# Staggered Price and Trend In‡ation: Some Nuisances[¤]

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Abstract

Most of the papers in the sticky-price literature are based on a log-linearisation around the zero in‡ation steady state, a simplifying but counterfactual assumption. This paper shows that when trend in‡ation is considered, both the long-run and the short run properties of time dependent staggered price models change dramatically. It follows that the results obtained by models log-linearised around a zero in‡ation steady state might be misleading.

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## 1 Introduction

"Macroeconomics is moving toward a New Neoclassical Synthesis" (Goodfriend and King (1998), p. 231). "Building on new classical macroeconomics and RBC analysis, it incorporates intertemporal optimization and rational expectations [...]. Building on New Keynesian economics, it incorporates imperfect competition and costly price adjustment [...]" (Goodfriend and King (1998), p. 255). Judging from the amount of recent paper on dynamic general equilibrium models of sticky prices, mainly time dependent staggered prices, the moving seems to be completed.[1] Given the aim to build quantitative models of economic fluctuations, the models are simulated and then, following the RBC tradition, compared with actual data.

The vast majority of the works in the literature log-linearise their model around a particular steady state: the zero inflation steady state.[2] This is due to reasons of simplicity, given that in actual data trend inflation in the developed world in the last forty/thirty years have been quite far from zero. The average inflation rates from the seventies onwards in major European countries range from approximately the 3% of Germany to the almost 10% of Spain with the U.S. around 5%. It is obvious that a time-dependent sticky-price framework is ill suited for describing economies with high rates of inflation, because in such an environment the sticky price assumption is unreasonable.[3] On the contrary, post world war data in developed economies show positive, but low levels of average inflation and thus the New Neoclassical Synthesis framework is applied to describe those data. The implicit assumption then must be that taking into account low levels of trend inflation would not matter anyway, because it would have a negligible effect both on the steady state (around which the model is log-linearised) and on the dynamic properties of the model.

This paper investigates this implicit assumption. It shows that is actually substantially faulty. It does that by analysing a standard sticky price dynamic general equilibrium model with the Calvo (1983)-Rotemberg (1982) sticky price specification, which is the most commonly employed in the literature. The structure is otherwise taken by the well-known paper of Chari et al. (2000b). It also analyses the case in which capital is treated as fixed (another common assumption in this literature, following an argument put forward by McCallum and Nelson (1999)). It turns out that when trend inflation is considered, both the long-run (i.e., steady state) and the short run (i.e., dynamics) properties of time dependent staggered price models change dramatically. First, using standard calibration values from

[1] This new workhorse model has been extensively used to investigate various issues: persistence (e.g., Jeanne (1998), Chari et al. (2000b), Ascari (2000)), monetary policy rule (e.g., Rotemberg and Woodford (1997), Clarida et al. (1999)), inflation dynamics (e.g., Gali and Gertler (1999)) and open economy (e.g., Chari et al. (2000a), Gali and Monacelli (1999)).

[2] Exceptions are King and Wolman (1996), Dotsey et al. (1999), Ascari (2000) and Chari et al. (2000a).

[3] Therefore, it would be pointless to show that for high average inflation rates time-dependent sticky price models deliver counterfactual results.

the literature, it is shown that the steady state output level is very much sensitive to the steady state rate of growth of money. Very mild level of trend inflation implies large, and unrealistic, changes in the steady state output level. Second, consequently, trend inflation matters for the dynamic properties of the log-linearised model. Indeed, the dynamics of the log-linearised model depends on the particular steady state around which it has been log-linearised. Since steady state differs a lot depending on the level of trend inflation, it comes as no surprise the fact that trend inflation matters for the dynamics of the log-linearised model. Finally, early old-fashioned sticky-price models has been extensively used to address a very important topic: disinflation (see, e.g., Blanchard and Fischer (1989), chp. 10). Again, the level of trend inflation from which the disinflation policy starts is extremely important for the dynamic behaviour of the model following a disinflation. In short, this paper shows that disregarding trend inflation is quite far away from being an innocuous assumption. As a consequence, the results obtained by models log-linearised around a zero inflation steady state might be misleading.

The issue of trend inflation has not been so far really tackled in the literature. Only very few papers mention it, namely King and Wolman (1996) and Ascari (1997). Both the papers, however, look only at the effects of trend inflation on the steady state, and this paper will consider their results in what follows. Sticky price models are certainly a very fruitful area of research, as witnessed by the great number of papers they have recently generated. They provide a framework that has very much increased our understanding of monetary policy and its trasmission mechanism. They have also revealed, however, potential problems, especially in explaining the dynamics of output both at business cycle frequencies (Chari et al. (2000b)) and at higher frequencies (Ellison and Scott (2000)). This paper points to a further nuisance challenging sticky price practitioners: the effect of trend inflation on the model long-run and short-run properties.

## 2 The model

The model is meant to be the most standard sticky price dynamic general equilibrium model. Thus we will use the Calvo (1983)-Rotemberg (1982) sticky price specification, which is the most commonly employed in the literature. The structure is otherwise taken by the well-known paper of Chari et al. (2000b), which is taken as the benchmark model. The model economy is therefore composed of a continuum of infinitely-lived consumers, producers of final and intermediate goods. The final good market is competitive, while the intermediate goods producers enjoy market power. The model is so familiar by now that does not need any detailed explanation[4]. The functional forms we use are also quite standard:

[4] The equations of the model are provided in Appendix 1.

| | |
|---|---|
| Instantaneous utility function | $\left\{ \left[ bC^{\frac{\eta-1}{\eta}} + (1-b)\left(\frac{M}{P}\right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} (1-L)^{e} \right\}^{1-\chi} / (1-\chi)$ |
| Production function of intermediate goods producers | $Y_i = AK_i^{1-\sigma} L_i^{\sigma}$ |
| Production function of final goods producers | $Y = \left[ \int Y_i^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}$ |

where $C$ = consumption, $M$ = money, $P$ = price of the final good, $Y_i$ = output of the intermediate good producer $i$; $K_i; L_i$ = capital and labour employed by the intermediate good producer $i$; $Y$ = final good output. The utility function is chosen because both is the same as Chari et al. (2000b) and it is quite general, encompassing most of the utility functions employed in the literature on sticky price models.

Moreover: (i) intermediate goods producers behave as Dixit-Stiglitz monopolistic competitors because they are facing a downward sloping factor demand from final good producers, with elasticity equal to $\theta$; (ii) they can change their price only in specific states of nature, and have to satisfy demand at the quoted price. The state of nature in which the firm can change its price will occur with probability $1-\alpha$; while with probability $\alpha$ the firm is stuck with the same price of the previous period. The problem of the intermediate goods producers can be defined as

$$\max_{\{p_{it}\}} E_t \left( \sum_{j=0}^{\infty} \alpha^j \Delta_{t;t+j} z_{t+j} \right) = E_t \left( \sum_{j=0}^{\infty} \alpha^j \Delta_{t;t+j} \left[ \left( \frac{P_{i;t}}{P_{t+j}} \right) Y_{i;t+j} - TC_{i;t+j}(Y_{i;t+j}) \right] \right) \tag{1}$$

where $\Delta_{t;t+j}$ represents the real discount factor from $t$ to $t+j$ applied by the firm to the stream of future real profits; $z$ = real profits, $P_i$ = price set by the firm, $TC_i$ = real total costs: Given the demand function, $Y_{i;t+j} = \left(\frac{P_{i;t}}{P_{t+j}}\right)^{-\theta} Y_{t+j}$, the optimal price fixed by re-setting firms in period $t$ is given by

$$P_{i;t}^{*} = \left( \frac{\theta}{\theta-1} \right) \frac{E_t \sum_{j=0}^{\infty} \alpha^j \Delta_{t;t+j} MC_{i;t+j} P_{t+j}^{\theta} Y_{t+j}}{E_t \sum_{j=0}^{\infty} \alpha^j \Delta_{t;t+j} P_{t+j}^{\theta-1} Y_{t+j}} \tag{2}$$

where $MC_i$ = real marginal cost of producer $i$: This equation represents the core of sticky price models, as thoroughly explained by King and Wolman (1996).

Finally, following an argument put forward by McCallum and Nelson (1999), it is also considered the case where capital is a fixed factor in the production function of intermediate goods producers (e.g., Rotemberg and Woodford (1997)).

