

Too much or too little? New tools for the static CCE Estimator.

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Motivation I

- In panel time series models with $(N, T) \Rightarrow \infty$ cross-section dependence via unobserved common factors (or heterogeneity) is likely to occur.
- Often modelled using interactive fixed effects

$$y_{i,t} = \beta_i x_{i,t} + u_{i,t} \quad (1)$$

$$u_{i,t} = \gamma_{1,i} f_{1,t} + \dots + \gamma_{M,i} f_{M,t} + \vartheta_{i,t} \quad (2)$$

- where we have M interactions between unit specific effects and time effects.
- $\gamma_{m,i}$ is the factor loading of factor $f_{m,t}$.

Motivation II

- The explanatory variables can also consist of a factor structure:

$$x_{i,t} = \gamma_{x,1,i} f_{x,1,t} + \dots + \gamma_{M_x,i} f_{M_x,t} + \epsilon_{i,t}$$

- Assumption: $\text{corr}(\vartheta_{i,t}, \epsilon_{i,t}) = 0!$
- This setting poses two challenges:
 - 1 Correlation across units, cross-sectional dependence
 - 2 If the factors in $x_{i,t}$ and $u_{i,t}$ overlap, the observables and unobservables are correlated
- Popular estimator: Common Correlated Effects Estimator (Pesaran, 2006)

(Static) Common Correlated Effects

- Pesaran (2006) proposes to span the space of the common factors using cross-section averages (CSA) in a static model:

$$y_{i,t} = \beta_i x_{i,t} + \psi_{x,i} \bar{x}_t + \psi_{y,i} \bar{y}_t + \epsilon_{i,t}$$

where $\bar{x}_t = 1/N \sum_{i=1}^N x_{i,t}$ and $\bar{y}_t = 1/N \sum_{i=1}^N y_{i,t}$ are the cross-section averages.

- Can be combined with a pooled or mean group estimator.
- Estimator is \sqrt{N} consistent.
- Proved to be versatile in many conditions (Kapetanios et al., 2011; Chudik et al., 2011; Westerlund, 2018).
- Extended to dynamic models (Chudik and Pesaran, 2015) and estimation of long run coefficients (Chudik et al., 2016).

Static CCE

Stata Implementations

- First implementation in `xtmg` (Eberhardt, 2012).
- Also implemented in `xtdcce2` (Ditzen, 2018a, 2021):
 - ▶ Static and dynamic models
 - ▶ Estimation of long run coefficients
 - ▶ Bootstrapping
 - ▶ Different types of cross-section averages
 - ▶ Estimation of degree and testing for cross-section dependence
 - ▶ Various `estat` and `predict` functions
- Syntax:

```
xtdcce2 depvar indepvars [if] , ... cr(varlist , options)
```

- ▶ `cr()` defines variables added as cross-section averages.

Static CCE

New developments

- In “early” years discussion on CCE focused on validity under stationary factors, autocorrelated factors, strong and semi strong cross-section dependence and dynamic models.
- A recurring topic is also bootstrapping.
- In past years discussion on what “spanning factor space” actually means intensified:
 - ① Regularized CCE
 - ② When does the “Rank Condition” hold?
 - ③ Information Criteria to select CSA.

Regularized CCE

- Large number of cross-section averages might only contain limited information, inducing non-trivial bias for pooled and mean group CCE, see Karabiyik et al. (2017).
- Juodis (2022) suggest rCCE approach:
 - ▶ Calculate cross-section averages (CSA)
 - ▶ Estimate number of common factors in CSA, \hat{m} , using ER or GR from Ahn and Horenstein (2013)
 - ▶ Replace cross-section averages with the first \hat{m} eigenvectors of CSA.
- Requires bootstrapping.
- Disadvantage: approach sensitive to estimation of factors and only for static panels.

Regularized CCE

```
xtdcce2, ... cr(rcce)
```

```
xtdcce2 depvar indepvars [if] , ... cr(varlist , rcce[(options)]))
```

- options are:
 - ▶ `criterion(er|gr)` specifies criterion to estimate number of common factors using the ER or GR criterion from Ahn and Horenstein (2013)
 - ▶ `scale` scales cross-section averages
 - ▶ `npc(real)` specifies number of eigenvectors without estimating it.
- Bootstrap is not automatically performed.
- Number of factors estimated based on `xtnumfac` (Ditzen and Reese, 2023).
- Unbalanced panels supported, then missing values in CSA are imputed.

