Interpreting and using heterogeneous choice & generalized ordered logit models

Richard Williams, University of Notre Dame, July 2006, http://www.nd.edu/~rwilliam/

I. Abstract

The assumptions of the ordered logit/probit models estimated by ologit and oprobit are often violated. When an ordinal regression model incorrectly assumes that error variances are the same for all cases, the standard errors are wrong and (unlike OLS regression) the parameter estimates are biased. Heterogeneous choice/ location-scale models, which can be estimated with the user-written program oglm, explicitly specify the determinants of heteroskedasticity in an attempt to correct for it. Further, these models can be used when the variance/variability of underlying attitudes is itself of substantive interest. In other instances, the parallel lines assumption of the ordered logit/probit model is violated; in such cases, a generalized ordered logit/probit model (estimated via gologit2) may be called for. This paper talks about how to interpret and use the models that are estimated by oglm and gologit2. We talk about key assumptions behind the models, when each type of model may be appropriate, when the models may be problematic, and how to interpret the results and make them easier to understand.

II. gologit formulas

a. Totally unconstrained gologit model:

$$P(Y_i > j) = g(X\beta_j) = \frac{\exp(\alpha_j + X_i\beta_j)}{1+[\exp(\alpha_j + X_i\beta_j)]}, \text{ j} = 1, 2, ..., \text{M} - 1$$

M is the number of categories of the ordinal dependent variable. When M = 2, the gologit model is equivalent to the logistic regression model. When M > 2, the gologit model becomes equivalent to a series of binary logistic regressions where categories of the dependent variable are combined, e.g. if M = 4, then for J = 1 category 1 is contrasted with categories 2, 3 and 4; for J = 2 the contrast is between categories 1 and 2 versus 3 and 4; and for J = 3, it is categories 1, 2 and 3 versus category 4.

b. Totally constrained gologit model (equivalent to ologit).

Betas, but not the Alphas, are the same for all values of j.

$$P(Y_i > j) = g(X\beta) = \frac{\exp(\alpha_j + X_i\beta)}{1+[\exp(\alpha_j + X_i\beta)]}, \text{ j} = 1, 2, ..., \text{M} - 1$$

c. Partially constrained gologit model, i.e. Partial Proportional Odds.

Some Betas are the same for all values of j, but others are free to differ, e.g.

$$P(Y_i > j) = \frac{\exp(\alpha_j + X1_i\beta1 + X2_i\beta2 + X3_i\beta3_j)}{1+[\exp(\alpha_j + X1_i\beta1 + X2_i\beta2 + X3_i\beta3_j)]}, \text{ j} = 1, 2, ..., \text{M} - 1$$

III. Using Stata & gologit2 to estimate gologit models

```
. use "http://www.indiana.edu/~jslsoc/stata/spex_data/ordwarm2.dta"
(77 & 89 General Social Survey)

. * Unconstrained gologit
. quietly gologit2  warm yr89 male white age ed prst, npl store(gologit) lrf

. * ologit-equivalent model as estimated by gologit2
. quietly gologit2  warm yr89 male white age ed prst, pl store(ologit) lrf

. * Partial proportional odds - free yr89 & male from parallel lines constraint
. qui gologit2  warm yr89 male white age ed prst, npl(yr89 male) store(gologit2) lrf

. outreg2 [gologit ologit gologit2] using mygologit, replace word long onecol nor2
addstat(LR Chi-Square, e(chi2), d.f., e(df_m))
```

COEFFICIENT	Unconstrained gologit	Totally constrained gologit (equivalent to ologit)	Partially constrained gologit (partial proportional odds)
Strongly Disagree			
yr89	0.956***	0.524***	0.984***
male	-0.301**	-0.733***	-0.333***
white	-0.529**	-0.391***	-0.383***
age	-0.0163***	-0.0217***	-0.0216***
ed	0.103***	0.0672***	0.0671***
prst	-0.00169	0.00607*	0.00591*
Constant	1.857***	2.465***	2.122***
Disagree			
yr89	0.536***	0.524***	0.534***
male	-0.718***	-0.733***	-0.693***
white	-0.349**	-0.391***	-0.383***
age	-0.0250***	-0.0217***	-0.0216***
ed	0.0559***	0.0672***	0.0671***
prst	0.00985***	0.00607*	0.00591*
Constant	0.720***	0.631***	0.602**
Agree			
yr89	0.331***	0.524***	0.326***
male	-1.086***	-0.733***	-1.098***
white	-0.378**	-0.391***	-0.383***
age	-0.0187***	-0.0217***	-0.0216***
ed	0.0567**	0.0672***	0.0671***
prst	0.00492	0.00607*	0.00591*
Constant	-1.002***	-1.262***	-1.048***
Observations	2293	2293	2293
LR Chi-Square	350.9	301.7	338.3
d.f.	18	6	10

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

IV. The Heterogeneous choice model (Keele & Park, 2006)

$$\Pr(y_i = 1) = g\left(\frac{x_i\beta}{\exp(z_i\gamma)}\right) = g\left(\frac{x_i\beta}{\exp(\ln(\sigma_i))}\right) = g\left(\frac{x_i\beta}{\sigma_i}\right)$$

- g stands for the link function (in this case logit; probit is also commonly used, and other options are possible, such as the complementary log-log, log-log and cauchit).

