Tools for Estimation of Grouped Conditional Logit Models

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McFadden's Discrete Choice Model

- There are N decisionmakers (i indexes decisionmakers).
- Each individual faces J_i choices (J_i is the choice set).
- Alternatives are finite, exhaustive, and mutually exclusive.
- Decisionmakers select option with highest value (utility, profit, etc).
- Examples:
  - Firm choice of location;
  - Immigrants residential choice;
  - Voter choice of candidate;
  - Patient choice of hospital.

McFadden's Discrete Choice Model

- p_{ij} - probability that individual i selects option j.

$$p_{ij} = \frac{\exp(\beta' \mathbf{x}_{ij})}{\sum_{j=1}^{J_i} \exp(\beta' \mathbf{x}_{ij})}$$

- The likelihood function is:

$$L_{DCM} = \prod_{i=1}^{N} \prod_{j=1}^{J_i} p_{ij}^{d_{ij}}$$

$$L_{DCM} = \underbrace{p_{11}^{d_{11}} p_{12}^{d_{12}} \ldots p_{1J_1}^{d_{1J_1}}}\, \underbrace{p_{21}^{d_{21}} p_{22}^{d_{22}} \ldots p_{2J_2}^{d_{2J_2}}} \cdots \underbrace{p_{N1}^{d_{N1}} p_{N2}^{d_{N2}} \ldots p_{NJ_N}^{d_{NJ_N}}}$$

d_{ij} is 1 if individual i picks choice j and 0 otherwise

Data Layout for `clogit`

$$
\begin{array}{ccc|cc|cc}
 & & & \mathbf{Y} & \mathbf{X} & & \\
i=1 & \left\{ \begin{array}{c} \text{Choice } 1 \\ \text{Choice } 2 \\ \dots \\ \text{Choice } J_1 \end{array} \right. & \begin{array}{c} p_{11} \\ p_{12} \\ \dots \\ p_{1J_1} \end{array} & \begin{array}{c} 0 \\ 1 \\ \dots \\ 0 \end{array} & \begin{array}{c} \mathbf{x}_{11} \\ \mathbf{x}_{21} \\ \dots \\ \mathbf{x}_{J_1 1} \end{array} & \mathbf{Y}_1 & \mathbf{X}_1 \\
 & & & & & & \\
i=2 & \left\{ \begin{array}{c} \text{Choice } 1 \\ \text{Choice } 2 \\ \dots \\ \text{Choice } J_2 \end{array} \right. & \begin{array}{c} p_{21} \\ p_{22} \\ \dots \\ p_{2J_2} \end{array} & \begin{array}{c} 1 \\ 0 \\ \dots \\ 0 \end{array} & \begin{array}{c} \mathbf{x}_{12} \\ \mathbf{x}_{22} \\ \dots \\ \mathbf{x}_{J_2 2} \end{array} & \mathbf{Y}_2 & \mathbf{X}_2
\end{array}
$$

The Grouped CLM

- Consider our examples again:
  - Patient choice of hospital;
  - Firm choice of location;
  - Immigrants residential choice;
  - Voter choice of candidate.
- Data may not change at the "individual" level.
- **Groups** of individuals face the same choices and attribute identical utility to each choice.
- The choice set may be very large.
- Data on choices may be summarized by vectors of counts.

The Likelihood of Grouped CLM

- If there are "groups" of individuals:

$$L_{DCM} = \underbrace{p_{11}^{d_{11}+d_{21}} p_{12}^{d_{12}+d_{22}} \ldots p_{1J_1}^{d_{1J_1}+d_{2J_2}}} \cdots \underbrace{p_{N1}^{d_{N1}} p_{N2}^{d_{N2}} \ldots p_{NJ_N}^{d_{NJ_N}}}$$

$$L_{GL} = \prod_{g=1}^{G} \prod_{j=1}^{J_i} p_{gj}^{n_{gj}}$$

- n_{gj} is the number of individuals from group g that select choice j.
- The likelihood function is the product of multinomial distributions for the groups.

