The persistence and rigidity of wages and prices*

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Abstract

We analyze welfare effects of monetary policy rules when both prices and wages are sticky using the linear-quadratic framework of Rotemberg and Woodford (1997). .... Among some simple rules that perform reasonably well across structural assumptions is a first difference version of the classical rule proposed by Henderson and McKibbin (1993) and Taylor (1993).

1 Introduction

The Philips Curve has been a central piece in macroeconomics for decades. The original work by Philips related wage inflation to unemployment. Starting from microfounded models with sluggish price adjustment, modern macroeconomics has derived the so called New Keynesian Philips curve relating price inflation to marginal cost and expected future price inflation. This relationship is at the center of large strand of literature, studying welfare based optimal monetary policy summarized in Woodford (2003, Chapter 6).

Since the seminal paper by Erceg, Henderson, and Levin (2000), it is well known that the joint analysis of sticky prices and wages has important implications for the conduct of monetary policy. In particular, a trade off between the stabilization of wage inflation, price inflation and the output gap exists for any kind of shock hitting the economy. Recently, a number of important contributions

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have pointed to wage stickiness as a crucial feature that allows monetary general equilibrium models to match the data. In this sense, empirical studies emphasizing wage rigidities, like Christiano, Eichenbaum, and Evans (2004) or Smets and Wouters (2003) render the joint analysis of sticky wages and sticky prices highly relevant.

Fuhrer and Moore (1995), Fuhrer (1997) and a number of other authors have criticized the New Keynesian Philips curve for its inability to capture the persistence in price inflation and for putting too much emphasis on forward looking behavior. In response Galí and Gertler (1999) have included a fraction of rule of thumb price setters into the standard New Keynesian setup, that gives rise to the so-called hybrid Philips curve. The hybrid Philips curve relates current inflation to last periods inflation, marginal cost and future inflation allowing for much more persistence in inflation.

An issue largely neglected until now is the joint analysis of hybrid wage and price Philips curves for optimal monetary policy.[1] What role does rule of thumb behavior in both wage and price setting play for optimal monetary policy? How is the performance of simple monetary policy rules affected by the fraction of backward looking agents in price and wage setting. We take up the question of optimal monetary policy with sticky wages and prices posed by Erceg, Henderson, and Levin (2000) and allow for backward looking rule of thumb behavior in both price and wage setting. Our hybrid price and wage Philips curves are estimated on Euro area and U.S. data via GMM. The estimates of the structural parameters are then used in a number of policy questions.[2]

We derive a purely quadratic welfare based loss function from the model and compute fully optimal monetary policy under commitment. We show analytically that a key parameter governing the relative weight on wage inflation variability versus price inflation variability is the Frish elasticity of labor supply. For a range of plausible values taken from micro-econometric estimates between 0.25 and 2, the weight on the variance of wage inflation relative to the weight on the variance of price inflation extends from 15 down to 1.5. Given that this parameter is crucial for optimal monetary policy, it is very unfortunate that it is typically very imprecisely estimated in macro models as well as in micro studies.

The paper proceeds as follows. Section 2 describes the setup of the model which is similar to Erceg, Henderson, and Levin (2000). Section 3 derives the hybrid wage and price Philips curves from a measure of backward looking wage and price setters and summarizes the key equations determining general equilib-

[1] Steinsson (2003) as well as Amato and Laubach (2003b) analyze optimal monetary policy with fully flexible wages and a hybrid price Philips curve.

[2] Amato and Laubach (2003a) have estimated parameters governing the wage and price Philips curves in a fully specified general equilibrium model, but did not allow for rule of thumb wage and price setters.

rium in the model. In section 4 we present our baseline calibration and discuss the crucial parameters that affect the loss function. Section 5 analyzes the welfare effects of fully optimal monetary policy as well as of certain popular simple rules for varying degrees of backward and forward looking wage and price setters. Section 7 summarizes the findings and concludes. Derivations of the proposition are deferred to the appendix.

2 Model

The model we consider is very similar in its key building blocks to the one in Erceg, Henderson, and Levin (2000), except for rule of thumb wage and price setters. In particular, capital is in fixed supply in the aggregate. We assume that there exists an economy wide rental market that allows capital to freely move between firms. We abstract from aggregate capital accumulation, because the derivation of a welfare based loss function for the central bank becomes extremely cumbersome with aggregate capital accumulation. Edge (2003) shows that the loss function in such a case additionally involves the variance of the investment gap, the covariance of the investment gap with the output gap and all future autocovariances of the investment gap. The assumption of perfect mobility of capital is not innocuous, either. It implies that real marginal cost is the same for all firms and independent of the quantity produced by any single firm. We relax this assumption in our robustness check. Finally, we assume that subsidies exists that completely offset the effects of monopolistic competition in the steady state. The assumption that the economy is efficient in the steady state is again chosen in order to avoid highly cumbersome derivations of the loss function that would arise with an inefficient steady state.[3] In the next subsection we start with a discussion of the households problem.

2.1 Households

There is a continuum of households with unit mass indexed by h. Households are infinitely lived, supply labor $N_t(h)$ and receive nominal wage $W_t(h)$, consume final goods $C_t(h)$, purchase state contigent securities $B_t(h)$. Furthermore, they are subject to lump sum transfers T_t, hold nominal money balances $M_t(h)$ and receive profits $\Gamma(h)_t$ from the monopolistic retailers. The utility function is assumed to be separable in consumption, real money balances and leisure. The representative

[3] This highly demanding task has recently accomplished by Benigno and Woodford (2004).

household's problems is:

$$
\begin{aligned}
&\max_{B_{t+1},M_{t+1},C_t,N_t} E_t \sum_{i=0}^{\infty} \beta^{t+i} \left[U(C_{t+i}(h)) + H\left(\frac{M_{t+i}(h)}{P_{t+i}}\right) + V(N_{t+i}(h)) \right] \\
&\text{s.t.} \quad C_t(h) = \frac{\delta_{t+1,t} B_t(h) - B_{t-1}(h)}{P_t} + (1+\tau_w)\frac{W_t(h)}{P_t} N_t(h) + T_t(h) + \Gamma_t(h) \\
&\qquad\qquad - \frac{M_{t+1}(h) - M_t(h)}{P_t}.
\end{aligned}
$$

Here B_t is a row vector of state contingent bonds, where each bond pays one unit in a particular state of nature in the subsequent period. The column vector $\delta_{t+1,t}$ represents the price of these bonds. Therefore, the inner product gives total expenditures for state contingent bonds. B_{t-1} is number of state contingent bonds that pay off in the particular state of nature at time t. The first order conditions for consumption and state contingent bond holdings give rise to the standard Euler equation. Note that consumption is perfectly insured against idiosyncratic labor income and therefore consumption is no longer indexed by h.

$$
U_C(C_t) = E_t \beta \left\{ \frac{P_t}{P_{t+1}} U_c(C_{t+1}) \right\} R_t^n. \tag{1}
$$

We follow the standard practice to omit the first-order condition for money holdings as this equation merely serves to back out the quantity of money that supports a given nominal interest rate.

The following functional forms are used in later parts of the analysis

$$
U(C_t) = \frac{C_t^{1-\sigma}}{1-\sigma} \tag{2}
$$

$$
V(N_t) = -\frac{N_t^{1+\chi}}{1+\chi} \tag{3}
$$

A continuum of households supply differentiated labor $N_t(h)$, which is aggregated according to the Dixit-Stiglitz form:

$$
L_t = \left[\int_0^1 [N_t(h)]^{\frac{\kappa-1}{\kappa}} \, dh \right]^{\frac{\kappa}{\kappa-1}}. \tag{4}
$$

Here W_t is the standard Dixit-Stiglitz index. The demand function for differentiated labor is:

$$
N_t(h) = \left[\frac{W_t(h)}{W_t} \right]^{-\kappa} L_t \tag{5}
$$

With probability θ_w a randomly chosen household is allowed to set its nominal wage in a given period. The household maximizes expected utility through choice of the nominal wage subject to the demand curve and the budget constraint. The FOC for this problem is:

$$E_t \sum_{j=0}^{\infty} (\theta_w \beta)^j N_{t+j}(h) U_C(C_{t+j}) \left[(1+\tau_w) \frac{W_t^*(h)}{P_{t+j}} + \frac{\kappa}{\kappa - 1} \frac{V_N(N_{t+j}(h))}{U_C(C_{t+j})} \right] = 0 \tag{6}$$

2.2 Production

Firms in the final good sector produce a homogeneous good, Y_t, using intermediate goods, $Y_t(z)$. There are a continuum of intermediate goods a measure unity. The production functions that transforms intermediate goods into final output is given by

$$Y_t = \left[\int_0^1 Y_t(z)^{\frac{\epsilon-1}{\epsilon}} dz \right]^{\frac{\epsilon}{\epsilon-1}} \tag{7}$$

where $\epsilon > 1$. The solution to the problem of optimal factor demand yields the following constant price elasticity demand function for variety z.

$$Y_t(z) = \left(\frac{P_t(z)}{P_t} \right)^{-\epsilon} Y_t \tag{8}$$

A continuum of monopolistically competitive intermediate goods firms owned by consumers indexed by $z \in [0,1]$ uses both labor $L_t(z)$ and capital $K_t(z)$ to produce output according to the following constant returns technology:

$$Y_t(z) = A_t L_t(z)^{1-\alpha} K_t(z)^{\alpha} \tag{9}$$

where A_t is a technology parameter. Capital is freely mobile across firms rather than being firm specific. Firms rent capital from households in a competitive market on a period by period basis after they observe the productivity shock. Firm z chooses $L_t(z)$ and $K_t(z)$ to minimize total cost subject to meeting demand

$$W_t P_t L_t(z) + Z_t K_t(z) \quad \text{s.t.} \quad A_t L_t(z)^{1-\alpha} K_t(z)^{\alpha} - Y_t = 0. \tag{10}$$

Let X_t denote the Lagrange multiplier with respect to the constraint and w_t^r the real wage. The first order conditions with respect to $L_t(z)$ and $K_t(z)$ are given by

$$w_t^r = (1-\alpha) X_t A_t K(z)_t^{\alpha} L(z)_t^{-\alpha} \tag{11}$$

$$Z_t = \alpha X_t A_t K(z)_t^{\alpha-1} L(z)_t^{1-\alpha} \tag{12}$$

The first order conditions imply that inputs adjust to equalize marginal cost across different factors, where the marginal cost of a factor is the ratio of the factor price to the marginal product. Since all firms choose the same capital to labor ratio, marginal cost is equalized across firms. This will not be true for the case of firm specific capital, where the immobility of capital across firms prevents firms from choosing equal capital to labor ratios.

For price setting, we follow the widely used time dependent pricing approach of Calvo (1983). In any give period, there is constant probability θ of receiving a signal that allows the firm to reset its price. Is the random signal not received, the firm carries on the price posted in the last period and satisfies any demand at that price.[4]

The problem of a firm that receives a signal to change its price in period t is to maximize expected real profits as valued by the household in those states of the world where the price remains fixed.[5]

$$\max_{P_t^*(z)} \mathrm{E}_t \sum_{i=0}^{\infty} (\theta\beta)^i \Lambda_{t+i} \left\{ (1+\tau_p) \left[\frac{P_t^*(z)}{P_{t+i}} \right]^{1-\epsilon} Y_{t+i} - X_{t+i} \left[\frac{P_t^*(z)}{P_{t+i}} \right]^{-\epsilon} Y_{t+i} \right\} \tag{13}$$

Here, Λ_t is the households marginal utility of consumption i periods from now and τ_t is sales subsidy suitably chosen as to offset the steady state effects of monopolistic competition $(1+\tau_p = \frac{\epsilon}{\epsilon-1})$. The first order condition is

$$\mathrm{E}_t \sum_{i=0}^{\infty} (\theta\beta)^i \Lambda_{t+i} \left\{ (1+\tau_p) P_t^*(z)(1-\epsilon) \left[P_{t+i} \right]^{\epsilon-1} Y_{t+i} + \epsilon X_{t+i} P_{t+i}^{\epsilon} Y_{t+i} \right\} \tag{14}$$

2.3 The flexible price solution and the gaps

In order to obtain welfare in terms of an output gap, it is useful to consider the solution under perfectly flexible prices. As shown in proposition 1 in the appendix, up to a first order approximation flexible price output is given by

$$\widehat{Y}_t^* = \left[\frac{1+\omega_2}{\omega_2 + \alpha - (1-\alpha)\omega_1} \right] \widehat{A}_t \tag{15}$$

Here $\omega_1 \equiv \frac{U_{CC}\bar{C}}{U_C}$ is the elasticity of the marginal utility of consumption evaluated at the steady state and $\omega_2 \equiv \frac{V_{NN}\bar{N}}{V_N}$ is the elasticity of the marginal utility of labor.

