

# Credit Rationing, Profit Accumulation and Economic Growth\*

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## Abstract

This paper studies how credit rationing affects endogenous growth when capital and debt are related to the firm's internal net worth, taken as collateral. The accumulation of firm's net worth determines the growth rate of capital and the growth rate of the economy. The relation between growth and interest rate is then negative without requiring convex adjustment costs on investment.

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## 1. Introduction

Empirical work has put forward the role of financial ratios on growth (Levine [1997]). The role of collateral requirements has been recently emphasized in empirical work on investment (see e.g. Whited [1992] test of credit rationing) and in business cycle macro-economic modelling (Bernanke and Gertler [1989], Kiyotaki and Moore [1997], Bernanke Gertler Gilchrist [1998]). If investment faces finance constraint related to internal net worth – as claimed by business cycle theorists – and if investment determines growth – as claimed by endogenous growth theorists –, one expects the growth rate of internal funds, driven by retained earnings of financially constrained firms, to have an impact on economic growth. Earnings are related to the gap between the marginal productivity of capital and the interest rate, due to credit rationing. This intertemporal effect of credit rationing leads to a negative relationship between interest rate and economic growth. It is a theoretical alternative to convex adjustment cost on investment in order to obtain this negative relationship in endogenous growth models (Barro and Sala-I-Martin [1992]).

This note is organised as follows. In section 2 are presented households' and entrepreneurs' behaviour facing a Kiyotaki and Moore [1997] financial constraint and financially constrained economic growth. Section 3 concludes.

## 2. The model

### 2.1. Households

A continuum of risk averse wage-earners, distributed on  $[0, L]$  maximize a constant intertemporal elasticity of substitution utility function over an infinite horizon.

$$U(c_t) = \sum_{\tau=0}^{+\infty} \frac{c_{t+\tau}^{1-\sigma} - 1}{1-\sigma} \cdot (1+\rho)^{-\tau} \text{ with } \sigma > 0, \sigma \neq 1. \quad (2.1)$$

$$U(c_t) = \sum_{\tau=0}^{+\infty} \ln(c_{t+\tau}) \cdot (1+\rho)^{-\tau} \text{ for } \sigma = 1. \quad (2.2)$$

Consumption at time  $t$  is  $c_t$ , the rate of time preference is  $\rho$ , and the inverse of the elasticity of substitution is  $\sigma$ . Agents supply inelastically one unit of labor and they have no disutility of labor. Households receive a real wage rate  $w_t$  from their work in firms. They lend to entrepreneurs and earn a rate of return  $r$  on their individual wealth  $b_{t-1}$ .

$$0 = -b_t + (1+r) \cdot b_{t-1} + w_t - c_t. \quad (2.3)$$

The initial wealth  $b_0$  is given and identical for all households. Then, optimal consumption growth is ( $C_t = L \cdot c_t$  denotes aggregate consumption):

$$\frac{c_{t+1}}{c_t} = \frac{C_{t+1}}{C_t} = 1 + g_c = \left( \frac{1+r}{1+\rho} \right)^{\frac{1}{\sigma}}. \quad (2.4)$$

## 2.2. Entrepreneurs

The Romer [1986] learning-by-doing model assumes that production of a single firm  $y_t$  is a linear homogeneous function  $F$  with constant return to scale of its capital input  $k_t$ , its labor input  $l_t$ . Production also depends on the aggregate capital stock  $K_t$ , which represent the uncompensated spillovers of knowledge or ideas from one producer to another. Production is subject to diminishing returns in  $k_t$  for fixed  $K_t$  but constant returns with respect to  $k_t$  and  $K_t$  together (the index for the firm is omitted):

$$y_t = F(k_t, K_t l_t) = k_t F(1, K_t \cdot (l_t/k_t)). \quad (2.5)$$

As in standard endogenous growth models, entrepreneurs are risk neutral and maximize the present value of net revenues (Barro and Sala-I-Martin [1992]). We consider a continuum of producers, distributed on  $[0, 1]$ . They can borrow or lend at the rate  $r$ . Then the present value of the firms dividends is  $\sum_{t=1}^{t=+\infty} d_t (1+r)^{-t}$ . The firm's flow of funds constraint states that cash inflows include net revenues  $y_t$  and net borrowing  $b_t - b_{t-1}$ , while cash outflows consist of dividends  $d_t$ , wage payment  $w_t l_t$ , interest payment  $r b_{t-1}$  and investment expenditures where  $p_t^I$  is the cost of capital in units of consumables and  $k_t - (1 - \delta) k_{t-1}$  is investment ( $\delta$  is the depreciation rate):