Regularized CCE

Example

- Dell et al. (2012) investigate effect of temperature ($wtem_{i,t}$) and precipitation ($wpre_{i,t}$) on economic growth ($g_{i,t}$).
- Balanced panel of 89 countries and over 42 years (1962-2003):¹

$$g_{i,t} = \mu_i + \beta_{1,i}wtem_{i,t} + \beta_{2,i}wpre_{i,t} + u_{i,t}$$

- Estimated number of common factors (output shortened):

```
. xtnumfac g wtem wpre , stand(5)
N   =      3738                T   =      42
N_g =       89                vars. =       3
```

IC	# factors	IC	# factors
...			
ER	1	GR	1
...			

8 factors maximally considered.

```
...
ER, GR from Ahn and Horenstein (2013)
...
```

¹This is only an example!

Regularized CCE

Example MG (no CSA)

```
. xtccce2 g wtem wpre, cr(g wtem wpre)
(Dynamic) Common Correlated Effects Estimator - Mean Group
Panel Variable (i): cc_num          Number of obs   =   3738
Time Variable (t): year            Number of groups =    89
Degrees of freedom per group:
  without cross-sectional averages = 39
  with cross-sectional averages    = 36
Number of                          F(534, 3204)     =   0.40
cross-sectional lags                0 to 0        =   1.00
variables in mean group regression = 178          R-squared      =   0.94
variables partialled out            = 356          R-squared (MG) =   0.10
                                   Root MSE       =   5.04
                                   CD Statistic    =   0.79
                                   p-value         =   0.4322
```

	g	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
Mean Group:						
	wtem	-.4927157	.3991215	-1.23	0.217	-1.27498 .289548
	wpre	.0794224	.0863427	0.92	0.358	-.0898062 .248651

```
Mean Group Variables: wtem wpre
Cross Sectional Averaged Variables: g wtem wpre
Heterogenous constant partialled out.
```

- `cr(g wtem wpre)` adds cross-section averages of `g`, `wtem` and `wpre`.
- No strong cross-section dependence left in residuals.
- No coefficients significant.
- Space of one common factor spanned by 3 cross-section averages.

Regularized CCE

Example

```
. xtdcce2 g wtem wpre, cr(g wtem wpre, rcce(npc(1)))
(Dynamic) Common Correlated Effects Estimator - Mean Group
Panel Variable (i): cc_num          Number of obs   =   3738
Time Variable (t): year            Number of groups =    89
Degrees of freedom per group:
  without cross-sectional averages = 39
  with cross-sectional averages    = 36
Number of
  cross-sectional lags              = 0
  variables in mean group regression = 178
  variables partialled out          = 356
F(534, 3204)                        =   0.44
Prob > F                             =   1.00
R-squared                             =   0.93
R-squared (MG)                       =  -0.04
Root MSE                              =   5.43
CD Statistic                          =  18.31
p-value                               =   0.0000
```

	g	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
Mean Group:							
	wtem	-.8734512	.2471953	-3.53	0.000	-1.357945	-.3889572
	wpre	.1390204	.0975042	1.43	0.154	-.0520843	.3301251

```
Mean Group Variables: wtem wpre
1 Regularized Cross-Section Averages from variables:
  g wtem wpre
Heterogenous constant partialled out.
```

- `cr(... , rcce(npc(1))` employs rCCE estimator with 1st eigenvalue instead of CSA.
- Some strong cross-section dependence left in residuals.
- Temperature significant.
- `xtdcce2` can also estimate number of common factors in CSA [Example](#).

Static CCE

The Rank Condition

- Key for consistent estimation is the rank condition.
- Rank condition implies that - loosely speaking (!!)- rank of average factor loadings has to be larger or equal than rank of factors.
- Karabiyik et al. (2017, 2021) show that if rank condition fails, CCE is inconsistent!
- Implies that adding CSAs with zero loadings, might lead to problems!
- In empirical practice: more CSA required than factors.

Rank Condition Classifier

- Reminder: Rank of unobserved factors (f), m , is not larger than the rank of unobserved average factor loadings (γ), g .
- Problem: both are unobserved! DeVos et al. (2024) propose an indicator if the rank condition holds:

$$\widehat{RC} = 1 - I(\hat{g} < \hat{m}) \quad (3)$$

where \hat{m} is the rank of the cross/product of the observed data estimated by ER or GR from Ahn and Horenstein (2013) and \hat{g} is rank of the unobserved factor loadings estimated from cross-section averages.