[4] A markup of price over marginal cost is necessary to ensure that for small positive demand shocks, the firm still makes positive profits on the marginal units demanded despite increasing marginal cost.

[5] Here we have made use of properties of the Cobb-Douglas Production function, rewriting total cost as marginal cost times production.

The subutility functions $U(C_t) \equiv \frac{C_t^{1-\sigma}}{1-\sigma}$ implies $\omega_1 = -\sigma$ and for $V(N_t) \equiv -\frac{N_t^{1+\chi}}{1+\chi}$ we have $\omega_2 = \chi$. Given these functional forms, the natural level of output in log-deviation is given by

$$\widehat{Y}_t^* = \left[\frac{1+\chi}{\chi + \alpha + (1-\alpha)\sigma}\right] \widehat{A}_t \tag{16}$$

One can use this equation together with the firm's first-order condition for labor demand, to derive a key equation for this model. That equation links marginal cost to the output gap and the gap between the average marginal rate of substitution between consumption and labor and the real wage.

$$\widehat{X}_t = \left[\frac{\chi+\alpha}{1-\alpha} + \sigma\right] \left(\widehat{Y}_t - \widehat{Y^*}_t\right) - \left[\chi \widehat{L}_t + \sigma \widehat{Y}_t - \widehat{w^r}_t\right] \tag{17}$$

When there are no nominal rigidities in the labor market, the marginal rate of substitution between consumption and labor is equal to the real wage and the last term in brackets vanishes. We then recover the condition from sticky price models that marginal cost is log-linearly related to the output gap. When additionally, prices are perfectly flexible, the marginal product of labor is equal to the real wage, i.e. log marginal cost is zero and it follows that the output gap is zero. These two gaps, the difference between the real wage and the marginal product of labor and the difference between the real wage and the marginal rate of substitution are at the center of the welfare analysis.[6]

3 Hybrid wage and price Philips curves

We allow for backward looking elements in the wage and price setting as proposed by Galí and Gertler (1999) and Galí, Gertler, and Lopez-Salido (2001). In particular, we assume that price setters that do receive a signal to re-set their price belong to one of two groups. A measure ω of backward looking firms set their price according to the following rule of thumb

$$P_t^b = \pi_{t-1} \left(P_{t-1}^*\right)^{1-\omega} \left(P_{t-1}^b\right)^{\omega} \tag{18}$$

The rule posits that these firms adjust prices according a geometric average of prices changed last period adjusted for last periods inflation rate. The consumption based price index is given by

$$P_t \equiv \left[\int_0^1 P_t(z)^{1-\epsilon} dz\right]^{\frac{1}{1-\epsilon}} \tag{19}$$

[6] See Gali, Gertler, and Lopez-Salido (2003) for estimating the cost of business cycle variations on the basis of these gaps.

Since the fraction of firms that can change the price is chosen randomly and by the law of large numbers, the aggregate price index evolves as

$$P_t^{1-\epsilon} = \theta P_{t-1}^{1-\epsilon} + (1-\theta)(1-\omega)(P_t^*)^{1-\epsilon} + (1-\theta)\omega(P_t^b)^{1-\epsilon} \tag{20}$$

As shown in proposition 3 in the appendix, this setup gives rise to the following hybrid new Keynesian Philips curve

$$\widehat{\pi}_t = \frac{(1-\omega)(1-\theta)(1-\beta\theta)}{\zeta}\widehat{X}_t + \frac{\beta\theta}{\zeta}\mathrm{E}_t\widehat{\pi}_{t+1} + \frac{\omega}{\zeta}\widehat{\pi}_{t-1} \tag{21}$$

Here $\zeta \equiv \theta + \omega[1-\theta(1-\beta)]$. If we assume that capital is no longer freely mobile, but firm specific, proposition 4 in the appendix shows that the Philips Curve is given by

$$\widehat{\pi}_t = \frac{(1-\omega)(1-\theta)(1-\beta\theta)}{\zeta}\frac{(1-\alpha)}{(1-\alpha+\alpha\epsilon)}\widehat{X}_t^a + \frac{\beta\theta}{\zeta}\mathrm{E}_t\widehat{\pi}_{t+1} + \frac{\omega}{\zeta}\widehat{\pi}_{t-1} \tag{22}$$

Here, $\hat{X}_t^a$ is average marginal cost. Since capital is fixed at the firm level, the capital-labor ratio differs across firms and so does marginal cost. For the optimal monetary policy analysis, we will consider both common and firm specific capital.

Similarly for wage setting, we assume that those households that do receive a signal to re-set their wages belong to one of two groups. A measure φ of backward looking households set their wage according to the following rule of thumb

$$W_t^b = \pi_{t-1}^w \left(W_{t-1}^*\right)^{1-\varphi} \left(W_{t-1}^b\right)^{\varphi} \tag{23}$$

The wage index is defined as

$$W_t \equiv \left[\int_0^1 W_t(h)^{1-\kappa} dh\right]^{\frac{1}{1-\kappa}} \tag{24}$$

Since the fraction of wage setters that receive the signal to change their wage is randomly chosen and by the law of large numbers, the aggregate wage index evolves according to the formula

$$W_t^{1-\kappa} = \theta_w W_{t-1}^{1-\kappa} + (1-\theta_w)(1-\varphi)\left(W_t^*\right)^{1-\kappa} + (1-\theta_w)\varphi\left(W_t^b\right)^{1-\kappa} \tag{25}$$

As shown in proposition 2 in the appendix This setup gives rise to a hybrid new Keynesian wage Philips curve

$$\widehat{\pi^w}_t = \frac{(1-\varphi)(1-\theta_w)(1-\beta\theta_w)}{(1+\kappa\chi)\zeta_w}\widehat{\mu}_t + \frac{\beta\theta_w}{\zeta_w}\mathrm{E}_t\widehat{\pi^w}_{t+1} + \frac{\varphi}{\zeta_w}\widehat{\pi^w}_{t-1} \tag{26}$$

Here $\zeta_w \equiv \theta_w + \varphi[1-\theta_w(1-\beta)]$ and $\widehat{\mu}_t \equiv \chi\widehat{L}_t + \sigma\widehat{C}_t - \widehat{w^r}_t$.

3.1 The key equations

The model has 6 endogenous variables: price inflation π_t, wage inflation π_t^w, labor L_t, output Y_t, marginal cost X_t, and the real wage w_t^r. We treat the rate of price inflation as the central bank's instrument. The policymakers problem is to maximize the welfare measure through choice of the inflation rate subject to the following constraints. [7]

$$\widehat{\pi^w}_t = \frac{(1-\varphi)(1-\theta_w)(1-\beta\theta_w)}{(1+\kappa\chi)\zeta_w}\widehat{\mu}_t + \frac{\beta\theta_w}{\zeta_w}\mathrm{E}_t\widehat{\pi^w}_{t+1} + \frac{\varphi}{\zeta_w}\widehat{\pi^w}_{t-1} \tag{27}$$

$$\widehat{\pi}_t = \frac{(1-\omega)(1-\theta)(1-\beta\theta)}{\zeta}\widehat{X}_t + \frac{\beta\theta}{\zeta}\mathrm{E}_t\widehat{\pi}_{t+1} + \frac{\omega}{\zeta}\widehat{\pi}_{t-1} \tag{28}$$

$$\widehat{Y}_t = \widehat{A}_t + (1-\alpha)\widehat{L}_t \tag{29}$$

$$\widehat{w^r}_t = \widehat{X}_t + \widehat{A}_t - \alpha\widehat{L}_t \tag{30}$$

$$\Delta\widehat{w^r}_t = \pi_t^w - \pi_t \tag{31}$$

$$\widehat{X}_t = \left[\frac{\chi+\alpha}{1-\alpha} + \sigma\right]\left(\widehat{Y}_t - \widehat{Y^*}_t\right) - \left[\chi\widehat{L}_t + \sigma\widehat{Y}_t - \widehat{w^r}_t\right] \tag{32}$$

$$\hat{A}_t = \rho\hat{A}_{t-1} + u_t \tag{33}$$

Here:

$$\begin{aligned}
\widehat{\mu}_t &\equiv \chi\widehat{L}_t + \sigma\widehat{Y}_t - \widehat{w^r}_t \\
\zeta_w &\equiv \theta_w + \varphi[1-\theta_w(1-\beta)] \\
\zeta &\equiv \theta + \omega[1-\theta(1-\beta)]
\end{aligned}$$

Upper case letters denote the aggregate of the respective lower case variables. (27) and (27) are the wage and price Philips curves. (29) is the log-linearized production function. It has been pointed out by Yun (1996) that the full non-linear aggregate production function depends on a price dispersion term.

$$Y_t = \frac{A_t}{D_t}\bar{K}^\alpha L_t^{1-\alpha} \quad \text{with:} \quad D_t \equiv \int_0^1 \left(\frac{P_t(z)}{P_t}\right)^{-\epsilon} dz \quad \text{and} \quad \hat{D}_t = \theta\hat{D}_{t-1} \tag{34}$$

[7] The consumption Euler equation does not impose a constraint on the policymaker. It serves merely to back out the path of the nominal instrument that supports the optimal allocation for given optimal paths of output and inflation.

Christiano, Eichenbaum, and Evans (2001) have shown that the price dispersion term can be ignored for a loglinear analysis around a steady state with zero price dispersion. One can further show that this term evolves as a univariate AR(1) regardless of the fraction of backward looking price setters by log-linearizing the price index and the price dispersion term. (30) is the firm's labor demand function. (31) is an identity defining the change the of the real wage. (31) links marginal cost to the output gap and the "wage gap". That wage gap is the difference between the average marginal rate of substitution between consumption and labor on the hand and the real wage on the other, see proposition 1 in the appendix. Finally, the last equation is the exogenous stochastic process is total factor productivity.

The welfare measure of the central bank is expected discounted lifetime utility of a randomly drawn household. As is common in the literature, we neglect the arbitrarily small utility flow from real money balances.

$$E_0 \sum_{j=0}^{\infty} \beta^j \mathbb{W}_{t+j} \equiv E_0 \sum_{j=0}^{\infty} \beta^j \left\{ U(C_{t+j}) + \int_0^1 V(N_{t+j}(h))dh \right\}. \tag{35}$$

where the unconditional expectation E averages across all possible histories of aggregate shocks. Let $\mathbb{W}_t^*$ denote period utility under perfectly flexible wages and prices. Following proposition 5 in the appendix, the consumption equivalent welfare measure $\mathbb{L} \equiv -\sum_{t=0}^{\infty} \beta^t \left(\mathbb{W}_t - \mathbb{W}_t^*\right) / \left(U_C \bar{C}\right)$ can be approximated up to second order by the following weighted sum of second moments.

$$\mathbb{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left[\tilde{\lambda}_0 \hat{\pi}_t^2 + \tilde{\lambda}_1 \left(\hat{Y}_t - \hat{Y}_t^* \right)^2 +^2 + \tilde{\lambda}_2 \left(\Delta \widehat{\pi}_t\right)^2 + \tilde{\lambda}_3 \widehat{\pi^w}_t^2 + \tilde{\lambda}_4 \left(\Delta \widehat{\pi^w}_t\right)^2 \right] \tag{36}$$

Since this loss function is free of first moments, it can be accurately evaluated by considering a linear approximation to the models equilibrium conditions. Here, the weights are given by

$$\tilde{\lambda}_0 = 0.5\epsilon \frac{\theta}{(1-\theta)(1-\theta\beta)} \tag{37}$$

$$\tilde{\lambda}_1 = 0.5 \left(\frac{\chi + \alpha}{1-\alpha} + \sigma \right) \tag{38}$$

$$\tilde{\lambda}_2 = \frac{\omega}{(1-\omega)\theta} \tilde{\lambda}_0 \tag{39}$$

$$\tilde{\lambda}_3 = 0.5\kappa^2 (1-\alpha)(\kappa^{-1} + \chi) \frac{\theta_w}{(1-\theta_w)(1-\theta_w\beta)} \tag{40}$$

$$\tilde{\lambda}_4 = \frac{\varphi}{(1-\varphi)\theta_w} \tilde{\lambda}_3 \tag{41}$$

For the case of immobile capital, $\tilde{\lambda}_0 = \frac{1}{2}\left[\frac{1}{1-\alpha} - \frac{\epsilon-1}{\epsilon}\right]\epsilon^2 \frac{\theta}{(1-\theta)(1-\theta\beta)}$ and $\tilde{\lambda}_2$ adjust accordingly. Any number for the loss function has no economic interpretation. However, the difference between two numbers when comparing alternative policy rules is an approximation of the one off increase in consumption (as a fraction of steady state consumption) necessary to make an average agent equally well off under both policies. [8]