Data Layout for the Grouped CLM

			$\mathbf{Y}$	$\mathbf{X}$
$g = 1$	Choice 1	p_{11}	n_{11}	$\mathbf{x}_{11}$
	Choice 2	p_{12}	n_{12}	$\mathbf{x}_{21}$
	...	...	...	...
	Choice J_1	p_{1J_1}	n_{1J_1}	$\mathbf{x}_{J_1 1}$
$g = 2$	Choice 1	p_{21}	n_{21}	$\mathbf{x}_{12}$
	Choice 2	p_{22}	n_{22}	$\mathbf{x}_{22}$
	...	...	...	...
	Choice J_2	p_{2J_2}	n_{2J_2}	$\mathbf{x}_{J_2 2}$

The `groupdata` command

- Description: converts the data set in memory to a new data set that can be used with commands that deal with multinomial type data. The dataset in memory must be in the format required by the command `clogit`.
- Syntax:
  ```
  groupdata varlist, dep(depvar)
  groupid(groupid) choiceid(choiceid)
  ```
- This command is useful only if you need to convert existing `clogit` type data to multinomial data.

The `multin` command

- Description: fits the grouped conditional logit regression model. It gives the same results as conditional logit regression but requires a smaller data set. The dependent variable is a count with the number of times each choice is selected.
- Syntax:
  `multin depvar [indepvars] [if] [in], group(varname) [options]`
- Estimates multinomial-likelihood regression models with a logit link function. With only two choices this is equivalent to a binomial regression model.

```
.
. * This example is taken from the Stata Manual
. *
. use http://www.stata-press.com/data/r9/choice, clear
. gen japan=(car==2)
. gen europe=(car==3)
. gen sexJap=sex*japan
. gen sexEur=sex*europe
. gen incJap=income*japan
. gen incEur=income*europe
. *
. * There are 295 individuals x 3 choices = 885 observations
. *
. sum
```

