Persistence and Nominal Inertia in a Generalised Taylor Economy: How Longer Contracts Dominate Shorter Contracts*

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Abstract

In this paper we develop the Generalise Taylor Economy *(GTE)* in which there are many sectors with overlapping contracts of different lengths. We are able to show that even in economies with the same average contract length, monetary shocks will be more persistent when there are longer contracts. In particular we are able to solve the puzzle of why Calvo contracts appear to be more persistent than simple Taylor contracts: it is because the standard calibration of Calvo contracts is not correct.

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1 Introduction

"There is a great deal of heterogeneity in wage and price setting. In fact, the data suggest that there is as much a difference between the average lengths of different types of price setting arrangements, or between the average lengths of different types of wage setting arrangements, as there is between wage setting and price setting. Grocery prices change much more frequently than magazine prices - frozen orange juice prices change every two weeks, while magazine prices change every three years! Wages in some industries change once per year on average, while others change per quarter and others once every two years. One might hope that a model with homogenous representative price or wage setting would be a good approximation to this more complex world, but most likely some degree of heterogeneity will be required to describe reality accurately."

Taylor (1999).

There are two main approaches to modelling nominal wage and price rigidity in the dynamic general equilibrium (DGE) macromodels: the staggered contract setting of Taylor (Taylor (1980)) and the Calvo model of random contract lengths generated by a constant hazard (reset) probability (Calvo (1983)). This paper proposes a generalization of the standard Taylor model to allow for an economy with many different contract lengths: we call this a Generalized Taylor Economy - *GTE* for short. The standard approach in the literature has been to adopt a simple Taylor economy, in which there is a single contract length in the economy: for example 2 or 4 quarters[1]. As the above quote from John Taylor indicates, in practice there is a wide range of wage and price setting behavior resulting in a variety of contract lengths. We can use the *GTE* framework to evaluate whether the hope expressed by John Taylor that a representative sector approach "is a good approximation to this more complex world".

An additional advantage of the *GTE* framework is that it includes the Calvo model as a special case, in the sense that we can set up the *GTE* to

[1] This is not to ignore some recent papers that have allowed for two sectors with different contract durations, such as Aoki (2001), Erceg and Levin (2002), Carlstrom, Fuerst and Ghironi (2003) or with multi-sectors such as Mankiw and Reis (2003). However, these studies are mainly concerned with computing optimal monetary policy in a Dynamic Equlibrium Setting.

have the same distribution of contract lengths as the Calvo model. This is an important contribution in itself since the two approaches have until now appeared to be distinct and incompatible at the theoretical level even if they are sometimes claimed to be empirically similar (see for example Kiley (2002) for a discussion). As we shall show, a simple Taylor economy can indeed be a good approximation to a Calvo model, but only if the two are calibrated in a consistent manner.

We develop our approach in a DGE setting following the approach of Ascari (2000). The issue we focus on is the way a monetary shock can generate changes in output through time, and in particular the degree of persistence of deviations of output from steady-state. Much recent attention has been devoted to the ability of the staggered contract approach of Taylor to generate enough persistence in the sense of being quantitatively able to generate the persistence observed in the data. Two influential papers in this are Chari, Kehoe and McGrattan (2000) and Ascari (2000). Both papers are pessimistic for staggered contracts. CKM develop a microfounded model of staggered price-setting and find that they do not generate enough persistence and conclude that the "mechanism to solve persistence problem must be found elsewhere". Ascari focusses on staggered wage setting, and finds that whilst nominal wage rigidities lead to more persistent output deviations than with price setting, they are still not enough to explain the data. Based on these conclusions, it is commonly inferred that in a dynamic equilibrium framework, the staggered contracts cannot generate enough persistence.

In this paper, we follow Ascari in focussing on staggered wage-contracts. However, we show that by allowing for an economy with a range of contract lengths, the presence of longer contracts can significantly increase the degree of persistence in output following a monetary shock. We calibrate the model in a way that in either the CKM or Ascari setting would not generate much persistence. We show that even a small proportion of longer contracts can significantly increase the degree of persistence. For example, we consider the case of a economy where 90% of the economy consist of simple 2-period Taylor contracts, and 10% have 8-period Taylor contracts (the average is 2.6 quarters) and show that the economy has a marked increase in output persistence. We also take an empirical distribution of contract lengths (from 1-8 quarters) for the US taken from Taylor (1993) and show that this will generate a significant degree of persistence.

It has long been observed that in the Calvo setting there can be a significant backlog of old contracts: for example, with a reset probability of

$\omega = 0.25$ (a common value used with quarterly data), there is a probability of over 10% that a contract will survive for 8 periods (see for example Erceg (1997), Wolman (1999)). We construct a *GTE* which has the same distribution of completed contract lengths as the Calvo distribution. We find that this Calvo-GTE has almost exactly the same persistence as the Calvo economy. This supports the idea that the persistence resulting from Calvo contracts is explained by the presence of longer-term contracts. However, it also shows that if we have the same distribution of contract lengths in a *GTE*, the persistence is the same. This indicates that the two approaches can be unified and are not so different. We also show that the view that Calvo is more persitent than the equivalent Taylor economy is based on a mis-calibration and inconsistent basis of comparison. The Taylor case is specified in terms of completed contract lengths: the Calvo is usually looked at in terms of contract age, which is very different from completed lifetime of a contract. As we show in Dixon and Kara (2004), a reset probability of $\omega = 0.25$ leads to an average contract lifetime of 7 periods, not 4. Hence simple Taylor economies of 4 quarters should be compared with Calvo reset probabilities of $\omega = 0.4$ for mean contract length to be equated.

The outline of the paper is as follows. In section 2 we outline the basic structure of the Economy. The main innovation here is to allow for the GTE contract structure. In section 3 we present the log-linearized general equilibrium and discuss the calibration of the model in relation to recent literature. In section 4 we explore the influence of longer term contracts on persistence as compared to the simple Taylor economy, and in section 5 we apply our methodology to evaluating persistence in the Calvo model.

2 The Model Economy

The approach of this paper is to model an economy in which there can be many sectors with different wage setting processes, which we denote a generalized Taylor economy.(*GTE*). As we will show later, an advantage of the *GTE* approach is that it includes as special cases not only the standard Taylor case of an economy where all wage contracts are of the same length, but also the Calvo process.

The model in this section is an extension of Ascari (2000) and includes a number of features essential to understanding the impact of monetary shock on output in a dynamic equilibrium setting. The exposition aims to outline the basic building blocks of the model. However, the novel aspects of this paper only begin with the wage setting process. Firstly, we describe the behavior of firms which is standard. Then we describe the structure of the contracts in a *GTE,* the wage-setting decision and monetary policy.

2.1 Firms

There is a continuum of firms $f \in [0,1]$, each producing a single differentiated good $Y(f)$, which are combined to produce a final consumption good Y. The production function here is CES with constant returns and corresponding unit cost function P

$$Y_t = \left[\int_0^1 Y_t(f)^{\frac{\theta-1}{\theta}} df\right]^{\frac{\theta}{\theta-1}} \tag{1}$$

$$P_t = \left[\int_0^1 P_{ft}^{1-\theta} df\right]^{\frac{1}{1-\theta}} \tag{2}$$

The demand for the output of firm f is

$$Y_{ft} = \left(\frac{P_{ft}}{P_t}\right)^{-\theta} Y_t \tag{3}$$

Each firm f sets the price P_{ft} and takes the firm-specific wage rate W_{ft} as given. Labor L_{ft} is the only input so that the inverse production function is

$$L_{ft} = \left(\frac{Y_{ft}}{\alpha}\right)^{\frac{1}{\sigma}} \tag{4}$$

Where $\sigma \leq 1$ represents the degree of diminishing returns, with $\sigma = 1$ being constant returns. The firm chooses $\{P_{ft,}Y_{ft}, L_{ft}\}$ to maximize profits subject to (3,4), yields the following solutions for price, output and employment at the firm level given $\{Y_t, W_{ft}, P_t\}$

$$P_{ft} = \left(\frac{\theta-1}{\theta}\right)\frac{\alpha^{-1/\sigma}}{\sigma} W_{ft} Y_{ft}^{\frac{1-\sigma}{\sigma}} \tag{5}$$

$$Y_{ft} = \kappa_1 \left(\frac{W_{ft}}{P_t}\right)^{-\sigma\varepsilon} Y_t^{\frac{\varepsilon\sigma}{\theta}} \tag{6}$$

$$L_{ft} = \kappa_2 \left(\frac{W_{ft}}{P_t}\right)^{-\varepsilon} Y_t^{\frac{\varepsilon}{\theta}} \tag{7}$$

where $\varepsilon = \frac{\theta}{\theta(1-\sigma)+\sigma} > 1 \quad \kappa_1 = \left(\frac{\theta-1}{\theta}\right)^{-\sigma\varepsilon} \sigma^{-\sigma\varepsilon}\alpha^{-\varepsilon} \quad \kappa_2 = \left(\frac{\theta-1}{\theta}\right)^{-\varepsilon} \sigma^{\varepsilon}\alpha^{\varepsilon(\frac{\theta-1}{\theta})}$

Price is a markup over marginal cost, which depends on the wage rate and the output level (when $\sigma < 1$): output and employment depend on the real wage and total output in the economy.