$$d_t = F(k_t, K_t l_t) - r b_{t-1} + b_t - b_{t-1} - p_t^I (k_t - (1 - \delta) k_{t-1}) - w_t l_t \quad (2.6)$$

We now introduce a financial market imperfection which may lead to credit ra-

tioning: households (creditors) take care never to let the size of the debt repayment to exceed the value of the collateral. The only collateralizable asset of a firm is its stock of capital. We assume that each firm's technology is idiosyncratic<sup>1</sup>. Once the productive activity has started, only the firm possesses the skills necessary to invent new designs from this input. Were the firm to withdraw its specific human capital, there would remain only physical capital  $K_t$ . We assume then that the firm has always the ability to threaten its creditors by withdrawing its capital input, and repudiate its debt contract. Creditors protect themselves by collateralizing the stock of capital, which may have a lower market value once liquidated than run by the entrepreneur. The firm may be able to negotiate the debt down to the liquidation value of his capital. We also assume that lenders lose a proportion  $(1 - \mu)$ , with  $0 < \mu < 1$ , of the collateral value, due to transaction costs. The credit constraint is then expressed as a constraint on leverage:

$$(1 + r)b_t \leq (1 - \mu)p_t^I k_t \Rightarrow \frac{b_t}{p_t^I k_t} \leq \frac{1 - \mu}{1 + r}. \quad (2.7)$$

We assume that producers consume at least a fraction  $d_m$  of their capital:

$$0 \leq d_t - d_m p_{t-1}^I k_{t-1}. \quad (2.8)$$

Producers have an identical initial endowment of wealth equal to  $p_0^I k_0 - b_0$ .

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<sup>1</sup>see e.g. Kiyotaki and Moore [1997]. In their setting, debt is a preferred means of finance rather than new share issues.

The entrepreneurs maximize the present value of dividends taking into account production possibilities and the flow of funds and credit constraints. They determine the optimal inputs of labor and capital and the optimal amount of debt. The Lagrangian for this problem is:

$$\sum_{t=0}^{t=+\infty} \left( \frac{1}{1+r} \right)^t \left( d_t + \lambda_t^d (d_t - d_m p_{t-1}^I k_{t-1}) + \lambda_t^b \left( (1-\mu) p_t^I k_t - (1+r) b_t \right) \right). \quad (2.9)$$

where  $d_t$  is given by the flow of funds constraint. The problem is convex at the individual level.  $\lambda_t^d$  is the Lagrangian multiplier associated with the constraint on producers' consumption and  $\lambda_t^b$  is the Lagrangian multiplier associated with the credit constraint.

The marginal condition on labor gives the firm optimal labour/capital ratio as a function of real wage and productivity parameters (e.g.  $K_t$ ):

$$F_{l_t}(k_t, K_t \cdot l_t) = F_{l_t}(1, K_t l_t / k_t) = w_t$$

The marginal condition on capital yields:

$$\left( 1 + \lambda_t^d \right) \left( F_{k_t}(k_t, K_t \cdot l_t) - p_t^I \right) + \lambda_t^b (1-\mu) p_t^I + \frac{1 + \lambda_{t+1}^d}{1+r} (1-\delta) p_{t+1}^I - \frac{\lambda_{t+1}^d d_m p_t^I}{1+r} = 0. \quad (2.10)$$

