

# rbicopula: Recursive bivariate copula estimation and decomposition of marginal effects

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# Motivation

## Effects of interest

### 1. What we assume

- ▶ Effect of a binary or treatment variable on a binary outcome variable
- ▶ Treatment variable itself is endogenous
- ▶ Unobservables may correlate with treatment and outcome equation
- ▶ Bivariate normal distribution may not fit our data

### 2. What we want

- ▶ Use different bivariate distributions and compare results
- ▶ Find the best-fitting bivariate distribution for our data
- ▶ Compute treatment effect and marginal effect of independent variables

### 3. What doesn't work:

- ▶ `bicop` doesn't allow `margins` as postestimation command
- ▶ `rbiprobit` only allows bivariate normal distribution
- ▶ `ivprobit` inappropriate; treatment variable is binary

```
ssc install rbiprobit
```

# Contribution

## A new Stata package

- ▶ `rbicopula` estimates RBMs like `bicop` or `rbiprobit`
  - ▶ allows different bivariate distributions or copulas
  - ▶ calculates Kendall's  $\tau$  as a comparison criterion of estimation results
  - ▶ allows weights (`pw`, `fw`, `iw`)
  - ▶ provides various variance estimators (`vce`)
  - ▶ `bootstrap`, `jackknife`, and `svy` prefix are allowed
- ▶ `rbicopula` accounts for recursive nature in postestimation
- ▶ Postestimation commands enable
  - ▶ Correct predictions
  - ▶ Computation of different treatment effects
  - ▶ Decomposition of average marginal effects of independent variables
  - ▶ Standard errors using the delta method or bootstrapping

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# Recursive bivariate model

## The Model

A structural model with endogenous explanatory treatment variable  $y_2$

$$y_1^* = x' \beta + \alpha y_2 + \epsilon_1 \quad , y_1 = 1 \left[ y_1^* > 0 \right] \quad (1)$$

$$y_2^* = z' \gamma + \epsilon_2 \quad , y_2 = 1 \left[ y_2^* > 0 \right] \quad (2)$$

$$\text{with } \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix} \sim F(\epsilon_1, \epsilon_2)$$

- ▶ dependence or correlation between  $\epsilon_1$  and  $\epsilon_2$  induces endogeneity
- ▶ flexible parametric distribution assumption for  $F(\epsilon_1, \epsilon_2)$
- ▶  $x'$  and  $z'$  can share some or all independent variables
- ▶ Greene (2018) notes that endogenous nature of  $y_2$  can be ignored
- ▶ Han and Lee (2019): estimates are at best weakly identified if  $x = z$

# Recursive bivariate model

Treatment Effects: ATE, ATET, and ATEC

## 1. Average treatment effect (ATE)

$$ATE = \Pr(y_1 = 1|x')|_{y_2=1} - \Pr(y_1 = 1|x')|_{y_2=0}$$

- ▶ Ceteris-paribus scenario over full sample
- ▶ Difference between marginal probabilities of  $y_1$
- ▶ Effect of discrete change in treatment holding all other observed and unobserved variables constant

## 2. Average treatment effect on the treated (ATET)

atet

## 3. Average treatment effect on conditional probability of outcome success (ATEC)

atec

# Decomposition of Marginal Effects

## Joint and Conditional Probabilities

- ▶ Independent variable  $d$  appears in both  $x'$  and  $z'$
- ▶ Decomposition of total marginal effects on the probabilities (except marginal probabilities) are then
  1. Continuous Variables (see Greene, 2018)

$$ME = \frac{\partial \Pr}{\partial \begin{pmatrix} x_d \\ z_d \end{pmatrix}} = \underbrace{\frac{\partial \Pr}{\partial x_d}}_{\text{direct effect}} + \underbrace{\frac{\partial \Pr}{\partial z_d}}_{\text{indirect effect}}$$

