3	4	5	6	7
Baseline	.000	.004	.053	.173	.351	.570	.784
$\alpha = 1.5$	.100	.189	.299	.428	.572	.726	.852
$\delta = 1.5$	.001	.028	.110	.238	.405	.603	.806
$\tau = 3$	.000	.018	.119	.294	.472	.595	.699
$K = 0.0075$	.000	.000	.014	.102	.272	.505	.756
$r_6 = 1.2$	.000	.000	.026	.133	.318	.557	.786
$r_1, .., r_7 = 1.2$	.000	.000	.002	.065	.260	.523	.768

Table D3. The coefficient of the supply shock in the pricing equation ($z_{s,t}$)

Time	1	2	3	4	5	6	7
Baseline	$4.1 \cdot 10^{19}$	$4.2 \cdot 10^{9}$	41623	131	7.40	1.71	.69
$\alpha = 1.5$	$2.2 \cdot 10^{32}$	$7.4 \cdot 10^{15}$	$4.3 \cdot 10^{7}$	3282	28.6	2.66	.71
$\delta = 1.5$	$3.8 \cdot 10^{27}$	$3.1 \cdot 10^{13}$	$2.8 \cdot 10^{6}$	832	14.4	1.88	.59
$\tau = 3$	18.07	3.88	1.80	1.18	.86	.61	.39
$K = 0.0075$	$8.8 \cdot 10^{21}$	$6.1 \cdot 10^{10}$	$1.6 \cdot 10^{5}$	257	10.4	2.05	.78
$r_6 = 1.2$	$2.2 \cdot 10^{17}$	$3.3 \cdot 10^{8}$	12919	80	6.35	1.74	.69
$r_1, .., r_7 = 1.2$	$7.7 \cdot 10^{11}$	$9.8 \cdot 10^{5}$	1006	29	4.60	1.61	.72

Table D4. The equilibrium prices of risky assets with true values $\theta = 1.2$.

Time	1	2	3	4	5	6	7
Baseline	1.000	1.001	1.011	1.035	1.070	1.114	1.161
$\alpha = 1.5$	1.020	1.038	1.060	1.086	1.114	1.146	1.176
$\delta = 1.5$	1.000	1.006	1.022	1.048	1.081	1.121	1.163
$\tau = 3$	1.000	1.004	1.024	1.059	1.101	1.141	1.174
$K = 0.0075$	1.000	1.000	1.003	1.020	1.054	1.101	1.154
$r_6 = 1.2$	1.000	1.000	1.005	1.027	1.064	1.112	1.161
$r_1, .., r_7 = 1.2$	1.000	1.000	1.000	1.013	1.052	1.105	1.158

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