The marginal condition on debt is:

$$\lambda_t^d - \lambda_{t+1}^d = \lambda_t^b(1 + r). \quad (2.11)$$

Taking into account the inequality conditions, we can distinguish 4 cases. The first case,  $\lambda_t^b = 0, \lambda_t^d = 0$ , reflects the fact that neither the credit constraint nor the constraint on producers' consumption is binding. Here the producer can borrow as much as necessary in order to undertake the optimal amount of investment. The second case,  $\lambda_t^b \neq 0, \lambda_t^d \neq 0$ , represents a situation in which both inequality constraints are binding. In the following we will analyze these two cases. The two remaining cases ( $\lambda_t^b = 0, \lambda_t^d \neq 0$ , or  $\lambda_t^b \neq 0, \lambda_t^d = 0$ ) are irrelevant and dealt with in the appendix. As all firms have an identical initial endowment of net worth, the credit and producers consumption constraints are binding for all firms or for none of them. The initial wealth distribution between wage-earners and entrepreneurs determine whether the credit constraint is initially -and subsequently- binding or not.

The first case refers to the neoclassical model. From the marginal condition on debt  $\lambda_t^d - \lambda_{t+1}^d = \lambda_t^b(1 + r)$  we can conclude that  $\lambda_{t+1}^d = 0$ , i.e. the constraint on producer's consumption does not bind in the next period. The amount of debt is indeterminate, as expected by the Modigliani-Miller theorem. The individual and aggregate marginal product of capital is equal to the cost of capital, so that the interest rate is equal to:

$$1 + r = \frac{p_{t+1}^I (1 - \delta)}{p_t^I - F_{k_t}(K_t, K_t L_t)} \quad (2.12)$$

When both constraints are binding ( $\lambda_t^b \neq 0, \lambda_t^d \neq 0$ ), dividends, capital and debt are given by the following conditions:

$$d_t = d_m p_{t-1}^I k_{t-1}, \quad (2.13)$$

$$d_t = F(k_t, K_t l_t) - r b_{t-1} + b_t - b_{t-1} - p_t^I (k_t - (1 - \delta) k_{t-1}) - w_t l_t, \quad (2.14)$$

$$b_t = \frac{1 - \mu}{1 + r} p_t^I k_t. \quad (2.15)$$

Taking into account that all firms are identical and distributed over  $[0,1]$  and constant returns to scale, we aggregate dividends and the flow of funds equation (aggregate variables are in capital letters):

$$d_m p_{t-1}^I K_{t-1} = F(K_t, K_t L_t) - r B_{t-1} + B_t - B_{t-1} - p_t^I (K_t - (1 - \delta) K_{t-1}) - w_t L_t,$$

The growth rate of aggregate and individual capital is equal to the growth rate of internal funds (retained earnings over equity):

$$g_K = \frac{K_t}{K_{t-1}} - 1 = \frac{1 - \delta + \left(\frac{p_{t-1}^I}{p_t^I}\right) \left(\frac{F(k_t, K_t l_t) - w_t L_t}{p_{t-1}^I K_{t-1}} - d_m - (1 + r) \frac{B_{t-1}}{p_{t-1}^I K_{t-1}}\right)}{1 - \frac{B_t}{p_t^I K_t}} - 1 \quad (2.16)$$

where  $\pi_t = (F(K_t, K_t L_t) - w_t L_t) / p_{t-1}^I K_{t-1} = F_{k_t}(K_t, K_t L_t) \cdot \frac{K_t}{p_{t-1}^I K_{t-1}}$  is the profit rate. At the aggregate and at the entrepreneur level, the profit rate depends on the optimal labor-capital ratio,  $\pi_t = (F(1, K_t \cdot (l_t/k_t)) - w_t (l_t/k_t)) \cdot \frac{k_t}{p_{t-1}^I k_{t-1}}$  which is



determined by the real wage rate and productivity parameters. We can aggregate the binding credit constraint, take as given that the credit constraint has been binding also the date before, and replace aggregate leverage to get the growth rate of aggregate capital:

$$g_K = \frac{K_t}{K_{t-1}} - 1 = \frac{1 - \delta + \left(\frac{p_{t-1}^I}{p_t^I}\right) (-d_m - (1 - \mu))}{1 - \frac{(1-\mu)}{(1+r)} - \frac{F_{k_t}(K_t, K_t L_t)}{p_t^I}} - 1. \quad (2.17)$$

The growth rate of capital is *decreasing* when the interest rate increases.