2. Discrete Variables (see Hasebe, 2013; Edwards et al., 2019)

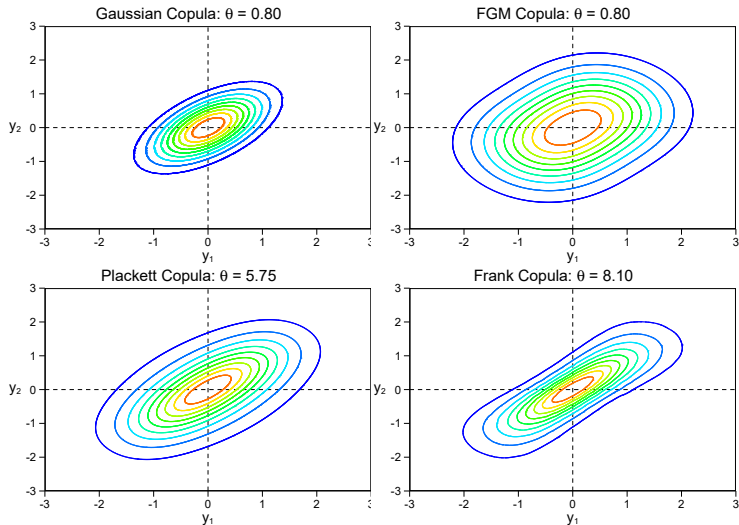
$$ME = \underbrace{[\Pr|_{x_d=1} - \Pr|_{x_d=0}]}_{\text{direct effect}} + \underbrace{[\Pr|_{z_d=1} - \Pr|_{z_d=0}]}_{\text{indirect effect}}$$



# Copula Functions

Basics

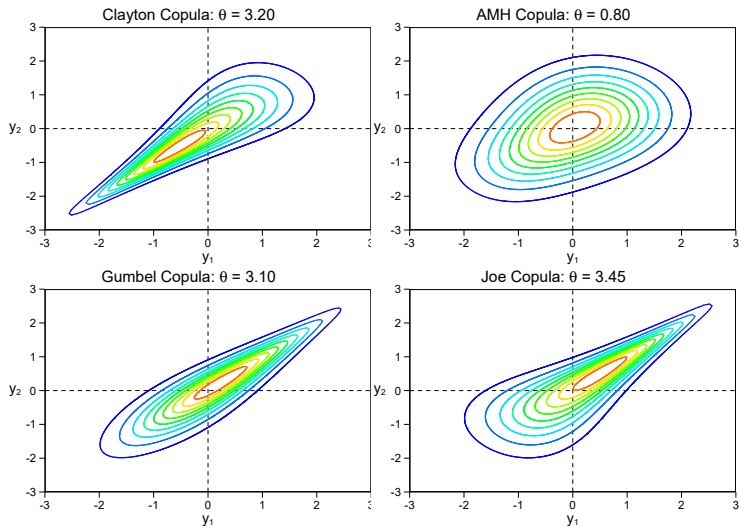
## Bivariate Density of Copulas



# Copula Functions

Copulas

## Bivariate Density of Copulas (con't)



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## Basic Syntax

```
rbicopula depvar [=] [indepvars] [if] [in] [weight]  
      , endogenous(depvar_en [=] [indepvars_en] [, enopts]) [options]
```

- ▶ *depvar* and *depvar\_en* have to be 0/1 variables
- ▶ *depvar\_en* automatically added to outcome equation as factor-variable
- ▶ `copula()` allows 9 different copula functions, e.g. `gaussian`, `fgm`, ...
- ▶ Factor variables and time-series operators allowed
- ▶ `bootstrap`, `jackknife`, and `svy` prefix are allowed
- ▶ Variance estimators: `robust`, `cluster robust`, `bootstrap`, ...
- ▶ Linear constraints are applicable

# Postestimation Commands

predict

## Margins and Treatment Effects

```
rbicopula margdec [if] [in] [weightf] [, response_options options]
```

```
rbicopula tmeffects [if] [in] [weightf] [, tmeffect(effecttype) options]
```

### rbiprobit margdec options

`effect(effecttype)` specify type of effect; *effecttype* may be total, direct, or indirect; default is total

`predict(pred_opt)` estimate margins for predict, *pred\_opt* ; multiple predict not applicable

`dydx(varlist)` estimate marginal effect of variables in *varlist*

...