2.2 The Structure of Contracts in a GTE

In this section we outline an economy in which there are potentially many sectors with different types of wage-setting processes. Within each sector there is a more or less standard Taylor process (i.e. overlapping contracts of a specified length). The economy is called a Generalized Taylor Economy (GTE).Corresponding to the continuum of firms f there is a unit interval of household-unions (one per firm). The economy consists N sectors $i = 1...N$. The budget shares of the N sectors with uniform prices (when prices p_f are equal for all $f \in [0, 1]$) are given by α_i with $\sum_{i=1}^{N} \alpha_i = 1$, the N vector $(\alpha_i)_{i=1}^{N}$ being denoted $\boldsymbol{\alpha}$,where $\boldsymbol{\alpha} \in \Delta^{N-1}$.

We can partition the unit interval into sub-intervals representing each sector. Let us define the cumulative budget share of sectors $k = 1...i$.

$$\hat{\alpha}_i = \sum_{k=1}^{i} \alpha_k$$

with $\hat{\alpha}_0 = 0$ and $\hat{\alpha}_N = 1$. The interval for sector i is then $[\hat{\alpha}_{i-1}, \hat{\alpha}_i]$.

Within each sector, each firm is matched with a firm-specific union: there are N_i cohorts of unions and firms in sector i. Again, we can partition the interval $[\hat{\alpha}_{i-1}, \hat{\alpha}_i]$ into cohort intervals: let the share of each cohort within the sector be λ_{ij} so that $\sum_{j=1}^{N_i} \lambda_{ij} = 1$, with the N_i-vector $\boldsymbol{\lambda}_i \in \Delta^{N_i-1}$ Again, we can define the cumulative share $\hat{\lambda}_{ij}$ analogously to $\hat{\alpha}_k$. The interval of firm-unions corresponding to cohort j in sector i is then

$$\left[\hat{\alpha}_{i-1} + \hat{\lambda}_{ij-1}\alpha_i, \hat{\alpha}_{i-1} + \hat{\lambda}_{ij}\alpha_i\right]$$

Clearly, if symmetry is assumed (cohorts are of equal size) $\lambda_{ij} = N_i^{-1}$ and $\hat{\lambda}_{ij} = jN_i^{-1}$.

The sectors are differentiated by the integer[2] contract length $T_i \in Z_{++}$, which is the same for all cohorts within a sector. The timing of the wage

[2]We work in discrete time in this paper, although the model obviously generalises to continuous time.

setting process within the sector can be summarized by an $N_i - 1$-tuple of integers $\{T_{ij}\}_{j=2}^{N_i}$ which specifies when in the wage-setting cycle cohort j moves . It is assumed that cohort 1 moves first (period 1): this defines the beginning of the cycle, so that $1 \leq T_{ij} \leq N_i$. If $T_{ij} = 3$, it means that cohort j sets its wage in period 3 periods after the first. By convention, we assume that the js are ordered so that T_{ij} is strictly increasing. Clearly, we have the restriction of $N_i \leq T_i$: there cannot be more cohorts than contract periods. If $N_i = T_i$, then one cohort moves in each period: if in addition the the cohorts are of equal size $\lambda_{ij} = N_i^{-1}$, we define a *uniform* wage setting process in sector i. If $N_i < T_i$, then there will be some periods when no cohort moves. For example, we can consider a sector with 8 period contracts in which there are two cohorts in which the second cohort moves 4 periods after the first, $T_{i2} = 4$. Alternatively, there might be three cohorts, with timing $\{2, 6\}$ so that the second cohort moves in period 2 and third in period 6.

In order to fully characterize the economy with non-uniform wage setting, we also need to specify the calender date t_i when the wage-setting process starts[3] for each contract length T_i. In the case of an economy with uniform wage setting processes in all sectors, the start dates are irrelevant, since each period is exactly the same in all sectors (i.e. the same proportion of wages are reset).

We can therefore characterize the wage setting process in a *GTE* by $(\mathbf{T}, \boldsymbol{\alpha}) \in Z_{++}^N \times \Delta^{N-1}$, which gives the contract lengths and sizes of the N sectors, $(N_{i,}\boldsymbol{\lambda}_i, t_i) \in Z_{++} \times \Delta^{N_i-1} \times Z_{++}$ which describes the number and relative size of the cohorts in each sector i, and the timing/synchronization of cohorts in that sector.

$$GTE := \left\{ (\boldsymbol{T}, \boldsymbol{\alpha}), \{N_{i,}\boldsymbol{\lambda}_i, t_i\}_{i=1}^N \right\}$$

In the case where each sector has a uniform wage setting process, we have a *uniform GTE* which is more simply parameterized by $(\mathbf{T}, \boldsymbol{\alpha})$since $(N_{i,}\boldsymbol{\lambda}_i) = (T_i, T_i^{-1})$ and t_i is irrelevant (each period looks the same). A *homogenous* or *simple* Taylor economy is one where there is just one sector with a uniform wage-setting process.

The general price index P can be defined in terms of sectors, or subinter-

[3] Of course, this is not unique: all that is required for each sector is the start date of one cycle, since then the start date of all cycles is given.

vals $[\hat{\alpha}_{i-1}, \hat{\alpha}_i]$ for each sector i.

$$P = \left[\sum_{i=1}^{N} \int_{\hat{\alpha}_{i-1}}^{\hat{\alpha}_i} P_f^{1-\theta} df \right]^{\frac{1}{1-\theta}}$$

This can be further broken down into intervals for each cohort, where we note that all firms in the same cohort face the same wage and hence set the same price $p_f = p_{ij}$ for $f \in \left[\hat{\alpha}_{i-1} + \hat{\lambda}_{ij-1}\alpha_i, \hat{\alpha}_{i-1} + \hat{\lambda}_{ij}\alpha_i \right]$

$$P = \left[\sum_{i=1}^{N} \sum_{j=1}^{N_i} \int_{\hat{\alpha}_{i-1}+\hat{\lambda}_{ij-1}\alpha_i}^{\hat{\alpha}_{i-1}+\hat{\lambda}_{ij}\alpha_i} P_{ij}^{1-\theta} df \right]^{\frac{1}{1-\theta}} \tag{8}$$

We can log linearize the price equations around the steady state , given the wages. All firms with the same wage will set the same price: define P_{ij} as the price set by firms in sector i cohort j. This yields the following log-linearization in terms of deviations from the steady state (where we assume $P^* = 1$):

$$p = \sum_{i=1}^{N} \sum_{j=1}^{N_i} \alpha_i \lambda_{ij} p_{ij} \tag{9}$$

Note that there is an important property of *CES* technology. The demand for an individual firm depends only on its own price and the general price index (see 3). There is no sense of location: whilst we divide the unit interval into segments corresponding to sectors and cohorts within sectors, this need not reflect any objective factor in terms of sector or cohort specific aspects of the technology. The sole communality within a sector is the length of the wage contract: the sole commonality within a cohort is the timing of the contract. The vectors $\boldsymbol{\alpha}$ and $\boldsymbol{\lambda}_i$ are best be thought of as simply measures of sector and cohort size. This is an important property which will become useful when we show that a Calvo economy can be represented by a *GTE*.