# 3 Trend Inflation and Steady State

In this section we perform an exercise similar to that of King and Wolman (1996), that is, we look at the effects of trend inflation on the steady state. While King and Wolman

(1996) concentrated on the mark-up, we will focus on the effects on steady state output.

Assume that $\gamma$ is the gross rate of growth of the money in steady state that is, $\gamma = \frac{M_t}{M_{t-1}}, \forall t.$ The steady state is then characterised by the constancy of the real variables and by a rate of growth of the nominal variables equal to $\gamma.$ There is broad agreement in the literature on the calibration values of most of the parameters. Calibrating a period as a quarter, then $\alpha$ is set to 0.75, which implies that prices are on average fixed for one year. $\theta$ is in most papers set to 10 (implying a mark-up of 1.1, in a zero-inflation steady state). The parameter for the money demand equation are taken from Chari et al. (2000b)[5], so $\eta = 0.39$ and $b$ is set so that the ratio $(M/PC) = 1.2.$ Then: $\beta = (0.965)^{1/4}, \sigma = 0.67$ and the depreciation rate $\delta = 1 - (0.92)^{1/4}$. The value of $e,$ instead, varies across papers, ranging from a value of 1 to values more in line with the microeconomic estimates as 6. $e$ is put equal to 1.5, again as Chari et al. (2000b).[6] With these numbers, in a zero-inflation steady state (ZISS henceforth) the model presents an annualised capital-output ratio of 2.5 and an investment-output ratio of 0.2, while households enjoy two thirds of their total endowment of time as leisure.

The steady state value of the optimal price set each period by the re-setting firms is

$$\frac{P_{i,t}}{P_t} = \left(\frac{\theta}{\theta - 1}\right)\left(\frac{1 - \alpha\beta\gamma^{\theta-1}}{1 - \alpha\beta\gamma^{\theta}}\right) MC \tag{3}$$

Two remarks follow. First, there is a maximum rate of growth of money supported by the steady state, because to get (3) the summations in (2) need to converge.[7] Hence it must be that $\alpha\beta\gamma^{\theta} < 1$, that is, trend inflation should be less than 12,6% annually. Unfortunately, this threshold number is not too far from the level of average inflation in the developed countries in the last thirty or forty years. Therefore, this first remark gives a first warning nuisance, since one wants to use these models to describe the behaviour of inflation in post-war data.

Second, a maximum level of sustainable trend inflation would not be worrying on the model performance, if trend inflation does not matter, that is, if it has only negligible effects. Unfortunately this does not seem to be the case. Figure 1 plots the percentage deviation of steady state output from output in a ZISS, as a function of the rate of growth of money (annualised in the graph). Steady state output decreases strongly with inflation. A steady state annual rate of inflation of 10% leads to a steady state output level 26% lower than in a ZISS. 8% trend inflation lowers output of 10% (with respect to ZISS) and 5% (= average inflation in the U.S. in the last forty years) of almost 3%. It is important to underline that

[5] Given that I employ the same utility function as Chari et al. (2000b), then I have the same money demand function.

[6] In any case, surprisingly enough, given the attention devoted to the parameter governing the elasticity of labour supply in the literature, all the presented results are very little sensitive to changes in the value of $e.$

[7] This point has already been acknowledged by King and Wolman (1996): see footnote 12 at p. 96.

instead, the capital/output ratio, the investment/output ratio and the steady state fraction of time devoted to work do not change very much with trend inflation.[8] Hence, calibrating the model one would not change the parameters value.

Third, as said above, following McCallum and Nelson (1999), capital is often treated as fixed (e.g., Rotemberg and Woodford (1997)). In this case, the steady state properties of such a model are even more disturbing. First, the maximum sustainable level of steady state inflation is now only 8% (because the marginal costs are now increasing depending on $\sigma$, see Appendix 1B). Second, again the steady state output level seems to be very sensitive to steady state inflation, as shown by Figure 2. In particular, for example, 5% trend inflation lowers output of 11.5% with respect to the ZISS, while 7% trend inflation cause output to be 39% lower than in a ZISS. There is, actually, a sort of 'continuity' between the two Figures, in the sense that as the value of $\sigma$ increases Figure 2 'tends' to Figure 1, as shown in Figure 3. If $\sigma = 1$, the behaviour of steady state output as a function of trend inflation is then similar to the case with capital. In other words, increasing $\sigma$ stretches out Figure 2, by pulling the asymptote (i.e., maximum level of sustainable trend inflation) to the right.

Admittedly, the results are somewhat sensitive to the value of $\theta$. Similarly to the increase in $\sigma$ in the previous Figure, a lower value of $\theta$ implies an higher value of sustanaible trend inflation; which in turn basically stretches out Figure 1 and 2, by shifting the vertical asymptot to the right (see Figure 4). For example, if $\theta = 4.3$, as in King and Wolman (1996), the maximum level of sustainable trend inflation is 32% and 19% in the model with capital and in the model with fixed capital respectively. In this case, 10% trend inflation would lower steady state output of 4% and 8% with respect to the ZISS, in the two different models respectively. In any case, most of the papers in the literature use $\theta = 10$, because $\theta = 4.3$ seems to result in an implausible high level of mark-up in a ZISS (i.e., 30%).[9]

To conclude, trend inflation has huge effects on the steady state properties of the model. The numbers above would imply enourmous costs of inflation in terms of loss in output. Moreover, the steady state properties of a sticky price model are also different depending on whether capital is treated as fixed or not. In any case, these properties are particularly embarrassing for anyone willing to use these models to analyse important facts as disinflations (see 4.2).[10]

[8] Except when trend inflation gets very close to its limiting upper value.

[9] Also the behaviour of the mark-up, on which King and Wolman (1996) focuses the analysis, is similarly very sensitive to trend inflation when $\theta = 10$. The steady state formula for marginal and average mark up are the same as in King and Wolman (1996) (in particular, see equations (18) at p. 92 and (19) at p. 93 therein), because of the same Calvo pricing framework. By considering only values of $\theta \leq 4.3$, King and Wolman (1996) overlooks the effect of trend inflation on the model when $\theta$ assumes higher values.

[10] This might be the reason why virtually no sticky price model has been devoted to such an issue, with the exception of some stilised models (i.e., Dazinger (1988), Ireland (1995), Ascari and Rankin (1997)).

# 4 Trend Inflation and Dynamics

## 4.1 Log-linearisation

Usually dynamic general equilibrium models are solved by log-linearising the models around a steady state. However, we saw in the previous section that different levels of trend inflation lead to very different steady states. In general, then, also the coefficients of the log-linearised equations would depend on the steady state level of inflation. Thus, an immediate and uncomfortable implication of the previous section is that the steady state around which one log-linearises should matter. Indeed it does.

To analyse how the dynamics of the model depend on trend inflation, the case with fixed capital and $\sigma = 1$ is examined: Figure 5 plots the impulse response of the model to a 1% rate of money growth shock, at different levels of trend inflation. When trend inflation is zero (see upper panel of Figure 5), the model has only real roots. Moreover, the response of output shows the known lack of persistence typical in the standard model.[11] Turning the steady state rate of growth of money to positive values very soon results in complex roots. As shown in Figure 5, the oscillation in the impulse responses typically induced by complex roots become more and more pronounced as trend inflation increases. As a result, persistence seems to increase. Moreover, as the value of trend inflation gets closer to the upper limit some puzzling features occur: (i) the size of the short-run effect becomes substantially larger; (ii) the impact effect of a positive money shock becomes negative (see the bottom panel in Figure 5); (ii) the model does not satisfy the Blanchard-Kahn conditions anymore and starts to produce explosive behaviour, by generating a number of explosive roots bigger than the number of non-predetermine variables. Therefore, it seems that not only the steady state, but also the dynamic properties of the standard model are very sensitive to the value of trend inflation.[12]

Analytical investigation sheds some light on this high sensistivity of the dynamic behaviour to trend inflation. Start with the well-known case where the log-linearisation is taken around the steady state with zero inflation (i.e., $\gamma = 1$). Define $\Pi_t = (P_t/P_{t-1})$ = gross inflation rate and use lower-case letters for the log-deviation of variables from their steady state values. The log-linearised version of (2) is

$$p_{it} - p_t = (1-\alpha\beta)E_t \sum_{j=0}^{\infty} (\alpha\beta)^j \left[\pi_{t,t+j} + mc_{t+j}\right] \tag{4}$$

[11] The process for the rate of growth of money supply used in this simulations is again mutuated from Chari et al. (2000b). Its autocorrelation term is 0.57. Hence, some persistence in the impulse response of output is due to the exogenous autocorrelation in the money supply process.