Variable	Obs	Mean	Std. Dev.	Min	Max
id	885	148	85.20683	1	295
sex	885	.7322034	.4430614	0	1
income	885	42.09661	12.42401	20.3	69.8
car	885	2	.8169583	1	3
size	885	2.623729	1.07612	1	4
choice	885	.3333333	.4716711	0	1
dealer	885	9.99322	7.145384	1	24
japan	885	.3333333	.4716711	0	1
europe	885	.3333333	.4716711	0	1
sexJap	885	.2440678	.429776	0	1
sexEur	885	.2440678	.429776	0	1
incJap	885	14.0322	21.11168	0	69.8
incEur	885	14.0322	21.11168	0	69.8

```
.
. * Estimation with clogit
. *
. clogit choice japan europe sexJap sexEur incJap incEur dealer, ///
> group(id) nolog

Conditional (fixed-effects) logistic regression   Number of obs   =        885
                                                  LR chi2(7)      =     146.62
                                                  Prob > chi2     =     0.0000
Log likelihood =  -250.7794                       Pseudo R2       =     0.2262

------------------------------------------------------------------------------
      choice |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       japan |  -1.352189   .6911829    -1.96   0.050    -2.706882    .0025049
      europe |  -2.355249   .8526681    -2.76   0.006    -4.026448   -.6840501
      sexJap |  -.5346039   .3141564    -1.70   0.089    -1.150339    .0811314
      sexEur |   .5704109   .4540247     1.26   0.209    -.3194612    1.460283
      incJap |   .0325318    .012824     2.54   0.011     .0073973    .0576663
      incEur |    .032042   .0138676     2.31   0.021      .004862    .0592219
      dealer |   .0680938   .0344465     1.98   0.048       .00058    .1356076
------------------------------------------------------------------------------
```

```
.
. * The command multin produces exactly the same results as clogit
. *
. multin choice japan europe sexJap sexEur incJap incEur dealer, ///
> group(id) nolog
Group is id

Grouped Conditional Logit Regression             Number of obs      =       885
Group variable: id                               Number of groups   =       295

                                                 Obs per group: min =         3
                                                                avg =       3.0
                                                                max =         3

                                                 Wald chi2(7)       =    120.19
Log likelihood  =  -250.7794                     Prob > chi2        =    0.0000

------------------------------------------------------------------------------
      choice |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       japan |  -1.352189   .6911829    -1.96   0.050    -2.706882    .0025049
      europe |  -2.355249   .8526681    -2.76   0.006    -4.026448   -.6840502
      sexJap |  -.5346039   .3141564    -1.70   0.089    -1.150339    .0811314
      sexEur |   .5704111   .4540247     1.26   0.209    -.3194611    1.460283
      incJap |   .0325318    .012824     2.54   0.011     .0073973    .0576663
      incEur |    .032042   .0138676     2.31   0.021      .004862    .0592219
      dealer |   .0680938   .0344465     1.98   0.048       .00058    .1356076
------------------------------------------------------------------------------
```

```
. *
. * Now we group the data with the groupdata command
. *
. groupdata japan europe sexJap sexEur incJap incEur dealer, ///
> dep(choice) groupid(id) choiceid(car)
Varlist  --> japan europe sexJap sexEur incJap incEur dealer
Dep Var  --> choice
Groupid  --> id
Choiceid  --> car
Getting there...
(735 observations deleted)

. *
. * We have 50 groups x 3 choices = 150 observations
. *
. sum

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
          id |       150       57.14    66.50933          1        242
         car |       150           2    .8192319          1          3
      dealer |       150    9.973333    7.057717          1         24
       japan |       150    .3333333    .4729838          0          1
      europe |       150    .3333333    .4729838          0          1
-------------+--------------------------------------------------------
      sexJap |       150    .2466667    .4325151          0          1
      sexEur |       150    .2466667    .4325151          0          1
      incJap |       150    13.94733    20.90216          0       69.8
      incEur |       150    13.94733    20.90216          0       69.8
      choice |       150    1.966667     3.04423          0          8
```

```
.
. * The multin command still produces the same results
. *
. multin choice japan europe sexJap sexEur incJap incEur dealer, ///
> group(id) nolog
Group is id

Grouped Conditional Logit Regression          Number of obs      =       150
Group variable: id                            Number of groups   =        50

                                              Obs per group: min =         3
                                                             avg =       3.0
                                                             max =         3

                                              Wald chi2(7)       =    120.19
Log likelihood  =  -250.7794                  Prob > chi2        =    0.0000

------------------------------------------------------------------------------
      choice |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       japan |  -1.352189   .6911829    -1.96   0.050    -2.706882    .0025049
      europe |  -2.355249   .8526681    -2.76   0.006    -4.026448   -.6840502
      sexJap |  -.5346039   .3141564    -1.70   0.089    -1.150339    .0811314
      sexEur |   .5704111   .4540247     1.26   0.209    -.3194611    1.460283
      incJap |   .0325318    .012824     2.54   0.011     .0073973    .0576663
      incEur |    .032042   .0138676     2.31   0.021      .004862    .0592219
      dealer |   .0680938   .0344465     1.98   0.048       .00058    .1356076
------------------------------------------------------------------------------
```

```
. * With two choices per groups multin performs binomial regression
. *
. drop if car==3
(50 observations deleted)

. sort id car

. bys id: egen n=sum(choice)

. drop if n==0
(14 observations deleted)

. *
. multin choice incJap sexJap japan, group(id) nolog
Group is id

Grouped Conditional Logit Regression                Number of obs      =        86
Group variable: id                                  Number of groups   =        43

                                                    Obs per group: min =         2
                                                                   avg =       2.0
                                                                   max =         2

                                                    Wald chi2(3)       =     59.53
Log likelihood  = -140.13099                        Prob > chi2        =    0.0000

------------------------------------------------------------------------------
      choice |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      incJap |   .0229655   .0113595     2.02   0.043     .0007014    .0452296
      sexJap |  -.4718934   .3119037    -1.51   0.130    -1.083214    .1394267
       japan |  -1.758435   .5847123    -3.01   0.003     -2.90445   -.6124198
------------------------------------------------------------------------------
```

```
. * The results are the same as for binomial regression
. *
. glm choice  incJap sexJap japan if car==2, family(bin n) nolog
note: japan dropped due to collinearity

Generalized linear models                          No. of obs      =        43
Optimization     : ML                              Residual df     =        40
                                                   Scale parameter =         1
Deviance         =  280.2619836                    (1/df) Deviance =   7.00655
Pearson          =  252.6354721                    (1/df) Pearson  =  6.315887

Variance function: V(u) = u*(1-u/n)                [Binomial]
Link function    : g(u) = ln(u/(n-u))              [Logit]
                                                   AIC             =  6.657255
Log likelihood   = -140.1309918                    BIC             =    30.729

------------------------------------------------------------------------------
             |                 OIM
      choice |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      incJap |   .0229655    .0113595     2.02   0.043     .0007014    .0452296
      sexJap |  -.4718934    .3119037    -1.51   0.130    -1.083214    .1394267
       _cons |  -1.758435    .5847123    -3.01   0.003     -2.90445   -.6124198
------------------------------------------------------------------------------

.
```