2.2.1 Household-Unions and Wage Setting

Households $h \in [0, 1]$ have preferences defined over consumption, labour, and real money balances. The expected life-time utility function takes the form

$$U_h = E_t \left[\sum_{t=0}^{\infty} \beta^t u(C_{ht}, \frac{M_{ht}}{P_t}, \underbrace{1 - H_{ht}}_{L_{ht}}) \right] \tag{10}$$

where C_{ht}, $\left(\frac{M_{ht}}{P_t}\right)$, H_{ht}, L_{ht} are household $h's$ consumption, end-of period real money balances, hours worked, and leisure respectively, t is an index for time, $0 < \beta < 1$ is the discount factor, and each household has the same flow utility function u, which is assumed to take the form

$$U(C_{ht}) + \delta \ln(\frac{M_{ht}}{P_t}) + V(1 - H_{ht}) \tag{11}$$

Each household-union belongs to a particular sector and wage-setting cohort within that sector (recall, that each household is twinned with firm $f = h$). Since the household acts as a monopoly union, hours worked are demand determined, being given by the (7).

The household's budget constraint is given by

$$P_t C_{ht} + M_{ht} + \sum_{s_{t+1}} Q(s^{t+1} \mid s^t) B_h(s^{t+1}) \leq M_{ht-1} + B_{ht} + W_{ht} H_{ht} + \pi_{ht} + T_{ht} \tag{12}$$

where $B_h(s^{t+1})$ is a one-period nominal bond that costs $Q(s^{t+1} \mid s^t)$ at state s^t and pays off one dollar in the next period if s^{t+1} is realized. B_{ht} represents the value of the household's existing claims given the realized state of nature. M_{ht} denotes money holdings at the end of period t. W_{ht} is the nominal wage, π_{ht} is the profits distributed by firms and $W_{ht}H_{ht}$ is the labour income. Finally, T_t is a nominal lump-sum transfer from the government.

The households optimization breaks down into two parts. First, there is the choice of consumption, money balances and one-period nominal bonds to be transferred to the next period to maximize expected lifetime utility (10) given the budget constraint (12). The first order conditions derived from the consumer's problem are as follows:

$$u_{ct} = \beta R_t E_t \left(\frac{P_t}{P_{t+1}} u_{ct+1} \right) \tag{13}$$

$$\sum_{s_{t+1}} Q(s^{t+1} \mid s^t) = \beta E_t \frac{u_{ct+1} P_t}{u_{ct} P_{t+1}} = \frac{1}{R_t} \tag{14}$$

$$\delta \frac{P_t}{M_t} = u_{ct} - \beta E_t \frac{P_t}{P_{t+1}} u_{ct+1} \tag{15}$$

Equation (13) is the Euler equation, (14) gives the gross nominal interest rate and (15) gives the optimal allocation between consumption and real balances. Note that the index h is dropped in equations (13) and (15), which reflects our assumption of complete contingent claims markets for consumption and implies that consumption is identical across all households in each period $(C_{ht} = C_t)^4$.

The reset wage is for household h in sector i is chosen to maximize lifetime utility given labour demand (7) and the additional constraint that nominal wage will be fixed for T_i periods in which the aggregate output and price level are given$\{Y_t, P_t\}$. From the unions point of view, we can collect together all of the terms in (7) which the union treats as exogenous by defining the constant K_t where:

$$K_t = \kappa_2 P_t^{\varepsilon} Y_t^{\frac{\varepsilon}{\theta}}$$

Since the reset wage at time t will only hold for T_i periods, we have the following first-order condition:

$$X_{it} = \left(\frac{\varepsilon}{\varepsilon - 1}\right)\left[\frac{E_t \sum_{s=0}^{T_i-1} \beta^s \left[V_L\left(1 - H_{t+s}\right)\left(K_{t+s}\right)\right]}{E_t \sum_{s=0}^{T_i-1} \beta^s \left[\frac{u_c(C_{t+s})}{P_{t+s}} K_{t+s}\right]}\right] \tag{16}$$

Where E_t represents the conditional expectation taken only over states of nature in which the household is unable to reset its wage contract. Equation (16) shows that the optimal wage is a constant "mark-up" (given by$\frac{\varepsilon}{\varepsilon-1}$) over the ratio of marginal utilities of leisure and marginal utility from consumption within the contract duration $s = t...t + T_{\cdot i} - 1$ When $T_i = 2$, this equation reduces to the fist order condition in Ascari (2000).

2.3 Government

There is a government conducts monetary policy via lump-sum transfer, that is,

$$T_t = M_t - M_{t-1} \tag{17}$$

The money supply M_t grows at a rate μ_t so that $M_t = \mu_t M_{t-1}$. To focus on the role of the GTE in generating the output persistence, following Huang and Liu (2001), we assume that there are no serial correlation in the money growth process and therefore $\ln(\mu_t)$ follows a white noise process, i.e.,

[4] See Ascari (2000).

$\ln(\mu_t) = \xi_t$, where ξ_t is a white noise process with a zero mean and a finite variance $\sigma^2_{\xi.}$ More specifically, we assume that the money supply follows a random walk, i.e., $m_t = m_{t-1} + \xi_t$.

3 General Equilibrium

In this section, we characterize equilibrium of the economy. We first describe the equilibrium conditions for sector i and then the equilibrium conditions for the aggregate economy. To compute an equilibrium, we reduced the equilibrium conditions to four equations, including the household's first order condition for setting its contract wage, the pricing equation, the household's money demand equation, and an exogenous law of motion for the growth rate of money supply. We then log-linearize this equilibrium conditions around a steady state. The steady state which we choose is the zero-inflation steady state, which is a standard assumption in this literature. The linearized version of the equations are listed and discussed below. We follow the notational convention that lower-case symbols represents log-deviations of variables from the steady state.

The linearized wage decision equation (16) for sector i is given by

$$x_{it} = \frac{1}{\sum_{s=0}^{T_i-1} \beta^s} \left[\sum_{s=0}^{T_i-1} \beta^s \left[p_{t+s} + \gamma y_{t+s} \right] \right] \tag{18}$$

The coefficients on output in the wage setting equation in all sectors is given by

$$\gamma = \frac{\eta_{ll} + \eta_{cc}(\sigma + \theta(1-\sigma))}{\sigma + \theta(1-\sigma) + \theta\eta_{ll}} \tag{19}$$

Where $\eta_{cc} = \frac{-U_{cc}C}{U_c}$ is the parameter governing risk aversion, $\eta_{ll} = \frac{-U_{ll}C}{U_l}$ is the inverse of the labour elasticity, θ is the elasticity of substitution of consumption goods.

Using equation (9) and aggregating for sector i, we get

$$p_{it} = w_{it} + \left(\frac{1-\sigma}{\sigma} \right) y_{it} \tag{20}$$

where

$$w_{it} = \sum_{j=1}^{N_i} \lambda_{ijt} w_{ijt}$$

Using equation (3) and aggregating for sector i yields

$$y_{it} = \theta(p_t - p_{it}) + y_t \tag{21}$$

Given the money demand equation (15), log-linerazing this equation yields the following;

$$y_t = m_t - p_t \tag{22}$$

Finally, the linearized price index in the economy is simply a weighted average of the ongoing prices in all sectors and is given by

$$p_t = \sum_{i=1}^{N} \alpha_i p_{it} \tag{23}$$

3.1 Simple Taylor Economies with Price and Wage setting

In this section, we examine whether our model can account for a contract multiplier. Since the novel aspect of our paper is the incorporation of generalized wage setting, it is useful to compare our results with identical models that makes the standard assumption of a simple Taylor economy. However, before presenting our main results by using the true parameter values, it useful to discuss the possible outcomes of the model. We first use the values of γ mentioned above for the simple Taylor case case and we then compare our model with generalized wage setting by way of numerical simulations. We later report our main results with true parameter values.

3.2 Calibration

The parameters of the model include the discount factor, β,the elasticity of substitution of labour,$\eta_{_{LL}}$,the elasticity of substitution of consumption,$\eta_{_{CC}}$,the elasticity of substitution of consumption goods, θ, the monetary policy parameter, ξ_t.

The utility is additively separable and for simplicity, we assume $\beta = 1$. Empirical studies reveal that intertemporal labour supply elasticity, $1/\eta_{LL}$, is low and is at most 1. In particular, the survey by Pencavel (1986) suggests that η_{LL} is between 2.2 and infinity. Therefore, we set $\eta_{LL} = 4.5$, which implies that intertemporal labour supply elasticity, $1/\eta_{LL}$, is 0.2. Following Ascari (2000), we set θ= 6. Finally, we set $\eta_{CC} = 1$ and $\sigma = 1$, which are all standard values used in the literature (see for example Huang and Liu (2002)).

