Mean Reversion in Equity Prices: The G-7 Evidence

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Abstract

In this paper we develop a two-factor continuous time model of stock prices in order to study stock returns predictability and reappraise the voluminous empirical literature. Using an exact discretization method, we show that focussing on the effects of the "intrinsic" continuous time parameters, in particular, the mean reversion parameter, produces pervasive support for mean reversion in the G-7 countries even at low frequencies and relatively short samples.

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1. Introduction

Mean reversion in stock prices still remains a rather controversial issue. Whereas theoretical justifications for the departure from the random walk model of equity prices have proliferated[1], the empirical evidence remains mixed and confusing. Fama and French (1988) and Poterba and Summers (1988) are the first to document the existence of negative correlation[2] between US equity portfolio returns over "medium" to "long" investment horizons, while Lehmann (1990) finds evidence in favor of return reversals in "winner" and "loser" portfolios even at the weekly frequency. On the contrary, Lo and MacKinlay (1988) report weak positive correlation between US portfolio returns over "short" investment horizons. Kim, Nelson, and Startz (1991) argue that the mean reversion results of Fama and French and Poterba and Summers are only detectable in prewar US data. In

[1]See, for example, the "fad variables" model of Shiller (1981, 1984) and Summers (1986), the "bandwagon effect" explanation of Poterba and Summers (1988), the "over-reaction" hypothesis of De Bondt and Thaler (1985, 1987), the "time-varying risk premium" explanation of Conrad and Kaul (1989), Conrad, Kaul and Nimalendran (1991), Fama and French (1988), and Keim and Stambaugh (1986), the information related (Hasbrouck (1991)) or strategic trading (Admati and Pfleiderer (1989)) market microstructure models, the " institutional structures " framework of Bessembinder and Hertzel (1993), and the "over-reaction and/or partial adjustment to new information" models of Brock, Lakonishok and LeBaron (1992), Jegadeesh (1990), Lehmann (1990), and Lo and MacKinlay (1990).

[2]Mean reversion implies that shocks to prices are temporary, i.e., returns are negatively autocorrelated at certain horizons.

turn, Richardson and Stock (1989) and Richardson (1993) report that correcting for small-sample bias problems could reverse the Fama and French and Poterba and Summers results.

Another strand of the literature deals with relative mean reversion in stock index data. Kasa (1992), in a multi-country study, reports that national stock indices are cointegrated and share one common stochastic trend which implies that the value of a properly weighted portfolio of shares in the markets of at least two countries that he examines is stationary, and thus will display mean reversion. Richards (1995) criticizes Kasa's results on the grounds that the use of asymptotic critical values in the cointegration tests is not appropriate. However, he detects a stationary component in relative prices of 16 OECD countries which implies relative mean reversion and reports that country specific returns relative to a world index are predictable. Finally, Balvers, Wu, and Gilliland (2000) report strong evidence of mean reversion over "long" investment horizons in relative stock index prices of 18 countries. Campbell, Lo, and MacKinlay (1997) summarize the debate concisely: "...we simply cannot tell " (p. 80).

The main objective of this paper is to attempt to "tell" more confidently about the existence of mean reversion in stock prices: Whilst maintaining the spirit and modeling assumptions of previous methodologies (in particular, Fama and French's (1988) approach), we aim to show that if the "intrinsic" behavior of stock prices is examined, which clearly was missing from earlier studies, then a reconciliation of the mixed empirical evidence is possible. Our motivation stems from a number of important points that emerge from the relevant literature: First, in contrast with the interest rate literature, mean reversion in stock prices arises as a result of the specification of different investment horizons, rather as an intrinsic property of the underlying stochastic model of equity prices. In their vast majority, the methodologies employed to examine mean reversion involve the use of a particular function of the sample autocorrelations between returns over different investment horizons. However, the theoretical justification of serial correlation in stock returns rests upon a number of theories (see footnote 1) which try to explain the various patterns in returns autocorrelations not in terms of the holding period, but as a result of the interaction between underlying economic factors. Moreover, the existing methodologies imply that the statistical properties of the underlying time series are a function of the investment horizon, which makes the detection of mean reversion a rather arbitrary issue. Second, a consequence of testing for mean reversion by returns autocorrelation tests is that long time series need to be employed. As Balvers, Wu, and Gilliland (2000) put it, "a serious obstacle in detecting mean reversion is the absence of reliable long time series, especially because mean reversion, if it exists, is thought to be slow and can only be picked up over long horizons." (p. 746).

In order to overcome these shortcomings we develop a two-factor continuous time model of stock prices that allows mean reversion and uncertainty in the equilibrium level to which prices revert. On theoretical grounds, this model is consistent with many of the proposed explanations of mean reversion in stock prices, such as " the over-reaction" hypothesis, the "bandwagon effect", the "time-varying risk premium", etc. (see Footnote 1). On empirical grounds, the choice of a continuous time framework attempts to rescue the confusion in the literature arising from the specification of the "holding time period" in stocks, a notion which becomes at least theoretically irrelevant in a continuous time setting. In other words, we are able to detect mean reversion as an "intrinsic" property of the underlying model for equity prices, that is, without explicit reference to the investment horizon over which price changes are measured. This obviates the need for employing long time series; in fact an advantage of our approach is that the recovery of the continuous-time parameters from discrete data sets can be achieved even from relatively small samples. Our continuous time model is tested in the G-7 national stock markets, US, UK, Japan, France, Canada, Germany, Italy, and is empirically supported. Finally, nesting mean reversion explicitly within the underlying stochastic process and thereby estimating the continuous time parameters directly from observables could be used for the more accurate valuation of equity derivatives in the spirit of Lo and Wang (1995), and the development of new trading strategies (for capitalizing on mean reversion) - possibly “contrarian”-, in the spirit of DeBondt and Thaler (1985), Richards (1995, 1997), and Balvers, Wu, and Gilliland (2000).

The maintained hypothesis in our paper is that the state variable, i.e., the (log) stock price is a difference stationary process in the spirit of Nelson and Plosser (1982). This approach was used by Fama and French (1988) and Poterba and Summers (1988) in their pioneering discrete-time models. Our continuous time framework assumes that (log) stock prices are generated by the mix of a nonstationary component modeled as an Arithmetic Brownian motion, and a stationary component modeled as an Ornstein-Uhlenbeck stochastic process. We recover the continuous time parameters, assess their statistical significance, and demonstrate that the mean reversion of the stationary component causes predictability even in daily stock returns which is opposed to the effect of the nonstationary price component which produces white noise in the continuously compounded returns.

