Constructing Krinsky and Robb Confidence Intervals for Mean and Median Willingness to Pay (WTP) Using Stata

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Motivation

- The ultimate goal pursued in most contingent valuation studies is to estimate willingness to pay (WTP) measures and confidence intervals.
- Because WTP measures are non-linear functions of estimated parameters, procedures such as the delta method (nlcom in Stata) are inappropriate as they yield symmetric confidence intervals (CI) .
- Non-symmetric CI obtained using Krinsky and Robb simulations are recommended (Park et al., 1991; Haab and McConnell, 2002; Creel and Loomis, 1991).
- Very recently, Arne Risa Hole (2007) has introduced the Krinsky and Robb procedure into Stata via the **wtp** command.
- However, this command does not feature mean and median WTP estimated from contingent valuation models.
- This void motivates me to develop a Stata program called **wtpcikr.**

Background

- National Oceanic Atmospheric Administration (NOAA) recommends the referendum (single bounded) format for eliciting WTP for non-market goods (Arrow et al., 1993).
- However, this approach yields inefficient welfare measures due to limited information obtained from each respondent.
- The referendum double bounded format (Hanemann et al., 1991) has emerged a means to improve efficiency in contingent valuation applications.
- Thus, double bounded models should provide narrower confidence intervals around welfare measures comparatively to single bounded ones.
- In other words, double bounded models (if more efficient) should yield lower ratios of confidence interval to mean (and/or median) WTP: (upper bound – lower bound)/mean.

Background Cont'd

- Example of referendum single bounded question format

  "Would you be willing to pay $X? Yes/No"

  WTP questions are also phrased as referendum vote questions:

  "If the proposed policy costs your household $X, would you vote in favor or against it?"

- Example of referendum double bounded question format

  "Would you be willing to pay $X? Yes/No
  - If Yes, would you be willing to pay $Z (where Z>X)? Yes/No
  - If No, would you be willing to pay $Y (where Y<X)? Yes/No"

Examples

- I illustrate the use of **wtpcikr** by two examples.
- In the first example, which replicates the CI results obtained by Haab and McConnell (2002), I use data from a contingent valuation survey with no follow-up question (single bounded format). The model estimated is an exponential probit where the log of the bid variable is taken.
- Data for the second example were collected from a contingent survey with a follow-up question (double bounded format). First, I estimate a (single bounded) probit model using responses to the first question only. Second, I estimate two (double bounded) bivariate probit models using answers to both the first and the follow-up questions (see Cameron and Quiggin, 1994). The second bivariate probit model is a restricted version of the first one.
- After estimating each model, **wtpcikr** is used to calculate mean/median WTP along with 95 percent confidence intervals based on Krinsky and Robb's procedure.
- In the next two slides, I present the formulas **wtpcikr** uses in the calculations and outline the Krinsky and Robb's procedure.

Formulas Used in the Calculations

Distribution	WTP or welfare Measure	Functional form	
		Linear	Exponential
Normal	Mean	$\frac{-\bar{X}\beta'}{\beta_0}$	$exp\left(\frac{-\bar{X}\beta'}{\beta_0}+0.5\sigma^2\right)$
	Median	$\frac{-\bar{X}\beta'}{\beta_0}$	$exp\left(\frac{-\bar{X}\beta'}{\beta_0}\right)$
Logistic	Mean	$\frac{-\bar{X}\beta'}{\beta_0}$	$\frac{\sigma\pi}{sin(\sigma\pi)}exp\left(\frac{-\bar{X}\beta'}{\beta_0}\right)$
	Median	$\frac{-\bar{X}\beta'}{\beta_0}$	$exp\left(\frac{-\bar{X}\beta'}{\beta_0}\right)$

$\bar{X}$ = row vector of sample mean including 1 for the constant term

$\beta'_{(k-1x1)}$ = column vector of estimated coefficients

β_0 = coefficient on the bid variable

In constant-only models, $\bar{X}=1$ and β' is the coefficient on the constant term