[12] Both the cases with varying capital and with fixed capital and $\sigma = 0.67$ present similar qualitative features, thus they are not reported. In the case with $\sigma = 0.67$, the puzzling features begin to appear at very low levels of inflation, because the upper bound is only 8%.

where $\pi_{t,t+j} = (\pi_{t+1} + \pi_{t+2} + ... + \pi_{t+j})$ and $\pi_{t,t} = 0$. This equation is usually combined with the log-linearised version of the general price level equation[13]

$$p_{it} - p_t = \frac{\alpha}{1-\alpha}\pi_t \tag{5}$$

in order to get the dynamics of inflation

$$\pi_t = \lambda mc_t + \beta E_t \pi_{t+1} \tag{6}$$

where $\lambda = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}$. As explained by Gali and Gertler (1999), among others, this is the so-called 'New Keynesian Phillips Curve'.[14] In other words, the inflation rate today depends just on the discounted sum of the future expected marginal costs, as can be easily found by iterating (6) forward

$$\pi_t = \lambda \sum_{j=0}^{\infty} \beta^j E_t mc_{t+j} \tag{7}$$

From a theoretical perspective, for a given expected future path of the marginal costs, the key parameter in the dynamics of inflation is therefore $\lambda$.

Again things are a bit different, however, when the log-linearisation is taken around a steady state with trend inflation (i.e., $\gamma > 1$); since it yields

$$\begin{aligned} p_{it} - p_t \;=\;& E_t \sum_{j=0}^{\infty} (\alpha\beta\gamma^{\theta})^j (1-\alpha\beta\gamma^{\theta}) \left[\theta\pi_{t,t+j} + y_{t+j} + mc_{t+j}\right] \\ & - E_t \sum_{j=0}^{\infty} (\alpha\beta\gamma^{(\theta-1)})^j (1-\alpha\beta\gamma^{(\theta-1)}) \left[(\theta-1)\pi_{t,t+j} + y_{t+j}\right] \end{aligned} \tag{8}$$

Combining this last equation with the log-linearised formula for the general price level, that is,

$$p_{it} - p_t = \frac{\alpha\gamma^{\theta-1}}{1-\alpha\gamma^{\theta-1}}\pi_t \tag{9}$$

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[13] The equation for the general price level in a standard Dixit-Stiglitz-monopolistic- competition framework is

$$P_t = \left(\int_0^1 P_{zt}^{1-\theta} dz\right)^{\frac{1}{1-\theta}} = \left[\alpha P_{t-1}^{1-\theta} + (1-\alpha) P_{it}^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

where we use $P_{it}$ and $P_{zt}$ to distinguish between respectively the new price set by the i re-setting firms and the price of all the firms indexed by z.

[14] In fact just assuming that the real marginal costs depend on output ($mc_t = \frac{1}{\phi} y_t$) and substituing, one gets

$$y_t = \frac{\alpha\phi}{(1-\alpha)(1-\alpha\beta)}\left[\pi_t - \beta E_t \pi_{t+1}\right]$$

yields the generalised version of (6), which can be written as

$$
\begin{aligned}
\pi_t = {} & \left(\frac{1-\alpha\gamma^{\theta-1}}{\alpha\gamma^{\theta-1}}\right)\Bigg[(1-\alpha\beta\gamma^{\theta})mc_t + \beta E_t\pi_{t+1} + \\
& +(1-\gamma)\beta(1-\alpha\gamma^{\theta-1})\left[y_t - \left(\theta+\frac{\alpha\gamma^{\theta-1}}{1-\alpha\gamma^{\theta-1}}\right)E_t\pi_{t+1} + \right. \\
& \left. +(1-\alpha\beta\gamma^{(\theta-1)})E_t\sum_{j=0}^{\infty}(\alpha\beta\gamma^{\theta})^j[(\theta-1)\pi_{t+1,t+1+j}+y_{t+1+j}]\right]\Bigg]
\end{aligned} \tag{10}
$$

Setting $\gamma = 1$, yields (6). Since $\gamma$ (gross trend inflation rate) is actually very close to one, one may approximate (??) by not considering the last additive term which is multiplied by $(\gamma - 1)$. In that case, an analytical expression very close to (6) is obtained

$$
\pi_t = \bar{\lambda}(\gamma)mc_t + \beta E_t\pi_{t+1} \qquad \text{or} \qquad \pi_t = \bar{\lambda}(\gamma)\sum_{j=0}^{\infty}\beta^j E_t mc_{t+j} \tag{11}
$$

where $\bar{\lambda}(\gamma) = \left(\frac{1-\alpha\gamma^{\theta-1}}{\alpha\gamma^{\theta-1}}\right)(1-\alpha\beta\gamma^{\theta})$. It is evident that trend inflation influences the behaviour of inflation, as shown in the following table.

| $\lambda = 0.086$ | $\gamma = (1.02)^{\frac{1}{4}}$ | $\gamma = (1.05)^{\frac{1}{4}}$ | $\gamma = (1.08)^{\frac{1}{4}}$ | $\gamma = (1.1)^{\frac{1}{4}}$ |
|---|---|---|---|---|
| $\bar{\lambda}(\gamma)$ | 0.06 | 0.031 | 0.012 | 0.0043 |
| $(\lambda - \bar{\lambda}(\gamma))/\lambda$ | 30% | 64% | 86% | 95% |

Table 1. Values of $\lambda$ as a function of trend inflation

The value of $\lambda$ is very much sensitive to the values trend inflation: Even for a small level of trend inflation, i.e., 2% annually, the value of $\lambda$ is reduced of 30% with respect to a log-linearisation around ZISS. This means that, for any given future expected path of the marginal costs, the dynamic response of inflation to marginal costs are overestimated, if trend inflation is not taken into account. Moreover, the higher the level of inflation, the further apart are the values of $\lambda$ and $\bar{\lambda}(\gamma)$. The model predicts that the dynamic response of inflation to marginal costs should be reduced of 64% if annualised trend inflation is 5%, up to 86% for 8% trend inflation and virtually zero for 10% trend inflation. Figure 6 visualises this effect.[15]

From the analysis above some important points follow. First, the model therefore implies that the log-linear approximation (7) which expresses the dynamics of inflation as a function of the future expected path of marginal costs in a ZISS gets substantially worse as trend inflation increases. It comes thus as no surprise that this fact is going to affect the dynamics of the model. Second, it does not seem to be appropriate to compare simulation data

[15] If $\theta = 4.3$, $\lambda$ is reduced of 30% if trend inflation is 5% and of 60% at 10% trend inflation, so the argument still holds.

obtained from a model with a ZISS with actual data (from VAR analysis, for example), where trend inflation is above zero. While the ZISS assumption tends to simplify the analysis giving neat results, the analysis above shows that disregarding the effects of trend inflation may lead to misleading results. Finally, in a quite influential paper Gali and Gertler (1999) propose an empirical formulation based on (6) to explain the dynamics of inflation.[16] Gali and Gertler (1999) argue that such a model could account for the behaviour of inflation in the last thirty years, and estimate the structural parameters of the model (i.e., $\alpha; \beta$). From what has just been said above, a model based on (6) is questionable when values of trend inflation are not only of two-digits, as in the pre-Volcker period, but just slightly above zero.

### 4.2 Disinflation

Not surprisingly, also the effect of a disinflationary policy would depend on the rate of steady state inflation. A log-linearised model is not suited to solve for the path of output following a sizeable disinflation, because a disinflation involves a move from one steady state to another. Hence we use the package for non-linear simulations DYNARE.[17] Figure 7 shows the path of output following a 4% disinflation, again in the model with fixed capital and constant return to scale. The upper panel shows the path of output after a disinflation from 4% to 0. At the beginning output decreases by more than 10% and so disinflation causes a substantial slump on impact. Then output starts increasing monotonically, untill it reaches its new, slighlty higher steady state level (recall Figure 3). In all the panels of Figure 7 the final steady state level is normalised to 1.[18] The second panel shows a 4% disinflation, from 6% to 2%. Qualitatively the path is very similar, but the impact effect is smaller while the steady state effect is bigger. And this features swiftly intensify as the starting rate of growth of money increases. As shown in the next panels, for a given size of the disinflationary policy (i.e., 4%), the higher the rate of growth of money, the smaller the negative impact effect and the bigger the positive steady state effect. Disinflating from 10% to 6% does not cause any decrease in output level, which is always above the starting steady state level. The long-run effect of the policy has taken over the short-run dynamics.