### rbiprobit tmeffects options

`tmeffect(effecttype)` specify type of effect; *effecttype* may be ate, atet, or atec; default is ate

...

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## rbicopula output table

empirical application

```
. use "https://cobanomics.github.io/rbicopula/data/ess7_uk.dta", clear
(Modified excerpt from European Social Survey Wave 7 for United Kingdom)

. global indeplist      c.age#c.age i.female i.urban educyrs rignore i.lbf

. rbicopula redist = $indeplist hhincdec hhmemb ///
>                , endog(imcult = $indeplist i.pareduc imcont) copula(frunk) nolog
```

Recursive Bivariate Copula Regression (Copula: FRANK)

```
Number of obs      =      1,256
Wald chi2(19)      =      402.80
Prob > chi2        =      0.0000

Log likelihood = -1118.4116
```

	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
redist						
imcult						
Culture undermined	-1.315288	.2477447	-5.31	0.000	-1.800859	-.8297176
age	.0387031	.0149494	2.59	0.010	.0094029	.0680033
c.age#c.age	-.000319	.0001386	-2.30	0.021	-.0005906	-.0000474
female						
Female	-.0133382	.0828927	-0.16	0.872	-.175805	.1491286
urban						
[1] (Sub)Urban	.0628647	.0908488	0.69	0.489	-.1151957	.2409252
educyrs	.0034365	.0112123	0.31	0.759	-.0185391	.0254122
rignore	.1461538	.0261515	5.59	0.000	.0948977	.1974098
lbf						
Employed	-.0534302	.1010681	-0.53	0.597	-.2515201	.1446597
hhincdec	-.0587576	.0162586	-3.61	0.000	-.0906238	-.0268913
hhmemb	-.0016695	.0383488	-0.04	0.965	-.0768317	.0734928
_cons	-1.369986	.6219236	-2.20	0.028	-2.588934	-.1510378

## rbicopula output table (con't)

-----							
imcult							
	age	.0048037	.01644	0.29	0.770	-.0274181	.0370254
	c.age#c.age	-.0000514	.0001572	-0.33	0.744	-.0003596	.0002568
	female						
	Female	.3517705	.0882759	3.98	0.000	.1787528	.5247881
	urban						
[1]	(Sub)Urban	-.1568613	.1011722	-1.55	0.121	-.3551552	.0414326
	educyrs	-.0594006	.0114453	-5.19	0.000	-.081833	-.0369682
	righleft	-.1364683	.021957	-6.22	0.000	-.1795033	-.0934333
	lbf						
	Employed	-.1580659	.1134969	-1.39	0.164	-.3805157	.0643839
	pareduc						
Academic	parent	-.1861031	.0952308	-1.95	0.051	-.3727521	.0005459
	imcont	-.0807959	.0254167	-3.18	0.001	-.1306116	-.0309801
	_cons	2.931108	.4854207	6.04	0.000	1.979701	3.882516
-----							
	/delta	5.312024	1.937433	2.74	0.006	1.514725	9.109323
-----							
	theta	5.312024	1.937433			1.514725	9.109323
-----							
	tau	.4757469					
-----							
Wald test of theta=0: chi2(1) = 7.51738				Prob > chi2 = 0.0061			

- ▶ In ML estimation dependence parameter  $\theta$  is not directly estimated, but the ancillary parameter  $\delta$
- ▶ estimated dependence between error terms is positive and significantly different from zero
- ▶ Kendall's  $\tau$  denoted by  $\tau_{\text{au}}$ ; there is no  $\tau$  for Plackett copula



# Copula Choice

Kendall's  $\tau$ 

## Comparison of Measures of Fit

Copula	$\theta$	$\tau$	Wald-test p-value	log-likelihood	AIC
<b>Gaussian</b>	0.540	0.363	0.001	-1120.12	2284.24
<b>FGM</b>	1.000	0.222	0.000	-1120.17	2284.35
<b>Plackett</b>	11.314	—	<b>0.175</b>	-1118.34	2280.68
<b>Clayton</b>	0.441	0.181	<b>0.123</b>	-1121.55	2287.10
<b>Frank</b>	5.312	0.476	0.006	-1118.41	2280.82
<b>Gumbel</b>	1.957	0.489	0.015	-1118.65	2281.30
<b>Joe</b>	3.840	0.601	0.026	-1117.84	<b>2279.69</b>
<b>AMH</b>	0.826	0.245	0.000	-1120.37	2284.75