Furthermore, we restrict the parameter values in such a way that this growth rate will be positive and the objective function of the firm  $\sum_{t=1}^{\infty} d_t(1+r)^{-t}$  be bounded. Dividends are given as  $d_t = d_m p_{t-1}^I k_{t-1}$ . Taking into account the growth rate of capital in the objective function, we get

$$V_0 = \frac{d_m k_0}{1+r} \sum_{t=0}^{\infty} p_t^I \cdot (1+r)^{-t} \cdot \left( \frac{1 - \delta + \left(\frac{p_{t-1}^I}{p_t^I}\right) (-d_m - (1 - \mu))}{1 - \frac{(1-\mu)}{(1+r)} - \frac{F_{k_t}(k_t, K_t l_t)}{p_t^I}} \right)^t. \quad (2.18)$$

If the relative prices are bounded, i.e. there exists  $p^I$ , such that  $p_t^I < p^I$ , for all t, the objective function is bounded, if

$$\frac{1}{1+r} \frac{1 - \delta + \left(\frac{p_{t-1}^I}{p_t^I}\right) (-d_m - (1 - \mu))}{1 - \frac{(1-\mu)}{(1+r)} - \frac{F_{k_t}(k_t, K_t l_t)}{p_t^I}} < 1.$$

The two conditions together – positive growth rate and bounded utility – yield conditions for the range of possible values of the interest rate  $1+r \in ]1+r_{\min}^K, 1+r_{\max}^K[$

where  $1 + r_{\min}^K$  is given as

$$1 + r_{\min}^K = \frac{2 - \delta - \mu + p_{t-1}^I (d_m + (1 - \mu)) / p_t^I}{1 - F_{k_t}(k_t, K_t l_t) / p_t^I}$$

and  $1 + r_{\max}^K$  as

$$1 + r_{\max}^K = \frac{1 - \mu}{\delta - F_{k_t}(k_t, K_t l_t) / p_t^I + p_{t-1}^I (d_m + (1 - \mu)) / p_t^I}. \quad (2.19)$$

### 2.3. Balanced growth

For a balanced growth to exist in the economy we have to find an interest rate, such that the growth rate of capital equals the growth rate of consumption:  $g^* = g_K = g_C$ .

We restrict the parameter values such that the utility at a balanced growth path of consumption and that the present value of net revenues at a balanced growth path are bounded. This requires that

$$U(c_0) = \sum_{\tau=0}^{+\infty} \frac{1}{1 - \sigma} \left( \left[ c_0 \left( \frac{1+r}{1+\rho} \right)^{\frac{\tau}{\sigma}} \right]^{1-\sigma} - 1 \right) \cdot (1 + \rho)^{-\tau} \quad (2.20)$$

is bounded, for  $\sigma \neq 1$ .<sup>2</sup> This is the case, if  $(1 + r)^{1-\sigma} < 1 + \rho$ . For  $\sigma \geq 1$  this condition is always fulfilled. For  $\sigma < 1$ , the interest rate has to be chosen such that  $1 + r \in ]1 + \rho, (1 + \rho)^{1/(1-\sigma)} [= ]1 + r_{\min}^c, 1 + r_{\max}^c[$ .

For Romer [1986] case of unconstrained growth, substituting the interest rate in

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<sup>2</sup>For  $\sigma = 1$  the utility is always bounded.

the condition for the growth rate of consumption yields:

$$\frac{c_{t+1}}{c_t} = \left( \frac{\frac{p_{t+1}^I(1-\delta)}{p_t^I - F_{k_t}(K_t, K_t L_t)}}{1 + \rho} \right)^{\frac{1}{\sigma}}. \quad (2.21)$$

This growth rate is positive when the marginal product of capital is be larger than the rate of time preference. Growth increases with the return on savings and decreases with the rate of time preference and the elasticity of substitution.