The remainder of this paper is organized as follows. In section 2 we present our two-factor continuous time stock price model, and develop reduced form expressions of the slope coefficients that embody the continuous time parameters without relying on crude approximations of the continuous time stochastic processes, thus avoiding temporal aggregation biases. In section 3 we show how the model can be tested and we propose a simple way to identify the continuous time parameters. Section 4 presents the data and our empirical results. Finally, Section 5 concludes the paper.

2. The Continuous Time Stock Price Model

2.1. The Model

Let $p(t)$ be the natural log of a stock price at time t. Following Fama and Frenh (1988), among others, we model $p(t)$ as the sum of a nonstationary component, $q(t)$, and a stationary component, $z(t)$, i.e.

$$p(t) = q(t) + z(t), \tag{1}$$

We assume that the permanent component follows an Arithmetic Brownian

Motion (ABM) process:

$$dq(t) = \alpha dt + \sigma dW_1(t), \tag{2}$$

where α and σ are constants, and $dW_1(t)$ is a standard Wiener process with mean zero and unit variance.

The temporary component is assumed to follow an Ornstein-Uhlenbeck stochastic process:

$$dz(t) = \beta(\gamma - z(t))\, dt + \rho dW_2(t), \tag{3}$$

where β is the speed-of-adjustment coefficient, γ is the long run mean of the process, ρ is the diffusion coefficient which allows the process to fluctuate around its long-run mean in a continuous but erratic way, and $dW_2(t)$ is a standard Wiener process independent of $dW_1(t)$.

The diffusion process in expression (3) is also known as a mean reverting elastic random walk; it is both Gaussian and Markovian but unlike the Wiener process, it does not have independent increments. Furthermore, when $t \to \infty$ we get an equilibrium stationary distribution. Negative correlation between returns can be explained intuitively as follows: for $\beta > 0$, and $z(t) > \gamma$, we would expect the change in the temporary component of the (log) stock price to be negative. This is clearly because$(\gamma - z(t)) < 0$ and hence the expected change, $E(dz(t))$, must be negative. Similarly, if $(\gamma - z(t)) > 0$, then we would expect that $E(dz(t))$ must be positive. Thus, the process always reverts to the mean γ with speed β. Finally, since $dW_1(t)$ and $dW_2(t)$ are independent Wiener processes, we assume (as in Fama and French (1988)) no correlation between the permanent and stationary components of the (log) stock price.

The general hypothesis in our continuous time stock price model in eq. (1)-(3) is that stock prices are nonstationary processes in which the permanent gain from each period's price shock is less than 1.0; the temporary shock will be gradually eliminated. However, a significant temporary part of the shock implies predictability of stock returns[3].

The solution to eq. (2) for $s > t$ is given by:

$$q(s) = q(t) + \alpha(s - t) + \sigma \int_t^s dW_1(\tau), \ s \geq t. \tag{4}$$

The scalar stochastic differential equation in (3) is narrow-sense linear and au-

[3] Schwartz and Smith (1997), independently to our work, develop a "Short-Term / Long-Term Model" for commodity prices which appears similar to ours. Our model, however dates earlier, see Hatgioannides (1995).

tonomous; it's solution is given (see Arnold (1973)) by:

$$z(s) = \gamma + e^{-\beta(s-t)} (z(t) - \gamma) + \rho e^{-\beta(s-t)} \int_t^s e^{\beta(\tau - t)} dW_2(\tau), \ s \geq t. \tag{5}$$

Taking Δ as an arbitrary time step, expressions (4) and (5) can be written in the following equivalent form:

$$q(t+\Delta) = q(t) + \alpha\Delta + \sigma \int_t^{t+\Delta} dW_1(\tau), \tag{6}$$

and

$$z(t+\Delta) = \gamma + e^{-\beta\Delta} (z(t) - \gamma) + \rho e^{-\beta\Delta} \int_t^{t+\Delta} e^{\beta(\tau-t)} dW_2(\tau). \tag{7}$$

If we interpret Δ as the time discretization interval, expression (7) implies an exact discrete time autoregressive process of order one (AR(1)):

$$z(t+\Delta) = \theta + \varphi z(t) + \epsilon_{t+\Delta}, \tag{8}$$

where

$$\theta = \gamma \left(1 - e^{-\beta\Delta}\right), \tag{9}$$

$$\varphi = e^{-\beta\Delta}, \tag{10}$$

$$\epsilon_{t+\Delta} = \rho e^{-\beta\Delta} \int_t^{t+\Delta} e^{\beta(\tau - t)} dW_2(\tau). \tag{11}$$

As in Fama and French (1988), the temporary component in expression (8) has an autoregressive structure. The parameter φ captures mean reversion in the temporary component and causes predictability in the form of negative correlation of returns. It is important to note that φ is not a constant but instead varies with any discrete investment horizon and depends explicitly on the intrinsic mean-reverting parameter β.

Since ρ and β are constants and $dW_2(\tau)$ is a standard Wiener process, it follows directly from eq. (11) that $\epsilon_{t+\Delta}$ is normally distributed, with mean

$$E(\epsilon_{t+\Delta}) = 0,$$

and variance

$$Var(\epsilon_{t+\Delta}) = \frac{\rho^2}{2\beta}\left(1 - e^{-2\beta\Delta}\right). \tag{12}$$

It is important to observe that the variance of $\epsilon_{t+\Delta}$ in eq. (12) is equal to the conditional variance (as of a generic time t) of the temporary component of the (log) stock price process, $z\left(t+\Delta\right)$, given by expression (7):

$$Var_t\left(z\left(t+\Delta\right)\right) = \frac{\rho^2}{2\beta}\left(1-e^{-2\beta\Delta}\right). \tag{13}$$

The conditional mean of $z\left(t+\Delta\right)$ in eq. (7) is given by:

$$E_t\left(z\left(t+\Delta\right)\right) = \gamma + e^{-\beta\Delta}\left(z\left(t\right)-\gamma\right) \tag{14}$$

The unconditional mean of $z\left(t+\Delta\right)$ in eq. (7) is given by:

$$E\left(z\left(t+\Delta\right)\right) = \gamma\left(1-e^{-\beta\Delta}\right) + e^{-\beta\Delta}E\left(z\left(t\right)\right),$$

which implies, given the stationarity of the z process, i.e. $E\left(z\left(t+\Delta\right)\right) = E\left(z\left(t\right)\right) = E\left(z\right)$, that:

$$E\left(z\right) = \gamma. \tag{15}$$

Finally, the unconditional variance of $z(t+\Delta)$ in eq. (7) is given by:

$$Var\left(z\left(t+\Delta\right)\right) = e^{-2\beta\Delta}Var\left(z\left(t\right)\right) + \rho^2 e^{-2\beta\Delta}\int_t^{t+\Delta} e^{2\beta(\tau-t)}d\tau,$$

which implies, for$Var\left(z\left(t+\Delta\right)\right)=Var\left(z\left(t\right)\right)=Var\left(z\right)$, that:

$$Var\left(z\right)=\frac{\rho^2}{2\beta} \tag{16}$$

Expressions (6)-(16) provide a complete statistical description over any discretization interval Δ of the continuous time stock price model in (1)-(3). Next, we present some of the key results in our paper by demonstrating how the unobserved continuous time parameters are embodied in the observed regression coefficients.