# Postestimation: Treatment effects

atet and atec

rbicopula tmeffects: Average treatment effects (ATE)

```
. rbicopula tmeffects, tmeffect(ate)
```

```
Treatment effect          Number of obs      =          1,256
Model VCE      : OIM
```

```
Expression   : Pr(redist=1), predict(pmarg1)
Effect       : Average treatment effect
dydx w.r.t.  : 1.imcult
```

```
-----+-----
```

		Delta-method				
	dy/dx	Std. Err.	z	P> z	[95% Conf. Interval]	
ate	-.4385982	.0902992	-4.86	0.000	-.6155814	-.2616151

```
-----+-----
```

## Postestimation: Marginal effects

rbicopula margdec: Average marginal effects (continuous independent variable)

```
. rbicopula margdec, dydx(rigleft) predict(p11) effect(direct)
```

```
Average marginal effects      Number of obs      =      1,256
Model VCE      : OIM
```

```
Expression      : Pr(redist=1,imcult=1), predict(p11)
dy/dx w.r.t.   : rigleft
```

```
-----+-----
           |              Delta-method
           |              dy/dx      Std. Err.      z    P>|z|      [95% Conf. Interval]
-----+-----
  rigleft |   .0327624   .0053263     6.15   0.000   .0223231   .0432017
-----+-----
```

```
. rbicopula margdec, dydx(rigleft) predict(p11) effect(indirect)
```

```
Average marginal effects      Number of obs      =      1,256
Model VCE      : OIM
```

```
Expression      : Pr(redist=1,imcult=1), predict(p11)
dy/dx w.r.t.   : rigleft
```

```
-----+-----
           |              Delta-method
           |              dy/dx      Std. Err.      z    P>|z|      [95% Conf. Interval]
-----+-----
  rigleft |  -.0015215   .0010085    -1.51   0.131  -.003498   .0004551
-----+-----
```

# Postestimation: Marginal effects

don't use margins

`rbicopula margdec`: Average marginal effects (continuous independent variable)

```
. rbicopula margdec, dydx(rigleft) predict(p11) effect(total)
```

```
Average marginal effects          Number of obs      =          1,256
```

```
Model VCE      : OIM
```

```
Expression    : Pr(redist=1,imcult=1), predict(p11)
```

```
dy/dx w.r.t. : rigleft
```

		Delta-method				[95% Conf. Interval]	
	dy/dx	Std. Err.	z	P> z			
rigleft	.0312409	.0049005	6.38	0.000	.0216362	.0408457	

- ▶ Direct effect of `rigleft` is positive
- ▶ Indirect effect of `rigleft` is negative
- ▶ Indirect effect doesn't offset direct effect entirely

## Postestimation: Plots

`rbicopula margdec` and `rbicopula tmeffects`: `Marginsplots`

▶ Marginsplot of total average marginal effects

```
. rbicopula margdec, dydx(rigleft hhincdec lbf) pr(p11) eff(total)
. marginsplot
```

▶ Marginsplot of indirect average marginal effects

```
. rbicopula margdec, dydx(female pareduc) pr(p10) eff(indirect)
. marginsplot
```

▶ Marginsplot of average treatment effect

```
. rbicopula tmeffects, tmeffect(ate)
. marginsplot
```

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# Conclusion and Future Work

## 1. Conclusion

- ▶ `rbicopula` identified even without IV (theoretically)
- ▶ Without IV: identification of `rbicopula` decisively based on parametric distribution assumption
- ▶ Three different treatment effects computable
- ▶ Decomposition of marginal effects gives insight about insignificant total marginal effects

## 2. Future Work

### 2.1 More options for postestimation commands

- ▶ `exp()`, `at()`, ...