The Krinsky and Robb's Procedure

1. Estimate the WTP model of interest
2. Obtain the vector of parameter estimates $\hat{\beta}$ and the variance-covariance (VCV) matrix $V(\hat{\beta})$
3. Calculate the Cholesky decomposition, C, of the VCV matrix such that $CC' = V(\hat{\beta})$
4. Randomly draw from standard normal distribution a vector x with k independent elements
5. Calculate a new vector of parameter estimates Z such that $Z = \hat{\beta} + C'x$
6. Use the new vector Z to calculate the WTP measures of interest
7. Repeat steps 4, 5, and 6 N(>=5000) times to obtain an empirical distribution of WTP
8. Sort the N values of the WTP function in ascending order
9. Obtain a 95% confidence interval around mean/median by dropping the top and bottom 2.5% of the observations

You estimate the WTP model of interest, and **wtpcikr** takes care of the rest.

Program Syntax and Overview

- Syntax
  - **wtpcikr** *varlist* [if] [in] , [*options*]

- Overview
  - **wtpcikr** computes mean and/or median WTP, Krinsky and Robb confidence intervals, achieved significance levels, and relative efficiency measures.
  - It works after estimation commands which should be probit, logit, biprobit, and user-written estimation commands, provided that the formulas apply.
  - **wtpcikr** uses the Stata matrix language, **Mata,** to take random draws from multivariate normal distribution with variance-covariance matrix **vmat** and the vector of parameter estimates **bmat** or the defaults to these options.
  - For exponential logit models, mean WTP is not defined if sigma>1. Stata will issue a warning and only median WTP will be computed if sigma>1.

WTPCIKR Options

- **reps**(#) sets the number of replications. The default value of reps() is 5000.
- **seed**(#) sets the random number seed. Seed(032007) is the default.
- **level**(#) sets the confidence level. The default is level(95).
- **equation**(*name*) specifies the name of the equation to be used in calculation, if there are multiple equations. The default is to use the first equation.
- **bmat**(*name*) specifies a vector of parameter estimates to be used. The default is e(b).
- **vmat**(*name*) specifies a VCV matrix to be used. The default is e(V).

WTPCIKR Options

- **mymean**(*name*) specifies a mean vector to be used. The default is a vector containing the sample means of the independent variables specified in *varlist* (except the bid variable). This option cannot be used for constant-only models.
- **exponential** specifies that the model functional form is exponential. Without this option, a linear functional form is assumed.
- **meanlist** displays the mean vector used in computation and may not be combined with mymean.
- **dots** requests that replication dots be displayed. By default, replication dots are suppressed.
- **saving**(filename, *savings_options*) requests that results be saved to a Stata data file (*filename*).

For more details, after installation, type in: **help wtpcikr**

wtpcikr can be installed from within Stata using the command line:

ssc install wtpcikr

Stata Code for Example 1 – Exponential probit


```
. drop _all
// May need to increase memory for large number of replications
. set memory 8m
. use south
. gen lbid=ln(bid)
. probit ypay lbid unlimwat govtpur environ waterbill urban
. wtpcikr lbid unlimwat govtpur environ waterbill urban, reps(50000) ///
   meanl expo
```

Results from example 1


```
Probit regression                               Number of obs   =         95
                                                LR chi2(6)      =      65.46
                                                Prob > chi2     =     0.0000
Log likelihood = -30.306675                     Pseudo R2       =     0.5192
```

ypay	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
lbid	-1.3007	.2714233	-4.79	0.000	-1.83268	-.7687205
unlimwat	-.9086091	.4507424	-2.02	0.044	-1.792048	-.0251703
govtpur	.9321542	.464851	2.01	0.045	.021063	1.843245
environ	1.826571	.5786197	3.16	0.002	.6924976	2.960645
waterbill	-.0444018	.0203859	-2.18	0.029	-.0843574	-.0044461
urban	1.033676	.4467691	2.31	0.021	.1580248	1.909327
_cons	3.529088	1.22522	2.88	0.004	1.1277	5.930476