4 Calibration

We use the baseline calibration from the sticky price model of Pappa (2004) and assume symmetric price and wage setting parameters. In particular, the markup is 14 % in both goods and labor market resulting in $\kappa = \epsilon = 7.88$. Furthermore assume that prices and wages are fixed on average 4 quarters, such that $\theta = \theta_w = \frac{3}{4}$. Set the fraction of backward looking agents in both price and wage setting to 0. Furthermore assume the coefficient of relative risk aversion $\sigma = 2$ and assume a Frisch (constant marginal utility of wealth) elasticity of labor supply of $\frac{1}{3}$, implying $\chi = 3$. The exogenous process for technology follows an AR(1) with autoregressive parameter equal to 0.906. The innovation has standard deviation equal to 0.00852. Finally the time preference rate is matched to yield an annual real interest rate of 1.03, i.e. $\beta = 1.03^{-0.25}$ These parameters give rise to the following weights in the loss function for the case of perfectly mobile capital:

$$\tilde{\lambda}_0 = 23.30, \quad \tilde{\lambda}_1 = 6.21, \quad \tilde{\lambda}_3 = 200.91 \tag{42}$$

Note that the weight on wage inflation is almost an order of magnitude larger than on price inflation for our benchmark calibration despite the fact that the average duration of wage contracts is the same as for price contracts. This is a result of a low wage elasticity. With stick wages, labor supply is demand determined. The inverse of the labor supply elasticity signals how much compensation in terms of real wage the household requires for supplying an extra unit of labor. With sticky wages households are induced to vary their labor supply without any such compensation taking place.[9] Therefore, it is clear that the inverse of the labor supply

[8]Note that by dividing by the marginal utility of consumption (which has dimension utils per unit of consumption) we are expressing welfare in terms of units of consumption. Further dividing by steady state consumption we express the measure as percentage compensation necessary to achieve the same level of welfare as under flexible prices and wages. It should be noted however, that the derivation of this consumption equivalent welfare measure involved dropping terms which are independent of policy. Therefore any single consumption equivalent number has no economic meaning. The difference between any two numbers is meaningful, as the omitted terms independent of policy would drop out anyways once we form the difference.

[9]The assumption of complete consumption insurance implies that household are free of any income risk stemming from wage stickiness. Dropping this assumption, would further increase

elasticity is closely related to the welfare cost of nominal wage stickiness. For instance, setting $\chi = 1$ brings the weight on wage inflation relative to price inflation down to 2.8 for our benchmark calibration. Another important parameter determining the relative weight is the wage elasticity of labor demand κ. The higher this parameter, the more substitutable are different varieties of labor in production. Differences in relative quantities of labor demanded by the labor aggregator are a function of differences in relative wages posted and that function is increasing in the substitutability (κ) of labor varieties in the aggregator. For instance, reducing the markup in both labor and goods market to 10% $(\kappa = \epsilon = 11)$ increases the weight on wage inflation stabilization to 15.75

5 Comparison of monetary policy rules

This section analyzes both simple monetary policy rules and fully optimal policy. The system of first-order conditions for the fully optimal policy problem can be found in the appendix on page 33. We restrict the analysis to monetary policy under commitment, because the rules we consider are very simple and therefore easy to communicate. Furthermore, the analysis of discretion vs. commitment with backward looking elements in price setting has already been conducted in Steinsson (2003), we expect results to be similar in this setup.

The measure of welfare we consider is the expectation as of time zero [10] of expected discounted lifetime utility as in (36). In the following table, for any variable x $\mathbb{V}[x] \equiv E_0 \sum_{j=0}^{\infty} \beta^j x_{t+j}^2$

We start the analysis by considering the fully optimal rule and three simple rules for a range of fractions of forward and backward looking wage and price setters. The simple rule we consider are complete output gap stabilization $\mathbb{V}[\hat{Y}_t - \hat{Y}_t^*] = 0$, complete wage inflation stabilization $\mathbb{V}[\widehat{\pi^w}_t] = 0$ and complete price inflation stabilization $\mathbb{V}[\widehat{\pi}_t] = 0$.

5.1 Optimal simple rules

In this subsection, we consider a class of simple interest rate rules of the following form

$$\hat{i}_t = \alpha_0 \hat{G}_t + \alpha_1 \hat{\pi}_t + \alpha_2 \widehat{\pi^w}_t + \alpha_3 \hat{i}_{t-1} \tag{43}$$

the welfare costs of wage stickiness.

[10] We assume the economy is in the steady at time zero.

We maximize welfare numerically subject to the condition that the equilibrium be determinate[11]. We consider how welfare is affected by successively restricting the rules to respond to less variables. We eliminate the output gap from the rule, as authors such as Schmitt-Grohé and Uribe (2004) and Neiss and Nelson (2003) have noted that the theoretically correct measure of output gap (actual minus flexible price output) is difficult to obtain in practice and may be badly proxied by non model based concepts such as detrended output. Finally, Schmitt-Grohé and Uribe (2004) have shown that responding to an incorrect measures of the gap, such as deviation from steady state, can involve large welfare losses . Given this risk, it seems natural ask how rules perform that neglect the output gap altogether. We furthermore eliminate the lagged interest rate from the reaction function in order to measure the gains from inertia. It has often been argued that reacting to the lagged interest rate is a simple way to introduce history dependence into the policy rate. Such inertia in simple rules may mimick the history dependence that is an important feature of the fully optimal plan under commitment with forward looking agents. Finally, we consider two rules that have often been proposed in the literature: The first one the classic Taylor (1993) rule:

$$\hat{i}_t = 0.5\hat{G}_t + 1.5\hat{\pi}_t \tag{44}$$

Finally, Amato and Laubach (2003a) found that a first difference version of this rule performed well in a model with rule of thumb price setters and consumers and it seems natural to consider it here as well. In that version of the rule, the rate of change of the nominal interest rate responds the output gap and price inflation with coefficients as suggested in Taylor (1993).

$$\Delta\hat{i}_t = 0.5\hat{G}_t + 1.5\hat{\pi}_t \tag{45}$$

Before discussing the performance of these simple rules, we undertake an even simpler analysis and compute the welfare costs of rules that fully stabilized one of three variables: price level, wage level and output gap. The detailed results are deferred for the appendix on page 35. Note that in the absence of wage rigidities, full stabilization of the price level or equivalently the output gap is the optimal rule. [12]

[11] Since equilibrium selection under indeterminacy is controversial, we require that the optimal rules implies determinacy. The algorithm assigns an arbitrarily large loss to rules that render the equilibrium indeterminate. Therefore the loss function is discontinous at the boundary of the determinacy region and standard MATLAB (Version 6.0) minimization routines are not applicable. We use the ”cliff-robust” minimization routine `csminwel.m`, that can deal with such setups. It is provided by C. Sims at `http://eco-072399b.princeton.edu/yftp/optimize`

[12] This follows from the absence of any time varying inefficiencies such that price inflation and the output gap are linearly related, see equation (17). Therefore, achieving zero variance for price inflation implies zero variance for the output gap and welfare under sticky prices is equal to welfare under flexible prices.

It turns out that the rules that are optimal when only prices are sticky, involve large losses when wages are sticky additionally. The loss from full price level stability ranges from XXX to XXX for varying degrees of backward looking agents in wage and price setting. ....

In the following table, coefficients with an asteriks as superscripts are imposed as a restriction, all other coefficients are optimized

6 Specific factor markets

So far we have assumed that capital is freely mobile across sectors and can be reallocated as to equalize the shadow value of capital across firms. It has been argued by Danthine and Donaldson (2002), Woodford (2003, p.166) and others that capital cannot be instantaneously be relocated across firms. In particular, it appears to be highly unreasonable that it is too costly to post a new price tag, but that it is costless to unbolt machinery and ship it between firms. Furthermore, Eichenbaum and Fischer (2004) show that departing from the assumption of perfect capital mobility is necessary to reconcile the Calvo (1983) model with the data. Sveen and Weinke (2004) further discuss the implications of modeling capital for the equilibrium dynamics in sticky prices models

For the purpose of business cycle analysis, capital might better be modeled as being firm specific. In this subsection, we solve the firms price setting problem when capital is fixed at the firm level. The problem of the firm is now to choose $P_t(z)$ subject to the demand curve and the production function to maximize

$$\max_{P_t^*(z)} \mathrm{E}_t \sum_{i=0}^{\infty} (\theta\beta)^i \Lambda_{t,t+i} \left\{ (1+\tau_p) \left[\frac{P_t^*(z)}{P_{t+i}} \right]^{1-\epsilon} Y_{t+i} - Z_{t+i} K(z) - w_{t+i}^r L_{t+i}(z) \right\}. \tag{46}$$

Noting that $\frac{\partial L_{t+j}(z)}{\partial P_t(z)} = \frac{\partial L_{t+j}(z)}{\partial Y_{t+j}(z)} \frac{\partial Y_{t+j}(z)}{\partial P_t(z)} = -\frac{\epsilon}{1-\alpha} \frac{L_{t+j}(z)}{P_t(z)}$, the first order condition for this problem is

$$\mathrm{E}_t \sum_{i=0}^{\infty} (\theta\beta)^i \Lambda_{t,t+i} \left\{ (1+\tau_p)(1-\epsilon) \left[\frac{P_t^*}{P_{t+i}} \right]^{1-\epsilon} Y_{t+i} + \frac{\epsilon}{1-\alpha} w_{t+i}^r L_{t+i}(z) \right\} \tag{47}$$

With firm specific capital, proposition 4 in the appendix shows that the Philips Curve is given by

$$\widehat{\pi}_t = \frac{(1-\omega)(1-\theta)(1-\beta\theta)}{\zeta} \frac{(1-\alpha)}{(1-\alpha+\alpha\epsilon)} \widehat{X}_t^a + \frac{\beta\theta}{\zeta} \mathrm{E}_t \widehat{\pi}_{t+1} + \frac{\omega}{\zeta} \widehat{\pi}_{t-1} \tag{48}$$

Table 1: Optimal and simple rules - mobile capital

(ω, φ)	rule	α_0	α_1	α_2	α_3	$\mathbb{L}$
$(0, 0)$	*optimal*	-	-	-	-	3.441
	rule 1	9.48	6.49	8.57	3.05	3.444
	rule 2	0^*	4.21	20.96	0.81	3.444
	rule 3	0^*	141.85	951.19	0^*	3.492
	Taylor	0.5^*	1.5^*	0^*	0^*	27.826
	Taylor FD	0.5^*	1.5^*	0^*	1^*	4.308
$(\frac{1}{2}, 0)$	*optimal*	-	-	-	-	3.678
	rule 1	5814.47	2937.22	-80.66	1392.04	3.680
	rule 2	0^*	9.56	37.45	-0.73	3.767
	rule 3	0^*	5.85	21.55	0^*	3.779
	Taylor	0.5^*	1.5^*	0^*	0^*	28.37
	Taylor FD	0.5^*	1.5^*	0^*	1^*	4.557
$(0, \frac{1}{2})$	*optimal*	-	-	-	-	3.557
	rule 1	0.11	1.66	7.75	0.98	3.561
	rule 2	0^*	1.60	7.70	0.96	3.561
	rule 3	0^*	4.56	28.16	0^*	4.021
	Taylor	0.5^*	1.5^*	0^*	0^*	32.488
	Taylor FD	0.5^*	1.5^*	0^*	1^*	4.657
$(\frac{1}{2}, \frac{1}{2})$	*optimal*	-	-	-	-	3.826
	rule 1	7.70	13.45	51.62	2.25	3.831
	rule 2	0^*	5.17	26.93	0.87	3.835
	rule 3	0^*	11.26	61.00	0^*	3.893
	Taylor	0.5^*	1.5^*	0^*	0^*	32.850
	Taylor FD	0.5^*	1.5^*	0^*	1^*	5.02

Here, $\hat{X}_t^a$ is average marginal cost. Since capital is fixed at the firm level, the capital-labor ratio differs across firms and so does marginal cost.

When we assume that capital is fixed at the level of an individual firm, the weight on price inflation in the loss function rises roughly by a factor 15 to 364.23, while the weights on the variability of the output gap and wage inflation remain the same. Price dispersion becomes much more costly now, since costs of output dispersion across producers rise. Price dispersion implies that the bundler demands different varieties in relative quantities that are socially inefficient. With firm specific capital, we have an additional inefficiency. Now each firm is producing the wrong quantities with the ”wrong” mix of factor inputs. For both common and specific capital, a given dispersion of relative prices leads to a the same dispersion of relative quantities. However, firm specific capital implies that a given dispersion in relative quantities results in a much bigger dispersion of labor across firms. That follows from firms inability to re-allocate capital to produce with the efficient capital labor ratio. Capital is fixed at the firm level, the firm can only adjust labor to vary production. Since labor has decreasing marginal product in production at the level of the individual firm, the dispersion of labor across firms is welfare reducing.[13] Therefore, the weight attached to price inflation rises strongly with firm specific capital.