Finally, we assume that at time t there is 1% shock to the distrubance term corresponding to the money growth rate, ξ_t, so that ,$\xi(t) = 1$, and $\xi(s) = 0$ for all $s > t$.

3.2.1 The Calibration of γ

The key parameter determining aggregate dynamics is γ. The magnitude of γ is important since it governs how responsive household-unions are to current and future changes in output (see equation 18). When there is an increase in aggregate demand, households face higher demand for their labour and therefore the marginal disutility of labour increases. With higher income they consume more and marginal utility of consumption falls. The increase combination of an increase in the marginal disutility of labour and the fall in the marginal utility of consumption leads household-unions to increase their wage[5]. The coefficient γ determines how wages change in response to changes in current and future output. If γ is large, then wages respond a lot to changes in output which implies faster adjustments and a short-lived response of output. On the other hand, if γ is small, then unions are not sensitive to changes in current and future output. In response to an increase in aggregate demand, the wage would not change very much and hence wages are more rigid. In the limit, if $\gamma = 0$, there will be no relationship between output and wages, so that shocks are permanent. Hence the smaller γ, the more rigid are wages and a smaller γ corresponds to a more persistent output.

Estimating γ as an unconstrained parameter, Taylor found that for the US γ is between 0.05 and 0.1. However, in a general equilibrium framework where we constrain γ to conform to micro-foundations. CKM find that with reasonable parameter values, γ will be bigger than one in a staggered price setting, whilst with staggered wage setting Ascari finds the value of γ to be

[5] In the context of price-setting, the coefficient reflects the slope of hte marginal cost curve.

0.2. Both CKM and Ascari argue that the microfounded value of γ is too high generate observed persistence following a monetary shock which raises doubts over the Taylor model in this respect. In a general equilibrium setting, γ is determined by the fundamental parameters of the model according to (19). In particular, its magnitude depends on the parameter governing risk aversion, $\eta_{cc,}$ the labour supply elasticity, η_{ll}^{-1} and the elasticity of substitution of consumption goods θ which determines the elasticity of firm demand and the markup from (3) and hence the markup (5).

With staggered price setting, Chari et al. (2000) find that with reasonable parameter values, the value of γ is bigger than one: in particular with $\sigma = 1$

$$\gamma^{CKM} = \eta_{ll} + \eta_{cc} = 1.2 > 1$$

However, for CKM the value of γ^{CKM} could reasonably be much higher[6]: for example with $\eta_{ll} = 4.5$ and $\eta_{cc} = 1$, $\gamma^{CKM} = 5.5$. Huang and Liu (2002) choose to set $\eta_{ll} = 2$, so that $\gamma^{CKM} = 2$.

As has been argued by Huang and Liu (2002) staggered price setting underestimates the persistence of output. The value of γ with wage-setting is much smaller. In our model, as in Ascari, with $\sigma = 1$,

$$\gamma^{Ascari} = \frac{\eta_{ll} + \eta_{cc}}{1 + \theta\eta_{ll}} = \frac{\gamma^{CKM}}{1 + \theta\eta_{ll}}$$

Under our preferred calibration, $\gamma^{CKM} = 5.5$, and $1 + \theta\eta_{ll} = 27$, so that $\gamma^{Ascari} = 0.2$. The value of γ under wage setting could arguably be much smaller: some authors set $\theta = 10$ and combined with a smaller $\eta_{ll} = 2$, $\gamma = 1/7 = 0.14$. Thus, our preferred calibration is almost the same as Ascari, so for wage-setting we use the Ascari value of $\gamma = 0.2$.

The lower value of γ is significant and means that with staggered wages the aggregate price level changes more slowly than with staggered prices. It is thus inferred that staggered wage setting has greater potential to generate persistence[7]. As a result, although in both mechanisms the equations are essentially identical, they have different implications on persistence in a general equilibrium framework. Whilst Ascari (2000) shows that output is more

[6]Since CKM were aiming to show that the staggered price model did not generate enough peristence, they chose a value of γ^{CKM} which was low to make the model as persistent as it could reasonably be.

[7]This contradicts with the common view that both settings have similar implications on persistence. Altough the equations are esentially the same. The the value of γ differs across settings. See Huang and Liu (2002) for further discussion.

persistent with the staggered wage setting, he shows it is still not persistent enough to generate the observed persistence in output. Therefore, in line with CKM, Ascari (2000) finds that staggered wage setting cannot generate enough output persistence.

Figure 1 illustrates how the magnitude of γ can affect the result by showing the impulse responses in two different cases. We use the value of γ originally used by CKM and Ascari (2000), which are $\gamma = 1.22$ and $\gamma = 0.20$ respectively. We assume as simple Taylor economy with $T = 2$ (wages last 6 months). All other decisions are made quarterly. We display impulse-response of output after a one percent monetary shock. As we can see from Figure 1, in response to the one percent monetary shock, output displays similar patterns in the case of $\gamma = 1.22$ and $\gamma = 0.20$. For both cases, output increases when the shock hits and quickly returns to its steady state level. For the case of $\gamma = 1.22$, output returns to steady state level when unions have had the opportunity to reset the wages. On the other hand, output is certainly more persistent with $\gamma = 0.20$, but not significantly. Finally, for the sake of comparison, we also include the impulse response of output in the case with $\gamma = 0.05$ originally used by Taylor (1980), which yields a level of persistence more in line with the evidence, but not the microfoundations.

4 Persistence in a GTE

The existing literature has tended to focus on the value γ in generating persistence. We want to explore another dimension: for a given γ, we allow for different contract lengths in the *GTE* framework we have developed. Having more than one type of contract length thus is necessary if the model is to generate output persistence beyond the initial contract period. In what follows, we show that including longer term contracts can significantly increase persistence. Of course, this is in a sense obvious: longer contracts lead to more persistence, and we can achieve any level of persistence if contracts are long enough (so long as $\gamma > 0$). However, we want to show that even small proportion of long-term contracts can lead to a significant increase. Throughout this section, we will take the value of $\gamma = 0.2$ and explore how persistence changes when we allow for a range of contract lengths. We do this in three stages: first we simply illustrate our case with a simple two sector example. Second, we use Taylor's 1993 calibrated model of the US economy allowing for contract lengths from 1-8 quarters. Lastly, we consider the Calvo contract process with the corresponding distribution of contract lengths from 1 to infinity.

First, let us take the simple case of a two sector uniform GTE, $\{\mathbf{T}, \boldsymbol{\alpha}\} = \{(2,8),(0.9,0.1)\}$: in sector 1 there are two period contracts, in sector 2 there are 8 period contracts: the short contract sectors produce 90% of the economies output, the long-contracts 10%. The average contract length in the whole economy (weighted by α_i) is 2.6 quarters.

In Figure 2 we show both the simple Taylor economy with only 2-period contracts alongside the GTE with 10% share of 8-period contracts. We report the impulse response of aggregate output after a one-percent shock in money supply as in Figure 1[8]. As can be seen from the Figure 2, the GTE and simple Taylor economy have dramatically different implications for persistence. In the simple Taylor economy with 2-quarter contracts, changes in money supply have a potentially large but short-lived effect on output. In the GTE , the presence of long-term contracts means that not only does aggregate output rise following a increase in the money supply, but it is considerably fore persistent.

4.0.2 Taylor's US Economy

The main question addressed in this section is whether the GTE can account for multiplier. To calibrate the share of each sector, N_i,which can be interpreted as the share of different contracts, we rely on the study by (Taylor (1993)), Taylor Calibrates the US economy as $\mathbf{T} = (1,2,3,4,5,6,7,8)$, with sector shares being: $\alpha_1 = 0.07$, $\alpha_2 = 0.19$, $\alpha_3 = 0.23, \alpha_4 = 0.21, \alpha_5 = 0.15, \alpha_6 = 0.08, \alpha_7 = 0.04, \alpha_8 = 0.03$. We can note that the largest sector is $3-$period contracts, the three contract lengths $(3,4,5)$ each have about 20%, with a fat tail of longer contracts (as many 7 and 8 quarter contracts as 1 quarter contracts).