Note: 0 failures and 1 success completely determined.

```
. wtpcikr lbid unlimwat govtpur environ waterbil urban, reps(50000) meanl expo
```

Krinsky and Robb Confidence Interval for WTP measures

MEASURES	WTP	LB	UB	ASL*	CI/MEAN
MEAN	17.97	13.18	37.61	0.0000	1.36
MEDIAN	13.37	10.05	18.80	0.0000	0.65

*: Achieved Significance Level for testing H0: WTP<=0 vs. H1: WTP>0
LB: Lower bound; UB: Upper bound

Sample mean of the variables used in the computation

	unlimwat	govtpur	environ	waterb~l	urban
Mean	0.45263	0.77895	0.18947	35.80000	0.74737

Results from Example 1 – Cont'd

Now supply a mean vector to be used in lieu of the default:

```
. matrix mat=[0.45, 0.78, 0.19, 35.80, 0.75] // use the the sample mean as in the book

. wtpcikr lbid unlimwat govtpur environ waterbil urban, reps(50000) mym(mat) expo


Krinsky and Robb Confidence Interval for WTP measures
```

MEASURES	WTP	LB	UB	ASL*	CI/MEAN
MEAN	18.07	13.25	37.90	0.0000	1.36
MEDIAN	13.45	10.11	18.90	0.0000	0.65

```
*: Achieved Significance Level for testing H0: WTP<=0 vs. H1: WTP>0
LB: Lower bound; UB: Upper bound
```

Krinsky and Robb CI for median WTP obtained by Haab and McConnell (2002) from the model is shown below (page 113):

Measure	**WTP**	**LB**	**UB**
Median	13.45	10.07	18.93

As can be seen, these results are similar to those calculated by **wtpcikr.**

Stata Code for Example 2- A Double Bounded Analysis


```
. drop _all
. set memory 8m // may need to increase memory for large number of replications
. use nasug07
* Single bounded model
. probit y1 bid1 inc1, cluster(cid) nolog    // model 1
. matrix mat=(41.852) // use median income (in $1000) in study area instead of sample mean
. wtpcikr bid1 inc1, reps(50000) mym(mat)

* Double bounded model
. biprobit (y1 bid1  inc1) (y2  bid2 inc1), cluster(cid) nolog  // model 2

/* Now test the null hypothesis that wtp1=wtp2 and sigma1=sigma2 using a Wald test, since LR test does not apply because of the
       cluster option */
. test ([y1]_cons=[y2]_cons) ([y1]bid1=[y2]bid2) ([y1]inc1=[y2]inc1)

* Now apply the restrictions since they cannot be rejected.
. constraint define 1 [y1]bid1=[y2]bid2
. constraint define 2 [y1]_cons=[y2]_cons
. constraint define 3 [y1]inc1=[y2]inc1
. biprobit (y1 bid1 inc1) (y2 bid2 inc1), const(1 2 3) nolog  cluster(cid) // model 3

. wtpcikr bid1 inc1, reps(50000) eq(y1) mym(mat)

* Comparison with the Delta method
. scalar cens=mat[1,1]
. nlcom (meanwtp_q1: -([y1]_cons+ [y1]inc1*cens)/[y1]bid1)  ///
 (meanwtp_q2: -([y2]_cons+ [y2]inc1*cens)/[y2]bid2)  // no need for this part
```

Results from the Double Bounded Analysis

- Confidence interval from the single bounded model 1:

Krinsky and Robb Confidence Interval for WTP measures

MEASURE	WTP	LB	UB	ASL*	CI/MEAN
MEAN/MEDIAN	322.65	195.99	572.76	0.0000	1.17

*: Achieved Significance Level for testing H0: WTP<=0 vs. H1: WTP>0
LB: Lower bound; UB: Upper bound