The Problem of Overdispersion

- What if there are unobserved group effects?
- For the examples considered groups could be:
  - SICs for firm choice of location;
  - Nationalities for immigrants' choices;
  - Precincts for voter choice;
  - DRGs for patient choice of hospital.
- The z-statistics are likely to be inflated.
- Similar to the effect of overdispersion in Poisson regression.

Dirichlet-Multinomial Regression

- A parametric alternative to deal with overdispersion in the grouped conditional logit model (or, more generally, multinomial- likelihood regression models).
- Assumes the existence of (gamma distributed) group random-effects and thus choices of individuals belonging to the same group are correlated.
- The likelihood has a closed form and thus estimation is fast and able to accommodate a large number of choices.
- Dirichlet-multinomial collapses to conditional logit if the random-effects have zero variance or if groups have just one individual.
- Dirichlet-multinomial regression is to grouped conditional logit what negative binomial is to poisson regression.

The `dirmul` command

- Description: fits the dirichlet-multinomial regression model. It is a parametric alternative to deal with overdispersed multinomial data. The data must be in the same format as for estimation with `multin`.
- Syntax:
  `dirmul depvar [indepvars] [if] [in], group(varname) [options]`
- The command accepts two different parameterizations for the dirichlet-multinomial distribution.
- Unlike `multin` it allows for the introduction of group level variables.
- With only two alternatives this command estimates the beta-binomial regression model.