In Figure 3, we report the response of output to innovations in monetary shock. We find persistent response in output. In particular, the effect of a one-percent monetary shock on output lasts roughly three years. It is evident that incorporating generalized wage setting into a dynamic equilibrium model has a significant effect on dynamic responses of output. The average contract length in this economy is 3.6 periods. We compare this economy with the corresponding simple Taylor Economy with an average contract length of 3.5

[8] We use Dynare to compute the impulse response functions. See Juillard (1996).

periods.

In Figure 4 we plot the output responses for 4 different *GTE*s. The responses are normalized in the sense that the impact is set at 1. We have the Taylor US economy as in Figure 3, we have the simple 2-period Taylor case, the GTE with 90% 2-period and 10% 8-period contracts ,and finally the GTE with 50% 3 period and 50% 4 period contracts (which gives the average 3.5 periods which is almost the same as Taylor's US economy. If we compare Taylor's US economy with the simpler economy with mean contract length 3.5, we can see that Taylor's US economy is more persistent: this is because it includes longer contracts despite having the same mean. We can also read off from Figure 4 the half lives of the impulse-response functions, which gives us a quantitative measure of the degree of persistence. For example, when there is simple Taylor economy with only 2-period contracts alongside the GTE with 10% share of 8-period contracts, the half-life increases from 0.74 periods to 1.25 periods. This is also the case when we compare Taylor's US economy with the corresponding Simple Taylor Economy. In particular, half-life increases from 3.5 periods to 4.4 periods.

5 Comparison with Calvo Economy

It has long been noted that Calvo contracts appear to be far more persistent than Taylor contracts. In this section, we will show that if we focus on the structure of contracts (as opposed to the wage-setting rule), the Calvo economy is a special case of the *GTE*. Kiley (2002) considers a setup very similar to ours (the main difference being that he focuses on price-setting) to compare a simple Taylor economy with a Calvo economy. His findings are that these two models imply very different dynamics both qualitatively and quantitatively. In particular, he compares a simple Taylor economy with 2 period contracts and a Calvo economy with a reset probability $\omega = 0.5$. He concludes that: "...Staggering imparts much less persistence than does partial adjustment for nearly all paramtererizations..." Kiley (2002).

What we show is that Kiley is making the wrong comparison. We can directly describe the Calvo contract structure in terms of a *GTE*. When we do this, the dynamics of the Calvo economy and the corresponding GTE are very similar (they differ only because of different wage-setting behavior). Indeed, the main reason for Kiley's conclusion is that he is not comparing like with like. The Taylor economy is described in terms of the distribution of *completed* contract lengths: the average completed contract in Calvo economy is $\frac{2}{\omega}-1$, which is almost twice the length of the expected contract duration $(1/\omega)$. Hence, we find that Kiley compares a Calvo economy with $\omega = 0.5$ which implies an average contract length of 3 periods with a 2-period Taylor contract. If we compare Calvo $\omega = 0.5$ with a Taylor economy with 3-period contracts, the persistence properties of the Calvo and Taylor are similar[9].

5.0.3 Calvo contracts as a special case of a *GTE*

Two main features of the Calvo setup stand out as different form the standard Taylor setup. First its "stochastic" nature: at the firm level, the length of the wage contract is random; second, that the model is described in terms of the "age" of contracts (which includes uncompleted durations) and the hazard rate (the reset probability ω). The second feature is easy to remedy: we can look at either Taylor or Calvo contracts and describe them in terms of either the distribution of completed contract durations (*lifespans*) or in terms of the distribution of *ages* all durations (complete and incomplete): it is simply two ways of describing the same process. On the first issue. the stochastic nature of the Calvo model at the firm level does affect the wage setting decision. However, apart form the wage setting decision we can describe the Calvo process in deterministic terms at the aggregate level because the firm level randomness washes out. At the aggregate level, the precise identity of individual firms does not matter: what matters is population demographics in terms of proportions of firms setting contracts of particular lengths at particular times. Because there is a continuum of firms, the behavior of contracts at the aggregate level can be seen as a purely deterministic process.

First, we set aside the precise level of wages and observe the duration of wages. We focus on the "demographics" of the contract lengths in a Calvo process[10]. If we take a snapshot of the economy in period t we will observe a proportion ω of wages changing of the remaining $(1-\omega)$ firm/unions do not. We observe the distribution of *durations*: a proportion $\omega(1-\omega)^{s-1}$ have been been in place for s periods. These are *non-completed* durations for $(1-\omega)$

[9] See Dixon and Kara (2004)

[10] For a discussion of "life span" and "age", in the unemployment duration literature, see Salant (1977), Carlson and Horrigan (1983). These studies present some examples and show that $E\left[T \mid s\right] = 2\omega^{-1}$. However, in discrete time specification needs to be a small adjustment in this formula, as pointed out by Carlson and Horrigan (1983) (cf. Luckett (1979))

firms. The expected completed duration T for a contract that has survived for s periods is $E[T \mid s] = s + \omega^{-1}$ for all $s \geq 0$ (the since the hazard rate ω is constant).

The steady-state cross-section of contract ages can be described by the proportions α_i^s of firms surviving at least s periods:

$$\alpha_i^s = \omega (1-\omega)^{i-1} : i = 1..\infty \tag{24}$$

with mean $\bar{s} = \omega^{-1}$. In demographic terms, s is the *age* of the contract: α_i^s is the proportion of the population of age s; $\bar{s}$ is the average age of the population.

The corresponding distribution of *completed* contract lengths i is given by[11]:

$$\alpha_i = \omega^2 i (1-\omega)^{i-1} \tag{25}$$

with mean $\bar{T} = \frac{2-\omega}{\omega}$. In demographic terms, T gives the distribution of ages at death (for example as reported by the registrar of deaths): α_i being the proportion of the steady state population who will live to die at age T.

Assuming that we are in steady state (which is implicit in the use of the Calvo model), we can assume that there are in fact ex ante *fixed* contract lengths. We can classify firms/unions by the duration of their "contract". The fact that the contract length of a firm/union is fixed is perfectly compatible with the notion of a reset probability if we assume that the firm does not know the contract length. We can think of the firm-union having a probability distribution over contract lengths given by α_i^s in (24): Nature chooses the contract length, but the firm-unions do not know when they have to set the price (when the contract begins).

Having redefined the Calvo economy in terms of completed contract lengths, we can now describe it as a *GTE*. There are an infinite number of sectors, each strictly positive integer corresponding to a contract length:

$$N = \infty \quad T_i = i, i = 1...\infty$$

The proportions of each completed contract length are given by (25) .The wage setting process in each sector is uniform: there are T_i^{-1} cohorts of equal size.

$$N_i = T_i^{-1}, \lambda_{ij} = \frac{1}{T_i}$$

[11] See Dixon and Kara (2004), Proposition 1.

Let us just check that this will yield a contract structure equivalent to the Calvo process. Since the wage setting process is uniform, we can consider the representative period. In the sector with T_i period contracts, a proportion α_i/T_i contracts come to an end. Hence, using (25) and summing across all sectors the total measure of all contracts in the economy coming to an end in any period is ω, since:

$$\sum_{i=1}^{\infty} \frac{\alpha_i}{T_i} = \sum_{i=1}^{\infty} \omega^2 (1-\omega)^{i-1} = \omega$$

As in the Calvo process, in the Calvo-GTE a proportion ω of the population comes to the end of a contract. If we look at this Calvo-GTE, the average observed duration of contacts (completed and non-completed) will be ω^{-1}. To see why, let us derive the average age from the distribution of contract lengths. The proportion of contracts age i is obtained by summing across all cohorts who reset wages i periods ago. Clearly, this means we sum only over sectors with completed contract lengths $T \geq i$

$$\begin{aligned} \alpha_i^s &= \sum_{T \geq i}^{\infty} \omega^2 (1-\omega)^{T-1} \\ &= \omega^2 (1-\omega)^{i-1} \sum_{T=1}^{\infty} (1-\omega)^{T-1} \\ &= \omega (1-\omega)^{i-1} \end{aligned}$$

Hence, as in Calvo, the average age of contracts in a *Calvo*-GTE is ω^{-1}. So, the average age of a contract in steady-state cross-section is ω^{-1}, and the average age of completed contracts is $2\omega^{-1} - 1$. This is because in a uniform wage process, on average each contract will be about half way through. In continuous time they would be exactly half way through: but because of the measurement of time in integers, it is not quite so: the average age is more than half completed contract length: in fact it is 0.5 more[12]. Thus for example, if we measure the average age of two period contracts in a uniform simple Taylor setting, the average age is $1.5 = \frac{1+2}{2}$, not 1 as it would be in

[12] In general,

$$\frac{1}{T} \sum_{i=1}^{T} i = \frac{T+1}{2}$$

continuous time. Hence the completed contract length is a little less than twice the age[13].