- If $\widehat{RC} = 1$, rank condition holds.
- Requires bootstrap to estimate variance of rank estimator of factor loadings.
- Consistency depends on fixed T . Trick: bound dimension with shrinkage.
- Indicator only valid for static panels.

Rank Condition Classifier

```
xtdcce2, ... cr(rccl)
```

```
xtdcce2 depvar indepvars [if] , ... cr(varlist , rcclassifier[(options)])
```

- options are:

- ▶ `er|gr` specifies criterion to estimate number of common factors using the ER or GR criterion from Ahn and Horenstein (2013)
- ▶ `standardize(integer)` standardize data prior to estimation of number of common factors.
- ▶ `replications(integer)` sets number of replication for bootstrapping variance of the rank estimator of the unobserved matrix of average factor loadings.
- ▶ `randomshrinkage` Instead of fold-over matrix, use matrix with entries drawn from random normal distribution.
- ▶ `noshrinkage` No shrinkage.

- Number of factors estimated based on `xtnumfac` (Ditzen and Reese, 2023).

Rank Condition Classifier

Example

```
. xtddce2 g wtem wpre, cr(g wtem wpre, rccl)
(Dynamic) Common Correlated Effects Estimator - Mean Group

Panel Variable (i): cc_num      Number of obs   =      3738
Time Variable (t): year        Number of groups =       89
Degrees of freedom per group:  Obs per group (T) =       42
without cross-sectional averages = 39
with cross-sectional averages  = 36
Number of cross-sectional lags 0 to 0      F(534, 3204)    =       0.40
variables in mean group regression = 178      Prob > F        =       1.00
variables partialled out       = 356      R-squared       =       0.94
                                R-squared (MG)    =       0.10
                                Root MSE        =       5.04
                                CD Statistic     =       0.79
                                p-value          =       0.4322
```

	g	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
Mean Group:							
	wtem	-.4927157	.3991215	-1.23	0.217	-1.27498	.289548
	wpre	.0794224	.0863427	0.92	0.358	-.0898062	.248651

Mean Group Variables: wtem wpre
 Cross Sectional Averaged Variables: g wtem wpre
 Heterogenous constant partialled out.

Classifier for Rank Condition (De Vos, Everaert and Sarafidis, 2024)

RC (1-I(g<m))*	Estimated	
	Rank (g)	Number of Factors (m)
1	2	1

* RC=1 indicates rank condition holds. g is rank of matrix of cross-sectional averages, m is number of factors in the data.

- `cr(... , rccl)` requests calculation of rank condition classifier.
- More common factors in CSA than in unobserved factors.

Information Criteria and CCE

- Long time no theory for use of information criteria and CCE
- Information criteria can be used for 2 purposes:
 - ▶ Selection of cross-section averages
 - ▶ Lag selection in dynamic models
- 1) received recently attention:
 - ▶ DeVos et al. (2024) propose a sequential method to identify the set of relevant cross-section averages.
 - ▶ Margaritella and Westerlund (2023) propose a similar criteria as in Bai and Ng (2002); Bai (2009).
- **No** guidance on **lag selection** in dynamic models - left for research!

Information Criteria and CCE

- Margaritella and Westerlund (2023) propose 4 criteria to identify the optimal set of cross-section averages:

$$IC_1(M) = \ln \hat{\sigma}^2(\hat{F}_M) + m \frac{N+T}{NT} \ln \left(\frac{NT}{N+T} \right), \quad IC_2(M) = \ln \hat{\sigma}^2(\hat{F}_M) + m \frac{N+T}{NT} \ln(C_{NT}^2)$$
$$PC_1(M) = \hat{\sigma}^2(\hat{F}_M) + m \hat{\sigma}^2(\hat{F}_{\bar{M}}) \frac{N+T}{NT} \ln \left(\frac{NT}{N+T} \right), \quad PC_2(M) = \hat{\sigma}^2(\hat{F}_M) + m \hat{\sigma}^2(\hat{F}_{\bar{M}}) \frac{N+T}{NT} \ln(C_{NT}^2)$$

with $\hat{\sigma}^2(\hat{F}_M)$ the error variance from a CCE estimation with m CSA and $\left(\hat{F}_{\bar{M}}\right)$ is the error variance with the full set of CSA.