2.2. Investment Horizon and Autocorrelation Coefficients

In Fama and French's (1988) study, a U-shaped pattern in autocorrelation coefficients over different investment horizons is expected theoretically when a temporary component exists. We show below that this is also a feature of our continuous time model in which, indeed the autocorrelation coefficient varies with the investment horizon - as in Fama and French -, but most importantly depends on the intrinsic continuous time parameters which we aim to recover.

The continuously compounded rate of return over a single holding period $\Delta,$ say from time t to $\left(t+\Delta\right),$ is $r\left(t,t+\Delta\right)=p\left(t+\Delta\right)-p\left(t\right),$ which can be written in view of eq. (1) as:

$$r\left(t,t+\Delta\right)=\left[q\left(t+\Delta\right)-q\left(t\right)\right]+\left[z\left(t+\Delta\right)-z\left(t\right)\right]. \tag{17}$$

The correlation coefficient between $r\left(t,t+\Delta\right)$ and $r\left(t-\Delta,t\right)$ is defined as:

$$\hat{\lambda}_{\Delta}=\frac{Cov\left(r\left(t,t+\Delta\right),r\left(t-\Delta,t\right)\right)}{Var\left(r\left(t-\Delta,t\right)\right)}. \tag{18}$$

We show in Appendix 1 how the above covariance and variance terms can be expressed in terms of the unobserved continuous time parameters of the model (1) - (3) to obtain, after simple rearrangements, the following reduced-form expression for the estimated slope coefficient $\hat{\lambda}_{\Delta}$:

$$\hat{\lambda}_{\Delta}=\frac{-\left(e^{-\beta\Delta}-1\right)^{2}\frac{\rho^{2}}{2\beta}}{-\frac{\rho^{2}}{\beta}\left(e^{-\beta\Delta}-1\right)+\sigma^{2}\Delta}. \tag{19}$$

Similarly, the autocorrelation coeffcient over n discrete periods is given by

$$\hat{\lambda}_{n\Delta}=\frac{-\left(e^{-\beta n\Delta}-1\right)^{2}\frac{\rho^{2}}{2\beta}}{-\frac{\rho^{2}}{\beta}\left(e^{-\beta n\Delta}-1\right)+\sigma^{2}n\Delta}, \tag{20}$$

Thus the correlation between returns defined over different investment horizons depends upon:

(a) the length of the investment horizon (n), and

(b) the properties of the stochastic process underlying stock returns, as expressed in this case by the sign and magnitude of the parameters $\beta, \rho,$ and σ.

In particular, the correlation coefficient $\hat{\lambda}_{\Delta}$ for given values of the parameters of the underlying stochastic process tends to zero for very small or very large investment horizons.

$$\lim_{n \to \infty} \hat{\lambda}_{n\Delta} = 0 \tag{21}$$

and

$$\lim_{n \to 0} \hat{\lambda}_{n\Delta} = 0 \tag{22}$$

This implies that the maximum (negative) value of the autocorrelation coefficient is attained at some point between the interval $0 < n < \infty$.[4] The value of the correlation coefficient for different values of β and over different investment horizons is evaluated using expression (20) and is shown in Figure 1.[5] To uncover the

[4] Partial differentiation of $\hat{\lambda}_{\Delta}$ with respect to n yields the value of n at which $\hat{\lambda}_{\Delta}$ is minimized. In turn, linearizing around $n = 1$ yields an expression for n in terms of β and Δ only.

[5] Results for individual countries are available upon request

importance of the mean reverting parameter in establishing the autocorrelation patterns of equity returns we fix the volatility parameters σ and ρ at the values of 0.15 and 0.13 respectively, which is approximately the average annualized value of each volatility coefficient across the stock markets and for the sample period covered in this study (see Tables 3a and 3b).[6]

[Insert Figure 1]

Figure 1 shows that the autocorrelation coefficient between returns exhibits the U-shaped pattern of Fama and French across investment horizons. The bigger the mean-reverting parameter β, the bigger the autocorrelation coefficient is. Furthermore, for different (theoretical) values of the mean reverting parameter β, the (theoretical) half-life of mean reversion ranges from one to three years. Note that when $\beta = 0$, $\hat{\lambda}_{n\Delta}$ is also equal to zero, which implies that if there is no "intrinsic" mean reversion in the stock price process, then the returns autocorrelation coefficient is zero irrespective of the investment horizon and the values of σ and ρ. We will evaluate next whether such a pattern in stock returns can be found empirically using the continuous time parameter estimates of the stock price model in (1) - (3) in the context of the G-7 national stock markets.

[6] The relative variability of the random walk and mean reverting components ($\frac{\sigma}{\rho}$) only affects the curvature of the U-shaped function.

3. Empirical Methodology

The core of our empirical methodology lies in the recovery of the "intrinsic" continuous-time parameters of our stock price model. It is well known that the form of a continuous time model does not depend on the unit of time or the frequency of observations. Therefore, the continuous time parameters will embody the "intrinsic" properties of the returns generating mechanism.

We propose a simple way to identify[7] the continuous time parameters of interest from: (i) the estimated slope coefficients in regressions of $r\left(t, t+\Delta\right)$ on $r\left(t-\Delta, t\right), \Delta$ being the discretization interval equal to the observation period, (ii) the autocovariances, and (iii) the unconditional means of the returns.