5.0.4 Pricing in the Calvo-*GTE*.

We have defined the Calvo-GTE in terms of the structure of completed contract lengths. The only difference between the Calvo economy and the Calvo-GTE is in the pricing decision. In the Calvo economy, the firm is uncertain of the contract length: the pricing decision must be made "ex ante", that is before the firm knows which length nature has choosen. This yields the standard Calvo pricing decision. Once the price is set, the firm finds out its contract length in due course[14]. By contrast, in the *Calvo-GTE*, the firm-unions know which sector they belong to when they set the wage. This has two implications. First, whereas in the Calvo process, all firm-unions will set the *same* price (since they have the same subjective probability distribution over durations), in the Taylor equivalent the firms-unions set a price conditional on the contract length. Hence, wages in each sector of the Calvo-GTE will be different. Taking the simple case of $\beta = 1$, from (18) the reset wage in sector i with a T_i contract is then the average "optimal" price over the T_i periods is

$$x_{it} = \frac{1}{T_i} \sum_{s=0}^{T_i} (p_{t+s} + \gamma y_{t+s})$$

Proposition 1 *Let $\beta = 1$. The mean reset wage at time t in the Calvo-GTE is*

$$\bar{x}_t = \sum_{s=0}^{\infty} \omega^2 (1-\omega)^{s-1} (p_{t+s} + \gamma y_{t+s})$$

Clearly, the *mean* reset wage in the Calvo-GTE is equal to the standard Calvo reset wage. In the Calvo-GTE, there is a distribution of sector specific reset wages x_{it} in each period. Hence, in addition to the distribution of prices across cohorts (defined by when they last reset prices) in the Calvo model, the GTE has a distribution across sectors within the cohort, but with same mean.

[13] See Luckett (1979).

[14] It does not matter when: either straight after the pricing decision or at the last moment when it gets the Calvo phone call that it is time to reset the wage.

5.0.5 Persistence in the Calvo and Calvo-GTE compared.

We now compare the Calvo-GTE and the standard Calvo economy. In theory, the Calvo-GTE and the Calvo economy are exactly the same except for the pricing decision. However, for computational purposes whilst the Calvo economy effectively has an infinite lag structure (via the Koyck transform), the Calvo-GTE has to be truncated. Hence we also introduce a Calvo-Calvo-GTE: that is the GTE with the same contract structure and pricing rule as the Calvo model, but truncated as in the Calvo-GTE. For the simulations, we truncated the distribution of contract lengths to 20 quarters $T = 1, ...20.$ with the 20 period contracts absorbing all of the weight from the longer contracts. When we apply the standard Calvo pricing rule to this truncated distribution, it yields a perceptible but negligible difference; hence all of the visually apparent differences between the Calvo-GTE and the standard Calvo model are due almost entirely to the difference in pricing behaviour.

In Figure 5 we compare the impulse response for the Calvo-GTE which has the same distribution of completed contract lengths as the Calvo distribution, with the standard Calvo economy for $\omega = 0.25$. We find that Calvo-GTE has almost exactly the same persistence as the Calvo economy. The effect is as little larger for 5 quarters and a little less subsequently. We also show the standard Taylor economy with the same mean contract length $\bar{T} = 7$. Although the effect is a greater for the first 5 quarters, the effect dies down and is significantly less thereafter. This reflects the fact that although the mean contract lengths are the same, the longer contracts in the Calvo and Calvo-GTE generate the extra persistence.

For comparison, Figure 6 shows the distributions of fractions of contracts for three different specifications; Calvo-GTE, the Taylor 1993 calibration of the US economy, and Calvo distribution of all durations (complete and incomplete).

6 Conclusions

In this paper we have developed a general framework, the GTE which unifies the previously disparate approaches of modelling dynamic price and wage setting: Calvo and Taylor. The approach is a generalization of the simple Taylor model to take into account the presence of a range of different contract lengths. We use this approach to focus on the effect of the presence longer term contracts on the persistence of impulse-response functions generated by a monetary shock. Our conclusions are the following:

- A small proportion of long-term contracts can generate a significant increase in persistence.
- The average length of contracts in the Calvo model has been seriously underestimated, because the age and life-time of contracts have been confused. If modelers want an average contract length of 4-quarters, they should choose a reset probability of $\omega = 0.4$. The often used value of 0.25 generates an average contract length of 7 quarters.
- When we compare the standard Calvo model with the corresponding Calvo-GTE, we find that although the wage-setting behavior differs, the persistence of the two is very similar.
- In general, if we want to model an economy with many different contract lengths using a simple Taylor economy, we should choose a contract length which is greater than the average. This is becuase the presence of contracts with longer duration leads to more persistence despite having a similar mean.

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7 Appendix

The mean reset wage at time t, $\bar{x}_t$ in the GTE is thus

$$\begin{aligned}
\bar{x}_t &= \sum_{T=1}^{\infty} \omega^2 T (1-\omega)^{T-1} \left[\sum_{s=0}^{T} \frac{(p_{t+s} + \gamma y_{t+s})}{T} \right] \\
&= \sum_{T=1}^{\infty} \omega^2 (1-\omega)^{T-1} \left[\sum_{s=0}^{T-1} (p_{t+s} + \gamma y_{t+s}) \right] \\
&= \omega (p_t + \gamma y_t) + \omega^2 (p_{t+1} + \gamma y_{t+1}) \sum_{T=2}^{\infty} (1-\omega)^{T-1} + \\
&\quad \omega^2 (p_{t+3} + \gamma y_{t+3}) \sum_{T=3}^{\infty} (1-\omega)^{T-1} + ... \\
&= \omega (p_t + \gamma y_t) + \omega^2 (1-\omega) (p_{t+1} + \gamma y_{t+1}) \sum_{T=2}^{\infty} (1-\omega)^{T-2} + \\
&\quad \omega^2 (1-\omega)^2 (p_{t+3} + \gamma y_{t+3}) \sum_{T=3}^{\infty} (1-\omega)^{T-3} + ... \\
&= \omega (p_t + \gamma y_t) + \omega^2 (1-\omega) (p_{t+1} + \gamma y_{t+1}) \sum_{T=1}^{\infty} (1-\omega)^{T-1} + \\
&\quad \omega^2 (1-\omega)^2 (p_{t+3} + \gamma y_{t+3}) \sum_{T=1}^{\infty} (1-\omega)^{T-1} + ... \\
&= \sum_{s=0}^{\infty} \omega^2 (1-\omega)^{s-1} (p_{t+s} + \gamma y_{t+s})
\end{aligned}$$