- Optimal set of CSA is then $\hat{M} = \text{argmin} IC(M)$.
- Integrated in `xtdcce2` as `estat ic`.
- Caveat: can only be applied to static models.

Information Criteria and CCE

Example

```
. estat ic
```

IC from Margaritella & Westerlund (2023)

Model	IC1	IC2	PC1	PC2
1	3.43*	3.48*	.	.

- ICs alone are not informative.
- Option sequential compares all permutations of CSA.

Information Criteria and CCE

Example

```
. estat ic, seq
Running 7 combinations of cross-section averages:
. . . . .
IC from Margaritella & Westerlund (2023)
```

Model	IC1	IC2	PC1	PC2
1	3.34	3.35	28	28
2	3.34	3.35	28	28
3	3.25*	3.26*	25.8*	25.8*
4	3.43	3.45	29.7	29.7
5	3.34	3.36	27.9	27.9
6	3.34	3.37	27.6	27.6
7	3.43	3.48	29.7	29.7

* indicates minimum.

Cross Section Averages:

Model 1: wpre

Model 2: wtem

Model 3: g

Model 4: wtem wpre

Model 5: g wpre

Model 6: g wtem

Model 7: g wtem wpre (Main Model)

Click on Model to run in xtdcce2.

- Model with 3 CSA has lowest IC.
- Possible to run models directly in Stata.
- Many combinations possible which can take time.

Taming the tests for cross-section dependence zoo

- Pesaran (2015, 2021) proposes a test for weak cross-section dependence, the CD-test:

H_0 weak dependence vs. H_1 strong dependence

- Test statistic:

$$CD = \sqrt{\frac{2T}{N(N-1)}} \sum_{i=1}^{N-1} \sum_{j=i+1}^N \rho_{i,j},$$

with $\rho_{i,j}$ is the correlation coefficient.

- Under the null hypothesis asymptotically: $CD \sim N(0, 1)$.
- In Stata: `xtcd2` (Ditzen, 2018b), `xtcd`, `xtcsd` (De Hoyos and Sarafidis, 2006),
- Problem of the CD test: tends to over reject.

Taming the tests for cross-section dependence zoo²

- Recent developments: CDw (Juodis and Reese, 2021), CDw with power enhancement (Fan et al., 2015) and CD* (Pesaran and Xie, 2021)
- CD* is complicated, but actually tests for independence against weak dependence in the presence of latent factors:

```
. xtcd2 g wtem wpre
Testing for cross-sectional dependence (CSD)
  CD CDw CDw+      CD*
H0: weak cross-section dependence      independence in presence of factors
H1: strong cross-section dependence    weak dependence in presence of factors
```

	CD	CDw	CDw+	CD*
g	29.35 (0.000)	-0.66 (0.508)	3519.69 (0.000)	2.82 (0.005)
wtem	149.24 (0.000)	-0.75 (0.452)	9856.16 (0.000)	5.08 (0.000)
wpre	22.18 (0.000)	-1.09 (0.275)	4035.09 (0.000)	4.89 (0.000)

p-values in parenthesis.

References

CD: Pesaran (2015, 2021)
 CDw: Juodis, Reese (2021)
 CDw+: CDw with power enhancement from Fan et al. (2015)
 CD*: Pesaran, Xie (2021) with 4 PC(s)

²Beta version of xtcdce2 4.9. Results may change.

Summary

- New developments in the CCE literature on **static** models:
 - ▶ Regularized CCE (rCCE)
 - ▶ Rank condition classifier
 - ▶ Information criteria to select CSA
- rCCE available with `xtdcce2` version 4.0.
- Rank condition classifier and IC will be available with version 4.7.
- How to install?

```
net install xtdcce2 , from("https://janditzen.github.io/xtdcce2/")
```

- More info:



jan.ditzen.net



[GitHub](https://github.com/janditzen)

References I

- Ahn, S. C., and A. R. Horenstein. 2013. Eigenvalue Ratio Test for the Number of Factors. Econometrica 81(3): 1203–1227.
- Bai, J. 2009. Panel Data Models With Interactive Fixed Effects. Econometrica 77(4): 1229–1279.
- Bai, J., and S. Ng. 2002. Determining the number of factors in approximate factor models. Econometrica 70(1): 191–221.
- Chudik, A., K. Mohaddes, M. H. Pesaran, and M. Raissi. 2016. Long-Run Effects in Large Heterogeneous Panel Data Models with Cross-Sectionally Correlated Errors. In Essays in Honor of Aman Ullah (Advances in Econometrics, Volumn 36), ed. R. C. Hill, G. González-Rivera, and T.-H. Lee, 85–135. Emerald Group Publishing Limited.