- Results from the Wald test reject model 2 in favor of model 3 (χ^2=4.06 and p-value=0.2551) . Estimation results for models 1 to 3 are not shown.
- Confidence interval from the double bounded model 3:

Krinsky and Robb Confidence Interval for WTP measures

MEASURE	WTP	LB	UB	ASL*	CI/MEAN
MEAN/MEDIAN	347.27	233.00	466.38	0.0000	0.67

*: Achieved Significance Level for testing H0: WTP<=0 vs. H1: WTP>0
LB: Lower bound; UB: Upper bound

- Now compare this with CI obtained from the delta method

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
meanwtp_q1	347.2651	57.78186	6.01	0.000	234.0147	460.5154
meanwtp_q2	347.2651	57.78186	6.01	0.000	234.0147	460.5154

Results from the Double Bounded Analysis, cont'd

- As can be seen, the relative efficiency measures calculated by **wtpcikr** indicate that the double bounded model 3 yields more efficient WTP measures than the single bounded model 1.

- As expected, the 95 percent confidence interval obtained from the delta method is symmetric around the mean/median WTP, making it inappropriate.

Data source for example 1

- Data for example 1 can be downloaded from the book (Haab and McConnell, 2002) website:
  http://www-agecon.ag.ohio-state.edu/people/haab.1/bookweb/

Conclusions

- Implementing the Krinsky and Robb (KR) procedure to estimate CI for mean and median WTP can be computationally challenging.
- Now through the **wtpcikr** command, the procedure is made available to contingent valuation practitioners.
- wtpcikr allows users to save to disk the generated empirical distribution that can be used for an external scope test of difference between WTP measures.
- The relative efficiency measure, which is the CI normalized by mean/median WTP, allows straightforward efficiency comparison across models.
- Also, the KR procedure is now easily implemented for revealed preference models such as conditional logit via the **wtp** command written by Hole.
- In the future (when I am no longer a post-doc), I might rewrite **wtpcikr** to make it handle large number of replications faster. But for now, I believe that it serves very well its intended purpose.

References

- Arrow, K.R., P.R. Solow, E.E. Portney, R. Leamer, R. Radner, and H. Schuman. 1993. "Report of the NOAA Panel on Contingent Valuation." *Federal Register* 58: 4601-4614.
- Creel, M.D. and J.B. Loomis. 1991. "Confidence Intervals for Welfare Measures with an Application to a Problem of Truncated Counts." *The Review of Economics and Statistics* 73: 370-373.
- Haab, T.C. and K.E. McConnell. 2002. *Valuing Environmental Natural Resources: The Econometrics of Non-Market Valuation*. Northampton, MA: Edward Elgar Publishing.
- Hanemann, W.M., J. Loomis, and B.J. Kanninen. 1991. "Statistical Efficiency of Double-bounded Dichotomous Choice Contingent Valuation." *American Journal of Agricultural Economics* 73: 1255-263.
- Hole, Arne R. 2007. *WTP: Stata module to Estimate Confidence Intervals for WTP Measures. http://econpapers.repec.org/software/bocbocode/s456808.htm*
- Cameron, T.A., J. Quiggin. 1994. "Estimation Using Contingent Valuation Data from a Dichotomous Choice with Follow-up Questionnaire."*Journal of Environmental and Economics Management*."27 218-34.
- Krinsky, I and A. L. Robb. 1986. "On Approximating the Statistical Properties of Elasticities." *Review of Economic and Statistics* 68: 715-719.
- Loomis, J. and E. Ekstrand. 1998. "Alternative Approach for Incorporating Uncertainty When Estimating Willingness to Pay: The Case of the Mexican Spotted Owl." *Ecological Economics* 27: 29-41
- Park, T, J.B. Loomis, and M. Creel. 1991. "Confidence Interval for Evaluating Benefit Estimates from Dichotomous Choice Contingent Valuation Studies." *Land Economics* 67: 64-73.

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Thanks !!!