A greater weight on the price inflation variability does not imply that price inflation targeting is more desirable with firm specific capital than with mobile capital. The reason is that the structural equations change, too. In particular, the slope of the price Philips curve falls by factor 15 from 0.0852 to 0.0055. A given disturbance to marginal cost results in much less price inflation with firm specific capital. Therefore, it may very well be the case that strong wage inflation targeting remains a desirable policy despite the fact that price inflation receives a much higher weight in the loss function with firm specific capital.

We now consider the performance of simple rules for the case of immobile capital. As noted earlier, immobile capital strongly raises the weight on price inflation variability in the loss function. At the same time it, the price Philips curve implies that a given disturbance to marginal cost has much less of an impact on price inflation. It is therefore a priori unclear how the results from the analysis of freely mobile capital will change.

[13]If production were linear in labor, the weights attached to price inflation variability would be the same across mobile and firm specific capital. Dispersion of labor across firms would still be welfare reducing, but only because it is identical to the dispersion of output across firms. Since each variety has decreasing marginal product in the bundler, dispersion of output is again welfare reducing.

Table 2: Optimal and simple rules - immobile capital

(ω, φ)	rule	α_0	α_1	α_2	α_3	$\mathbb{L}$
$(0, 0)$	optimal	-	-	-	-	2.636
	rule 1	11.95	4.82	5.55	1.97	2.658
	rule 2	0^*	231,697.27	299,570.44	4,816.31	2.661
	rule 3	0^*	54,935.89	69,025.37	0^*	2.682
	Taylor	0.5^*	1.5^*	0^*	0^*	21.746
	Taylor FD	0.5^*	1.5^*	0^*	1^*	2.855
$(\frac{1}{2}, 0)$	optimal	-	-	-	-	3.177
	rule 1	124.28	318.82	344.55	21.70	3.219
	rule 2	0^*	32.23	25.64	-0.84	3.446
	rule 3	0^*	18.41	14.13	0^*	3.465
	Taylor	0.5^*	1.5^*	0^*	0^*	26.047
	Taylor FD	0.5^*	1.5^*	0^*	1^*	3.401
$(0, \frac{1}{2})$	optimal	-	-	-	-	2.749
	rule 1	143.39	26.77	43.87	2.86	2.754
	rule 2	0^*	6.40	8.50	1.19	2.783
	rule 3	0^*	19.65	23.09	0^*	3.14
	Taylor	0.5^*	1.5^*	0^*	0^*	24.869
	Taylor FD	0.5^*	1.5^*	0^*	1^*	3.115
$(\frac{1}{2}, \frac{1}{2})$	optimal	-	-	-	-	3.349
	rule 1	29.09	11.49	4.02	3.10	3.365
	rule 2	0^*	453.98	543.12	2.80	3.379
	rule 3	0^*	351.36	409.14	0^*	3.381
	Taylor	0.5^*	1.5^*	0^*	0^*	29.410
	Taylor FD	0.5^*	1.5^*	0^*	1^*	3.694

7 Summary and Conclusion

This paper has evaluated the welfare effects of monetary policy rules in a simple general equilibrium model with sticky wages and prices. It has analyzed fully optimal policy and simple monetary policy rules in variants of the baseline model of Erceg, Henderson, and Levin (2000). We chose our departures from the seminal paper at modeling points with very little consensus among macro-economists.

First, we depart from the assumption of full rationality and purely forward looking behavior by allowing a fraction of wage and price setters to be backward looking. This is in the spirit of rule of thumb consumers as in Campbell and Mankiw (1989) test of the life cycle hypothesis and was first suggested for price setters by Galí and Gertler (1999). Second, following the criticism in Danthine and Donaldson (2002), we depart from the assumption of a frictionless rental market for capital that instantaneously and costlessly allows to reallocate capital across firms . Instead we model capital as fixed at business cycle frequency. Finally, we scrutinize Calvo (1983) price and wage contracts. It has been pointed out by Kiley (2002) and Ascari (2004) that Calvo (1983) contracts imply much more price dispersion than comparable schemes with finite horizon. We allow for a general contract scheme as suggested in Wolman (1999) that encompasses Taylor (1980) contracts as a special case and can pick up some salient features of state dependent pricing of Dotsey, King, and Wolman (1999).

Introducing backward looking wage and price setters increases the cost of any given monetary policy rules, but only moderately. When some contracts are determined by backward looking agents, inflation is more persistent. A shock to inflation today affects inflation in future periods, because some agents are backward looking. While inflation is more persistent it is also less variable.

Appendix

Proposition 1 (marginal cost). *Up to first order, marginal costs is related to the gaps driving the wage and price Philips curves in the following way*

$$\widehat{X}_t = \left[\frac{\chi+\alpha}{1-\alpha}+\sigma\right]\left(\widehat{Y}_t - \widehat{Y^*}_t\right) - \left[\chi\widehat{L}_t + \sigma\widehat{Y}_t - \widehat{w^r}_t\right] \tag{49}$$

Proof of proposition 1: Log-linearize the first order condition for labor demand

$$\widehat{w^r}_t = -\alpha\widehat{L}_t + \widehat{A}_t + \widehat{X}_t \tag{50}$$

Define $\widehat{\mu}_t \equiv \chi\widehat{L}_t + \sigma\widehat{C}_t - \widehat{w^r}_t$. Subtracting the marginal rate of substitution, $\chi\hat{L}_t + \sigma\hat{Y}_t$, from both sides of the above expression, one obtains

$$-\hat{\mu}_t = [-\chi-\alpha]\,\widehat{L}_t - \sigma\widehat{Y}_t + \widehat{A}_t + \widehat{X}_t \tag{51}$$

Log-linearizing the production function, we can replace $\widehat{L}_t$ with $(1-\alpha)^{-1}(\widehat{Y}_t - \widehat{A}_t)$ and arrive at

$$-\widehat{\mu}_t = \left[\frac{-\chi-\alpha}{1-\alpha}-\sigma\right]\widehat{Y}_t + \left[1+\frac{\alpha+\chi}{1-\alpha}\right]\widehat{A}_t + \widehat{X}_t \tag{52}$$

$$-\widehat{\mu}_t = -\left[\frac{\chi+\alpha}{1-\alpha}+\sigma\right]\widehat{Y}_t + \left[\frac{\chi+\alpha}{1-\alpha}+\sigma\right]\left[\frac{1+\chi}{\chi+\alpha+\sigma(1-\alpha)}\right]\widehat{A}_t + \widehat{X}_t \tag{53}$$

Recalling the definition of output under flexible prices (16), one can write

$$\widehat{X}_t = \left[\frac{\chi+\alpha}{1-\alpha}+\sigma\right]\left(\widehat{Y}_t - \widehat{Y^*}_t\right) - \widehat{\mu}_t \tag{54}$$

$\square$

Proposition 2 (hybrid wage Philips curve). *Under Calvo wage setting with a measure of backward looking firms, the hybrid new wage Keynesian Philips curve is*

$$\widehat{\pi^w}_t = \frac{(1-\varphi)(1-\theta_w)(1-\beta\theta_w)}{(1+\kappa\chi)\zeta_w}\widehat{\mu}_t + \frac{\beta\theta_w}{\zeta_w}E_t\widehat{\pi^w}_{t+1} + \frac{\varphi}{\zeta_w}\widehat{\pi^w}_{t-1}$$

Here $\zeta_w \equiv \theta_w + \varphi[1-\theta_w(1-\beta)]$.

Proof of proposition 2: We assume the wage subsidy is set to exactly offset impact of the monopolistic competition in the steady state $1+\tau = \frac{\kappa}{\kappa-1}$. We can rewrite (6) using the demand function $N_t(h) = \left(\frac{W_t(h)}{W_t}\right)^{-\kappa} L_t$ and $V_N(N_t(h))_t = -N_t(h)^{\chi}$ as well as $U_C(C_t(h)) = C_t^{-\sigma}$ to express it in terms of the optimal nominal wage W_t^* and aggregate variables only:

$$E_t \sum_{j=0}^{\infty} (\theta_w \beta)^j \left(\frac{W_t^*}{W_{t+j}}\right)^{-\kappa} L_{t+j} C_{t+j}^{-\sigma} \left[\frac{W_t^*}{P_{t+j}} - \frac{\left(\frac{W_t^*}{W_{t+j}}\right)^{-\kappa\chi} L_{t+j}^{\chi}}{C_{t+j}^{-\sigma}}\right] = 0 \tag{55}$$

We can solve for the nominal wage W_t^*

$$W_t^* = \left(\frac{\sum_{j=0}^{\infty} (\theta_w \beta)^j W_{t+j}^{\kappa(1+\chi)} L_{t+j}^{1+\chi}}{\sum_{j=0}^{\infty} (\theta_w \beta)^j W_{t+j}^{\kappa} L_{t+j} C_{t+j}^{-\sigma} P_{t+j}^{-1}}\right)^{\frac{1}{1+\kappa\chi}} \tag{56}$$

Define $w_t^* \equiv \frac{W_t^*}{W_t}$, $w_t^r \equiv \frac{W_t}{P_t}$ and $\pi_{t,t+j}^w \equiv \frac{W_{t+j}}{W_t}$. We can write the above condition as

$$w_t^* = \left(\frac{\sum_{j=0}^{\infty} (\theta_w \beta)^j \left(\pi_{t,t+j}^w\right)^{\kappa(1+\chi)} L_{t+j}^{1+\chi}}{\sum_{j=0}^{\infty} (\theta_w \beta)^j \left(\pi_{t,t+j}^w\right)^{\kappa} L_{t+j} C_{t+j}^{-\sigma} \frac{w_{t+j}^r}{\pi_{t,t+j}^w}}\right)^{\frac{1}{1+\kappa\chi}} \tag{57}$$

Rewrite this expression defining the following two auxiliary variables.

$$D_t \equiv \sum_{j=0}^{\infty} (\theta_w \beta)^j \left(\pi_{t,t+j}^w\right)^{\kappa(1+\chi)} L_{t+j}^{1+\chi} \tag{58}$$

$$G_t \equiv \sum_{j=0}^{\infty} (\theta_w \beta)^j \left(\pi_{t,t+j}^w\right)^{\kappa} L_{t+j} C_{t+j}^{-\sigma} \frac{w_{t+j}^r}{\pi_{t,t+j}^w} \tag{59}$$

These infinite sums have a recursive representation. The behavior of optimizing firms is fully described by the following three equations

$$w_t^* = \left(\frac{D_t}{G_t}\right)^{\frac{1}{1+\kappa\chi}} \tag{60}$$

$$D_t = L_t^{1+\chi} + \beta\theta \mathrm{E}_t \left(\pi_{t+1}^w\right)^{\kappa(1+\chi)} D_{t+1} \tag{61}$$

$$G_t = L_t C_t^{-\sigma} w_t^r + \beta\theta \mathrm{E}_t \left(\pi_{t+1}^w\right)^{\kappa-1} G_{t+1} \tag{62}$$

Define $w_t^* = \frac{W_t^*}{W_t}$ and $w_t^b = \frac{W_t^b}{W_t}$. Log-linearize the definition of the wage index (25) and the rule of thumb (23), respectively:

$$\widehat{w^b}_t = \frac{\theta_w}{(1-\theta_w)\varphi} \widehat{\pi^w}_t - \frac{(1-\varphi)}{\varphi} \widehat{w^*}_t \tag{63}$$

$$\widehat{w^b}_t = (1-\varphi)\widehat{w^*}_{t-1} + \varphi \widehat{w^b}_{t-1} - \widehat{\pi^w}_t + \widehat{\pi^w}_{t-1} \tag{64}$$

Use the first equation to eliminate $\widehat{w^b}_t$ from the second and solve for $\widehat{w^*}_t$ to obtain

$$\widehat{w^*}_t = \frac{(1-\theta_w)\varphi + \theta_w}{(1-\varphi)(1-\theta_w)}\widehat{\pi^w}_t - \frac{\varphi}{(1-\varphi)(1-\theta_w)}\widehat{\pi^w}_{t-1} \tag{65}$$

Log-Linearizing the auxiliary equations defining recursively the condition for optimal wage setting around a steady state with zero wage inflation yields

$$\widehat{D}_t = (1-\beta\theta_w)(1+\chi)\widehat{L}_t + \beta\theta\kappa(1+\chi)\widehat{\pi^w}_{t+1} + \beta\theta_w\widehat{D}_{t+1} \tag{66}$$

$$\widehat{G}_t = (1-\beta\theta_w)\left[w_t^r + \widehat{L}_t - \sigma\widehat{C}_t\right] + \beta\theta(\kappa-1)\widehat{\pi^w}_{t+1} + \beta\theta_w\widehat{G}_{t+1} \tag{67}$$

Substituting these two equations into the log-linearized first order condition for wage setting (60) yields

$$\widehat{w^*}_t = \frac{(1-\theta_w\beta)}{(1+\kappa\chi)}\left[\chi\widehat{L}_t + \sigma\widehat{C}_t - \widehat{w^r}_t\right] + \beta\theta_w\left(\widehat{\pi^w}_{t+1} + \widehat{w^*}_{t+1}\right) \tag{68}$$

Substituting out $\widehat{w^*}_t$ one arrives at

$$\widehat{\pi^w}_t = \frac{(1-\varphi)(1-\theta_w)(1-\beta\theta_w)}{(1+\kappa\chi)\zeta_w}\widehat{\mu}_t + \frac{\beta\theta_w}{\zeta_w}\mathrm{E}_t\widehat{\pi^w}_{t+1} + \frac{\varphi}{\zeta_w}\widehat{\pi^w}_{t-1} \tag{69}$$

Here, $\widehat{\mu}_t \equiv \chi\widehat{L}_t + \sigma\widehat{C}_t - \widehat{w^r}_t$ and $\zeta_w \equiv \theta_w + \varphi[1-\theta_w(1-\beta)]$. □

Proposition 3 (hybrid price Philips curve). *Under Calvo price setting with a measure ω of backward looking firms, the hybrid new Keynesian price Philips curve is*

$$\widehat{\pi}_t = \frac{(1-\omega)(1-\theta)(1-\beta\theta)}{\zeta}\widehat{X}_t + \frac{\beta\theta}{\zeta}E_t\widehat{\pi}_{t+1} + \frac{\omega}{\zeta}\widehat{\pi}_{t-1}$$

Here $\zeta \equiv \theta + \omega[1-\theta(1-\beta)]$.