As a conclusion, trend inflation is found to matter a lot, not only for the steady state properties of the model but also, if not even more, for the effects on its dynamic properties.

[16] Gali and Gertler (1999) model is slightly different since it includes also a fraction of backward-looking price setters.

[17] This package has been elaborated by Michel Juillard at CEPREMAP (see Juillard (1996)) based on the algorithm in Boucekkine (1995).

[18] So one can easily read on the vertical axis scale on the left the difference between the starting and the final steady state.

## 5 Ways out

It has been shown above that trend inflation has some disturbing effects both on the steady state and on the dynamics of a standard staggered price model, with Calvo-Rotemberg pricing. Is there a possible way out?

First, one may think that most of the nuisances come from the particular price contract structure has been analysed in this paper, and that most of these problems would not be present in a Taylor (1980) type of model. For example, a Taylor (1980) contract structure would not impose any upper bound on the steady state rate of money growth. In this case, in fact, the first order condition for price re-setting firms would present a ratio between finite summations, and so there would be no issues of convergence of infinite sums. This is certainly true, but that seems the only real difference. As shown in Ascari (1997), one can get similar steady state effects also in a simple Taylor (1980) type of model, and hence one would expect the dynamic properties of the model to be affected.[19]

There are however two possible ways out. The first one is to use a sort of Calvo-Fischer type of rigidity (see, e.g, Yun (1996) and Jeanne (1998)). To get rid of the trend inflation effects, one can incorporate it in the prices which cannot be reset, that is to use the so-called Fischer (1977) or 'predetermined' contracts, within the Calvo setup. This can be shown (see Appendix 2) to cancel the effects of trend inflation: both the steady state and the dynamic equations of the optimal reset price are the same with positive or with zero money growth.[20]

However, there are some difficulties in assuming this kind of automatic adjustment to trend inflation. The first obvious one is that in reality we do not observe such contracts, because most prices and wages are fixed within a year (see Taylor (1998)). What we observe sometimes are multiperiod indexed contracts, which are actually quite different. Indexed contracts are: (i) adjusted to inflation ex-post and not ex-ante; (ii) adjusted not to trend inflation but to actual inflation in the previous period.[21] Second, in terms of microfoundations, one of the rationales given for the directly postulated Calvo contract structure is that it is analytically equivalent to the Rotemberg (1982) model of quadratic cost of changing price (e.g., Ireland (1997)). This would imply, however, that the microeconomic rationale

[19] See Ascari (2000). However, quite interstingly, in Ascari (2000) trend inflation has a definite negative impact on the persistence of the effects of money shocks on output. As shown above, this does not seem to be the case in a Calvo type of model, because the roots become complex, and this appears to increase persistence (see Figure 5).

[20] Obviously here we are just referring to the equations regarding the behaviour of inflation (pricing rule and price index). In general, other equations as well would depend on steady state inflation (e.g., money demand, possibly leisure decisions etc.)

[21] Moreover, it would be easier to defend indexed contracts in a staggered wage model rather than a staggered price one, since indexed wage contracts are indeed observed in reality, and they can be easily justified by the willingness of the workers to defend their real income.

for keeping the price fixed for a certain amount of time is a quadratic 'menu cost' of changing price, and it would be difficult then to justify a costless automatic 'menu' adjustment to trend inflation. As a conclusion, the idea to use Fischer (1977) contracts to get around the problem does not seem a winner.

Yet, this is the solution actually employed, somewhat 'by accident', in most of the literature, in the following sense. A ZISS is the same whatever kind of rigidity is assumed (flex price, fixed or predetermined contracts). As we saw above, in a Calvo-Taylor type of setup, the steady state would depend on trend inflation and so would also the coefficients of the log-linearised dynamic equations. Thus, trend inflation, which in actual data is different from zero, should be taken into account and this would affect the results. In a Calvo-Fischer setup, instead, the steady state and the log-linearised dynamic equations would be the same as in the ZISS, whatever the level of trend inflation. Hence, focusing only on ZISS, it is as if this type of price rigidity has been employed.

As well known, the only alternative is state-dependent models. A remarkable example is the model in Dotsey et al. (1999). In a state-dependent model, in fact, the duration of contracts depends on the state of the economy and should respond to trend inflation. In other words, $\alpha$ should decrease with $\gamma$ counteracting the effect of trend inflation, as it does in Dotsey et al. (1999). Indeed, suppose that at 10% trend inflation $\alpha$ were equal to 0.5, implying that prices are fixed for one semester on average. Then the percentage deviation of steady state output from ZISS in a model with capital would be 2.1%, which may be considered high or low, but surely more reasonable than 26%, as before.[22] If prices are fixed only for 4 months (i.e., $\alpha = 0.25$), then the deviation would be 1%. Figure 8 shows the deviation of steady state output from ZISS as a function of trend inflation and of $\alpha$.[23] It is evident then the changes in $\alpha$ would mitigate the steady state effects of trend inflation and presumably, also the effects on the dynamics. Figure 9 shows the contour levels which gives an idea about how $\alpha$ should vary with trend inflation in order to keep output at the same level (that is, in order to deliver superneutrality[24]). It is then evident that changes in $\alpha$ can alleviate the nuisances. In other words, and as a bottom line, the Lucas critique seems to be really biting in these models.

[22] It is worth noting however that the changes in $\alpha$ reported here are very much bigger than the one predicted by the Dotsey et al. (1999) model.

[23] Note that in the white parts of Figure 8 and 9, the model is not defined, because the level of trend inflation is higher than its upper value.

[24] As said above, however, it is good to keep in mind that in this microfounded model non-superneutrality is induced also by some other well known effects of trend inflation on money demand, capital and leisure choices.

## 6 Conclusion

To conclude, one of the most fruitful recent area of research in macroeconomics is certainly the so-called New Neoclassical Synthesis. Our understanding of monetary policy and its e¤ects on the macroeconomy has greatly improved thanks to the numerous contributions in this literature. Most of the papers in this literature, however, use time-dependent staggered price models and assume zero trend in‡ation. It can hardly be justi…ed to assume zero trend in‡ation to describe and model the data of post-war in‡ation.

This paper shows that unfortunately in these models trend in‡ation matters. If it is considered, then time dependent staggered price models demonstrate some limits: several nuisances appear both regarding their long-run (i.e., steady state) and the short run (i.e., dynamics) properties. Indeed, this paper shows that: (i) very mild level of trend in‡ation implies huge, and unrealistic, changes in the steady state output level; (ii) trend in‡ation changes the dynamic properties of the log-linearised model; (iii) the level of trend in‡ation is also extremely important for the dynamic behaviour of the model following a disin‡ation. In short, this paper shows that disregarding trend in‡ation is very far away from being an innocuous assumption. The results obtained by models log-linearised around a zero in‡ation steady state might therefore be misleading.

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# Appendix 1. The Model

## (A) The Model with variable capital

1) Household

Given the utility function

$$U = \left\{ \left[ bC^{\frac{\eta-1}{\eta}} + (1-b)\left(\frac{M}{P}\right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} (1-L)^{e} \right\}^{1-\chi} / (1-\chi) \qquad (12)$$

the first order condition for the representative households are the following:

$$\frac{W_t}{P_t} = \frac{eC_t\left[1+\bar{b}\left(\frac{m_t}{C_t}\right)^{\frac{\eta-1}{\eta}}\right]}{1-L_t} \qquad (13)$$

$$\frac{U_m(t)}{U_C(t)} = \bar{b}\left(\frac{C_t}{m_t}\right)^{\frac{1}{\eta}} = \frac{i_t}{1+i_t} \qquad (14)$$

$$E_t\left(\frac{U_C(t)}{U_C(t+1)}\beta(1+r_t)\right) = E_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\frac{-1}{\eta}}\left(\frac{cm_t}{cm_{t+1}}\right)^{\frac{1}{\eta}-\chi}\left(\frac{1-L_t}{1-L_{t+1}}\right)^{e(1-\chi)}\beta(1+r_t)\right] = 1 \qquad (15)$$

where $W_t$ = nominal wage; $P_t$ = general price level; $C_t$ = consumption; $m_t = \left(\frac{M_t}{P_t}\right)$ = real money balances; $\bar{b} = \frac{1-b}{b}$; $cm_t = \left[bC_t^{\frac{\eta-1}{\eta}} + (1-b)m_t^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$; $L_t$ = labour supply; $U_X(t)$ = marginal utility with respect to the argument $X$ ( for $X = C; m; L$); $i_t$ = nominal interest rate; $r_t$ = real interest rate.