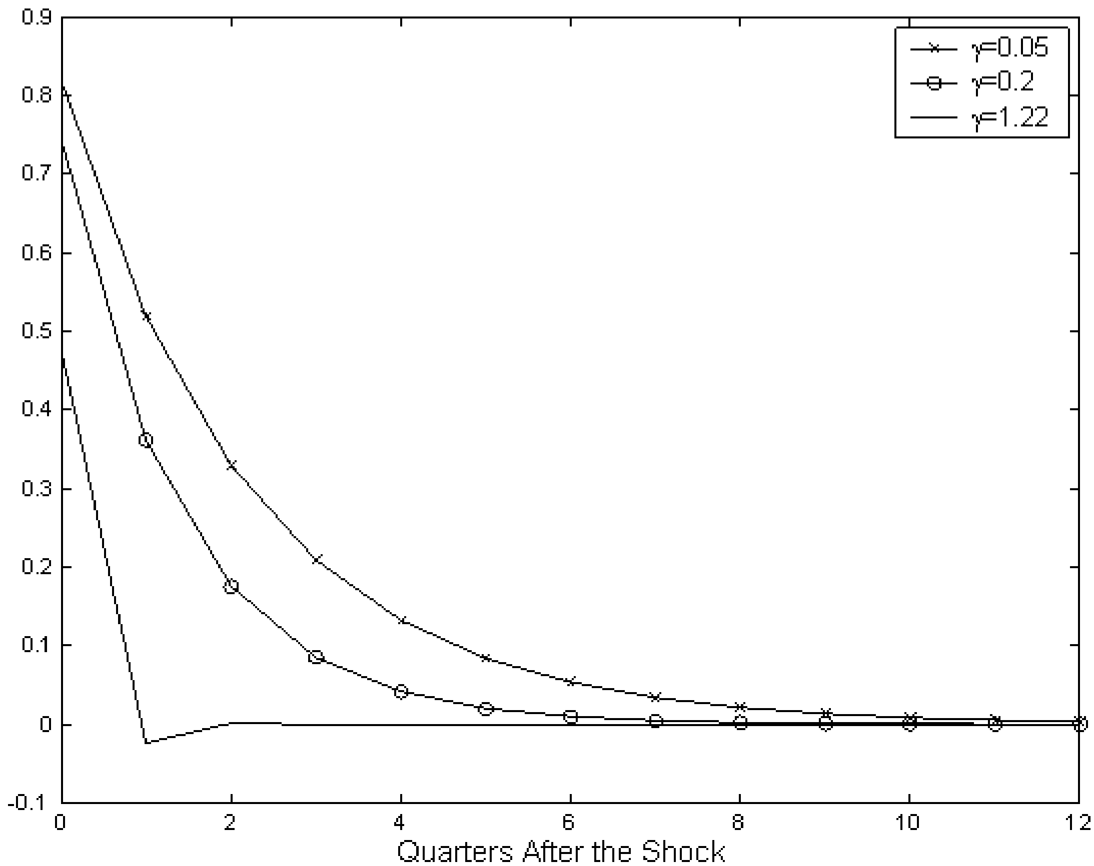

Figure 1: Output Response for Alternative $\gamma' s$

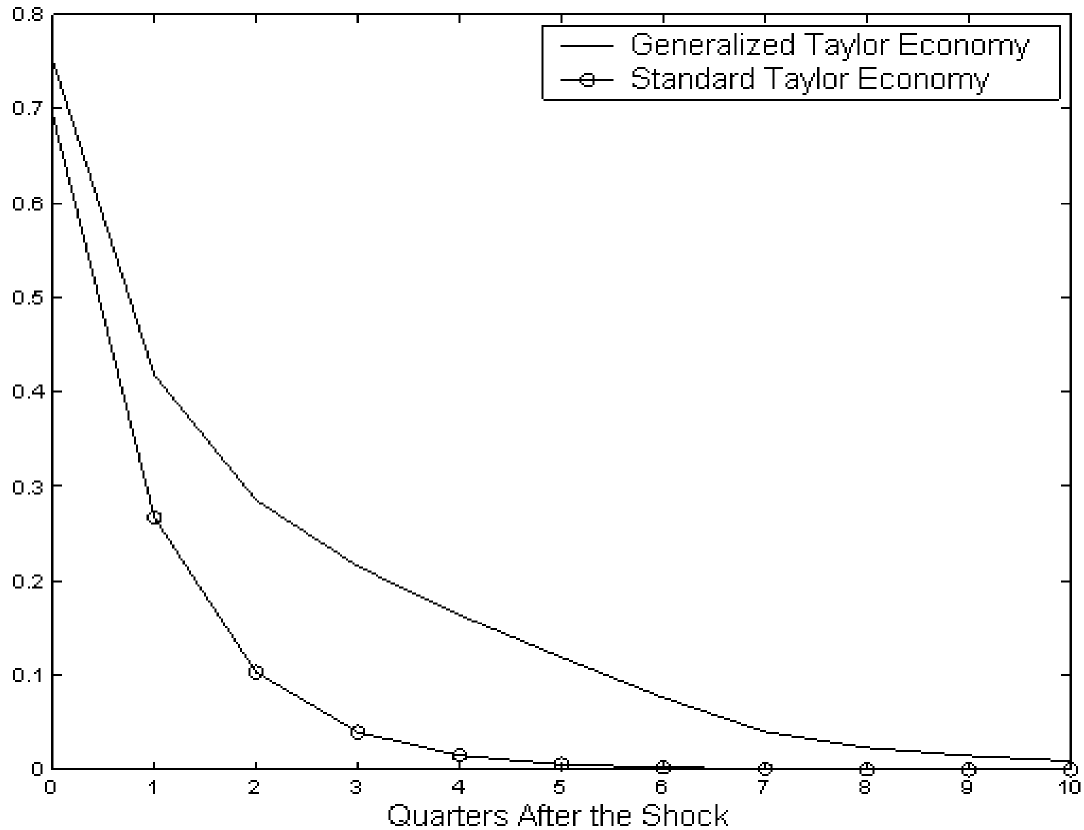

Figure 2: Output Response in Different Settings

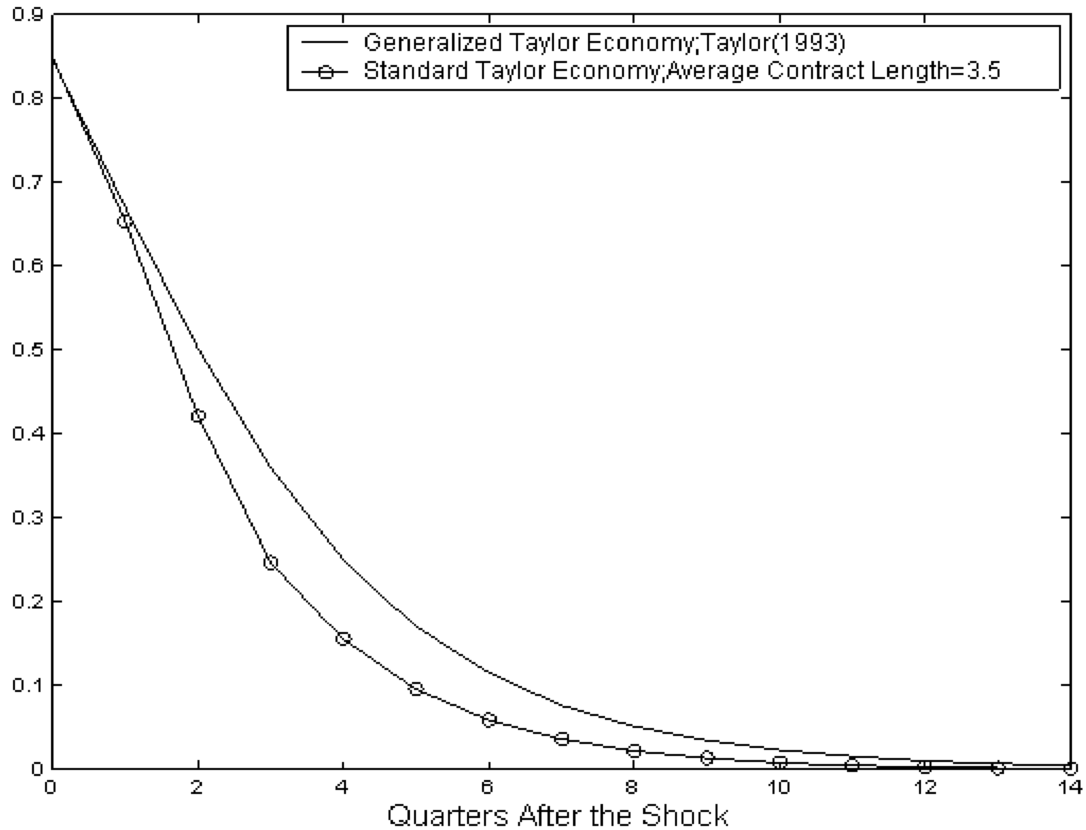

Figure 3: Output response in Taylor's US economy