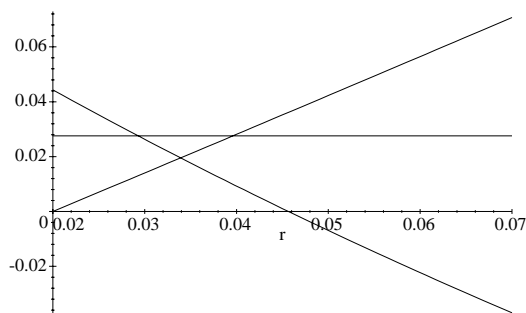
In case of financially constrained growth, the interest rate has to be chosen such that  $r \in ]r_{\min}^c, r_{\max}^c[$  and  $r \in ]r_{\min}^K, r_{\max}^K[$  and the growth rate of capital has to equal the growth rate of consumption. For this we need

$$g_c(r_{\min}) < g_K(r_{\min}), \text{ with } r_{\min} = \max(r_{\min}^c, r_{\min}^K)$$

$$g_c(r_{\max}) > g_K(r_{\max}), \text{ with } r_{\max} = \min(r_{\max}^c, r_{\max}^K).$$

This is indeed feasible if we choose the production function and the utility function in the appropriate way. In the Cobb Douglas case ( $y_t = Ak_t^{1-\alpha}(K_t l_t)^\alpha$ , with  $0 < \alpha < 1$ ), one has  $F_{k_t}(k_t, K_t l_t) = (1 - \alpha) A \left( \frac{w}{\alpha AK} \right)^{\frac{\alpha}{\alpha-1}}$ . Once the externality is taken into account,  $F_{k_t}(K_t, K_t L_t) = (1 - \alpha) AL^\alpha$ . A numerical example is given by the following choice of parameter values:  $\sigma = 0.7$ ,  $\rho = 0.02$ ,  $A = 0.2$ ,  $L = 2$ ,  $\alpha = 0.3$ ,  $p^I = 2$ ,  $\mu = 0.4$ ,  $d_m = 0.01$ , and  $\delta = 0.05$ . Then  $r_{\min}^c = 0.02$ ,  $r_{\max}^c = 0.068$ ,  $r_{\min}^K = 0.028$ , and  $r_{\max}^K = 0.046$ . The interest rate in the unconstrained case  $r^* = 0.04$ . The growth rates are  $g_c = \left( \frac{1+r}{1.02} \right)^{1.428} - 1$  and  $g_K = \frac{0.34}{0.914 - 0.6/(1+r)} - 1$  and they are depicted in the

following figure. In this example, the growth rate with financial constraints is smaller than the growth rate without financial constraints which is given by the horizontal line.



### 3. Conclusion

This note shows how the growth rate of internal funds drives the growth rate of capital and economic growth when firms are facing credit rationing and a Romer like externality. An extension could model cyclical endogenous growth with credit rationing.

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## APPENDIX:

**Case 3: Only the constraint on the producer’s consumption is binding:**

$\lambda_t^b = 0, \lambda_t^d \neq 0$ . The following two conditions on dividends have to be fulfilled:

$$d_t = d_m \cdot p_{t-1}^I \cdot k_{t-1}, \quad (3.1)$$

$$d_t = F(k_t, K_t l_t) - r b_{t-1} + b_t - b_{t-1} - p_t^I (k_t - (1 - \delta)k_{t-1}) - w_t l_t. \quad (3.2)$$

Suppose that both conditions hold true and that the credit constraint is not bind-

ing. Define

$$a = (1 - \mu)p_t^I k_t - (1 + r)b_t > 0$$

as the difference between what the firm can borrow and what the firm actually does borrow which is greater than zero since the credit constraint does not hold. Since the firm is free to choose its level of debt, a level of  $B_t + a/2$  is also possible. It satisfies the credit constraint, but increases the value of net revenues such that the constraint on the consumption of the producer does no longer apply. Therefore, we can exclude the case where only the constraint on producer's consumption is binding.

**Case 4: Only the credit constraint is binding:**  $\lambda_t^b \neq 0, \lambda_t^d = 0$ . The marginal condition on debt becomes:

$$-\lambda_{t+1}^d = (1 + r)\lambda_t^b.$$

We know therefore that in the next period the condition on the consumption of the producer will be binding. In the next period we either have a situation in which the credit constraint is no longer binding which corresponds to the case we excluded before or we have a situation in which both constraints are binding. Thus, a situation in which only the credit constraint is binding can be only transitory.