We use non-overlapping data throughout our estimation procedures. Richardson and Stock (1989) point out that assessing the significance of variance ratios and autocorrelation statistics using standard asymptotic theory may provide a poor approximation to the sampling distribution, especially with overlapping data. In particular, Valkanov (2003) shows that in long-horizon regressions with overlapping data the stochastic order of the variables is altered, resulting in unorthodox

[7] Schwartz and Smith (1997) use Kalman filtering procedures to estimate the continuous time parameters. Alternatively, a Generalized Method of Moments estimation technique can be employed.

limiting distributions of the slope estimator and its t-statistic.[8] More intuitively, Richardson (1993) argues that the Fama and French (1988) autocorrelation estimates and corresponding serial correlation patterns should be expected even if the true underlying model is a random walk. Estimation with overlapping data causes multiperiod autocorrelation estimators to have many sample autocovariances in common, picking up much of the same spurious autocorrelation at "close" horizons. If two coefficient estimates are far apart in terms of periods they refer to, then they have very little in common, and they are close to their unconditional average of zero. This may be a valid explanation for the observed by Fama and French (1988) U-shaped pattern in stock-return data, consistent with a random walk model in equity prices. Our estimation procedure obviates the need for long time series, thus allowing us to use non-overlapping data and clarify whether the regularities of equity returns documented by previous empirical studies exist, or are merely induced by overlapping data series.

The continuous time unknown parameters in equation (19) are: (i) the speed-of-adjustment coefficient of the temporary component β, (ii) the instantaneous variance of the temporary component ρ^2, and (iii) the instantaneous variance of the

[8] In a rolling summation of series integrated of order zero (or (I(0)), the new long-horizon variable behaves asymptotically as a series integrated of order one (or I(1)). Thus long-horizon regressions will always produce significant results.

permanent component σ^2. It is obvious that none of these parameters is identifiable from eq. (19) alone. However, we can identify the speed-of-adjustment coefficient, β, by focusing on the unconditional covariance of non-overlapping returns: The numerator of (19) is the covariance between $r(t, t+\Delta)$ and $r(t-\Delta, t)$, the sum of expressions (A4) and (A8) in Appendix 1:

$$Cov\left(r\left(t, t+\Delta\right), r\left(t-\Delta, t\right)\right) = -\left(e^{-\beta\Delta} - 1\right)^2 \frac{\rho^2}{2\beta}. \tag{23}$$

Similarly, choosing 2Δ to be the discretization interval:

$$Cov\left(r\left(t, t+2\Delta\right), r\left(t-2\Delta, t\right)\right) = -\left(e^{-2\beta\Delta} - 1\right)^2 \frac{\rho^2}{2\beta}. \tag{24}$$

Generally, it is straightforward to prove that for arbitrary non-overlapping discretization intervals the covariances between returns are given by the following formula:

$$Cov\left(r\left(t, t+n\Delta\right), r\left(t-n\Delta, t\right)\right) = -\left(e^{-\beta n\Delta} - 1\right)^2 \frac{\rho^2}{2\beta}, \text{ for } n = 1, 2, ... \tag{25}$$

Dividing equation (23) by equation (24) we can identify β[9]. Substituting the value

[9]Call $Cov\left(r\left(t, t+\Delta\right), r\left(t-\Delta, t\right)\right) = X$, and $Cov\left(r\left(t+2\Delta\right), r\left(t-2\Delta, t\right)\right) = Y$. It follows

of β back in (23) we can identify ρ^2. In turn, using the values of β, ρ^2,and $\hat{\lambda}_\Delta$ we can identify σ^2 from equation (19). Finally, the unconditional mean of $r(t, t+\Delta)$ was found in section 2 to be equal to:

$$E(r(t, t+\Delta)) = \gamma + \alpha \tag{26}$$

Similarly,

$$E(r(t, t+2\Delta)) = \gamma + 2\alpha. \tag{27}$$

It is clear from expressions (26) and (27) that we can identify uniquely - for given Δ - the remaining continuous time parameters of interest γ (i.e. the long run mean of the temporary component) and α (i.e. the instantaneous mean of the permanent component). Table 1 collects the formulae used for identification of the continuous-time parameters.

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from eq.(23) and eq.(24) that $\frac{X}{Y} = \frac{\left(e^{-\beta}-1\right)^2}{\left(e^{-2\beta}-1\right)^2}$. In turn, $\left(\frac{X}{Y}\right)^{\frac{1}{2}} = \frac{e^{-\beta}-1}{e^{-2\beta}-1}$. Call $z = e^{-\beta}$; then $z^2 = e^{-2\beta}$. Therefore, $\left(\frac{X}{Y}\right)^{\frac{1}{2}} = \frac{z-1}{z^2-1}$, which implies that $z^2\sqrt{X} - z\sqrt{Y} + \left(\sqrt{Y} - \sqrt{X}\right) = 0$, which implies that $z_{1,2} = \frac{\sqrt{Y} \pm \sqrt{Y} \mp 2\sqrt{X}}{2\sqrt{X}}$. Then, $z_1 = \sqrt{\frac{Y}{X}} - 1$, and $z_2 = 1$. Finally, since $z = e^{-\beta}$, it follows that $\beta = -\ln z_1$.

4. Data and Empirical Results

4.1. Description of the Data

Daily data are obtained from *Datastream* for stock market indices of the G-7 countries, i.e. US, UK, Japan, France, Canada, Germany, Italy. The sample covers the period from 01/01/1983 to 01/01/2001, for a total of 4695 observations. The data used are value-weighted indices constructed by *Datastream.* Closing index prices are used which initially do not include dividends. The daily dividend yield corresponding to each stock index is also obtained and added to closing prices to generate another set of index prices including dividends.[10]

We generate continuously compounded daily returns (close-to-close) for all indices, and by summing the daily returns over 5 trading days we generate weekly returns (in the case of the United States). Since the primary objective of this paper is to nest mean reversion within the underlying continuous time stochastic process for equity indices, we use primarily "short" holding period returns - up to 1 week -, although our estimation methodology can be easily extended to "longer"

[10]The *Datastream* indices represent to a large extent the stock markets in the different countries and provide consistency, transparency, and international comparability. They also tend to be highly correlated with other well-known indices. For instance, the *Datastream* index for the London Stock Exchange has a correlation coefficient with the FTSE ALL SHARE of 0.99 over our sample period.

investment horizons.

Table 2 presents summary statistics for our data set. Following the critique by Richardson and Stock (1989) and Richardson (1993) we use non-overlapping returns (See the discussion in Section 3 of the paper). As can be seen from Table 2 all equity indices are negatively skewed and leptokurtic. Application of standard unit root tests indicates that our equity index series can be treated as integrated of order one, I(1), processes.