Proof of proposition 3: Focussing on a symmetric equilibrium and therefore dropping z, (14) can be expressed as

$$P_t^* = \frac{E_t\sum_{j=0}^{\infty}(\theta\beta)^i\Lambda_{t,t+i}X_{t+i}P_{t+i}^{\epsilon}Y_{t+i}}{E_t\sum_{j=0}^{\infty}(\theta\beta)^i\Lambda_{t,t+i}P_{t+i}^{\epsilon-1}Y_{t+i}} \tag{70}$$

It is again convenient to rewrite the Calvo price setting condition in terms of stationary variables using these auxiliary equations

$$p_t^* = \frac{B_t}{F_t}, \text{ with:} \tag{71}$$

$$B_t \equiv E_t\sum_{i=0}^{\infty}(\theta\beta)^i\Lambda_{t+i}X_{t+i}\left(\frac{P_{t+i}}{P_t}\right)^{\epsilon}Y_{t+i}, \tag{72}$$

$$F_t \equiv E_t\sum_{i=0}^{\infty}(\theta\beta)^i\Lambda_{t+i}\left(\frac{P_{t+i}}{P_t}\right)^{\epsilon-1}Y_{t+i}. \tag{73}$$

p_t^* stands for the optimal nominal price P_t^* divided by the current period price index P_t. The infinite discounted sums B_t and F_t have a recursive representation, where we have expressed the stochastic discount factor $\Lambda_{t,t+i}$ by the ratio of the marginal utilities of consumption:

$$B_t = X_t Y_t + \beta\theta E_t C_{t+1}^{-\sigma} C_t^{\sigma} \pi_{t+1}^{\epsilon} B_{t+1}, \tag{74}$$

$$F_t = Y_t + \beta\theta E_t C_{t+1}^{-\sigma} C_t^{\sigma} \pi_{t+1}^{\epsilon-1} F_{t+1}. \tag{75}$$

Define $p_t^* = \frac{P_t^*}{P_t}$ and $p_t^b = \frac{P_t^b}{P_t}$. Log-linearize the definition of the price index (20) and the rule of thumb (18), respectively:

$$\widehat{p^b}_t = \frac{\theta}{(1-\theta)\omega}\widehat{\pi}_t - \frac{(1-\omega)}{\omega}\widehat{p^*}_t \tag{76}$$

$$\widehat{p^b}_t = (1-\omega)\widehat{p^*}_{t-1} + \omega\widehat{p^b}_{t-1} - \widehat{\pi}_t + \widehat{\pi}_{t-1} \tag{77}$$

Use the first equation to eliminate $\widehat{p}_t^b$ from the second and solve for $\widehat{p}_t^*$ to obtain

$$\widehat{p^*}_t = \frac{(1-\theta)\omega + \theta}{(1-\omega)(1-\theta)}\widehat{\pi}_t - \frac{\omega}{(1-\omega)(1-\theta)}\widehat{\pi}_{t-1} \tag{78}$$

The log-linearized first order condition for price setting of forward looking firms (71), (74) and (75) around a steady state with zero price inflation yields

$$\widehat{p^*}_t = (1-\theta\beta)\,\widehat{X}_t + \beta\theta\left(\widehat{\pi}_{t+1} + \widehat{p^*}_{t+1}\right) \tag{79}$$

Substituting out $\widehat{p^*}_t$ one arrives at

$$\widehat{\pi}_t = \frac{(1-\omega)(1-\theta)(1-\beta\theta)}{\zeta}\widehat{X}_t + \frac{\beta\theta}{\zeta}\mathrm{E}_t\widehat{\pi}_{t+1} + \frac{\omega}{\zeta}\widehat{\pi}_{t-1} \tag{80}$$

Here $\zeta \equiv \theta + \omega[1-\theta(1-\beta)]$. □

Proposition 4 (Price Philips curve with specific factor markets). *Under Calvo price setting with a measure ω of backward looking firms, the hybrid new Keynesian price Philips curve for the case of specific factor markets is*

$$\widehat{\pi}_t = \frac{(1-\omega)(1-\theta)(1-\beta\theta)}{\zeta}\widehat{X}_t + \frac{\beta\theta}{\zeta}E_t\widehat{\pi}_{t+1} + \frac{\omega}{\zeta}\widehat{\pi}_{t-1}$$

Here $\zeta \equiv \theta + \omega[1-\theta(1-\beta)]$.

Proof of proposition 4: Using the production function to express hours in terms of output, using the demand function, recalling that the subsidy offsets the steady state effects of monopolistic competition and defining $\Psi_t \equiv Y_{t+i}^{\frac{1}{1-\alpha}} A_{t+i}^{-\frac{1}{1-\alpha}} K^{\frac{-\alpha}{1-\alpha}}$ we can rewrite the first order condition for price setting as

$$\mathrm{E}_t \sum_{i=0}^{\infty} (\theta\beta)^i \Lambda_{t,t+i} \left\{ \left[\frac{P_t^*}{P_{t+i}}\right]^{1-\epsilon} Y_{t+i} + \frac{1}{1-\alpha} w_{t+i}^r \left[\frac{P_t^*}{P_{t+i}}\right]^{-\frac{\epsilon}{1-\alpha}} \Psi_{t+i} \right\} \tag{81}$$

Solving for the optimal nominal price yields

$$P_t^* = \left[\frac{\mathrm{E}_t \sum_{i=0}^{\infty} (\theta\beta)^i \Lambda_{t,t+i} \frac{1}{1-\alpha} w_{t+i}^r P_{t+i}^{\frac{\epsilon}{1-\alpha}} \Psi_{t+i}}{\mathrm{E}_t \sum_{i=0}^{\infty} (\theta\beta)^i \Lambda_{t,t+i} P_{t+i}{}^{\epsilon-1} Y_{t+i}}\right]^{\frac{1-\alpha}{1-\alpha+\epsilon\alpha}} \tag{82}$$

Dividing trough by the current price level, we can express this in terms of stationary variables[14]

$$p_t^* = \left[\frac{\mathrm{E}_t \sum_{i=0}^{\infty} (\theta\beta)^i \Lambda_{t,t+i} \frac{1}{1-\alpha} w_{t+i}^r \left(\Pi_{j=1}^i \pi_{t+j}\right)^{\frac{\epsilon}{1-\alpha}} \Psi_{t+i}}{\mathrm{E}_t \sum_{i=0}^{\infty} (\theta\beta)^i \Lambda_{t,t+i} \left(\Pi_{j=1}^i \pi_{t+j}\right)^{\epsilon-1} Y_{t+i}}\right]^{\frac{1-\alpha}{1-\alpha+\epsilon\alpha}} \tag{83}$$

Rewriting these expressions recursively, by use of auxiliary variables

$$p_t^* = \left(\frac{B_t}{F_t}\right)^{\frac{1-\alpha}{1-\alpha+\epsilon\alpha}}, \text{ with:} \tag{84}$$

$$B_t \equiv \mathrm{E}_t \sum_{i=0}^{\infty} (\theta\beta)^i \Lambda_{t,t+i} \frac{1}{1-\alpha} w_{t+i}^r \left(\Pi_{j=1}^i \pi_{t+j}\right)^{\frac{\epsilon}{1-\alpha}} \Psi_{t+i} \tag{85}$$

$$F_t \equiv \mathrm{E}_t \sum_{i=0}^{\infty} (\theta\beta)^i \Lambda_{t,t+i} \left(\Pi_{j=1}^i \pi_{t+j}\right)^{\epsilon-1} Y_{t+i}. \tag{86}$$

The infinite discounted sums B_t and F_t have the following recursive representation

$$B_t = \frac{1}{1-\alpha} w_t^r \Psi_t + \beta\theta E_t C_{t+1}^{-\sigma} C_t^{\sigma} \pi_{t+1}^{\frac{\epsilon}{1-\alpha}} B_{t+1}, \tag{87}$$

$$F_t = Y_t + \beta\theta E_t C_{t+1}^{-\sigma} C_t^{\sigma} \pi_{t+1}^{\epsilon-1} F_{t+1}. \tag{88}$$

Log-linearizing (84), (87), and (88) yields the following equation

$$\widehat{p^*}_t = \left(\frac{1-\alpha}{1-\alpha+\alpha\epsilon}\right)(1-\theta\beta)\left[\widehat{w^r}_t + \frac{1}{1-\alpha}\left(\alpha\hat{Y}_t - \hat{A}_t\right)\right] + \beta\theta\left(\widehat{\pi}_{t+1} + \widehat{p^*}_{t+1}\right) \tag{89}$$

[14] $\Pi_{j=1}^0 \pi_{t+j} \equiv 1$.

Note that for any firm z loglinear marginal cost (i.e the ratio of the real wage to the marginal product of labor) is $\widehat{w^r}_t + \frac{1}{1-\alpha}\left(\alpha\widehat{Y}(z)_t - \hat{A}_t\right)$. Since firms post difference prices, they sell different quantities, and marginal cost differs across firms. Up to log-linearization, the output aggregator equals the average over its individual components, i.e $\int_0^1 \hat{Y}(z)_t dz = \hat{Y}_t + \mathcal{O}(||\xi||^2)$. Therefore, $\hat{X}^a_t \equiv \widehat{w^r}_t + \frac{1}{1-\alpha}\left(\alpha\hat{Y}_t - \hat{A}_t\right)$ is a first order approximation of average marginal cost.