Dirichlet-Multinomial: An Example

- Choice of hospital by patient in the Tampa-St. Petersburg market (1998 data).
- 13079 patients choose one of 25 hospitals.
- Patients are grouped by DRG $\times$ zipcode.
  - Hospital (choice) characteristics:
    - PROFIT - hospital is for profit;
    - TEACH - hospital is teaching;
    - NURSE - Nursing intensity (nursing hours per inpatient day);
    - CIRC - hospital has specialized services in circulatory diseases;
    - DVTIME - Drive time from zipcode area to hospital;
  - Group characteristics:
    - ZIPINC - Average zipcode level household income;
    - Dummies for DRG.

```
.
. * This dataset is already grouped
. *
. multin count profit teach nurseday dv* ii_circ, group(group) nolog
Group is group

Grouped Conditional Logit Regression              Number of obs      =     14950
Group variable: group                             Number of groups   =       598

                                                  Obs per group: min =        25
                                                                 avg =      25.0
                                                                 max =        25

                                                  Wald chi2(8)       =  17647.23
Log likelihood  = -14559.186                      Prob > chi2        =    0.0000

------------------------------------------------------------------------------
       count |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      profit |  -.6536845   .0389896   -16.77   0.000    -.7301028   -.5772663
       teach |  -1.874995   .0924765   -20.28   0.000    -2.056245   -1.693744
    nurseday |   .0684215   .0214293     3.19   0.001     .0264209    .1104222
      dvtime |  -.1509747   .0037079   -40.72   0.000    -.1582421   -.1437073
    dvprofit |  -.0094552   .0021184    -4.46   0.000    -.0136072   -.0053032
     dvteach |    .037509    .003817     9.83   0.000     .0300278    .0449902
  dvnurseday |   .0023778   .0010908     2.18   0.029     .0002398    .0045157
     ii_circ |   .8962367   .0397494    22.55   0.000     .8183294     .974144
------------------------------------------------------------------------------
```

```
.
. * The variance of group random effects is identical for all groups (Par1)
. *
. dirmul count profit teach nurseday dv* ii_circ, group(group) nolog
Dirichlet-Multinomial - Par1                    Number of obs      =     14950
Group variable: group                           Number of groups   =       598

                                                Obs per group: min =        25
                                                               avg =      25.0
                                                               max =        25

                                                Wald chi2(8)       =   3075.13
Log likelihood  = -7465.8792                    Prob > chi2        =    0.0000

------------------------------------------------------------------------------
       count |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      profit |  -.6555403   .0873018    -7.51   0.000    -.8266487   -.4844319
       teach |  -1.868412   .1802051   -10.37   0.000    -2.221607   -1.515216
    nurseday |   .1980528    .045711     4.33   0.000     .1084609    .2876448
      dvtime |  -.0850945   .0066882   -12.72   0.000    -.0982031   -.0719858
    dvprofit |   .0041665   .0039122     1.07   0.287    -.0035012    .0118342
     dvteach |   .0589334   .0068375     8.62   0.000     .0455323    .0723346
  dvnurseday |  -.0076552    .002036    -3.76   0.000    -.0116456   -.0036648
     ii_circ |   .6434551   .0595928    10.80   0.000     .5266553    .7602549
       _cons |    .005837   .1542142     0.04   0.970    -.2964173    .3080914
------------------------------------------------------------------------------
```

```
.
. * Choices for individuals in the same group are equicorrelated (Par2)
. *
. dirmul count profit teach nurseday dv* ii_circ, group(group) eqcorr nolog

Dirichlet-Multinomial - Par2                    Number of obs      =     14950
Group variable: group                           Number of groups   =       598

                                                Obs per group: min =        25
                                                               avg =      25.0
                                                               max =        25

                                                Wald chi2(8)       =   3454.59
Log likelihood  =   -7226.58                    Prob > chi2        =    0.0000

------------------------------------------------------------------------------
       count |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
eq1          |
      profit |  -.6600816   .0839166    -7.87   0.000    -.8245551    -.495608
       teach |  -1.753955   .1708162   -10.27   0.000    -2.088749   -1.419162
    nurseday |   .1907597   .0448168     4.26   0.000     .1029203    .2785991
      dvtime |  -.1003966   .0065316   -15.37   0.000    -.1131984   -.0875948
    dvprofit |   .0027863   .0036976     0.75   0.451    -.0044608    .0100334
     dvteach |   .0501731   .0062628     8.01   0.000     .0378982     .062448
  dvnurseday |  -.0064536   .0019445    -3.32   0.001    -.0102646   -.0026425
     ii_circ |   1.081281   .0852516    12.68   0.000     .9141911    1.248371
-------------+----------------------------------------------------------------
rho          |
       _cons |   .2320623   .0063269    36.68   0.000     .2196618    .2444627
------------------------------------------------------------------------------