### 2.2 More Measures of Dependence

- ▶ Blomqvist's  $\beta$  and Spearman's  $\rho$

### 2.3 Goodness-of-fit-tests

- ▶ Vuong test, Clarke test, and further model selection methods


# Thank you

Version 1.1.0 available

```
net install rbicopula, from("https://cobanomics.github.io/rbicopula/")
```

For Frank copula you have to additionally install integrate

```
ssc install integrate, replace
```

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## Formula of Treatment Effects: ATET

Following Chiburis et al. (2012) the average treatment effect on the treated is defined by

$$\begin{aligned} \text{ATET} &= \Pr(y_1 = 1, y_2 = 1 | x', z')|_{y_2=1} - \Pr(y_1 = 1, y_2 = 1 | x', z')|_{y_2=0} \\ &= \left\{ 1 - \Phi(-x'\beta - \alpha) - \Phi(-z'\gamma) + C\left[\Phi(-x'\beta - \alpha), \Phi(-z'\gamma); \theta\right] \right\} \\ &\quad - \left\{ 1 - \Phi(-x'\beta) - \Phi(-z'\gamma) + C\left[\Phi(-x'\beta), \Phi(-z'\gamma); \theta\right] \right\} \quad \forall y_{2i} = 1 \end{aligned}$$

- ▶ Ceteris-paribus scenario over sub-sample of treated
- ▶ Effect of discrete change in treatment on the adjusted joint probability

### Formula of Treatment Effects: ATEC

Following Alrasheed (2019) the average treatment effect on conditional probability of outcome success (ATEC) is defined by

$$\text{ATEC} = \frac{\Pr(y_1 = 1, y_2 = 1|x', z')}{\Pr(y_2 = 1|z')} - \frac{\Pr(y_1 = 1, y_2 = 0|x', z')}{\Pr(y_2 = 0|z')}$$

- ▶ Accounts for selection on unobservables
- ▶ Utilizes conditional probabilities of the outcome  $\Pr(y_{1i} = 1|y_{2i} = s)$  for  $s = 0, 1$  over full sample.
- ▶ Effect of a discrete change in treatment, holding only the observed variables constant
- ▶ Imposing no constraint on dependence between equations to account for changes in unobserved variables as a consequence of the treatment
- ▶ ATEC collapses to ATE if equations are independent

$$F(\epsilon_1, \epsilon_2) = C [F_1(\epsilon_1), F_2(\epsilon_2); \theta] = C [u, v; \theta]$$

- ▶ allows non-normal dependence between error terms
- ▶ binds univariate marginal distributions  $u$  and  $v$  to generate a bivariate distribution
- ▶ Depending on copula, the dependence parameter  $\theta$  has different intervals
- ▶ Implementation in `rbicopula`
  - ▶ univariate marginal distribution functions are identical
  - ▶ univariate marginal distributions are normal

$$F_j(\epsilon_j) = \Phi(\epsilon_j)$$

# Appendix

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## Available Copulas in `rbicopula`

Copula	Function $C(u, v)$	Range of $\theta$	Independence
Product	$u \cdot v$	N/A	N/A
Gaussian	$\Phi_2 [\Phi^{-1}(u), \Phi^{-1}(v); \theta]$	$-1 < \theta < +1$	$\theta = 0$
FGM	$uv \cdot [1 + \theta \cdot (1 - u) \cdot (1 - v)]$	$-1 \leq \theta \leq +1$	$\theta = 0$
Plackett	$\frac{r - \sqrt{r^2 - 4uv\theta \cdot (\theta - 1)}}{2(\theta - 1)}$	$\theta \in (0, +\infty)$	$\theta = 1$
Clayton	$(u^{-\theta} + v^{-\theta} - 1)^{-1/\theta}$	$\theta \in (0, +\infty)$	$\theta = 0$
Frank	$-\frac{1}{\theta} \cdot \ln \left[ 1 + \frac{(e^{-\theta u} - 1) \cdot (e^{-\theta v} - 1)}{e^{-\theta} - 1} \right]$	$\theta \in (-\infty, +\infty) \setminus \{0\}$	$\theta = 0$
Gumbel	$\exp \left\{ - \left[ (-\ln(u))^\theta + (-\ln(v))^\theta \right]^{1/\theta} \right\}$	$1 \leq \theta < \infty$	$\theta = 1$
Joe	$1 - \left[ (\tilde{u})^\theta + (\tilde{v})^\theta - (\tilde{u}\tilde{v})^\theta \right]^{1/\theta}$	$1 \leq \theta < \infty$	$\theta = 1$
AMH	$uv \cdot [1 - \theta \cdot (1 - u) \cdot (1 - v)]^{-1}$	$-1 \leq \theta \leq +1$	$\theta = 0$