4.2. Empirical Results

Section 3 demonstrates that we can test for mean reversion by identifying the continuous-time parameters of the stochastic stock price model (1)-(3) using equations (23)-(27). Using the Ordinary Least Squares (OLS) estimation procedure, we first estimate slope coefficients in regressions of $r(t, t+\Delta)$ on $r(t-\Delta, t)$ for discretization intervals $\Delta = 1$ day for all the countries in our sample except for the US (where we use $\Delta = 1$ week). Throughout we use non-overlapping data on continuously compounded returns to avoid inducing spurious correlation and serious biases in our continuous-time coefficient estimates. It should be noted that in contrast to Fama and French (1988) and the other empirical literature , we do not assess the overall performance of our mean reverting stock price model by evaluating the return correlation coefficient across different investment horizons. Rather, the important point in our testing methodology is to extract the continuous time parameters from the estimated discrete time equations, notably, the speed-of-adjustment coefficient of the temporary component β which induces "intrinsic" mean reversion in the stock price process. Initial estimation of the autocorrelation coefficients for the discretization intervals mentioned above serves merely the purpose of recovering the volatility parameter σ and the standard errors of the continuous-time parameters.

The statistical significance of the continuous time parameters was evaluated by invoking large sample theory and using a simple application of the log-linearization process known as the delta method. The unknown parameters were expressed as functions of the estimated autoregressive coefficients $\hat{\lambda}_{\Delta}$ - in particular, the autocovariances of returns which appear in the identifying formulas for $\beta, \rho,$and σ were formulated as the product of the estimated autocorrelation coefficients and return variances -, and the standard errors obtained as log-linear functions of the standard errors of $\hat{\lambda}_{\Delta}$. Asymptotic normality is assumed throughout and standard errors are corrected for the heteroskedasticity observed in returns using White's correction (1980).

Tables 3a and 3b show the estimated continuous time parameters for the seven national stock market indices together with their standard errors. Table 3a ignores dividends while Table 3b presents results inclusive of dividends sampled at daily frequencies. It is well known that by ignoring dividends a spurious pattern of mean reversion may be generated, especially at the higher frequencies. If dividends are paid out but ignored in the data, we may expect a sudden negative return at the time that dividends are paid. Over time this negative return will be reversed as the payment date for the next dividend comes nearer and becomes incorporated in prices. The positive point estimate and the t-ratio for the all-important speed-of-adjustment coefficient β, both with and without dividends, demonstrate strong and statistically significant evidence for mean reversion even at the daily frequency for five countries (Canada, France, Germany, Italy, UK) and at the weekly frequency for the US. We had to change the discretization period for the US since convergence in our numerical and statistical estimation procedures could not be achieved for daily data. In particular, values for the dividend inclusive β are smaller in magnitude (except for the UK and France, where they are marginally higher) than corresponding estimates from Table 3a, as expected, but only marginally so. In the case of Japan, a negative, but insignificant, β is obtained both with and without dividends.[11]

[11]Given the historical performance of the Japanese equity markets during the sample, with

Naturally, given the maintained hypothesis of mean reversion at all horizons according to the model of equations (1)-(3), it is appropriate to infer correlations at long horizons from correlations at short horizons, as in figure 1. It is true, however, that our findings may be attributed to spurious mean reversion caused by the bid-ask bounce, especially when one uses - as we do - daily observations. Our indices for the G7 economies are constructed from the last recorded trade of each day and one cannot assess whether it is a bid or ask price. We acknowledge that closing prices, as compared, for example, to midpoints of bid-ask prices may cast doubt on the intrinsic nature of our mean reverting results. We have experimented, though, with index data for the UK alone for which the bid-ask price was available, and with several individual stock series for the G7 countries for which again we had access to bid and ask daily closing prices, and still found evidence of statistically significant mean reversion.[12] Furthermore, since the indices are value-weighted, the effect of infrequent or non-synchronous trading (e.g. Lo and Mackinlay (1988), Lehmann (1990)) on our results, which is concentrated in small

the prolonged boom period in the 1980s, and the bust period of the 1990s, it does not come as a surprise that we report a negative and insignificant value for the mean reverting coefficient. Also Table 4 shows that effectively no temporary component exists in Japanese stock prices (around 1% of the variation in returns is accounted for by the stationary component).

[12]Results for bid-to-bid, ask-to-ask, and the midpoint of bid-ask closing returns are available upon request. We have also investigated the effect on our results of "dead stocks" in an index, by using value-weighted recalculated index data which only account for the historical performance of the index constituents at 01/01/2001 over the sample period. Results are quite similar to Tables 3a and 3b and are not reported to conserve space.

stocks, is mitigated. What's more to the purpose, such effects have been shown to induce positive serial correlation in stock portfolios (e.g. Lo and Mackinlay (1988), Bessembinder and Hertzel (1993)), and if anything, should bias our results against mean reversion.

The results suggest a half-life of mean reversion for all markets involved of between one-half and two years (the minimum of the U-shaped curve, see figure 1). Note that markets seem to react faster to temporary shocks than other studies have suggested. For example, Balvers et al (2000) in their multi-country study report a speed of mean-reversion with a half-life of three to three and one-half years. However, we use more recent data at higher frequencies than previous studies to find that the speed of mean reversion towards the specified stochastic trend path of stock prices has risen, which implies lower degree of persistence in the temporary component of stock prices. It seems that stock markets are becoming more efficient over time, reaping the benefits of globalization.

4.3. Dynamic Simulations

Dynamic simulations for equity returns are carried out in order to evaluate our theoretical mean-reverting model using the estimated continuous time parameters for all countries. To start the simulations, we need an initial value for the temporary component, $z(t)$. Following Poterba and Summers (1988), this is estimated as the share of return variation over the sample period due to the transitory component (see Table 4) multiplied by the initial sample price. One thousand replications of equations (2) and (3) are carried out and the Mean-Squared-Error (MSE) was calculated by comparing the average return path from the simulations to the actual returns of the seven stock market indices. For all markets, the low MSE values indicate that the proposed theoretical model is consistent with the empirical behavior of stock returns.

5. Conclusion

In this paper we develop a continuous time stock price model with the intension to study stock returns predictability and reappraise the voluminous empirical literature. Mean reversion in stock returns is better examined within a continuous time framework since most of the conflicting results in the literature arise from the specification of the "holding time period" in stocks, a notion which becomes at least theoretically irrelevant in a continuous time setting. Our theoretical framework nests with the modeling philosophies of earlier studies and assumes that stock returns are generated by the joint effect of a stationary component, modelled as an Ornstein-Uhlenbeck process, and a nonstationary component, modelled by an Arithmetic Brownian motion process. The general hypothesis in our model is that stock prices are nonstationary processes in which the permanent gain from each period's shock is less than 1.0; the temporary shock will be gradually eliminated.