References II

- Chudik, A., and M. H. Pesaran. 2015. Common correlated effects estimation of heterogeneous dynamic panel data models with weakly exogenous regressors. Journal of Econometrics 188(2): 393–420.
- . 2019. Mean group estimation in presence of weakly cross-correlated estimators. Economics Letters 175: 101–105.
- Chudik, A., M. H. Pesaran, and E. Tosetti. 2011. Weak and strong cross-section dependence and estimation of large panels. The Econometrics Journal 14(1): C45–C90.
- De Hoyos, R. E., and V. Sarafidis. 2006. Testing for cross-sectional dependence in panel-data models. The Stata Journal 6(4): 482–496.
- Dell, B. M., B. F. Jones, and B. A. Olken. 2012. Temperature Shocks and Economic Growth: Evidence from the Last Half Century. American Economic Journal: Macroeconomics 4(3): 66–95.

References III

- DeVos, I., G. Everaert, and V. Sarafidis. 2024. A method to evaluate the rank condition for CCE estimators A method to evaluate the rank condition for CCE estimators. Econometric Reviews 0(0): 1–34. URL <https://doi.org/10.1080/07474938.2023.2292383>.
- Ditzen, J. 2018a. Estimating Dynamic Common-Correlated Effects in Stata. The Stata Journal 18(3): 585–617.
- . 2018b. Estimating dynamic common-correlated effects in Stata. The Stata Journal 18(3): 585 – 617.
- . 2021. Estimating long run effects and the exponent of cross-sectional dependence: an update to xtdcce2. The Stata Journal 21(3): 687–707. URL <https://ideas.repec.org/p/bzn/wpaper/bemps81.html>.

References IV

- Ditzen, J., and S. Reese. 2023. A battery of estimators for the number of common factors in time series and panel data models. The Stata Journal 23(2): 438–454. URL <https://journals.sagepub.com/doi/abs/10.1177/1536867X231175305>.
- Eberhardt, M. 2012. Estimating panel time series models with heterogeneous slopes. The Stata Journal 12(1): 61–71.
- Fan, J., Y. Liao, and J. Yao. 2015. Power Enhancement in High-Dimensional Cross-Section Tests. Econometrica 83(4): 1497–1541.
- Juodis, A. 2022. A regularization approach to common correlated effects estimation. Journal of Applied Econometrics (October 2021): 788–810.
- Juodis, A., and S. Reese. 2021. The Incidental Parameters Problem in Testing for Remaining Cross-Section Correlation. Journal of Business Economic Statistics .

References V

- Kapetanios, G., M. H. Pesaran, and T. Yamagata. 2011. Panels with non-stationary multifactor error structures. Journal of Econometrics 160(2): 326–348.
- Karabiyik, H., S. Reese, and J. Westerlund. 2017. On the role of the rank condition in CCE estimation of factor-augmented panel regressions. Journal of Econometrics 197(1): 60–64. URL <http://dx.doi.org/10.1016/j.jeconom.2016.10.006>.
- Karabiyik, H., J. Westerlund, and A. Juodis. 2021. On the Robustness of the Pooled CCE Estimator. Journal of Econometrics 220(2): 325–348.
- Margaritella, L., and J. Westerlund. 2023. Using Information Criteria to Select Averages in CCE. The Econometrics Journal .
- Pesaran, M. H. 2006. Estimation and inference in large heterogeneous panels with a multifactor error structure. Econometrica 74(4): 967–1012.

References VI

- . 2015. Testing Weak Cross-Sectional Dependence in Large Panels. Econometric Reviews 34(6-10): 1089–1117.
- . 2021. General diagnostic tests for cross-sectional dependence in panels. Empirical Economics 60(1): 13–50. URL <https://doi.org/10.1007/s00181-020-01875-7>.
- Pesaran, M. H., and R. Smith. 1995. Estimating long-run relationships from dynamic heterogeneous panels. Journal of Econometrics 68(1): 79–113.
- Pesaran, M. H., and Y. Xie. 2021. A Bias-Corrected CD Test for Error Cross-Sectional Dependence in Panel Data Models with Latent Factors. Cambridge Working Papers in Economics 2158.
- Westerlund, J. 2018. CCE in panels with general unknown factors. Econometrics Journal 21(3): 264–276.