where  $r = 1 + (\theta - 1)(u + v)$  for Plackett copula and  $\tilde{u} = 1 - u$ ,  $\tilde{v} = 1 - v$  for Joe copula

```
predict [type] newvar [if] [in] [, statistic]
```

### ***statistic***

p11	Pr(depvar = 1, depvar_en = 1); the default
p10	Pr(depvar = 1, depvar_en = 0)
p01	Pr(depvar = 0, depvar_en = 1)
p00	Pr(depvar = 0, depvar_en = 0)
pmarg1	Pr(depvar = 1); marginal success probability for outcome eq.
pmarg2	Pr(depvar_en = 1); marginal success probability for treatment eq.
pcond1	Pr(depvar = 1   depvar_en = 1)
pcond2	Pr(depvar_en = 1   depvar = 1)
xb1	linear prediction for outcome eq.
xb2	linear prediction for treatment eq.
...	

### 1. Joint Probabilities

For  $s = 0, 1$  and  $t = 0, 1$  joint probabilities are given by

$$\Pr(y_1 = s, y_2 = t | x, z) = st - tq_1 \cdot u - sq_2 \cdot v + q_1 q_2 \cdot C(u, v; \theta)$$

where

$$q_1 = 2s - 1$$

$$q_2 = 2t - 1$$

$$v = \Phi(-z'\gamma)$$

$$u = \begin{cases} \Phi(-x'\beta - \alpha) & \text{if } t = 1 \\ \Phi(-x'\beta) & \text{if } t = 0 \end{cases}$$

### 2. Conditional Probabilities

$$\Pr(y_1 = 1 | y_2 = 1, x, z) = \frac{\Pr(y_1 = 1, y_2 = 1 | x, z)}{\Phi(z'\gamma)}$$

$$\Pr(y_2 = 1 | y_1 = 1, x, z) = \frac{\Pr(y_1 = 1, y_2 = 1 | x, z)}{\Phi(x'\beta + \alpha)}$$

### 3. Marginal Probabilities

$$\Pr(y_1 = 1 | x) = \Phi(x'\beta + \alpha y_2)$$

$$\Pr(y_2 = 1 | z) = \Phi(z'\gamma)$$



### 4. Unconditional Mean Function (see Blasch et al., 2019; Alrasheed, 2019)

$$\begin{aligned} E[y_1|x, z] &= \Pr(y_2 = 1|z) \cdot E[y_1|y_2 = 1, x, z] \\ &\quad + \Pr(y_2 = 0|z) \cdot E[y_1|y_2 = 0, x, z] \\ &= \Pr(y_1 = 1, y_2 = 1|x, z) + \Pr(y_1 = 1, y_2 = 0|x, z) \\ &= \Phi_2(x'\beta + \alpha, z'\gamma, \rho) + \Phi_2(x'\beta, -z'\gamma, -\rho) \end{aligned}$$

## An empirical application

### 1. Research question

Does the perception of immigrants as a hazard of national culture effect natives' preference for redistribution?