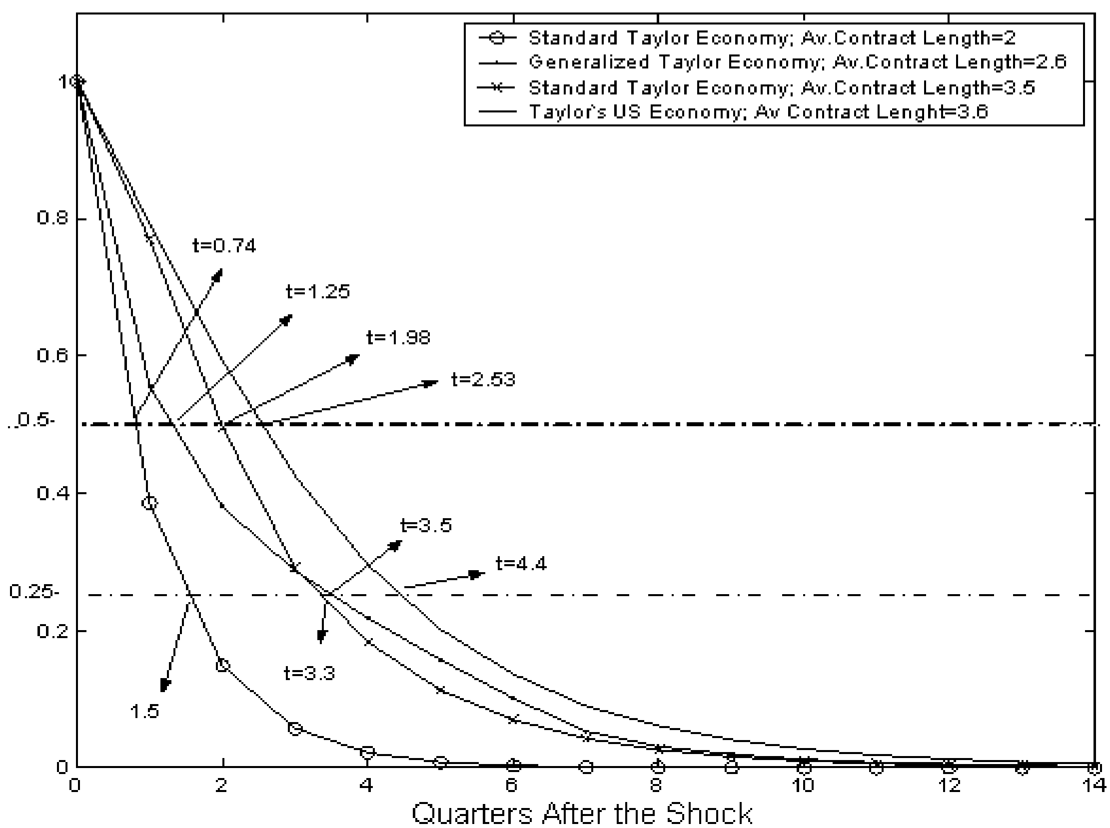

Figure 4: Normalized Responses

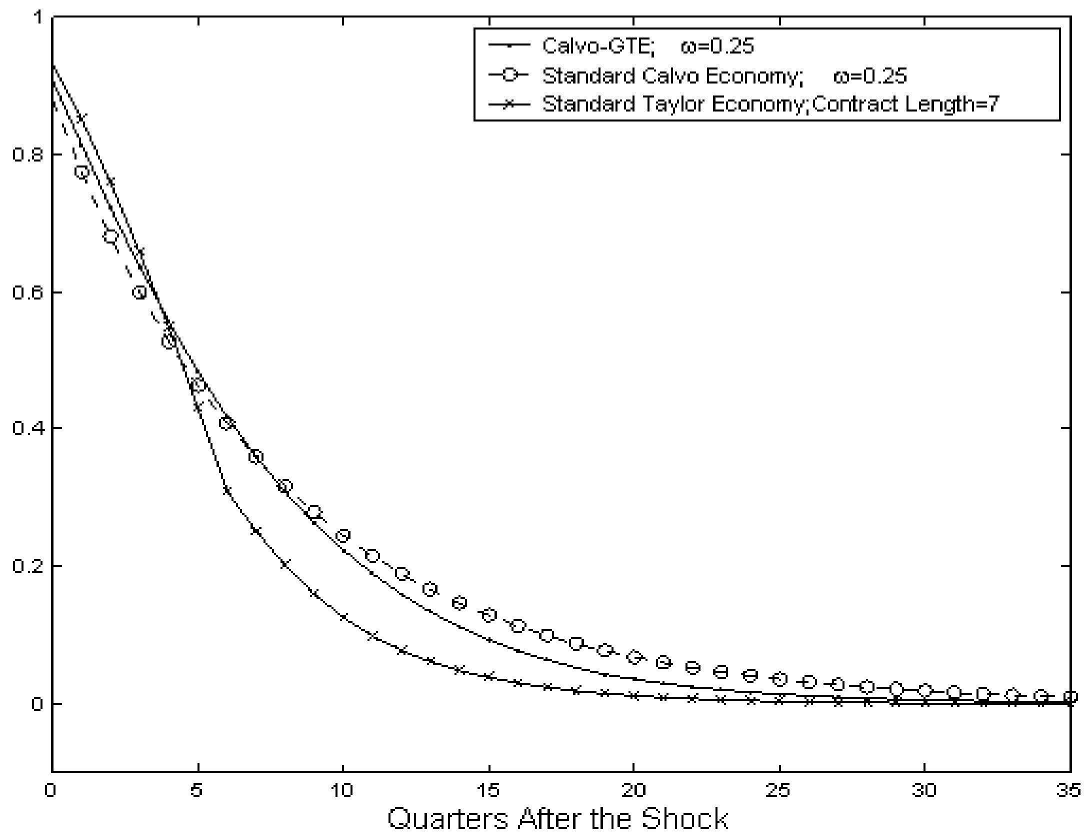

Figure 5: Output response of the Calvo Economy and the corresponding GTE

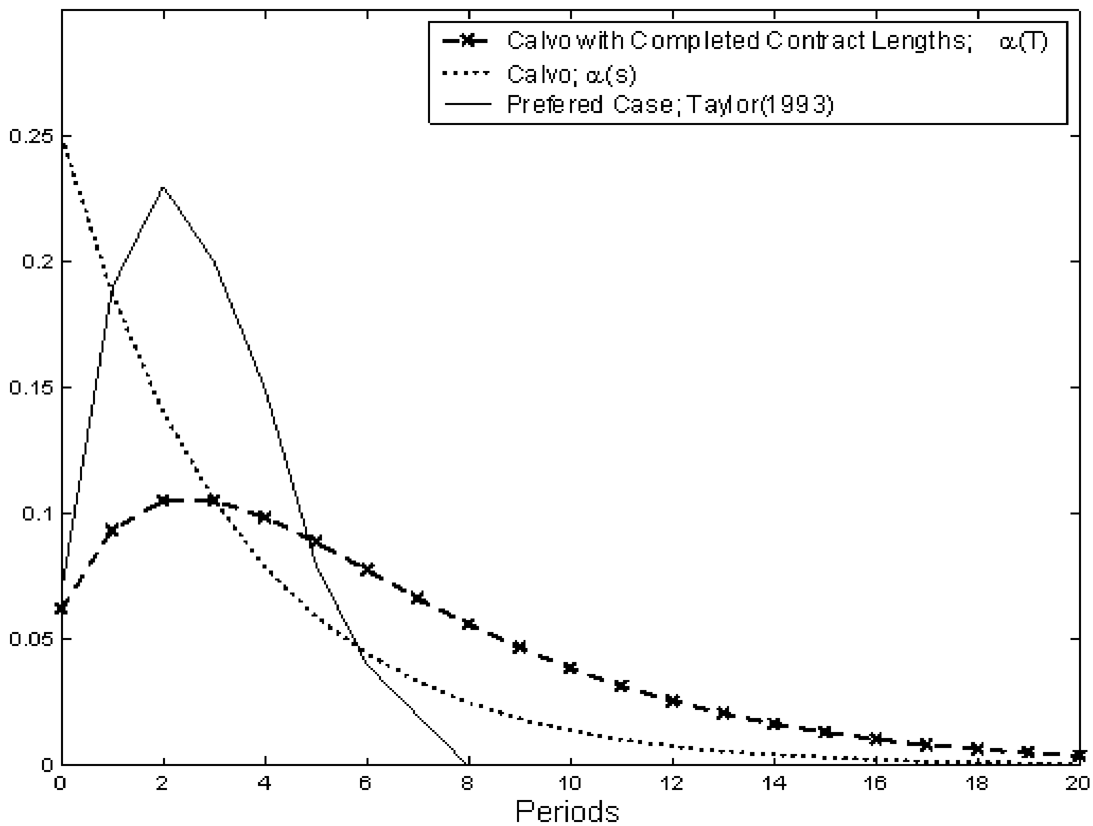

Figure 6: The Distribution of Contract Lengths(α) in Different Settings