2) Pricing equations

Final good producers use the following technology

$$Y_t = \left[\int_0^1 Y_{i;t}^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}} \qquad (16)$$

Their demand for intermediate inputs is therefore equal to

$$Y_{i;t+j} = \left(\frac{P_{i;t}}{P_{t+j}}\right)^{-\theta} Y_{t+j} \qquad (17)$$

The problem of the representative intermediate goods producer firms that reset the price is

$$\underset{\{p_{it}\}}{Max} \quad E_t\left(\sum_{j=0}^{\infty} \alpha^j \Delta_{t;t+j} z_{t+j}\right) = E_t\left(\sum_{j=0}^{\infty} \alpha^j \Delta_{t;t+j}\left[\left(\frac{P_{i;t}}{P_{t+j}}\right) Y_{i;t+j} - TC_{i;t+j}(Y_{i;t+j})\right]\right) \qquad (18)$$

$$\text{s.t.} \qquad Y_{i,t+j} = \left(\frac{P_{i,t}}{P_{t+j}}\right)^{-\theta} Y_{t+j}$$

where $\Delta_{t,t+j}$ represents the real discount factor from $t$ to $t+j$ applied by the firm to the stream of future real profits[25]; $z$ = real profits, $P_i$ = price set by the firm, $TC_i$ = real total costs. The optimal price fixed by re-setting firms in period $t$ is

$$P^*_{i,t} = \left(\frac{\theta}{\theta-1}\right)\frac{E_t \sum_{j=0}^{\infty} \alpha^j \Delta_{t,t+j} MC_{i,t+j} P^{\theta}_{t+j} Y_{t+j}}{E_t \sum_{j=0}^{\infty} \alpha^j \Delta_{t,t+j} P^{\theta-1}_{t+j} Y_{t+j}} \tag{19}$$

where $MC_i$ = real marginal cost of producer $i$. Note that (19) can be wriiten as $P^*_{i,t} = \frac{\theta}{\theta-1}\frac{\Psi(t)}{\Phi(t)}$, where

$$\Psi(t) = MC_{i,t} P^{\theta}_t Y_t + \alpha\beta E_t\left[\Psi(t+1)\right] \tag{20}$$

$$\Phi(t) = P^{\theta-1}_t Y_t + \alpha\beta E_t\left[\Phi(t+1)\right] \tag{21}$$

The price of the final good is given by

$$P_t = \left[\int_0^1 P_{i,t}^{1-\theta} di\right]^{\frac{1}{1-\theta}} \tag{22}$$

3) Technology

Denoting by $q_t$ the real user cost of capital, the cost minimisation problem of a representative intermediate goods producer firm $i$ is

$$\underset{\{K_{i,t-1},L_t\}}{MIN} \qquad q_t K_{i,t-1} + \overbrace{w_t}^{W_t/P_t} L_{i,t}$$

$$\text{s.t.} \qquad Y_{i,t} = A_t (K_{i,t-1})^{1-\sigma}(L_{i,t})^{\sigma}$$

which gives the following usual first order conditions

$$q_t = A_t(1-\sigma)\left(\frac{L_{i,t}}{K_{i,t-1}}\right)^{\sigma} MC_{i,t} \tag{23}$$

$$w_t = A_t\sigma\left(\frac{K_{i,t-1}}{L_{i,t}}\right)^{1-\sigma} MC_{i,t} \tag{24}$$

Combining these two equations with the production function yields the equations for the demand of labour and capital and for the marginal cost

$$L^d_{i,t} = \frac{Y_{i,t}}{A_t}\left[\frac{\sigma}{1-\sigma}\frac{q_t}{w_t}\right]^{1-\sigma} \tag{25}$$

[25] For simplicity, we will put that equal to $\beta$, the real discount factor in the utility function.

$$K^d_{i,t-1} = \frac{Y_{i,t}}{A_t}\left(\frac{1-\sigma}{\sigma}\frac{w_t}{q_t}\right)^{\sigma} \tag{26}$$

$$MC_{i,t} = \frac{1}{A_t}\left[\frac{w_t}{\sigma}\right]^{\sigma}\left(\frac{q_t}{1-\sigma}\right)^{1-\sigma} \tag{27}$$

4) Market clearing

The aggregate resource constraint is

$$Y_t = C_t + X_t \tag{28}$$

where $X_t = \left[\int_0^1 X_{z,t}dz\right]$ and $X_t$ = aggregate investment while $X_{i,t}$ = investment of the intermediate goods producer i: $X_{i,t}$ is given by the following capital accumulation equation for the single intermediate goods producer i

$$K_{i,t} = (1-\delta)K_{i,t-1} + X_{i,t} \tag{29}$$

where $\delta$ = depreciation rate. This linear equation can be aggregated over all the intermediate goods producers and then substituted into the aggregate resource constraint to get

$$Y_t = C_t + K_t - (1-\delta)K_{t-1} \tag{30}$$

Market clearing on the capital and labour markets require

$$K_{t-1} = \left(\int_0^1 K^d_{i,t-1}di\right) \tag{31}$$

$$L^d_t = \left(\int_0^1 L^d_{i,t}di\right) = L^s_t \tag{32}$$

Following Yun (1996) the equation to link intermediate goods output and final good output is given by

$$IO_t = \left(\int_0^1 Y_{i,t}di\right) = \left(\frac{P_t}{\tilde{P}_t}\right)^{\theta} Y_t \tag{33}$$

where $\tilde{P}_t = \left[\int_0^1 P_{i,t}^{-\theta}di\right]^{-\frac{1}{\theta}}$ and $IO_t$ = 'aggregator' of intermediate goods output.

Finally, exploiting the property that, given the Cobb-Douglas production function for intermediate goods producer, the ratio $\left[\frac{K_{i,t-1}}{L_{i,t}}\right]$ is the same across all firm i; it is possible to aggregate to obtain:

$$IO_t = A_t K_{t-1}^{1-\sigma} L_t^{\sigma} \tag{34}$$

$$L_t^d = \frac{IO_t}{A_t}\left(\frac{\sigma}{1-\sigma}\frac{q_t}{w_t}\right)^{1-\sigma} \tag{35}$$

$$K_{t-1}^d = \frac{IO_t}{A_t}\left(\frac{1-\sigma}{\sigma}\frac{w_t}{q_t}\right)^{\sigma} \tag{36}$$

$$MC_t = \frac{1}{A_t}\left[\frac{w_t}{\sigma}\right]^{\sigma}\left(\frac{q_t}{1-\sigma}\right)^{1-\sigma} \tag{37}$$

The model is closed by the equation $r = q - \delta$.

## (B) The Model with fixed capital

Both the household problems and the pricing problem of the resetting firms do not change and so the first order conditions. The difference is given by the technology of intermediate goods producers, now given by

$$Y_{i;t} = A_t L_{i;t}^{\sigma} \tag{38}$$

The labour demand and the real marginal cost of firm i is therefore

$$L_{i;t}^d = \left(\frac{Y_{i;t}}{A_t}\right)^{\frac{1}{\sigma}} \tag{39}$$

$$MC_{i;t} = \frac{1}{\sigma} A_t^{-\frac{1}{\sigma}} w_t Y_{i;t}^{\frac{1}{\sigma}-1} \tag{40}$$

The aggregate resource constraint is now simply given by

$$Y_t = C_t \tag{41}$$

and the link between aggregate labour demand and aggregat output is provided by

$$L_t^d = \left(\int_0^1 L_{i;t}^d di\right) = \left(\frac{Y_t}{A_t}\right)^{\frac{1}{\sigma}}\left(\frac{P_t}{\overline{P_t}}\right)^{\frac{\theta}{\sigma}}$$

where $\overline{P_t} = \left(\int_0^1 P_{i;t}^{-\frac{\theta}{\sigma}} di\right)^{-\frac{\sigma}{\theta}}$:

Note that now marginal costs depend upon the quantity produced by the single firm, given the decreasing returns to scale. In other words, different firms charging different prices would produce different levels of output and hence have different marginal costs. Consider the optimal reset price formula in a non-stochastic steady state. This is still described by

$$P^*_{i,t} = \left(\frac{\theta}{\theta-1}\right)\frac{\Psi(t)}{\Phi(t)} \tag{42}$$

$$\Phi(t) = P_t^{\theta-1}Y_t + \alpha\beta E_t\left[\Phi(t+1)\right]$$

$$\Psi(t) = MC_{i,t}P_t^{\theta}Y_t + \alpha\beta E_t\left[\Psi(t+1)\right]$$

The $MC_{i,t}$ in $\Psi(t)$ is now increasing over time, since

$$MC_{i,t+j} = \frac{1}{\sigma}A_{t+j}^{-\frac{1}{\sigma}}w_{t+j}\left(\frac{P^*_{i,t}}{P_{t+j}}\right)^{-\theta(\frac{1}{\sigma}-1)}Y_{t+j}^{(\frac{1}{\sigma}-1)}$$

and $P^*_{i,t}$ is fixed untill the new resetting. The variable $\Psi(t)$ needs therefore to be deflated accordingly to make it stationary. In a non-stochastic environment,

$$\Phi(t) = \sum_{j=0}^{\infty}(\alpha\beta)^j P_{t+j}^{\theta-1}Y_{t+j} \tag{43}$$

$$\Psi(t) = \sum_{j=0}^{\infty}(\alpha\beta)^j MC_{i,t+j}P_{t+j}^{\theta}Y_{t+j} = \sum_{j=0}^{\infty}(\alpha\beta)^j\frac{1}{\sigma}A_{t+j}^{-\frac{1}{\sigma}}w_{t+j}\left(\frac{P^*_{i,t}}{P_{t+j}}\right)^{-\theta(\frac{1}{\sigma}-1)}Y_{t+j}^{(\frac{1}{\sigma}-1)}P_{t+j}^{\theta}Y_{t+j} \tag{44}$$

Substituing (43) and (44) in (42) yields a dynamic equation that links $P^*_{i,t}$ to aggregate variables.

$$P_{i,t}^{*1+-\theta(\frac{1}{\sigma}-1)} = \left(\frac{\theta}{\theta-1}\right)\frac{\sum_{j=0}^{\infty}(\alpha\beta)^j\frac{1}{\sigma}A_{t+j}^{-\frac{1}{\sigma}}w_{t+j}Y_{t+j}^{\frac{1}{\sigma}}P_{t+j}^{\frac{\theta}{\sigma}}}{\sum_{j=0}^{\infty}(\alpha\beta)^j P_{t+j}^{\theta-1}Y_{t+j}} \tag{45}$$

In a non-stochastic steady state $A_t, Y_t$ and $w_t$ are constant over time, while $P_{t+1}/P_t = \gamma$; hence substituting it yields

$$\Phi(t) = P_t^{\theta-1}Y\sum_{j=0}^{\infty}(\alpha\beta\gamma^{\theta-1})^j \tag{46}$$

$$\Psi(t) = \frac{1}{\sigma}A^{-\frac{1}{\sigma}}wY^{\frac{1}{\sigma}}P_t^{\theta/\sigma}P_{i,t}^{*-\theta(\frac{1}{\sigma}-1)}\sum_{j=0}^{\infty}(\alpha\beta\gamma^{\theta/\sigma})^j \tag{47}$$

Substituting the expression for $\Phi(t)$ and $\Psi(t)$ in (42) then one can obtain a formula that links the reset price with the aggregate variables in the non-stochastic steady state and then solve for $Y.$ It is clear, however, that the two summations in (46) and (47) need to converge. In particular, it needs to be: $\alpha\beta\gamma^{\theta/\sigma} < 1$ ,i.e., $\gamma < (\alpha\beta)^{-\sigma/\theta}$. Putting $\alpha = 0.75, \beta = 0.99, \sigma = 0.67, \theta = 10$, it yields $\gamma < 1.02$, which means an annual rate of grwoth of money lower than 8%.

# Appendix 2. The Calvo-Fischer Case

Yun (1996) and Jeanne (1998) assume that the new price set in a generic period t is actually indexed to trend inflation. Hence, even if the firm is not allowed to revise its price, the latter grows at the same rate as trend inflation. Then the problem of the firm is

$$\max_{\{P_{it}\}} E_t\left(\sum_{j=0}^{\infty} \alpha^j \Delta_{t,t+j} z_{t+j}\right) = E_t\left(\sum_{j=0}^{\infty} \alpha^j \Delta_{t,t+j}\left[\left(\frac{P_{i,t}\Pi^j}{P_{t+j}}\right)^{1-\theta} Y_{t+j} - TC_{i,t+j}(Y_{i,t+j})\right]\right) \tag{48}$$

where $Y_{i,t+j} = \left(\frac{P_{i,t}\Pi^j}{P_{t+j}}\right)^{-\theta} Y_{t+j}$ and the optimal price is

$$P_{it}^{*} = \left(\frac{\theta}{\theta-1}\right)\frac{E_t\sum_{j=0}^{\infty}\alpha^j \Delta_{t,t+j} MC_{t+j}\left(\frac{P_{t+j}}{\gamma^j}\right)^{\theta} Y_{t+j}}{E_t\sum_{j=0}^{\infty}\alpha^j \Delta_{t,t+j}\left(\frac{P_{t+j}}{\gamma^j}\right)^{\theta-1} Y_{t+j}} \tag{49}$$

The steady state value is

$$\frac{P_{i,t}}{P_t} = \left(\frac{\theta}{\theta-1}\right) MC \tag{50}$$

which coincides with the flexible price steady state. Moreover, note that there is not any upper value for the steady state rate of growth of money.

The log-linearised optimal price setting rule equation coincides with the log-linearisation of a typical Calvo framework around a zero money growth steady state

$$p_{it} - p_t = (1-\alpha\beta)E_t\sum_{j=0}^{\infty}(\alpha\beta)^j\left[\pi_{t,t+j} + mc_{t+j}\right] \tag{51}$$

and so it is also for the log-linearised general price level equation

$$p_{it} - p_t = \frac{\alpha}{1-\alpha}\pi_t \tag{52}$$

Putting them together one gets the usual New Keynesian Phillips Curve. Hence, a Calvo-Fischer structure delivers exactly the kind of equations used in most models in the literature.

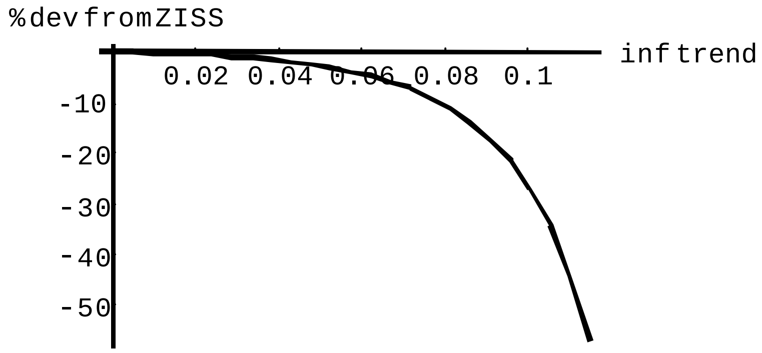


Figure 1. Percentage deviation from zero-in‡ation steady state output

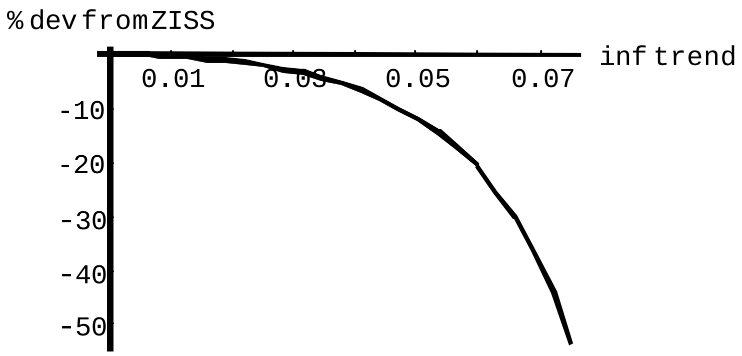


Figure 2. Percentage deviation from zero-in‡ation steady state output
in the …xed capital model