Using conventional return autocorrelation tests, we develop reduced form expressions of the slope coefficient that embodies the continuous time parameters without relying on crude approximations of the continuous time stochastic processes that typically lead to temporal aggregation biases. In turn, we develop a methodology for the identification of the continuous-time parameters of interest from unconditional covariances over non-overlapping intervals, slope coefficients, and unconditional means of stock returns. Finally, we use the identified parameters to examine how they cause the autocorrelation coefficient between stock returns to vary with the investment horizon. Not surprisingly, we are able to confirm that the famous U-shaped pattern in returns autocorrelations is an empirical phenomenon.

For the first time in the literature we report statistically significant evidence of mean reversion in daily data for Canada, France, Germany, Italy, and the UK, and in weekly data for the US. Dynamic simulation experiments suggest that our theoretical model is consistent with the empirical behavior of stock returns.

An obvious extension of our work is to utilize Lo and Wang's (1995) framework for pricing index options in a mean reverting framework. This is easily accomplished since we estimate the continuous time volatility parameters.

Up to now, the common wisdom in the literature was that mean reversion, if it exists, is thought to be slow and can only be picked up over long horizons. We believe that our paper contributes to the finance literature through our findings in the context of seven national stock markets. To paraphrase Campbell, Lo, andMacKinlay(1997), we "can tell" that mean reversion exists in stock prices.

APPENDIX 1

Substituting expression (17) into (18) in the main text we obtain the one-period autocorrelation coefficient.

$$\begin{aligned} \hat{\lambda}_{\Delta} &= \frac{Cov\left[q\left(t+\Delta\right)-q(t)\right)+\left(z\left(t+\Delta\right)-z\left(t\right)\right),\left(q\left(t\right)-q\left(t-\Delta\right)\right)+\left(z\left(t\right)-z\left(t-\Delta\right)\right)]}{Var[(q(t)\ -\ q(t-\Delta))\ +\ (z(t)\ -\ z(t-\Delta))]} \\ &= \frac{Cov\left[q\left(t+\Delta\right)-q\left(t\right),q\left(t\right)-q\left(t-\Delta\right)\right]+Cov\left[z\left(t+\Delta\right)-z\left(t\right),z\left(t\right)-z\left(t-\Delta\right)\right]}{Var\left(q\left(t\right)-q\left(t-\Delta\right)\right)+Var\left(z\left(t\right)-z\left(t-\Delta\right)\right)}, \end{aligned} \tag{A1}$$

where the last equality follows from the assumption that the q and z processes are uncorrelated.

We first evaluate the covariance and variance terms of the temporary component in expression (A1). Expression (7), using the definitions in (9) and (11), implies that:

$$z\left(t+\Delta\right)-z\left(t\right)=\theta+z\left(t\right)\left(e^{-\beta\Delta}-1\right)+\epsilon_{t+\Delta}.$$

Therefore, the second covariance term in the numerator of expression (20) becomes after substitutions:

$$
\begin{aligned}
& Cov\left(z\left(t+\Delta\right)-z\left(t\right),z\left(t\right)-z\left(t-\Delta\right)\right)= \\
= \;& Cov(\theta+z\left(t\right)\left(e^{-\beta\Delta}-1\right)+\epsilon_{t+\Delta},z\left(t\right)-z(t-\Delta)) \\
= \;& \left(e^{-\beta\Delta}-1\right)Var\left(z\left(t\right)\right)-\left(e^{-\beta\Delta}-1\right)Cov\left(z\left(t\right),z\left(t-\Delta\right)\right). \qquad \text{(A2)}
\end{aligned}
$$

We evaluate next the $Cov\left(z\left(t\right),z\left(t-\Delta\right)\right)$ term in the last equality of expression (A2): First, due to the (weak) stationarity of the $z\left(t\right)$ process, it follows that $Cov\left(z\left(t\right),z\left(t-\Delta\right)\right) = Cov\left(z\left(t\right),z\left(t+\Delta\right)\right)$, which in turn is equal to: $Cov\left(z\left(t\right),z\left(t+\Delta\right)\right) = E\left(z\left(t\right)z\left(t+\Delta\right)\right)-E\left(z\left(t\right)\right)E\left(z\left(t+\Delta\right)\right)$. Second, substituting in the last equation the solution for $z\left(t+\Delta\right)$ in eq.(7) after multiplying it by $z\left(t\right)$, using the definition for $\epsilon_{t+\Delta}$ in eq. (11), and observing that the unconditional mean $E\left(z\left(t\right)\right)=E\left(z\left(t+\Delta\right)\right)=\gamma$ from eq.(15), we obtain:

$$
\begin{aligned}
& Cov\left(z\left(t\right),z\left(t+\Delta\right)\right)= \\
= \;& E\left[\gamma\left(1-e^{-\beta\Delta}\right)z\left(t\right)+e^{-\beta\Delta}\left(z\left(t\right)\right)^2+z\left(t\right)\epsilon_{t+\Delta}\right]-\gamma^2 \\
= \;& \gamma^2\left(1-e^{-\beta\Delta}\right)+e^{-\beta\Delta}\left(\frac{\rho^2}{2\beta}+\gamma^2\right)-\gamma^2 \\
= \;& e^{\beta\Delta}\frac{\rho^2}{2\beta}, \qquad \text{(A3)}
\end{aligned}
$$

where in the second equality above we used the simple result: $E\left(z\left(t\right)^2\right) = Var\left(z\left(t\right)\right)+\left[E\left(z\left(t\right)\right)\right]^2$, and we substituted for the unconditional variance of

$z(t)$ given by expression (16). We also used the fact that $E(\epsilon_{t+\Delta}) = 0$.

Substituting expressions (16) and (A3) for $Var(z(t))$ and $Cov(z(t), z(t-\Delta))$ respectively, in the last equality of eq. (A2) we obtain:

$$
\begin{aligned}
& Cov(z(t+\Delta) - z(t), z(t) - z(t-\Delta)) = \\
= \quad & (e^{-\beta\Delta} - 1)\frac{\rho^2}{2\beta} - \left(e^{-\beta\Delta} - 1\right) e^{-\beta\Delta}\frac{\rho^2}{2\beta} \\
= \quad & -\left(e^{-\beta\Delta} - 1\right)^2 \frac{\rho^2}{2\beta},
\end{aligned}
\tag{A4}
$$

which is the second covariance term of the numerator in expression (A1).