### 2. Data

- ▶ European Social Survey (Wave 7, 2014)
- ▶ Individual Data from the United Kingdom
- ▶ Data adjusted for demonstration purposes
- ▶ Sample restricted to respondents with no migration background

### 3. The Model

- ▶ Binary outcome variable: `redist`  
*Should the government reduce difference in income levels?*  
(Agree = 1, Disagree = 0)
- ▶ Binary treatment variable: `imcult`  
*Do immigrants undermine or enrich country's cultural life?*  
(Undermine = 1, Enrich = 0)

## Varlist of independent variables

- ▶ Independent variables common to both equations
  - ▶ Age (`age`)
  - ▶ Gender (`female`)
  - ▶ Place of residence (`urban`)
  - ▶ Years of education (`educyrs`)
  - ▶ Main activity, last 7 days (`lbf`)
  - ▶ Self-placement on political left-right scale (`rightleft`)
- ▶ Independent Variables only in treatment equation
  - ▶ At least one parent is academic (`pareduc`)
  - ▶ Frequency of contact with immigrants beyond workplace and friendships (`imcont`)
- ▶ Independent Variables only in outcome equation
  - ▶ Household income (`hhincdec`)
  - ▶ Number of household members (`hhmemb`)

### Kendall's Rank Correlation or Kendall's $\tau$

$$\tau = \Pr \left[ (X_1 - X_2)(Y_1 - Y_2) > 0 \right] - \Pr \left[ (X_1 - X_2)(Y_1 - Y_2) < 0 \right]$$

where  $(X_1, Y_1)$  and  $(X_2, Y_2)$  are independent pairs of random variables from  $C$

- ▶ Measure of degree of dependence
- ▶ Allows comparison of dependence pattern between different copulas
- ▶ Limited to a range of  $[-1, 1]$
- ▶ Negative (positive) values indicate negative (positive) dependence
- ▶  $\tau = 0$  indicates independence

**Important:** For Frank copula you must additionally install `integrate`

```
ssc install integrate, replace
```

## rbicopula tmeffects: ATET and ATEC

```
. rbicopula tmeffects, tmeffect(atet)
```

```
Treatment effect          Number of obs    =      1,026
Model VCE      : OIM
```

```
Expression   : Pr(redist=1,imcult=1|imcult=1) - Pr(redist=1,imcult=1|imcult=0)
Effect       : Average treatment effect on the treated
dydx w.r.t.  : 1.imcult
```

```
-----+-----
          |              Delta-method
          |              dy/dx   Std. Err.      z    P>|z|    [95% Conf. Interval]
-----+-----
      atet |   -0.3977149   .0867208   -4.59   0.000   -0.5676845   -0.2277453
-----+-----
```

```
. rbicopula tmeffects, tmeffect(atec)
```

```
Treatment effect          Number of obs    =      1,256
Model VCE      : OIM
```

```
Expression   : Pr(redist=1|imcult=1)-Pr(redist=1|imcult=0), predict(pcond1)-predict(pc
> ond10)
Effect       : Average treatment effect on conditional probability
dydx w.r.t.  : 1.imcult
```

```
-----+-----
          |              Delta-method
          |              dy/dx   Std. Err.      z    P>|z|    [95% Conf. Interval]
-----+-----
      atec |  -0.0184466   .0302367   -0.61   0.542   -0.0777095   .0408163
-----+-----
```

## Incorrect standard errors using margins

```
. margins, dydx(rigleft) predict(p11)
```

```
Average marginal effects          Number of obs    =      1,256
Model VCE      : OIM
```

```
Expression      : Pr(redist=1,imcult=1), predict(p11)
dy/dx w.r.t.   : rigleft
```

```
-----+-----
```

		Delta-method				
	dy/dx	Std. Err.	z	P> z	[95% Conf. Interval]	
rigleft	.0312409	.004587	6.81	0.000	.0222505	.0402314

```
-----+-----
```

```
. rbicopula margdec, dydx(rigleft) predict(p11) effect(total)
```

```
Average marginal effects          Number of obs    =      1,256
Model VCE      : OIM
```

```
Expression      : Pr(redist=1,imcult=1), predict(p11)
dy/dx w.r.t.   : rigleft
```

```
-----+-----
```

		Delta-method				
	dy/dx	Std. Err.	z	P> z	[95% Conf. Interval]	
rigleft	.0312409	.0049005	6.38	0.000	.0216362	.0408457

```
-----+-----
```