```

```
.
. * The correlation coefficient is a function of other variables (Par2)
. *
. xi: dirmul count profit teach nurseday  dv* ii_circ, group(group) eqcorr var2
> (zipinc i.drg) nolog
i.drg             _Idrg_88-373        (naturally coded; _Idrg_88 omitted)

Dirichlet-Multinomial - Par2                    Number of obs      =     14950
Group variable: group                           Number of groups   =       598

                                                Obs per group: min =        25
                                                               avg =      25.0
                                                               max =        25

                                                Wald chi2(8)       =   3491.04
Log likelihood  = -7167.5924                    Prob > chi2        =    0.0000

------------------------------------------------------------------------------
       count |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
eq1          |
      profit |  -.6035418   .0837984    -7.20   0.000    -.7677836      -.4393
       teach |  -1.654932   .1707307    -9.69   0.000    -1.989558   -1.320306
    nurseday |   .1391581   .0450294     3.09   0.002     .0509021    .2274141
      dvtime |  -.1046364   .0065669   -15.93   0.000    -.1175073   -.0917654
    dvprofit |   .0015502    .003721     0.42   0.677    -.0057429    .0088432
     dvteach |   .0497152   .0062924     7.90   0.000     .0373824     .062048
  dvnurseday |  -.0055291    .001951    -2.83   0.005     -.009353   -.0017051
     ii_circ |   1.038091   .0853201    12.17   0.000     .8708668    1.205316
-------------+----------------------------------------------------------------
rho          |
      zipinc |   .0025323   .0011553     2.19   0.028     .0002679    .0047967
   _Idrg_116 |   .0433963   .0224043     1.94   0.053    -.0005152    .0873078
   _Idrg_127 |   -.012599   .0217572    -0.58   0.563    -.0552423    .0300443
   _Idrg_209 |  -.0832532   .0188009    -4.43   0.000    -.1201023   -.0464041
   _Idrg_373 |   .0890644   .0222103     4.01   0.000      .045533    .1325958
       _cons |   .1856912   .0239115     7.77   0.000     .1388255    .2325569
------------------------------------------------------------------------------
```

```
.
. * dirmul can be used to estimate the Beta-Binomial regression model
. *
. * This example is from Stata J. vol5, n.3 pp385-394
. *
. use williams
(Data from Williams (1975) paper)

. gen trt_cov=trt*(class==1)

. *
. * The Beta-Binomial with the Heckman&Willis parameterization (Par1)
. *
. dirmul y class1 trt_cov, group(group) nolog

Dirichlet-Multinomial - Par1                Number of obs      =        64
Group variable: group                       Number of groups   =        32

                                            Obs per group: min =         2
                                                           avg =       2.0
                                                           max =         2

                                            Wald chi2(2)       =     43.70
Log likelihood  = -54.046101                Prob > chi2        =    0.0000

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      class1 |   3.345722   .8159031     4.10   0.000     1.746581    4.944862
     trt_cov |  -1.161821   .4998502    -2.32   0.020    -2.141509   -.1821328
       _cons |  -.2026163   .4152369    -0.49   0.626    -1.016466    .6112331
------------------------------------------------------------------------------
```

```
. * The Beta-Binomial with equicorrelation (Par2)
. *
. dirmul y class1 trt_cov, group(group) eqcorr nolog

Dirichlet-Multinomial - Par2                    Number of obs      =        64
Group variable: group                           Number of groups   =        32

                                                Obs per group: min =         2
                                                               avg =       2.0
                                                               max =         2

                                                Wald chi2(2)       =     36.85
Log likelihood  = -55.608221                    Prob > chi2        =    0.0000

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
eq1          |
      class1 |   2.528208   .7673721     3.29   0.001     1.024186     4.03223
     trt_cov |  -.6651523   .4596397    -1.45   0.148    -1.566029    .2357249
-------------+----------------------------------------------------------------
rho          |
       _cons |   .1927195   .0725971     2.65   0.008     .0504317    .3350073
------------------------------------------------------------------------------
```

Conclusion

- `multin` estimates multinomial-likelihood models with logit link.
- `multin` produces the same results as clogit (for conditional logit or conditional logistic with one case and multiple controls) but may require a much smaller data set.
- `groupdata` converts individual level data (clogit) to multinomial grouped data.
- `dirmul` estimates a parametric alternative for overdispersed multinomial-likelihood models. It accepts two alternative parameterizations.
- `dirmul` may be used to estimate the beta-binomial regression model.

Additional Information

- To download these commands:
  `net from http://people.musc.edu/~guimarap/stata`
- For a reference:
  "Dirichlet-Multinomial Regression" by Paulo Guimaraes and Richard Lindrooth, Economics Working Paper Archive at WUSTL, Econometrics, no. 0509001, Available from IDEAS at:
  http://ideas.repec.org/p/wpa/wuwpem/0509001.html