The second variance term in the denominator of expression (A1) is evaluated as follows: First, due to the stationarity of the $z(t)$ process, it follows that: $Var(z(t) - z(t-\Delta)) = Var(z(t+\Delta) - z(t))$, which after substitution from expression (7) and using the definition of $\epsilon_{t+\Delta}$ in (11) becomes

$$
Var(z(t+\Delta) - z(t)) = \left(e^{-\beta\Delta} - 1\right)^2 Var(z(t)) + Var(\varepsilon_{t+\Delta}).
$$

Second, substituting in the equation above the expressions for $Var(z(t))$ and

$Var\left(\epsilon_{t+\Delta}\right)$ given by eq. (16) and (12) respectively, we obtain:

$$\begin{aligned} Var\left(z\left(t+\Delta\right)-z\left(t\right)\right) &= \left(e^{-\beta\Delta}-1\right)^2\frac{\rho^2}{2\beta}+\frac{\rho^2}{2\beta}\left(1-e^{-\beta\Delta}\right) \\ &= -\frac{\rho^2}{\beta}\left(e^{-\beta\Delta}-1\right). \end{aligned} \tag{A5}$$

Now we concentrate on the evaluation of the terms

$Cov\left(q\left(t+\Delta\right)-q\left(t\right), q\left(t\right)-q\left(t-\Delta\right)\right)$ and $Var(q\left(t\right)-q\left(t-\Delta\right))$ which are related to the random walk (permanent) component of the returns process. Using expression (6) we obtain:

$$q\left(t+\Delta\right)-q\left(t\right)=\alpha\Delta+\sigma\int_t^{t+\Delta}dW_1\left(\tau\right) \tag{A6}$$

and

$$q\left(t\right)-q\left(t-\Delta\right)=\alpha\Delta+\sigma\int_{t-\Delta}^{t}dW_1\left(\tau\right). \tag{A7}$$

Substituting expressions (A6) and (A7) in $Cov\left(q\left(t+\Delta\right)-q\left(t\right), q\left(t\right)-q\left(t-\Delta\right)\right)$

it follows that:

$$\begin{aligned}
& Cov\left(q(t+\Delta)-q\left(t\right),q\left(t\right)-q\left(t-\Delta\right)\right) \\
= \; & Cov\left(\alpha\Delta+\sigma\int_{t}^{t+\Delta}dW_{1}\left(\tau\right),\alpha\Delta+\sigma\int_{t-\Delta}^{t}dW_{1}\left(\tau\right)\right) \\
= \; & Cov\left(\sigma\int_{t}^{t+\Delta}dW_{1}\left(\tau\right),\sigma\int_{t-\Delta}^{t}dW_{1}\left(\tau\right)\right)=0, \qquad \text{(A8)}
\end{aligned}$$

since non-overlapping increments of standard Brownian motion are independent.

Next, using expression (A7) we have

$$Var\left(q\left(t\right)-q\left(t-\Delta\right)\right)=Var\left(\alpha\Delta+\sigma\int_{t-\Delta}^{t}dW_{1}\left(\tau\right)\right)=\sigma^{2}\Delta \qquad \text{(A9)}$$

Substituting expressions (A4), (A5), (A8), and (A9) in eq. (A1), we obtain after simple rearrangements:

$$\hat{\lambda}_{\Delta}=\frac{-\left(e^{-\beta\Delta}-1\right)^{2}\frac{\rho^{2}}{2\beta}}{-\frac{\rho^{2}}{\beta}\left(e^{-\beta\Delta}-1\right)+\sigma^{2}\Delta}$$

■

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Figure 1: This figure depicts the effect of the mean reverting parameter on the autocorrelation coefficient as the latter varies with the investment horizon

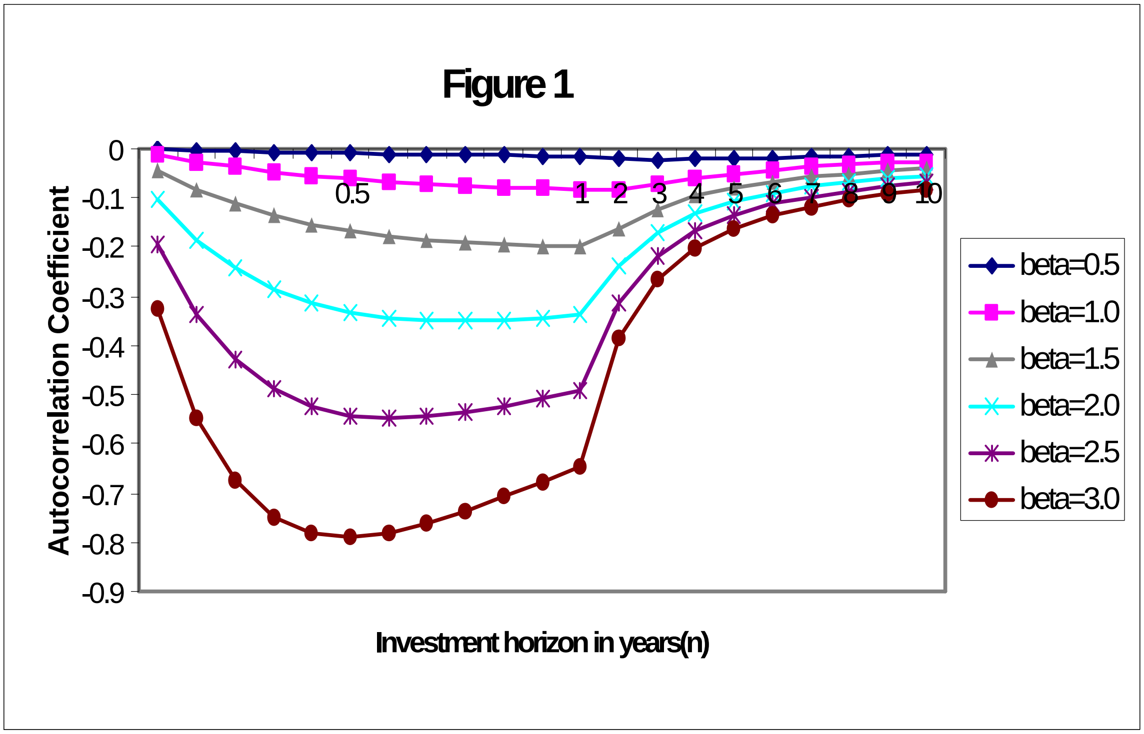

Table 1: Formulae for the Recovery of the Continuous Time Parameters

The continuous time parameters of the model (1)-(3) are reported below together with their descriptions and the formulae used for their identification and recovery.