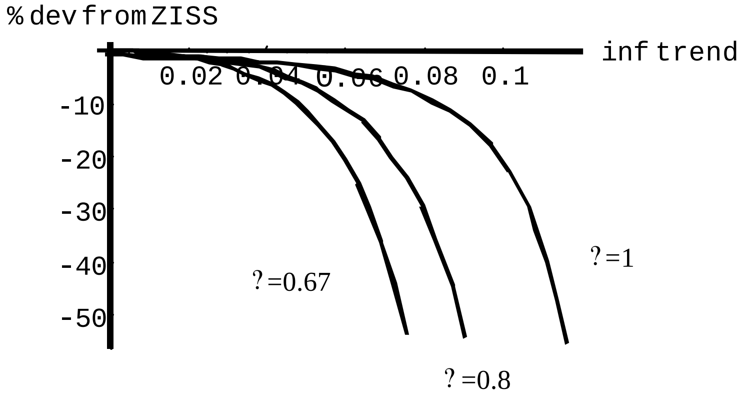


Figure 3. Percentage deviation from zero-in‡ation steady state output, as ¾ varies in the …xed capital model

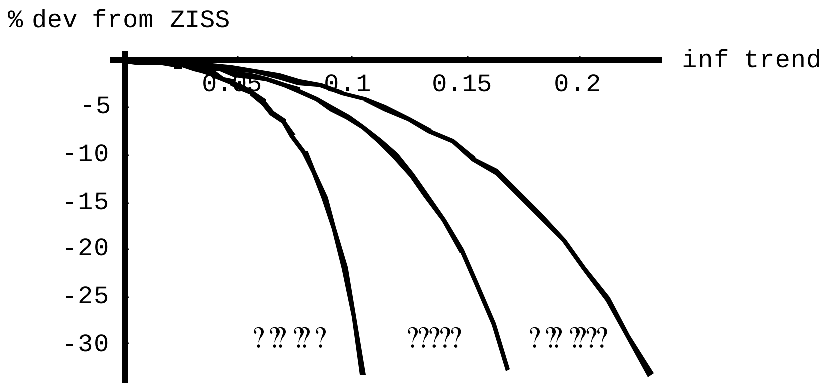


Figure 4. Percentage deviation from zero-in‡ation steady state output, as µ varies

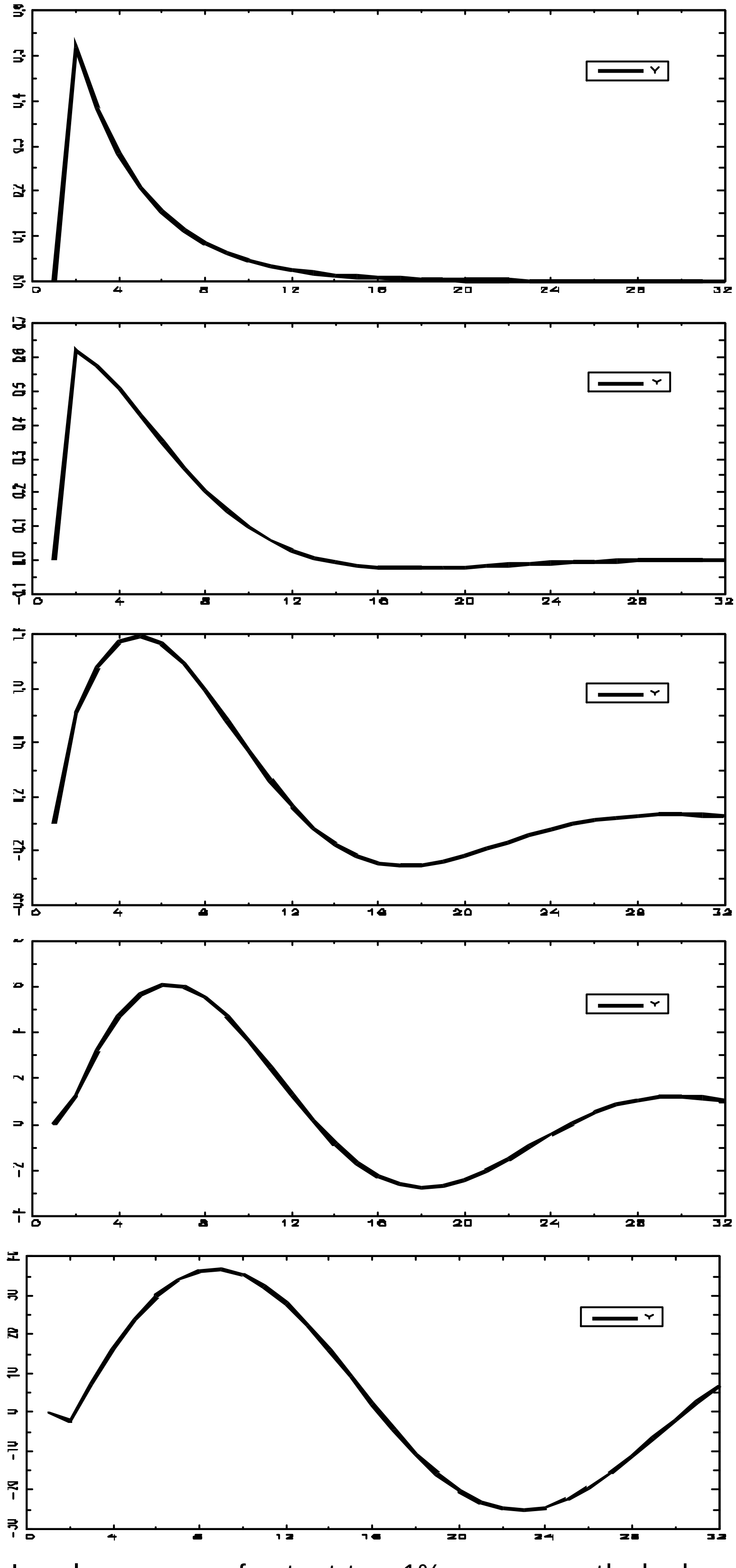


Fig. 5 Impulse response of output to a 1% money growth shock.
Trend in‡ation: (i) 0; (ii) 2.5%; (iii) 5%; (iv) 7.5%; (v) 10%
(…xed capital model and ¾ = 1)