Substituting out $\widehat{p^*}_t$ and using steps similar as for specific factor markets, one arrives at

$$\widehat{\pi}_t = \frac{(1-\omega)(1-\theta)(1-\beta\theta)}{\zeta}\frac{(1-\alpha)}{(1-\alpha+\alpha\epsilon)}\widehat{X}^a_t + \frac{\beta\theta}{\zeta}\mathrm{E}_t\widehat{\pi}_{t+1} + \frac{\omega}{\zeta}\widehat{\pi}_{t-1} \tag{90}$$

□

Proposition 5 (loss function). *Define average utility across households*

$$\mathbb{W}_t = U(C_{t+j}) + \int_0^1 V(N_{t+j}(h))dh \tag{91}$$

Let $\mathbb{W}^*_t$ denote average utility under flexible prices and wages. With Calvo wage and price setting, we can approximate the loss function $\mathbb{L} \equiv -\sum_{t=0}^{\infty}\beta^t\left(\mathbb{W}_t - \mathbb{W}^*_t\right)/U_C\bar{C}$ up to second order and neglecting terms independent of policy by the term

$$\mathbb{L} = E_0\sum_{t=0}^{\infty}\beta^t\left[\tilde{\lambda}_0\hat{\pi}^2_t + \tilde{\lambda}_1\left(\hat{Y}_t - \hat{Y}^*_t\right)^2 +^2 + \tilde{\lambda}_2\left(\Delta\widehat{\pi}_t\right)^2 + \tilde{\lambda}_3\widehat{\pi^w}^2_t + \tilde{\lambda}_4\left(\Delta\widehat{\pi^w}_t\right)^2\right] \tag{92}$$

The weights are given by

$$\begin{aligned}
\tilde{\lambda}_0 &= \frac{1}{2}\epsilon\frac{\theta}{(1-\theta)(1-\theta\beta)} \\
\tilde{\lambda}_1 &= \frac{1}{2}\left(\frac{\chi+\alpha}{1-\alpha} + \sigma\right) \\
\tilde{\lambda}_2 &= \frac{\omega}{(1-\omega)\theta}\tilde{\lambda}_0 \\
\tilde{\lambda}_3 &= \frac{1}{2}(1-\alpha)(\kappa^{-1}+\chi)\kappa^2\frac{\theta_w}{(1-\theta_w)(1-\theta_w\beta)} \\
\tilde{\lambda}_4 &= \frac{\varphi}{(1-\varphi)\theta_w}\tilde{\lambda}_3
\end{aligned}$$

Proof of proposition 5 (following Erceg, Henderson, and Levin (2000)): Some relations are used repeatedly throughout this text. For a generic variable X_t, let

$\tilde{X}_t \equiv X_t - \bar{X}$ denote arithmetic deviation from the steady state $\bar{X}$, let $\widehat{X_t} \equiv logX_t - log\bar{X}$ denote logarithmic deviation. Up to second order, the relation between the two is

$$\tilde{X}_t \approx \bar{X}\left(\widehat{X}_t + \frac{1}{2}\widehat{X}_t^2\right).$$

For the often used Dixit-Stiglitz type agrregators of the form

$$X_t = \left[\int_0^1 X_t(j)^{\phi} dj\right]^{\frac{1}{\phi}}$$

the logarithmic approximation is

$$\widehat{X}_t \approx \mathcal{E}_j \widehat{X}_t(j) + \frac{1}{2}\phi \mathcal{VAR}_j \widehat{X}_t(j). \tag{93}$$

Here, the cross-sectional mean is denoted by $\mathcal{E}_j$ and the cross-sectional variance is denoted by $\mathcal{VAR}_j$. Finally, let $||\xi||$ denote an upper bound to the exogenous disturbances. The goal of the following paragraphs is to approximate all expressions involving integrals across households indexed by h or across firms indexed by z, in terms of aggregate variables. Approximations are second order Taylor expansions, i.e. terms of order higher than two are omitted.[15]

Equipped with these simple tools, (35) can be approximated in the following way. The second order approximation to the utility of consumption is

$$U(C_t) = U_C\bar{C}\widehat{C}_t + \frac{1}{2}\left(U_C\bar{C} + U_{CC}\bar{C}^2\right)\widehat{C}_t^2 + \mathcal{O}(||\xi||^3). \tag{94}$$

Here, $\mathcal{O}(||\xi||^3)$ denotes a residual that is third order or higher in the bound on the exogenous disturbance. Taking unconditional expectation [16]

$$\mathcal{E}U(C_t) = U_C\bar{C}\left[\mathcal{E}\widehat{C}_t + \frac{1}{2}(1+\omega_1)\mathcal{VAR}\,\widehat{C}_t\right] + \mathcal{O}(||\xi||^3). \tag{95}$$

Here $\omega_1 = \frac{U_{CC}\bar{C}}{U_C}$ is the elasticity of marginal utility of consumption evaluated at the steady state. The second order approximation to the utility of labor is

$$V(N_t(h)) = V_N\bar{N}\widehat{N}_t(h) + \frac{1}{2}\left(V_N\bar{N} + V_{NN}\bar{N}^2\right)\widehat{N}_t^2(h) + \mathcal{O}(||\xi||^3)$$

[15] Omitting all terms of order higher than two implies that when substitutions into squares of variables are undertaken, only the first order terms of the Taylor expansion of these variables are substituted.

[16] We are using the fact that up to first order C_t has mean zero, so that up to second order the centered and uncentered second moments are equal, i.e. $\mathcal{E}\widehat{C}_t^2 = (\mathcal{E}\widehat{C})^2 + \mathcal{VAR}\widehat{C}_t = \mathcal{VAR}\widehat{C}_t + \mathcal{O}(||\xi||^3)$.

Integrating this expression over h yields

$$\frac{\int_0^1 V(N_t(h))dh}{V_N\bar{N}} = \mathcal{E}_h\widehat{N}_t(h) + \frac{1}{2}(1+\omega_2)\left(\left[\mathcal{E}_h\widehat{N}_t(h)\right]^2 + \mathcal{VAR}_h\widehat{N}_t(h)\right) + \mathcal{O}(||\xi||^3)$$

The term $\omega_2 \equiv \frac{V_{NN}\bar{N}}{V_N}$ is the elasticity of marginal utility of labor evaluated at the steady state. From total labor supply $L_t \equiv \left[[N_t(h)]^{\frac{\kappa-1}{\kappa}} dh\right]^{\frac{\kappa}{\kappa-1}}$ using the results stated in (93)

$$\widehat{L}_t = \mathcal{E}_h\widehat{N}_t(h) + \frac{\kappa-1}{2\kappa}\mathcal{VAR}_h\widehat{N}_t(h) + \mathcal{O}(||\xi||^3) \tag{96}$$

Solve (96) to eliminate $\mathcal{E}_h\widehat{N}_t(h)$. This yields the following approximation to the utility of labor

$$\int_0^1 V(N_t(h))dh = V_N\bar{N}\left[\widehat{L}_t + \frac{1}{2}(1+\omega_2)\widehat{L}_t^2\right] \tag{97}$$

$$+\frac{1}{2}V_N\bar{N}\left[(\kappa^{-1}+\omega_2)\mathcal{VAR}_h\widehat{N}_t(h)\right] + \mathcal{O}(||\xi||^3) \tag{98}$$

We need to eliminate L_t in order to arrive at an output gap term, consider the second order approximation to total demand for labor $L_t = \int_0^1 L_t(z)dz$

$$\widehat{L}_t \equiv log\mathcal{E}_z L_t(z) - log\bar{L} = \mathcal{E}_z\widehat{L}_t(z) + \frac{1}{2}\mathcal{VAR}_z\widehat{L}_t(z) + \mathcal{O}(||\xi||^3) \tag{99}$$

Since all firms face the same relative price of capital and the labor bundler, they have the same capital-labor ratio. This fact, together with a fixed aggregate capital stock gives rise to the following exact loglinear relation derived from the production function.

$$\mathcal{E}_z\widehat{L}_t(z) = \mathcal{E}_z\widehat{Y}_t(z) - \widehat{A}_t + \alpha\widehat{L}_t \tag{100}$$

$$\mathcal{VAR}_z\widehat{L}_t(z) = \mathcal{VAR}_z\widehat{Y}_t(z) \tag{101}$$

From the definition of the output bundler, we have

$$\mathcal{E}_z\widehat{Y}_t(z) = \widehat{Y}_t - \frac{1}{2}\frac{\epsilon-1}{\epsilon}\mathcal{VAR}_z\widehat{Y}_z(z) + \mathcal{O}(||\xi||^3) \tag{102}$$

Substituting (100) into yields (99)

$$\widehat{L}_t = \mathcal{E}_z\widehat{Y}_t(z) - \widehat{A}_t + \alpha\widehat{L}_t + \frac{1}{2}\mathcal{VAR}_z\widehat{Y}_t(z) + \mathcal{O}(||\xi||^3) \tag{103}$$

Using (102) to eliminate $\mathcal{E}_z\widehat{Y}_t(z)$ and rearranging yields

$$\widehat{L}_t = \frac{1}{1-\alpha}\left(\widehat{Y}_t - \widehat{A}_t + \frac{1}{2\epsilon}\mathcal{VAR}_z\widehat{Y}_t(z)\right) + \mathcal{O}(||\xi||^3) \tag{104}$$

Substituting for $\widehat{L}_t$ and $\widehat{L}_t^2$ into (97) yields

$$\begin{aligned}\frac{2}{V_N\bar{N}}\int_0^1 V(N_t(h))dh = &\frac{2}{1-\alpha}\left(\widehat{Y}_t - \widehat{A}_t\right) + \frac{1+\omega_2}{(1-\alpha)^2}\left(\widehat{Y}_t - \widehat{A}_t\right)^2 \\ &+ \frac{1}{(1-\alpha)\epsilon}\mathcal{VAR}_z\widehat{Y}_t(z) \\ &+ (\kappa^{-1}+\omega_2)\mathcal{VAR}_h\widehat{N}_t(h) + \mathcal{O}(||\xi||^3)\end{aligned} \tag{105}$$

Combining the approximations of the sub-utility function for consumption and for labor, yields

$$\begin{aligned}\mathbb{W} = &U_C\bar{C}\left[\mathcal{E}\widehat{C}_t + \frac{1}{2}(1+\omega_1)\mathcal{E}\widehat{C}_t^2\right] + \frac{V_N\bar{N}}{1-\alpha}\mathcal{E}\left(\widehat{Y}_t - \widehat{A}_t\right) \\ &+ \frac{V_N\bar{N}}{2}\frac{1+\omega_2}{(1-\alpha)^2}\mathcal{E}\left(\widehat{Y}_t - \widehat{A}_t\right)^2 + \frac{V_N\bar{N}}{2}\frac{1}{(1-\alpha)\epsilon}\mathcal{VAR}_z\widehat{Y}_t(z) \\ &+ \frac{V_N\bar{N}}{2}(\kappa^{-1}+\omega_2)\mathcal{VAR}_h\widehat{N}_t(h) + \mathcal{O}(||\xi||^3)\end{aligned}$$

The model features no aggregate capital accumulation, hence $C_t = Y_t$. Furthermore, since the labor-leisure decision is not distorted in the steady state, we have that $(1-\alpha)U_C\bar{Y} = -V_N\bar{N}$.[17] Together these two conditions ensure that all linear terms cancel, except terms which are independent of policy and therefore do not affect the optimal policy. These terms are denoted $t.i.p.$

$$\begin{aligned}\frac{\mathbb{W}}{0.5U_C\bar{C}} = &\left((1+\omega_1) - \frac{1+\omega_2}{(1-\alpha)}\right)\mathcal{E}\widehat{Y}_t^2 + 2\frac{1+\omega_2}{(1-\alpha)}\mathcal{E}\widehat{Y}_t\widehat{A}_t - \frac{1}{2\epsilon}\mathcal{VAR}_z\widehat{Y}_t(z) \\ &- 2(1-\alpha)(\kappa^{-1}+\omega_2)\mathcal{VAR}_h\widehat{N}_t(h) + t.i.p. + \mathcal{O}(||\xi||^3)\end{aligned}$$

As such the above model is free of first order terms and can be accurately evaluated using a linear approximation to the models equilibrium conditions. It has become customary in the literature to rewrite the expressions in terms of the output gap $Y_t - Y_t^*$. This aids in our economic interpretation of the policy problem. Subtracting from this equation $\mathbb{W}^*$, the second order approximation to the utility

[17] This follows from the first order condition for labor supply $U_C\frac{w}{p} = -V_N$ and $\frac{w}{p} = (1-\alpha)\frac{Y}{N}$

function evaluated in a model with completely flexible prices and wages and an efficient level of output Y_t^*, one arrives at[18]

$$\frac{\mathbb{W}-\mathbb{W}^*}{U_C\bar{C}} = \frac{1}{2}\left((1+\omega_1)-\frac{1+\omega_2}{(1-\alpha)}\right)\mathcal{E}\left[\widehat{Y}_t^2-(\widehat{Y}_t^*)^2\right] + \frac{1+\omega_2}{(1-\alpha)}\mathcal{E}\widehat{A}_t\left(\widehat{Y}_t-\widehat{Y}_t^*\right)-\frac{1}{2\epsilon}\mathcal{VAR}_z\widehat{Y}_t(z) - \frac{1}{2}(1-\alpha)(\kappa^{-1}+\omega_2)\mathcal{VAR}_h\widehat{N}_t(h)+t.i.p.+\mathcal{O}(||\xi||^3)$$

To rewrite the expression in terms of an output gap, we proceed in the following way. Using the definition of the natural level of output, and defining the term $\Lambda \equiv \omega_2+\alpha-(1-\alpha)\omega_1$ we have the following two relations

$$\frac{1}{2}\left((1+\omega_1)-\frac{1+\omega_2}{(1-\alpha)}\right)\mathcal{E}\left[\widehat{Y}_t^2-(\widehat{Y}_t^*)^2\right] = -\frac{\Lambda}{2(1-\alpha)}\left[\widehat{Y}_t^2-(\widehat{Y}^*)^2\right], \tag{106}$$

$$\frac{1+\omega_2}{(1-\alpha)}\widehat{A}_t\left(\widehat{Y}_t-\widehat{Y}_t^*\right) = \frac{\Lambda}{1-\alpha}\widehat{Y}_t^*(\widehat{Y}_t-\widehat{Y}_t^*). \tag{107}$$

Finally note that

$$-\frac{1}{2}\left(\widehat{Y}_t-\widehat{Y}^*\right)^2 = -\frac{1}{2}\left[\widehat{Y}_t^2-(\widehat{Y}^*)^2\right]+\widehat{Y}_t^*(\widehat{Y}_t-\widehat{Y}_t^*) \tag{108}$$