Parameter	**Description**	**Identifying Formula**
β	adjustment speed of temporary component	$-\ln\left\{\left[\frac{Cov(r(t,t+2\Delta),r(t-2\Delta,t))}{Cov(r(t,t+\Delta),r(t-\Delta,t))}\right]^{\frac{1}{2}}-1\right\}$
ρ	instantaneous stdev of temporary component	$\left\{\left[\frac{2\beta Cov(r(t,t+\Delta),r(t-\Delta,t))}{(e^{-\beta\Delta}-1)^2}\right]^{\frac{1}{2}}\right\}$
σ	instantaneous stdev of permanent component	$\left\{\frac{Cov(r(t,t+\Delta),r(t-\Delta,t))}{\hat{\lambda}_\Delta}+\frac{\rho^2}{\beta}(e^{-\beta\Delta}-1)\right\}^{\frac{1}{2}}$
α	instantaneous mean of permanent component	$E\left(r\left(t,t+2\Delta\right)\right)-E\left(r\left(t,t+\Delta\right)\right)$
γ	long-run mean of temporary component	$E\left(r\left(t,t+\Delta\right)\right)-\alpha$

TABLE 2: Summary Statistics

Summary statistics are reported for non-overlapping continuously compunded returns for all equity indices included in our sample. Daily data are used for all countries from 01/01/1983 to 01/01/2001 except for the US where weekly returns are employed.

	Mean	**Min**	**Max**	**Stdev.**	**Skew.**	**Kurt.**	**DF**
CANADA	0.0004	−0.1165	0.0876	0.0079	−1.1824	22.274	−45.68*
FRANCE	0.0007	−0.0986	0.0806	0.0111	−0.5431	6.1185	−61.62*
GERMANY	0.0005	−0.1264	0.0670	0.0118	−0.6840	7.9162	−65.00*
ITALY	0.0005	−0.0843	0.0840	0.0127	−0.2054	3.8559	−60.44*
JAPAN	0.0001	−0.1614	0.1243	0.0128	−0.1933	10.862	−60.51*
SINGAP.	0.0002	−0.2640	0.1399	0.0125	−2.4456	58.685	−56.63*
SPAIN	0.0006	−0.0973	0.0694	0.0111	−0.4304	6.4243	−55.96*
SWITZ.	0.0005	−0.1231	0.0662	0.0091	−1.6377	21.221	−62.36*
UK	0.0005	−0.1301	0.0649	0.0088	−1.1416	16.099	−59.78*
US	0.0028	−0.3049	0.1158	0.0242	−2.3588	28.942	−49.73*

Note: The DF statistic in the last collumn refers to the Augmented Dickey Fuller statistic which tests for stationarity of equity index returns. * indicates rejection of the null hypothesis of non-stationarity at the 1% significance level.

TABLE 3a: Continuous-Time Parameters (dividend exclusive)

The continuous-time parameters for the seven national stock indices when index returns do not include dividends are reported below. T-ratios are given in parenthesis below the estimated coefficients. Standard errors were calculated using the Delta Method and are adjusted for heteroscedasticity.

	$\boldsymbol{\beta}$	$\boldsymbol{\rho}$	$\boldsymbol{\sigma}$	$\boldsymbol{\alpha}$	γ
CANADA	2.2319*** (4.718)	0.0074*** (4.841)	0.0064*** (15.56)	0.0003	0.0000
FRANCE	2.6839** (3.577)	0.0082* (1.810)	0.0093*** (19.45)	0.0007	0.0000
GERMANY	1.9308** (2.124)	0.0052* (1.747)	0.0113*** (19.85)	0.0005	0.0000
ITALY	3.7051*** (6.596)	0.0116 (0.804)	0.0114*** (25.51)	0.0004	0.0000
JAPAN	−0.8432 (−0.082)	0.0009 (1.300)	0.0128*** (3.792)	0.0002	0.0000
UK	2.0539*** (2.734)	0.0063*** (3.998)	0.0078*** (18.93)	0.0003	0.0000
US	1.5325*** (3.408)	0.0171 (0.594)	0.0217*** (18.92)	0.0028	0.0000

Note: ***, **, and * denote signifiance at the 1, 5, and 10 percent levels respectively.

TABLE 3b: Continuous-Time Parameters (dividend inclusive)

The continuous-time parameters for the seven national stock indices when index returns include dividends are reported below. T-ratios are given in parenthesis below the estimated coefficients. Standard errors were calculated using the Delta Method and are adjusted for heteroscedasticity.

	$\boldsymbol{\beta}$	$\boldsymbol{\rho}$	$\boldsymbol{\sigma}$	$\boldsymbol{\alpha}$	$\boldsymbol{\gamma}$
CANADA	2.1964*** (4.638)	0.0072*** (5.090)	0.0062*** (15.47)	0.0003	0.0000
FRANCE	2.7021*** (3.542)	0.0080* (1.924)	0.0110*** (19.57)	0.0007	0.0000
GERMANY	1.8608** (2.080)	0.0051* (1.741)	0.0111*** (19.80)	0.0005	0.0000
ITALY	3.6010*** (6.390)	0.0112 (0.898)	0.0110*** (25.51)	0.0004	0.0000
JAPAN	−0.7627 (−0.084)	0.0011 (1.219)	0.0127*** (4.080)	0.0002	0.0000
UK	2.1600*** (2.880)	0.0061*** (3.391)	0.0075*** (19.24)	0.0003	0.0000
US	1.4927*** (3.410)	0.0169 (0.594)	0.0219*** (18.93)	0.0028	0.0000

Note: ***, **, and * denote signifiance at the 1, 5, and 10 percent levels respectively.

TABLE 4: Dynamic Simulations

Dynamic Simulation Results for the seven national stock indices are reported below. The percentage of return variation attributable to the stationary component for the relevant countries is reported, as well as the mean squared error when actual returns are compared with returns simulated using the model $(1)-(3)$.

	%of return variation due to stationary component	**Mean Squared Error**
CANADA	34.8	0.0018
FRANCE	21.3	0.0002
GERMANY	8.57	0.0012
ITALY	21.4	0.0014
JAPAN	0.01	0.0003
UK	21.7	0.0009
US	24.3	0.0001

Note: Since $r(t, t+\Delta =) [q(t+\Delta) - q(t)] + [z(t+\Delta) - z(t)]$ (see expression (17)), then $Var[r(t,t+\Delta)] = Var[q(t+\Delta) - q(t)] + Var[z(t+\Delta) - z(t)] = \sigma^2\Delta - \frac{\rho^2}{\beta}\left(e^{-\beta\Delta} - 1\right)$

from expressions (A4) and (A9) respectively. Therefore, the share of return variation due to the stationary component is equal to $1 - \frac{\sigma^2\Delta}{\sigma^2\Delta - \frac{\rho^2}{\beta}\left(e^{-\beta\Delta}-1\right)}$