MG with no CSA [back](#)

```
. xtccce2 g wtem wpre, nocross
(Dynamic) Common Correlated Effects Estimator - Mean Group
Panel Variable (i): cc_num          Number of obs   =       3738
Time Variable (t): year            Number of groups =        89
Degrees of freedom per group:
  without cross-sectional averages  = 39
  with cross-sectional averages     = 39
Number of                          F(267, 3471)     =       0.87
cross-sectional lags                none            Prob > F        =       0.93
variables in mean group regression = 178           R-squared       =       0.94
variables partialled out            = 89           R-squared (MG) =       0.01
                                      Root MSE       =       5.28
                                      CD Statistic   =       22.42
                                      p-value       =       0.0000
```

	g	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
Mean Group:							
	wtem	-.6109638	.2163648	-2.82	0.005	-1.035031	-.1868967
	wpre	.1805911	.088414	2.04	0.041	.0073029	.3538793

Mean Group Variables: wtem wpre
Heterogenous constant partialled out.

rCCE - number of factors estimated [back](#)

```
. xtccce2 g wtem wpre, cr(g wtem wpre, rcce)
(Dynamic) Common Correlated Effects Estimator - Mean Group
Panel Variable (i): cc_num          Number of obs      =      3738
Time Variable (t): year            Number of groups   =       89
Degrees of freedom per group:      Obs per group (T)  =       42
  without cross-sectional averages = 39
  with cross-sectional averages    = 36
Number of                          F(534, 3204)       =      0.47
cross-sectional lags                = 0                Prob > F           =      1.00
variables in mean group regression = 178              R-squared          =      0.93
variables partialled out            = 356              R-squared (MG)    =      0.07
                                   Root MSE           =      5.11
                                   CD Statistic       =      0.43
                                   p-value            =      0.6704
```

	g	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
Mean Group:							
	wtem	-.0618716	.3554096	-0.17	0.862	-.7584617	.6347185
	wpre	.1070671	.0918071	1.17	0.244	-.0728715	.2870057

Mean Group Variables: wtem wpre
 2 Regularized Cross-Section Averages from variables:
 g wtem wpre
 Heterogenous constant partialled out.

Information Criteria and CCE

Example

```
. estat ic, model((g wtem wpre) (g) (g wtem))  
Running 3 combinations of cross-section averages:
```

```
IC from Margaritella & Westerlund (2023)
```

Model	IC1	IC2	PC1	PC2
1	3.43	3.48	29.5	29.5
2	3.25*	3.26*	25.4*	25.4*
3	3.34	3.37	27.7	27.7

* indicates minimum.

Cross Section Averages:

Model 1: g wtem wpre (Main Model)

Model 2: g

Model 3: g wtem

Click on Model to run in xtdcce2.

- Compare specific models.
- Syntax: `model((model1) (model2) ...)`.
- (model1) is the reference model.

Static CCE

Mean Group Estimator

- Main contributions: Pesaran and Smith (1995); Pesaran (2006); Chudik and Pesaran (2019)
- $\hat{\beta}_{MG} = \frac{1}{N} \sum_{i=1}^N \hat{\beta}_i$
- Asymptotic variance estimator $V(\hat{\beta}_{MG}) = \frac{1}{N(N-1)} \sum_{i=1}^N (\hat{\beta}_i - \hat{\beta}_{MG})^2$
- Individual coefficients asymptotically normal with $(N, T) \xrightarrow{j} \infty$ with no particular order. Rank condition requires $\sqrt{T}/N \rightarrow 0$.
- Mean group asymptotically unbiased if $N \rightarrow \infty$ and T fixed and $(N, T) \xrightarrow{j} \infty$. Also needs $T > K$.

Static CCE

Pooled Estimator

- Main contribution: Pesaran (2006)
- Estimate β_p directly with the condition $\beta_i = \beta_p$.
- Various variance estimators, such as $V(\hat{\beta}_p)_{np} = f(\hat{\beta}_i, \hat{\beta}_{MG}, \tilde{X}'\tilde{X})$ or $V(\hat{\beta}_p)_{hac} = f(\hat{\beta}_p, \tilde{X}'\tilde{X}, \hat{\epsilon}_{i,t})$.
- Depending on the estimator, we need to make sure that the residuals are cross-section dependence free!
- Asymptotically normal with $(N, T) \xrightarrow{j} \infty$ with no particular order.