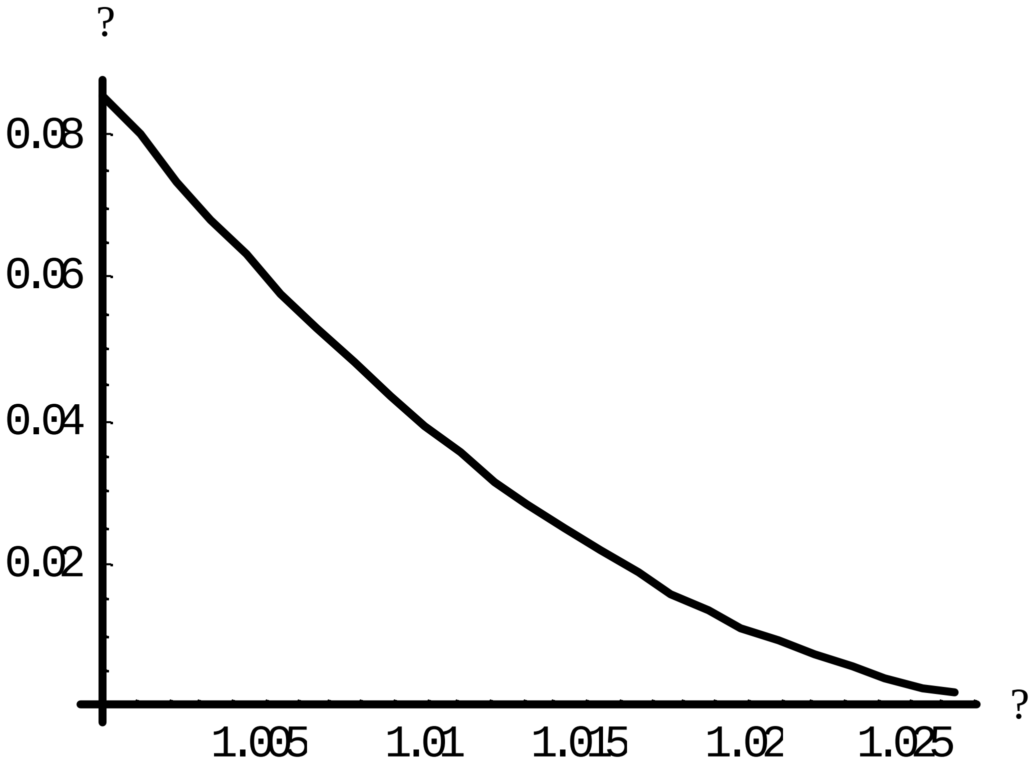


Fig 6. ¸ as a function of °

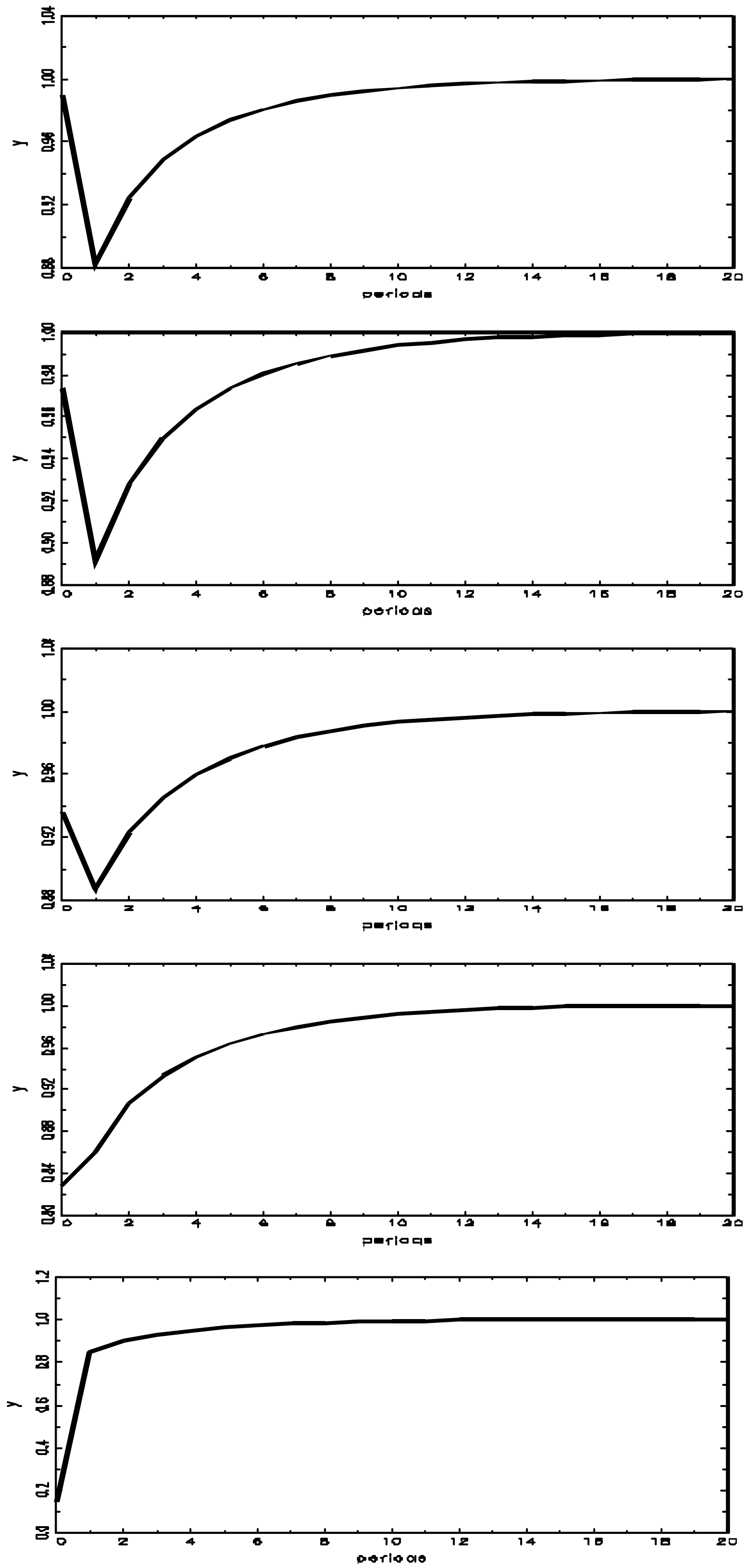

Fig. 7 Dynamics of output after a 4% disin‡ation, starting from:
(i) 4%; (ii) 6%; (iii) 8%; (iv) 10%; (v) 12%
(…xed capital model and ¾ = 1)

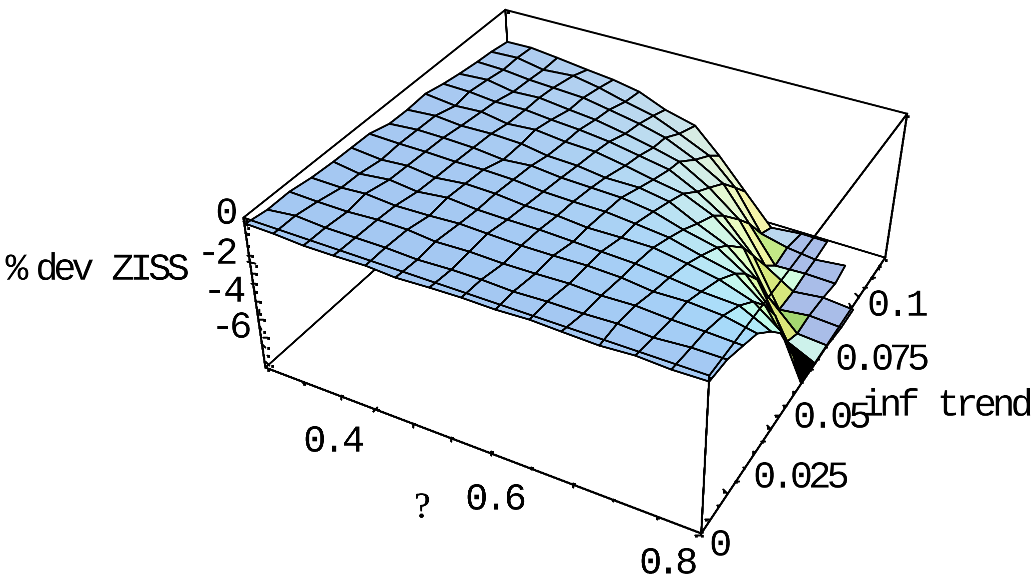


Figure 8. Percentage deviation from ZISS as a function
of trend in‡ation and of ® (model with capital)

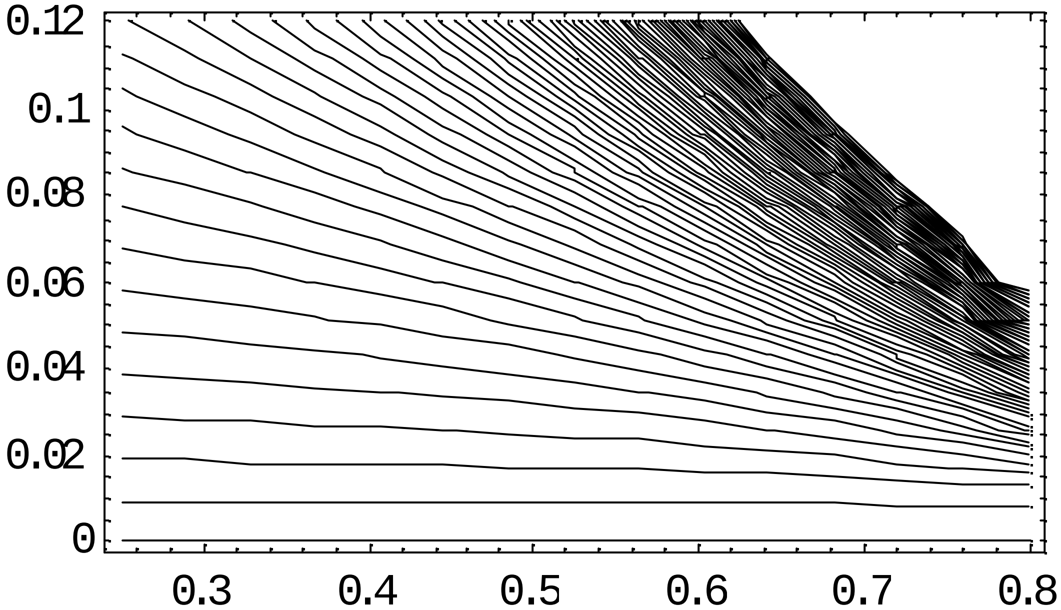


Figure 9. Contour levels of the previous Figure