Therefore, we can rewrite the welfare criterion compactly as

$$\mathbb{W}-\mathbb{W}^* = \lambda_1\mathcal{VAR}\left(\widehat{Y}_t-\widehat{Y}_t^*\right)+\lambda_2\mathcal{VAR}_h\widehat{N}_t(h)+\lambda_3\mathcal{VAR}_z\widehat{Y}_t(z)+t.i.p+\mathcal{O}(||\xi||^3) \tag{109}$$

Here, the weights are given by:

$$\lambda_1 = -\frac{1}{2}\frac{U_C\bar{C}\Lambda}{(1-\alpha)},\ \lambda_2 = -\frac{1}{2}(1-\alpha)(\kappa^{-1}+\omega_2)U_C\bar{C}, \lambda_3 = -\frac{1}{2}\frac{U_C\bar{C}}{\epsilon}. \tag{110}$$

Define a loss function as $\mathbb{L} \equiv -\sum_{j=0}^{\infty}\beta^j\left(\mathbb{W}_{t+j}-\mathbb{W}_{t+j}^*\right)/(U_C\bar{C})$. Further, define $\Delta_t^N \equiv \mathcal{VAR}_h \log N_t(h)$ and similarly $\Delta_t^Y \equiv \mathcal{VAR}_z \log Y_t(z)$. Note that it follows from proposition 8 that the infinite discounted sum of cross-sectional dispersion of output across producers can be rewritten as

$$\sum_{j=0}^{\infty}\beta^j\Delta_{t+j}^Y = \sum_{j=0}^{\infty}\beta^j\frac{\epsilon^2}{(1-\beta\theta)(1-\theta)}\left[\theta\left(\log\pi_t\right)^2+\frac{\omega}{(1-\omega)}\left(\Delta\log\pi_t\right)^2\right] + \mathcal{O}(||\xi||^3)+t.i.p. \tag{111}$$

[18] Since policy has no effect on welfare with completely flexible prices, this is equivalent to adding a constant in any maximization problem: It will change the value of the objective function, but not the value of the maximand.

Similarly, it follows from proposition 7 that

$$\sum_{j=0}^{\infty}\beta^j\Delta_{t+j}^N = \sum_{j=0}^{\infty}\beta^j\frac{\kappa^2}{(1-\beta\theta_w)(1-\theta_w)}\left[\theta_w\left(\log\pi_t^w\right)^2+\frac{\varphi}{(1-\varphi)}\left(\Delta\log\pi_t^w\right)^2\right] + \mathcal{O}(||\xi||^3)+t.i.p. \tag{112}$$

Using these expressions, we arrive at (92). $\square$

Proposition 6 (loss function with immobile capital). *When capital is immobile at the firm level, we can approximate the loss function* $\mathbb{L}\equiv-\sum_{t=0}^{\infty}\beta^t\left(\mathbb{W}_t-\mathbb{W}_t^*\right)/U_C\bar{C}$ *by a loss function of the same form as with freely mobile capital. The weights in the loss function attached to the variance of price inflation and the variance of the rate of change of price inflation now change to*

$$\tilde{\lambda}_0 = \frac{1}{2}\left[\frac{1}{1-\alpha}-\frac{\epsilon-1}{\epsilon}\right]\epsilon^2\frac{\theta}{(1-\theta)(1-\theta\beta)}$$

$$\tilde{\lambda}_2 = \frac{\omega}{(1-\omega)\theta}\tilde{\lambda}_0$$

Proof of proposition 6: Here we sketch on the part of the derivation that is different from the case with mobile capital. The following loglinear relation derived from the production function holds exactly

$$\mathcal{E}_z\hat{Y}_t(z) = \hat{A}_t+(1-\alpha)\mathcal{E}_z\hat{L}_t(z) \tag{113}$$

It follows that $\mathcal{VAR}_z\hat{L}_t(z)=\frac{1}{(1-\alpha)^2}\mathcal{VAR}_z\hat{Y}_t(z)$. Solving for $\hat{L}(z)_t$, substituting into (99) and making use of (102) yields

$$\hat{L}_t = \frac{1}{1-\alpha}\left[\hat{Y}_t-\hat{A}_t\right]+\frac{1}{2}\left[\frac{1}{(1-\alpha)^2}-\frac{\epsilon-1}{\epsilon(1-\alpha)}\right]\mathcal{VAR}_z\hat{Y}_t(z) \tag{114}$$

Substituting this into (97) yields

$$\begin{aligned}\frac{2}{V_N\bar{N}}\int_0^1 V(N_t(h))dh = &\frac{2}{1-\alpha}\left(\widehat{Y}_t-\widehat{A}_t\right)+\frac{1+\omega_2}{(1-\alpha)^2}\left(\widehat{Y}_t-\widehat{A}_t\right)^2\\ &+\frac{1}{1-\alpha}\left[\frac{1}{1-\alpha}-\frac{\epsilon-1}{\epsilon}\right]\mathcal{VAR}_z\widehat{Y}_t(z)\\ &+(\kappa^{-1}+\omega_2)\mathcal{VAR}_h\widehat{N}_t(h)+\mathcal{O}(||\xi||^3)\end{aligned} \tag{115}$$

Following similar steps as before one can show that the weight on dispersion of output across producers is now given by $\lambda_3=-\frac{1}{2}U_C\bar{C}\left[\frac{1}{1-\alpha}-\frac{\epsilon-1}{\epsilon}\right]$. Following the same steps as for mobile capital we arrive at the new weights. $\square$

Proposition 7 (wage dispersion with backward looking wage setters). *With a measure of backward looking wage setters, the cross sectional dispersion of labor* $\Delta_t^N \equiv \mathcal{VAR}_h \log N_t(h)$ *is related to wage inflation* $\log \pi_t^w \equiv \log W_t - \log W_{t-1}$ *in the following way*

$$\Delta_t^N = \theta_w \Delta_{t-1}^N + \kappa^2 \left[\frac{\theta_w}{(1-\theta_w)} (\log \pi_t^w)^2 + \frac{\varphi}{(1-\theta_w)(1-\varphi)} (\Delta \log \pi_t^w)^2 \right] + \mathcal{O}(||\xi||^3) \tag{116}$$

Proof of proposition 7: From the demand function for labor $N_t(h) = \left(\frac{W_t(h)}{W_t}\right)^{-\kappa} L_t$ we have that

$$\mathcal{VAR}_h \log N_t(h) = \kappa^2 \mathcal{VAR}_h \log W_t(h). \tag{117}$$

Proceed further in the following steps. Define $\bar{W}_t \equiv \mathcal{E}_h \log W_t(h)$. Note that

$$\bar{W}_t - \bar{W}_{t-1} = \mathcal{E}_h \left[\log W_t(h) - \bar{W}_{t-1}\right] \tag{118}$$

$$= \theta_w \mathcal{E}_h \left[\log W_{t-1}(h) - \bar{W}_{t-1}\right] \tag{119}$$

$$+ (1-\theta_w) \left[(1-\varphi)(logW_t^*(h) - \bar{W}_{t-1}) + \varphi \left(\log W_t^b - \bar{W}_{t-1}\right)\right]$$

$$= (1-\theta) \left[(1-\varphi)(\log W_t^*(h) - \bar{W}_{t-1}) + \varphi \left(\log W_t^b - \bar{W}_{t-1}\right)\right] \tag{120}$$

Note that, the difference between $\bar{W}_t$ and $\log W_t$ is second order and therefore up to first order, the left hand side of (121) is the log of wage inflation.

$$\log \pi_t^w = (1-\theta_w) \left[(1-\varphi)(logW_t^*(h) - \bar{W}_{t-1}) + \varphi \left(\log W_t^b - \bar{W}_{t-1}\right)\right] + \mathcal{O}(||\xi||^2) \tag{121}$$

Note further that taking logs of the definition of the rule of thumb, recalling that $\log \pi_{t-1}^w - \bar{W}_{t-1} = -\bar{W}_{t-2} + \mathcal{O}(||\xi||^2)$ and using (121) we have immediately

$$\log W_t^b(h) - \bar{W}_{t-1} = \frac{1}{(1-\theta_w)} \log \pi_{t-1}^w + \mathcal{O}(||\xi||^2) \tag{122}$$

Similarly take logs of the rule of thumb, solve for $\log W_{t-1}^*$ and subtract $\bar{W}_{t-2}$ on both sides to arrive at

$$\log W_{t-1}^*(h) - \bar{W}_{t-2} = \frac{1}{(1-\varphi)} \left\{ \left[\log W_t^b - \log \pi_{t-1}^w - \bar{W}_{t-2}\right] + \varphi \left(\log W_{t-1}^b - \bar{W}_{t-2}\right) \right\} \tag{123}$$

Forwarding this equation one period recalling that the difference between $\log W_t$ and $\bar{W}_t$ is second order and making use of (122) twice, we arrive at

$$\log W_t^*(h) - \bar{W}_{t-1} = \frac{1}{(1-\theta_w)(1-\varphi)} \left[\log \pi_t^w - \varphi \log \pi_{t-1}^w\right] + \mathcal{O}(||\xi||^2) \quad (124)$$

Next, consider the measure of wage dispersion $\Delta_t^W \equiv \mathcal{VAR}_h \log W_t(h)$. Sine W_{t-1} is independent of h we can write

$$\begin{aligned}
\Delta_t^W &= \mathcal{VAR}_h \left[\log W_t(h) - \bar{W}_{t-1}\right] && (125)\\
&= \mathcal{E}_h \left\{\left[\log W_t(h) - \bar{W}_{t-1}\right]^2\right\} - (\mathcal{E}_h \log W_t(h) - \bar{W}_{t-1})^2 && (126)\\
&= \theta_w \mathcal{E}_h \left\{\left[\log W_{t-1}(h) - \bar{W}_{t-1}\right]^2\right\} - (\bar{W}_t - \bar{W}_{t-1})^2 && (127)\\
&\quad + (1-\theta_w)\left[(1-\varphi)(\log W_t^*(h) - \bar{W}_{t-1})^2 + \varphi(\log W_t^b(h) - \bar{W}_{t-1})^2\right] &&\\
&= \theta_w \Delta_{t-1}^W + (1-\theta_w)(1-\varphi)(\log W_t^*(h) - \bar{W}_{t-1})^2 &&\\
&\quad + (1-\theta_w)\varphi(\log W_t^b(h) - \bar{W}_{t-1})^2 - (\bar{W}_t - \bar{W}_{t-1})^2 && (128)
\end{aligned}$$

Substituting (122) and (124) and simplifying, one arrives at

$$\Delta_t^W = \theta_w \Delta_{t-1}^W + \frac{\theta_w}{(1-\theta_w)} (\log \pi_t^w)^2 + \frac{\varphi}{(1-\theta_w)(1-\varphi)} (\Delta \log \pi_t^w)^2 + \mathcal{O}(||\xi||^3) \quad (129)$$

Finally solving for Δ_t^N by making use of (117) we arrive at (121). □

Proposition 8 (price dispersion with backward looking price setters). *With a measure of backward looking price setters, the cross sectional dispersion of output* $\Delta_t^Y \equiv \mathcal{VAR}_z \log Y_t(z)$ *is related to price inflation* $\log \pi_t \equiv \log P_t - \log P_{t-1}$ *in the following way*

$$\Delta_t^Y = \theta \Delta_{t-1}^Y + \epsilon^2 \left[\frac{\theta}{(1-\theta)} (\log \pi_t)^2 + \frac{\omega}{(1-\theta)(1-\omega)} (\Delta \log \pi_t)^2\right] + \mathcal{O}(||\xi||^3) \quad (130)$$

Proof of proposition 8: The proof follows similar steps as for wage dispersion and is omitted. □

Proposition 9 (output and labor dispersion with Wolman (1999) pricing). *Under the Wolman (1999) pricing scheme with a maximum of J cohorts of firms charging identical prices, whose fraction of the overall price index are denoted by*

ω_j, we have the following approximation of dispersion of output across producers and of labor across households

$$\mathcal{VAR}_z \log Y(z)_t = \epsilon^2 \sum_{j=0}^{J^p-1} \omega_j^p \left(\log P_{t,j} - \log P_t\right)^2 + \mathcal{O}(||\xi||^3) \tag{131}$$

$$\mathcal{VAR}_h \log N(h)_t = \kappa^2 \sum_{j=0}^{J^w-1} \omega_j^w \left(\log W_{t,j} - \log W_t\right)^2 + \mathcal{O}(||\xi||^3) \tag{132}$$