- x is a vector of values for the ith observation. The x's are the explanatory variables and are said to be the determinants of the choice, or outcome.

- z is a vector of values for the ith observation. The z's define groups with different error variances in the underlying latent variable. The z's and x's need not include any of the same variables, although they can.

- Beta & Gamma are vectors of coefficients. They show how the x's affect the choice and the z's affect the variance (or more specifically, the log of sigma).

- The numerator in the above formula is referred to as the choice equation, while the denominator is the variance equation. These are also referred to as the location and scale equations. Also, the choice equation includes a constant term but the variance equation does not.

V. Stata example using oglm

```
. oglm warm yr89 male white age ed prst, het( yr89 male)  hc store(oglm)

Heteroskedastic Ordered Logistic Regression       Number of obs   =       2293
                                                  LR chi2(8)      =     331.03
                                                  Prob > chi2     =     0.0000
Log likelihood = -2830.2563                       Pseudo R2       =     0.0552

------------------------------------------------------------------------------
        warm |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
choice       |
        yr89 |   .4531574   .0686839     6.60   0.000     .3185394    .5877755
        male |  -.6345402   .0697638    -9.10   0.000    -.7712748   -.4978057
       white |  -.3087676    .102739    -3.01   0.003    -.5101323   -.1074029
         age |  -.0186098   .0021728    -8.56   0.000    -.0228684   -.0143512
          ed |   .0535685   .0135944     3.94   0.000     .0269239     .080213
        prst |   .0052866     .00278     1.90   0.057    -.0001622    .0107353
-------------+----------------------------------------------------------------
variance     |
        yr89 |  -.1486188   .0458169    -3.24   0.001    -.2384183   -.0588192
        male |  -.1909211    .044807    -4.26   0.000    -.2787412   -.1031011
-------------+----------------------------------------------------------------
       /cut1 |  -2.151122   .2114069   -10.18   0.000    -2.565472   -1.736772
       /cut2 |  -.5696264   .1992724    -2.86   0.004    -.9601932   -.1790596
       /cut3 |   1.066508   .2022099     5.27   0.000     .6701839    1.462832
------------------------------------------------------------------------------
```

VI. Comparison of Marginal Effects Across Models

Note: The `mfx2` command makes it easy to compute marginal effects in multiple-outcome models and to store them in a format that can be easily used by routines like `outreg2` and `estout`.

```
. quietly estimates restore ologit
. quietly mfx2, stub(ologit)
. quietly estimates restore oglm
. quietly mfx2, stub(oglm)
. quietly estimates restore gologit2
. quietly mfx2, stub(gologit2)
. outreg2 [ologit_mfx oglm_mfx gologit2_mfx] using mymfx, replace word long onecol
alpha(0.001, 0.01, 0.05) nor2
```

Marginal Effects for the ordered logit, heterogeneous choice, and gologit models

COEFFICIENT	Ordered Logit	Heterogeneous Choice	Gologit
Strongly Disagree			
yr89	-0.0499***	-0.0786***	-0.0896***
male	0.0746***	0.0355***	0.0326***
white	0.0345***	0.0319***	0.0332***
age	0.00214***	0.00213***	0.00209***
ed	-0.00664***	-0.00613***	-0.00650***
prst	-0.000600	-0.000605	-0.000573
Disagree			
yr89	-0.0775***	-0.0618***	-0.0404***
male	0.105***	0.137***	0.137***
white	0.0594***	0.0543***	0.0590***
age	0.00319***	0.00318***	0.00324***
ed	-0.00990***	-0.00916***	-0.0100***
prst	-0.000895	-0.000904	-0.000885
Agree			
yr89	0.0539***	0.0995***	0.0860***
male	-0.0814***	-0.0344***	-0.0270
white	-0.0356***	-0.0333***	-0.0363***
age	-0.00241***	-0.00240***	-0.00247***
ed	0.00746***	0.00691***	0.00767***
prst	0.000675	0.000682	0.000676
Strongly Agree			
yr89	0.0735***	0.0409***	0.0441***
male	-0.0979***	-0.138***	-0.143***
white	-0.0583***	-0.0529***	-0.0558***
age	-0.00293***	-0.00291***	-0.00286***
ed	0.00908***	0.00839***	0.00886***
prst	0.000821	0.000828	0.000782
Observations	2293	2293	2293

Standard errors in parentheses
*** p<0.001, ** p<0.01, * p<0.05