Proof of proposition 9: Note again from the demand function faced by an individual producer that $\mathcal{VAR}_z \log Y(z)_t = \epsilon^2 \mathcal{VAR}_z \log P(z)_t$. Using that the difference between $\log P_t$ and $\mathcal{E}_z \log P_t(z)$ is second order and that P_t is independent of z, we have that

$$\begin{aligned}
\mathcal{VAR}_z \log Y(z)_t &= \epsilon^2 \mathcal{VAR}_z \left(\log P(z)_t - \log P_t\right) && (133)\\
&= \epsilon^2 \mathcal{E}_z \left(\log P(z)_t - \log P_t\right)^2 - \epsilon^2 \left(\mathcal{E}_z \log P(z)_t - P_t\right)^2 && (134)\\
&= \epsilon^2 \mathcal{E}_z \left(\log P(z)_t - \log P_t\right)^2 + \mathcal{O}(||\xi||^3) && (135)\\
&= \epsilon^2 \sum_{j=0}^{J} \omega_j \left(\log P_{t,j} - \log P_t\right)^2 + \mathcal{O}(||\xi||^3) && (136)
\end{aligned}$$

The last equality follows from the fact that all firms in cohort j charge the same price and that these cohorts have weights in the price index are also the the probabilities of any firm z charging price $P_{t,j}$. Analogous steps can be done to prove the second part of the proposition. □

Corollary. *It follows immediately, that for N period overlapping Taylor (1980) contracts, output dispersion can be approximated by*

$$\mathcal{VAR}_z \log Y(z)_t = \frac{\epsilon^2}{N} \sum_{j=0}^{J-1} E\left(\log P_{t,j} - \log P_t\right)^2 + \mathcal{O}(||\xi||^3) \tag{137}$$

Proposition 10 (loss function with Wolman (1999) pricing). *Under the Wolman (1999) pricing scheme with a maximum of J cohorts of firms (assuming only forward looking wage and price setters) we have the following loss function*

$$\mathbb{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left[\tilde{\lambda}_0 \sum_{j=0}^{J^p-1} \omega_j^p \left(\widehat{p^*}_{t-j}\right)^2 + \tilde{\lambda}_1 \left(\hat{Y}_t - \hat{Y}_t^*\right)^2 + \tilde{\lambda}_3 \sum_{j=0}^{J^w-1} \omega_j^w \left(\widehat{w^*}_{t-j}\right)^2 \right] \tag{138}$$

with: $\tilde{\lambda}_0 = \frac{1}{2}\epsilon, \quad \tilde{\lambda}_1 = \frac{1}{2}\left(\frac{\chi+\alpha}{1-\alpha} + \sigma\right), \quad \tilde{\lambda}_3 = \frac{1}{2}(1-\alpha)(\kappa^{-1} + \omega_2)\kappa^2$

Proof of proposition 10: This follows immediately by plugging in the results from proposition 9 on page 31 into (109). □

The Lagrangian of the policy problem:

Define $G_t \equiv \hat{Y}_t - \hat{Y}_t^*$ substitute out the Y_t from the policy problem to reduce the dimension of the system

$$
\begin{aligned}
&E_0 \sum_{t=0}^{\infty} \beta^t \left[\hat{\pi}_t^2 + \tilde{\lambda}_1 \hat{G}_t^2 + \tilde{\lambda}_2 \left(\Delta\widehat{\pi}_t\right)^2 + \tilde{\lambda}_3 \widehat{\pi^w}_t^2 + \tilde{\lambda}_4 \left(\Delta\widehat{\pi^w}_t\right)^2\right] \\
&+ \sum_{t=0}^{\infty} \psi_t^1 \beta^t \left[-\widehat{\pi^w}_t + \frac{(1-\varphi)(1-\theta_w)(1-\beta\theta_w)}{(1+\kappa\chi)\zeta_w}\left[-\widehat{X}_t + \left(\frac{\chi+\alpha}{1-\alpha}+\sigma\right)\hat{G}_t\right] + \frac{\beta\theta_w}{\zeta_w}\widehat{\pi^w}_{t+1} + \frac{\varphi}{\zeta_w}\widehat{\pi^w}_{t-} \right. \\
&+ \sum_{t=0}^{\infty} \psi_t^2 \beta^t \left[-\widehat{\pi}_t + \frac{(1-\omega)(1-\theta)(1-\beta\theta)}{\zeta}\widehat{X}_t + \frac{\beta\theta}{\zeta}\widehat{\pi}_{t+1} + \frac{\omega}{\zeta}\widehat{\pi}_{t-1}\right] \\
&+ \sum_{t=0}^{\infty} \psi_t^3 \beta^t \left[-\widehat{w^r}_t + \widehat{X}_t + \widehat{A}_t - \alpha\widehat{L}_t\right] \\
&+ \sum_{t=0}^{\infty} \psi_t^4 \beta^t \left[-\Delta\widehat{w^r}_t + \pi_t^w - \pi_t\right] \\
&+ \sum_{t=0}^{\infty} \psi_t^5 \beta^t \left[-\widehat{X}_t + \left[\frac{\chi+\alpha}{1-\alpha}+\sigma\right]\hat{G}_t - \left[\left(\chi+\sigma(1-\alpha)\right)\widehat{L}_t + \sigma\hat{A}_t - \widehat{w^r}_t\right]\right]
\end{aligned}
$$

Here:

$\zeta_w \equiv \theta_w + \varphi[1-\theta_w(1-\beta)]$ and $\zeta \equiv \theta + \omega[1-\theta(1-\beta)]$.

Write the problem more compactly

$$
\begin{aligned}
&E_0 \sum_{t=0}^{\infty} \beta^t \left[\hat{\pi}_t^2 + \tilde{\lambda}_1 \hat{G}_t^2 + \tilde{\lambda}_2 \left(\Delta \widehat{\pi}_t \right)^2 + \tilde{\lambda}_3 \widehat{\pi^w}_t^2 + \tilde{\lambda}_4 \left(\Delta \widehat{\pi^w}_t \right)^2 \right] \\
+ &\sum_{t=0}^{\infty} \psi_t^1 \beta^t \left[-\widehat{\pi^w}_t + \xi_1 \widehat{X}_t + \xi_2 \hat{G}_t + \xi_3 \widehat{\pi^w}_{t+1} + \xi_4 \widehat{\pi^w}_{t-1} \right] \\
+ &\sum_{t=0}^{\infty} \psi_t^2 \beta^t \left[-\widehat{\pi}_t + \xi_5 \widehat{X}_t + \xi_6 \widehat{\pi}_{t+1} + \xi_7 \widehat{\pi}_{t-1} \right] \\
+ &\sum_{t=0}^{\infty} \psi_t^3 \beta^t \left[-\widehat{w^r}_t + \widehat{X}_t + \widehat{A}_t - \alpha \widehat{L}_t \right] \\
+ &\sum_{t=0}^{\infty} \psi_t^4 \beta^t \left[-\widehat{w^r}_t + \widehat{w^r}_{t-1} + \pi_t^w - \pi_t \right] \\
+ &\sum_{t=0}^{\infty} \psi_t^5 \beta^t \left[-\widehat{X}_t + \xi_8 \hat{G}_t - \xi_9 \widehat{L}_t - \sigma \hat{A}_t + \widehat{w^r}_t \right]
\end{aligned}
$$

Here, the coefficients are given

$$
\begin{aligned}
\xi_1 &= -\frac{(1-\varphi)(1-\theta_w)(1-\beta\theta_w)}{(1+\kappa\chi)\zeta_w} \\
\xi_2 &= -\xi_1 \left(\frac{\chi+\alpha}{1-\alpha} + \sigma \right) \\
\xi_3 &= \frac{\beta\theta_w}{\zeta_w} \\
\xi_4 &= \frac{\varphi}{\zeta_w} \\
\xi_5 &= \frac{(1-\omega)(1-\theta)(1-\beta\theta)}{\zeta} \\
\xi_6 &= \frac{\beta\theta}{\zeta} \\
\xi_7 &= \frac{\omega}{\zeta} \\
\xi_8 &= \frac{\chi+\alpha}{1-\alpha} + \sigma \\
\xi_9 &= \chi + \sigma(1-\alpha)
\end{aligned}
$$

The first-order conditions for this problem are

$$
\begin{aligned}
\hat{L}_t: &\quad 0 = -\alpha\psi_t^3 - \xi_9\psi_t^5 \\
\hat{X}_t: &\quad 0 = \xi_1\psi_t^1 + \xi_5\psi_t^2 + \psi_t^3 - \psi_t^5 \\
\widehat{w^r}_t: &\quad 0 = -\psi_t^3 - \psi_t^4 + \beta\psi_{t+1}^4 \\
\hat{G}_t: &\quad 0 = 2\tilde{\lambda}_1\hat{G}_t + \xi_2\psi_t^1 + \xi_8\psi_t^5 \\
\hat{\pi}_t: &\quad 0 = 2\hat{\pi}_t + 2\tilde{\lambda}_2\left[(1+\beta)\hat{\pi}_t - \hat{\pi}_{t-1} - \beta\hat{\pi}_{t+1}\right] - \psi_t^2 + \beta\xi_7\psi_{t+1}^2 + \frac{1}{\beta}\xi_6\psi_{t-1}^2 - \psi_t^4 \\
\widehat{\pi^w}_t: &\quad 0 = 2\tilde{\lambda}_3\widehat{\pi^w}_t + 2\tilde{\lambda}_4\left[(1+\beta)\widehat{\pi^w}_t - \widehat{\pi^w}_{t-1} - \beta\widehat{\pi^w}_{t+1}\right] - \psi_t^1 + \beta\xi_4\psi_{t+1}^1 + \frac{1}{\beta}\xi_3\psi_{t-1}^1 + \psi_t^4
\end{aligned}
$$

Some simple policy rules

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Table 3: Optimal and simple rules for varying degrees of backward looking agents

(ω,φ)	rule	$\mathbb{V}[\hat{G}_t]$	$\mathbb{V}[\hat{\pi}_t]$	$\mathbb{V}[\Delta\hat{\pi}_t]$	$\mathbb{V}[\widehat{\pi^w}_t]$	$\mathbb{V}[\Delta\widehat{\pi^w}_t]$	$\mathbb{L}$
$(0,0)$	optimal	0.351	0.431	0.271	0.026	0.015	3.252
	P stabil	2.619	0	0	0.255	0.118	11.802
	W stabil	1.786	1.055	0.746	0	0	5.990
	G stabil	0	0.414	0.157	0.294	0.065	13.641
$(\frac{1}{4},0)$	optimal	0.340	0.381	0.147	0.028	0.016	3.400
	P stabil	2.619	0	0	0.255	0.118	11.802
	W stabil	1.806	0.957	0.419	0	0	6.412
	G stabil	0	0.401	0.090	0.294	0.065	13.767
$(\frac{1}{2},0)$	optimal	0.316	0.346	0.077	0.030	0.016	3.457
	P stabil	2.619	0	0	0.255	0.118	11.802
	W stabil	1.844	0.879	0.220	0	0	6.568
	G stabil	0	0.397	0.049	0.294	0.065	13.874
$(0,\frac{1}{4})$	optimal	0.388	0.454	0.283	0.022	0.008	3.370
	P stabil	2.755	0	0	0.249	0.066	12.840
	W stabil	1.786	1.055	0.746	0	0	5.990
	G stabil	0	0.468	0.162	0.334	0.038	16.151
$(\frac{1}{4},\frac{1}{4})$	optimal	0.375	0.404	0.154	0.024	0.008	3.528
	P stabil	2.775	0	0	0.249	0.066	12.840
	W stabil	1.806	0.957	0.419	0	0	6.412
	G stabil	0	0.456	0.094	0.334	0.038	16.293
$(\frac{1}{2},\frac{1}{4})$	optimal	0.346	0.368	0.081	0.026	0.009	3.593
	P stabil	2.775	0	0	0.249	0.066	12.840
	W stabil	1.844	0.879	0.220	0	0	6.568
	G stabil	0	0.454	0.052	0.334	0.038	16.427
$(0,\frac{1}{2})$	optimal	0.463	0.475	0.287	0.019	0.004	3.447
	P stabil	3.149	0	0	0.254	0.035	13.965
	W stabil	1.786	1.055	0.746	0	0	5.990
	G stabil	0	0.646	0.174	0.464	0.022	22.666
$(\frac{1}{4},\frac{1}{2})$	optimal	0.450	0.425	0.157	0.021	0.004	3.613
	P stabil	3.149	0	0	0.254	0.035	13.965
	W stabil	1.806	0.957	0.419	0	0	6.412
	G stabil	0	0.638	0.103	0.465	0.022	22.870
$(\frac{1}{2},\frac{1}{2})$	optimal	0.413	0.391	0.083	0.022	0.004	3.694
	P stabil	3.149	0	0	0.254	0.035	13.965
	W stabil	1.844	0.879	0.220	0	0	6.568
	G stabil	0	0.646	0.059	0.467	0.022	23.115